DICCA
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Slender fibers for measuring flow properties

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with:

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Markus Holzner and Stefano Brizzolara (ETHZ, Zurich)

Complex Lagrangian Problems of Particles in Flows (ICTS, March 14-18, 2022)

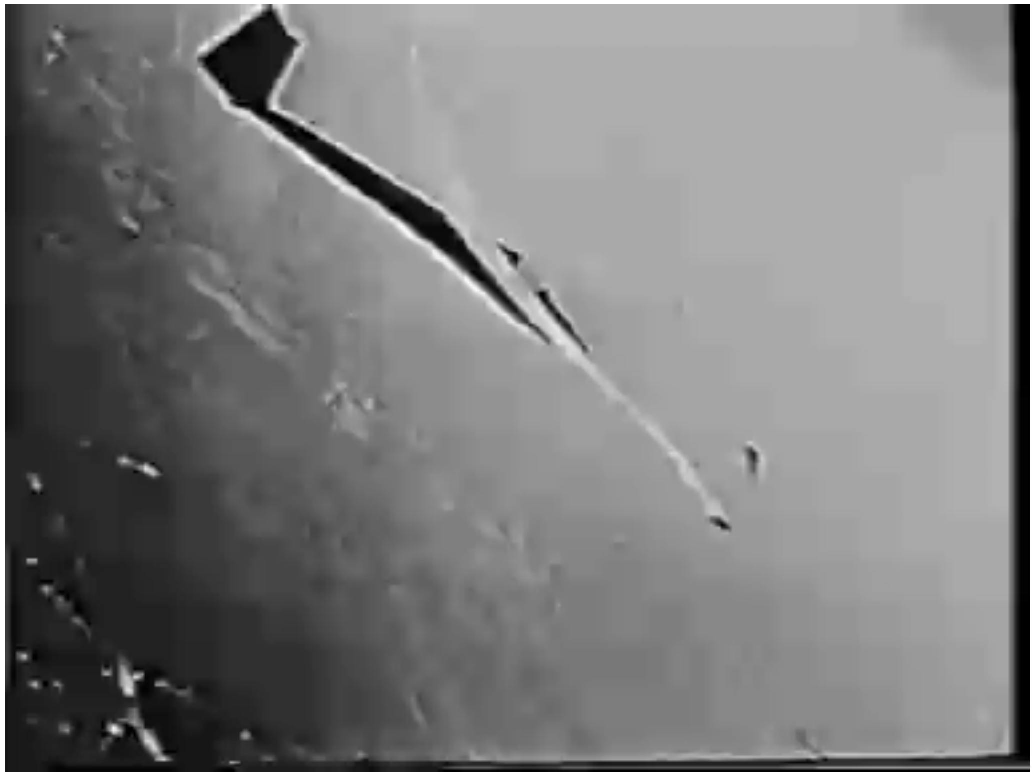
It is a problem of fluid-structure interaction

Tacoma bridge collapse (1940)

Torsional modes became resonant due to fluttering instability

It is a problem of fluid-structure interaction

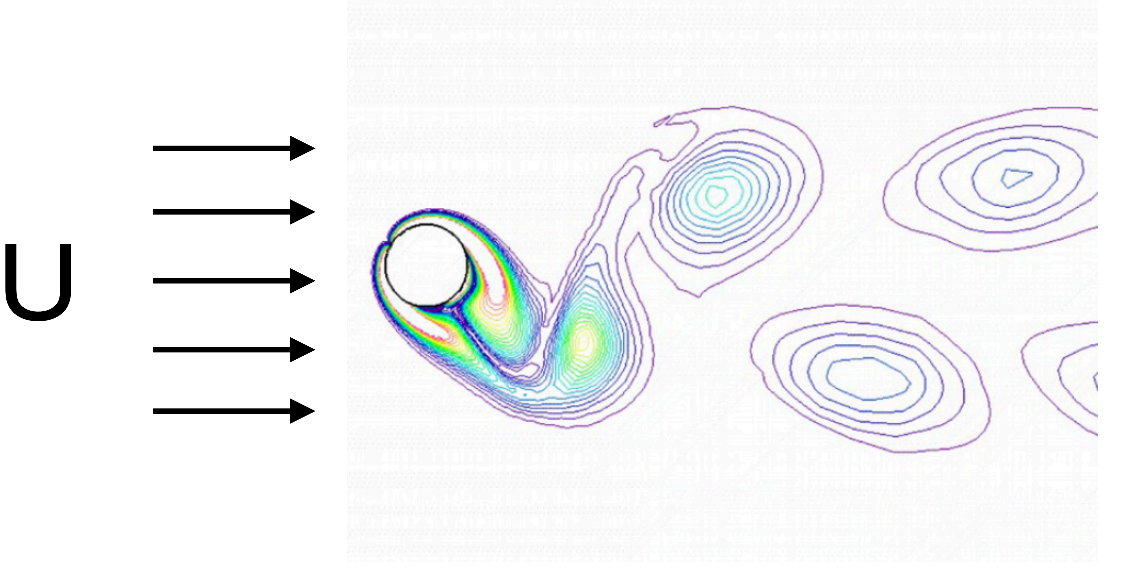
Resonance with aerodynamics loads



Fluttering

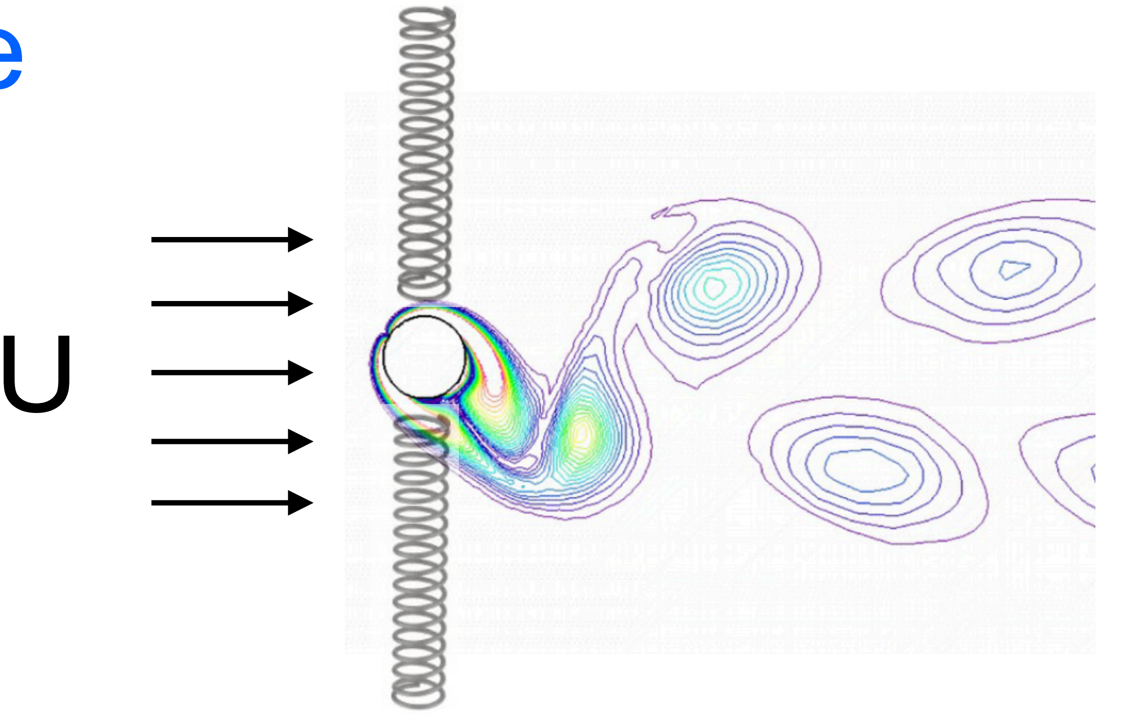
Sometimes fluttering is very welcome

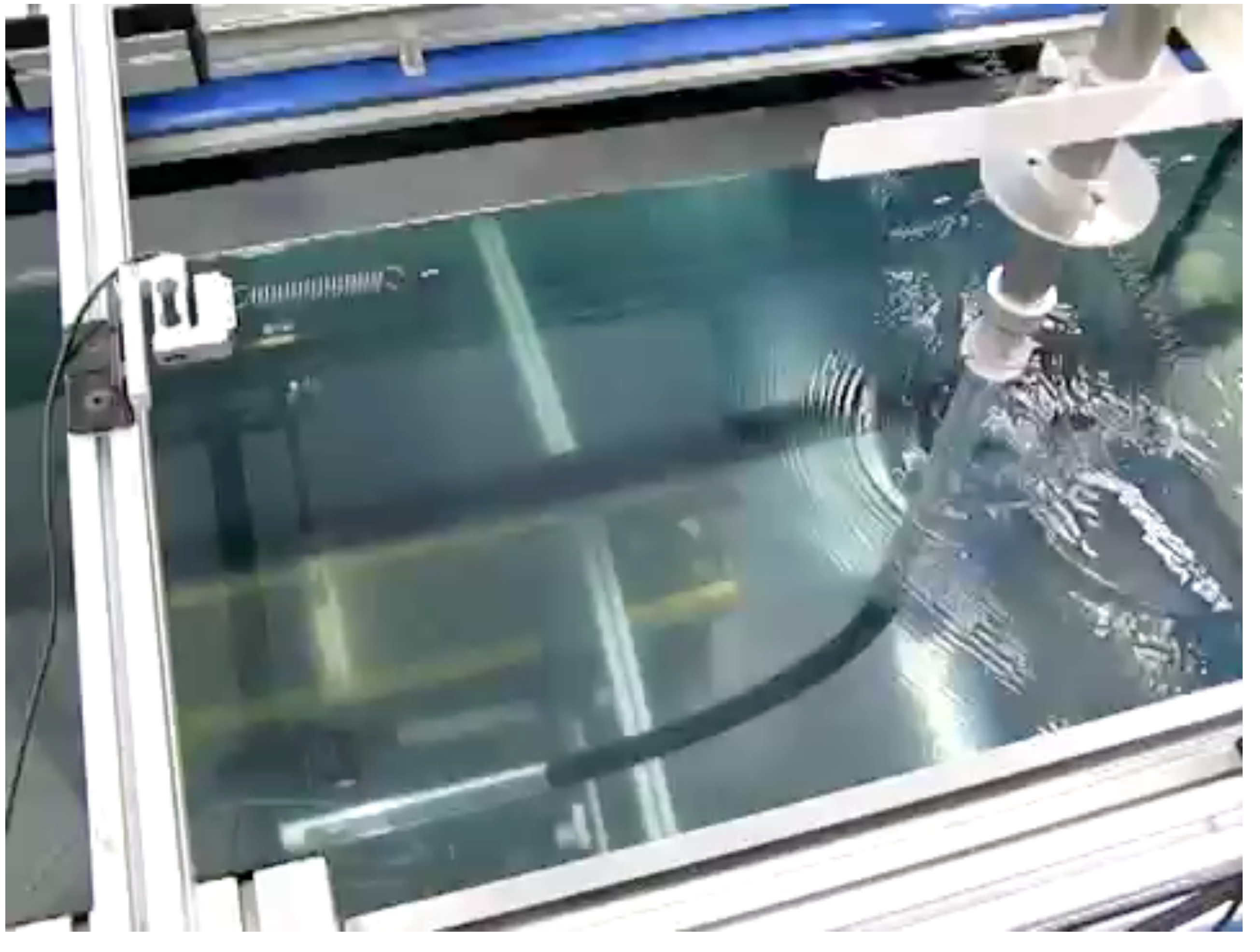
Self-sustained vibrations can be induced by fluid flows



Sometimes resonances are very welcome

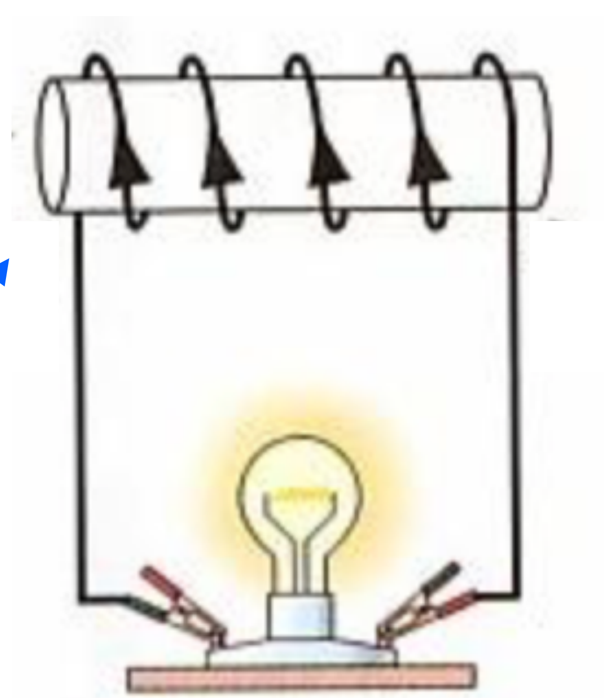
Self-sustained vibrations can be induced by fluid flows



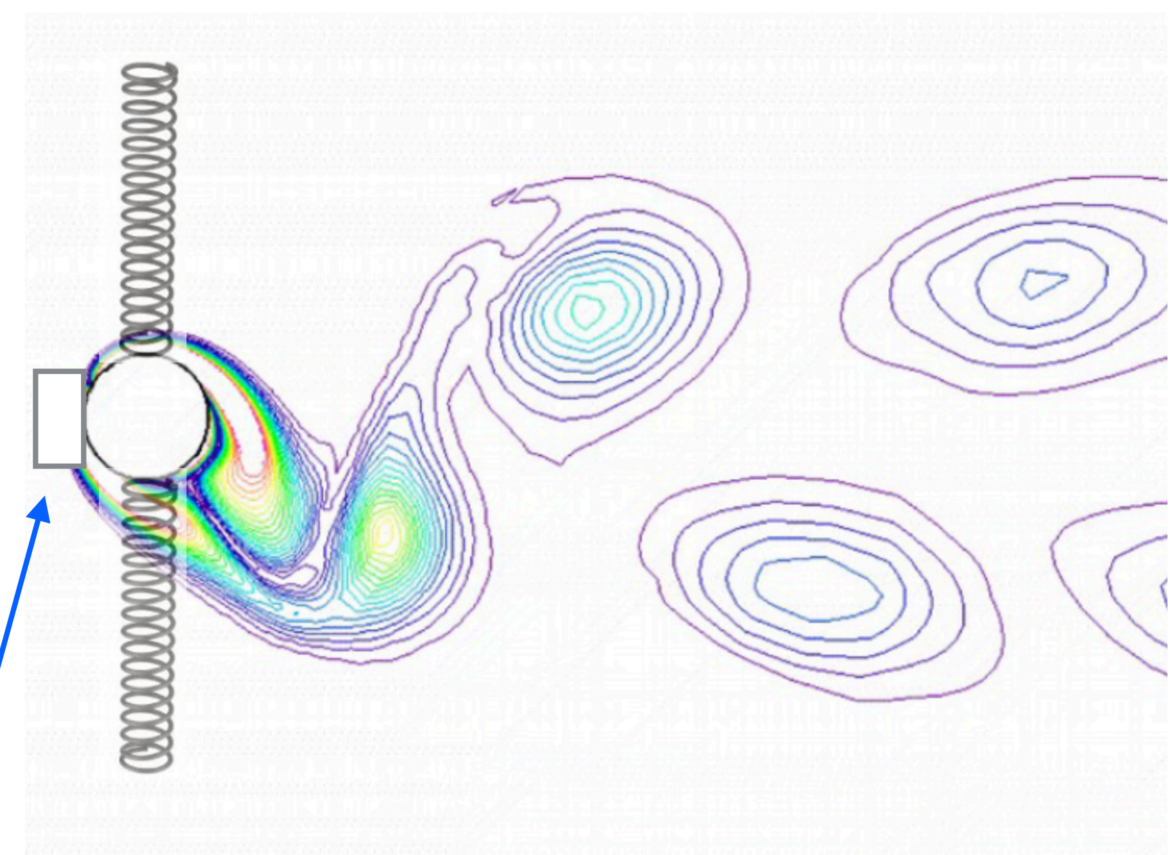


EH by fluid-structure interaction

Once oscillations are generated energy can be extracted via Faraday effect



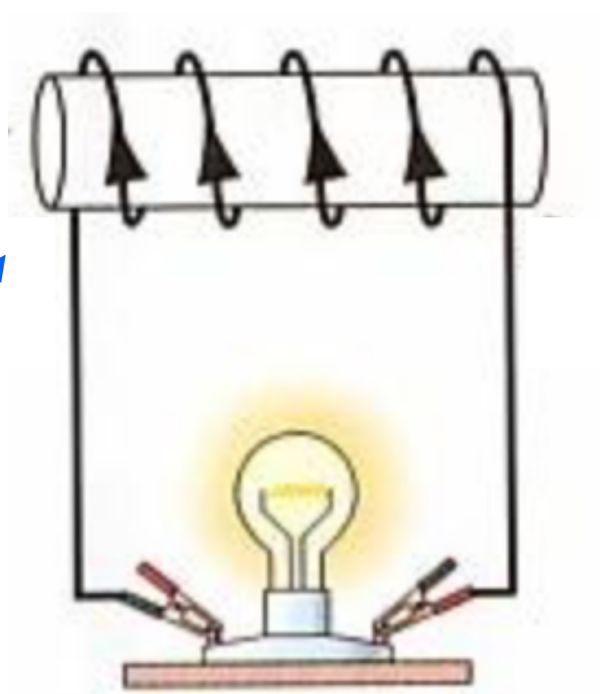
coil



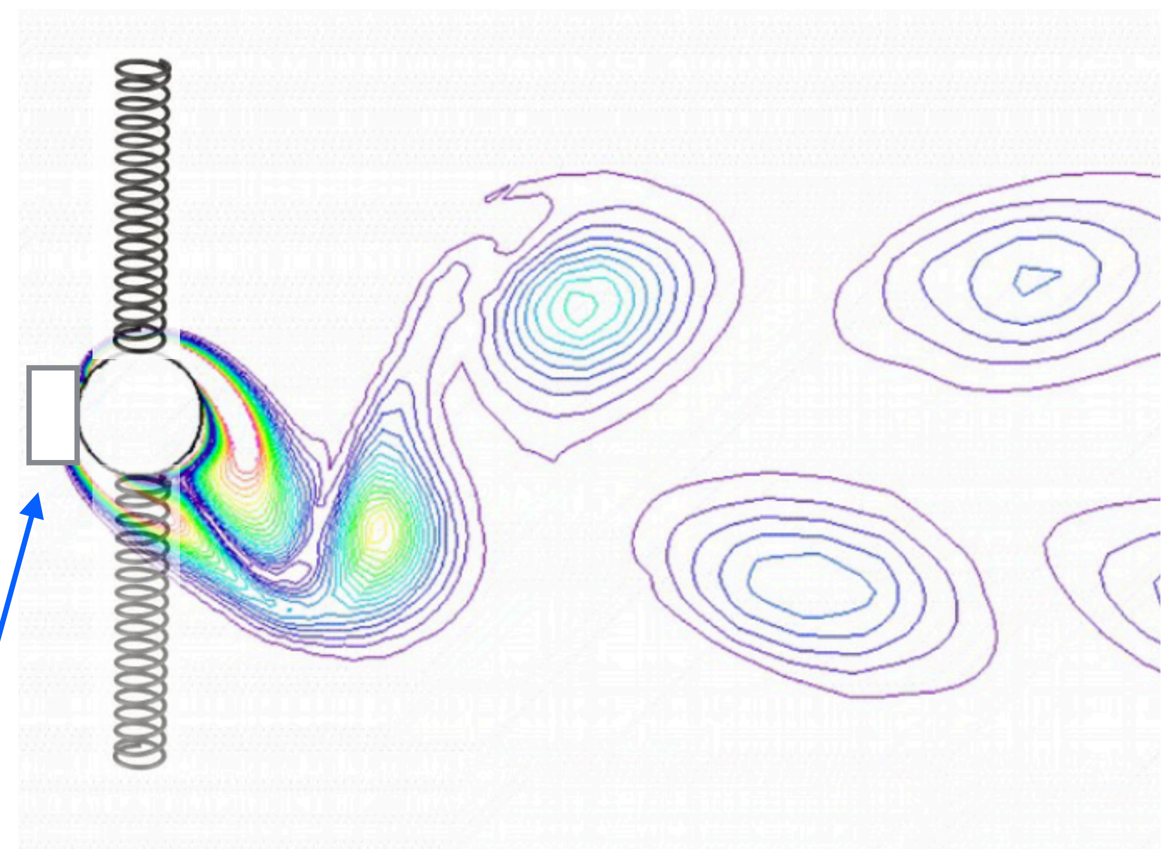
magnet

EH by fluid-structure interaction

Once oscillations are generated energy can be extracted via Faraday effect



coil

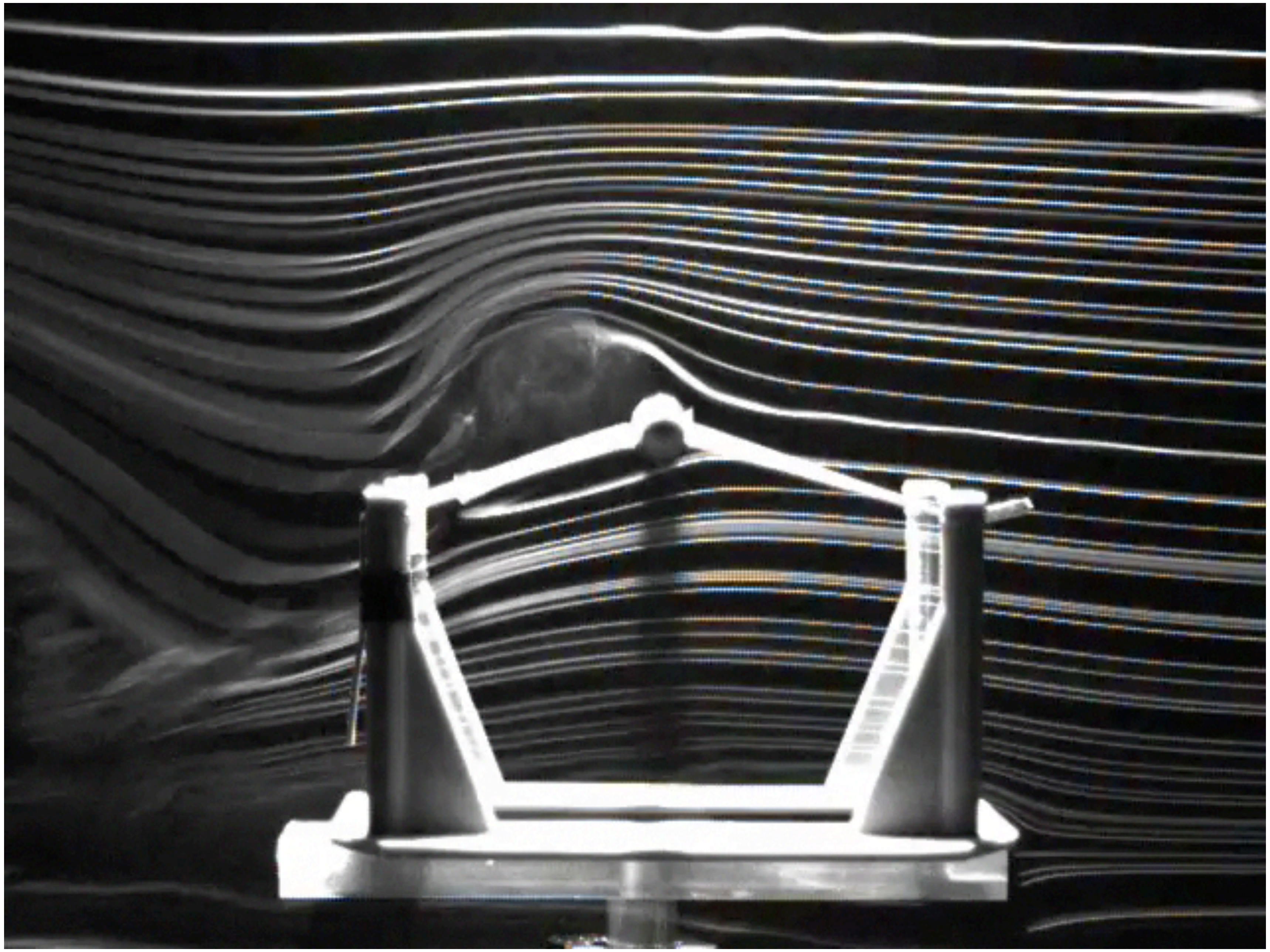


magnet

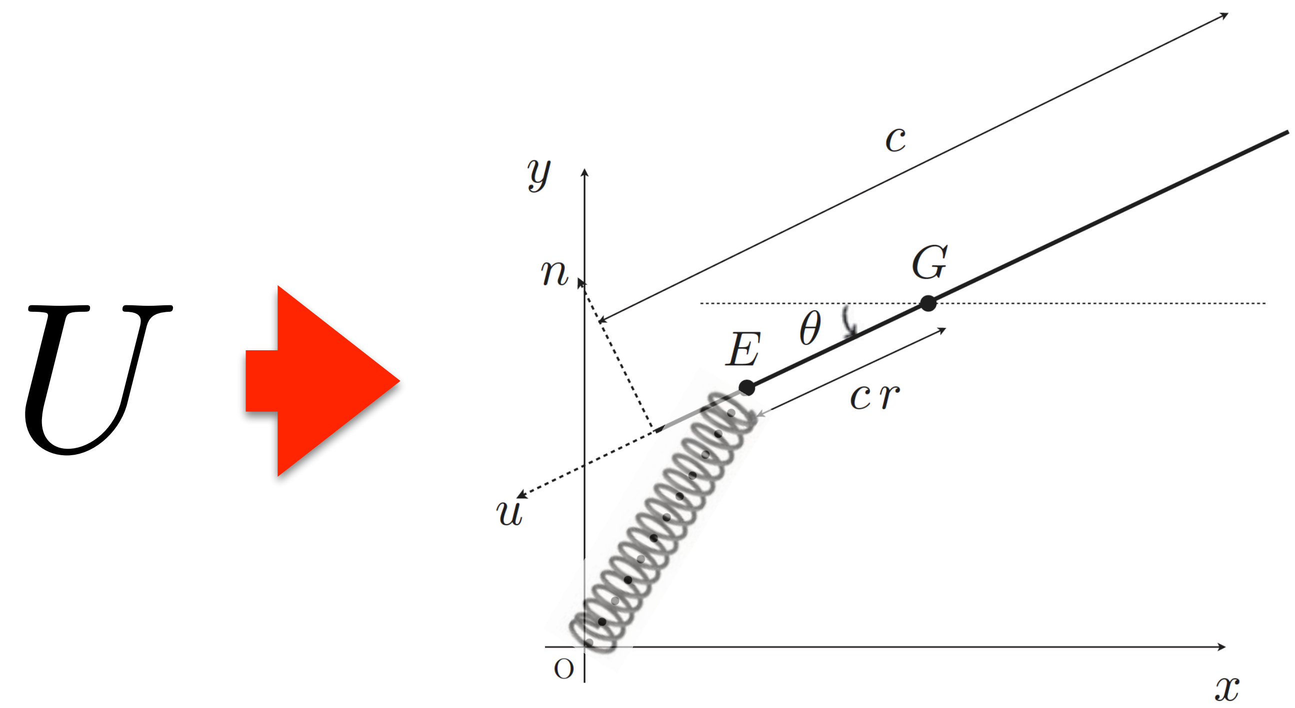
High frequency/amplitude are desirable

Flapping in air is much more efficient

A bluff body is not the best idea to generate relevant lift: a wing is much better !



Resonance condition for flapping



Two frequencies are playing:

wind vane: $I_E\ddot{\theta} + \rho U^2 c^2 \theta = 0 \longrightarrow f \sim \sqrt{\rho \frac{U^2}{m}}$

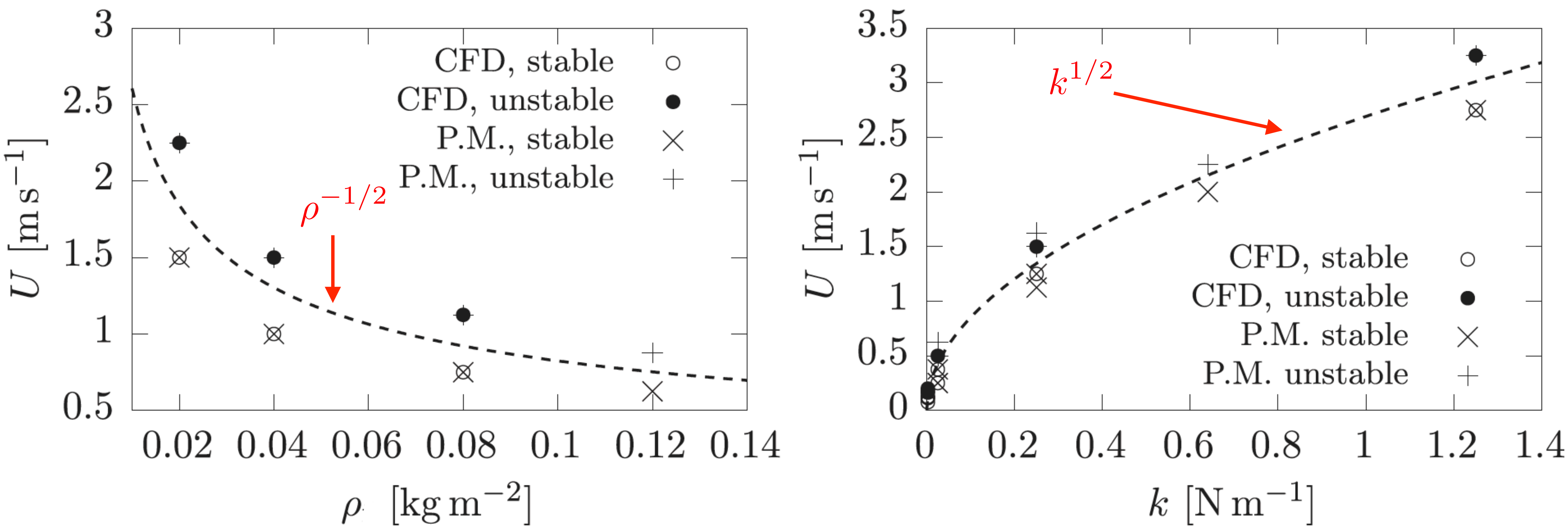
elastic oscillations: $f_{el} \sim \sqrt{\frac{k}{m}}$

resonance: $U\sqrt{\frac{\rho}{m}} \sim \sqrt{\frac{k}{m}}$ → $U_{crit} \sim k^{1/2} \rho^{-1/2}$

It works ….

Numerical simulations

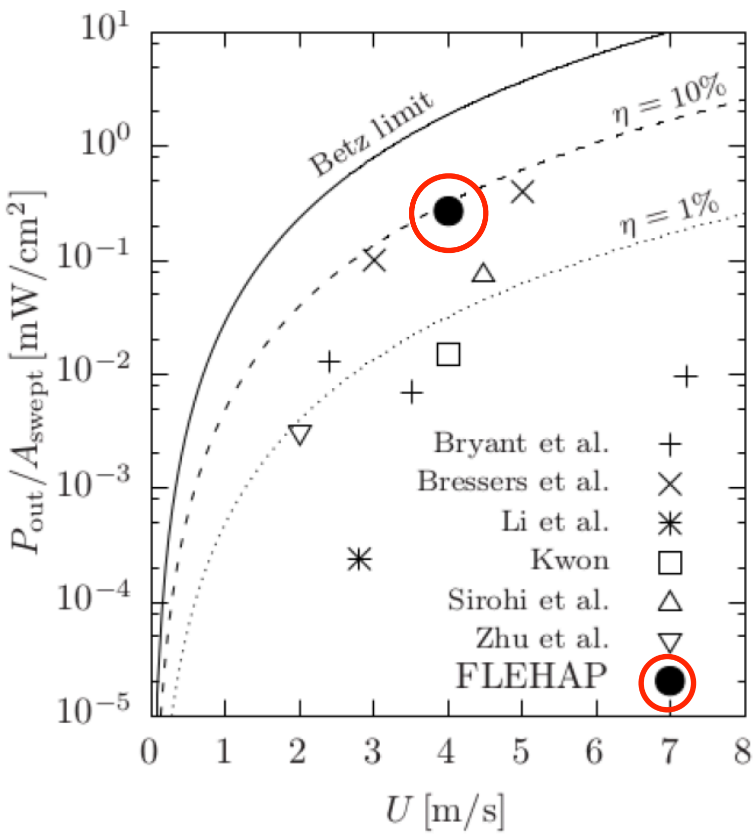
$$U_{crit} \sim k^{1/2}\rho^{-1/2}$$

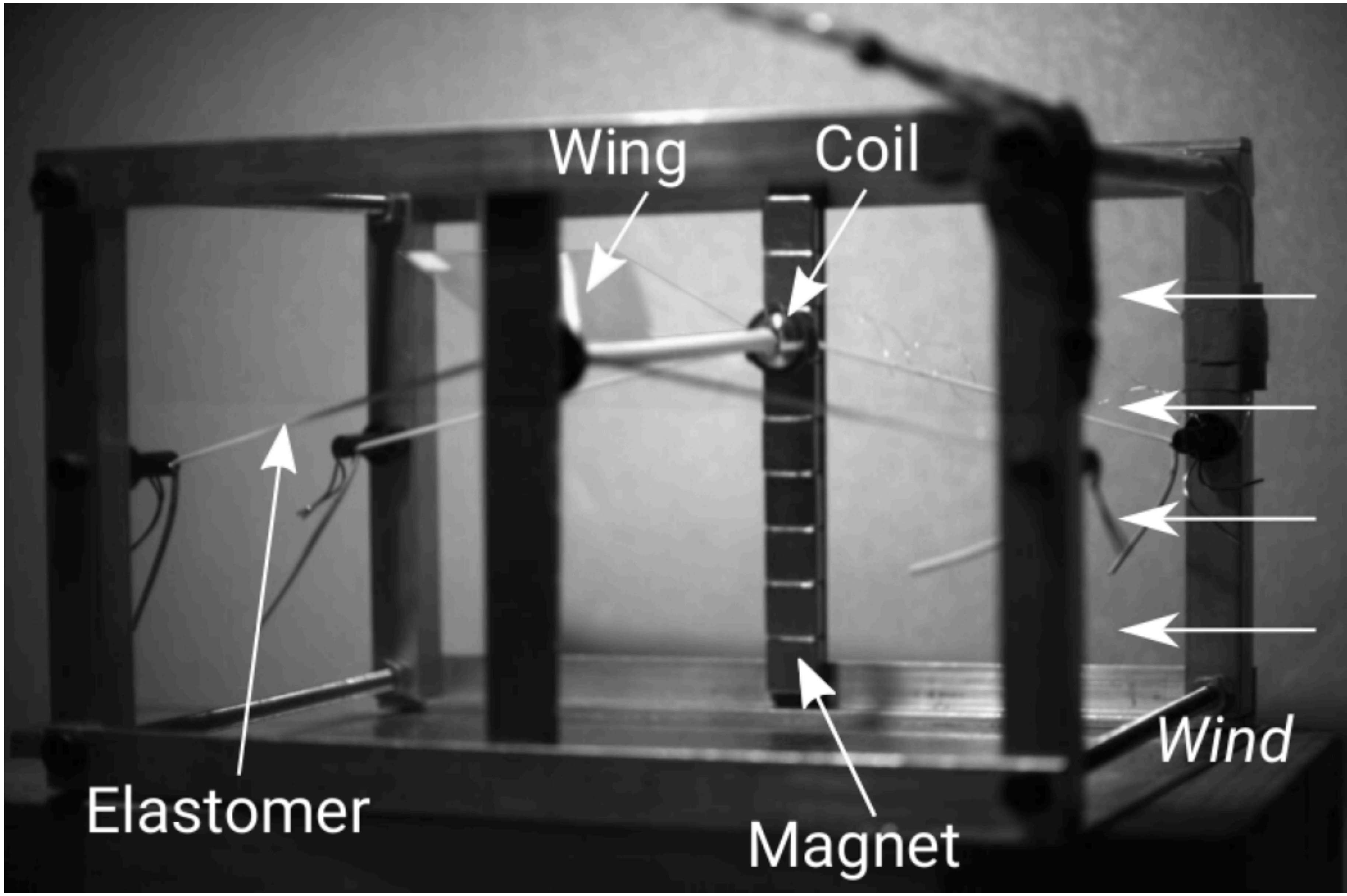


Energy can be really harvested!

The FLEHAP device

(FLuttering Energy Harvester for Autonomous Powering)
UNIGE patent



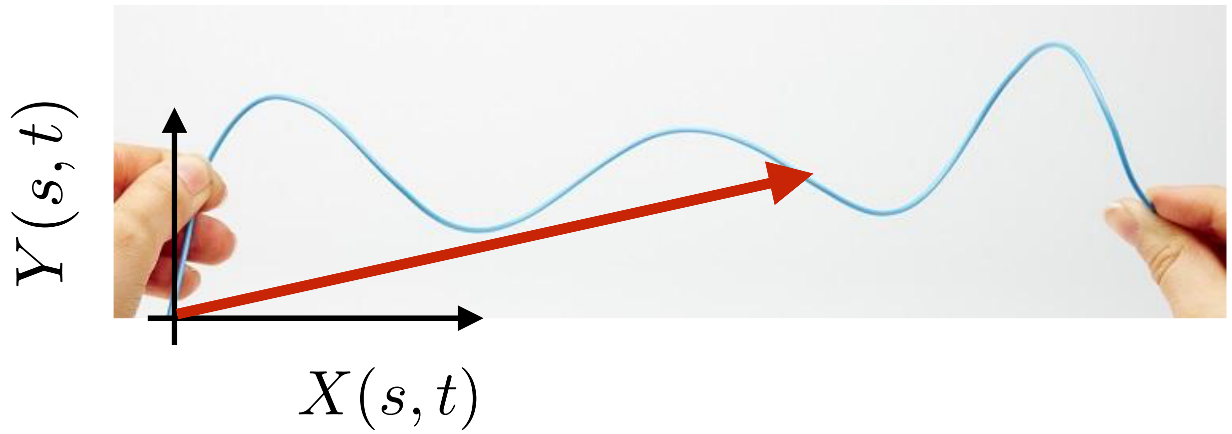


Back to fibers

? Can a fiber of length c be used as a proxy of turbulence statistics at scales c in the inertial range ?

van Gogh (1889)

Back to fibers …



length c

governed by the Euler-Bernoulli equation

$$\rho_1 \frac{\partial^2 \boldsymbol{X}}{\partial t^2} = \frac{\partial}{\partial s}\left(T\frac{\partial \boldsymbol{X}}{\partial s}\right) - \frac{\partial^2}{\partial s^2}\left(\gamma\frac{\partial^2 \boldsymbol{X}}{\partial s^2}\right) - \mathbf{F}$$

density (ρ_1)

tension (T)

bending rigidity (γ)

flow field contribution ($\mathbf{F}$)

fiber inextensibility: $\partial_s \mathbf{X} \cdot \partial_s \mathbf{X} = 1$

Fiber dynamics in a nutshell

$$\rho_1 \frac{\partial^2 \boldsymbol{X}}{\partial t^2} + \eta \left(\frac{\partial \boldsymbol{X}}{\partial t} - \boldsymbol{u} \right) = \frac{\partial}{\partial s} \left(T \frac{\partial \boldsymbol{X}}{\partial s} \right) - \frac{\partial^2}{\partial s^2} \left(\gamma \frac{\partial^2 \boldsymbol{X}}{\partial s^2} \right)$$

$$\tau_B = \alpha \left(\frac{\rho_1 c^4}{\gamma} \right)^{1/2}$$

$\alpha = \pi/22.3733 \sim 0.14$ (from normal-mode analysis)

Fiber dynamics in a nutshell

$$\rho_1 \frac{\partial^2 \boldsymbol{X}}{\partial t^2} + \eta \left(\frac{\partial \boldsymbol{X}}{\partial t} - \boldsymbol{u} \right) = \frac{\partial}{\partial s} \left(T \frac{\partial \boldsymbol{X}}{\partial s} \right) - \frac{\partial^2}{\partial s^2} \left(\gamma \frac{\partial^2 \boldsymbol{X}}{\partial s^2} \right)$$

$$\tau_\nu = \frac{2\rho_1}{\eta}$$

$$\tau_B = \alpha \left(\frac{\rho_1 c^4}{\gamma} \right)^{1/2}$$

$\alpha = \pi/22.3733 \sim 0.14$ (from normal-mode analysis)

$$\zeta = \frac{\tau_B}{\tau_\nu} = \frac{\alpha c^2 \eta}{2\rho_1^{1/2} \gamma^{1/2}}$$

$\zeta \ll 1$ under-damped case

$\zeta \gg 1$ over-damped case

Fiber dynamics in a nutshell

$$\rho_1 \frac{\partial^2 \boldsymbol{X}}{\partial t^2} + \eta \left(\frac{\partial \boldsymbol{X}}{\partial t} - \boldsymbol{u} \right) = \frac{\partial}{\partial s} \left(T \frac{\partial \boldsymbol{X}}{\partial s} \right) - \frac{\partial^2}{\partial s^2} \left(\gamma \frac{\partial^2 \boldsymbol{X}}{\partial s^2} \right)$$

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$$\zeta = \frac{\tau_B}{\tau_\nu} = \frac{\alpha c^2 \eta}{2\rho_1^{1/2}\gamma^{1/2}}$$

$\zeta \ll 1$ under-damped case $\qquad c \ll \gamma^{1/4}\rho_1^{1/4}\eta^{-1/2}$

$\zeta \gg 1$ over-damped case $\qquad c \gg \gamma^{1/4}\rho_1^{1/4}\eta^{-1/2}$

Resonance condition and consequences

$\zeta \ll 1$ under-damped case $\quad c \ll \gamma^{1/4} \rho_1^{1/4} \eta^{-1/2}$

For the most efficient flapping we assert:

$$\tau_B \sim \tau(c)$$

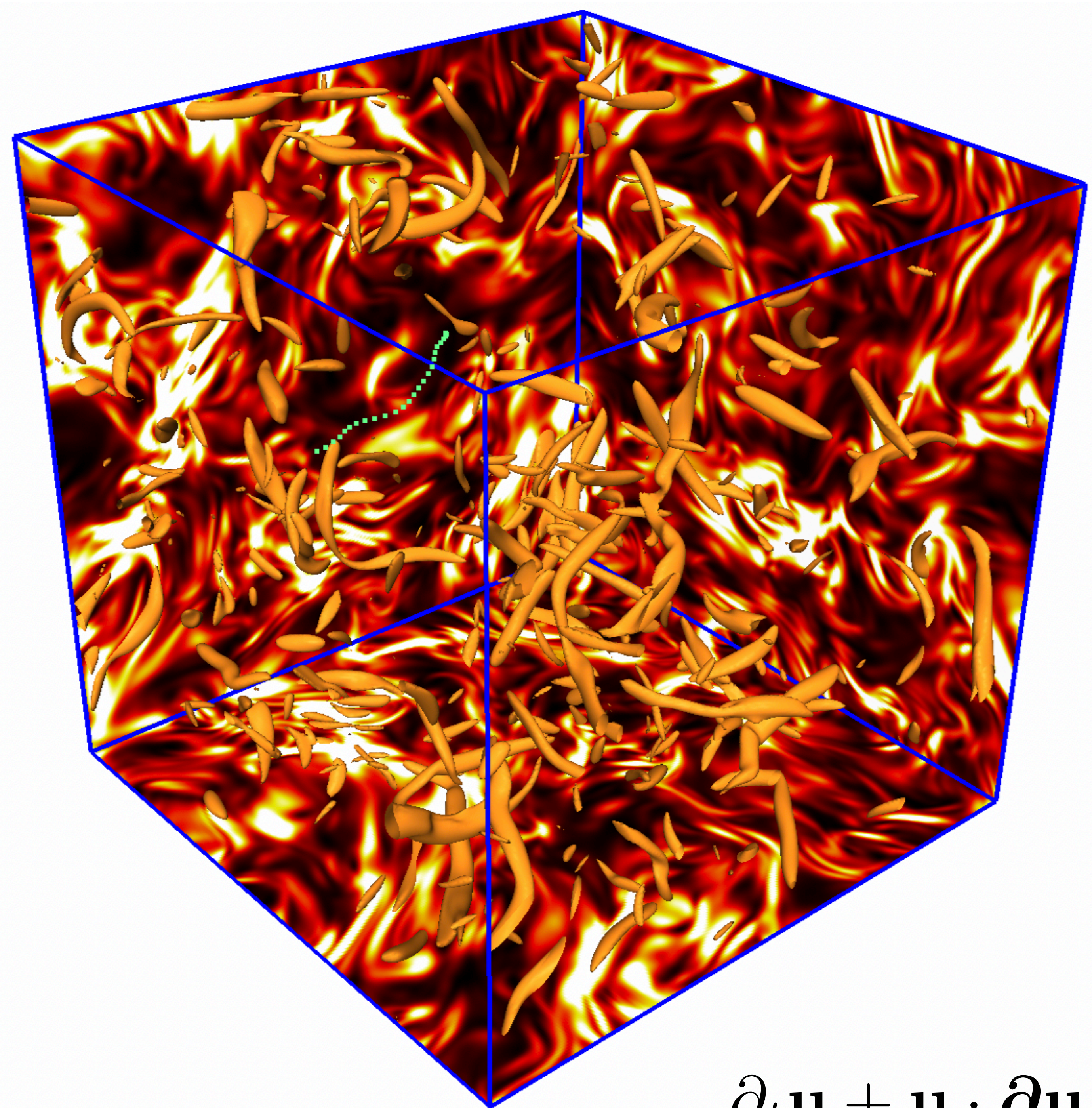
elastic time (τ_B)

turbulence time at the scale of the fiber ($\tau(c)$)

$$\alpha \left(\frac{\rho_1 c^4}{\gamma} \right)^{1/2} \sim c^{2/3} \epsilon^{-1/3}$$

$$\Rightarrow \quad \gamma_{crit} \sim c^{8/3} \epsilon^{2/3} \rho_1 \alpha^2$$

Fiber fully-coupled to turbulence



Rosti, Banaei, Brandt, A.M., PRL (2018)

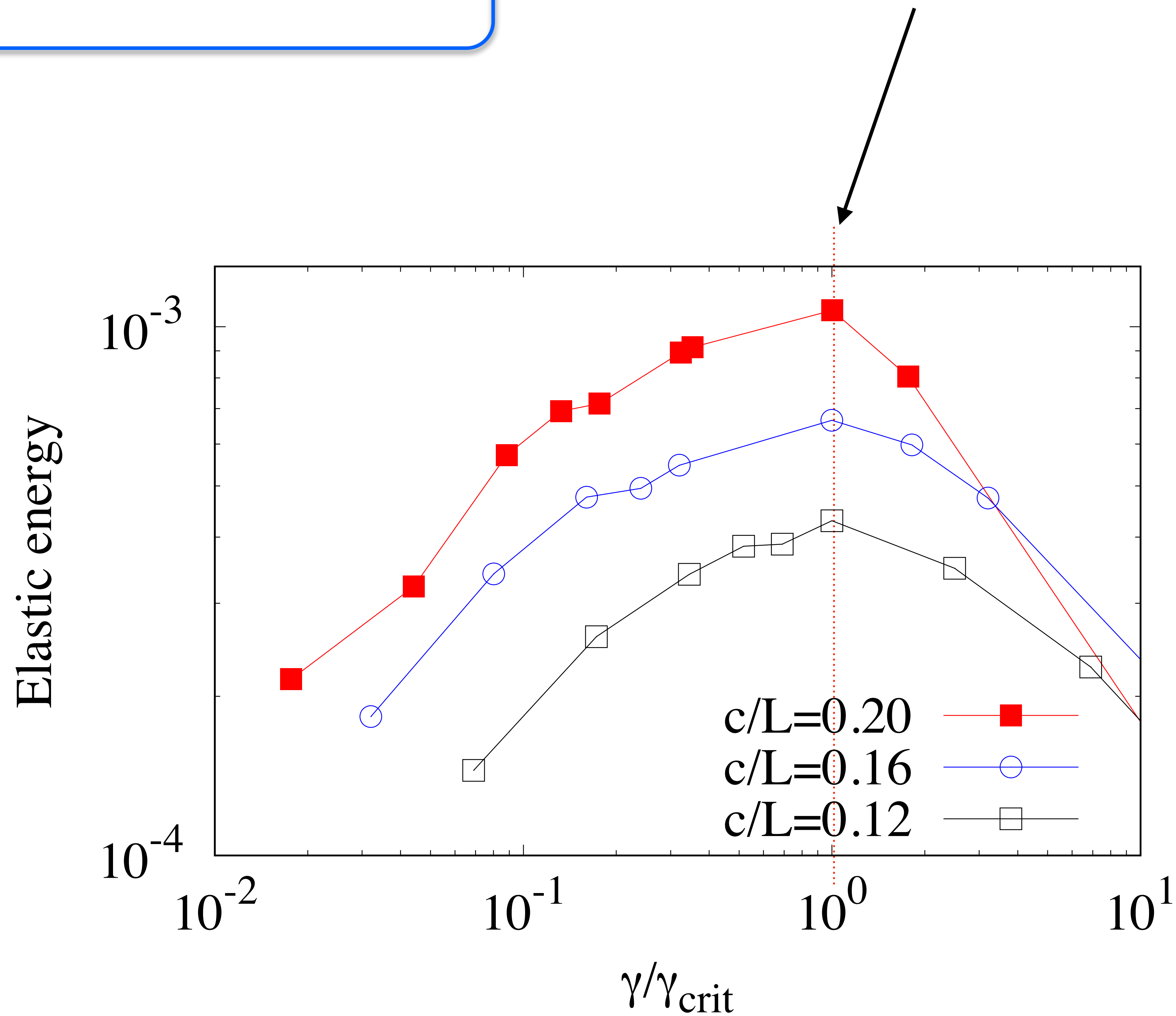
Stationary, homogeneous and isotropic turbulence ruled by the NS equations

$$
\begin{aligned}
\partial_t \mathbf{u} + \mathbf{u} \cdot \boldsymbol{\partial} \mathbf{u} &= -\boldsymbol{\partial} p/\rho_0 + \nu \partial^2 \mathbf{u} + \mathbf{f} + \mathbf{f}^T \\
\boldsymbol{\partial} \cdot \mathbf{u} &= 0, \\
\rho_1 \ddot{\mathbf{X}} &= \partial_s (T \partial_s \mathbf{X}) - \gamma \partial_s^4 \mathbf{X} + \mathbf{F}
\end{aligned}
$$

Resonance condition and consequences

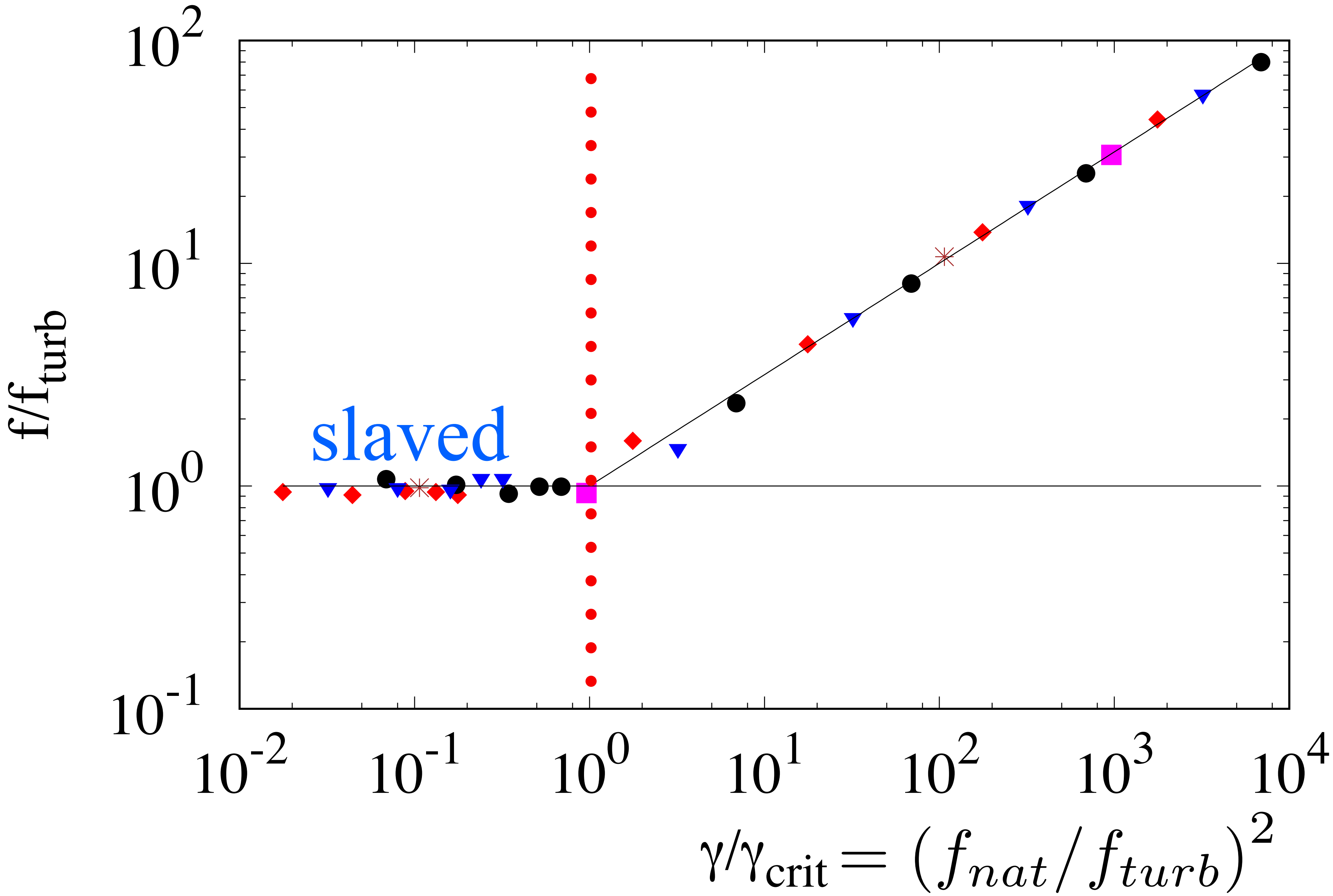
$$\gamma_{crit} \sim c^{8/3}\epsilon^{2/3}\rho_1\alpha^2$$

maximum at $\gamma \sim \gamma_{\rm crit}$



Resonance condition and consequences

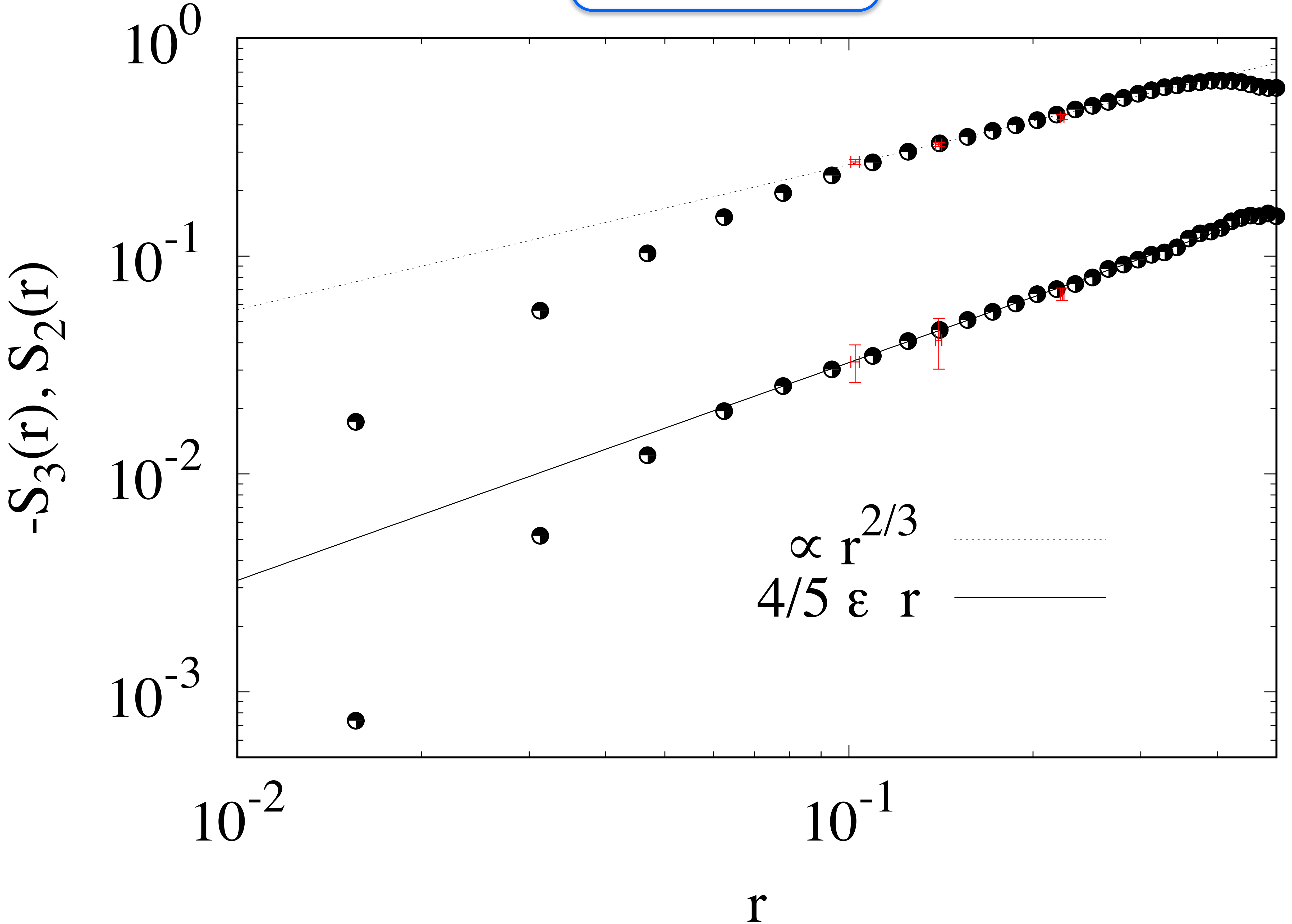
$$\gamma_{crit} \sim c^{8/3}\epsilon^{2/3}\rho_1$$



$$f_{\rm turb} \equiv 1/\tau(c)$$

Can the fiber be used to reveal turbulence statistics?

$\gamma/\gamma_{\mathrm{crit}} = 1/2$



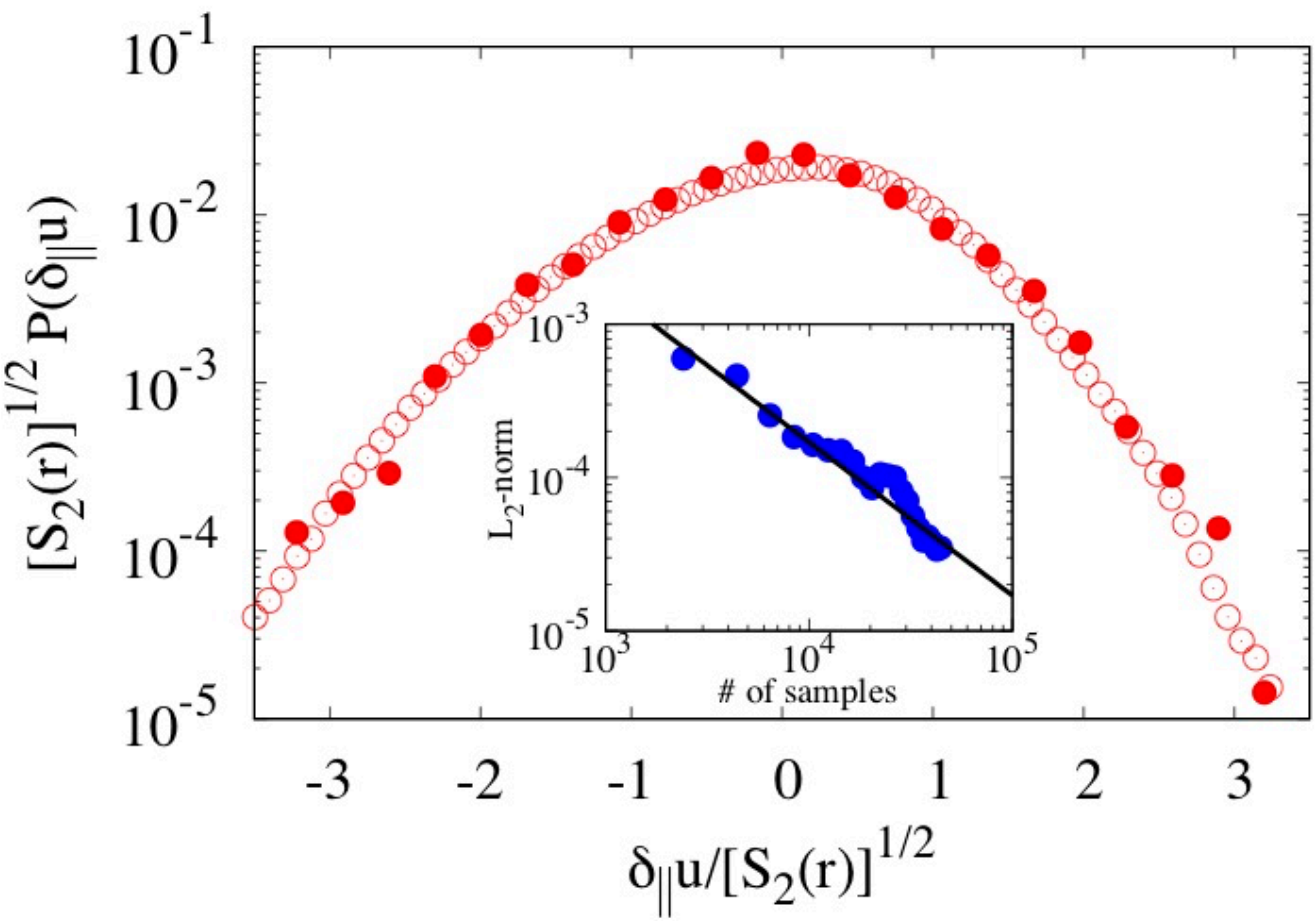
Can the fiber be used to reveal turbulence statistics?

$$\gamma/\gamma_{\mathrm{crit}} = 1/2$$

○ standard Eulerian

● from the fiber

Pdf of velocity increments

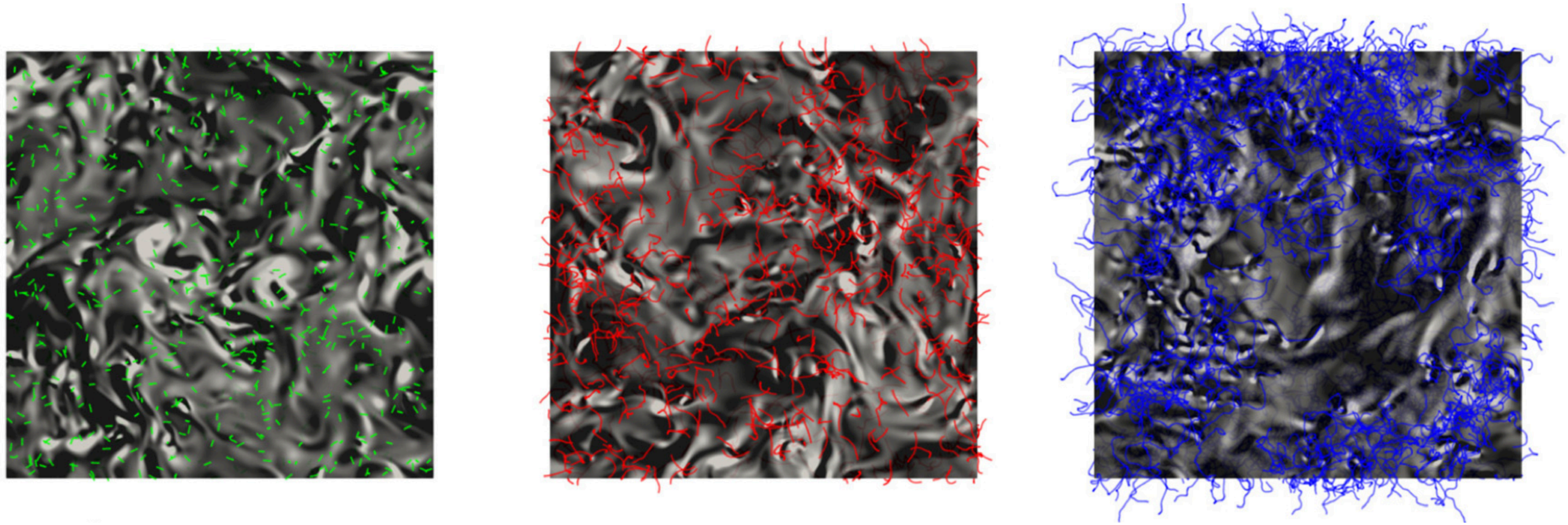


$$r = 0.14\,L$$

The non dilute case

Olivieri, A.M., Rosti, POF (2021)

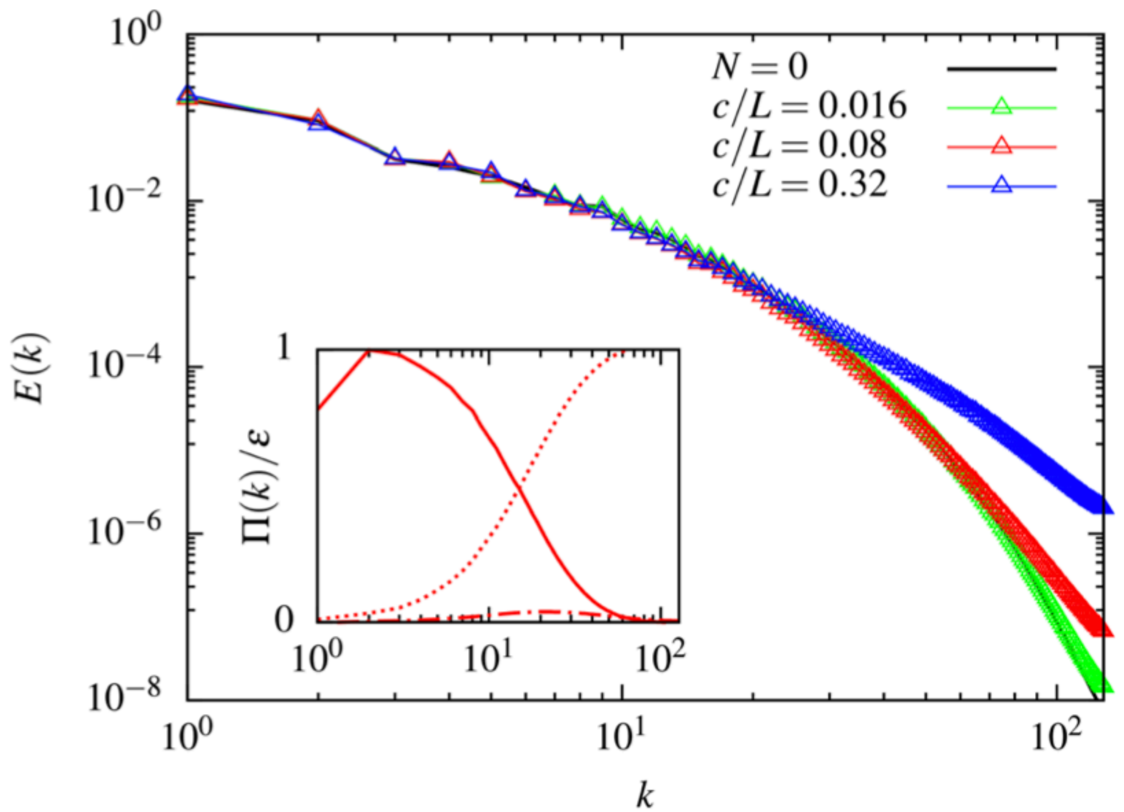
N=1000 fibers of different lengths: c/L=0.016, 0.08, 0.32



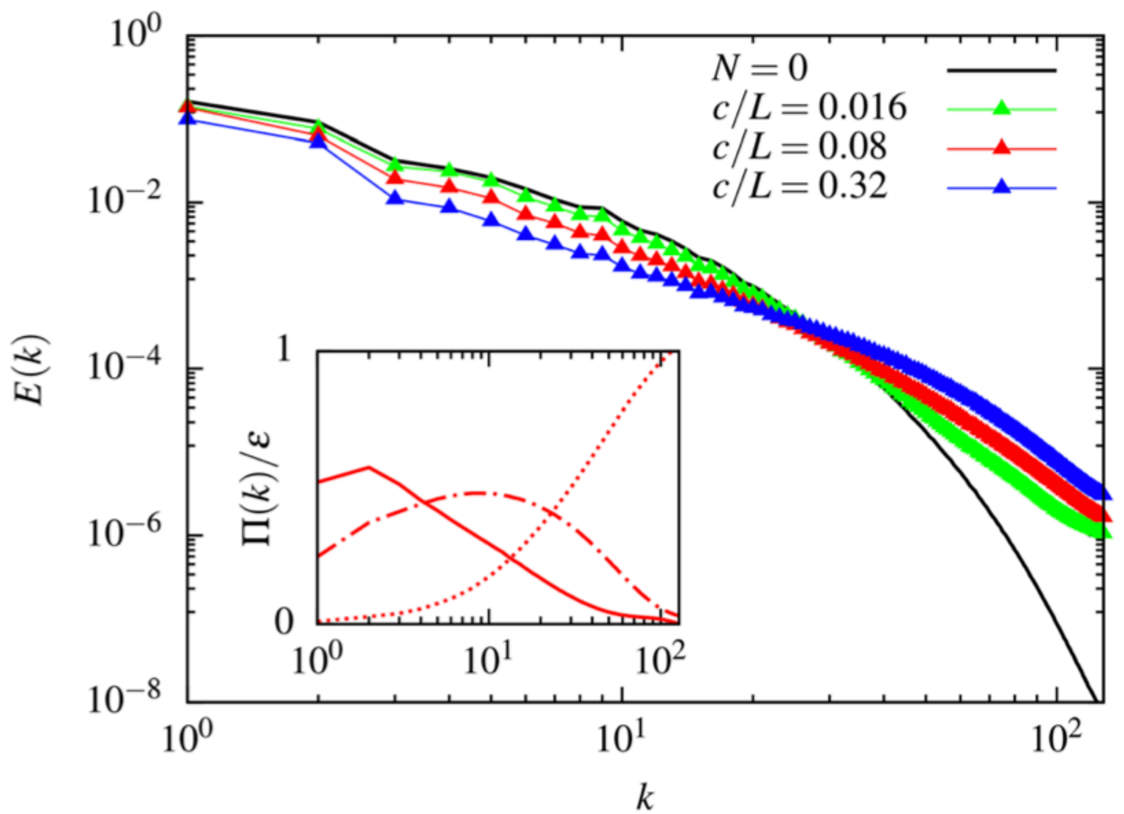
here fiber back-reaction is not negligible

The non dilute case

Fibers of small inertia



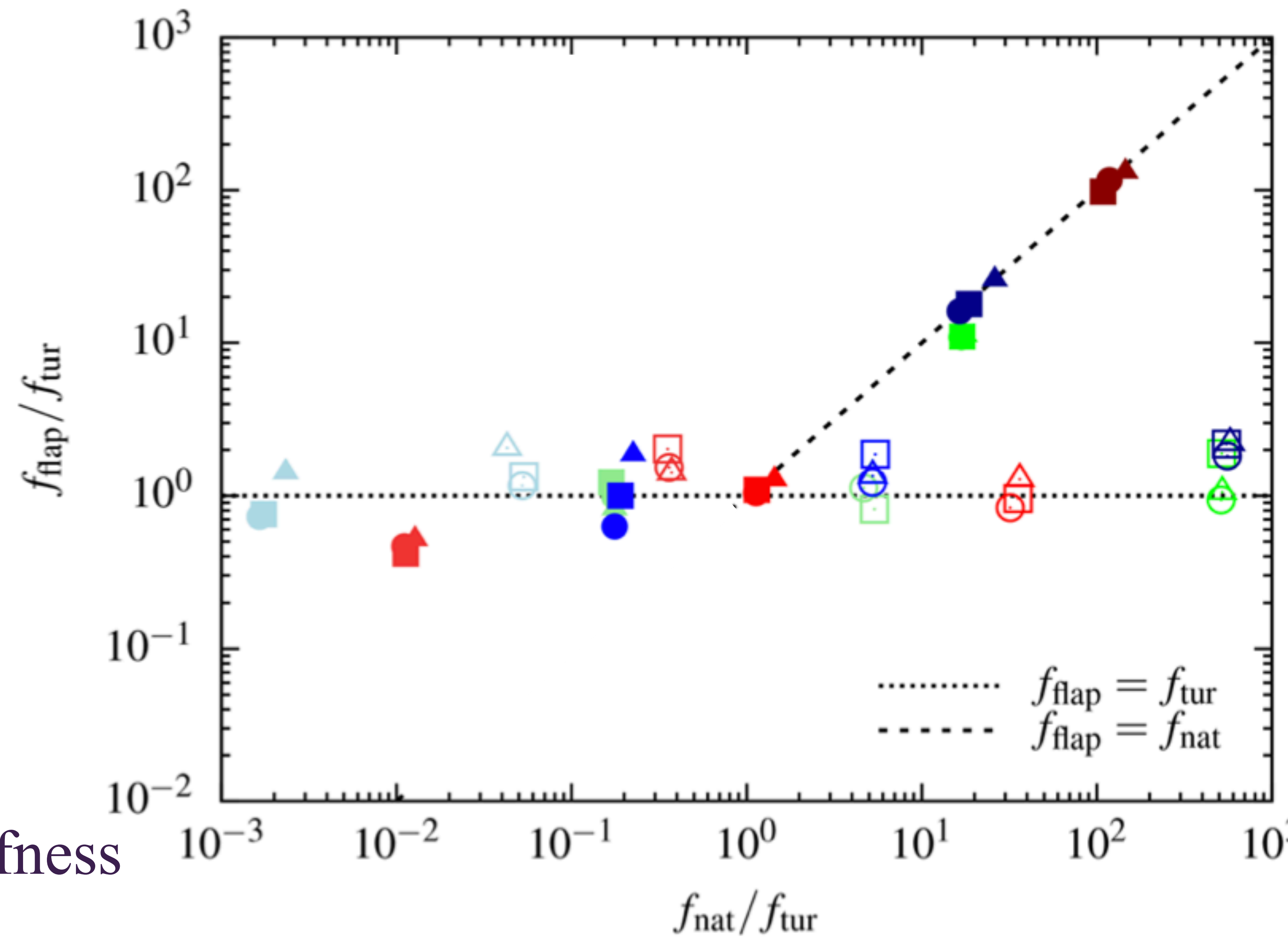
Larger inertia



The non dilute case

$$f_{tur} \sim \sqrt{S_2(c)}/c$$

filled: inertial fibers
empty: neutrally b.
circle: N=10
square: N=100
triangle: N=1000
colour: different lengths
brightness: different stiffness



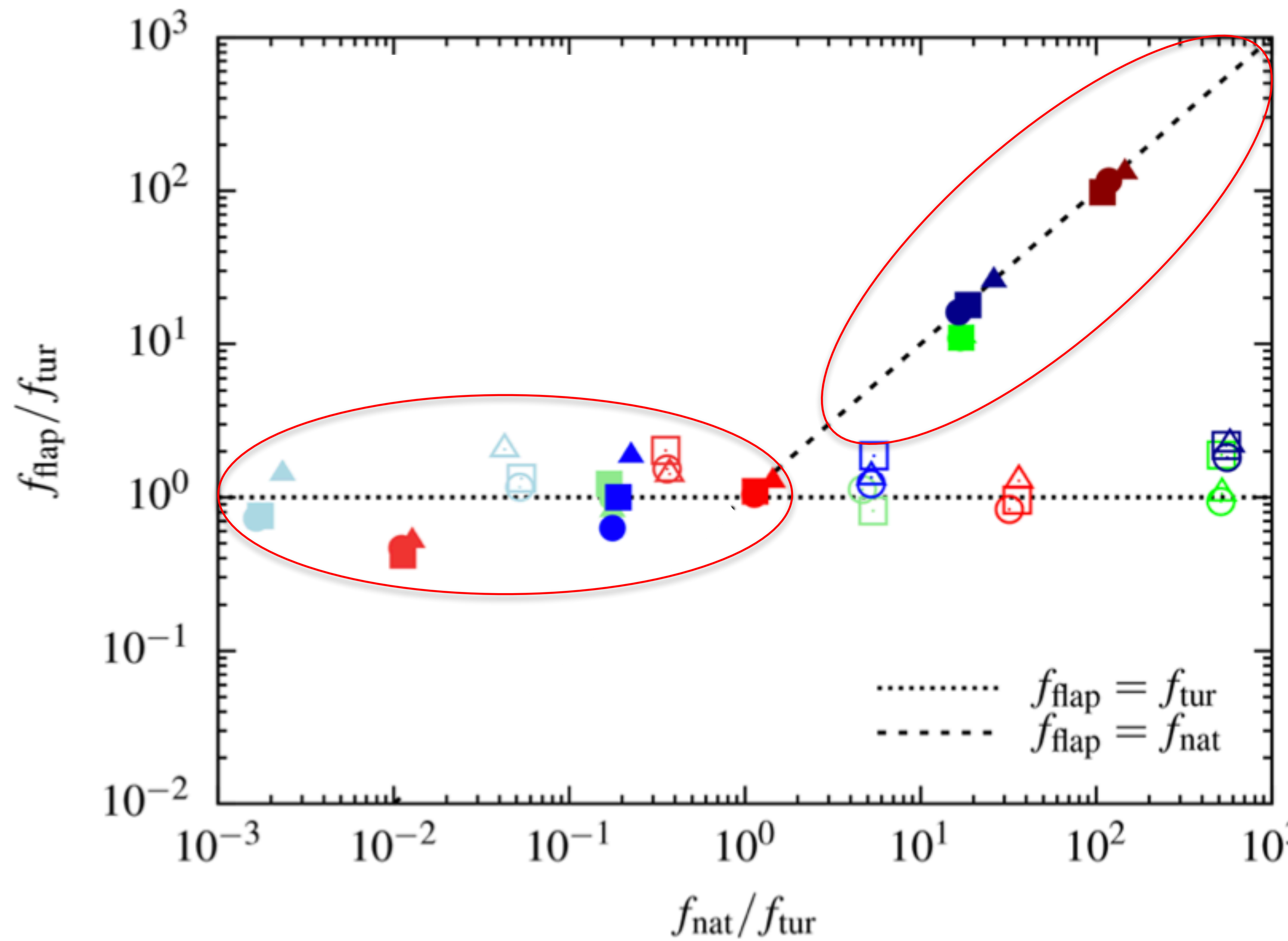
$$= (\gamma/\gamma_{crit})^{1/2}$$

for turbulence à la K41

The non dilute case

$$f_{tur} \sim \sqrt{S_2(c)}/c$$

underdamped



$$= (\gamma/\gamma_{crit})^{1/2}$$

for turbulence à la K41

The non dilute case

$$f_{tur} \sim \sqrt{S_2(c)}/c$$

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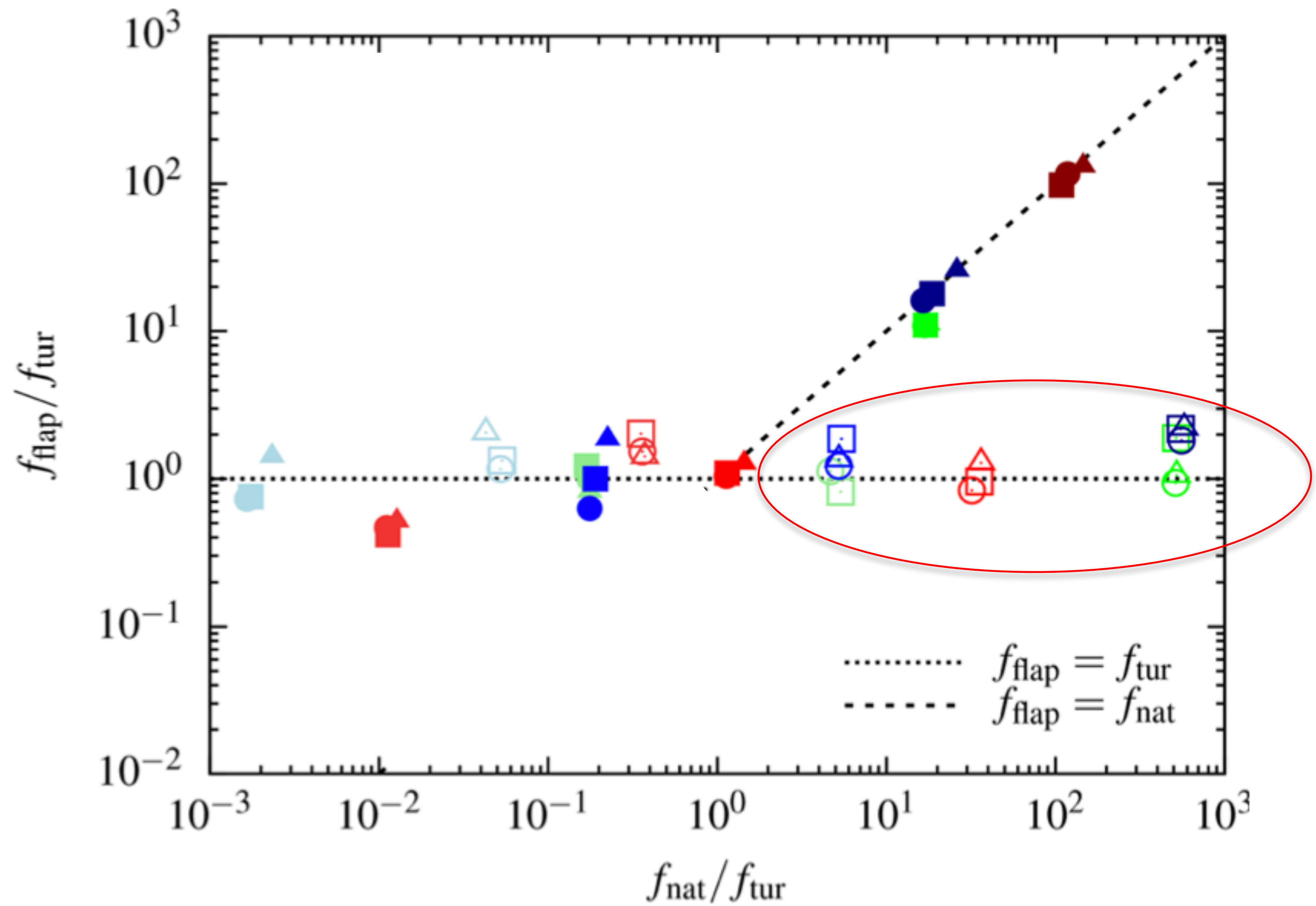


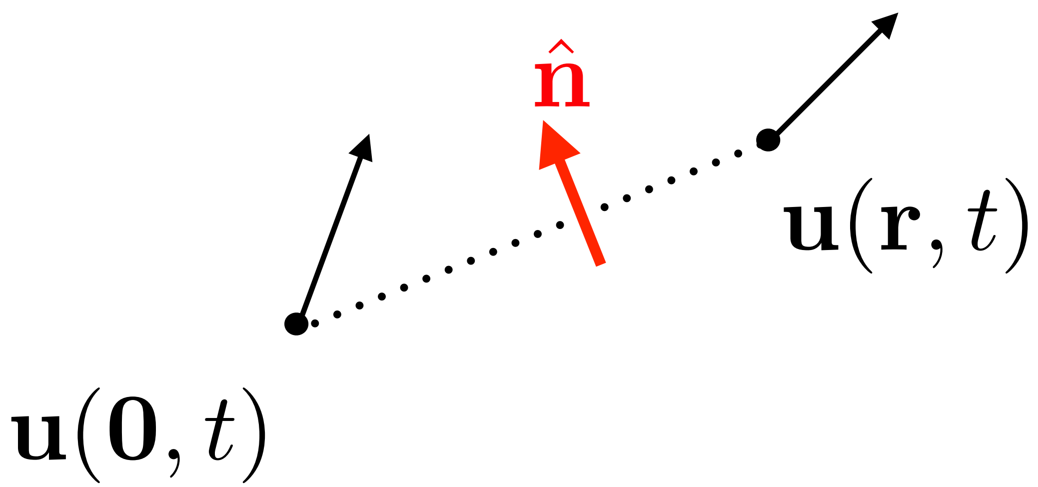
$$= (\gamma/\gamma_{crit})^{1/2}$$

for turbulence à la K41

Rigid fibers for transverse statistics

Relevant observables:

$$\delta u_\perp \equiv [\mathbf{u}(\mathbf{r},t) - \mathbf{u}(\mathbf{0},t)] \cdot \hat{\mathbf{n}}$$



No theory for even moments (odd moments are zero)

Experimental measures do exist (e.g. Noullez et al JFM 1997)

The case of rigid fibers

Of course: $$\delta u_{\parallel} = \delta v_{\parallel}$$ NO

fluid fiber

Relevant questions:

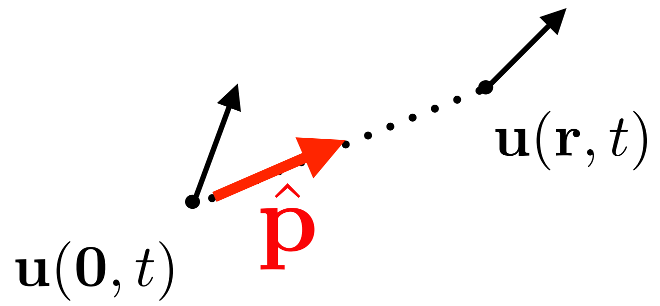
? $$\delta u_{\perp} = \delta v_{\perp}$$?

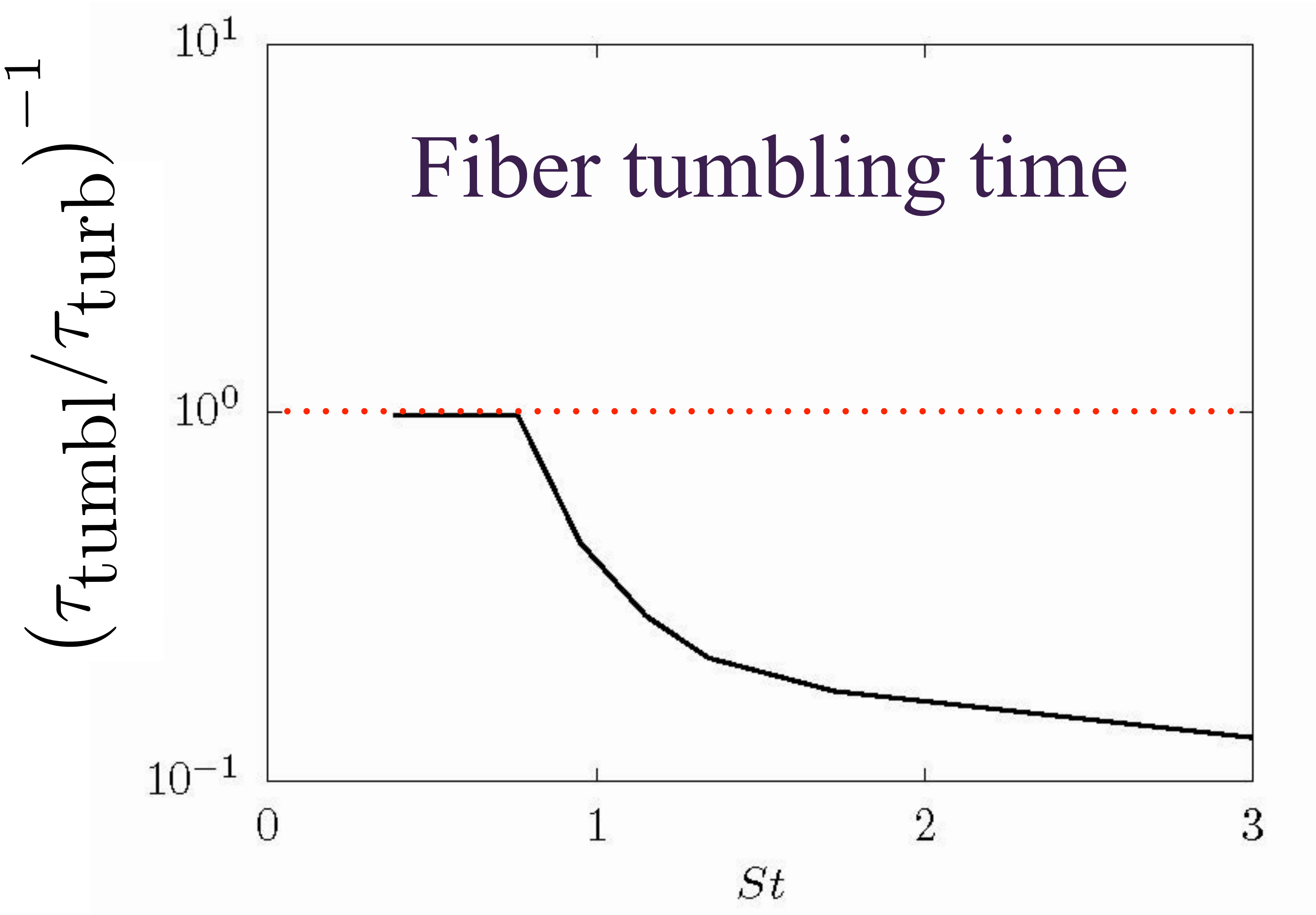
$$\tau_{\rm turb} = \tau_{\rm tumbl}$$

The case of rigid fibers (numerics)

$$\tau_{\mathrm{turb}} = c^{2/3}\epsilon^{-1/3}$$

$$\tau_{tumbl} = \langle \dot{\hat{\mathbf{p}}} \cdot \dot{\hat{\mathbf{p}}} \rangle^{-1/2}$$

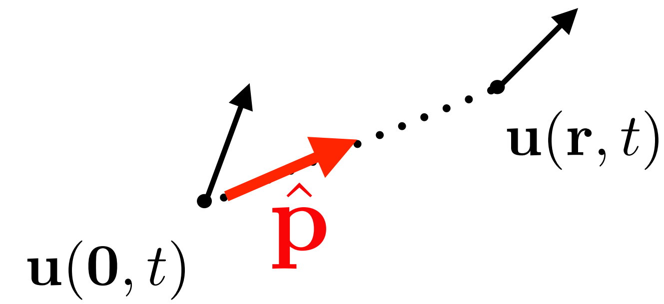




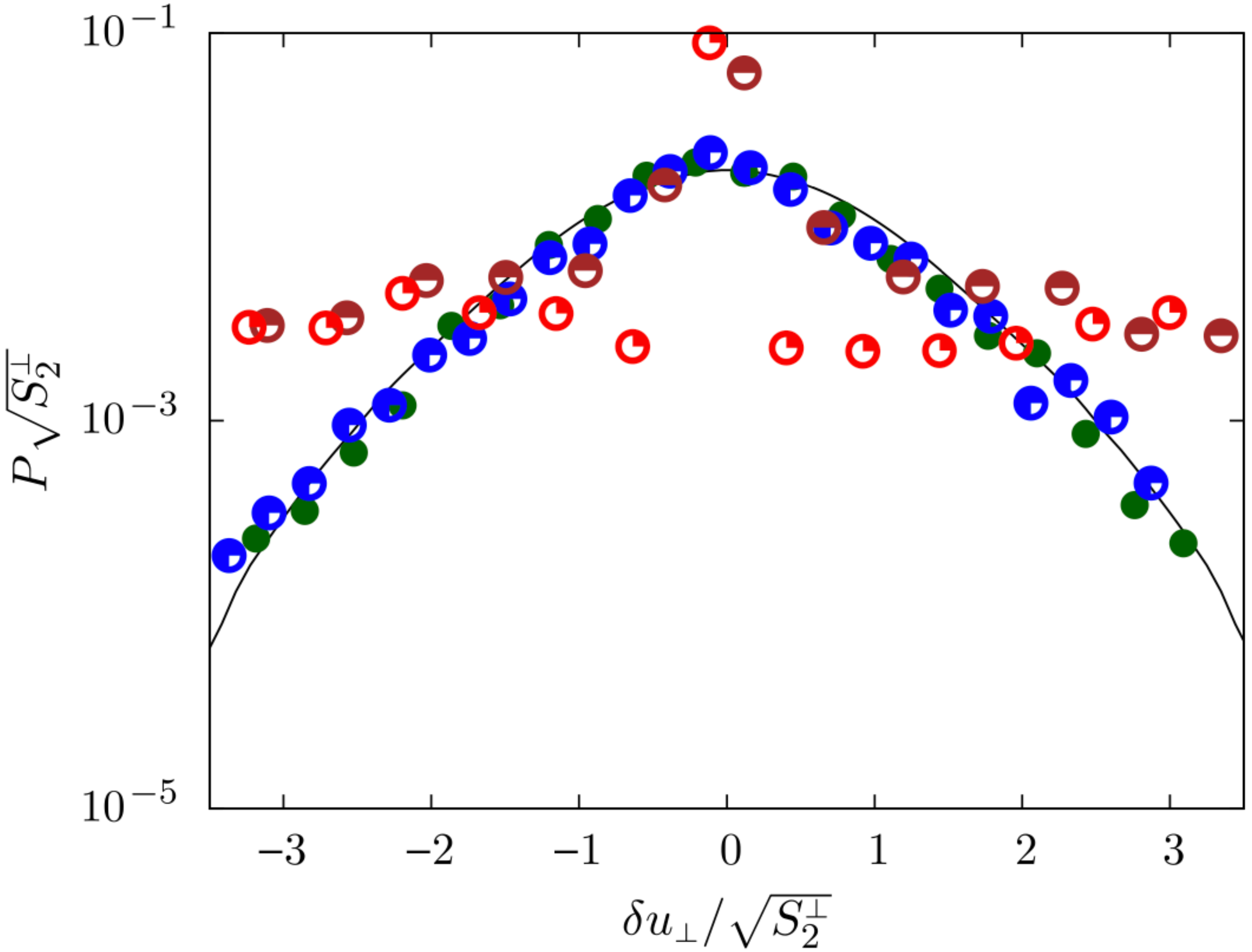
fiber measures turbulence eddy-turnover time at small St

The case of rigid fibers (numerics)

$$\tau_{\text{turb}} = c^{2/3}\epsilon^{-1/3}$$
$$\tau_{tumbl} = \langle \dot{\hat{\mathbf{p}}} \cdot \dot{\hat{\mathbf{p}}} \rangle^{-1/2}$$



Pdf of transverse velocity increments

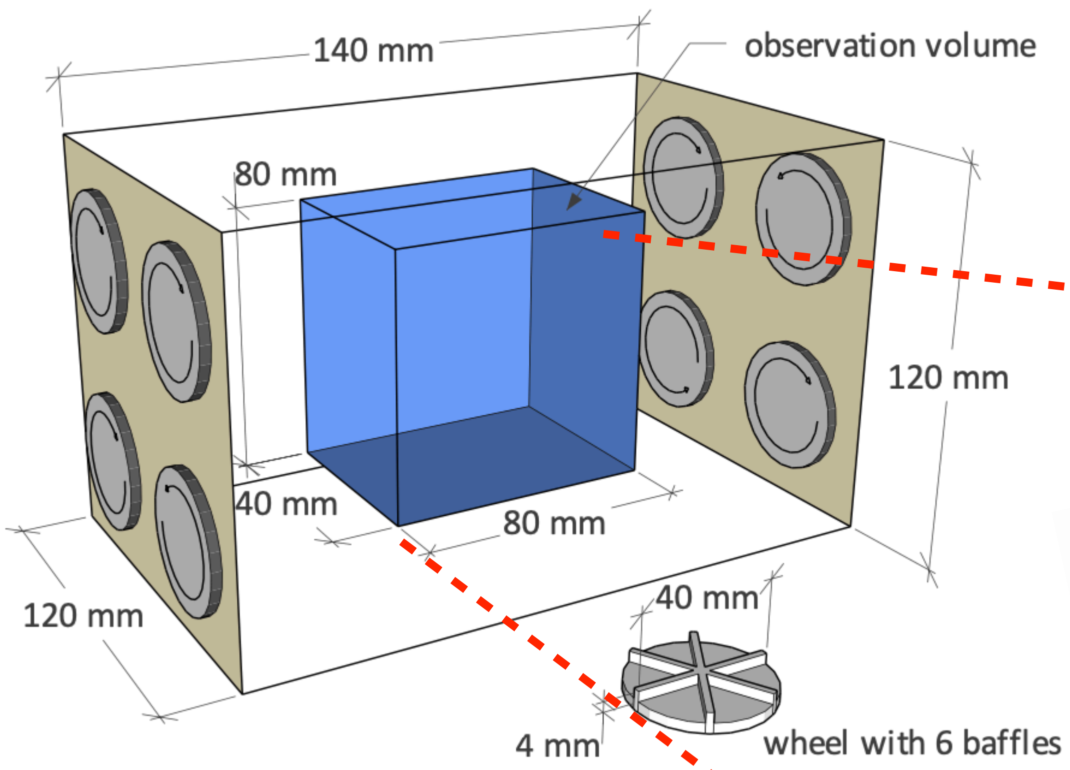


fiber measures transverse fluctuations at small St

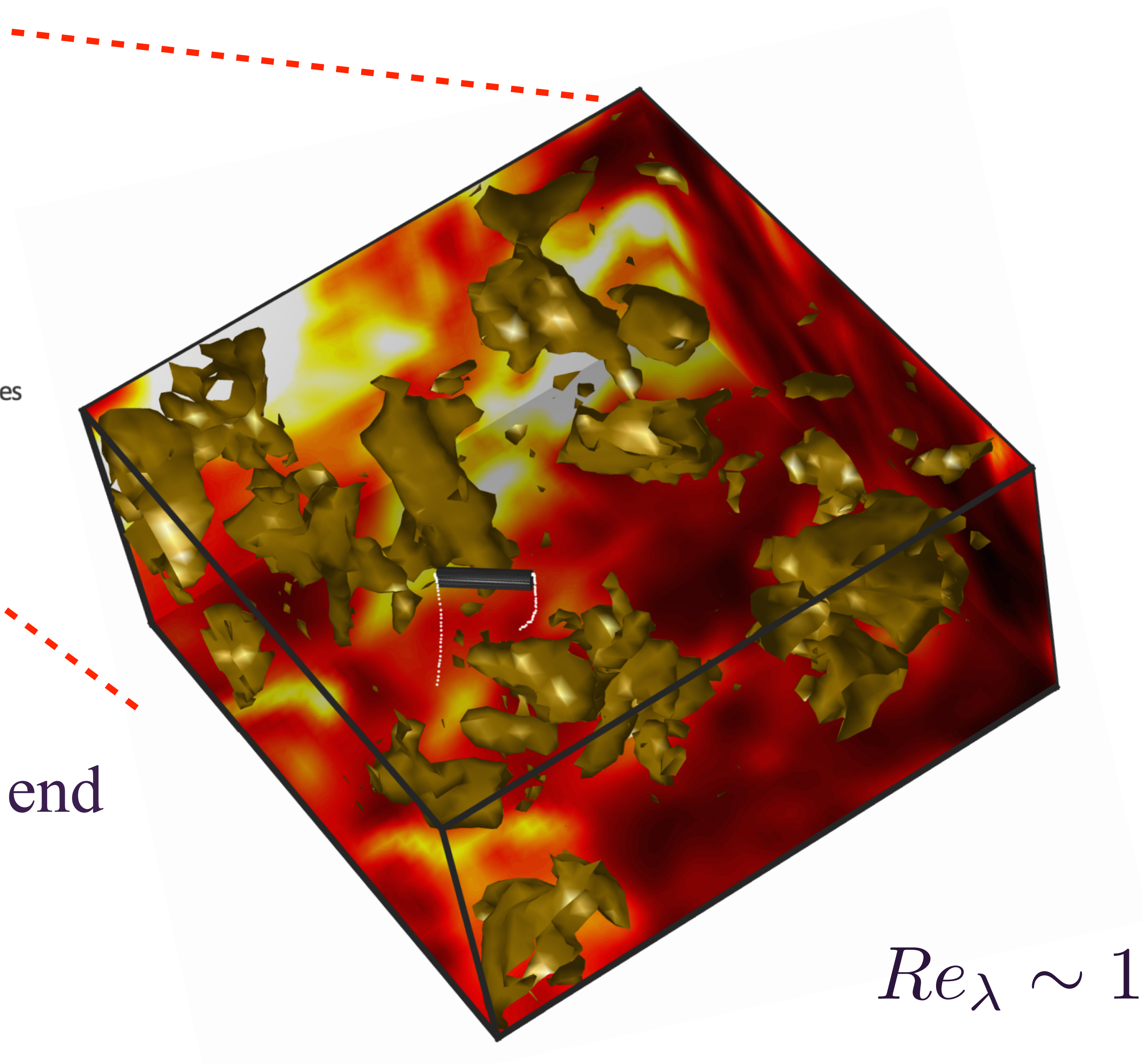
From the world of simulations to real life

Brizzolara, Rosti, Olivieri, Brandt, Holzner, A.M., PRX (2021)

The ETHZ aquarium



Fibers: hand-crafted with Polydimethylsiloxane (neutrally buoyant and fluorescent)



Fiber tracking to assess positions/velocity of fiber end points

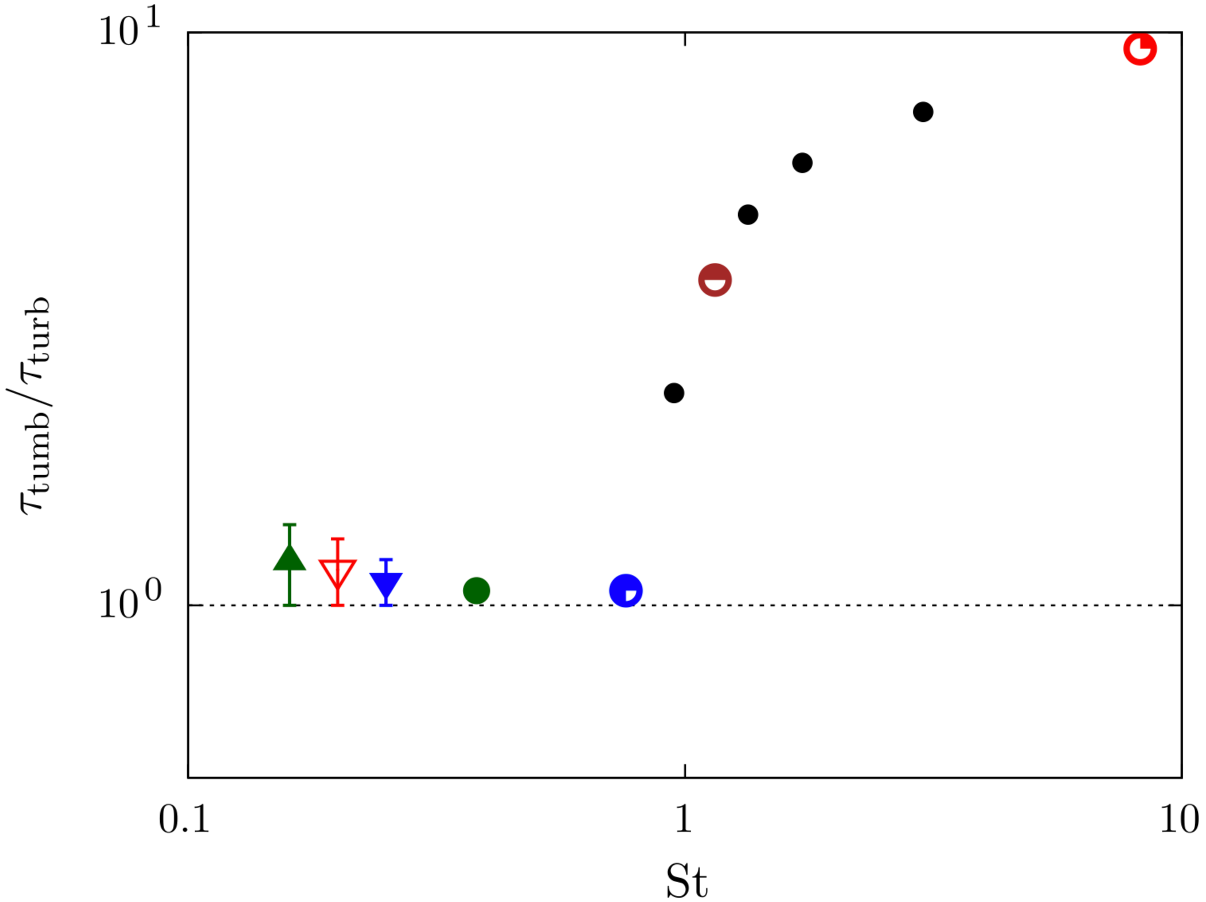
$Re_\lambda \sim 145$

The case of rigid fibers (experiments): IR statistics

$$\tau_{\text{turb}}(c) = \frac{c}{\sqrt{\frac{15}{2} S_2^{\perp}}}$$

bullets: from DNS

triangles: from FTV

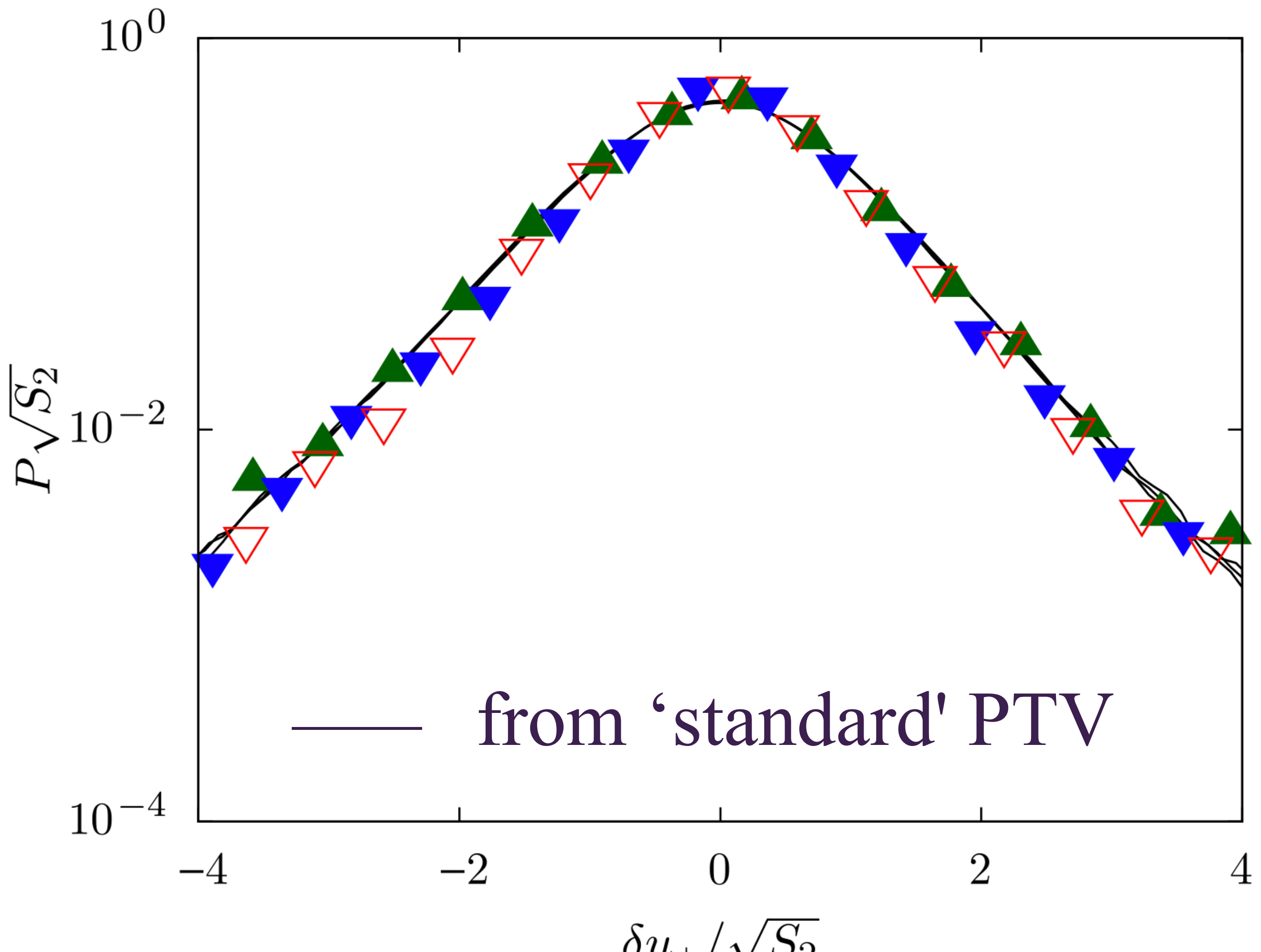


St from: Bounoua, Bouchet, Verhille, PRL (2018)

$c/L = 0.71$

$c/L = 0.45$

$c/L = 0.57$

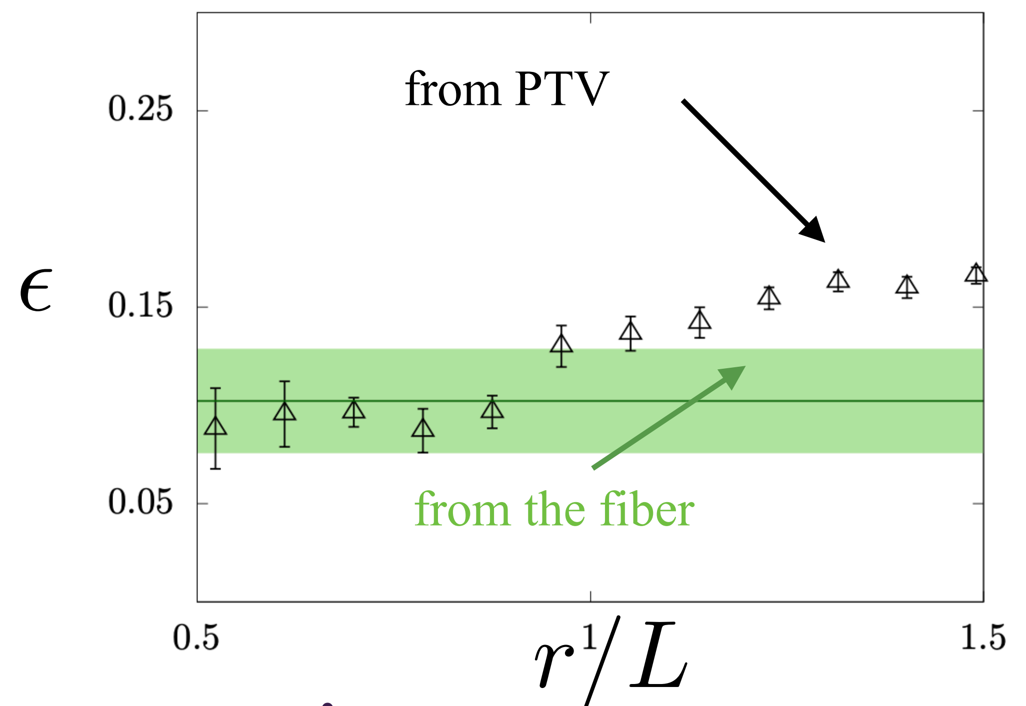


Measuring gradients

with smart particles: Hejazi, Krellenstein, Voth, APS Meeting (2017)
Hejazi, Krellenstein, Voth, Exp. in Fluids (2019)

energy dissipation rate $\epsilon = \frac{15}{2}\nu\left\langle\left(\frac{\partial u_1}{\partial x_2}\right)^2\right\rangle \approx \frac{15}{2}\nu\left\langle\left(\frac{\delta u_\perp}{c}\right)^2\right\rangle$

our fiber: Nylon, length = 8η



Rigid fibers are a proxy of two-point transverse statistics of turbulence

Fiber Tracking Velocimetry

Brizzolara, Rosti, Olivieri, Brandt, Holzner, A.M. PRX (2021)

Back to numerics: assembly of fibers for the full gradient tensor

In 2D: three (hinged) fibers to measure the full gradient tensor

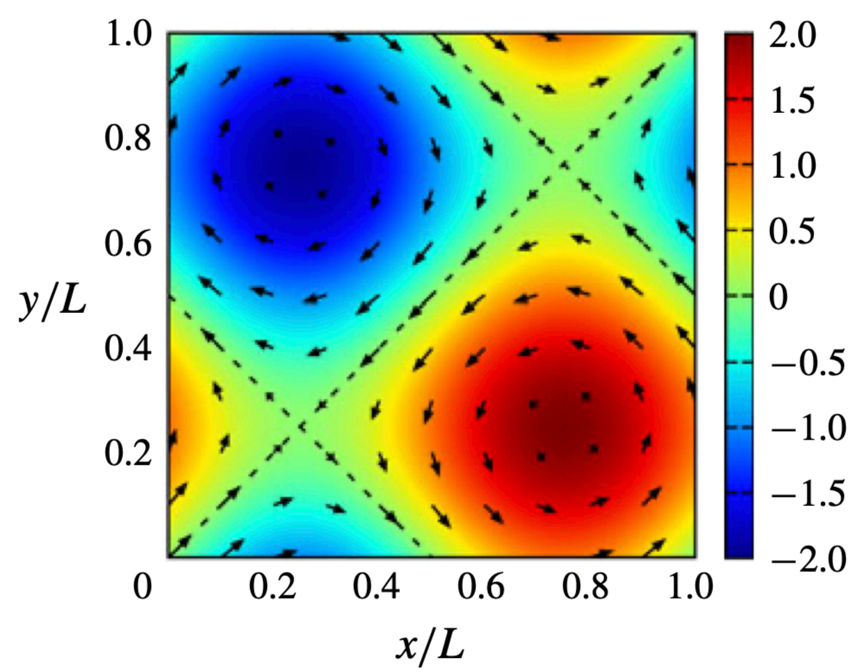
$$\delta V_\perp = \delta \boldsymbol{V} \cdot \hat{\boldsymbol{r}}^\perp$$ ⟵ for the fiber

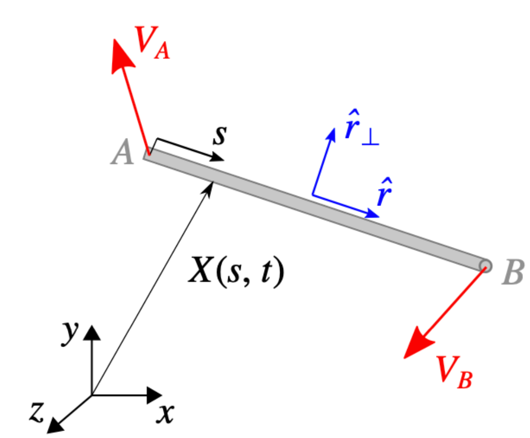
$$\delta u_\perp = \delta \boldsymbol{u} \cdot \hat{\boldsymbol{r}}^\perp$$ ⟵ for the flow velocity

$$\delta V_\perp^{(1)} = \partial_j u_i \hat{r}_j^{(1)} \hat{r}_i^{\perp(1)} \mathrm{c}$$

$$\delta V_\perp^{(2)} = \partial_j u_i \hat{r}_j^{(2)} \hat{r}_i^{\perp(2)} \mathrm{c}$$

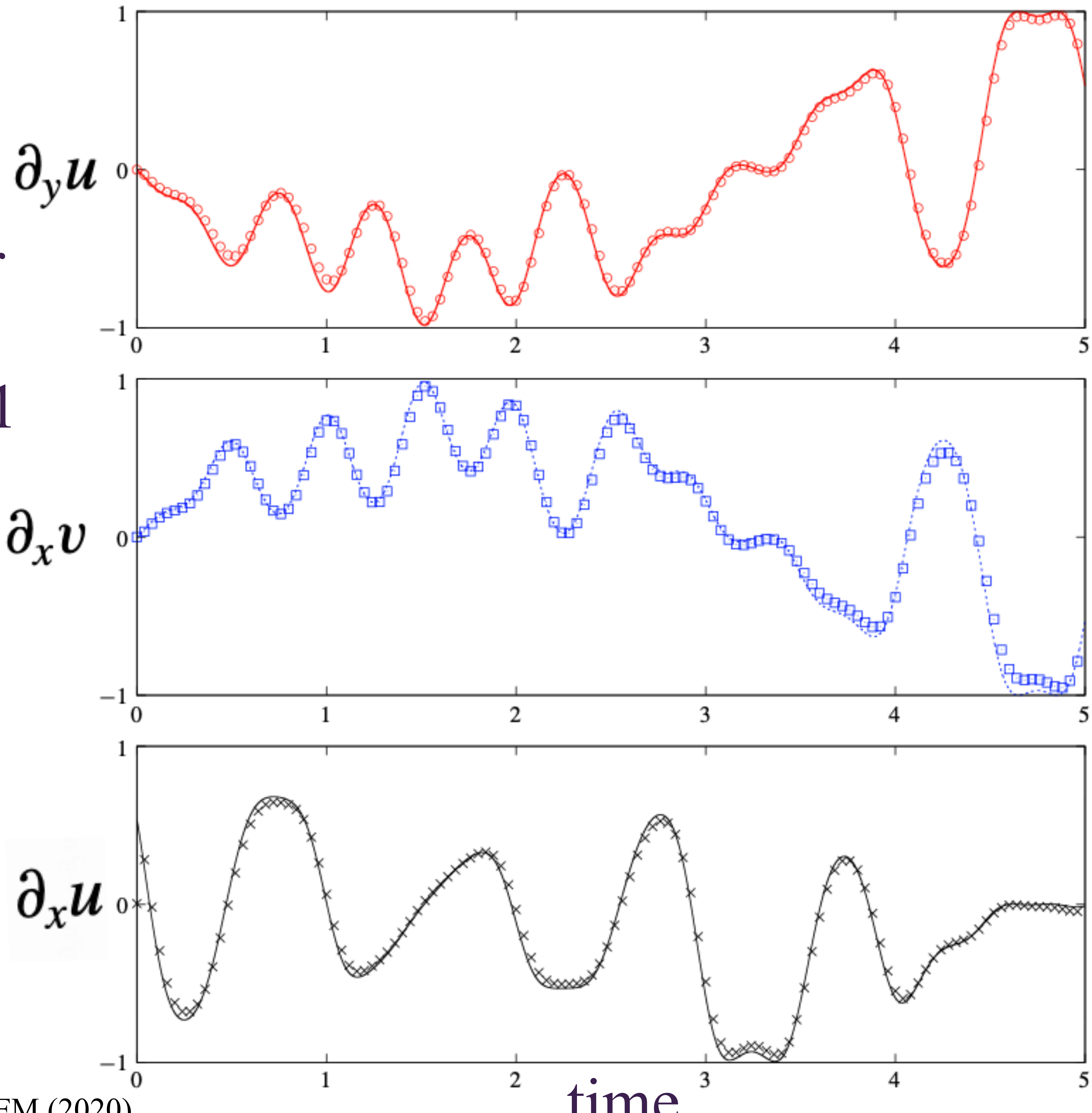
$$\delta V_\perp^{(3)} = \partial_j u_i \hat{r}_j^{(3)} \hat{r}_i^{\perp(3)} \mathrm{c}$$





relative error smaller than 1% for St~0.1

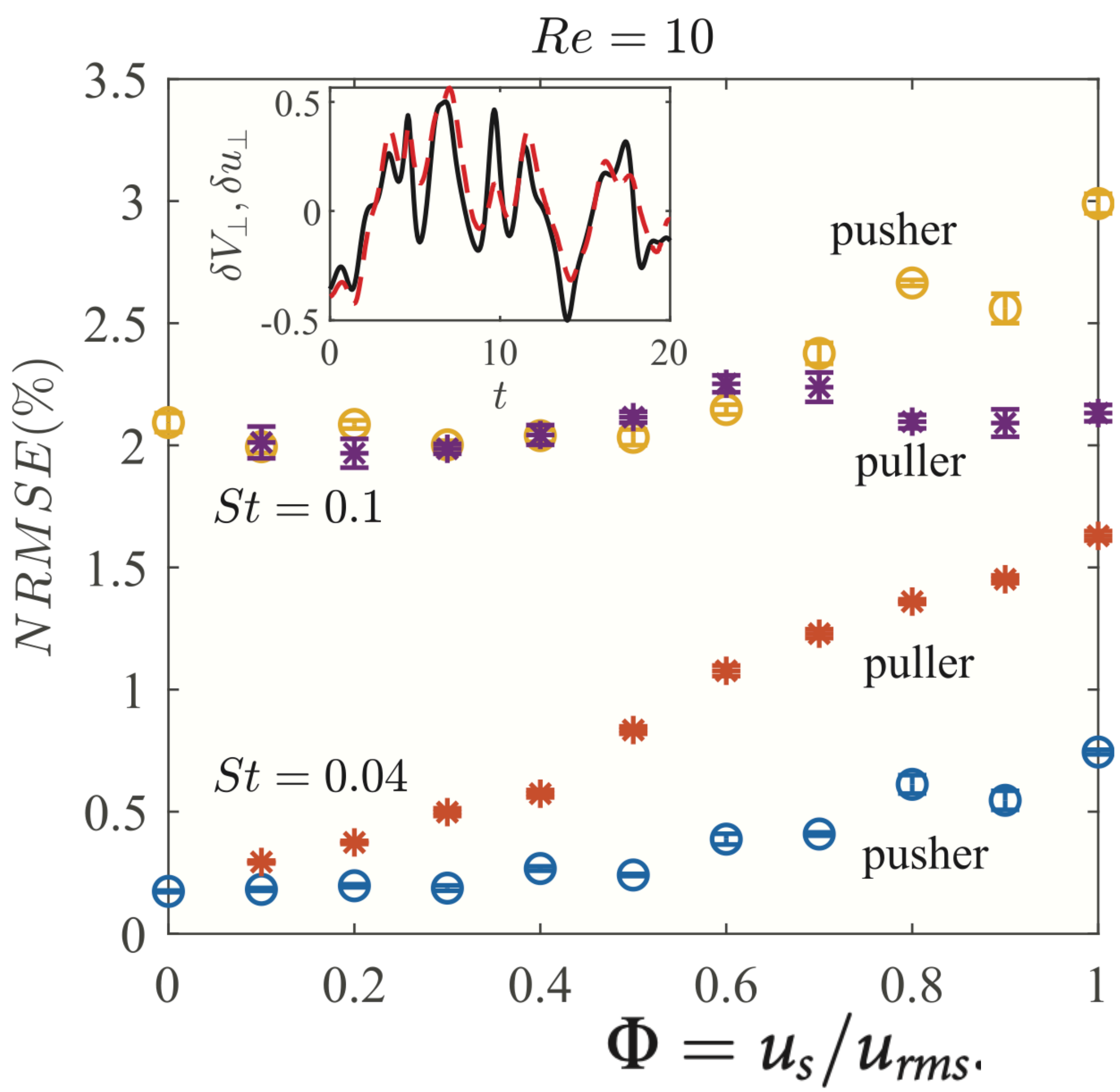
It does not work if the three fibers are clamped instead of hinged



Cavaiola, Olivieri, A.M., JFM (2020)

Self-propelled fibres: may they measure flow properties net of self motion?

At least for weak propulsions the conclusion is expected to hold also for `swimmers'



NS are forced by:

$$\mathbf{f}^V = \partial_t \mathbf{u} - (1/Re)\partial^2 \mathbf{u}$$

where

$$u = \sin(z + \varepsilon \sin(\Omega t)) + \cos(y + \varepsilon \sin(\Omega t)),$$
$$v = \sin(x + \varepsilon \sin(\Omega t)) + \cos(z + \varepsilon \sin(\Omega t)),$$
$$w = \sin(y + \varepsilon \sin(\Omega t)) + \cos(x + \varepsilon \sin(\Omega t))$$

$$NRMSE = \frac{\left(\frac{1}{t_{\max}} \int_{t=0}^{t_{\max}} (\delta V_\perp - \delta u_\perp)^2 \, dt\right)^{\frac{1}{2}}}{\pi G}$$

$$G = \sqrt{\langle \dot{\gamma}^2 \rangle} \qquad \langle \dot{\gamma}^2 \rangle = \langle e_{ij} e_{ij} \rangle$$

Cavaiola, A.M., POF (2021)

Conclusions

Both fully-resolved numerical simulations and experiments show the ability of fibers (rigid or elastic) to measure flow properties

↓

Fiber Tracking Velocimetry

Brizzolara, Rosti, Olivieri, Brandt, Holzner, A.M. PRX (2021)

To do: non ideal turbulence