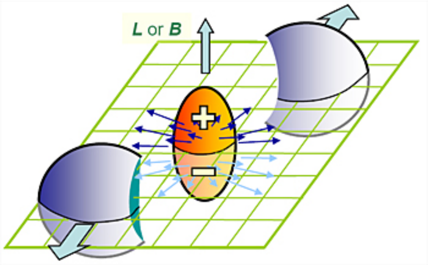
Is there a holographic description of Heavy-Ion Collisions beyond the Page time?

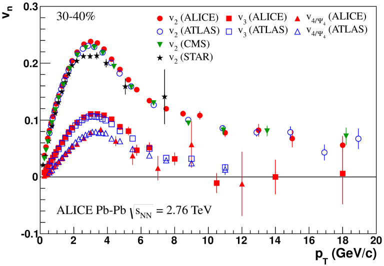
Andreas Schäfer
in collaboration with B. Müller, S. Waeber, L. Yaffe etc.

Institute for Theoretical Physics
University of Regensburg

- We explored numerically the early phase of high energy heavy ion collisions in recent years using AdS/CFT duality
- We now try to extend the dual description to cover hadronization
- There are many open questions

Key question of relativistic heavy ion physics: Does the quark gluon plasma realy thermalize? Is “hydrodynamization” equivalent to thermalization?





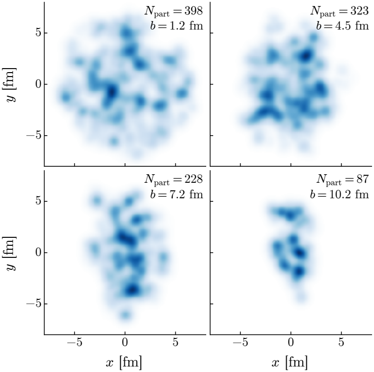
Observable: Elliptic flow $v_n \sim \cos(n\phi)$ with $n = 2$

How can transverse communication happen in less than 1fm/c?

$\gamma(Pb) > 2500$ giving it a width of $11\text{fm}/2500 = 0.004\text{fm}$

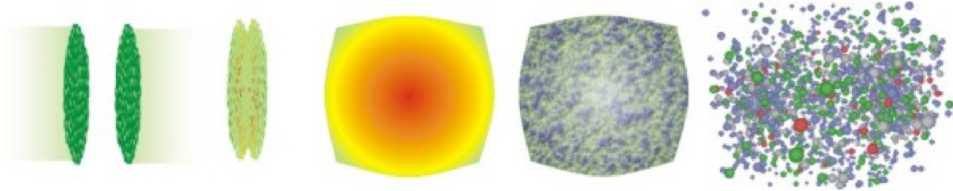
transverse color coherence length much smaller than transverse size $1/Q_s < 0.2\text{fm}$

very inhomogeneous energy density arXiv:1605.03954



But: Entropy cannot be produced because QCD is T-invariant! The apparent hydrodynamization must be observable dependent.$\Rightarrow$ ETH “Eigenstate Thermalization Hypothesis”

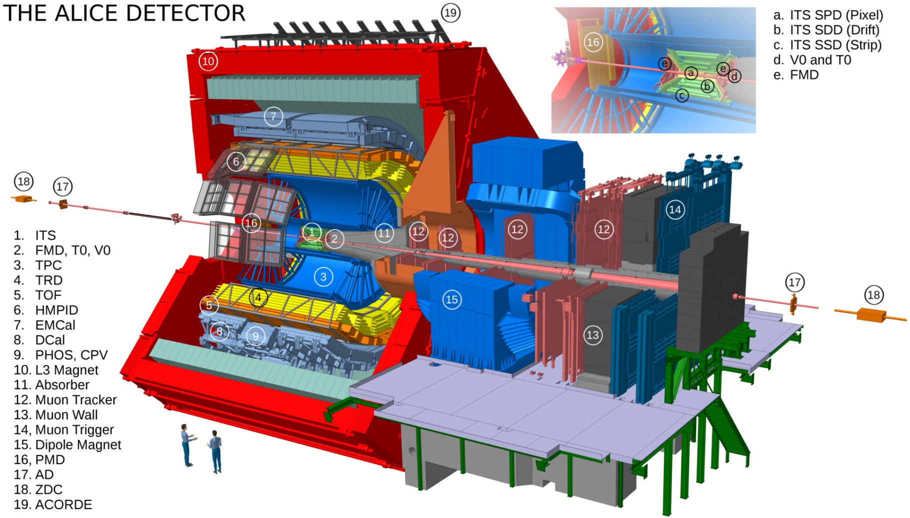
Just one example, the hadron yields: arXiv:1809.04681, ALICE, CERN

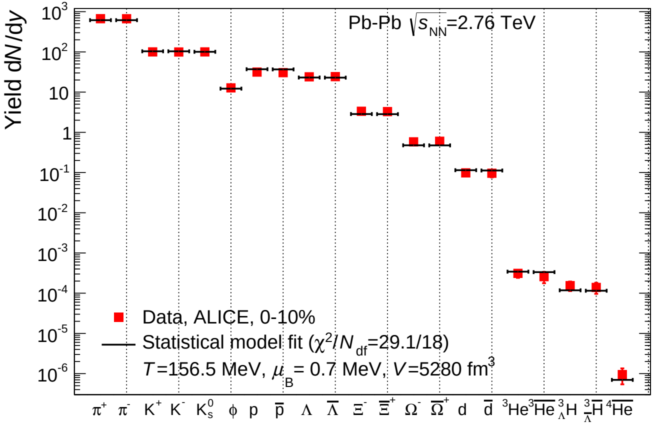


AdS/CFT clarified that hydrodynamization (local obervables) is fast

ETH might require much longer to apply (system wide correlations)

There is very much high precision data, e.g. from ALICE





But: R(rms , $^3_\Lambda H$)=10.6 fm~ $2R_{\rm Pb}$;

$-B = 0.4 MeV << 156$ MeV   the yield should be suppressed

Thus one has two convincingly motivated interpretations which seem to be contradictory

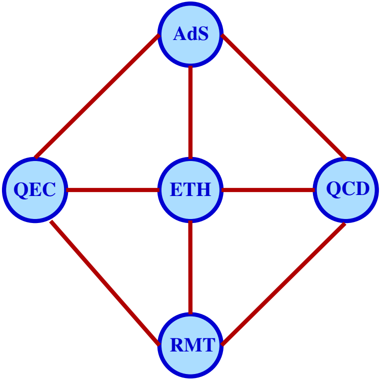
- Hundreds of detailed measurements support the fireball interpretation, i.e. entropy production, hydrodynamics etc.
- General T-invariance suggest a microcanonical picture with highly entangled many particle quark-gluon and hadronic states.

Proponents of both lines of argument seem to be correct. How can this be?

One needs two standard elements of quantum information theory: Page curve plus ETH. The Page curve reminds of the information problem of BH physics and in fact it is argued that both are very similar.

ETH could explain the ${}^{3}_{\Lambda}H$ puzzle.

ETH predicts that small probes should look thermal, where "small" can be half of the system size. (This limit of "half of the system size" occurs again in BH physics and Quantum Error Correction.)



ETH: D'Alesio, Kafri, Polkovnikov, Rigol 1509.06411

$$O_{mn} = \langle m|\hat{O}|n\rangle = O(\bar{E})\delta_{mn} + e^{-S(\bar{E})/2} f_O(\bar{E},\omega) R_{mn}$$

$\bar{E} = (E_m + E_n)/2$, $\omega = E_m - E_m)$, $S(\bar{E})$ thermodynamic entropy at energy $\bar{E}$, $O(\bar{E})$ and $f_O(\bar{E},\omega)$ are smooth functions, $O(\bar{E})$ is identical to the expectation value of the microcanonical ensemble at energy $\bar{E}$, and R_{mn} is a random matrix.

Questions: Does RMT apply to QCD?

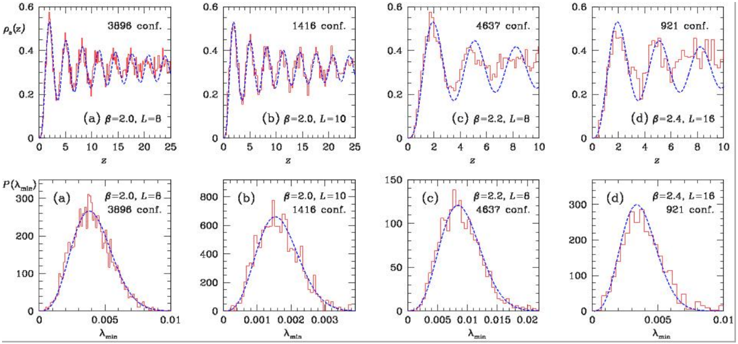
Does ETH apply to QCD?

QCD is a prime example for an ergodic theory.

A HIC in the ultra vacuum of the LHC is a prime example for an isolated system.

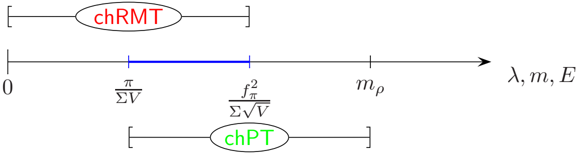
A long story: Berbenni-Bitsch, Meyer, AS, Verbaarschot and Wettig, “Microscopic universality in the spectrum of the lattice Dirac operator,” hep-lat/9704018

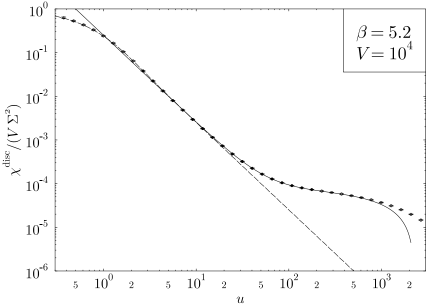
Comparison of microscopic level spacing for LQCD (red) and RMT(blue)



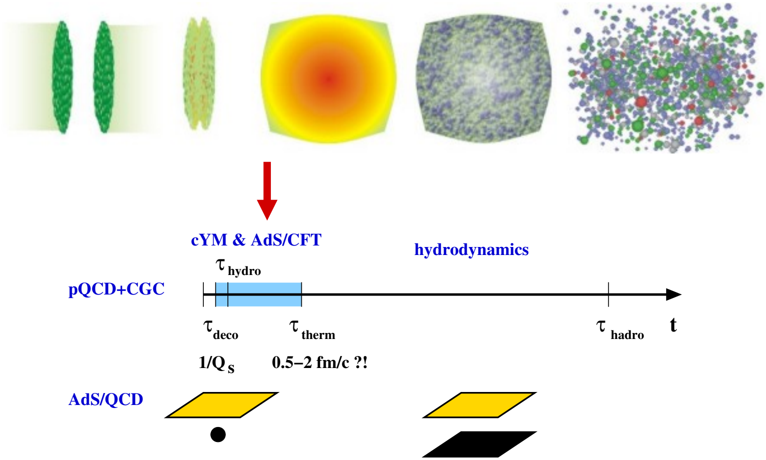
Simulations with quenched SU(3) Kogut-Susskind fermions

M. Göckeler, H. Hehl, P. Rakow, AS, T. Wettig hep-lat/0105011





The AdS/CFT picture of HICs

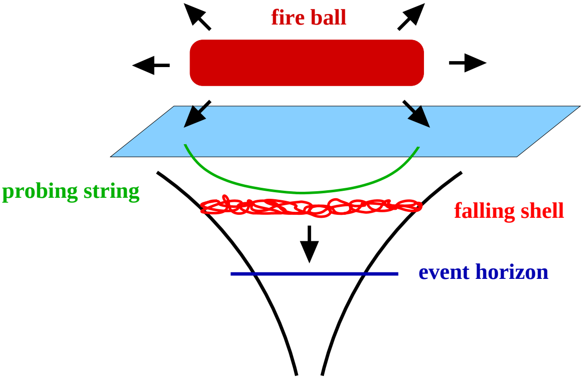


entropy production = information loss

Equilibration times from AdS/CFT:

Idea: Probe black brane formation with a string or membrane, breaking conformal invariance by a “quench”.

Balasubramanian, Bernamonti, de Boer, Copland, Craps, Keski-Vakkuri, Muller, AS, Shigemori, and Staessens; arXiv:1012.4753; 1103.2683; 1307.1487; 1307.7086



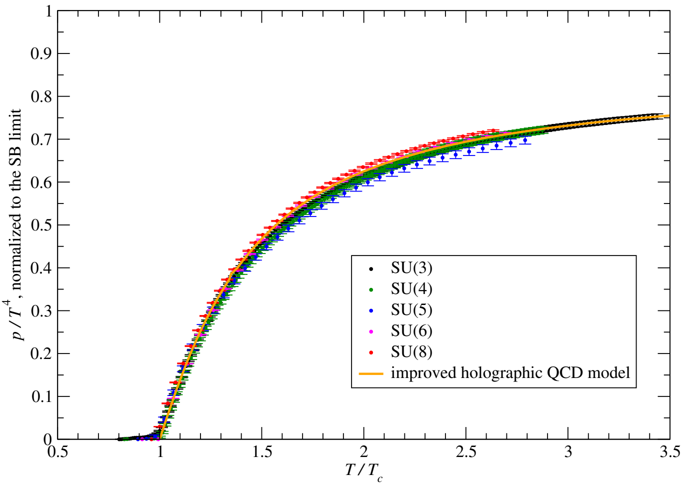
Two major results:

- Equilibration happens extremely fast $O(0.2\mathrm{fm}/c)$
- Equilibration happens first on short distances (top-down).

QCD is neither $SU(\infty)$ nor supersymmetric nor conformal nor infinite $\lambda = g^2_{YM} N$ but the differences are calculable:

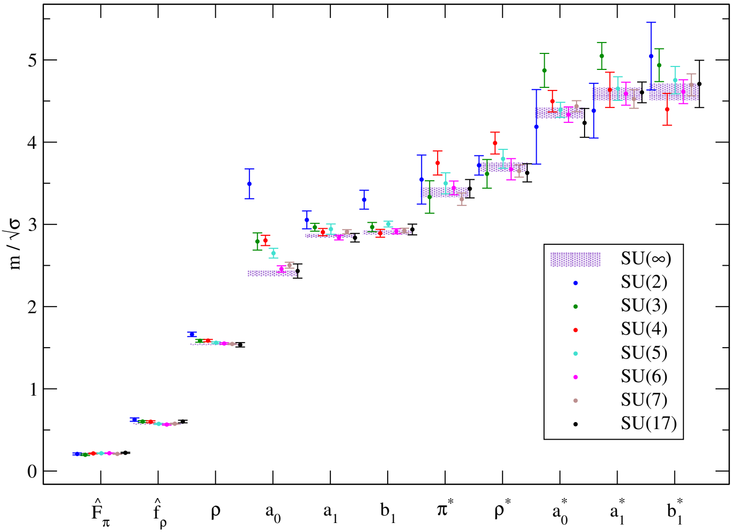
- $N = 3$
  dedicated lattice calculations
- effects of non-conformality of QCD
  dedicated pQCD and lattice calculations
  conformal perturbation theory e.g. Kumericki, Mueller, K. Passek-Kumericki, AS arXiv:hep-ph/0605237 NLO→NNLO for GPDs
  ϵ_{cr}: Braun, Manashov, Moch, and Strohmaier, arXiv:1810.04993.

Pressure



SU(*N*) pure gauge theory in 1+3 dimensions

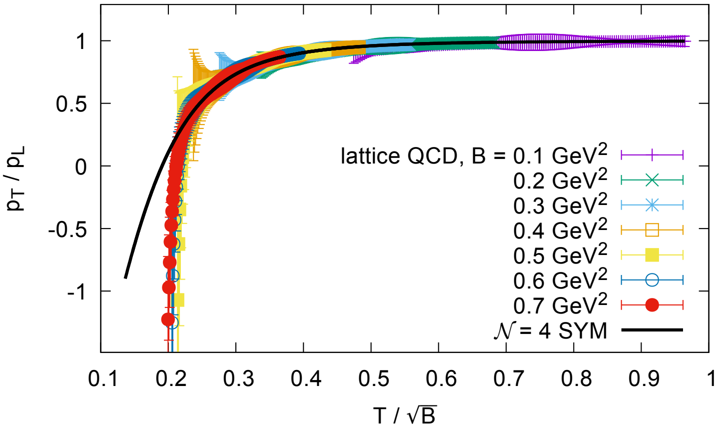
M. Panero, 0907.3719



$T = 0$ meson spectrum and decay constants

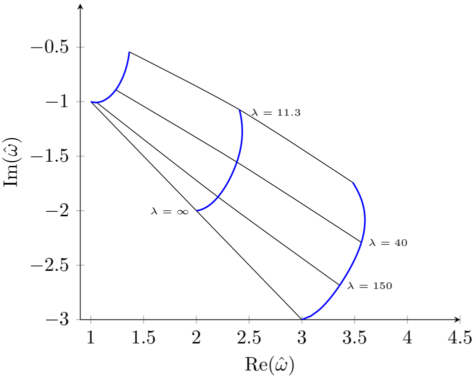
G. Bali et. al, 1304.4437

What happens if you break conformal symmetry explicitly by a background magnetic field? Endrodi, Kaminski, AS, Wu and Yaffe, [arXiv:1806.09632].



Finite coupling (QFT) corrections correspond to weak coupling quantum corrections in string theory

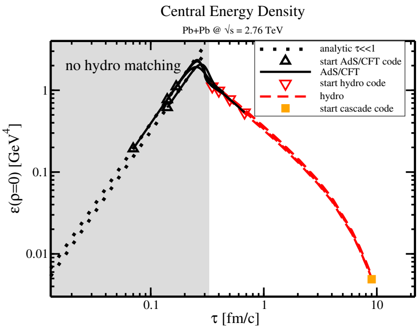
Waeber and AS, arXiv:1804.01912, The Quasi Normal Mode (QNM) spectrum.

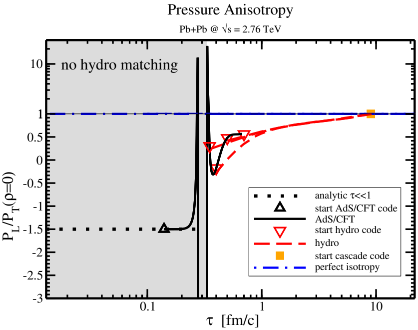


$\tau_{hydro} = -1/(Im\ \omega_{QNM})$ confirms earlier result and makes it more precise ($\tau_{hydro} \sim 0.2$ fm/c)

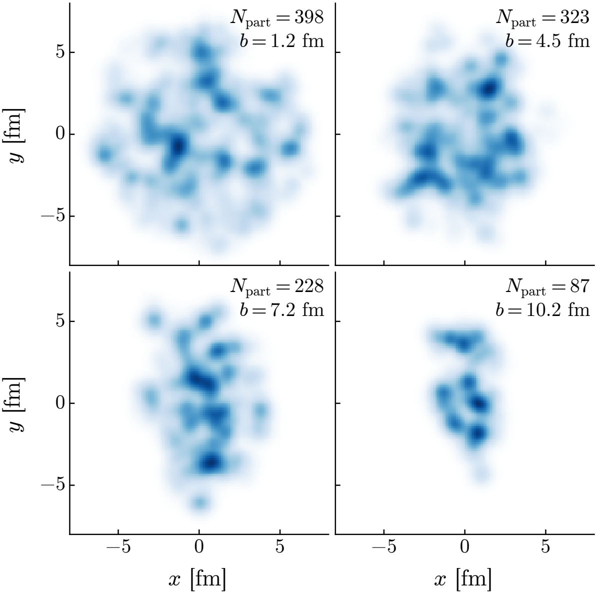
The AdS gravity equations result in a smooth transition to hydrodynamics. Viscous relativistic hydrodynamics is a gradient expansion which fails at early times. The late time behavior seems to be very stable and confirms perfect thermal and hydrodynamic behavior from 1fm/c on. Heller, Chesler, Berges and many, many more

Hydrodynamics must, in fact, already apply at 1 fm/*c* to describe v_2 etc. This can be explained by AdS/CFT.



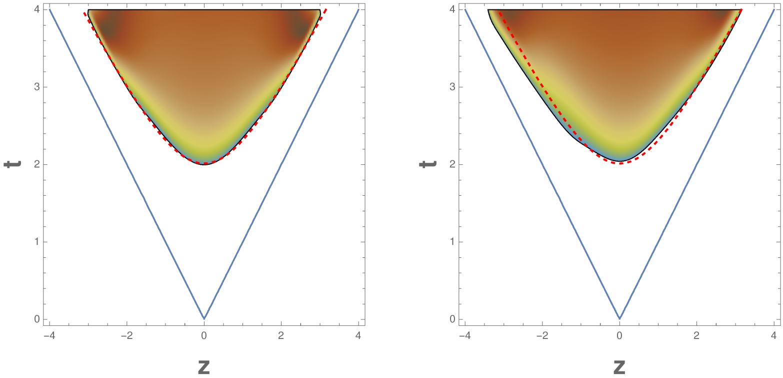


Remember



$N_{\text{part}} = 398$
$b = 1.2$ fm
$N_{\text{part}} = 323$
$b = 4.5$ fm
$N_{\text{part}} = 228$
$b = 7.2$ fm
$N_{\text{part}} = 87$
$b = 10.2$ fm
y [fm]
x [fm]
5
0
−5

Also this can be described by AdS/CFT 1906.05086 equilibration time $O(1 fm/c)$ (remember top-down)

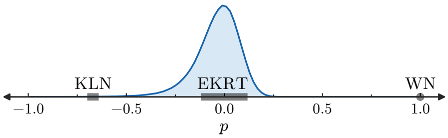


answer: Hydrodynamization occurs at **fixed eigenzeit** $\Rightarrow$ basically not boost dependent, geometric mean criterium: $\Delta = \frac{1}{p}\sqrt{\delta T^{\mu\nu}\, \delta T_{\mu\nu}} < 0.15$ with $\delta T^{\mu\nu} = T^{\mu\nu} - T^{\mu\nu}_{\text{hydro}}$

Bernhard, Moreland, Bass Liu, Heinz arXiv:1605.03954 Fit result: parameterization of combined entropy density:

$$s \sim \left(\frac{\tilde{T}_A^p + \tilde{T}_B^p}{2}\right)^{1/p}, \quad -\infty \leq p \leq \infty$$

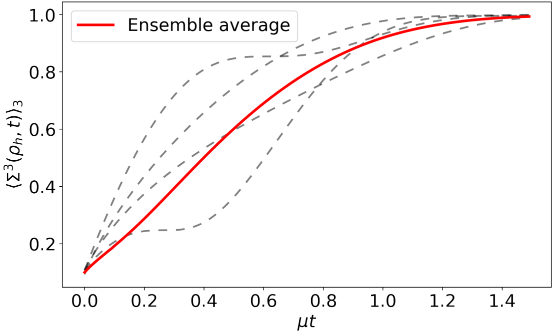
$$\tilde{T}(x_{perp}) = \sum_{i=1}^{N_{part}} \frac{\gamma_i}{2\pi w^2} \exp\left(-\frac{(x_\perp - x_{i,\perp})^2}{2w^2}\right)$$



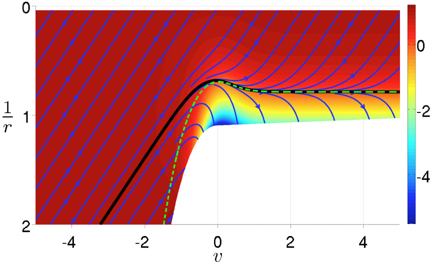
By construction the hydro initialization time must be identical for each transverse pixel. Both features are reproduced by AdS/CFT 1906.05086

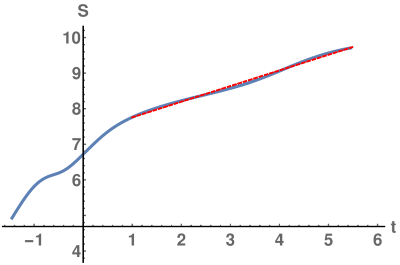
Lyapunov spectra and entropy generation

The generic classical picture



In the linear phase: $\frac{dS}{dt} = h_{KS} = \sum_{\lambda>0} \lambda$ sum over positive Lyapunov exponents



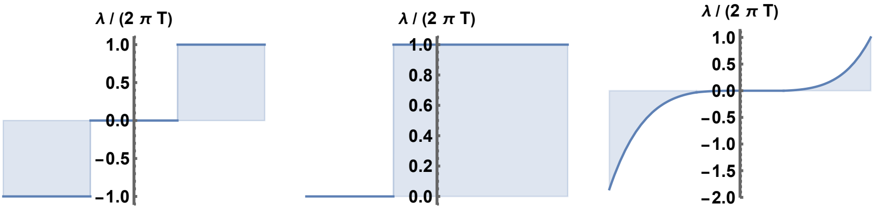


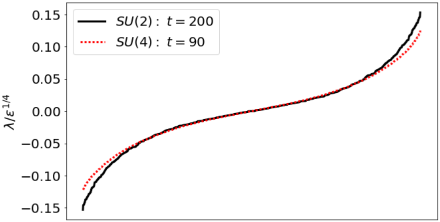
Left: Chesler and Yaffe, arXiv:0812.2053 symmetric

Right: Waeber, Rabenstein, Muller, AS, Yaffe; arXiv:2001.07161 realistic asymmetries

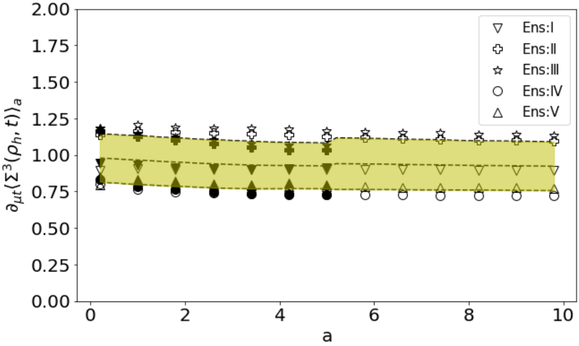
Note: $Re(\omega_{QNM}) \neq 0$

This allows to determine the Lyapunov spectrum of SU($N \to \infty$) gauge theory





maximum λ from: Maldacena, Shenker, Stanford “A bound on chaos”; arXiv:1503.01409



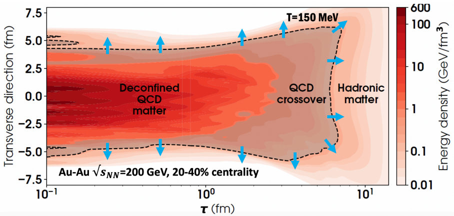
Georg Mayer analysed millions of cases numerically

arXiv:2107.01300

The leftmost possibility is realized, Liouvilles theorem is fulfilled

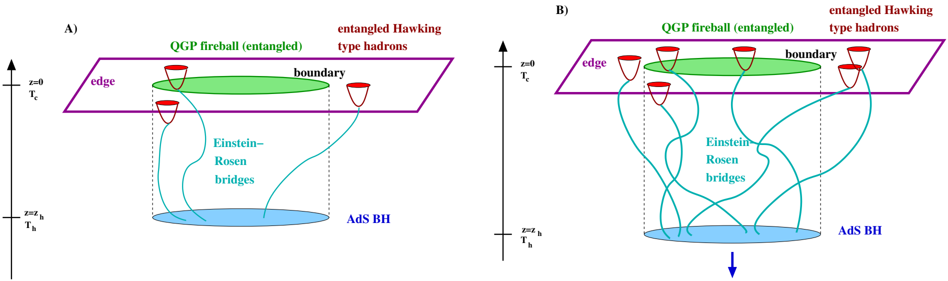
Speculations on hadronisation in AdS/QCD

The schematic model we advertise fits hydrodynamics results and HIC phenomenology



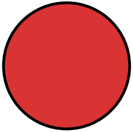
the radial size of the QGP fireball is roughly constant
the time till hadronization is roughly 10fm/c
Gale, Paquet, Schenke, Shen; arXiv:2009.07841

hadronization happens by two distinct mechanisms: Hadron emission from the surface ($\sim$ 15%) and a smooth but rapid confinement/deconfinement cross-over transition of the bulk ($\sim$ 85%). The latter we identify with a Hawking-Page like transition modified by finite volume and QCD$\neq$ CFT effects.

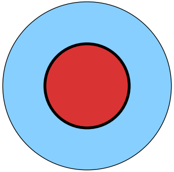


We call the AdS boundary “edge” and the surface of the collision system “boundary”. The dual of the fireball is an AdS BH. No entropy is produced, the Hawking-like hadron radiation is entangled with the BH. According to Maldacena and Susskind arXiv:1306.0533 and May and Van Raamsdonk arXiv: 2011.14258 entanglement can be represented by AdS Einstein-Rosen bridges

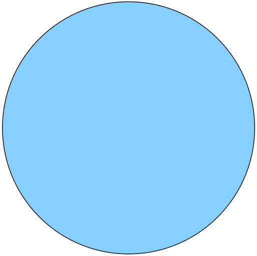
The analogy:



Fully entangled QGP



Entangled QGP plus hadrons

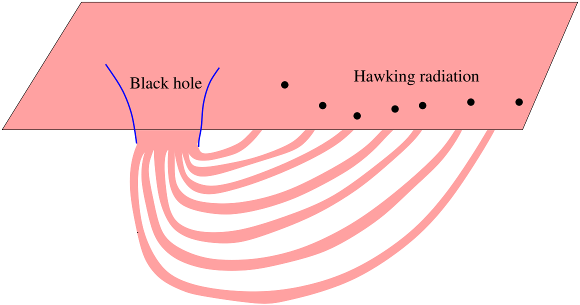


Fully entangled hadron gas

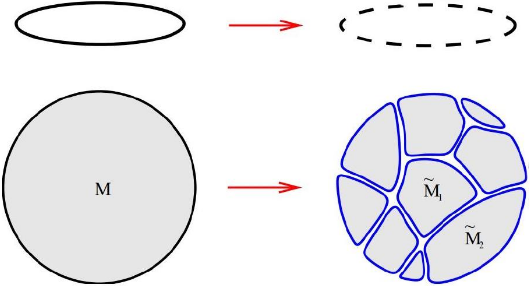
Hadron-hole production at the boundary is treated in analogy

Note that for BH: infalling particles move balistically
and for QGP: Infalling holes in a medium → rather inward propagating entanglement wave with $v_E \leq c$.

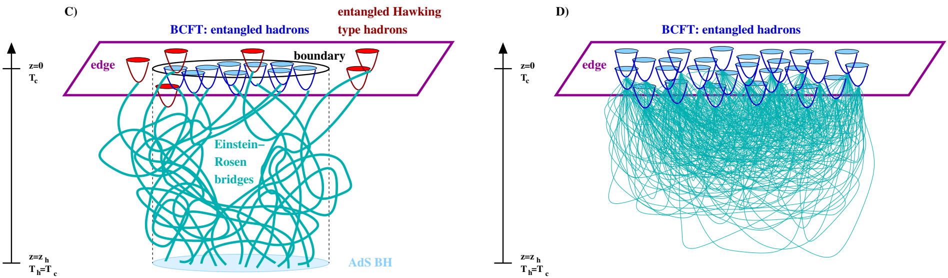
entangled CFT's in the boundary = Einstein-Rosen bridges in the holographical dual (EPR=ER). Maldacena and Susskind 1306.0533



Mary and Van Raamsdonk 2011.14258

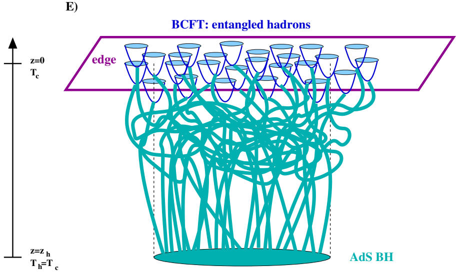


A system of entangled BCFT-bits (Boundary CFT) is nearly holographically equivalent to a CFT (examples for 1+1 and 2+1 dimensions). QCD $\rightarrow$ BCFT.



In the HP transition the remaining fireball hadronizes and the AdS BH disappears. As no entropy is produced the final hadrons are entangled, illustrated by many ER bridges

Monogamy implies that on average any two hadrons are only entangled by $\tau(\rho_{A_iA_j}) \sim 1/N_h$ and thus look thermal in very good approximation



As the HP transition is smoothed out and the difference between **C)** and **D)** is only due to $O(1/N_h)$ effects **E)** at $T_c + \epsilon$ is a very good approximation of **D)** at $T_c - \epsilon$.

ETH or monogamy of entanglement?

monogamy is well defined for qbits: quantum entanglement cannot be freely shared among many objects Wootters:1982zz, Wootters:1997id,Coffman:1999jd,Osborne2006xx. A quantity $\tau(\rho_{AB})$ called "tangle" quantifies entanglement between the elements of bi-partitions of multi-particle quantum states

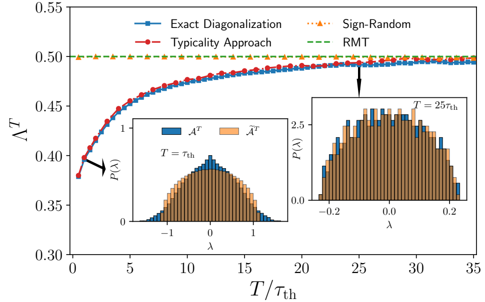
$$0 \leq \sum_{k=2}^{n} \tau(\rho_{A_1,A_k}) \leq \tau(\rho_{A_1,(A_2 A_3 \ldots A_n)}) \leq 1$$

Is 10 fm/c long enough to establish ETH behavior?

How fast does entanglement or decoherence propagate? Couch, Eccles, Nguyen, Swingle, Xu; arXiv:1908.06993 The information velocity $v_I = \min\left(\frac{v_E}{1-f}, v_B\right) \leq c$ with entanglement fraction f

Wang, Lamann, Richter, Steinigeweg, Dymarsky 2110.04085

The time needed to establish ETH behavior depends on the observable



$$\Lambda^T = \frac{\mathcal{M}_2^2}{\mathcal{M}_4}$$

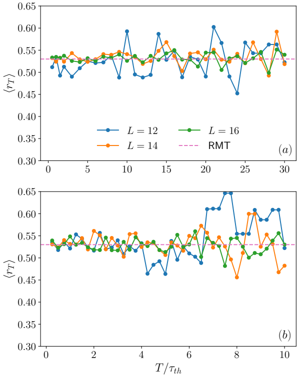
$$\mathcal{M}_k = \mathrm{Tr}[(\mathcal{O}_c^T)^k]/d$$

$$\mathcal{O}_c^T = \mathcal{O}^T - \mathrm{Tr}(\mathcal{O}^T)/d \quad \text{energy window} \quad \left[-\frac{\pi}{T}, \frac{\pi}{T}\right]$$

the mean ratio of adjacent level spacings

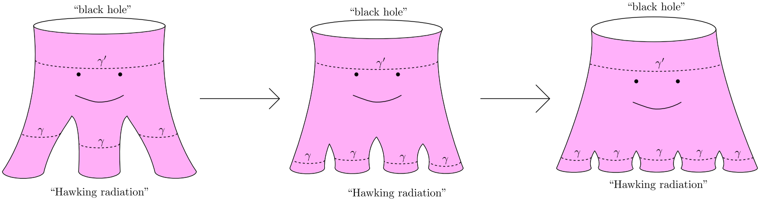
$$\langle r_T \rangle = \frac{1}{d} \sum_{\alpha} \frac{\min(\Delta_\alpha, \Delta_{\alpha+1})}{\max(\Delta_\alpha, \Delta_{\alpha+1})}$$

gap between two adjacent eigenvalues $\Delta_\alpha = |\lambda^T_{\alpha+1} - \lambda^T_\alpha|$ of $\mathcal{O}^T$



Many Questions

- Is AdS/BCFT the adequate framework to describe hadronization?
- Do quantum gravity correction smooth our the HP transition to a cross-over? Does it describe the real QCD deconfinement/confinement cross-over?
- There exist toy model calculations for ER bridge formation for two horizons, e.g. Shimaji, Takayanagi, and Wei; arXiv: 1812.01176 and Anderson, Parrikar, and Soni; arXiv:2103.14746. Generalized to many hadrons e.g. Akers, Engelhardt, Harlow 1910.00972



and 1+3/1+4 dimensions

- Can one calculate the geometric form of an ER bridge explicitely for the thermofield double state?
- Is there any realistic experiment which can differentiate between an entangled hadron gas with close to zero entropy and a thermal hadron gas with large entropy?
- Does QCD show ETH behavior?
- Does already monogamy of entanglement describe phenomenology?
- etc., please send comments to: andreas.schaefer@physik.uni-r.de

Conclusions

- ETH, monogamy of entanglement, decoherence and thermalization of isolated quantum systems are topics of universal interest.
- Heavy Ion Collisions in the ultra-high vacuum of, e.g. the LHC, offer an ideal situation to study them. There are many Pbyte of data, the question is how to interpret it.
- A QFT treatment seems to be unfeasible. The only chance is a holographic treatment.
- We suggest a highly speculative model, combining various ideas found in the literature, which fits HIC phenomenology.
- My question to you is: Does it make sense? If is does we would start with detailed numerical studies, if not, this would be a waste of time.