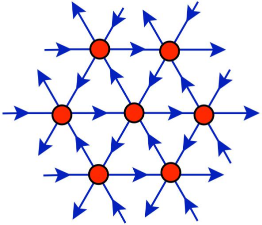
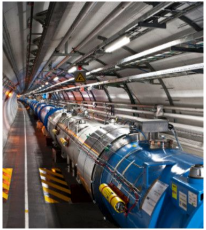
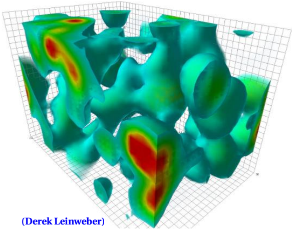
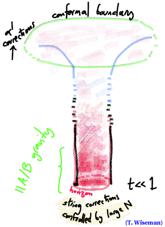
Lattice supersymmetric field theories — Part 2

David Schaich (University of Liverpool)









Nonperturbative and Numerical Approaches
to Quantum Gravity, String Theory and Holography

International Centre for Theoretical Sciences, Bangalore, 24 August 2021

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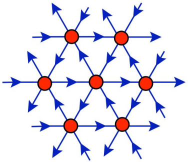
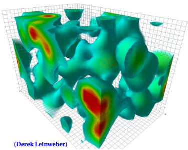
Any questions about last time?



**Overcoming challenges opens many opportunities
for lattice studies of supersymmetric QFTs**

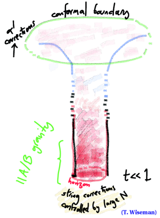
Motivation, background, formulation

- ✓ Supersymmetry breaking in discrete space-time
- Supersymmetry preservation — wrap up



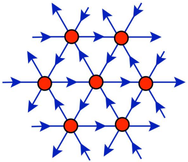
Applications with significant recent progress

- Maximal $\mathcal{N} = 4$ super-Yang–Mills
- Lower dimensions $d < 4$
- Minimal $\mathcal{N} = 1$ super-Yang–Mills



Remaining challenges: Super-QCD; Sign problems

Any questions about last time?



Overcoming challenges opens many opportunities
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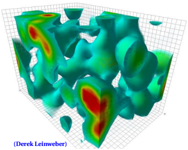
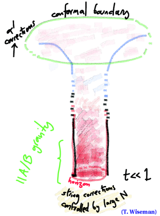
Motivation, background, formulation — wrap up



Applications with significant recent progress

Remaining challenges



Conceptual focus with interaction encouraged

"It's better to uncover a little than to cover a lot" (V. Weisskopf)

Lattice $\mathcal{N} = 4$ SYM — recap

Lattice action $S_{\text{lat}} = \frac{N}{4\lambda_{\text{lat}}} \text{Tr}\left[\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\overline{\mathcal{D}}_a\mathcal{U}_a - \frac{1}{2}\eta d\right) - \frac{1}{4}\epsilon_{abcde}\ \chi_{ab}\overline{\mathcal{D}}_c\ \chi_{de}\right]$

Gauge invariance $\longleftrightarrow$ trace over closed loops

Fixes orientations of lattice variables and finite-difference operators

Site variables	**Link variables**	**Plaquette variables**
$G(n)\ \eta(n)\ G^{\dagger}(n)$	$G(n)\ \psi_a(n)\ G^{\dagger}(n+\widehat{\mu}_a)$	$G(n+\widehat{\mu}_a+\widehat{\mu}_b)\ \chi_{ab}(n)\ G^{\dagger}(n)$
	$G(n)\ \mathcal{U}_a(n)\ G^{\dagger}(n+\widehat{\mu}_a)$	
	$G(n+\widehat{\mu}_a)\ \overline{\mathcal{U}}_a(n)\ G^{\dagger}(n)$	

Lattice $\mathcal{N} = 4$ SYM — recap

Site variables

$G(n)\,\eta(n)\,G^\dagger(n)$

Link variables

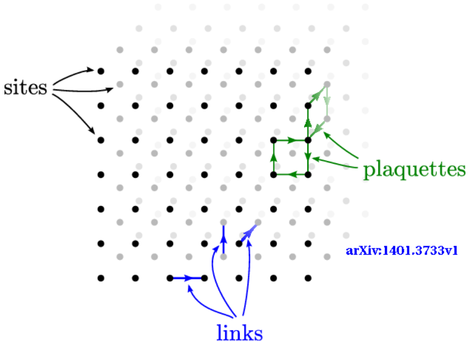
$G(n)\,\psi_a(n)\,G^\dagger(n+\widehat{\mu}_a)$

$G(n)\,\mathcal{U}_a(n)\,G^\dagger(n+\widehat{\mu}_a)$

$G(n+\widehat{\mu}_a)\,\overline{\mathcal{U}}_a(n)\,G^\dagger(n)$

Plaquette variables

$G(n+\widehat{\mu}_a+\widehat{\mu}_b)\,\chi_{ab}(n)\,G^\dagger(n)$



Examples:

$\mathrm{Tr}\left[\eta\overline{\mathcal{U}}_a\psi_a\right]$

$\mathrm{Tr}\left[\chi_{ab}\mathcal{U}_a\psi_b\right]$

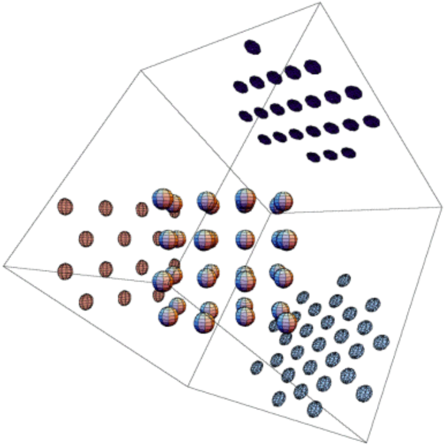
Lattice $\mathcal{N} = 4$ SYM — geometric structure

Return to dimensional reduction, treating all five $\mathcal{U}_a$ symmetrically

Start with hypercubic lattice
in 5d momentum space

Symmetric constraint $\sum_a \partial_a = 0$
projects to 4d momentum space

Result is A_4 lattice
$\longrightarrow$ dual A_4^* lattice in position space



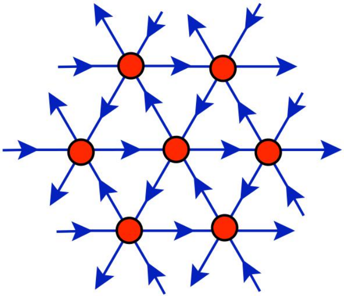
A_4^* lattice of five links spanning four dimensions

Return to dimensional reduction, treating all five $\mathcal{U}_a$ symmetrically

$A_4^* \sim$ 4d analog of 2d triangular lattice

Basis vectors linearly dependent and non-orthogonal

Large S_5 point group symmetry



S_5 point group symmetry

S_5 irreps precisely match onto irreps of twisted SO(4)$_{\text{tw}}$

$$\psi_a \longrightarrow \psi_\mu, \quad \overline{\eta} \qquad \text{is} \qquad \mathbf{5} \longrightarrow \mathbf{4} \oplus \mathbf{1}$$

$$\chi_{ab} \longrightarrow \chi_{\mu\nu}, \quad \overline{\psi}_\mu \qquad \text{is} \qquad \mathbf{10} \longrightarrow \mathbf{6} \oplus \mathbf{4}$$

More explicitly,

$$\psi_\mu = P_{\mu a}\psi_a \qquad \chi_{\mu\nu} = P_{\mu a}P_{\nu b}\chi_{ab}$$

$$\overline{\eta} = P_{5a}\psi_a \qquad \overline{\psi}_\mu = P_{\mu a}P_{5b}\chi_{ab}$$

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & 0 & 0 \\ \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} & -\frac{3}{\sqrt{12}} & 0 \\ \frac{1}{\sqrt{20}} & \frac{1}{\sqrt{20}} & \frac{1}{\sqrt{20}} & \frac{1}{\sqrt{20}} & -\frac{4}{\sqrt{20}} \\ \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$$

Projection matrix, $P^{-1} = P^T$

$P_{\mu a} = (\widehat{e}_a)_\mu$ are basis vectors of A_4^* lattice

Restoration of $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$ supersymmetries

S_5 irreps precisely match onto irreps of twisted $\text{SO}(4)_{\text{tw}}$

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$S_5 \longrightarrow \text{SO}(4)_{\text{tw}}$ in continuum limit restores $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$ [arXiv:1306.3891]

Continuum limit (I)

Assuming RG blocking transformation that preserves $\mathcal{Q}$ and S_5
compare lattice action and most general long-range S_{eff} allowed by symmetries

$$S_{\text{lat}} \sim \text{Tr}\left[\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\overline{\mathcal{D}}_a\mathcal{U}_a - \frac{1}{2}\eta d\right) - \frac{1}{4}\epsilon_{abcde}\ \chi_{ab}\overline{\mathcal{D}}_c\ \chi_{de}\right]$$

$$S_{\text{eff}} \sim \text{Tr}\left[\mathcal{Q}\left(\alpha_1\chi_{ab}\mathcal{F}_{ab} + \alpha_2\eta\overline{\mathcal{D}}_a\mathcal{U}_a - \frac{\alpha_3}{2}\eta d\right) - \frac{\alpha_4}{4}\epsilon_{abcde}\chi_{ab}\overline{\mathcal{D}}_c\chi_{de}\right]$$
$$+ \gamma\mathcal{Q}\left\{\text{Tr}\left[\eta\mathcal{U}_a\overline{\mathcal{U}}_a\right] - \frac{1}{N}\text{Tr}\left[\eta\right]\text{Tr}\left[\mathcal{U}_a\overline{\mathcal{U}}_a\right]\right\}$$

Eliminate three α_i by rescaling fields and 't Hooft coupling [arXiv:1408.7067]

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Eliminate three α_i by rescaling fields and 't Hooft coupling [arXiv:1408.7067]

$$\longrightarrow S_{\text{eff}} \sim \text{Tr}\left[\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\overline{\mathcal{D}}_a\mathcal{U}_a - \frac{\alpha_1\alpha_3}{2\alpha_2^2}\eta d\right) - \frac{1}{4}\epsilon_{abcde}\chi_{ab}\overline{\mathcal{D}}_c\chi_{de}\right]$$
$$+ \gamma'\mathcal{Q}\left\{\text{Tr}\left[\eta\mathcal{U}_a\overline{\mathcal{U}}_a\right] - \frac{1}{N}\text{Tr}\left[\eta\right]\text{Tr}\left[\mathcal{U}_a\overline{\mathcal{U}}_a\right]\right\}$$

Moduli space

$$S_{\text{eff}} \sim \text{Tr}\left[\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\overline{\mathcal{D}}_a\mathcal{U}_a - \frac{\alpha_1\alpha_3}{2\alpha_2^2}\eta d\right) - \frac{1}{4}\epsilon_{abcde}\chi_{ab}\overline{\mathcal{D}}_c\chi_{de}\right]$$
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γ' terms $\longrightarrow$ scalar mass and cubics, lifting moduli space [arXiv:1408.7067]

Lattice action is 'topological' ($\mathcal{Q}$-invariant) observable, $\mathcal{QO} = 0$
$\longrightarrow$ can be analyzed semi-classically

Moduli space

Lattice action is 'topological' ($\mathcal{Q}$-invariant) observable, $\mathcal{Q}\mathcal{O} = 0$

$\longrightarrow$ can be analyzed semi-classically

Field rescalings $\longrightarrow$ $S_{\text{lat}} = g^{-2}\mathcal{Q}\Lambda + S_{\text{closed}}$

$$\frac{\partial}{\partial g^{-2}}\langle\mathcal{O}\rangle = \frac{\partial}{\partial g^{-2}}\frac{\int \mathcal{O}\ e^{-g^{-2}\mathcal{Q}\Lambda - S_{\text{closed}}}}{\int e^{-g^{-2}\mathcal{Q}\Lambda - S_{\text{closed}}}}$$

$$= -\langle\mathcal{O}\mathcal{Q}\Lambda\rangle + \langle\mathcal{O}\rangle\langle\mathcal{Q}\Lambda\rangle = -\langle\mathcal{Q}(\mathcal{O}\Lambda)\rangle = 0$$

$\Longrightarrow Z_{\text{lat}} = \int e^{-S_{\text{lat}}}$ independent of coupling,

so perturbatively compute to one loop for $g^2 \to 0$

A bit of lattice perturbation theory

Propagators and vertices affected by space-time discretization

Example (Feynman gauge): [arXiv:1102.1725]

$$\langle \overline{\mathcal{A}}(k_\mu)\mathcal{A}(-k_\mu)\rangle = \frac{1}{k^2} = \frac{1}{\sum_\mu k_\mu^2} \longrightarrow \frac{a^2}{\sum_\mu 4\sin^2(ak_\mu/2)}$$

Aside: Up to one loop, all divergences occur for $|ak_\mu| \ll 1$
where lattice and continuum results coincide

$\Longrightarrow$ Lattice $\beta = 0$ to one loop (but not topological)

A bit of lattice perturbation theory

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One-loop partition function: [arXiv:1102.1725]

$$Z_{\text{lat}} = \frac{\det\left[\overline{\mathcal{D}}_a\mathcal{D}_a\right]\det^4\left[\overline{\mathcal{D}}_a\mathcal{D}_a\right]}{\det^5\left[\overline{\mathcal{D}}_a\mathcal{D}_a\right]} = 1$$

Cancellation between ghosts & fermions vs. bosons

$\Longrightarrow$ quantum moduli space protected to all orders in lattice perturbation theory

Continuum limit (II)

$$S_{\text{eff}} \sim \text{Tr}\left[\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\overline{\mathcal{D}}_a\mathcal{U}_a - \frac{\alpha_1\alpha_3}{2\alpha_2^2}\eta d\right) - \frac{1}{4}\epsilon_{abcde}\chi_{ab}\overline{\mathcal{D}}_c\chi_{de}\right]$$
$$+ \gamma\mathcal{Q}\left\{\text{Tr}\left[\eta\mathcal{U}_a\overline{\mathcal{U}}_a\right] - \frac{1}{N}\text{Tr}\left[\eta\right]\text{Tr}\left[\mathcal{U}_a\overline{\mathcal{U}}_a\right]\right\}$$

Protected moduli space perturbatively forces $\gamma = 0$

Assuming non-perturbative effects (e.g., instantons) also preserve moduli space,
only one log. tuning to recover full continuum symmetries
$\text{SO}(4)_{\text{tw}}, \;\; \mathcal{Q}_a, \;\; \mathcal{Q}_{ab}$

Real-space RG for lattice $\mathcal{N} = 4$ SYM

Above also assumed RG blocking transformation
that preserves $\mathcal{Q}$ and S_5 symmetries $\longleftrightarrow$ geometric structure

Simple transformation constructed in arXiv:1408.7067

$$\mathcal{U}'_a(n') = \xi\,\mathcal{U}_a(n)\mathcal{U}_a(n+\widehat{\mu}_a) \qquad\qquad \eta'(n') = \eta(n)$$

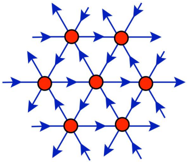
$$\psi'_a(n') = \xi\left[\psi_a(n)\mathcal{U}_a(n+\widehat{\mu}_a) + \mathcal{U}_a(n)\psi_a(n+\widehat{\mu}_a)\right] \qquad\qquad d'(n') = d(n)$$

$$\chi'_{ab}(n') = \xi^2\left[\text{six permutations of } \chi_{ab}\overline{\mathcal{U}}_a\overline{\mathcal{U}}_b\right]$$

Doubles lattice spacing $a \longrightarrow a' = 2a$, with tunable rescaling factor ξ

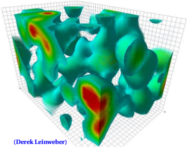
$$G(n)\;\psi'_a(n')\;G^{\dagger}(n+2\widehat{\mu}_a) \qquad\qquad G(n+2\widehat{\mu}_a+2\widehat{\mu}_b)\;\chi'_{ab}(n')\;G^{\dagger}(n)$$

Checkpoint



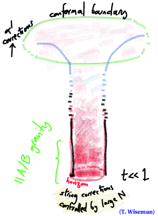
✓ Motivation, background, formulation

- ✓ Supersymmetry breaking in discrete space-time
- ✓ Supersymmetry preservation in discrete space-time



Applications with significant recent progress

- Maximal $\mathcal{N} = 4$ super-Yang–Mills
- Lower dimensions $d < 4$
- Minimal $\mathcal{N} = 1$ super-Yang–Mills



Remaining challenges: Super-QCD; Sign problems

Questions?

"It's better to uncover a little than to cover a lot"

Moving towards practical lattice calculation

Analytic results for twisted $\mathcal{N} = 4$ SYM on A_4^* lattice

U(N) gauge invariance + $\mathcal{Q}$ + S_5 lattice symmetries

$\longrightarrow$ Moduli space preserved to all orders

$\longrightarrow$ One-loop lattice β function vanishes

$\longrightarrow$ Only one log. tuning to recover continuum $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$

[arXiv:1102.1725, arXiv:1306.3891, arXiv:1408.7067]

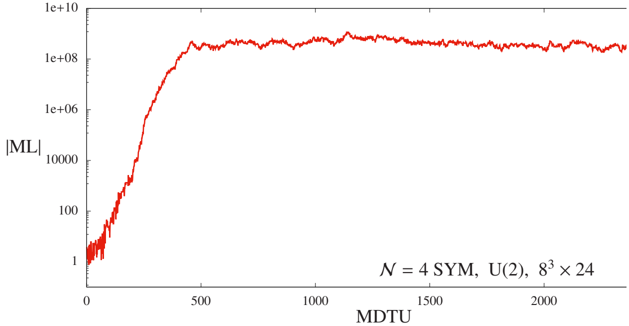
Not yet practical for numerical calculations

Must regulate zero modes and flat directions, in both SU(N) and U(1) sectors

Problem with SU(N) flat directions

Recall $\mathcal{U}_a \to \mathbb{I}_N + \mathcal{A}_a$ needed to recover continuum covariant derivative

Links can wander far away when doing Markov-chain importance sampling via rational hybrid Monte Carlo (RHMC) algorithm



Complexified Polyakov loop ('Maldacena loop', ML)

$$\text{ML} = \frac{1}{L^3} \sum_{x,y,z} \text{Tr} \left[\prod_{t=0}^{N_t - 1} \mathcal{U}_t(x, y, z, t) \right]$$

Should have $|\text{ML}| \approx 1$ for all λ_{lat} ($\mathcal{Q}\overline{\text{ML}} = 0$)

Regulating SU(N) flat directions

Add SU(N) scalar potential to lattice action — multiple options, similar behavior

$$S_{\text{lat}} = \frac{N}{4\lambda_{\text{lat}}} \text{Tr}\left[\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\overline{\mathcal{D}}_a\mathcal{U}_a - \frac{1}{2}\eta d\right) - \frac{1}{4}\epsilon_{abcde}\,\chi_{ab}\overline{\mathcal{D}}_c\,\chi_{de}\right] + \frac{N}{4\lambda_{\text{lat}}}\mu^2 V$$

$$V = \sum_a \left(\frac{1}{N}\text{Tr}\left[\mathcal{U}_a\overline{\mathcal{U}}_a\right] - 1\right)^2 \qquad\qquad V = \sum_a \frac{1}{N}\text{Tr}\left[\left(\mathcal{U}_a\overline{\mathcal{U}}_a - \mathbb{I}_N\right)^2\right]$$

Gauge-invariant but explicitly breaks $\mathcal{Q}$

Continuum limit requires $\mu^2 \to 0$ to restore $\mathcal{Q}$ and recover physical moduli space

Soft $\mathcal{Q}$ breaking

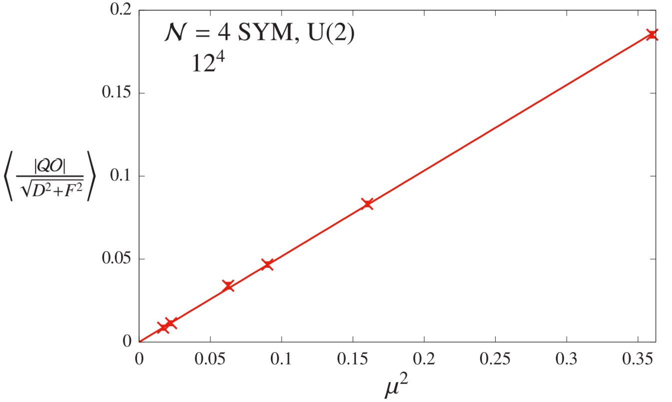
SU(N) scalar potential $\propto \mu^2 \sum_a \left(\text{Tr}\left[\mathcal{U}_a\overline{\mathcal{U}}_a\right] - N\right)^2$ breaks $\mathcal{Q}$ **softly**

$\longrightarrow$ $\mathcal{Q}$-violating operators vanish $\propto \mu^2 \to 0$

Check via Ward identity violations

$$\left\langle \text{Tr}\ \mathcal{Q}\left[\eta\mathcal{U}_a\overline{\mathcal{U}}_a\right]\right\rangle \neq 0$$

$$\mathcal{Q}\eta\mathcal{U}_a\overline{\mathcal{U}}_a = \left(\overline{\mathcal{D}}_b\mathcal{U}_b\right)\mathcal{U}_a\overline{\mathcal{U}}_a - \eta\psi_a\overline{\mathcal{U}}_a$$



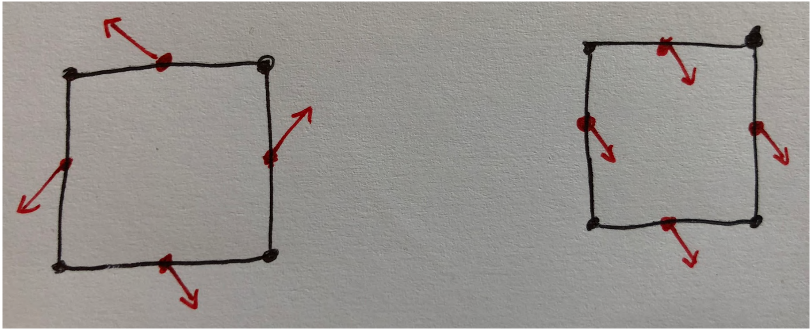
Problem with U(1) flat directions

$\text{U}(N) = \text{SU}(N) \otimes \text{U}(1)$ includes compact U(1) lattice gauge theory

$\longrightarrow$ confinement transition via monopole condensation

Count monopole worldlines from phases of $\det \mathcal{U}$ in plaquettes bounding cells

[DeGrand–Toussaint, 1980]

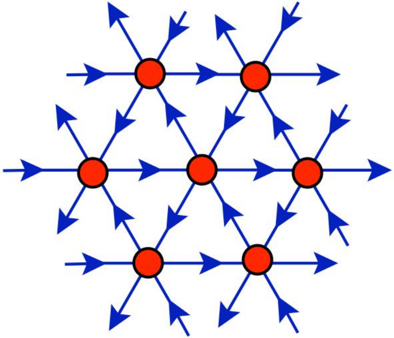


Counting monopole worldlines

A_4^* lattice complicates monopole worldline counting [arXiv:1405.0644]

Represent A_4^* as hypercube plus backwards diagonal link

Merge cells into hypercubes to count — neighboring M_μ–$\overline{M}_\mu$ pairs annihilate



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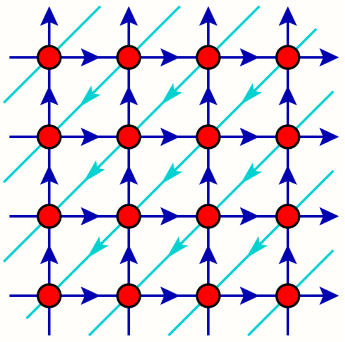
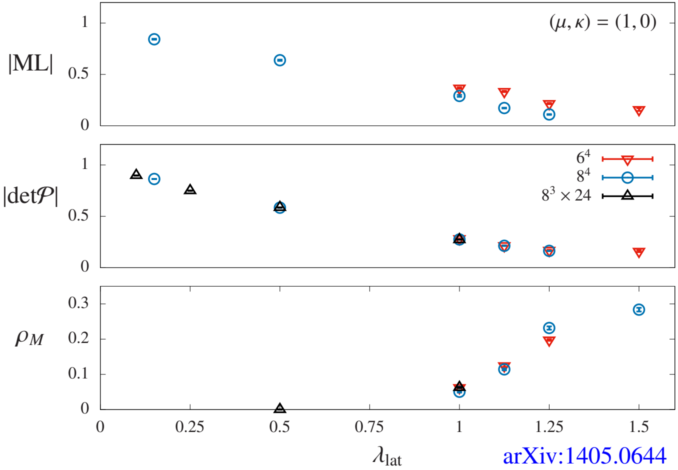


U(1) confinement transition



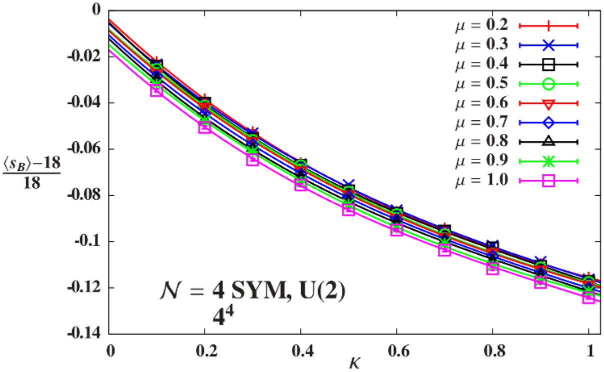
Monopole condensation $\longrightarrow$ confined lattice phase not present in continuum

Naively regulating U(1) flat directions

Can add another soft $\mathcal{Q}$-breaking term depending on plaquette determinant

$$S_{\text{soft}} = \frac{N}{4\lambda_{\text{lat}}}\mu^2 \sum_a \left(\frac{1}{N}\text{Tr}\left[\mathcal{U}_a\overline{\mathcal{U}}_a\right] - 1\right)^2 + \kappa \sum_{a<b} \left|\det \mathcal{P}_{ab} - 1\right|^2$$

Ward identity violations
more sensitive to κ than to μ^2



Here checking bosonic action

$$\mathcal{Q}S_{\text{lat}} = 0 \longrightarrow \langle s_B \rangle = 9N^2/2$$

Better regulating U(1) flat directions

Possible to impose $\mathcal{Q}$-invariant constraints on generic site operator $\mathcal{O}(n)$

$$S_{\text{lat}} \propto \text{Tr}\left[\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\left\{\overline{\mathcal{D}}_a\mathcal{U}_a + G\mathcal{O}\right\} - \frac{1}{2}\eta d\right) - \frac{1}{4}\epsilon_{abcde}\,\chi_{ab}\overline{\mathcal{D}}_c\,\chi_{de} + \mu^2 V\right]$$

Modifies auxiliary field equations of motion $\longrightarrow$ moduli space [arXiv:1505.03135]

$$d(n) = \overline{\mathcal{D}}_a\mathcal{U}_a(n) \qquad \longrightarrow \qquad d(n) = \overline{\mathcal{D}}_a\mathcal{U}_a(n) + G\mathcal{O}$$

Choose $\mathcal{O} = \sum_{a \neq b}\left[\det \mathcal{P}_{ab} - 1\right]\mathbb{I}_N$ to lift U(1) zero mode & flat directions

U(1) decouples in continuum $\longrightarrow$ no need to tune parameter G

Better regulating U(1) flat directions

$$S_{\text{lat}} \propto \text{Tr}\left[\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\left\{\overline{\mathcal{D}}_a\mathcal{U}_a + G\sum_{a<b}\left[\det\mathcal{P}_{ab} - 1\right]\mathbb{I}_N\right\} - \frac{1}{2}\eta d\right) - \frac{1}{4}\epsilon_{abcde}\,\chi_{ab}\overline{\mathcal{D}}_c\,\chi_{de} + \mu^2 V\right]$$

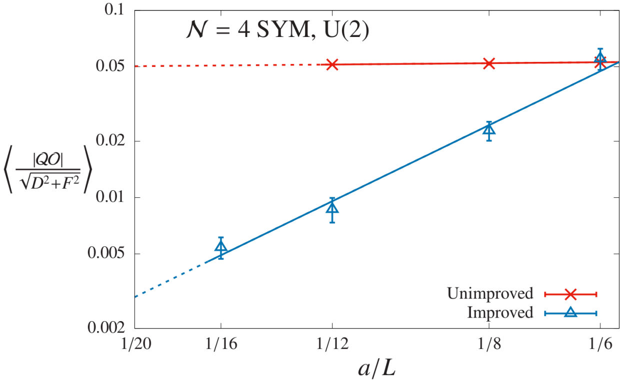
Much better

approach to continuum

Ward ident. violations $\propto (a/L)^2$

Effective $\mathcal{O}(a)$ improvement

since $\mathcal{Q}$ forbids all dim.-5 ops

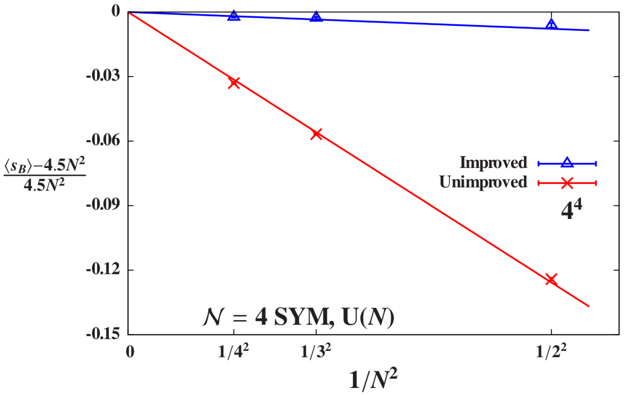


Larger N improves soft $\mathcal{Q}$ breaking

$$S_{\text{lat}} \propto \text{Tr}\left[\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\left\{\overline{\mathcal{D}}_a\mathcal{U}_a + G\sum_{a<b}\left[\det\mathcal{P}_{ab} - 1\right]\mathbb{I}_N\right\} - \frac{1}{2}\eta d\right) - \frac{1}{4}\epsilon_{abcde}\,\chi_{ab}\overline{\mathcal{D}}_c\,\chi_{de} + \mu^2 V\right]$$

Larger N also helps
for both actions

Ward ident. violations $\propto 1/N^2$

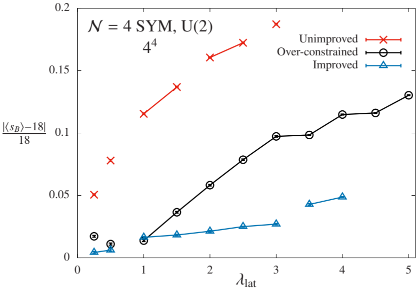


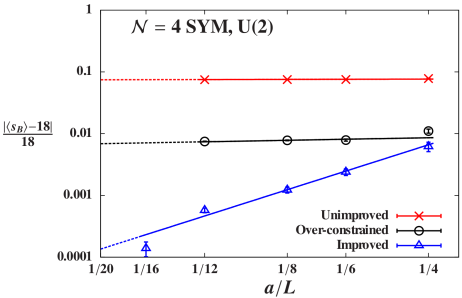
Can we do even better?

What if we include both SU(N) and U(1) deformations in $\mathcal{O}(n)$? [arXiv:1505.03135]

$$S_{\text{lat}} \propto \text{Tr}\left[\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\left\{\overline{\mathcal{D}}_a\mathcal{U}_a + G\mathcal{O}\right\} - \frac{1}{2}\eta d\right) - \frac{1}{4}\epsilon_{abcde}\,\chi_{ab}\overline{\mathcal{D}}_c\,\chi_{de}\right]$$

Over-constrains system $\longrightarrow$ Ward ident. violations without explicit $\mathcal{Q}$ breaking

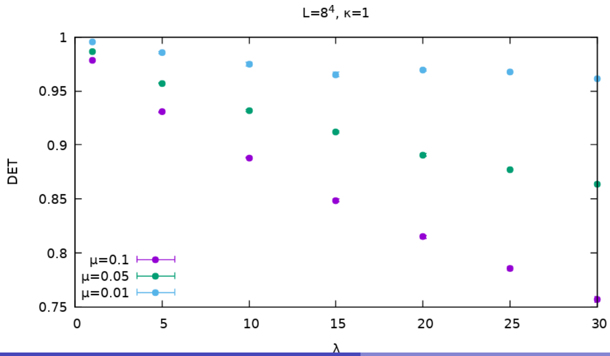




Ongoing experimentation

What if $\mathcal{O} = \sum_a [\text{Re}\det\mathcal{U}_a - 1]\,\mathbb{I}_N$? [Catterall–Giedt–Toga, arXiv:2009.07334]

$$S_{\text{lat}} \propto \text{Tr}\left[\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\left\{\overline{\mathcal{D}}_a\mathcal{U}_a + G\mathcal{O}\right\} - \frac{1}{2}\eta d\right) - \frac{1}{4}\epsilon_{abcde}\,\chi_{ab}\overline{\mathcal{D}}_c\,\chi_{de} + \mu^2 V\right]$$



U(1) gauge dependent!

U(1) decouples as $a \to 0$

$\longrightarrow$ irrelevant for $a > 0$?

Results look reasonable,

reach strong $\lambda_{\text{lat}} = 30$

The cost of twisted lattice $\mathcal{N} = 4$ SYM

so that the full improved action becomes

$$S_{\text{imp}} = S'_{\text{exact}} + S_{\text{closed}} + S'_{\text{soft}} \tag{18}$$

$$S'_{\text{exact}} = \frac{N}{4\lambda_{\text{lat}}} \sum_n \text{Tr}\Bigg[-\overline{\mathcal{F}}_{ab}(n)\mathcal{F}_{ab}(n) - \chi_{ab}(n)\mathcal{D}^{(+)}_{[a}\psi_{b]}(n) - \eta(n)\overline{\mathcal{D}}^{(-)}_a\psi_a(n) + \frac{1}{2}\left(\overline{\mathcal{D}}^{(-)}_a\mathcal{U}_a(n) + G\sum_{a\neq b}\left(\det\mathcal{P}_{ab}(n) - 1\right)\mathbb{I}_N\right)^2\Bigg] - S_{\text{det}}$$

$$S_{\text{det}} = \frac{N}{4\lambda_{\text{lat}}} G \sum_n \text{Tr}\left[\eta(n)\right] \sum_{a\neq b}\left[\det\mathcal{P}_{ab}(n)\right] \text{Tr}\left[\mathcal{U}_b^{-1}(n)\psi_b(n) + \mathcal{U}_a^{-1}(n+\widehat{\mu}_b)\psi_a(n+\widehat{\mu}_b)\right]$$

$$S_{\text{closed}} = -\frac{N}{16\lambda_{\text{lat}}} \sum_n \text{Tr}\left[\epsilon_{abcde}\ \chi_{de}(n+\widehat{\mu}_a+\widehat{\mu}_b+\widehat{\mu}_c)\overline{\mathcal{D}}^{(-)}_c\chi_{ab}(n)\right],$$

$$S'_{\text{soft}} = \frac{N}{4\lambda_{\text{lat}}}\mu^2 \sum_n\sum_a\left(\frac{1}{N}\text{Tr}\left[\mathcal{U}_a(n)\overline{\mathcal{U}}_a(n)\right] - 1\right)^2$$

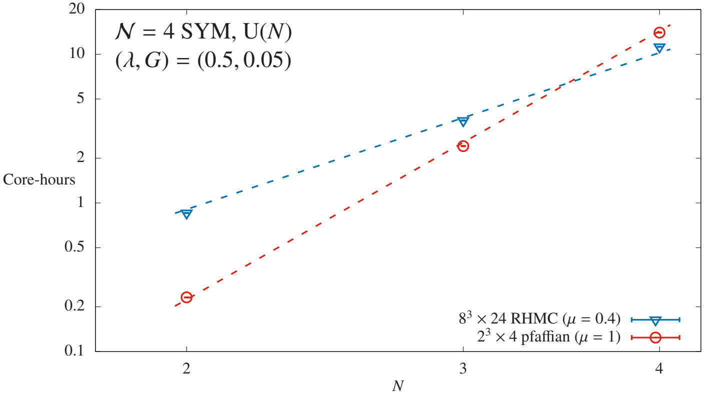
Computationally challenging, e.g. $\gtrsim$100 gathers per fermion matrix–vector op.

Public parallel code github.com/daschaich/susy [arXiv:1410.6971]
actively developed for improved performance and new applications

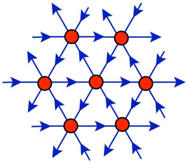
Computational cost scaling

Blue: RHMC cost scaling $\sim N^{3.5}$ since condition number increases [and $\sim V^{5/4}$]

Red: Pfaffian cost scaling $\sim N^6$ as expected

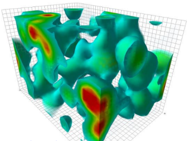


Next time



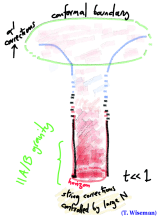
**Overcoming challenges opens many opportunities
for lattice studies of supersymmetric QFTs**

✓ Motivation, background, formulation
- ✓ Supersymmetry breaking in discrete space-time
- ✓ Supersymmetry preservation



(Derek Leinweber)

Applications with significant recent progress
- Maximal $\mathcal{N} = 4$ super-Yang–Mills — wrap up
- Lower dimensions $d < 4$
- Minimal $\mathcal{N} = 1$ super-Yang–Mills



(T. Wiseman)

Remaining challenges: Super-QCD; Sign problems