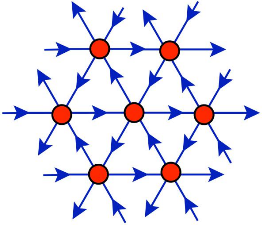
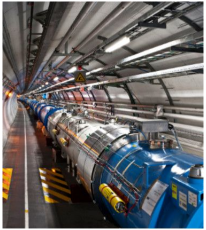
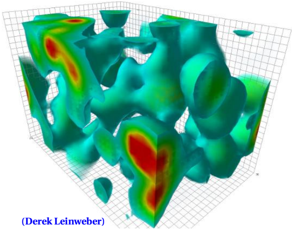
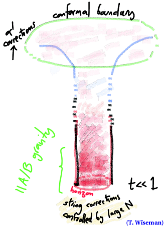
Lattice supersymmetric field theories — Part 3

David Schaich (University of Liverpool)









Nonperturbative and Numerical Approaches
to Quantum Gravity, String Theory and Holography

International Centre for Theoretical Sciences, Bangalore, 25 August 2022

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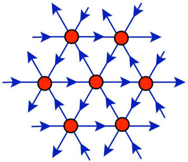
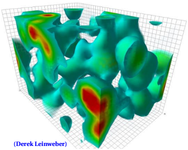
Any questions about last time?

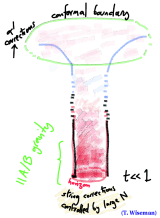


Overcoming challenges opens many opportunities for lattice studies of supersymmetric QFTs

✓ Motivation, background, formulation
- ✓ Supersymmetry breaking in discrete space-time
- ✓ Supersymmetry preservation in discrete space-time



Applications with significant recent progress
- Maximal $\mathcal{N} = 4$ super-Yang–Mills — wrap up
- Lower dimensions $d < 4$
- Minimal $\mathcal{N} = 1$ super-Yang–Mills



Remaining challenges: Super-QCD; Sign problems

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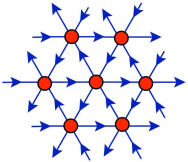

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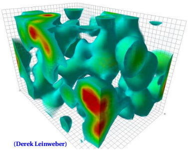

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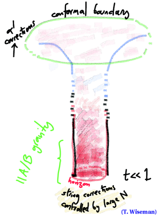


Conceptual focus with interaction encouraged

"It's better to uncover a little than to cover a lot" (V. Weisskopf)

Twisted lattice $\mathcal{N} = 4$ SYM — recap

so that the full improved action becomes

$$S_{\text{imp}} = S'_{\text{exact}} + S_{\text{closed}} + S'_{\text{soft}} \tag{18}$$

$$S'_{\text{exact}} = \frac{N}{4\lambda_{\text{lat}}} \sum_n \text{Tr}\left[-\overline{\mathcal{F}}_{ab}(n)\mathcal{F}_{ab}(n) - \chi_{ab}(n)\mathcal{D}^{(+)}_{[a}\psi_{b]}(n) - \eta(n)\overline{\mathcal{D}}^{(-)}_a\psi_a(n) + \frac{1}{2}\left(\overline{\mathcal{D}}^{(-)}_a\mathcal{U}_a(n) + G\sum_{a\neq b}\left(\det\mathcal{P}_{ab}(n) - 1\right)\mathbb{I}_N\right)^2\right] - S_{\text{det}}$$

$$S_{\text{det}} = \frac{N}{4\lambda_{\text{lat}}} G \sum_n \text{Tr}\left[\eta(n)\right] \sum_{a\neq b}\left[\det\mathcal{P}_{ab}(n)\right] \text{Tr}\left[\mathcal{U}_b^{-1}(n)\psi_b(n) + \mathcal{U}_a^{-1}(n+\widehat{\mu}_b)\psi_a(n+\widehat{\mu}_b)\right]$$

$$S_{\text{closed}} = -\frac{N}{16\lambda_{\text{lat}}} \sum_n \text{Tr}\left[\epsilon_{abcde}\ \chi_{de}(n+\widehat{\mu}_a+\widehat{\mu}_b+\widehat{\mu}_c)\overline{\mathcal{D}}^{(-)}_c\chi_{ab}(n)\right],$$

$$S'_{\text{soft}} = \frac{N}{4\lambda_{\text{lat}}}\mu^2 \sum_n \sum_a \left(\frac{1}{N}\text{Tr}\left[\mathcal{U}_a(n)\overline{\mathcal{U}}_a(n)\right] - 1\right)^2$$

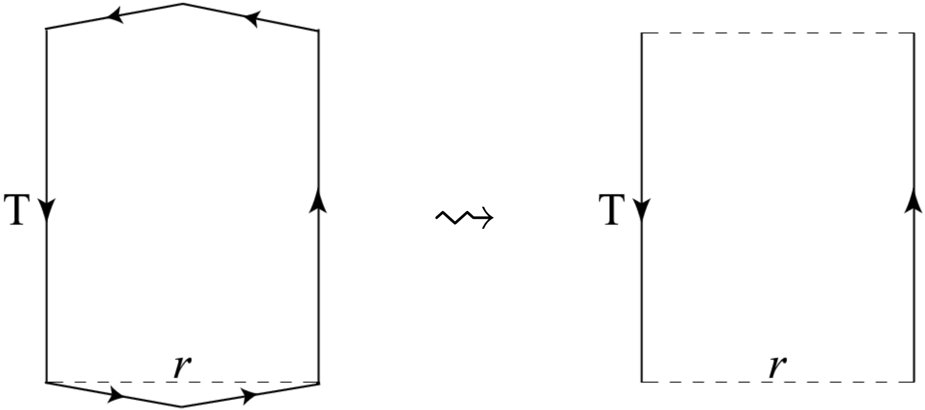
Computationally challenging, e.g. $\gtrsim$100 gathers per fermion matrix–vector op.

Public parallel code github.com/daschaich/susy [arXiv:1410.6971]
actively developed for improved performance and new applications

Static potential $V(r)$ for 4d $\mathcal{N} = 4$ SYM

Static probes $\longrightarrow$ $r \times T$ Wilson loops $W(r, T) \propto e^{-V(r)\,T}$

Coulomb gauge trick reduces A_4^* lattice complications



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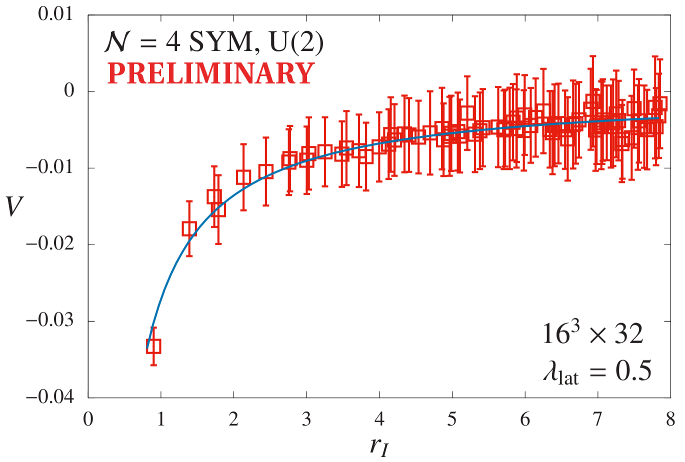
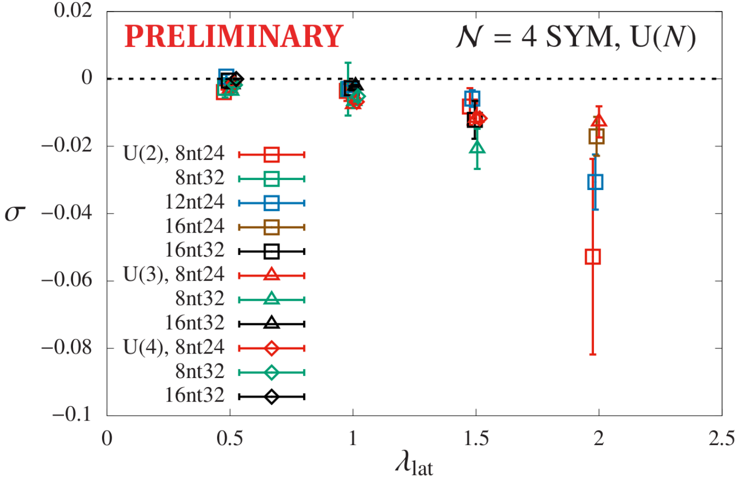


Static potential is Coulombic at all λ

String tension σ from fits to confining form $V(r) = A - C/r + \sigma r$

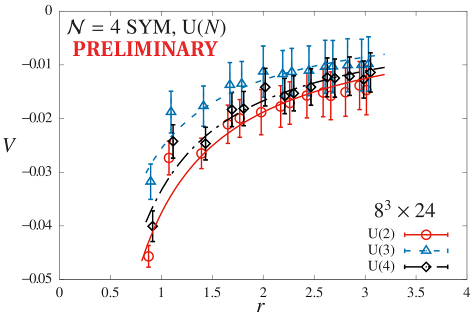


Slightly negative values flatten $V(r_I)$ for $r_I \lesssim L/2$

$\sigma \to 0$ as accessible range of r_I increases on larger volumes

Perturbative improvement for the static potential

Results above are improved to reduce short-distance discretization artifacts



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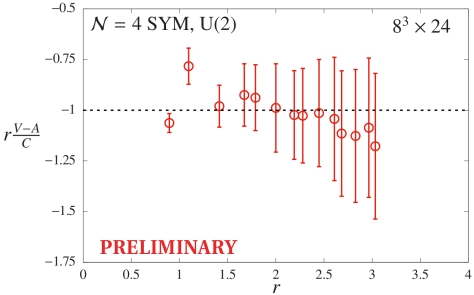


Danger of distorting Coulomb coefficient C from fits to $V(r) = A - C/r$

Tree-level improvement

Classic trick to reduce discretization artifacts in static potential

Associate $V(r_\nu)$ data with 'r_I' from Fourier transform of gluon propagator

Recall $\displaystyle \frac{1}{4\pi^2 r^2} = \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{e^{ir_\nu k_\nu}}{k^2}$ where $\displaystyle \frac{1}{k^2} = G(k_\nu)$ in continuum

$$A_4^* \text{ lattice} \longrightarrow \frac{1}{r_I^2} \equiv 4\pi^2 \int_{-\pi}^{\pi} \frac{d^4\widehat{k}}{(2\pi)^4} \frac{\cos\left(r_\nu \widehat{k}_\nu\right)}{4\sum_{\mu=1}^{4} \sin^2\left(\widehat{k}\cdot\widehat{e}_\mu \;/\; 2\right)}$$

Tree-level lattice propagator from arXiv:1102.1725

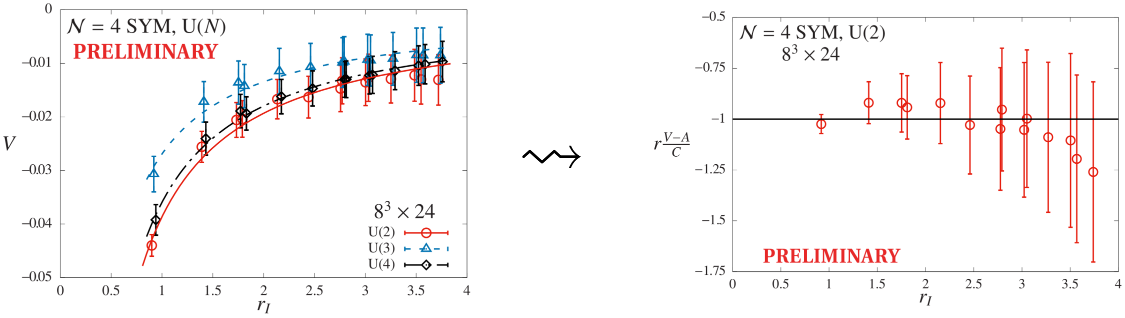
$\widehat{e}_\mu$ are A_4^* lattice basis vectors;

momenta $\widehat{k} = \frac{2\pi}{L}\sum_{\mu=1}^{4} n_\mu \widehat{g}_\mu$ depend on dual basis vectors

Tree-level-improved static potential

$$\frac{1}{r_I^2} \equiv 4\pi^2 \int_{-\pi}^{\pi} \frac{d^4\widehat{k}}{(2\pi)^4} \frac{\cos\left(r_\nu \widehat{k}_\nu\right)}{4\sum_{\mu=1}^{4} \sin^2\left(\widehat{k} \cdot \widehat{e}_\mu \;/\; 2\right)}$$

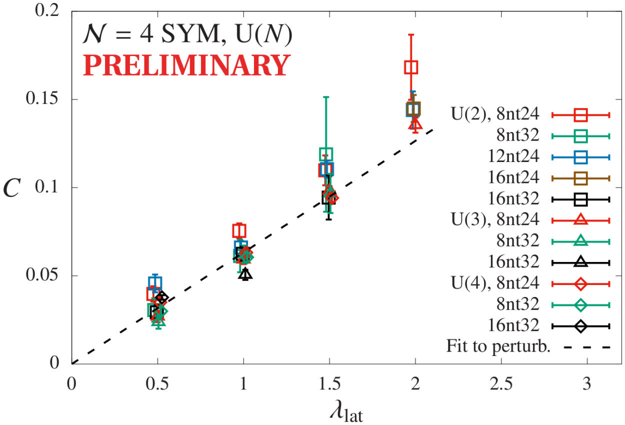
$\longrightarrow$ significantly reduced discretization artifacts



Coupling dependence of Coulomb coefficient

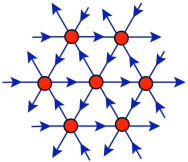
Continuum perturbation theory $\longrightarrow$ $C(\lambda) = \lambda/(4\pi) + \mathcal{O}(\lambda^2)$

Holography $\longrightarrow$ $C(\lambda) \propto \sqrt{\lambda}$ for $N \to \infty$ and $\lambda \to \infty$ with $\lambda \ll N$



For $\lambda_{\text{lat}} \leq 2$, consistent with leading-order perturbation theory

Checkpoint



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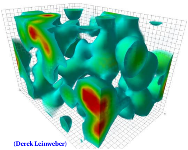
Applications with significant recent progress



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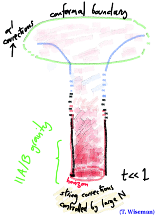
Lower dimensions $d < 4$

Minimal $\mathcal{N} = 1$ super-Yang–Mills

Remaining challenges: Super-QCD; Sign problems



Questions?

"It's better to uncover a little than to cover a lot"

Lots of opportunities in lower dimensions $d < 4$

Significant simplifications

$\sharp$ degrees of freedom $\propto L^d$

Fewer interactions $\longrightarrow$ less computational work

Dimensionful 't Hooft coupling $[\lambda] = 4 - d$, super-renormalizable in some cases

Three dimensions — research talk next week

Two dimensions — some highlights here

One dimension — few highlights here $\longrightarrow$ lectures by Georg Bergner next week

2d maximal SYM thermodynamics

arXiv:1709.07025

Naive dimensional reduction

$\longrightarrow$ skewed $r_L \times r_\beta$ torus with four scalar $\mathcal{Q}$

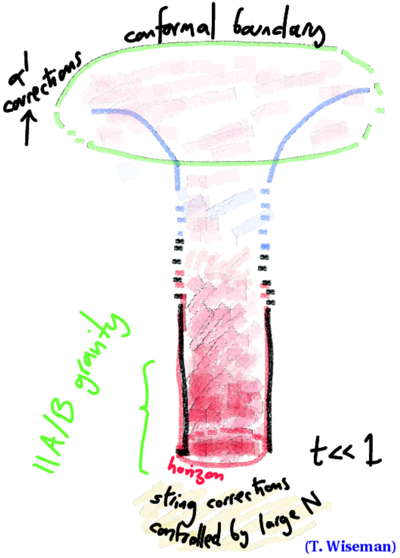
Thermal boundary conditions

$\longrightarrow$ dim'less temperature $t = 1/r_\beta = T/\sqrt{\lambda}$

Low temperatures t at large N

$\updownarrow$

Black branes in dual supergravity



Phase diagram expectations

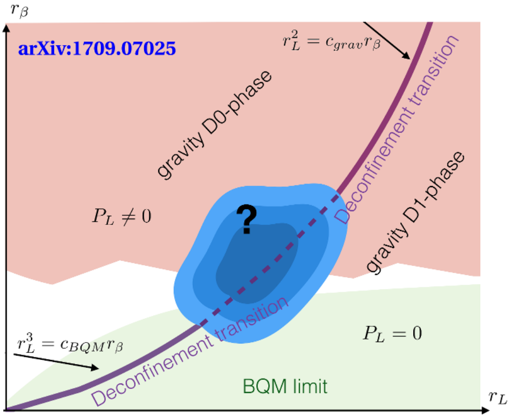
First-order transitions predicted from bosonic QM at high t $(r_\beta \ll 1)$
from holography at low t $(r_\beta \gg 1)$

For decreasing r_L at large N

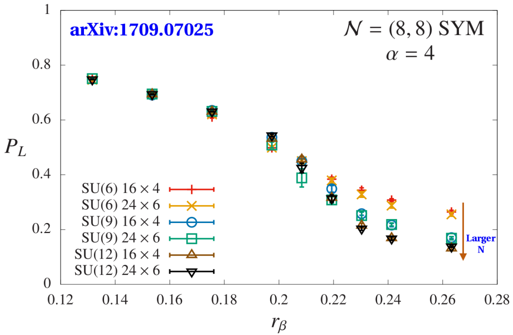
homogeneous black string (D1)
$\longrightarrow$ localized black hole (D0)

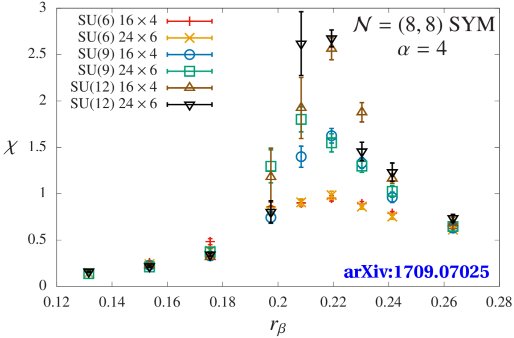
$\Updownarrow$

"spatial deconfinement"
signalled by Wilson line P_L



Spatial deconfinement transition signals — high-t example





Peaks in Wilson line susceptibility match change in its magnitude |PL|,

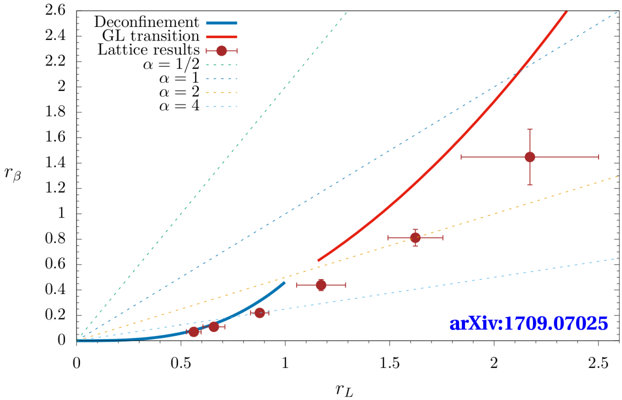
grow with size of SU(N) gauge group, comparing $N = 6, 9, 12$

Agreement for 16×4 vs. 24×6 lattices (aspect ratio $\alpha = r_L / r_\beta = 4$)

Lattice results for 2d $\mathcal{N} = (8, 8)$ SYM phase diagram

Good agreement with bosonic QM at high temperatures

Harder to control low-temperature uncertainties (larger $N > 16$ should help)



Overall consistent with holography

Comparing multiple lattice sizes and $6 \leq N \leq 16$

Controlled extrapolations are work in progress

A different twist in two dimensions

The "A twist" doesn't complexify links [Sugino, Matsuura, Hanada, Ohta, ...]

$\longrightarrow$ SU(N) gauge invariance but $\mathcal{Q}_A^2 =$ gauge transformation

Suffers from exponentially many degenerate vacua

Matsuura & Sugino [arXiv:1402.0952] resolve this problem in two dimensions but not in higher dimensions

In two dimensions, can formulate A-twist $\mathcal{N} = (2,2)$ SYM on arbitrary polygon decompositions of Riemann surfaces

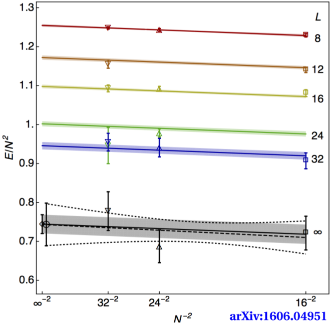
Matsuura–Misumi–Ohta, arXiv:1408.6998

Super-Yang–Mills quantum mechanics

4d SU(N) SYM $\longrightarrow$ quantum mechanics of $N \times N$ matrices [G. Bergner lectures]

Predict corrections to SUGRA result through large-N continuum extrapolations

Monte Carlo String/M-Theory Collaboration, arXiv:1606.04951



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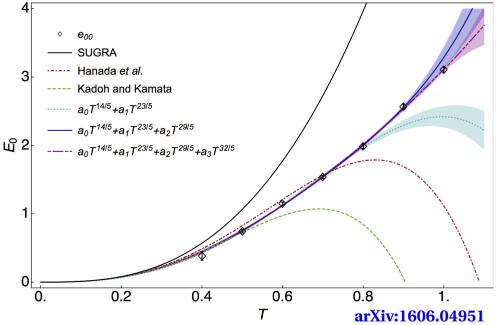


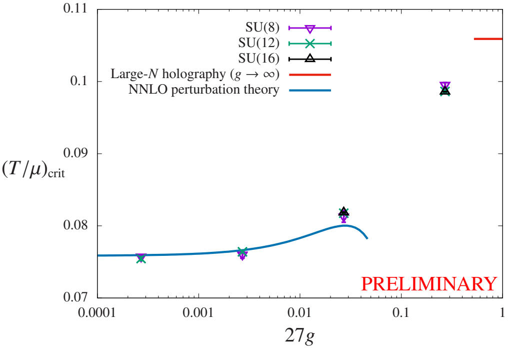
Supersymmetric mass deformation

Berenstein–Maldacena–Nastase, hep-th/0202021

Generalize SYM QM while preserving maximal supersymmetry

$\longrightarrow$ more interesting features including phase transition at critical T/mu

Jha–Joseph–DS, 2201.03097 & to appear

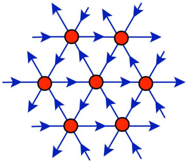


Phase diagram of critical T/μ
vs. dimensionless coupling g

For small $g \lesssim 10^{-3}$, agree with
NNLO perturbation theory

Approach leading-order holography
as g increases

Checkpoint



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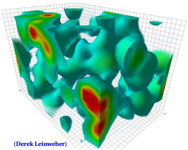

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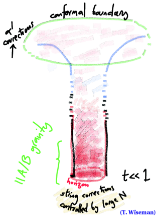


Remaining challenges: Super-QCD; Sign problems

Questions?

"It's better to uncover a little than to cover a lot"

$\mathcal{N} = 1$ SYM is special case with no scalars

SU(N) gauge theory with single massless Majorana fermion in adjoint rep.

Straightforward lattice fermion formulations explicitly break chiral symmetry
$\longrightarrow$ large additive gluino mass renormalization

Chiral ('overlap' or 'domain-wall') lattice fermions numerically expensive

$\mathcal{N} = 1$ SYM is special case with no scalars

SU(N) gauge theory with single massless Majorana fermion in adjoint rep.

1) Fine-tune gluino mass $\longrightarrow$ supersymmetry in chiral continuum limit

2) Overlap or domain-wall fermions

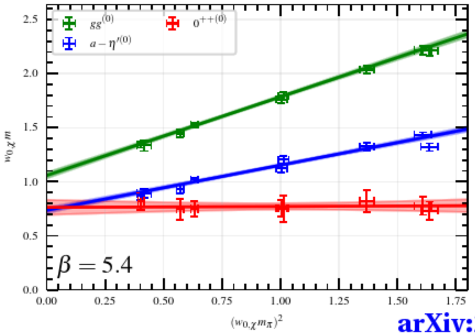
$\longrightarrow$ automatic (accidental) supersymmetry in continuum limit

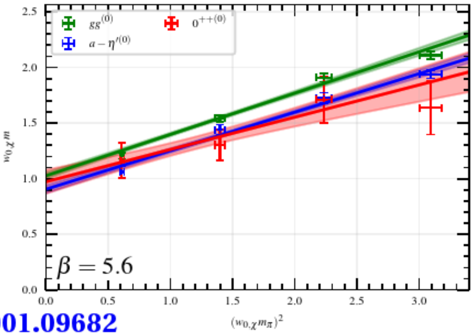
Selected recent progress fine-tuning gluino mass

Scalar, pseudoscalar and fermionic partner
approach degenerate supermultiplet for massless gluino

Smaller lattice spacing 'a' (larger β) $\longrightarrow$ improved supermultiplet formation

Desy–Münster–Regensburg–Jena, arXiv:1902.11127 & arXiv:2001.09682





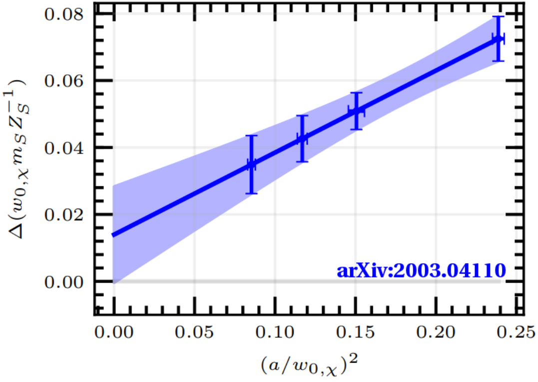
arXiv:2001.09682

Selected recent progress fine-tuning gluino mass

Measure of supersymmetry breaking from Ward identities

vanishes in chiral continuum limit, $a^2 \to 0$

Desy–Münster–Regensburg–Jena, arXiv:2003.04110



Extrapolation consistent

with $\mathcal{O}(a^2)$ discretization artifacts

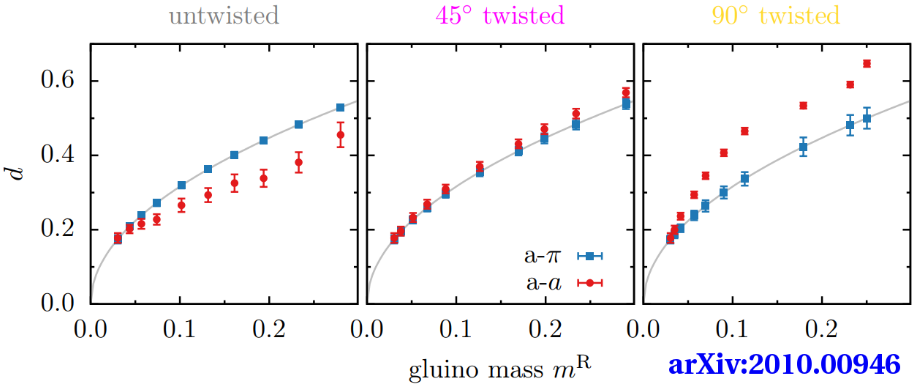
expected for this lattice action

Selected recent progress fine-tuning gluino mass

Alternate ‘twisted-mass’ action provides extra ‘twist angle’ parameter

$\longrightarrow$ tune this to improve approach to continuum limit

Steinhauser–Sternbeck–Wellegehausen–Wipf, arXiv:2010.00946



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Lattice chiral symmetry

Chiral symmetry means $\{D, \gamma_5\} = \gamma_5 D + D\gamma_5 = 0$ for massless fermion operator

Only a 'remnant' can be realized on the lattice [Ginsparg–Wilson, 1982]

$$D\gamma_5 + \gamma_5 D = aD\gamma_5 D \qquad \longrightarrow \qquad \left(\mathbb{I} - \frac{a}{2}D\right)\gamma_5 D + D\gamma_5\left(\mathbb{I} - \frac{a}{2}D\right) = 0$$

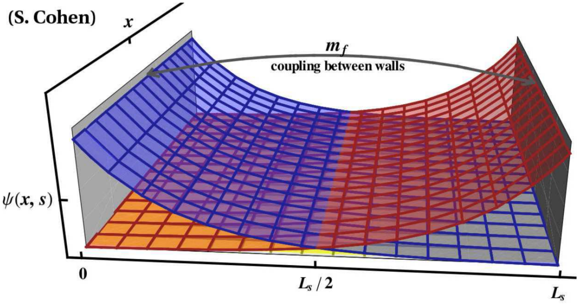
Difficult to construct fermion operators that obey the Ginsparg–Wilson relation

'Overlap' operator $aD_{\text{ov}} = \mathbb{I} + \gamma_5 \text{sign}\left[\gamma_5 D_W(\kappa)\right]$

requires computing $\text{sign}\left[H\right] = \dfrac{H}{\sqrt{H \cdot H}}$ for large matrix

'Domain-wall' operator introduces extra direction. . .

Domain-wall fermions



$L_s \sim \mathcal{O}(10)$ copies of 4d gauge fields — expensive! [Used by 0810.5746, 0902.4267]

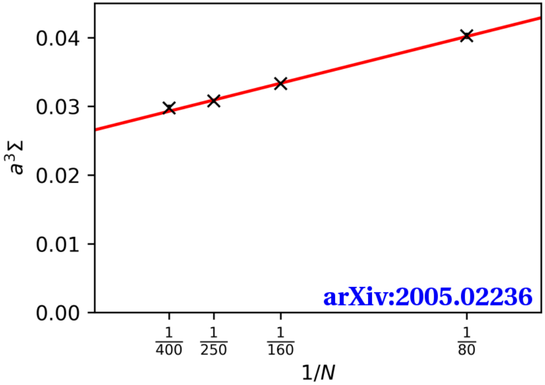
Localized fermions have renormalized mass $m = m_f + m_{\text{res}}$
with residual mass $m_{\text{res}} \ll m_f$ from overlap around $L_s/2$

$L_s \to \infty$ allows exact chiral symmetry at non-zero lattice spacing

Recent progress with overlap $\mathcal{N} = 1$ super-Yang–Mills

N-order polynomial approximation to compute matrix sign function

Piemonte–Bergner–López, arXiv:2005.02236



Bare gluino condensate from 12^4 lattices

$N \to \infty$ gives chiral limit

Only multiplicative renormalization

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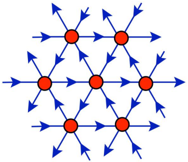

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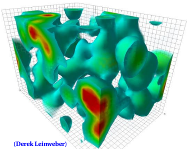

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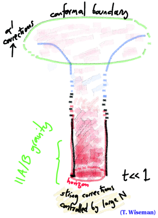

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Future frontier: Supersymmetric QCD

Add 'quarks' and squarks $\longrightarrow$ investigate electric–magnetic dualities,
dynamical supersymmetry breaking and more

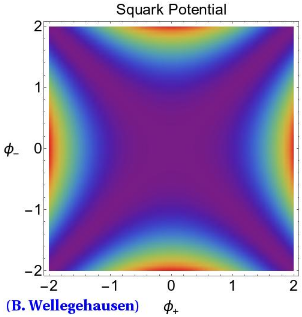
Fine-tuning back with a vengeance

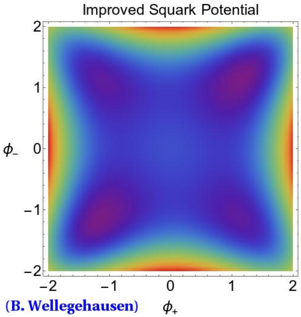
$\mathcal{O}(10)$ parameters, even using overlap or domain-wall fermions [arXiv:0903.2443]

Pursuing superQCD with full fine-tuning

First step: Lattice perturbation theory as guide for future fine-tuning

Wellegehausen–Wipf, arXiv:1811.01784; Costa–Panagopoulos, arXiv:1812.06770





Alternately include only fundamental + adjoint fermions, leave scalars for future

Bergner–Piemonte, arXiv:2008.02855

Simplify superQCD: Twisted theories in 2d or 3d

Quiver construction preserves susy sub-algebra [arXiv:0805.4491, arXiv:0807.2683]

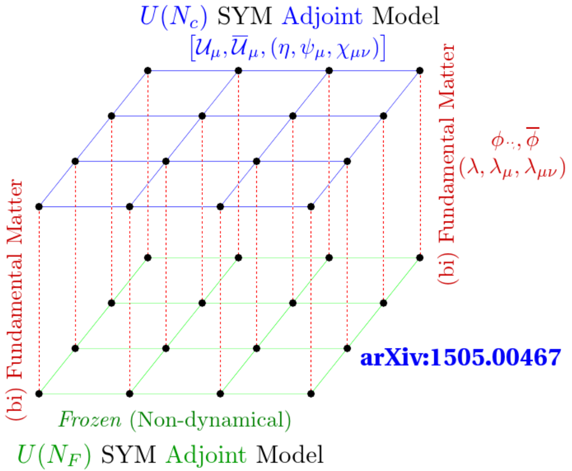
2-slice lattice SYM
with U(N) $\times$ U(F) gauge group

Adj. fields on each slice

Bi-fundamental in between

Decouple U(F) slice

$\longrightarrow$ U(N) SQCD in $d-1$ dims.
with F fund. hypermultiplets

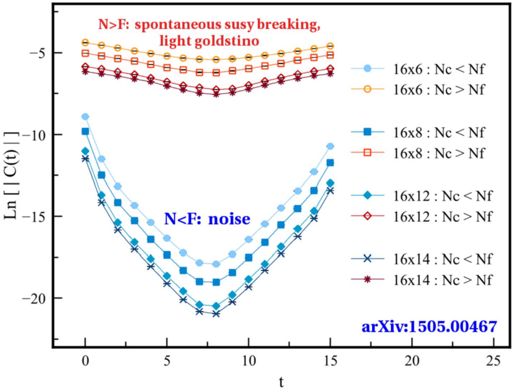


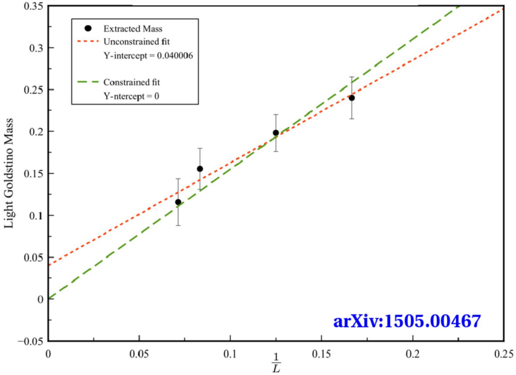
Dynamical susy breaking in 2d lattice superQCD

U(N) superQCD with F fundamental hypermultiplets

Observe spontaneous susy breaking only for $N > F$, as expected

Catterall–Veernala, arXiv:1505.00467





Future frontier: Sign problems

Recall typical algorithms sample field configurations Φ with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi]}$

$\longrightarrow$ "sign problem" if action $S[\Phi]$ can be negative or complex

Example: Spontaneous susy breaking needs vanishing Witten index

Witten index is just $\mathcal{Z} = \int \mathcal{D}\Phi \; e^{-S[\Phi]}$ $\longrightarrow$ severe sign problem to have $\mathcal{Z} = 0$

Motivates alternative approaches to be discussed:

Complex Langevin — A. Kumar [Sat]

Tensor networks — D. Kadoh lectures; R. Jha [Fri]; R. Sakai [Mon]

Quantum simulation — S. Chandrasekharan [Sat]; I. Raychowdhury [Tue]; Y. Meurice [Tue]; E. Zohar [Thu]

Future frontier: Sign problems

Recall typical algorithms sample field configurations Φ with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi]}$

$\longrightarrow$ "sign problem" if action $S[\Phi]$ can be negative or complex

Example: $\mathcal{N} = 4$ SYM has complex pfaffian $\text{pf}\,\mathcal{D} = |\text{pf}\,\mathcal{D}| e^{i\alpha}$

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int [d\mathcal{U}][d\overline{\mathcal{U}}] \;\; \mathcal{O} \;\; e^{-S_B[\mathcal{U},\overline{\mathcal{U}}]} \;\; \text{pf}\,\mathcal{D}[\mathcal{U},\overline{\mathcal{U}}]$$

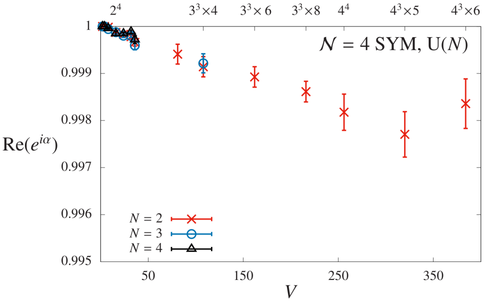
We phase quench $\text{pf}\,\mathcal{D} \longrightarrow |\text{pf}\,\mathcal{D}|$, need to reweight $\langle \mathcal{O} \rangle = \dfrac{\left\langle \mathcal{O} e^{i\alpha} \right\rangle_{\text{pq}}}{\left\langle e^{i\alpha} \right\rangle_{\text{pq}}}$

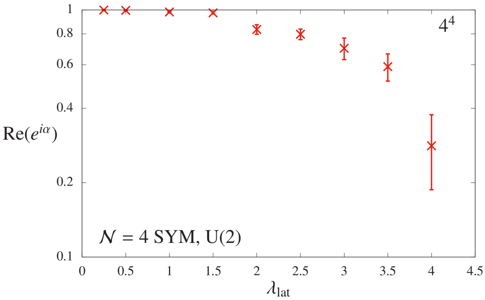
$$\Longrightarrow \left\langle e^{i\alpha} \right\rangle_{\text{pq}} = \frac{\mathcal{Z}}{\mathcal{Z}_{\text{pq}}}$$ quantifies severity of sign problem

$\mathcal{N} = 4$ SYM sign problem

Fix $\lambda_{lat} = g^2_{lat}N = 0.5$

Pfaffian nearly real positive
for all accessible volumes



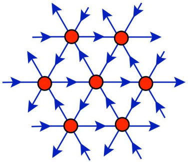


Fix 4^4 volume

Fluctuations increase with coupling

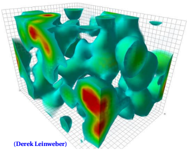
Signal-to-noise
becomes obstruction for $\lambda_{lat} \gtrsim 4$

Wrap up

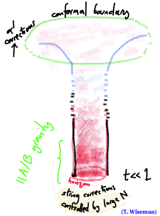


Overcoming challenges opens many opportunities
for lattice studies of supersymmetric QFTs

✓ Motivation, background, formulation
- ✓ Supersymmetry breaking in discrete space-time
- ✓ Supersymmetry preservation in discrete space-time



✓ Applications with significant recent progress
- ✓ Maximal $\mathcal{N} = 4$ super-Yang–Mills
- ✓ Lower dimensions $d < 4$
- ✓ Minimal $\mathcal{N} = 1$ super-Yang–Mills



✓ Remaining challenges: Super-QCD; Sign problems

Further resources

Lattice studies of supersymmetric gauge theories

David Schaich*

Department of Mathematical Sciences,
University of Liverpool, Liverpool L69 7ZL, United Kingdom

(Dated: 17 August 2022)

Updated version of arXiv:2208.03580 at icts.res.in/program/numstrings2022/talks

arXiv:0903.4881 by Catterall, Kaplan and Ünsal remains most detailed review

Expect connections with lectures by:

Anna Hasenfratz — Introduction to Lattice Field Theory

Georg Bergner — Matrix Models, Gauge-Gravity Duality, and Simulations…

Supplement: Scaling dimensions from lattice $\mathcal{N} = 4$ SYM

Arguably simplest non-trivial 4d QFT $\longrightarrow$ dualities, amplitudes, ...

SU(N) gauge theory with $\mathcal{N} = 4$ fermions Ψ^{I} and 6 scalars Φ^{IJ},
all massless and in adjoint rep.

Maximal 16 supersymmetries Q_α^{I} and $\overline{Q}_{\dot{\alpha}}^{\mathrm{I}}$ $\quad \mathrm{I} = 1, \cdots, 4$
transform under global SU(4) $\sim$ SO(6) R symmetry

Conformal $\longrightarrow$ β function is zero for all values of $\lambda = g^2 N$
Exact, perturbative, holographic & bootstrap results
for spectrum of **scaling dimensions** $\Delta(\lambda)$

Anomalous dim. from fermion operator eigenmodes

arXiv:2102.06775

Conformality broken by finite volume and non-zero lattice spacing

Consider analogue of mass anomalous dimension,

$\gamma_*(\lambda) = 0$ for continuum $\mathcal{N} = 4$ SYM

Antisymmetric fermion operator $\longrightarrow$ paired eigenvalues $\pm\lambda_k$

$$\Psi^T D\Psi = \chi_{ab}\mathcal{D}^{(+)}_{[a}\psi_{b]} + \eta\mathcal{D}^{\dagger(-)}_{a}\psi_a + \frac{1}{2}\epsilon_{abcde}\chi_{ab}\mathcal{D}^{\dagger(-)}_{c}\chi_{de}$$

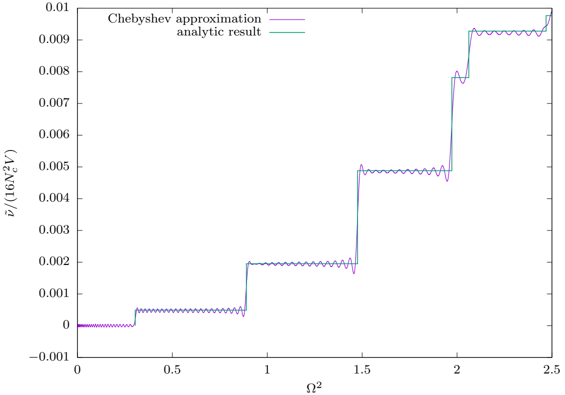
Anomalous dimension related to mode number of $D^\dagger D$

$$\nu(\Omega^2) = \int_0^{\Omega^2}\rho(\omega^2)\,d\omega^2 \propto \left(\Omega^2\right)^{2/(1+\gamma_*)} \qquad \rho(\omega^2) = \frac{1}{V}\sum_k \left\langle\delta(\omega^2 - \lambda_k^2)\right\rangle$$

Chebyshev expansion for mode number

Stochastically estimate Chebyshev expansion
[Fodor et al., arXiv:1605.08091]

$$\rho_r(x) \approx \sum_{n=0}^{P} \frac{2-\delta_{n0}}{\pi\sqrt{1-x^2}} c_n T_n(x)$$



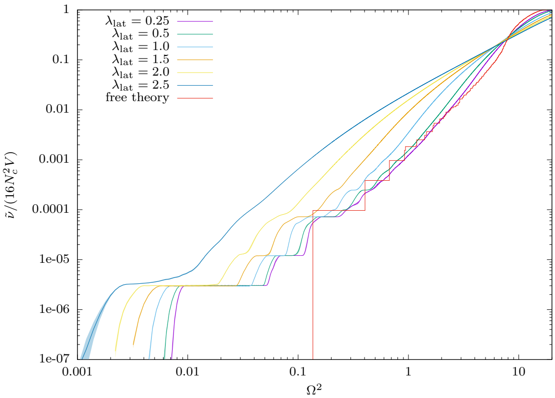
⟵ Example mode number for U(2) 8^4 free theory, $P = 1000$

$5000 \le P \le 10000$ for $N = 2, 3, 4$ volumes up to 16^4

Checked vs. direct eigensolver and stochastic projection

Mode number scale dependence

Anomalous dimension from $D^\dagger D$ mode number $\quad \nu(\Omega^2) \propto \left(\Omega^2\right)^{2/(1+\gamma_*)}$



U(2) 16^4 lattices with $0.25 \leq \lambda_{\text{lat}} \leq 2.5$

Free theory also shows lattice effects

Power law varies with scale Ω^2

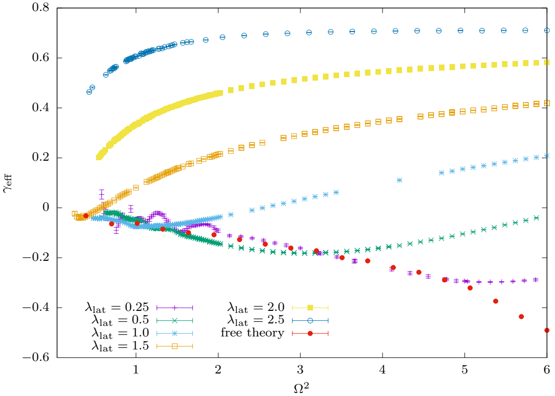
$\longrightarrow$ scale-dependent effective $\gamma_{\text{eff}}(\Omega^2)$

Extract by fitting in windows $\left[\Omega^2, \Omega^2 + \ell\right]$

with fixed $\ell \in [0.03, 1]$

Convergence to continuum $\gamma_* = 0$

Broken conformality $\longrightarrow$ scale-dependent effective anomalous dim. $\gamma_{\text{eff}}(\Omega^2)$



U(2) 16^4 lattices with $0.25 \leq \lambda_{\text{lat}} \leq 2.5$

Free theory also shows lattice effects

Recover true $\gamma_* = 0$ in IR, $\Omega^2 \ll 1$

Stronger couplings $\longrightarrow$ larger artifacts

Konishi operator scaling dimension Δ_K

$\mathcal{O}_K(x) = \sum_{\mathrm{I}} \mathrm{Tr}\left[\Phi^{\mathrm{I}}(x)\Phi^{\mathrm{I}}(x)\right]$ is simplest conformal primary operator

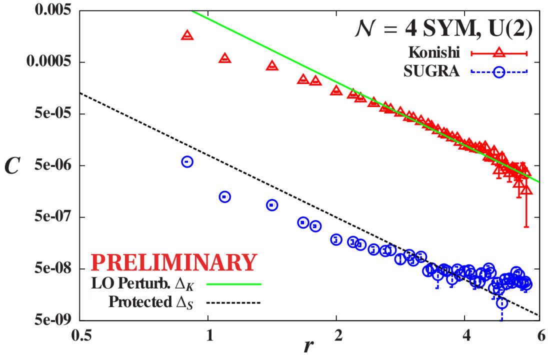
Scaling dimension $\Delta_K(\lambda) = 2 + \gamma_K(\lambda)$ investigated through
perturbation theory (& S duality), holography, conformal bootstrap

$C_K(r) \equiv \mathcal{O}_K(x+r)\mathcal{O}_K(x) \propto r^{-2\Delta_K}$

20′ ‘SUGRA’ op. has $\Delta_S = 2$

Work in progress to compare:

- Direct power-law decay
- Finite-size scaling
- Monte Carlo RG



Konishi operator scaling dimension Δ_K

Lattice scalars $\varphi(n)$ from polar decomposition $\mathcal{U}_a(n) = e^{\varphi_a(n)} U_a(n)$

$$\mathcal{O}_K^{\text{lat}}(n) = \sum_a \text{Tr}\left[\varphi_a(n)\varphi_a(n)\right] - \text{vev} \qquad \mathcal{O}_S^{\text{lat}}(n) \sim \text{Tr}\left[\varphi_a(n)\varphi_b(n)\right]$$

$$C_K(r) \equiv \mathcal{O}_K(x+r)\mathcal{O}_K(x) \propto r^{-2\Delta_K}$$

$20'$ ‘SUGRA’ op. has $\Delta_S = 2$

Work in progress to compare:
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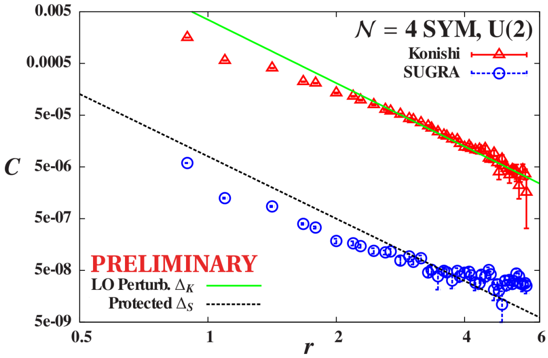


Scaling dimensions from MCRG stability matrix

Lattice system: $H = \sum_i c_i \mathcal{O}_i$ (infinite sum)

Couplings flow under RG blocking $\longrightarrow$ $H^{(n)} = R_b H^{(n-1)} = \sum_i c_i^{(n)} \mathcal{O}_i^{(n)}$

Conformal fixed point $\longrightarrow$ $H^* = R_b H^*$ with couplings c_i^*

Linear expansion around fixed point $\longrightarrow$ stability matrix T_{ik}^*

$$c_i^{(n)} - c_i^* = \sum_k \left.\frac{\partial c_i^{(n)}}{\partial c_k^{(n-1)}}\right|_{H^*} \left(c_k^{(n-1)} - c_k^*\right) \equiv \sum_k T_{ik}^* \left(c_k^{(n-1)} - c_k^*\right)$$

Correlators of $\mathcal{O}_i$, $\mathcal{O}_k$ $\longrightarrow$ elements of stability matrix [Swendsen, 1979]

Eigenvalues of T_{ik}^* $\longrightarrow$ scaling dimensions of corresponding operators

Smearing for Konishi analyses

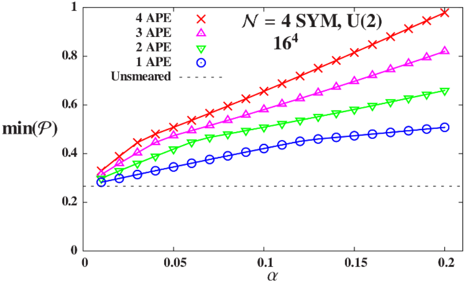
Smear to enlarge (MCRG or variational) operator basis

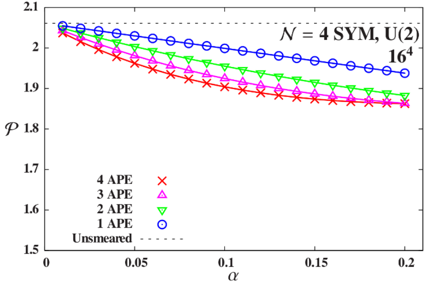
APE-like smearing: $\quad — \quad \longrightarrow \quad (1-\alpha) — \quad + \quad \frac{\alpha}{8} \sum \sqcap,$

staples built from unitary parts of links but no final unitarization

Average plaquette stable upon smearing (**right**),

minimum plaquette steadily increases (**left**)



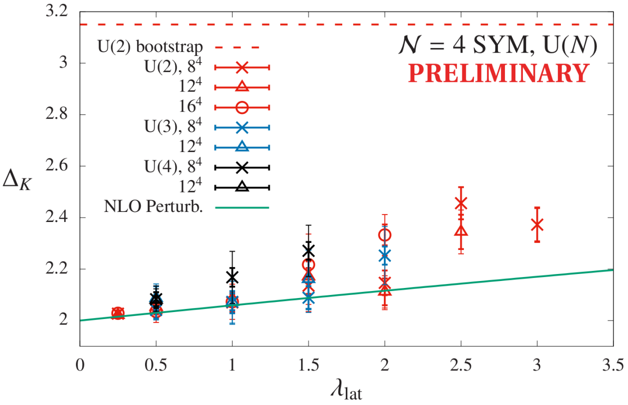


Preliminary Δ_K results from Monte Carlo RG

Both Konishi and SUGRA in T^*_{ik}

Impose protected $\Delta_S = 2$
$\longrightarrow$ Δ_K consistent with pert. theory

Systematic uncertainties from
different amounts of smearing



Complication from twisting $SO(4)_R \subset SO(6)_R$

$\mathcal{O}_K^{\text{lat}}$ mixes with $SO(4)_R$-singlet part of $SO(6)_R$-nonsinglet $\mathcal{O}_S$
$\longrightarrow$ disentangle via variational analyses

Backup: Dimensional reduction to 2d $\mathcal{N} = (8,8)$ SYM

Naive for now: 4d $\mathcal{N} = 4$ SYM code with $N_x = N_y = 1$

$A_4^* \longrightarrow A_2^*$ (triangular) lattice

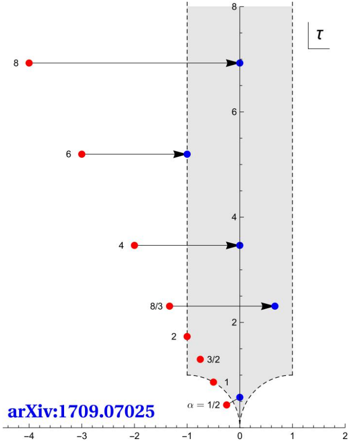
Torus **skewed** depending on $\alpha = L/N_t$

Modular transformation into fundamental domain

$\longrightarrow$ some skewed tori actually rectangular

Also need to stabilize compactified links

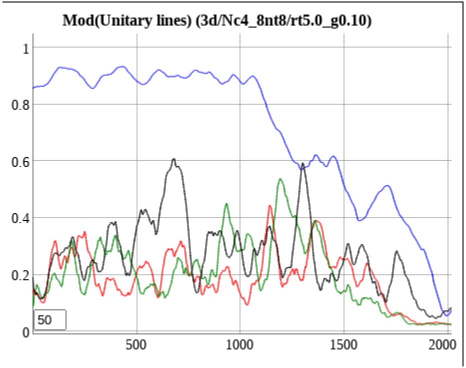
to ensure broken center symmetries

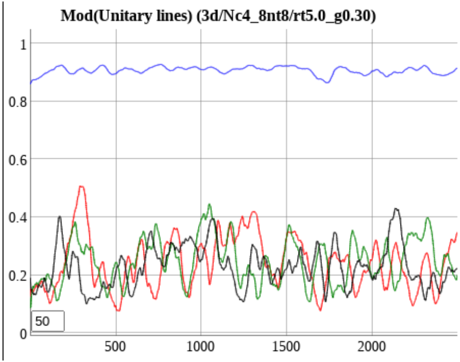


Backup: Stabilizing compactified links

Add potential $\propto \text{Tr}\left[(\varphi - \mathbb{I}_N)^\dagger (\varphi - \mathbb{I}_N)\right]$ to break center symmetry in reduced dir(s)

($\sim$Kaluza–Klein rather than Eguchi–Kawai reduction)



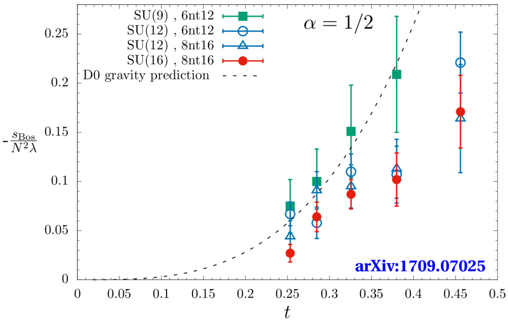


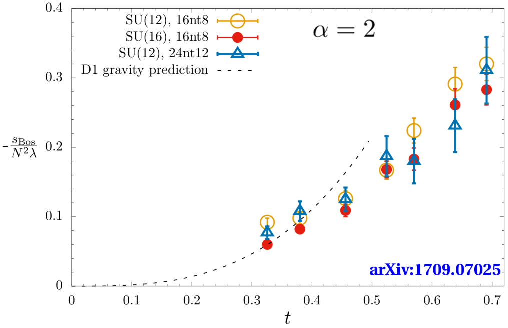
Backup: Check holographic black hole energies

Lattice results consistent with leading expectation for sufficiently low $t \lesssim 0.4$

Similar behavior $\longrightarrow$ difficult to distinguish phases

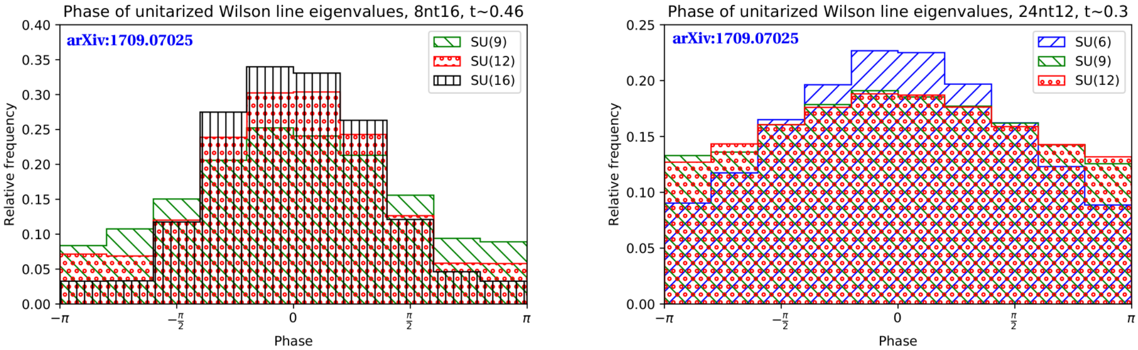
$\propto t^{3.2}$ for small-r_L D0 phase $\propto t^{3}$ for large-r_L D1 phase





Backup: 2d Wilson line eigenvalues for low t

Large-N eigenvalue phase distribution also signals spatial deconfinement



Left: $\alpha = 1/2$ distributions more localized as N increases $\longrightarrow$ D0 black hole

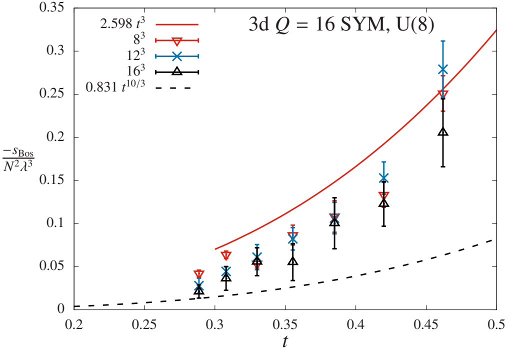
Right: $\alpha = 2$ distributions more uniform as N increases $\longrightarrow$ D1 black string

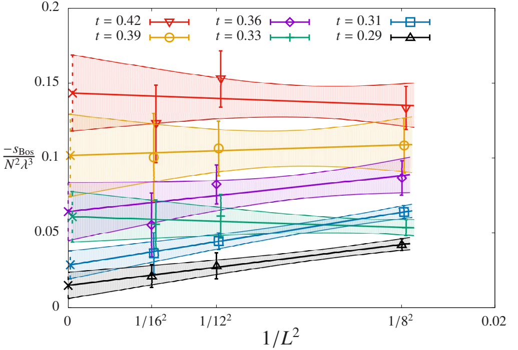
Backup: 3d thermodynamics and continuum extrapolation

Dimensional reduction to 3d $\mathcal{N} = 8$ SYM with two scalar $\mathcal{Q}$ [arXiv:2010.00026]

Approach leading holographic expectation $\propto t^{10/3}$ for low $t \lesssim 0.3$

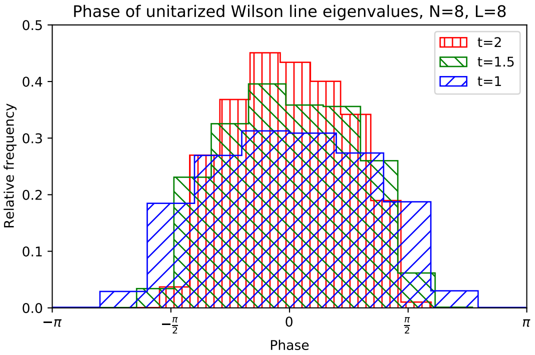
Carry out continuum extrapolations for fixed aspect ratio $\alpha = 1$ and $N = 8$

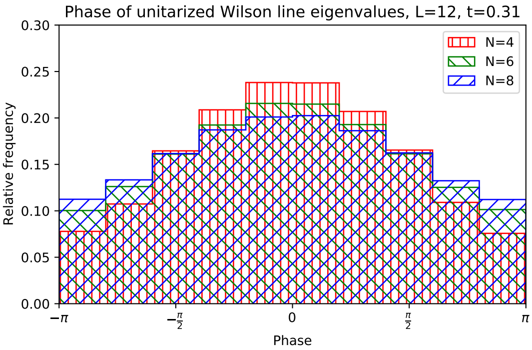




Backup: 3d $\mathcal{N} = 8$ SYM Wilson line eigenvalues

Large-N eigenvalue phase distribution also signals spatial deconfinement





Left: High-temperature U(8) 8^3 distributions more compact as t increases

Right: Low-temperature U(N) 12^3 distributions more uniform as N increases

Backup: More on dynamical susy breaking

Spontaneous susy breaking means $\langle 0\,|H|\,0\rangle > 0$ or equivalently $\langle \mathcal{Q}\mathcal{O}\rangle \neq 0$

Twisted superQCD auxiliary field e.o.m. $\longleftrightarrow$ Fayet–Iliopoulos D-term potential

$$d = \overline{\mathcal{D}}_a \mathcal{U}_a + \sum_{i=1}^{F} \phi_i \overline{\phi}_i - r\mathbb{I}_N \qquad \longleftrightarrow \qquad \text{Tr}\left[\left(\sum\nolimits_i \phi_i \overline{\phi}_i - r\mathbb{I}_N\right)^2\right] \in H$$

Have $F \times N$ scalar vevs to zero out $N \times N$ matrix

$\longrightarrow$ $N > F$ suggests susy breaking, $\langle 0\,|H|\,0\rangle > 0 \longleftrightarrow \langle \mathcal{Q}\eta\rangle = \langle d\rangle \neq 0$