INTERNATIONAL CENTRE *for* THEORETICAL SCIENCES

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

NONPERTURBATIVE AND NUMERICAL APPROACHES TO QUANTUM GRAVITY, STRING THEORY AND HOLOGRAPHY (HYBRID)

Loop-String-Hadron framework for a SU(3) lattice gauge theory

Indrakshi Raychowdhury

BITS-Pilani, K K Birla Goa Campus

30 August, 2022

Classical Computation Era

Change of Paradigm

Quantum Computation Era

Classical Computation Era

Change of Paradigm

Quantum Computation Era

Our role

- Identifying the physics problem that would benefit from quantum computation
- Reformulating the problem suitable for quantum computation
- NISQ-era quantum simulation algorithms: analog and digital
- Think beyond NISQ era…

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Lattice gauge theory calculations without sign problem:
Real time dynamics

Classical Computation Era

Change of Paradigm

Quantum Computation Era

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- Think beyond NISQ era…

Lattice gauge theory calculations without sign problem: Real time dynamics

Classical Computation Era

Lattice gauge theory calculations

Quantum Computation Era

- Requires different theoretical framework.
- Addressed different objectives
- Computational Methods are entirely different.

Ultimate goal: performing LATTICE-QCD calculations using Quantum Computer

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Too complicated to start with!

Simpler, yet similar models:

Quantum Link Models

Schwinger Model: QED in 1+1d

Simple theory: discrete gauge theories

$\mathbb{Z}_N$ **gauge theory;** $\mathbb{Z}_2$ **gauge theory in 2+1 dimensions**

Simplest, non-abelian gauge theory:

SU(2) gauge theory

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Emergent research directions:

Analog quantum simulation protocols,
Digital algorithms,
Tensor network calculations
Hybrid analog-digital algorithm

Ultimate goal: performing LATTICE-QCD calculations using Quantum Computer

State of the art:

PRL 110, 125304 (2013) PHYSICAL REVIEW LETTERS week ending 22 MARCH 2013

Cold-Atom Quantum Simulator for SU(2) Yang-Mills Lattice Gauge Theory

Erez Zohar,[1] J. Ignacio Cirac,[2] and Benni Reznik[1]

PRL 110, 125303 (2013) PHYSICAL REVIEW LETTERS week ending 22 MARCH 2013

Atomic Quantum Simulation of U(N) and SU(N) Non-Abelian Lattice Gauge Theories

D. Banerjee,[1] M. Bögli,[1] M. Dalmonte,[2] E. Rico,[2,3] P. Stebler,[1] U.-J. Wiese,[1] and P. Zoller[2,3]

Ann. Phys. (Berlin) 525, No. 10–11, 777–796 (2013) / DOI 10.1002/andp.201300104 annalen der physik

Ultracold quantum gases and lattice systems: quantum simulation of lattice gauge theories**

Uwe-Jens Wiese[1,2,*]

Quantum Algorithms for Simulating the Lattice Schwinger Model

Alexander F. Shaw[1,5], Pavel Lougovski[1], Jesse R. Stryker [2], and Nathan Wiebe[3,4]

PHYSICAL REVIEW D 100, 034518 (2019)

General methods for digital quantum simulation of gauge theories

Henry Lamm,[*] Scott Lawrence,[†] and Yukari Yamauchi[‡]

(NuQS Collaboration)

Simulating lattice gauge theories within quantum technologies

Mari Carmen Bañuls[1,2], Rainer Blatt[3,4], Jacopo Catani[5,6,7], Alessio Celi[3,8], Juan Ignacio Cirac[1,2], Marcello Dalmonte[9,10], Leonardo Fallani[5,6,7], Karl Jansen[11], Maciej Lewenstein[8,12,13], Simone Montangero[14,15,a], Christine A. Muschik[3], Benni Reznik[16], Enrique Rico[17,18], Luca Tagliacozzo[19], Karel Van Acoleyen[20], Frank Verstraete[20,21], Uwe-Jens Wiese[22], Matthew Wingate[23], Jakub Zakrzewski[24,25], and Peter Zoller[3]

PRL 115, 240502 (2015) PHYSICAL REVIEW LETTERS week ending 11 DECEMBER 2015

Non-Abelian SU(2) Lattice Gauge Theories in Superconducting Circuits

A. Mezzacapo,[1,2] E. Rico,[1,3] C. Sabín,[4] I. L. Egusquiza,[5] L. Lamata,[1] and E. Solano[1,3]

Quantum-classical computation of Schwinger model dynamics using quantum computers

N. Klco, E. F. Dumitrescu, A. J. McCaskey, T. D. Morris, R. C. Pooser, M. Sanz, E. Solano, P. Lougovski, and M. J. Savage

Phys. Rev. A 98, 032331 – Published 28 September 2018

PHYSICAL REVIEW X 10, 021041 (2020)

Lattice Gauge Theories and String Dynamics in Rydberg Atom Quantum Simulators

Federica M. Surace,[1,2] Paolo P. Mazza,[1,3] Giuliano Giudici,[1,2,3] Alessio Lerose,[1,3] Andrea Gambassi,[1,3] and Marcello Dalmonte[1,2]

SU(2) hadrons on a quantum computer

Yasar Atas *,[1,2,†] Jinglei Zhang *,[1,2,‡] Randy Lewis,[3] Amin Jahanpour,[1,2] Jan F. Haase,[1,2,§] and Christine A. Muschik[1,2,4]

PHYSICAL REVIEW RESEARCH 2, 023015 (2020)

Towards analog quantum simulations of lattice gauge theories with trapped ions

Zohreh Davoudi,[1,2] Mohammad Hafezi,[3,4] Christopher Monroe,[3,5] Guido Pagano,[3,5,6] Alireza Seif,[3] and Andrew Shaw[1]

PHYSICAL REVIEW D 104, 034501 (2021)

SU(2) lattice gauge theory on a quantum annealer

Sarmed A Rahman, Randy Lewis, Emanuele Mendicelli, and Sarah Powell

Department of Physics and Astronomy, York University, Toronto, Ontario M3J 1P3, Canada

SU(2) non-Abelian gauge field theory in one dimension on digital quantum computers

Natalie Klco,[*] Martin J. Savage,[†] and Jesse R. Stryker[‡]

Toward simulating quantum field theories with controlled phonon-ion dynamics: A hybrid analog-digital approach

Zohreh Davoudi,[1,*] Norbert M. Linke,[2] and Guido Pagano[3]

PHYSICAL REVIEW LETTERS 129, 051601 (2022)

Improved Hamiltonians for Quantum Simulations of Gauge Theories

Marcela Carena,[1,2,3,4,*] Henry Lamm,[1,†] Ying-Ying Li,[1,‡] and Wanqiang Liu[4,§]

Review articles

Quantum simulation of lattice gauge theories in more than one space dimension—requirements, challenges and methods

Erez Zohar

PHYSICAL REVIEW A 105, 023322 (2022)

Cold-atom quantum simulator for string and hadron dynamics in non-Abelian lattice gauge theory

Raka Dasgupta[1,*] and Indrakshi Raychowdhury[2,3,†]

and many more..

Ultimate goal: performing LATTICE-QCD calculations using Quantum Computer

State of the art:

For a SU(3) gauge theory

PHYSICAL REVIEW D **103**, 094501 (2021)

Editors' Suggestion

Trailhead for quantum simulation of SU(3) Yang-Mills lattice gauge theory in the local multiplet basis

Anthony Ciavarella,[1,*] Natalie Klco,[2,†] and Martin J. Savage[1,‡]

arXiv:2207.03473v1

Real-time evolution of SU(3) hadrons on a quantum computer

Yasar Y. Atas,[1,2,*] Jan F. Haase,[1,2,3,†] Jinglei Zhang,[1,2,‡] Victor Wei,[1,4]
Sieglinde M.-L. Pfaendler,[5] Randy Lewis,[6] and Christine A. Muschik[1,2,7]

IQuS@UW-21-027, NT@UW-22-05

Preparations for Quantum Simulations of Quantum Chromodynamics in $1+1$ Dimensions: (I) Axial Gauge

Roland C. Farrell,[1,*] Ivan A. Chernyshev,[1,†] Sarah J. M. Powell,[2,‡]
Nikita A. Zemlevskiy,[1,§] Marc Illa,[1,¶] and Martin J. Savage[1,$]

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limited progress so far!

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limited progress so far!

Actually, true for any non-Abelian gauge group

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Experimental demonstration

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer 2016

Esteban A. Martinez, Christine A. Muschik, Philipp Schindler, Daniel Nigg, Alexander Erhard, Markus Heyl, Philipp Hauke, Marcello Dalmonte, Thomas Monz, Peter Zoller & Rainer Blatt

Floquet approach to $\mathbb{Z}_2$ lattice gauge theories with ultracold atoms in optical lattices 2019

Christian Schweizer, Fabian Grusdt, Moritz Berngruber, Luca Barbiero, Eugene Demler, Nathan Goldman, Immanuel Bloch & Monika Aidelsburger

Self-verifying variational quantum simulation of lattice models 2019

C. Kokail[1,2,3], C. Maier[1,2,3], R. van Bijnen[1,2,3], T. Brydges[1,2], M. K. Joshi[1,2], P. Jurcevic[1,2], C. A. Muschik[1,2], P. Silvi[1,2], R. Blatt[1,2], C. F. Roos[1,2] & P. Zoller[1,2*]

A scalable realization of local U(1) gauge invariance in cold atomic mixtures 2020

Alexander Mil[1,*], Torsten V. Zache[2], Apoorva Hegde[1], Andy Xia[1], Rohit P. Bhatt[1], Markus K. Oberthaler[1], Philipp Hauke[1,2,3], Jürgen Berges[2], Fred Jendrzejewski[1]

Observation of gauge invariance in a 71-site Bose–Hubbard quantum simulator 2020

Bing Yang, Hui Sun, Robert Ott, Han-Yi Wang, Torsten V. Zache, Jad C. Halimeh, Zhen-Sheng Yuan, Philipp Hauke & Jian-Wei Pan

Engineering a U(1) lattice gauge theory in classical electric circuits 2022

Hannes Riechert[1,2], Jad C. Halimeh,[3] Valentin Kasper,[4,5] Landry Bretheau,[2] Erez Zohar,[6] Philipp Hauke,[3] and Fred Jendrzejewski[1]

Thermalization dynamics of a gauge theory on a quantum simulator 2022

ZHAO-YU ZHOU, GUO-XIAN SU, JAD C. HALIMEH, ROBERT OTT, HUI SUN, PHILIPP HAUKE, BING YANG, ZHEN-SHENG YUAN, JÜRGEN BERGES, AND JIAN-WEI PAN

fewer Authors Info & Affiliations

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Q. How to make non-Abelian gauge theories accessible on quantum computer?

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fewer Authors Info & Affiliations

Framework: Hamiltonian Formalism

PHYSICAL REVIEW D VOLUME 11, NUMBER 2 15 JANUARY 1975

Hamiltonian formulation of Wilson's lattice gauge theories

John Kogut*
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

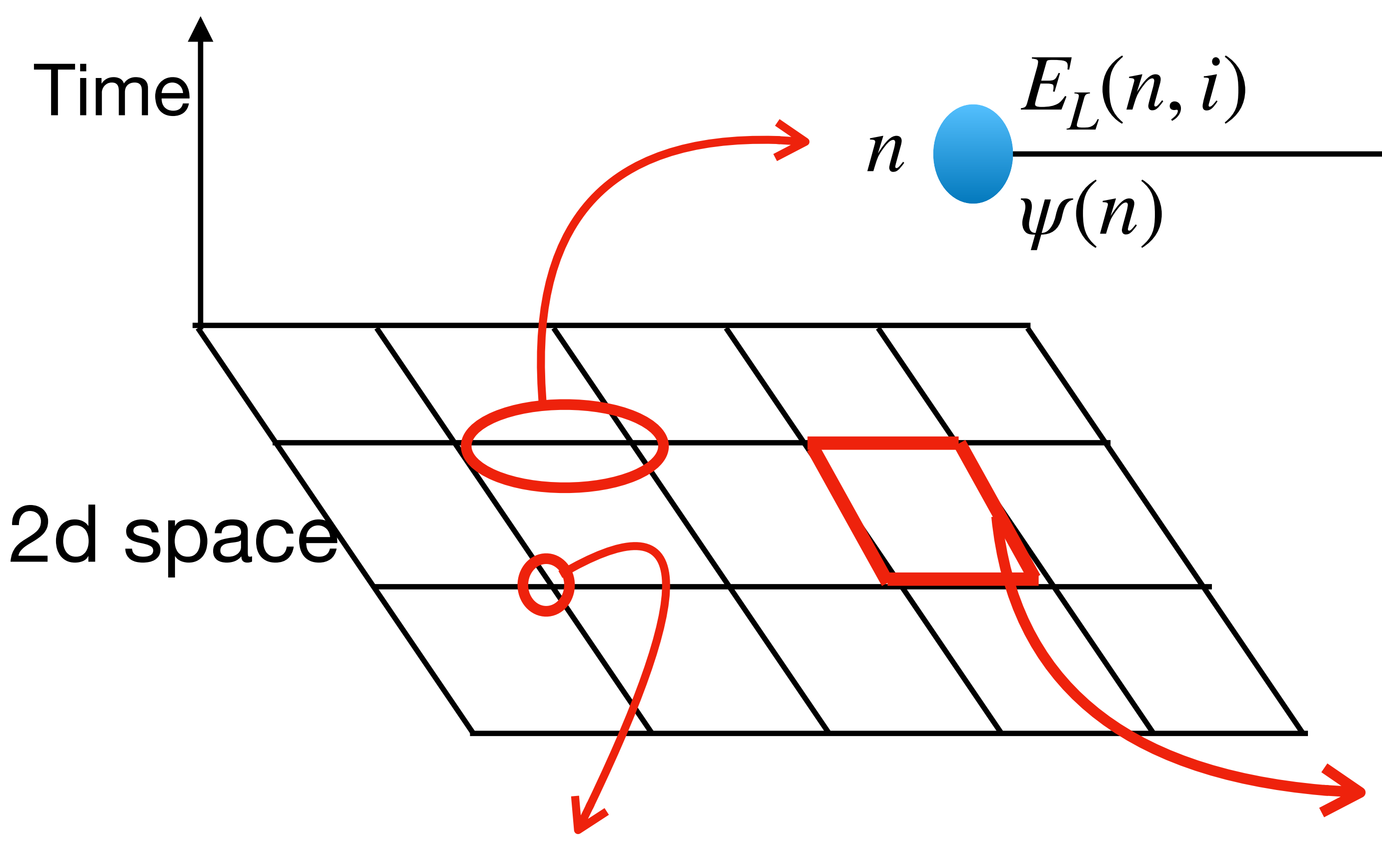
Leonard Susskind†
Belfer Graduate School of Science, Yeshiva University, New York, New York
and Tel Aviv University, Ramat Aviv, Israel
and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

(Received 9 July 1974)

Wilson's lattice gauge model is presented as a canonical Hamiltonian theory. The structure of the model is reduced to the interactions of an infinite collection of coupled rigid rotators. The gauge-invariant configuration space consists of a collection of strings with quarks at their ends. The strings are lines of non-Abelian electric flux. In the strong-coupling limit the dynamics is best described in terms of these strings. Quark confinement is a result of the inability to break a string without producing a pair.

Framework: Hamiltonian Formalism

Kogut-Susskind '74



n $E_L(n,i)$ $U(n,i)$ $E_R(n,i)$ $n+i$

$\psi(n)$ $\psi(n+i)$

$$[E_L, U] = U$$
$$[E_R, U] = -U$$
$$E_L^2 = E_R^2 = E^2$$
$$U_{plaquette} = U_1 U_2 U_3^\dagger U_4^\dagger$$

Gauss' law constraint:

$$G(n)|\Psi_{phys}\rangle = 0$$

$$[H, G(n)] = 0 \quad \forall n$$

$$G(n) = \sum_I \left[E_L(n,I) - E_R(n-I,I)\right] - \rho(n)$$

$$H = H_E + H_M + H_I + H_B$$

$$\frac{g^2 a}{2} \sum_{n,I} E^2(n,I)$$

$$m \sum (-1)^n \, \psi^\dagger(n)\psi(n)$$

Staggered fermion

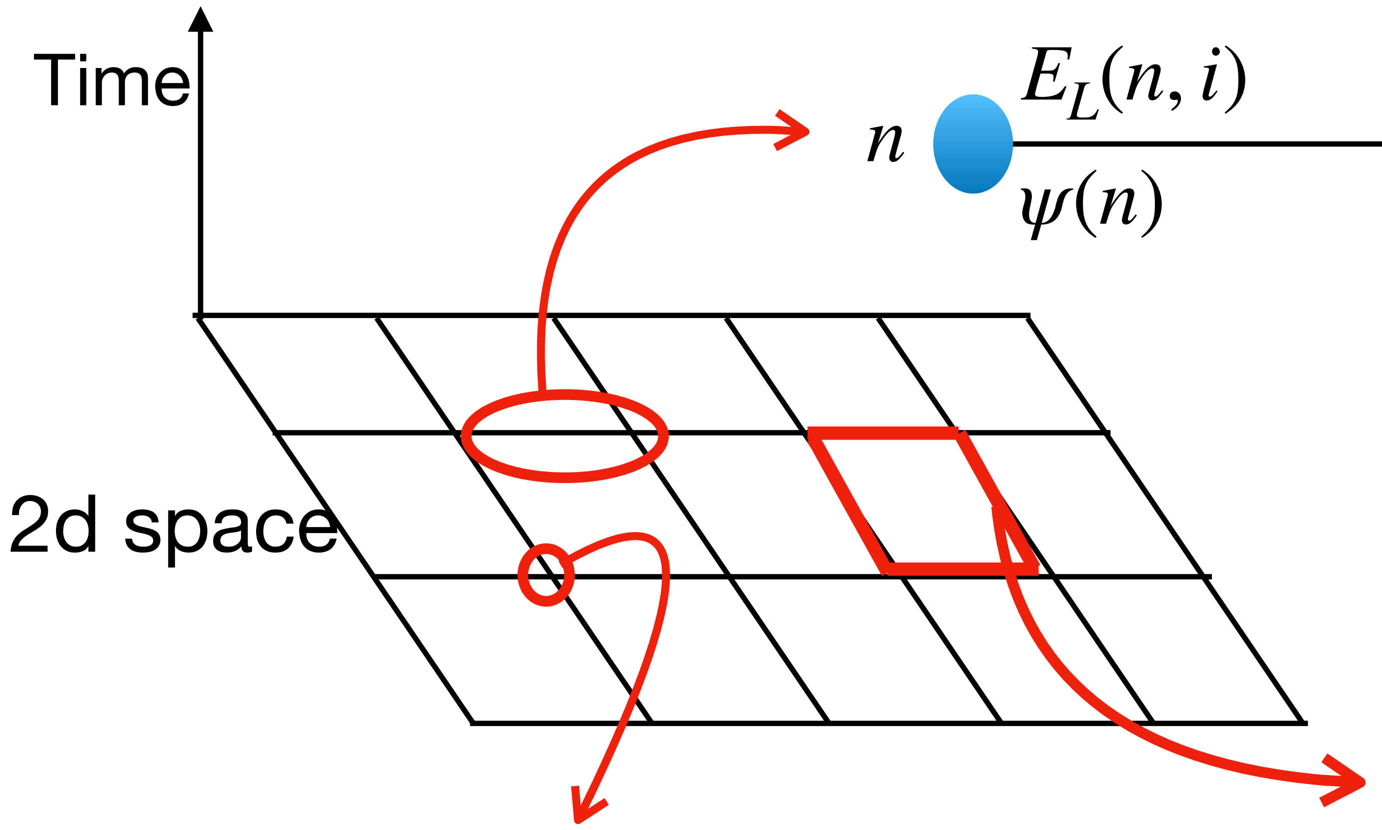
$$\frac{1}{2a} \sum_{n,I} (-1)^n \, \psi^\dagger(n) U(n,I) \psi(n+I)$$

$$\frac{2a}{g^2} \sum_{plaquettes} \left[\mathrm{Tr} U_{plaquette} + h.c\right]$$

Framework: Hamiltonian Formalism

Kogut-Susskind '74

$$n \quad \overset{E_L(n,i)}{\underset{\psi(n)}{\bullet}} \text{———} U(n,i) \text{———} \overset{E_R(n,i)}{\underset{\psi(n+i)}{\bullet}} \quad n+i$$



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$$H = H_E + H_M + H_I + H_B$$

$$H_E: \quad \frac{g^2 a}{2} \sum_{n,I} E^2(n,I)$$

$$H_M: \quad m \sum (-1)^n \, \psi^\dagger(n)\psi(n)$$

Staggered fermion

$$H_I: \quad \frac{1}{2a} \sum_{n,I} (-1)^n \, \psi^\dagger(n) U(n,I) \psi(n+I)$$

$$H_B: \quad \frac{2a}{g^2} \sum_{plaquettes} \left[\mathrm{Tr} U_{plaquette} + h.c\right]$$

U(1): $U(n,I) = e^{i\theta(n,I)}$

Schwinger Model : U(1) in 1+1d, H_B term absent

SU(2):

$$E \to E^a, \quad a = 1,2,3$$
$$U \to U_{\alpha\beta}, \quad \alpha, \beta = 1,2$$
$$\psi \to \psi_\alpha, \quad \alpha = 1,2$$
$$G(n) \to G^a(n) = \sum_I \left[E_L^a(n,I) + E_R^a(n-I,I)\right] + \psi(n)^\dagger \frac{\sigma^a}{2} \psi(n)$$

SU(3): $a = 1,2,3,\ldots,8.$

PHYSICAL REVIEW D **104,** 074505 (2021)

Search for efficient formulations for Hamiltonian simulation of non-Abelian lattice gauge theories

Zohreh Davoudi,[1,2] Indrakshi Raychowdhury,[1] and Andrew Shaw[1]

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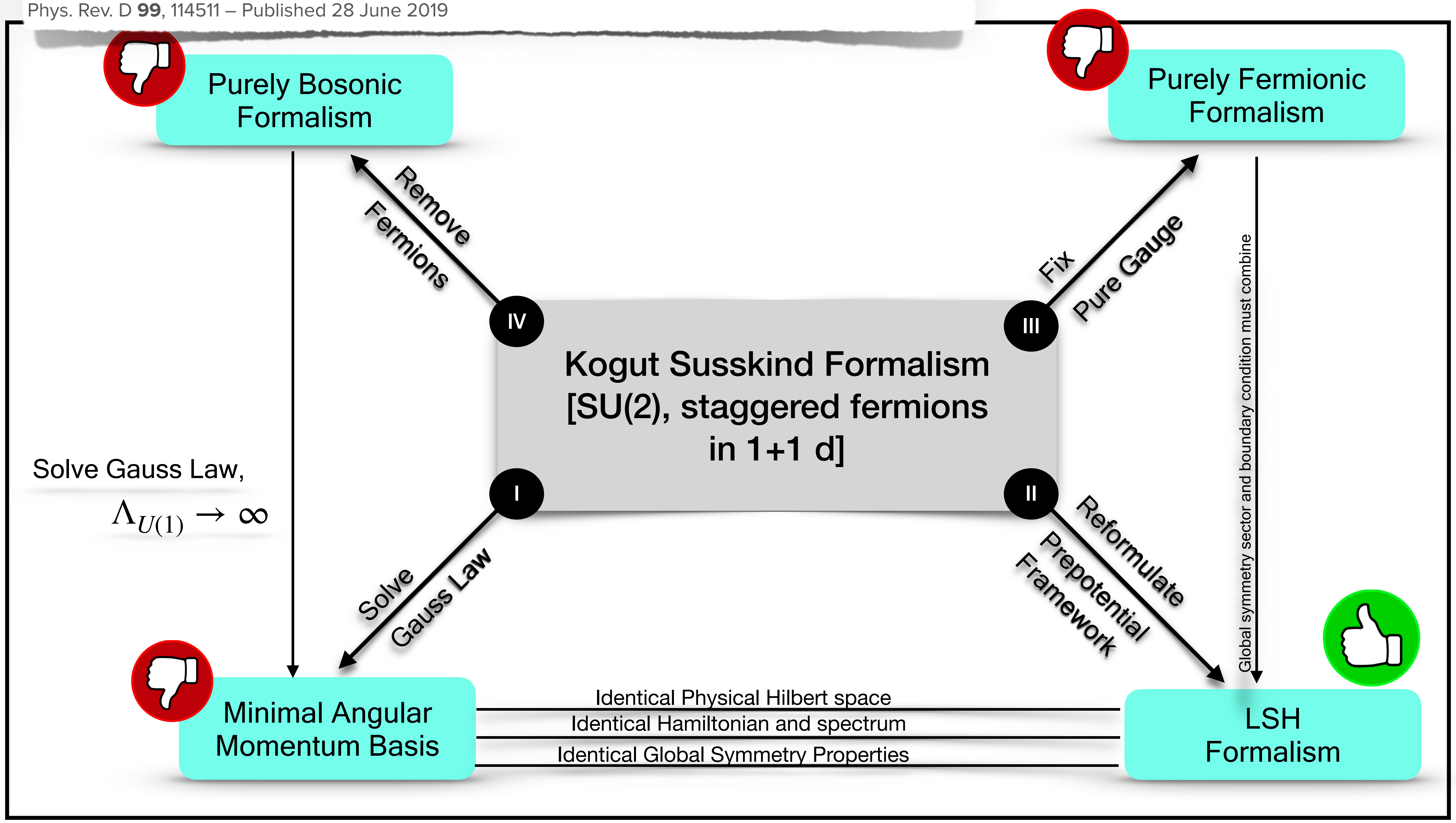
Removing staggered fermionic matter in $U(N)$ and $SU(N)$ lattice gauge theories

Erez Zohar and J. Ignacio Cirac
Phys. Rev. D **99**, 114511 – Published 28 June 2019



Another (also most popular) candidate:
Quantum Link Model

QCD as a quantum link model

R. Brower, S. Chandrasekharan, and U.-J. Wiese
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SU(2) rishon representation of gauge fields

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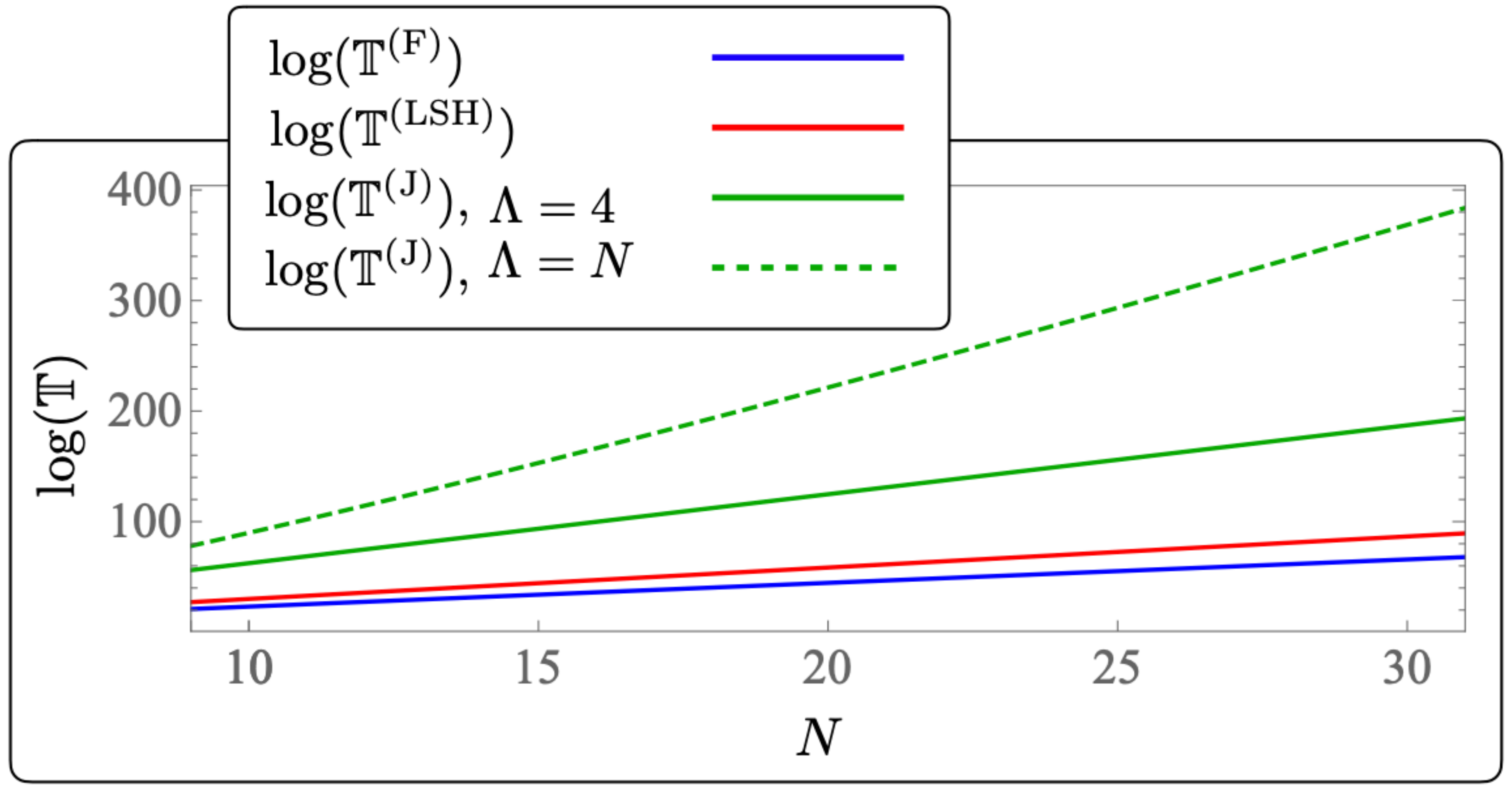

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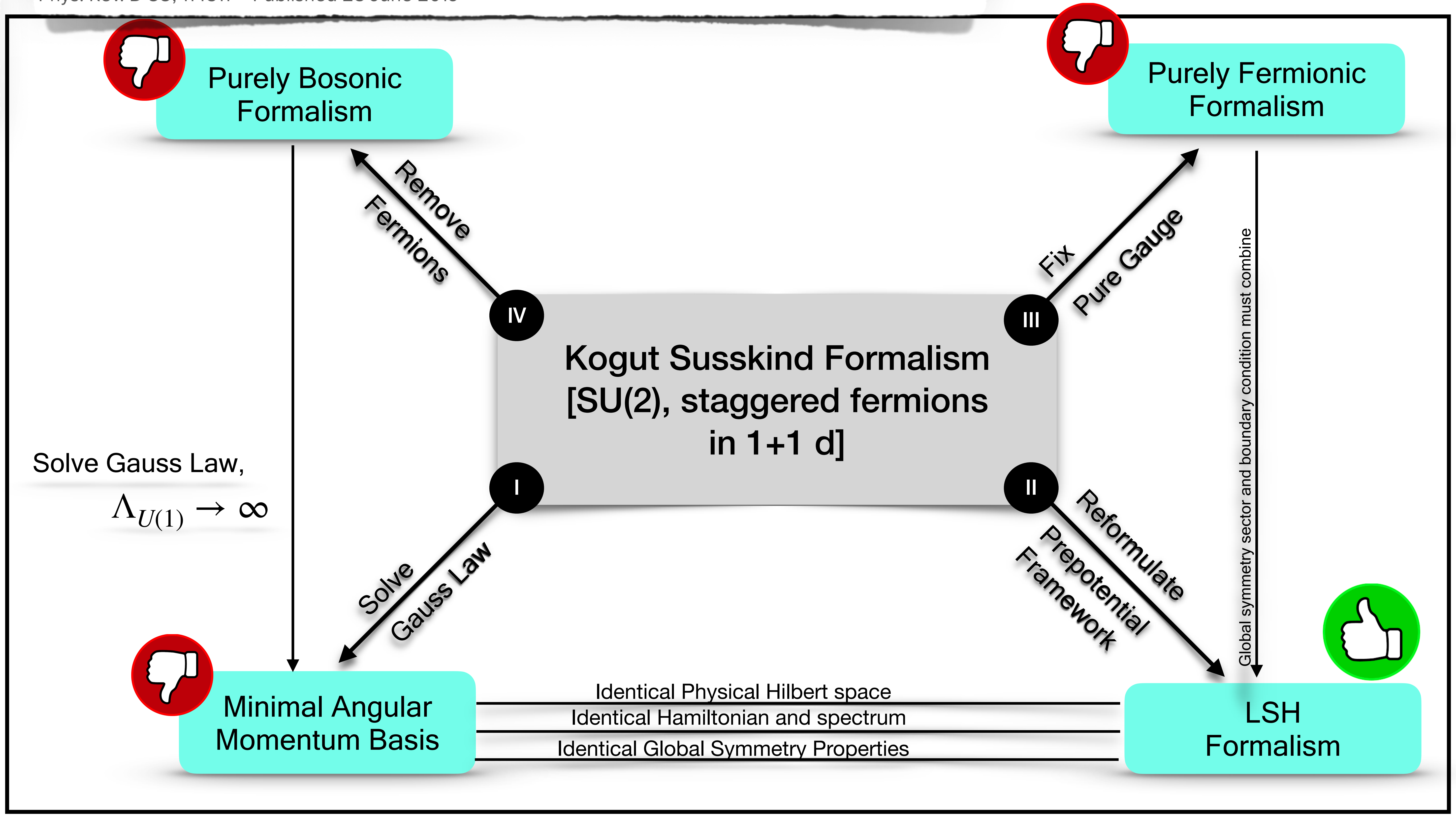

- LSH Hamiltonian dynamics
- No need to impose Gauss law constraint: Significant reduction in the cost of Hilbert space generation, 1-sparse basis.

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Collaborators:

Manu Mathur, SNBNCBS, India

Ref:
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JPA 2009; JPA 2010; JMP 2009; JMP 2010; JMP 2011
***IR**, PhD Thesis, 2014;*
*Ramesh Anishetty, **IR**, PRD 2014;*
***IR**, arXiv: 1507.07305; EPJC 2019;*

Ramesh Anishetty, IMSc, India

Reformulation of the original Kogut-Susskind Formalism in terms of Schwinger bosons

Formulated for SU(2), SU(3) and arbitrary SU(N)

Formulated for any dimension

GAUGE INVARIANCE + PROLIFERATION OF LOOP DEGREES OF FREEDOM

Describes dynamics of only physical degrees of freedom

Prepotential Formulation of Gauge Theories + Staggered fermions

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(former grad student at INT),
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postdoc at UMD

Ramesh Anishetty, IMSc, India

Reformulation of the original Kogut-Susskind Formalism in terms of Schwinger bosons

Formulated for SU(2), SU(3) and arbitrary SU(N)

Formulated for any dimension

GAUGE INVARIANCE + PROLIFERATION OF LOOP DEGREES OF FREEDOM

Describes dynamics of only physical degrees of freedom

Loop String Hadron (LSH) Formulation for SU(2) gauge theory

PHYSICAL REVIEW D **101,** 114502 (2020)

Loop, string, and hadron dynamics in SU(2) Hamiltonian lattice gauge theories

Indrakshi Raychowdhury[1,*] and Jesse R. Stryker[2,†]

Prepotential Formulation

$$\hat{E}^{\mathrm{a}}_{L/R} \equiv \hat{a}^\dagger(L/R) T^{\mathrm{a}} \hat{a}(L/R)$$

$$[\hat{E}^{\mathrm{a}}_L, \hat{E}^{\mathrm{b}}_L] = i\epsilon^{\mathrm{abc}} \hat{E}^{\mathrm{c}}_L,$$

$$[\hat{E}^{\mathrm{a}}_R, \hat{E}^{\mathrm{b}}_R] = i\epsilon^{\mathrm{abc}} \hat{E}^{\mathrm{c}}_R,$$

$$[\hat{E}^{\mathrm{a}}_L, \hat{E}^{\mathrm{b}}_R] = 0.$$

$$[\hat{E}^{\mathrm{a}}_L, \hat{U}] = -T^{\mathrm{a}} \hat{U},$$

$$[\hat{E}^{\mathrm{a}}_R, \hat{U}] = +\hat{U} T^{\mathrm{a}},$$

$$[\hat{U}_{\alpha\beta}, \hat{U}_{\gamma\delta}] = [\hat{U}_{\alpha\beta}, (\hat{U}_{\gamma\delta})^\dagger] = 0$$

$$\hat{U}_L \equiv \frac{1}{\sqrt{\hat{N}_L + 1}} \begin{pmatrix} \hat{a}^\dagger_2(L) & \hat{a}_1(L) \\ -\hat{a}^\dagger_1(L) & \hat{a}_2(L) \end{pmatrix},$$

$$\hat{U}_R \equiv \begin{pmatrix} \hat{a}^\dagger_1(R) & \hat{a}^\dagger_2(R) \\ -\hat{a}_2(R) & \hat{a}_1(R) \end{pmatrix} \frac{1}{\sqrt{\hat{N}_R + 1}}.$$

$$\hat{E}^2 \equiv \hat{E}^{\mathrm{a}}_L \hat{E}^{\mathrm{a}}_L = \hat{E}^{\mathrm{a}}_R \hat{E}^{\mathrm{a}}_R$$

$$\hat{N}_{L/R} = \hat{a}^\dagger(L/R) \cdot \hat{a}(L/R)$$

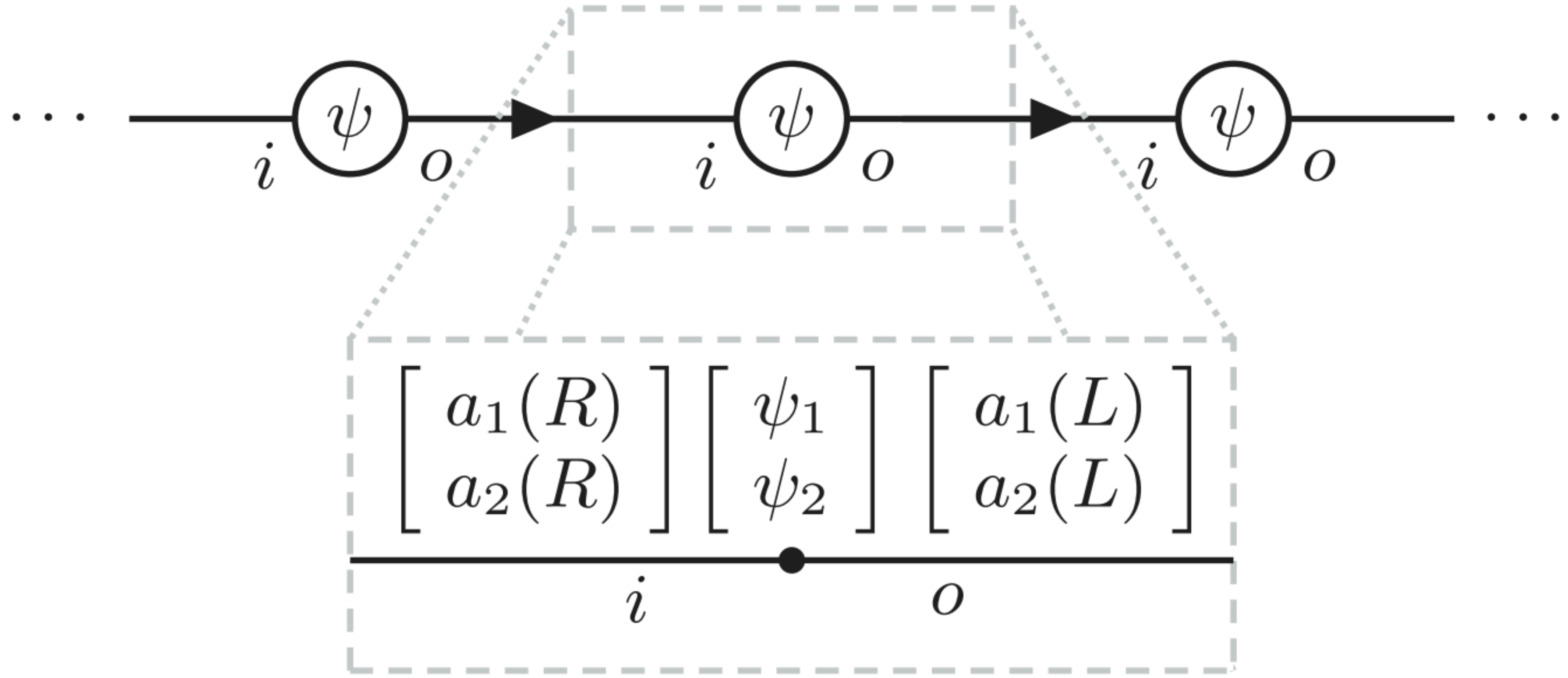
Abelian Gauss' Law

$$N_L(x) = N_R(x)$$

Local SU(2) Invariant Operators in 1d: loops-strings- hadrons

(i) *Pure gauge loop operators.*—$\mathcal{L}^{\sigma,\sigma'}$:

Gauge singlets constructed out of left and right bosons



(ii) *Incoming string operators.*—$\mathcal{S}_{\text{in}}^{\sigma,\sigma'}$:

Gauge singlets constructed out of left bosons and fermions

Outgoing string operators.—$\mathcal{S}_{\text{out}}^{\sigma,\sigma'}$:

Gauge singlets constructed out of Right boson and fermions

Hadron operators.—$\mathcal{H}^{\sigma,\sigma}$:

Gauge singlets constructed out of two fermions

$$(1/2)\mathcal{L}^{--}(\mathcal{S}_{\text{in}}^{++})^{n_i}(\mathcal{S}_{\text{out}}^{++})^{n_o}|0\rangle = \delta_{n_i,1}\delta_{n_o,1}\mathcal{H}^{++}|0\rangle$$

Local SU(2) Invariant Operators in 1d: loops-strings- hadrons

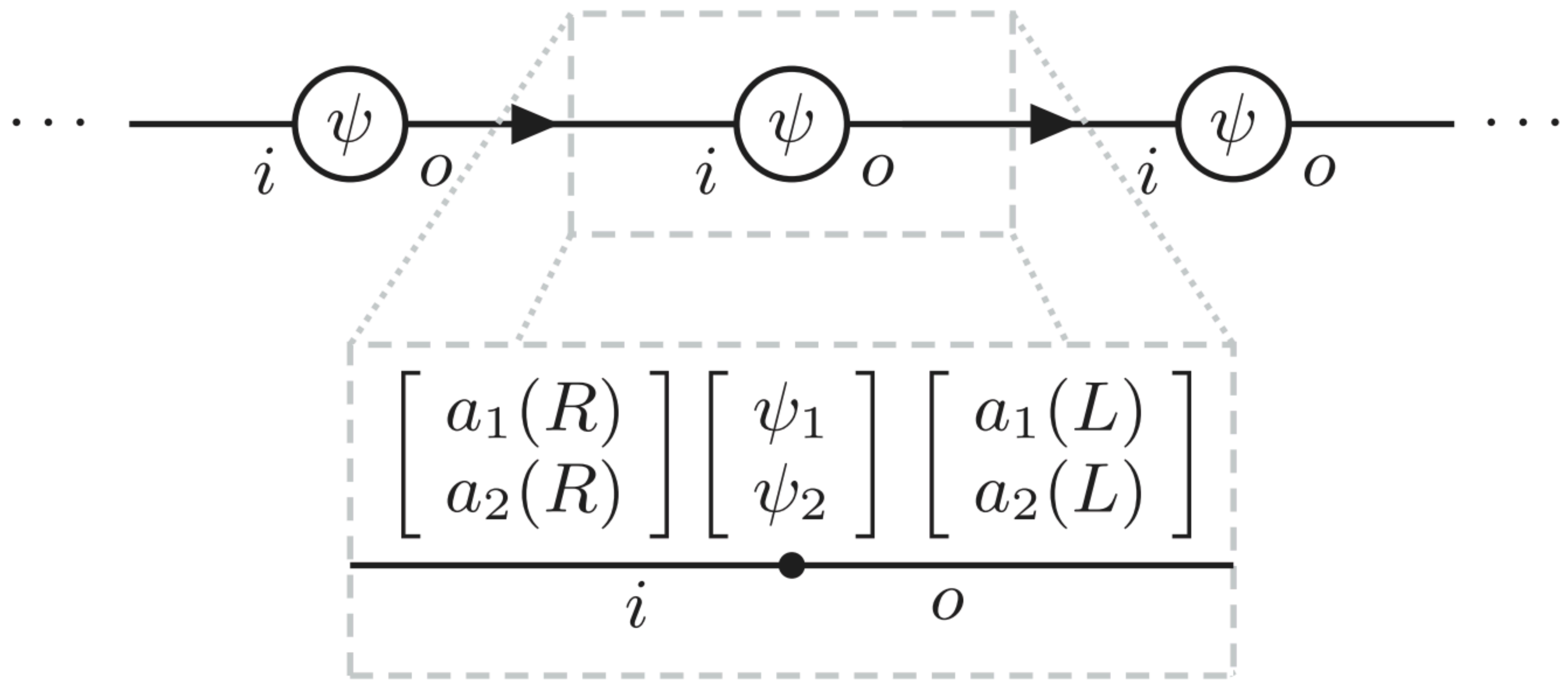
(i) *Pure gauge loop operators.*—$\mathcal{L}^{\sigma,\sigma'}$:

$$\mathcal{L}^{++} = a(R)^{\dagger}_{\alpha} a(L)^{\dagger}_{\beta} \epsilon_{\alpha\beta}$$

$$\mathcal{L}^{--} = a(R)_{\alpha} a(L)_{\beta} \epsilon_{\alpha\beta} = (\mathcal{L}^{++})^{\dagger}$$

$$\mathcal{L}^{+-} = a(R)^{\dagger}_{\alpha} a(L)_{\beta} \delta_{\alpha\beta}$$

$$\mathcal{L}^{-+} = a(R)_{\alpha} a(L)^{\dagger}_{\beta} \delta_{\alpha\beta} = (\mathcal{L}^{+-})^{\dagger}.$$



(ii) *Incoming string operators.*—$\mathcal{S}^{\sigma,\sigma'}_{\text{in}}$:

$$\mathcal{S}^{++}_{\text{in}} = a(R)^{\dagger}_{\alpha} \psi^{\dagger}_{\beta} \epsilon_{\alpha\beta}$$

$$\mathcal{S}^{--}_{\text{in}} = a(R)_{\alpha} \psi_{\beta} \epsilon_{\alpha\beta} = (\mathcal{S}^{++}_{\text{in}})^{\dagger}$$

$$\mathcal{S}^{+-}_{\text{in}} = a(R)^{\dagger}_{\alpha} \psi_{\beta} \delta_{\alpha\beta}$$

$$\mathcal{S}^{-+}_{\text{in}} = a(R)_{\alpha} \psi^{\dagger}_{\beta} \delta_{\alpha\beta} = (\mathcal{S}^{+-}_{\text{in}})^{\dagger}.$$

Outgoing string operators.—$\mathcal{S}^{\sigma,\sigma'}_{\text{out}}$:

$$\mathcal{S}^{++}_{\text{out}} = \psi^{\dagger}_{\alpha} a(L)^{\dagger}_{\beta} \epsilon_{\alpha\beta}$$

$$\mathcal{S}^{--}_{\text{out}} = \psi_{\alpha} a(L)_{\beta} \epsilon_{\alpha\beta} = (\mathcal{S}^{++}_{\text{out}})^{\dagger}$$

$$\mathcal{S}^{+-}_{\text{out}} = \psi^{\dagger}_{\alpha} a(L)_{\beta} \delta_{\alpha\beta}$$

$$\mathcal{S}^{-+}_{\text{out}} = \psi_{\alpha} a(L)^{\dagger}_{\beta} \delta_{\alpha\beta} = (\mathcal{S}^{+-}_{\text{out}})^{\dagger}.$$

Hadron operators.—$\mathcal{H}^{\sigma,\sigma}$:

$$\mathcal{H}^{++} = -\frac{1}{2!} \psi^{\dagger}_{\alpha} \psi^{\dagger}_{\beta} \epsilon_{\alpha\beta}$$

$$\mathcal{H}^{--} = \frac{1}{2!} \psi_{\alpha} \psi_{\beta} \epsilon_{\alpha\beta} = (\mathcal{H}^{++})^{\dagger}.$$

$$(1/2)\mathcal{L}^{--}(\mathcal{S}^{++}_{\text{in}})^{n_i}(\mathcal{S}^{++}_{\text{out}})^{n_o}|0\rangle = \delta_{n_i,1}\delta_{n_o,1}\mathcal{H}^{++}|0\rangle$$

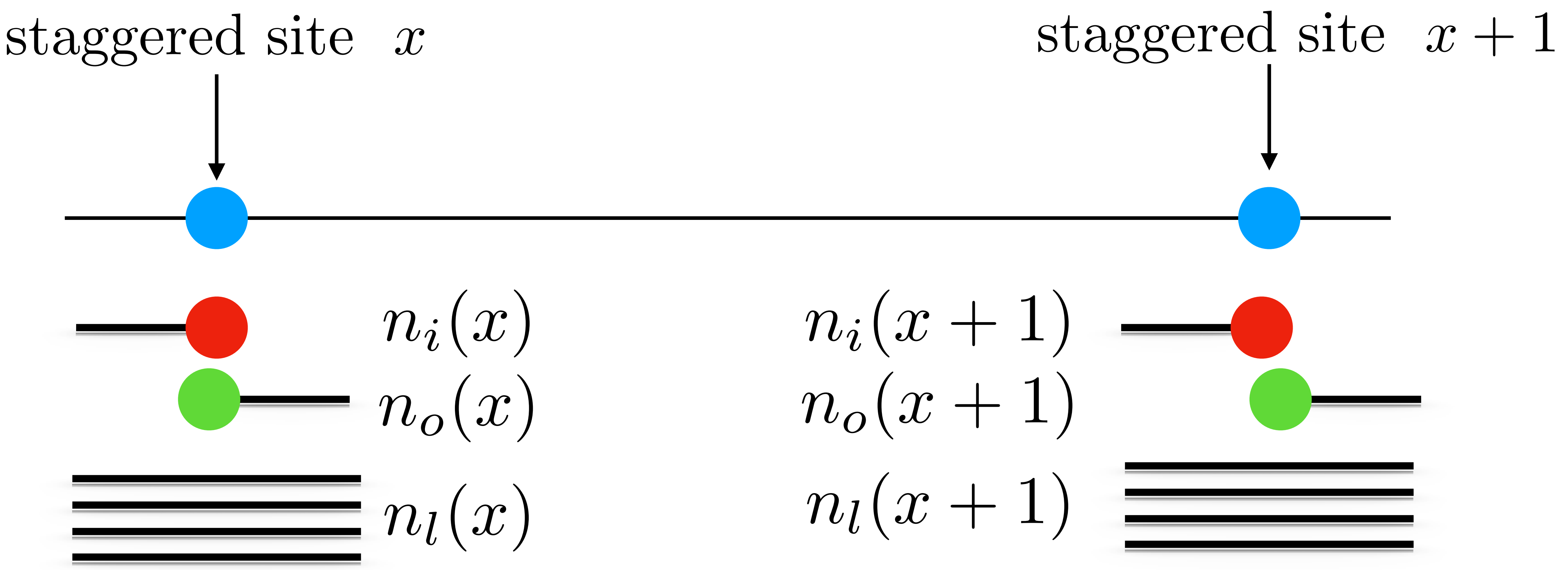
LSH Formulation: local LSH basis for SU(2) in 1+1 dimension

At each site define: $n_l(x), n_i(x), n_o(x)$,

$$|n_l, n_i, n_o\rangle = \left(\mathscr{L}^{++}\right)^{n_l}\left(\mathcal{S}_i^{++}\right)^{n_i}\left(\mathcal{S}_o^{++}\right)^{n_o}|0\rangle$$

$$0 \leq n_l(x) \leq \infty,$$
$$n_i(x) \in \{0,1\},$$
$$n_o(x) \in \{0,1\}.$$

Abelian weaving along the link: $\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x)) = \hat{n}_l(x+1) + \hat{n}_i(x+1)(1-\hat{n}_o(x+1))$



LSH Formulation: key ingredients

Local gauge invariant Hilbert space

Local constraint on each link: Abelian Gauss' law

LSH Formulation: key ingredients

Generalization to QCD

First attempt: SU(3) gauge theory in 1+1 dimension

Local gauge invariant Hilbert space

Local constraint on each link: Abelian Gauss' law

Starting point: Prepotential formulation of SU(3) gauge theory

Prepotential formulation of SU(3) gauge theory

Ramesh Anishetty, Manu Mathur, IR, (2009), (2010)

Not a trivial generalisation of SU(2)

Kogut-Susskind Formulation

Schwinger boson representation of SU(3)

SU(3): rank 2

Fundamental and anti-fundamental representation:
Two Schwinger boson triplets $a^\dagger_\alpha, b^{\dagger\alpha}$

$\equiv$

Prepotential formulation for SU(3)

$A^\dagger_\alpha, B^{\dagger\alpha}$: Irreducible Schwinger boson representation of SU(3)
$A^\dagger_\alpha \cdot B^{\dagger\alpha} \approx 0$

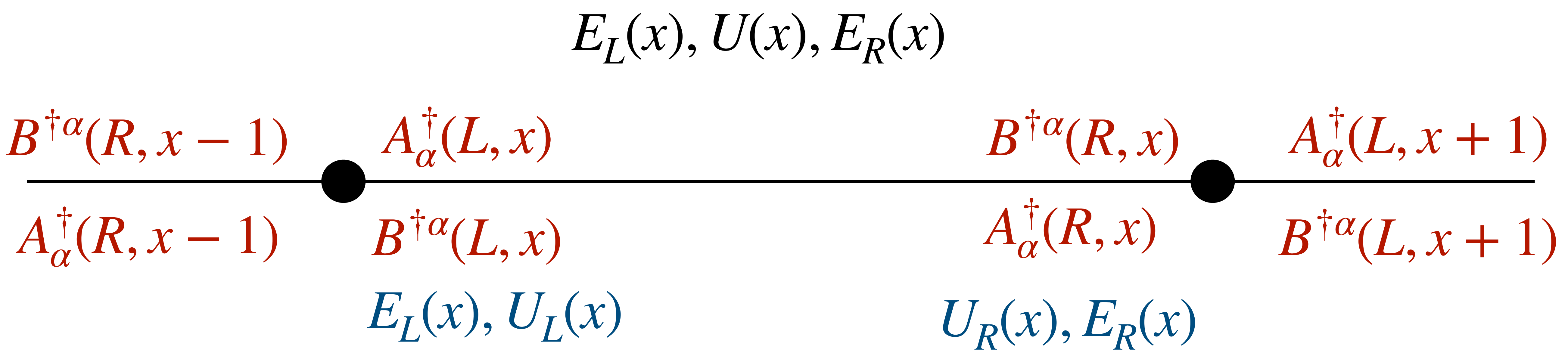
Solved

Multiplicity problem caused by: $\sum_{\alpha=1}^{3} a^\dagger_\alpha b^{\dagger\alpha} \Rightarrow \equiv a^\dagger \cdot b^\dagger$

SU(3) singlets

Chaturvedi and Mukunda J. Math. Phys. 43, 5262 (2002)

Prepotential formulation of SU(3) gauge theory



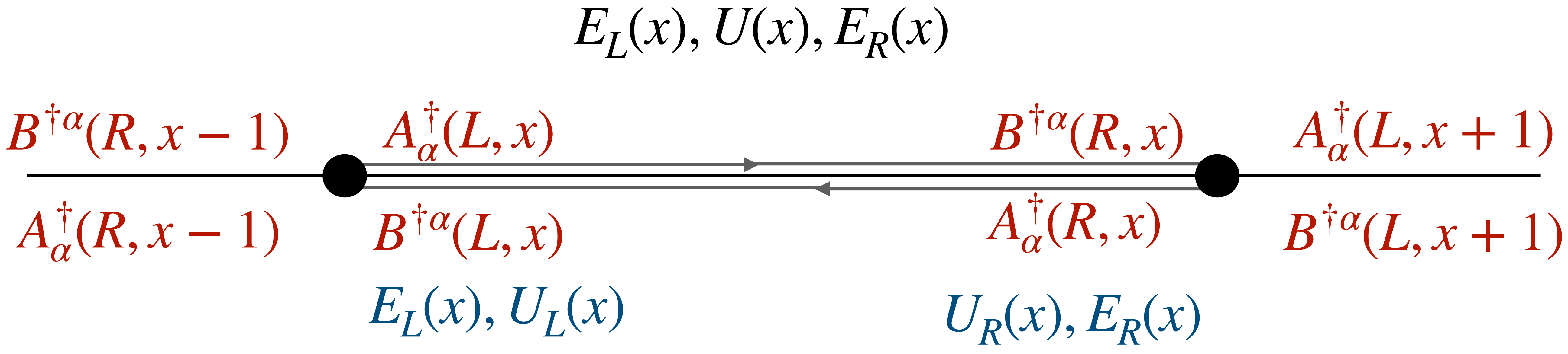
Abelian Gauss' Law

$$\boxed{\begin{array}{c} N_A(L,x) = N_B(R,x) \\ N_B(L,x) = N_A(R,x) \end{array}}$$

Imposes continuity of the flux lines

Directed flow of electric flux on a link: From triplet to anti-triplet

Prepotential formulation of SU(3) gauge theory



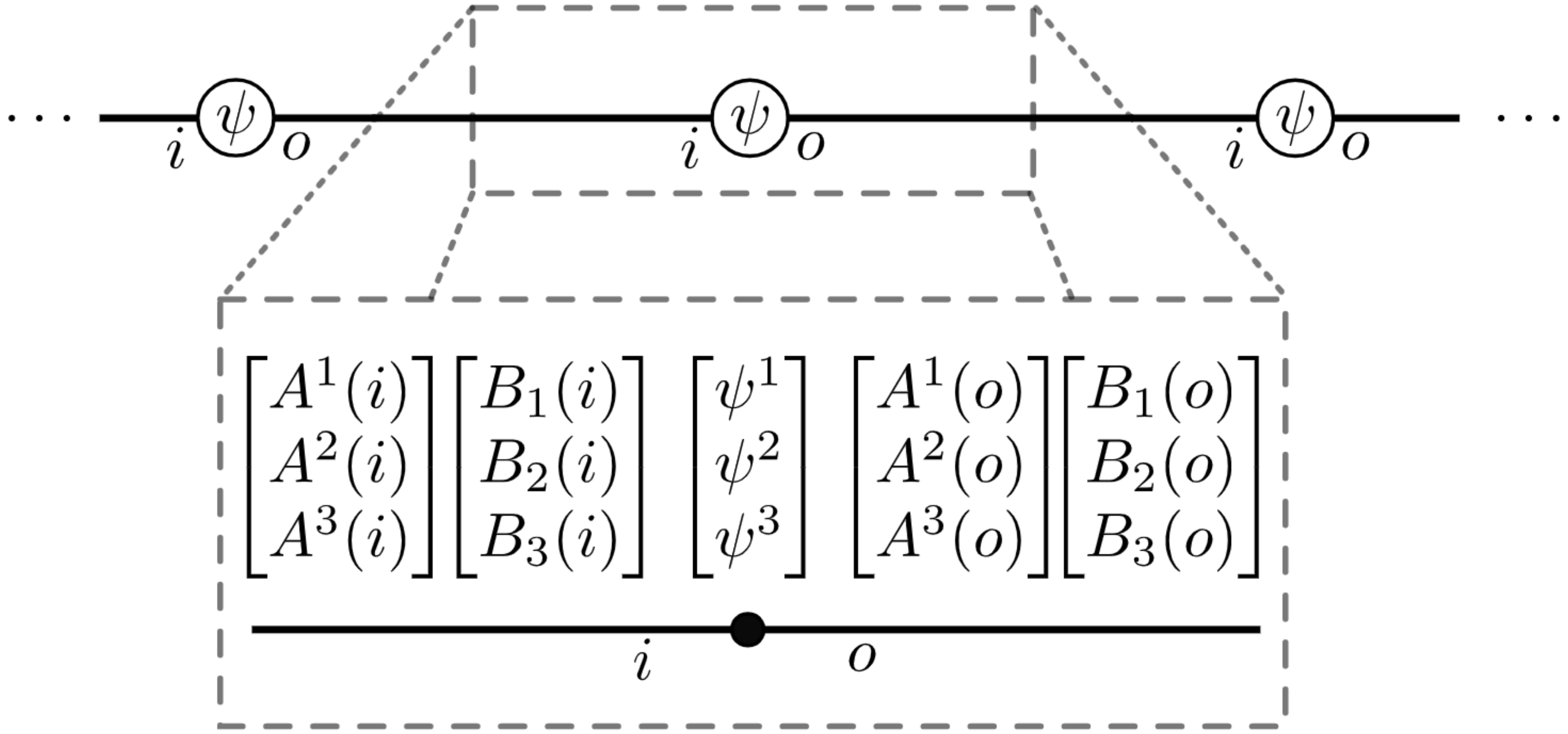
Abelian Gauss' Law

$$\boxed{\begin{gathered} N_A(L, x) = N_B(R, x) \\ N_B(L, x) = N_A(R, x) \end{gathered}}$$

Imposes continuity of the flux lines

Directed flow of electric flux on a link: From triplet to anti-triplet

Loop-String-Hadron formulation of SU(3) gauge theory



Collaborators:

Jesse Stryker

Saurabh Kadam

Local ingredients:

$\underline{3}$	$\underline{3^*}$
$A^\dagger_\alpha(i)$	$A^\alpha(i)$
$A^\dagger_\alpha(o)$	$A^\alpha(o)$
$B_\alpha(i)$	$B^{\dagger\alpha}(i)$
$B_\alpha(o)$	$B^{\dagger\alpha}(o)$
$\psi^\dagger_\alpha$	ψ^α

Singlets can be formed using:

$\delta^\alpha{}_\beta \equiv \cdot$ $\quad\quad$ $\epsilon^{\alpha\beta\gamma}$ or $\epsilon_{\alpha\beta\gamma} \equiv \wedge$

Loop-String-Hadron basis: onsite SU(3) invariant basis

Local ingredients:

$\underline{3}$	$\underline{3^*}$
$A^\dagger_\alpha(i)$	$A^\alpha(i)$
$A^\dagger_\alpha(o)$	$A^\alpha(o)$
$B_\alpha(i)$	$B^{\dagger\alpha}(i)$
$B_\alpha(o)$	$B^{\dagger\alpha}(o)$
$\psi^\dagger_\alpha$	ψ^α

Singlets can be formed using:

$\delta^\alpha{}_\beta \equiv \cdot$ $\qquad$ $\epsilon^{\alpha\beta\gamma}$ or $\epsilon_{\alpha\beta\gamma} \equiv \wedge$

Bosonic: $\quad A^\dagger(i) \cdot B^\dagger(o)$

$B^\dagger(i) \cdot A^\dagger(o)$

Fermionic + Bosonic : $\quad \psi^\dagger \cdot B^\dagger(i)$

$\psi^\dagger \cdot B^\dagger(o)$

$\psi^\dagger \cdot \left(A^\dagger(i) \wedge A^\dagger(o)\right)$

$\psi^\dagger \cdot \left(\psi^\dagger \wedge A^\dagger(i)\right)$

$\psi^\dagger \cdot \left(\psi^\dagger \wedge B^\dagger(o)\right)$

Fermionic: $\quad \psi^\dagger \cdot \left(\psi^\dagger \wedge \psi^\dagger\right)$

LSH state:

$$|n_P, n_Q, \nu_i, \nu_m, \nu_o\rangle = \mathcal{N}^{n_P, n_Q}_{\nu_i, \nu_m, \nu_o} \left(A^\dagger(i) \cdot B^\dagger(o)\right)^{n_P} \left(B^\dagger(i) \cdot A^\dagger(o)\right)^{n_Q} |0, 0, \nu_i, \nu_m, \nu_o\rangle$$

$\underline{3}$	$\underline{3^*}$
$A^\dagger_\alpha(i)$	$A^\alpha(i)$
$A^\dagger_\alpha(o)$	$A^\alpha(o)$
$B_\alpha(i)$	$B^{\dagger\alpha}(i)$
$B_\alpha(o)$	$B^{\dagger\alpha}(o)$
$\psi^\dagger_\alpha$	ψ^α

LSH state:

$$|n_P, n_Q, \nu_i, \nu_m, \nu_o\rangle = \mathcal{N}^{n_P, n_Q}_{\nu_i, \nu_m, \nu_o} \left(A^\dagger(i)\cdot B^\dagger(o)\right)^{n_P} \left(B^\dagger(i)\cdot A^\dagger(o)\right)^{n_Q} |0, 0, \nu_i, \nu_m, \nu_o\rangle$$

$\underline{3}$	$\underline{3^*}$
$A^\dagger_\alpha(i)$	$A^\alpha(i)$
$A^\dagger_\alpha(o)$	$A^\alpha(o)$
$B_\alpha(i)$	$B^{\dagger\alpha}(i)$
$B_\alpha(o)$	$B^{\dagger\alpha}(o)$
$\psi^\dagger_\alpha$	ψ^α

Bosonic:

$\left[A^\dagger(i)\cdot B^\dagger(o)\right]^{n_P} \rightarrow$

$\left[A^\dagger(o)\cdot B^\dagger(i)\right]^{n_Q} \rightarrow$

$|\Omega\rangle \rightarrow$

Fermionic + Bosonic :

$\psi^\dagger \cdot B^\dagger(o)|\Omega\rangle \rightarrow$

$\psi^\dagger \cdot A^\dagger(i) \wedge A^\dagger(o)|\Omega\rangle \rightarrow$

$\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(o)|\Omega\rangle \rightarrow$

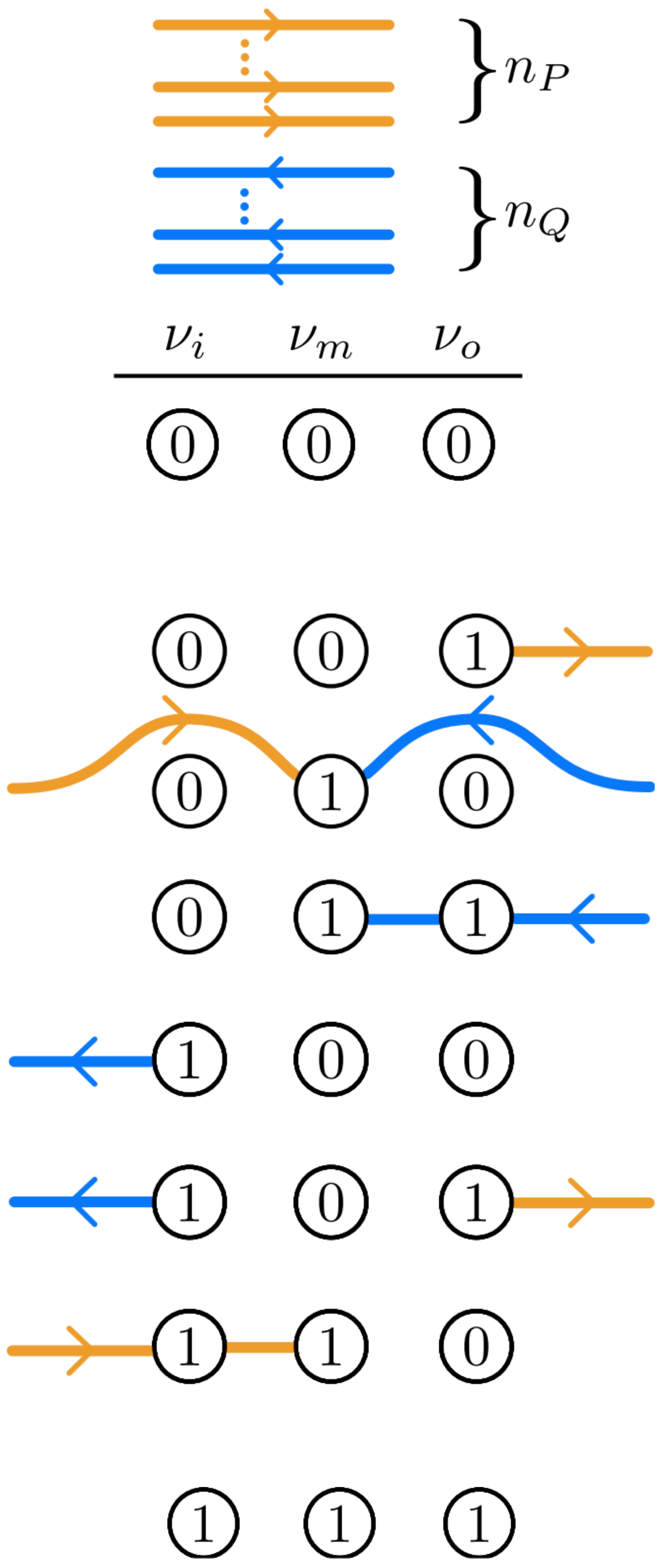
$\psi^\dagger \cdot B^\dagger(i)|\Omega\rangle \rightarrow$

$\psi^\dagger \cdot B^\dagger(i)\ \psi^\dagger \cdot B^\dagger(o)|\Omega\rangle \rightarrow$

$\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(i)|\Omega\rangle \rightarrow$

Fermionic:

$\psi^\dagger \cdot \psi^\dagger \wedge \psi^\dagger|\Omega\rangle \rightarrow$



Loop-String-Hadron basis: Pictorial Representation

LSH state:

$$|n_P, n_Q, \nu_i, \nu_m, \nu_o\rangle = \mathcal{N}^{n_P,n_Q}_{\nu_i,\nu_m,\nu_o} \left(A^\dagger(i)\cdot B^\dagger(o)\right)^{n_P} \left(B^\dagger(i)\cdot A^\dagger(o)\right)^{n_Q} |0,0,\nu_i,\nu_m,\nu_o\rangle$$

$\underline{3}$	$\underline{3^*}$
$A^\dagger_\alpha(i)$	$A^\alpha(i)$
$A^\dagger_\alpha(o)$	$A^\alpha(o)$
$B_\alpha(i)$	$B^{\dagger\alpha}(i)$
$B_\alpha(o)$	$B^{\dagger\alpha}(o)$
$\psi^\dagger_\alpha$	ψ^α

Bosonic:

$[A^\dagger(i)\cdot B^\dagger(o)]^{n_P} \rightarrow$

$[A^\dagger(o)\cdot B^\dagger(i)]^{n_Q} \rightarrow$

$|\Omega\rangle \rightarrow$

Fermionic + Bosonic :

$\psi^\dagger \cdot B^\dagger(o)|\Omega\rangle \rightarrow$

$\psi^\dagger \cdot A^\dagger(i) \wedge A^\dagger(o)|\Omega\rangle \rightarrow$

$\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(o)|\Omega\rangle \rightarrow$

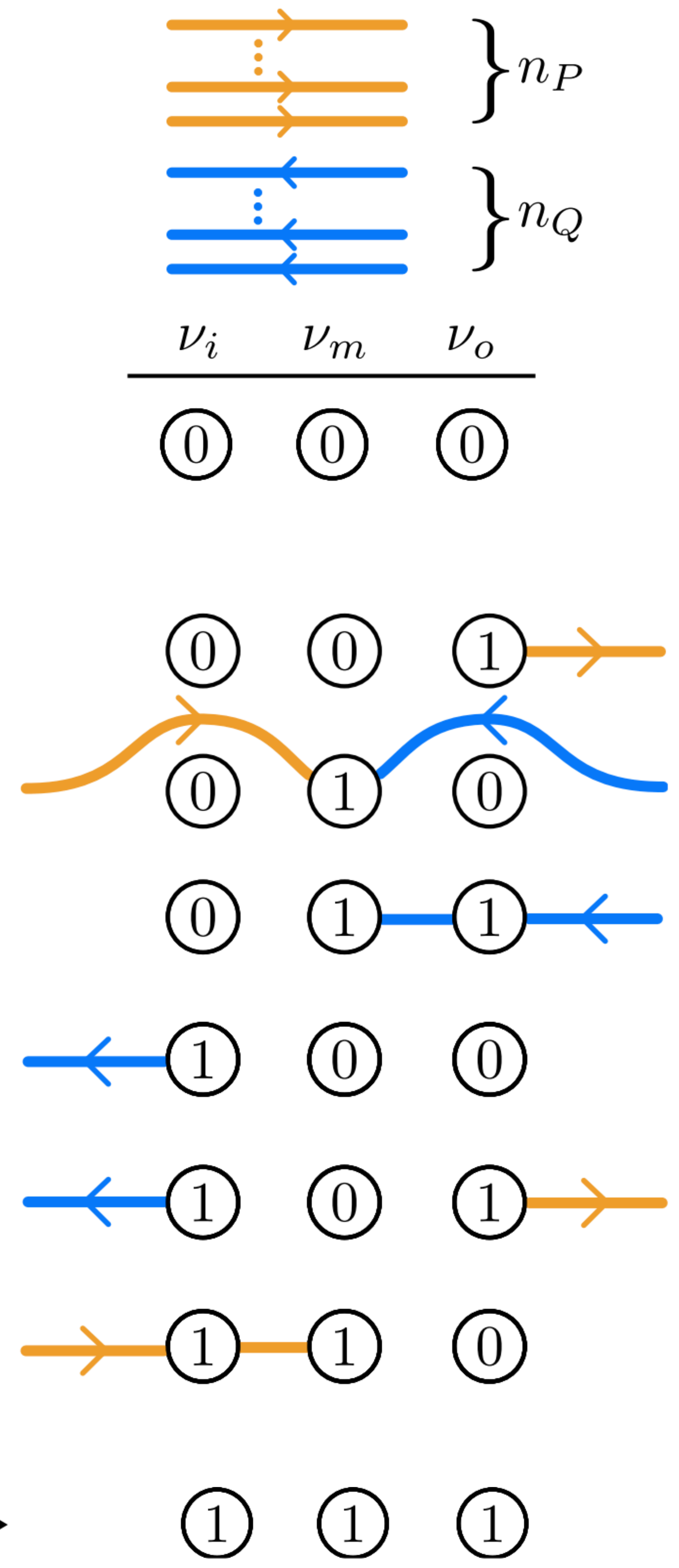
$\psi^\dagger \cdot B^\dagger(i)|\Omega\rangle \rightarrow$

$\psi^\dagger \cdot B^\dagger(i)\ \psi^\dagger \cdot B^\dagger(o)|\Omega\rangle \rightarrow$

$\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(i)|\Omega\rangle \rightarrow$

Fermionic:

$\psi^\dagger \cdot \psi^\dagger \wedge \psi^\dagger|\Omega\rangle \rightarrow$



Snapshots of loops-strings-hadron configurations at each site

We further need to weave these along links

↓

Abelian Gauss laws

Loop-String-Hadron basis: Pictorial Representation

Local LSH state:

$$\boxed{|n_P, n_Q, \nu_i, \nu_m, \nu_o\rangle}$$

$$n_P,\ n_Q = 0,\ 1,\ 2,\ \cdots$$
$$\nu_i,\ \nu_m,\ \nu_o = 0, 1$$

Abelian Gauss laws

$$\boxed{Q_o(r) = P_i(r+1) \quad \& \quad P_o(r) = Q_i(r+1)}$$

$$\begin{aligned} n_Q(r) + \nu_m(r)\,(1-\nu_i(r)) &= n_P(r+1) + \nu_m(r+1)\,(1-\nu_o(r+1)) \\ n_P(r) + \nu_o(r)\,(1-\nu_m(r)) &= n_Q(r+1) + \nu_i(r+1)\,(1-\nu_m(r+1)) \end{aligned}$$

LSH Formulation: key ingredients for SU(3) in 1+1 dimension

Local LSH state:

$$\boxed{|n_P, n_Q, \nu_i, \nu_m, \nu_o\rangle}$$

$$n_P,\, n_Q = 0,\ 1,\ 2,\ \cdots$$
$$\nu_i,\, \nu_m,\, \nu_o = 0, 1$$

Abelian Gauss laws

$$\boxed{Q_o(r) = P_i(r+1) \quad \& \quad P_o(r) = Q_i(r+1)}$$

$$\begin{aligned} n_Q(r) + \nu_m(r)\,(1-\nu_i(r)) &= n_P(r+1) + \nu_m(r+1)\,(1-\nu_o(r+1)) \\ n_P(r) + \nu_o(r)\,(1-\nu_m(r)) &= n_Q(r+1) + \nu_i(r+1)\,(1-\nu_m(r+1)) \end{aligned}$$

Towards building the LSH Hamiltonian

Kogut-Susskind Hamiltonian → Irreducible Schwinger boson representation of SU(3) coupled to on-site staggered fermions → Local LSH operators weaved together by the Abelian Gauss law

Local building blocks

One-quark operators

$\psi^\dagger \cdot B(0)^\dagger \equiv \widehat{o}$	$\psi^\dagger \cdot B(i) \wedge A(i)^\dagger \equiv \widehat{m}$
$\psi \cdot B(o) \equiv \widehat{o}$	$\psi \cdot A(i) \wedge B(i)^\dagger \equiv \widehat{m}$
$\psi^\dagger \cdot B(i)^\dagger \equiv \widehat{i}$	$\psi^\dagger \cdot B(o) \wedge A(o)^\dagger \equiv \widehat{m}$
$\psi \cdot B(i) \equiv \widehat{i}$	$\psi \cdot A(o) \wedge B(o)^\dagger \equiv \widehat{m}$
$\psi^\dagger \cdot A(o) \equiv \widehat{i}$	$\psi^\dagger \cdot A(i)^\dagger \wedge A(o)^\dagger \equiv \widehat{m}$
$\psi \cdot A(o)^\dagger \equiv \widehat{i}$	$\psi \cdot A(i) \wedge A(o) \equiv \widehat{m}$
$\psi^\dagger \cdot A(i) \equiv \widehat{o}$	
$\psi \cdot A(i)^\dagger \equiv \widehat{o}$	

Purely bosonic operators

$A(i)^\dagger \cdot B(o)^\dagger \equiv$	$A(i) \cdot B(o) \equiv$
$B(i)^\dagger \cdot A(o)^\dagger \equiv$	$B(i) \cdot A(o) \equiv$
$A(i)^\dagger \cdot A(o) \equiv$	$A(i) \cdot A(o)^\dagger \equiv$

Two-quark operators

$\psi^\dagger \cdot \psi^\dagger \wedge A(o)^\dagger \equiv \widehat{m}\widehat{o}$	$\psi \cdot \psi \wedge A(o) \equiv \widehat{m}\widehat{o}$
$\psi^\dagger \cdot \psi^\dagger \wedge A(i)^\dagger \equiv \widehat{i}\widehat{m}$	$\psi \cdot \psi \wedge A(i) \equiv \widehat{i}\widehat{m}$

Three-quark operators

$\psi^\dagger \cdot \psi^\dagger \wedge \psi^\dagger \equiv \widehat{i}\widehat{m}\widehat{o}$	$\psi \cdot \psi \wedge \psi \equiv \widehat{i}\widehat{m}\widehat{o}$

One-quark operators

$\psi^\dagger \cdot B(0)^\dagger \equiv \widehat{o}$	$\psi^\dagger \cdot B(i) \wedge A(i)^\dagger \equiv \widehat{m}$
$\psi \cdot B(o) \equiv \widehat{o}$	$\psi \cdot A(i) \wedge B(i)^\dagger \equiv \widehat{m}$
$\psi^\dagger \cdot B(i)^\dagger \equiv \widehat{i}$	$\psi^\dagger \cdot B(o) \wedge A(o)^\dagger \equiv \widehat{m}$
$\psi \cdot B(i) \equiv \widehat{i}$	$\psi \cdot A(o) \wedge B(o)^\dagger \equiv \widehat{m}$
$\psi^\dagger \cdot A(o) \equiv \widehat{i}$	$\psi^\dagger \cdot A(i)^\dagger \wedge A(o)^\dagger \equiv \widehat{m}$
$\psi \cdot A(o)^\dagger \equiv \widehat{i}$	$\psi \cdot A(i) \wedge A(o) \equiv \widehat{m}$
$\psi^\dagger \cdot A(i) \equiv \widehat{o}$	
$\psi \cdot A(i)^\dagger \equiv \widehat{o}$	

Purely bosonic operators

$A(i)^\dagger \cdot B(o)^\dagger \equiv$	$A(i) \cdot B(o) \equiv$
$B(i)^\dagger \cdot A(o)^\dagger \equiv$	$B(i) \cdot A(o) \equiv$
$A(i)^\dagger \cdot A(o) \equiv$	$A(i) \cdot A(o)^\dagger \equiv$

Two-quark operators

$\psi^\dagger \cdot \psi^\dagger \wedge A(o)^\dagger \equiv \widehat{m}\widehat{o}$	$\psi \cdot \psi \wedge A(o) \equiv \widehat{m}\widehat{o}$
$\psi^\dagger \cdot \psi^\dagger \wedge A(i)^\dagger \equiv \widehat{i}\widehat{m}$	$\psi \cdot \psi \wedge A(i) \equiv \widehat{i}\widehat{m}$

Three-quark operators

$\psi^\dagger \cdot \psi^\dagger \wedge \psi^\dagger \equiv \widehat{i}\widehat{m}\widehat{o}$	$\psi \cdot \psi \wedge \psi \equiv \widehat{i}\widehat{m}\widehat{o}$

acting on

$$|n_P, n_Q, \nu_i, \nu_m, \nu_o\rangle$$

One-quark operators
$\psi^\dagger \cdot B(0)^\dagger \equiv$
$\psi \cdot B(o) \equiv$
$\psi^\dagger \cdot B(i)^\dagger \equiv$
$\psi \cdot B(i) \equiv$
$\psi^\dagger \cdot A(o) \equiv$
$\psi \cdot A(o)^\dagger \equiv$
$\psi^\dagger \cdot A(i) \equiv$
$\psi \cdot A(i)^\dagger \equiv$
$\psi^\dagger \cdot B(i) \wedge A(i)^\dagger \equiv$
$\psi \cdot A(i) \wedge B(i)^\dagger \equiv$
$\psi^\dagger \cdot B(o) \wedge A(o)^\dagger \equiv$
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$\psi \cdot \psi \wedge A(o) \equiv$
$\psi^\dagger \cdot \psi^\dagger \wedge A(i)^\dagger \equiv$
$\psi \cdot \psi \wedge A(i) \equiv$
Three-quark operators
$\psi^\dagger \cdot \psi^\dagger \wedge \psi^\dagger \equiv$
acting on
$|n_P, n_Q, \nu_i, \nu_m, \nu_o\rangle$
$\equiv n_P \to n_P + 1$
$\equiv n_P \to n_P - 1$
$\equiv n_Q \to n_Q + 1$
$\equiv n_Q \to n_Q - 1$
$\equiv \nu_i \to \nu_i + 1$
$\equiv \nu_i \to \nu_i - 1$
$\equiv \nu_m \to \nu_m + 1$
$\equiv \nu_m \to \nu_m - 1$
$\equiv \nu_o \to \nu_o + 1$
$\equiv \nu_o \to \nu_o - 1$
$\equiv \begin{pmatrix}\nu_m\\ \nu_o\end{pmatrix} \to \begin{pmatrix}\nu_m+1\\ \nu_o+1\end{pmatrix}$
$\equiv \begin{pmatrix}\nu_m\\ \nu_o\end{pmatrix} \to \begin{pmatrix}\nu_m-1\\ \nu_o-1\end{pmatrix}$
$\equiv \begin{pmatrix}\nu_i\\ \nu_m\end{pmatrix} \to \begin{pmatrix}\nu_i+1\\ \nu_m+1\end{pmatrix}$
$\equiv \begin{pmatrix}\nu_i\\ \nu_m\end{pmatrix} \to \begin{pmatrix}\nu_i-1\\ \nu_m-1\end{pmatrix}$
$\equiv \begin{pmatrix}\nu_i\\ \nu_m\\ \nu_o\end{pmatrix} \to \begin{pmatrix}\nu_i+1\\ \nu_m+1\\ \nu_o+1\end{pmatrix}$
$\equiv \begin{pmatrix}\nu_i\\ \nu_m\\ \nu_o\end{pmatrix} \to \begin{pmatrix}\nu_i-1\\ \nu_m-1\\ \nu_o-1\end{pmatrix}$

Towards building the LSH Hamiltonian

We formalize the quantum numbers with number operators $\hat{n}_P$, $\hat{n}_Q$, $\hat{\nu}_i$, $\hat{\nu}_m$, and $\hat{\nu}_o$:

$$\hat{n}_P = \sum_{n_P,n_Q,\nu_i,\nu_m,\nu_o} |n_P\, n_Q\,;\, \nu_i\, \nu_m\, \nu_o\rangle \langle n_P\, n_Q\,;\, \nu_i\, \nu_m\, \nu_o|\, n_P$$

$$\hat{n}_Q = \sum_{n_P,n_Q,\nu_i,\nu_m,\nu_o} |n_P\, n_Q\,;\, \nu_i\, \nu_m\, \nu_o\rangle \langle n_P\, n_Q\,;\, \nu_i\, \nu_m\, \nu_o|\, n_Q$$

$$\hat{\nu}_i = \sum_{n_P,n_Q,\nu_i,\nu_m,\nu_o} |n_P\, n_Q\,;\, \nu_i\, \nu_m\, \nu_o\rangle \langle n_P\, n_Q\,;\, \nu_i\, \nu_m\, \nu_o|\, \nu_i$$

$$\hat{\nu}_m = \sum_{n_P,n_Q,\nu_i,\nu_m,\nu_o} |n_P\, n_Q\,;\, \nu_i\, \nu_m\, \nu_o\rangle \langle n_P\, n_Q\,;\, \nu_i\, \nu_m\, \nu_o|\, \nu_m$$

$$\hat{\nu}_o = \sum_{n_P,n_Q,\nu_i,\nu_m,\nu_o} |n_P\, n_Q\,;\, \nu_i\, \nu_m\, \nu_o\rangle \langle n_P\, n_Q\,;\, \nu_i\, \nu_m\, \nu_o|\, \nu_o$$

$$\boxed{|n_P\, n_Q\,;\, \nu_i\, \nu_m\, \nu_o\rangle = (\hat{\Lambda}_P^\dagger)^{n_P} (\hat{\Lambda}_Q^\dagger)^{n_Q} (\hat{\chi}_i^\dagger)^{\nu_i} (\hat{\chi}_m^\dagger)^{\nu_m} (\hat{\chi}_o^\dagger)^{\nu_o}\, |0\,0\,;\,0\,0\,0\rangle}$$

To describe transitions, we also introduce normalized ladder operators

$$\hat{\Lambda}_P = \sum_{n_P=1}^{\infty} \sum_{n_Q,\nu_i,\nu_m,\nu_o} |n_P - 1,\, n_Q\,;\, \nu_i\, \nu_m\, \nu_o\rangle \langle n_P\, n_Q\,;\, \nu_i\, \nu_m\, \nu_o|$$

$$\hat{\Lambda}_Q = \sum_{n_Q=1}^{\infty} \sum_{n_P,\nu_i,\nu_m,\nu_o} |n_P,\, n_Q - 1\,;\, \nu_i\, \nu_m\, \nu_o\rangle \langle n_P\, n_Q\,;\, \nu_i\, \nu_m\, \nu_o|$$

$$\hat{\chi}_i = \sum_{n_P,n_Q,\nu_m,\nu_o} |n_P\, n_Q\,;\, 0, \nu_m, \nu_o\rangle \langle n_P\, n_Q\,;\, 1, \nu_m, \nu_o|$$

$$\hat{\chi}_m = \sum_{n_P,n_Q,\nu_i,\nu_o} |n_P\, n_Q\,;\, \nu_i, 0, \nu_o\rangle \langle n_P\, n_Q\,;\, \nu_i, 1, \nu_o|\, (-1)^{\nu_i}$$

$$\hat{\chi}_o = \sum_{n_P,n_Q,\nu_i,\nu_m} |n_P\, n_Q\,;\, \nu_i, \nu_m, 0\rangle \langle n_P\, n_Q\,;\, \nu_i, \nu_m, 1|\, (-1)^{\nu_i+\nu_m}$$

KS Hamiltonian in LSH basis:

re-written in terms of LSH operators

The LSH Hamiltonian for (1+1)d SU(3) gauge theory

$$H = H_E + H_M + H_I$$

$$H_E = \sum_r H_E(r) = \sum_r \frac{1}{3}\left(P(r)^2 + Q(r)^2 + P(r)Q(r)\right) + P(r) + Q(r)$$

$$H_M = \sum_r H_M(r) = \sum_r \mu(-)^r\left(\hat{\nu}_i(r) + \hat{\nu}_m(r) + \hat{\nu}_o(r)\right)$$

$$\begin{aligned}
H_I(r, r+1) = &\left\{\hat{\chi}_o^\dagger (\hat{\Lambda}_P^+)^{\hat{\nu}_m} \sqrt{1 - \frac{\hat{\nu}_m}{\hat{n}_P + 2}} \sqrt{1 - \frac{\hat{\nu}_i}{\hat{n}_P + \hat{n}_Q + 3}}\right\}_r \left\{\sqrt{1 + \frac{\hat{\nu}_m}{\hat{n}_P + 1}} \sqrt{1 + \frac{\hat{\nu}_i}{\hat{n}_P + \hat{n}_Q + 2}}\, \hat{\chi}_o (\hat{\Lambda}_P^+)^{1-\hat{\nu}_m}\right\}_{r+1} + \text{H.c.} \\
&+ \left\{\hat{\chi}_i^\dagger (\hat{\Lambda}_Q^-)^{1-\hat{\nu}_m} \sqrt{1 + \frac{\hat{\nu}_m}{\hat{n}_Q + 1}} \sqrt{1 + \frac{\hat{\nu}_o}{\hat{n}_P + \hat{n}_Q + 2}}\right\}_r \left\{\sqrt{1 - \frac{\hat{\nu}_m}{\hat{n}_Q + 2}} \sqrt{1 - \frac{\hat{\nu}_o}{\hat{n}_P + \hat{n}_Q + 3}}\, \hat{\chi}_i (\hat{\Lambda}_Q^-)^{\hat{\nu}_m}\right\}_{r+1} + \text{H.c.} \\
&+ \left\{\hat{\chi}_m^\dagger (\hat{\Lambda}_P^-)^{1-\hat{\nu}_o} (\hat{\Lambda}_Q^+)^{\hat{\nu}_i} \sqrt{1 + \frac{\hat{\nu}_o}{\hat{n}_P + 1}} \sqrt{1 - \frac{\hat{\nu}_i}{\hat{n}_Q + 2}}\right\}_r \left\{\sqrt{1 - \frac{\hat{\nu}_o}{\hat{n}_P + 2}} \sqrt{1 + \frac{\hat{\nu}_i}{\hat{n}_Q + 1}}\, \hat{\chi}_m (\hat{\Lambda}_P^-)^{\hat{\nu}_o} (\hat{\Lambda}_Q^+)^{1-\hat{\nu}_i}\right\}_{r+1} + \text{H.c.}
\end{aligned}$$

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& + \left\{\hat{\chi}_i^\dagger(\hat{\Lambda}_Q^-)^{1-\hat{\nu}_m}\sqrt{1 + \frac{\hat{\nu}_m}{\hat{n}_Q + 1}}\sqrt{1 + \frac{\hat{\nu}_o}{\hat{n}_P + \hat{n}_Q + 2}}\right\}_r \left\{\sqrt{1 - \frac{\hat{\nu}_m}{\hat{n}_Q + 2}}\sqrt{1 - \frac{\hat{\nu}_o}{\hat{n}_P + \hat{n}_Q + 3}}\,\hat{\chi}_i(\hat{\Lambda}_Q^-)^{\hat{\nu}_m}\right\}_{r+1} + \text{H.c.} \\
& + \left\{\hat{\chi}_m^\dagger(\hat{\Lambda}_P^-)^{1-\hat{\nu}_o}(\hat{\Lambda}_Q^+)^{\hat{\nu}_i}\sqrt{1 + \frac{\hat{\nu}_o}{\hat{n}_P + 1}}\sqrt{1 - \frac{\hat{\nu}_i}{\hat{n}_Q + 2}}\right\}_r \left\{\sqrt{1 - \frac{\hat{\nu}_o}{\hat{n}_P + 2}}\sqrt{1 + \frac{\hat{\nu}_i}{\hat{n}_Q + 1}}\,\hat{\chi}_m(\hat{\Lambda}_P^-)^{\hat{\nu}_o}(\hat{\Lambda}_Q^+)^{1-\hat{\nu}_i}\right\}_{r+1} + \text{H.c.}
\end{aligned}$$

Structurally identical to the SU(2) LSH construction

The LSH Hamiltonian for (1+1)d SU(3) gauge theory

$$H = H_E + H_M + H_I$$

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&+ \left\{\hat{\chi}_i^\dagger (\hat{\Lambda}_Q^-)^{1-\hat{\nu}_m} \sqrt{1 + \frac{\hat{\nu}_m}{\hat{n}_Q + 1}} \sqrt{1 + \frac{\hat{\nu}_o}{\hat{n}_P + \hat{n}_Q + 2}}\right\}_r \left\{\sqrt{1 - \frac{\hat{\nu}_m}{\hat{n}_Q + 2}} \sqrt{1 - \frac{\hat{\nu}_o}{\hat{n}_P + \hat{n}_Q + 3}}\, \hat{\chi}_i (\hat{\Lambda}_Q^-)^{\hat{\nu}_m}\right\}_{r+1} + \text{H.c.} \\
&+ \left\{\hat{\chi}_m^\dagger (\hat{\Lambda}_P^-)^{1-\hat{\nu}_o} (\hat{\Lambda}_Q^+)^{\hat{\nu}_i} \sqrt{1 + \frac{\hat{\nu}_o}{\hat{n}_P + 1}} \sqrt{1 - \frac{\hat{\nu}_i}{\hat{n}_Q + 2}}\right\}_r \left\{\sqrt{1 - \frac{\hat{\nu}_o}{\hat{n}_P + 2}} \sqrt{1 + \frac{\hat{\nu}_i}{\hat{n}_Q + 1}}\, \hat{\chi}_m (\hat{\Lambda}_P^-)^{\hat{\nu}_o} (\hat{\Lambda}_Q^+)^{1-\hat{\nu}_i}\right\}_{r+1} + \text{H.c.}
\end{aligned}$$

Structurally identical to the SU(2) LSH construction

Numerically benchmarked with completely gauge fixed (pure gauge) Hamiltonian

The LSH Hamiltonian

Numerically benchmarked with completely gauge fixed (pure gauge) Hamiltonian

Q	(P_{out}, Q_{out})	$d(P_{out}, Q_{out})$	Eigenvalue
0	(0, 0)	1	0.000
1	(1, 0)	3	-0.387 1.721
2	(0, 1)	3	-1.535 0.868 3.333
2	(2, 0)	6	1.333
3	(0, 0)	1	-3.858 -0.497 2.137 4.884
3	(1, 1)	8	-0.081 2.747
4	(1, 0)	3	-2.562 1.089 4.140
4	(0, 2)	6	1.333
5	(0, 1)	3	-1.277 2.610
6	(0, 0)	1	0.000

$$d_{(P_{out},Q_{out})} = \frac{1}{2}(P_{out}+1)(Q_{out}+1)(P_{out}+Q_{out}+2)$$

$$\psi(r) \to \psi'(r) = \left[\prod_{y<r} U(y)\right] \psi(r) \qquad \psi^\dagger(r) \to \psi^{\dagger\prime}(r) = \psi^\dagger(r) \left[\prod_{y<r} U(y)\right]^\dagger$$

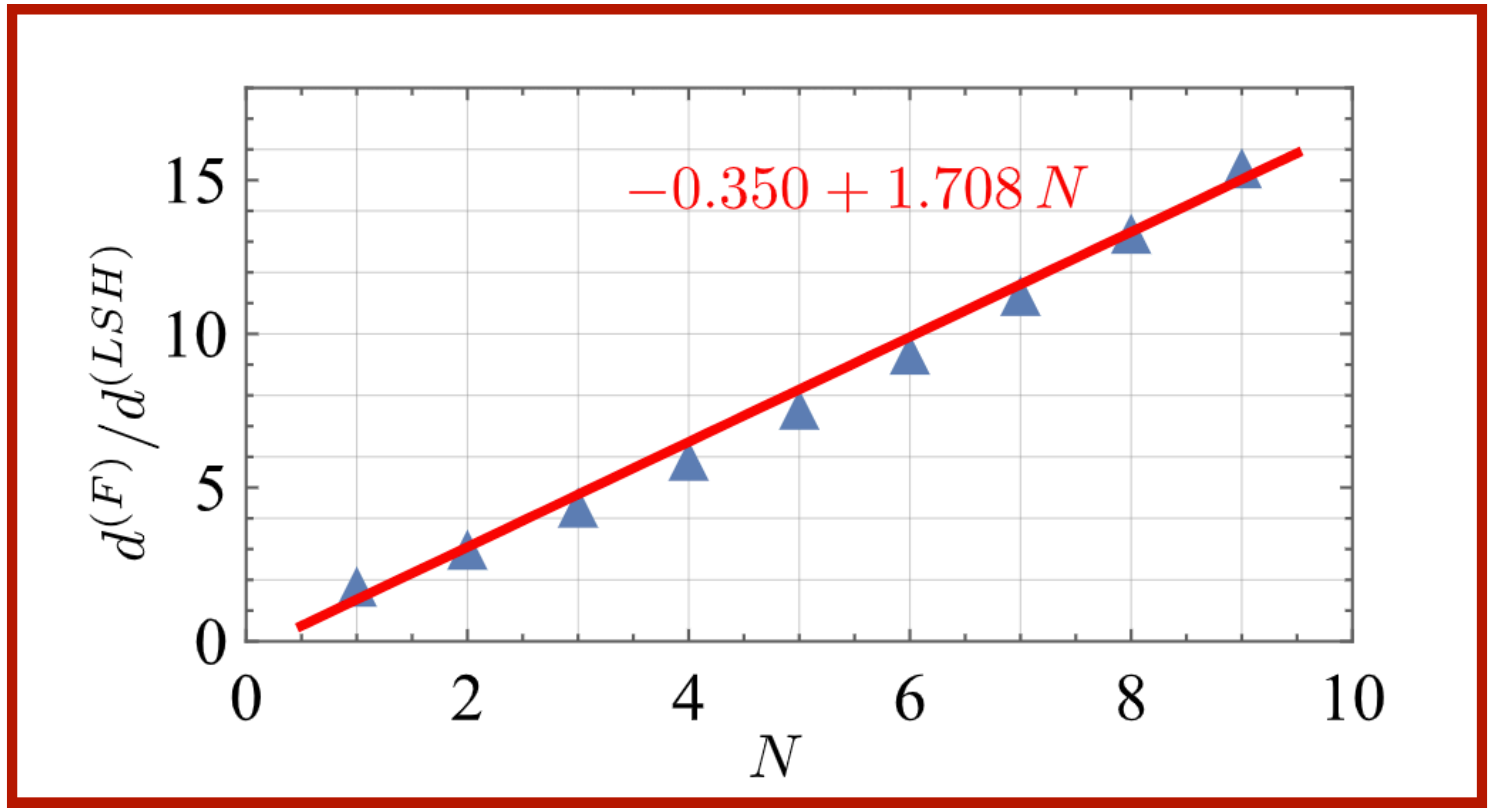
$$U(r) \to U'(r) = \left[\prod_{y<r} U(y)\right] U(r) \left[\prod_{z<r+1} U(z)\right]^\dagger$$

$$\boxed{|\Psi\rangle^{(F)} = \prod_{x=0}^{N-1} |f_1, f_2, f_3\rangle_{(x)}}$$

Contains degeneracy

Due to global symmetries

Fermionic Hamiltonian with long range interaction



Benefits of working in the LSH framework: Applications in quantum simulation

Already demonstrated for SU(2)

Benefits of working in the LSH framework: Applications in quantum simulation

Symmetry protection protocol:

Already demonstrated for SU(2)

Accepted Paper

Protecting local and global symmetries in simulating 1+1D non-Abelian gauge theories

Phys. Rev. D

Emil Mathew and Indrakshi Raychowdhury

Accepted 24 August 2022

ABSTRACT

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Efficient quantum simulation protocols for any quantum theories demand efficient protection protocols for its underlying symmetries. This task is nontrivial for gauge theories as it involves local symmetry/invariance. For non-Abelian gauge theories, protecting all the symmetries generated by a set of mutually non-commuting generators, is particularly difficult. In this letter, a global symmetry-protection protocol is proposed. Using the novel loop-string-hadron formalism of non-Abelian lattice gauge theory, we numerically demonstrate that all of the local symmetries get protected even for large time by this global symmetry protection scheme. With suitable protection strength, the dynamics of a (1+1)-dimensional SU(2) lattice gauge theory remains confined in the physical Hilbert space of the theory even in presence of explicit local symmetry violating terms in the Hamiltonian that may occur in both analog and digital simulation schemes as an error. The whole scheme holds for SU(3) gauge theory as well.

Emil Mathew,
Grad. Student,
BITS-Pilani, Goa

Benefits of working in the LSH framework: Applications in quantum simulation

Symmetry protection protocol:

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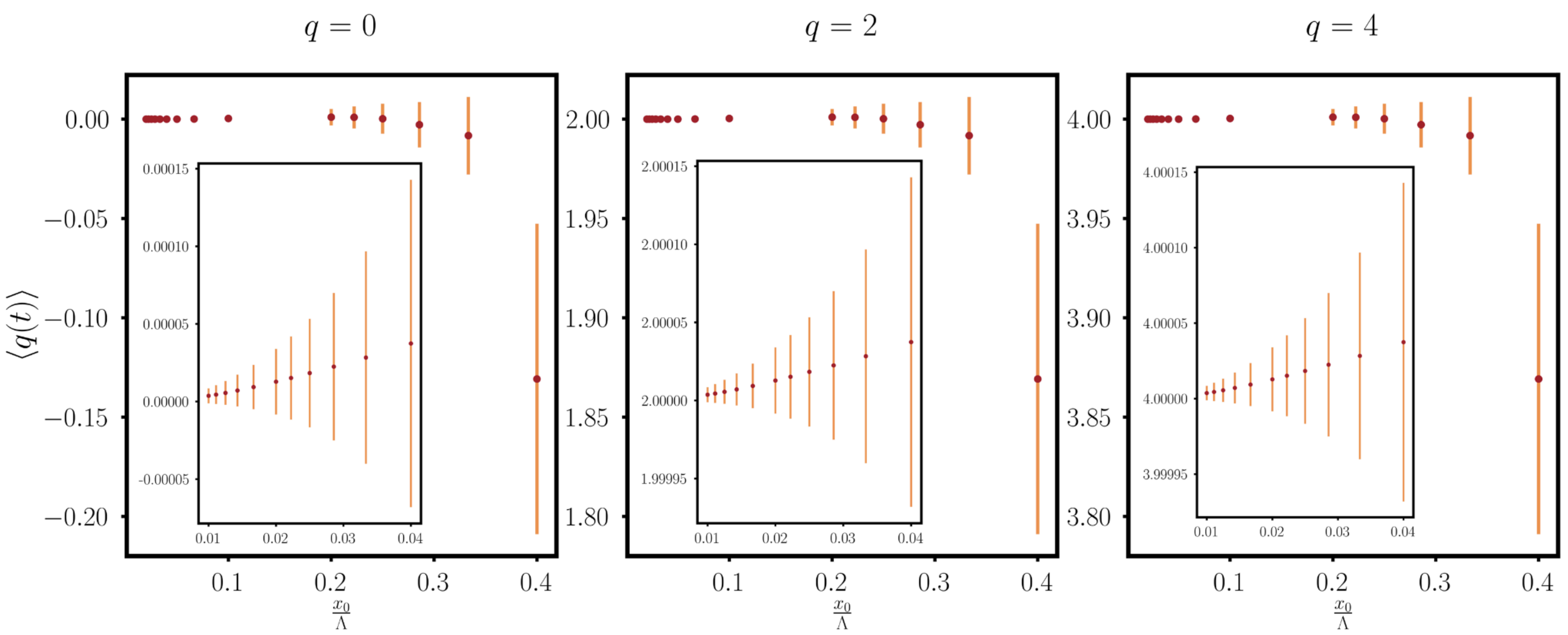

Protection of global symmetries

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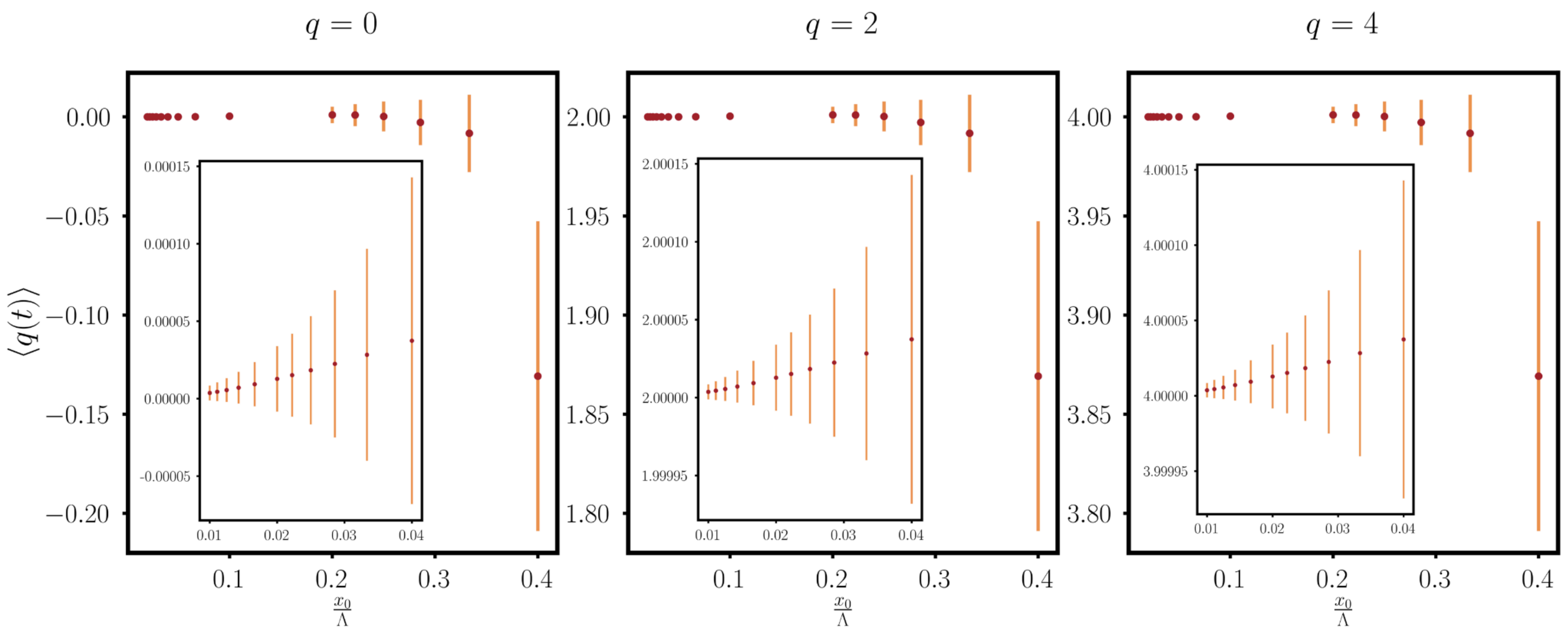
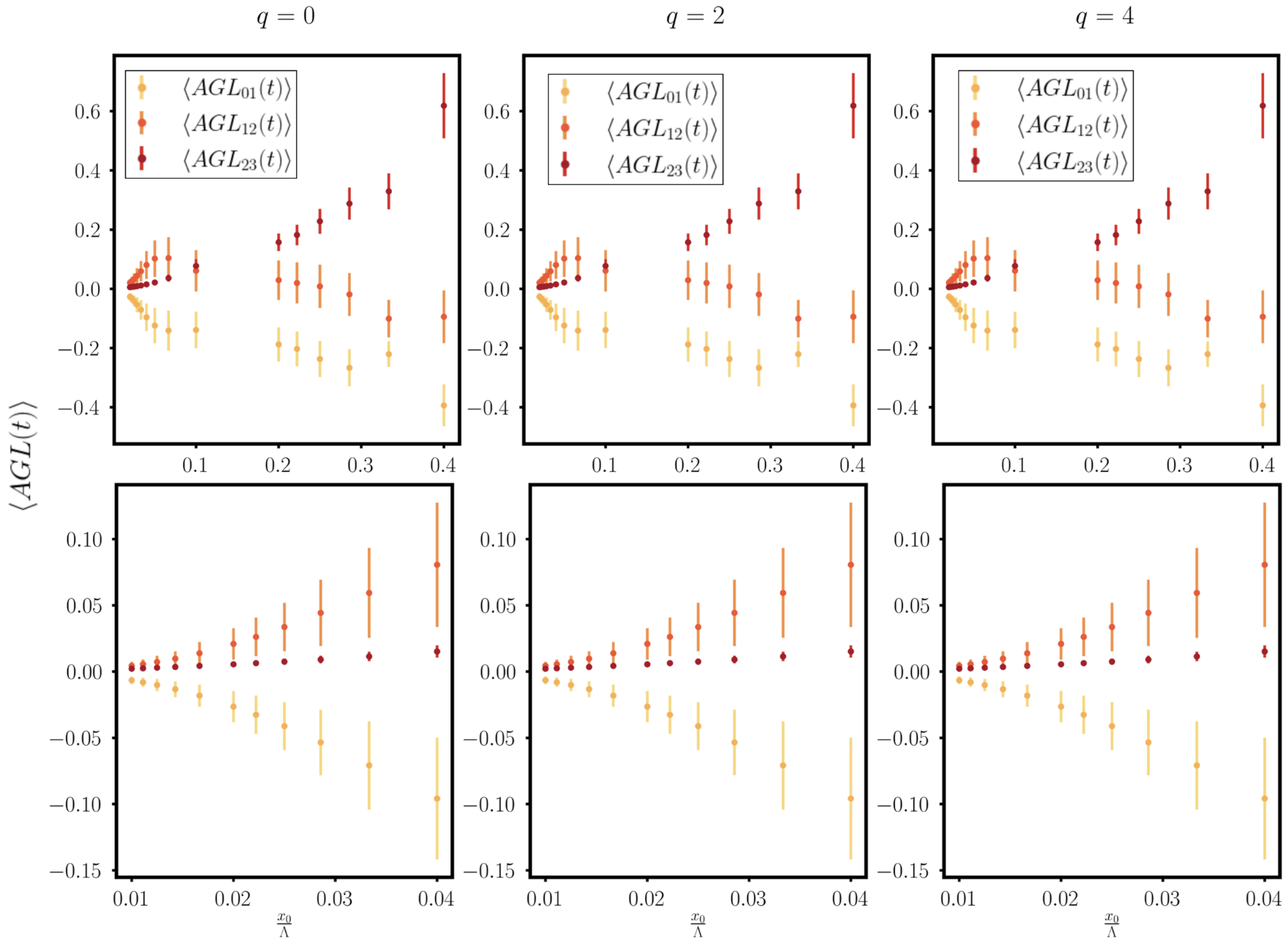


Protection of global symmetries

Complete protection of all the local symmetries



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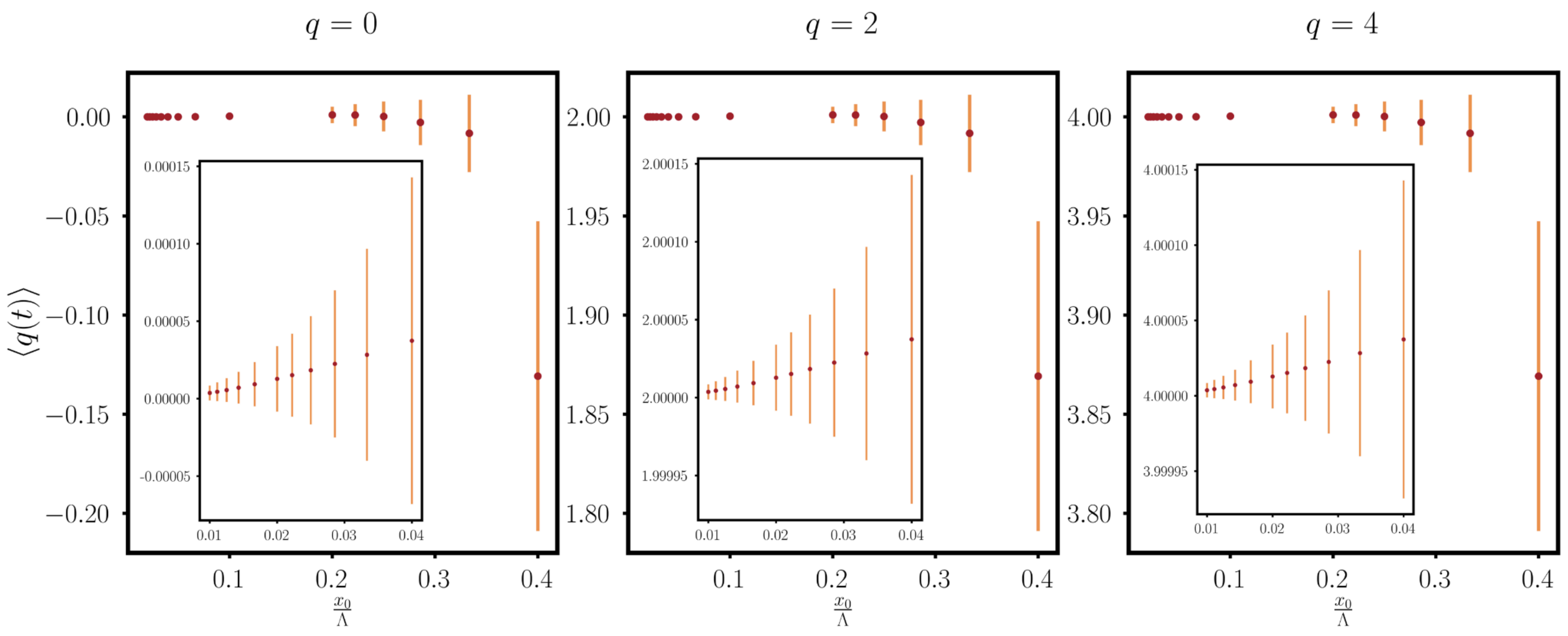
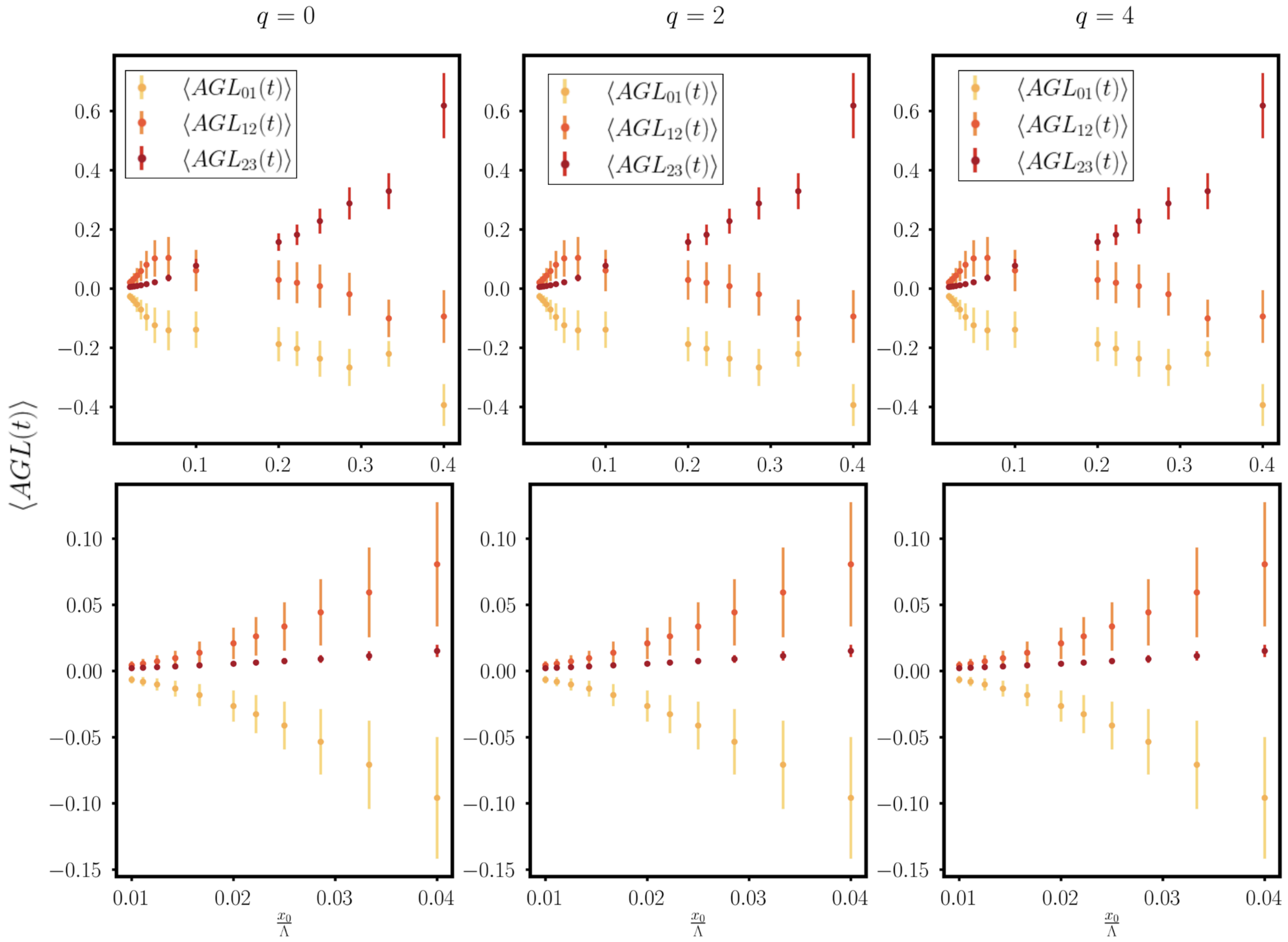


Protection of global symmetries

Quantum simulation of non-Abelian gauge theory without imposing any local constraint

Complete protection of all the local symmetries



Benefits of working in the LSH framework: Applications in quantum simulation

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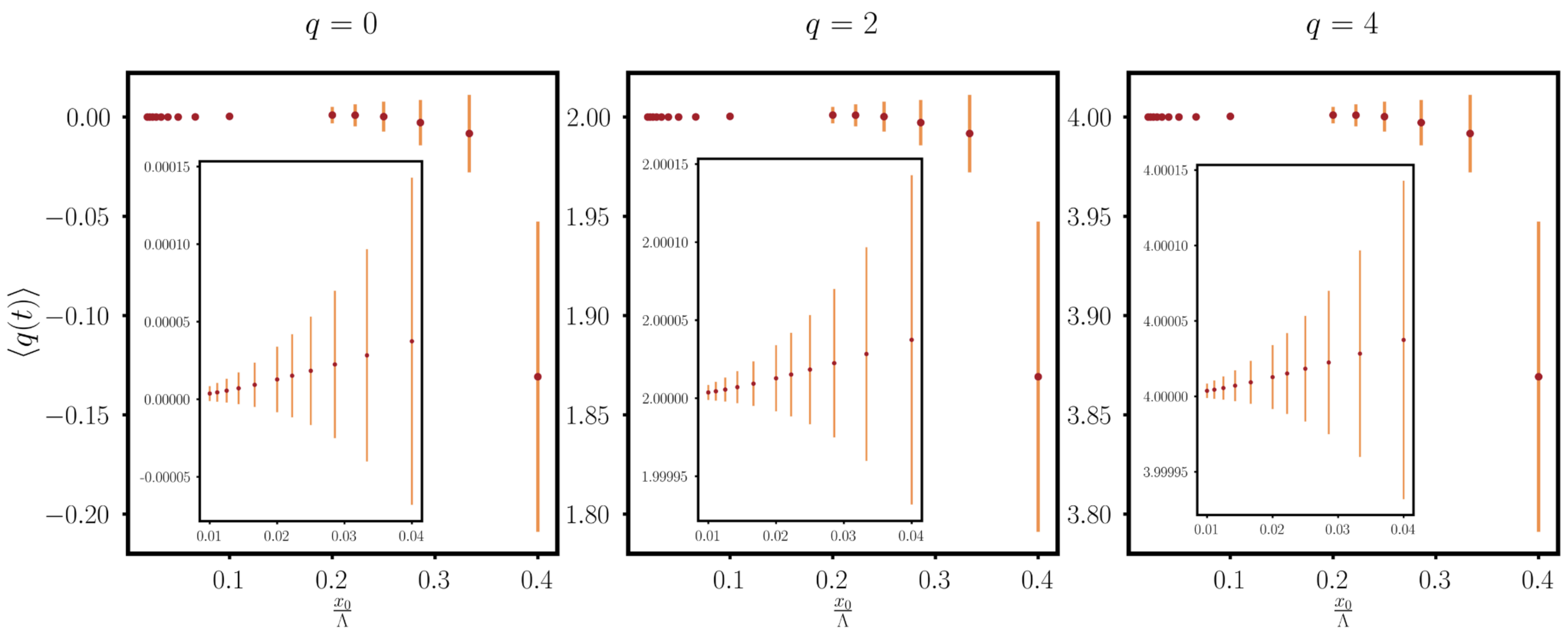
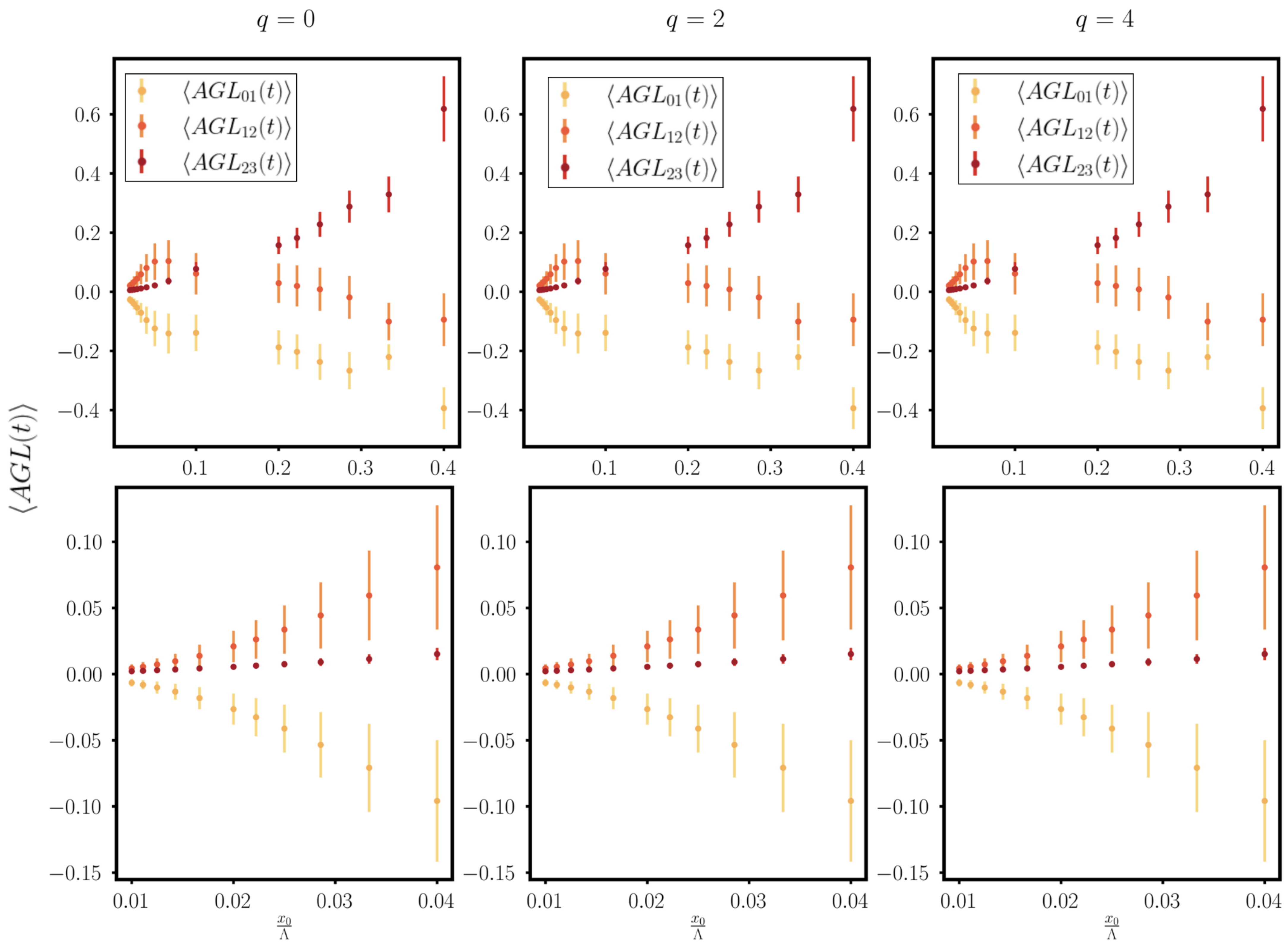


Protection of global symmetries

Complete protection of all the local symmetries

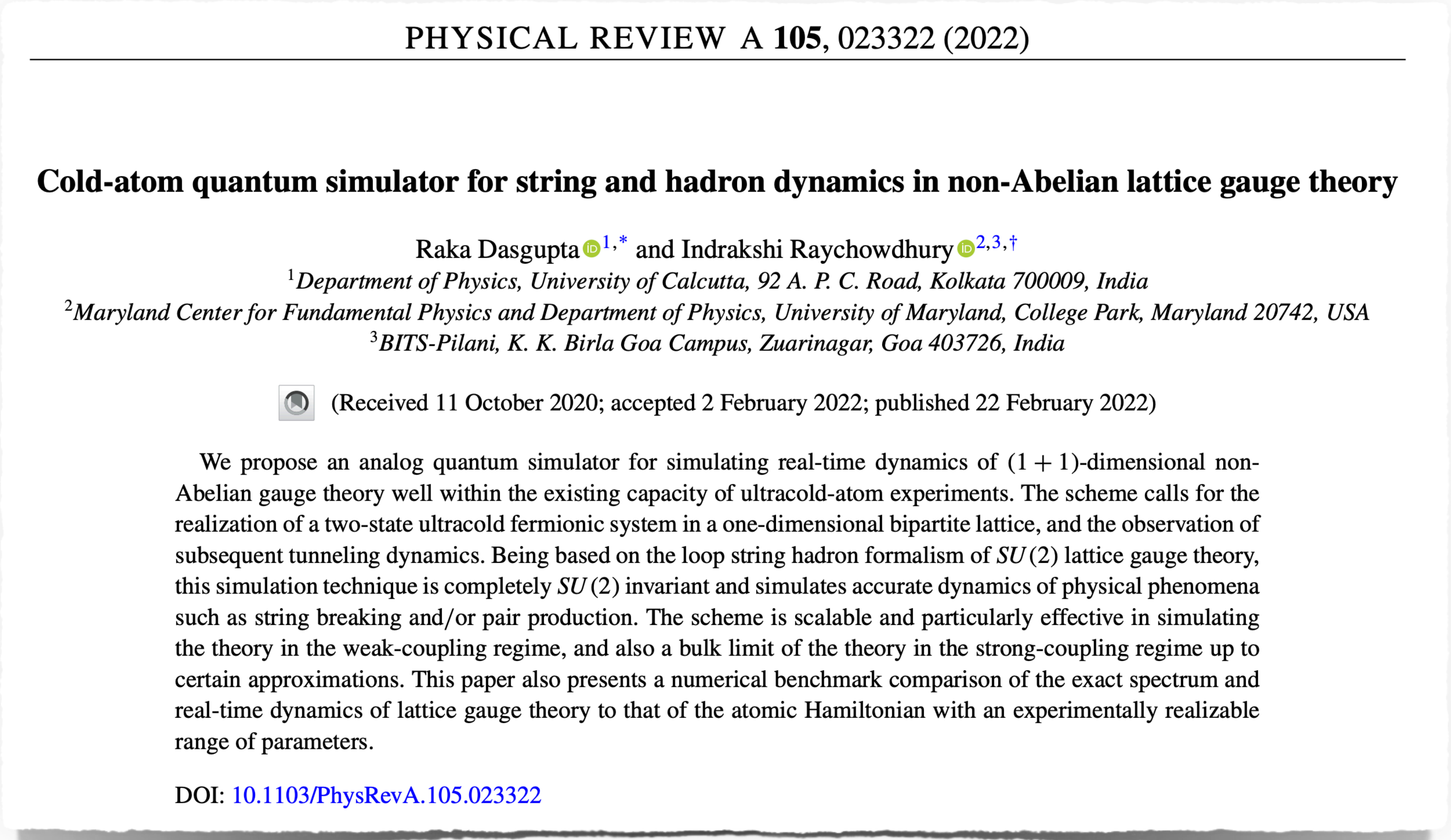


Quantum simulation of non-Abelian gauge theory without imposing any local constraint

Also crucial for tensor network calculations

Benefits of working in the LSH framework: Applications in quantum simulation

Already demonstrated for SU(2)



PHYSICAL REVIEW A **105**, 023322 (2022)

Cold-atom quantum simulator for string and hadron dynamics in non-Abelian lattice gauge theory

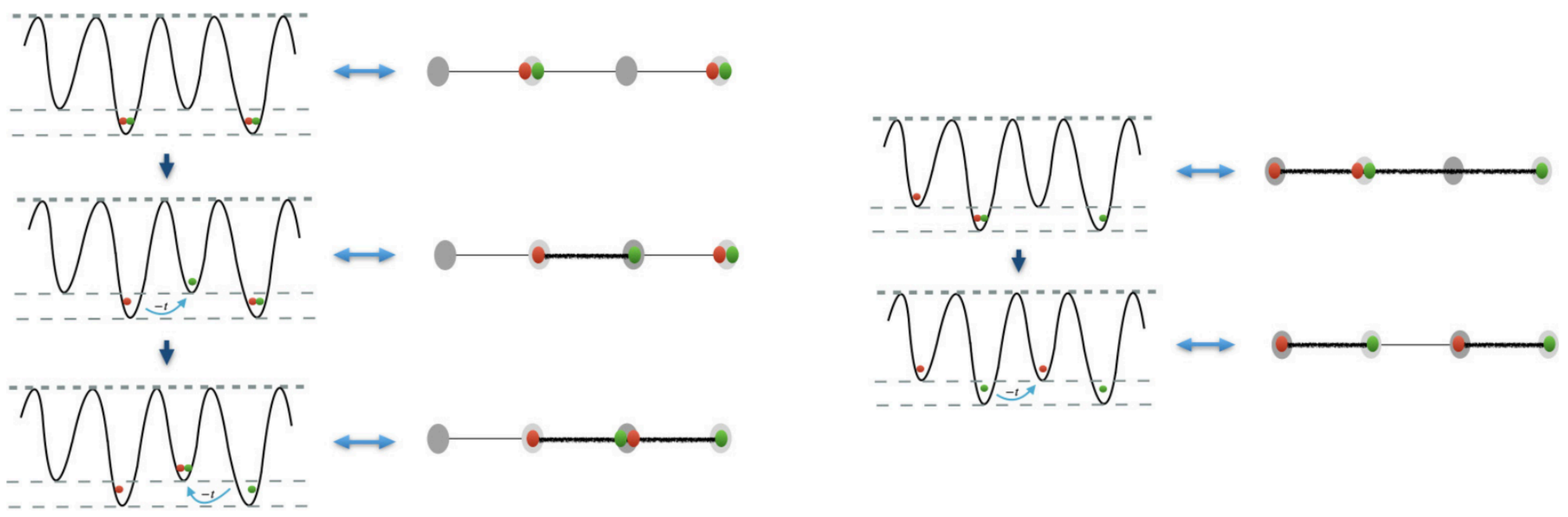
Raka Dasgupta[1,*] and Indrakshi Raychowdhury[2,3,†]
[1]*Department of Physics, University of Calcutta, 92 A. P. C. Road, Kolkata 700009, India*
[2]*Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, Maryland 20742, USA*
[3]*BITS-Pilani, K. K. Birla Goa Campus, Zuarinagar, Goa 403726, India*

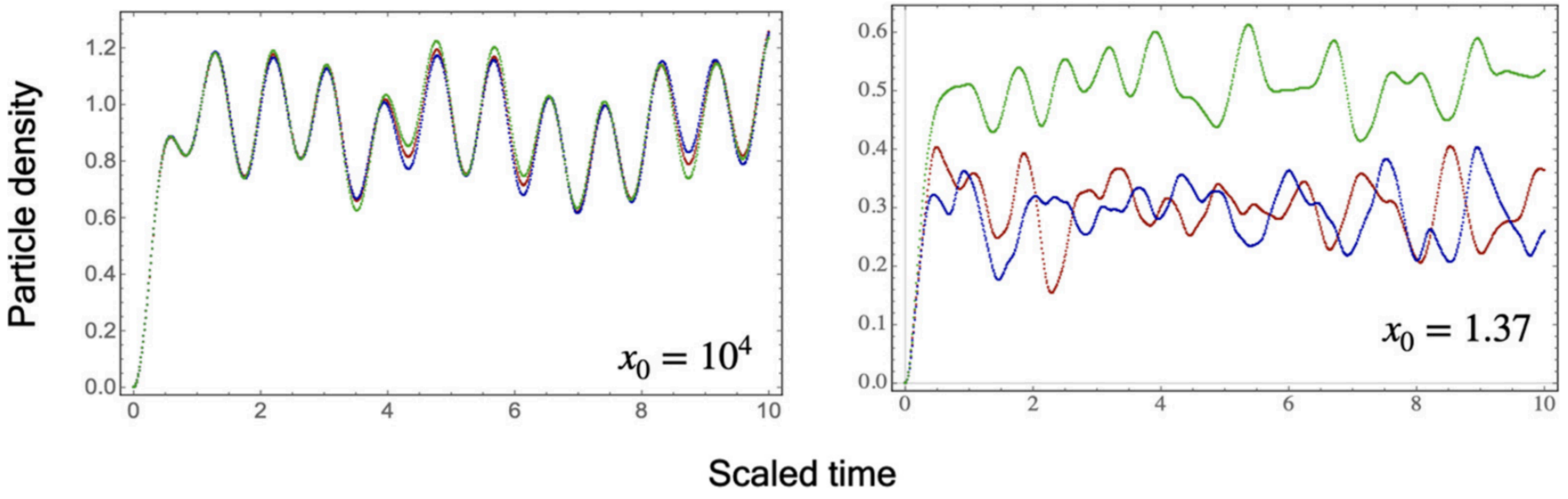
(Received 11 October 2020; accepted 2 February 2022; published 22 February 2022)

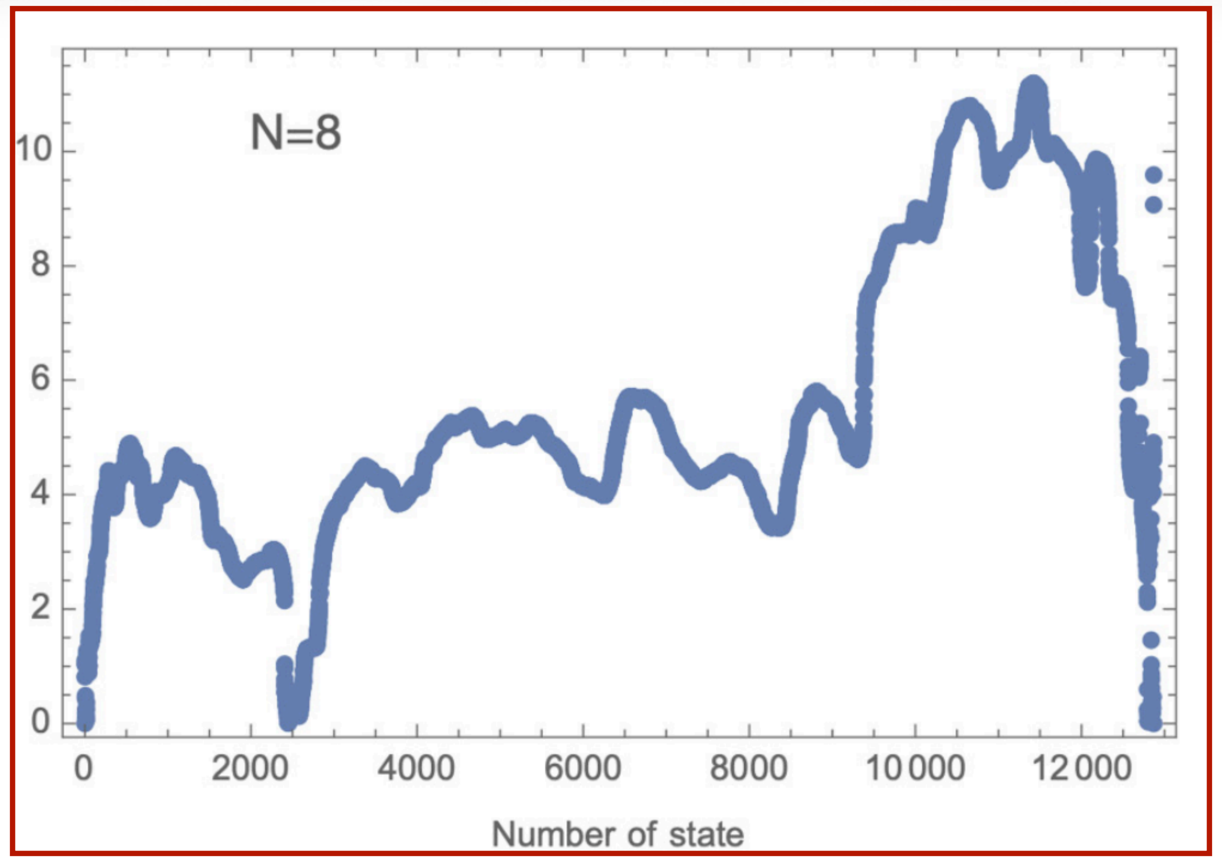
We propose an analog quantum simulator for simulating real-time dynamics of (1 + 1)-dimensional non-Abelian gauge theory well within the existing capacity of ultracold-atom experiments. The scheme calls for the realization of a two-state ultracold fermionic system in a one-dimensional bipartite lattice, and the observation of subsequent tunneling dynamics. Being based on the loop string hadron formalism of $SU(2)$ lattice gauge theory, this simulation technique is completely $SU(2)$ invariant and simulates accurate dynamics of physical phenomena such as string breaking and/or pair production. The scheme is scalable and particularly effective in simulating the theory in the weak-coupling regime, and also a bulk limit of the theory in the strong-coupling regime up to certain approximations. This paper also presents a numerical benchmark comparison of the exact spectrum and real-time dynamics of lattice gauge theory to that of the atomic Hamiltonian with an experimentally realizable range of parameters.

DOI: 10.1103/PhysRevA.105.023322

Analog Quantum Computation





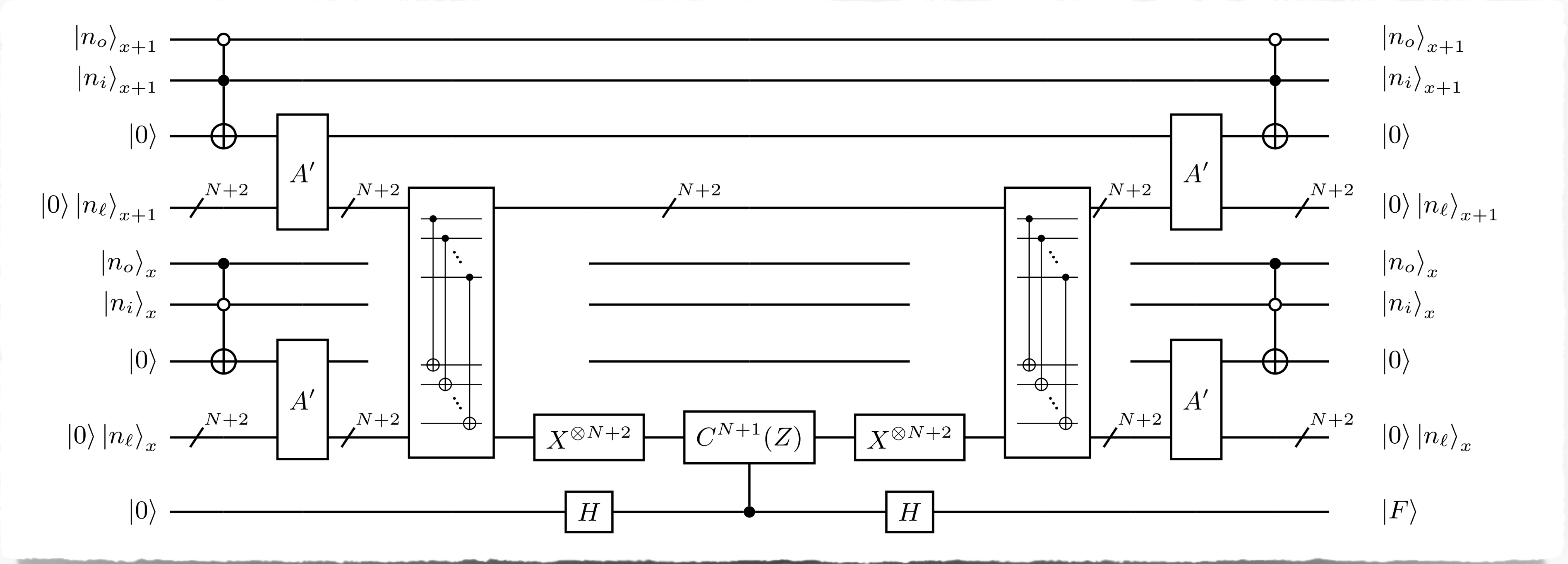


Percentage shift of the spectrum of simulated Hamiltonian from the original Hamiltonian

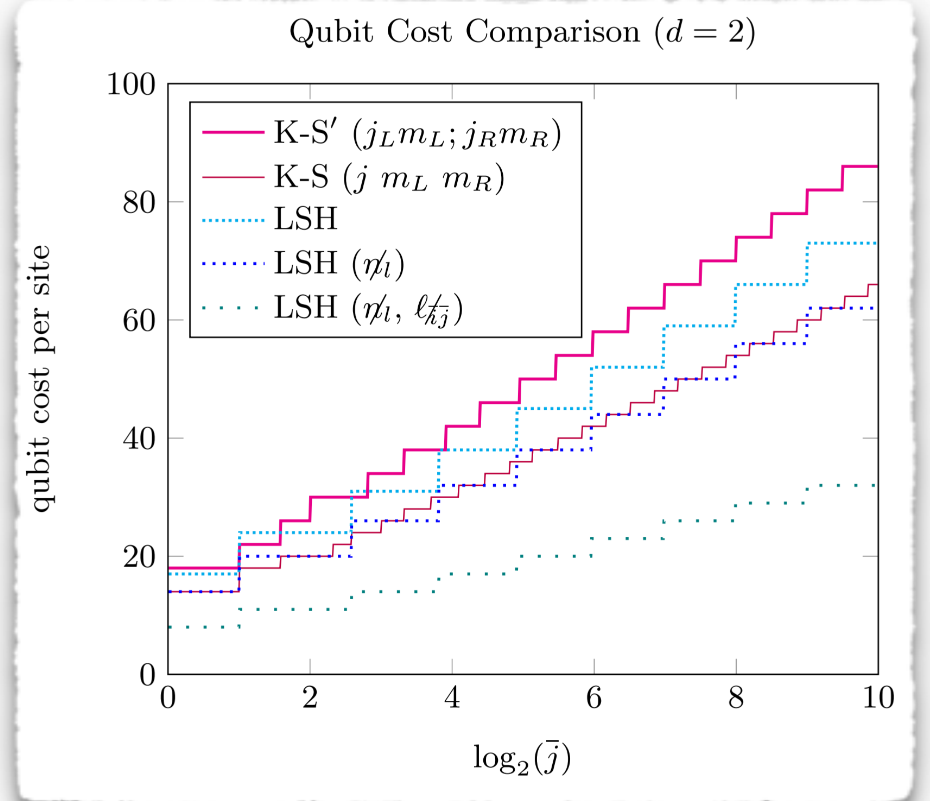
Benefits of working in the LSH framework: Applications in quantum simulation

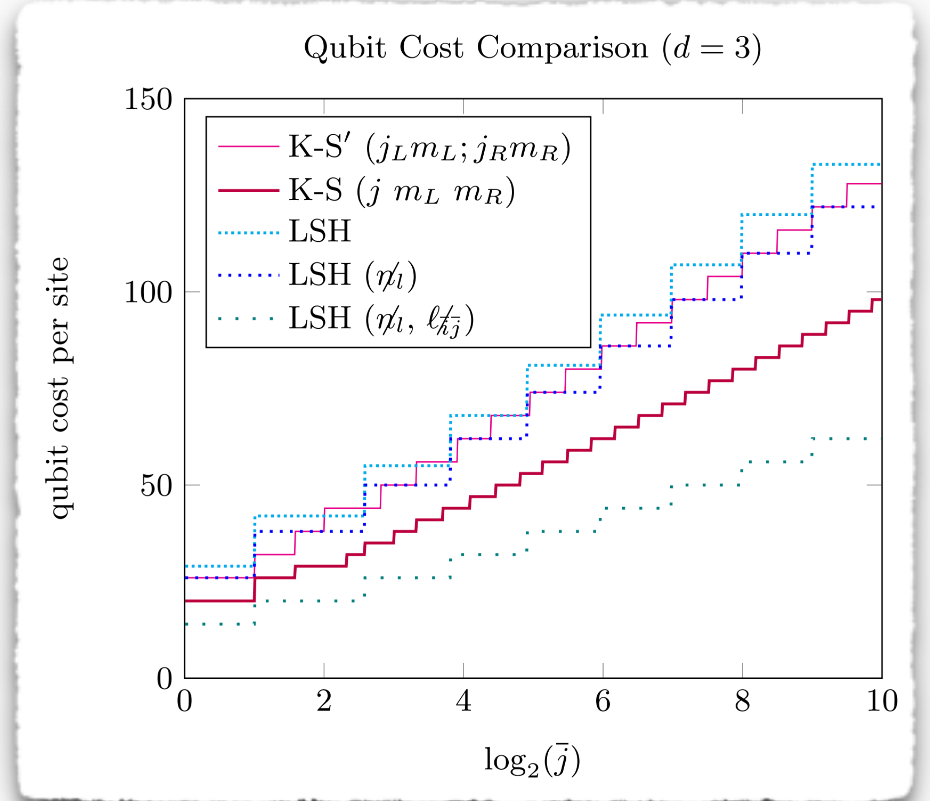
Already demonstrated for SU(2)

Analog Quantum Computation



Qubit Cost Analysis





PHYSICAL REVIEW RESEARCH **2**, 033039 (2020)

Solving Gauss's law on digital quantum computers with loop-string-hadron digitization

Indrakshi Raychowdhury*
Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, Maryland 20742, USA

Jesse R. Stryker†
Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195, USA

(Received 22 April 2020; accepted 4 June 2020; published 9 July 2020)

We show that using the loop-string-hadron (LSH) formulation of SU(2) lattice gauge theory (I. Raychowdhury and J. R. Stryker, Phys. Rev. D **101**, 114502 (2020)) as a basis for digital quantum computation easily solves an important problem of fundamental interest: implementing gauge invariance (or Gauss's law) exactly. We first discuss the structure of the LSH Hilbert space in d spatial dimensions, its truncation, and its digitization with qubits. Error detection and mitigation in gauge theory simulations would benefit from physicality "oracles," so we decompose circuits that flag gauge-invariant wave functions. We then analyze the logical qubit costs and entangling gate counts involved with the protocols. The LSH basis could save or cost more qubits than a Kogut-Susskind-type representation basis, depending on how the bases are digitized as well as the spatial dimension. The numerous other clear benefits encourage future studies into applying this framework.

DOI: 10.1103/PhysRevResearch.2.033039

Also upcoming results by Jesse Stryker, Alex Shaw and Zohreh Davoudi, UMD

Benefits of working in the LSH framework: Applications in quantum simulation

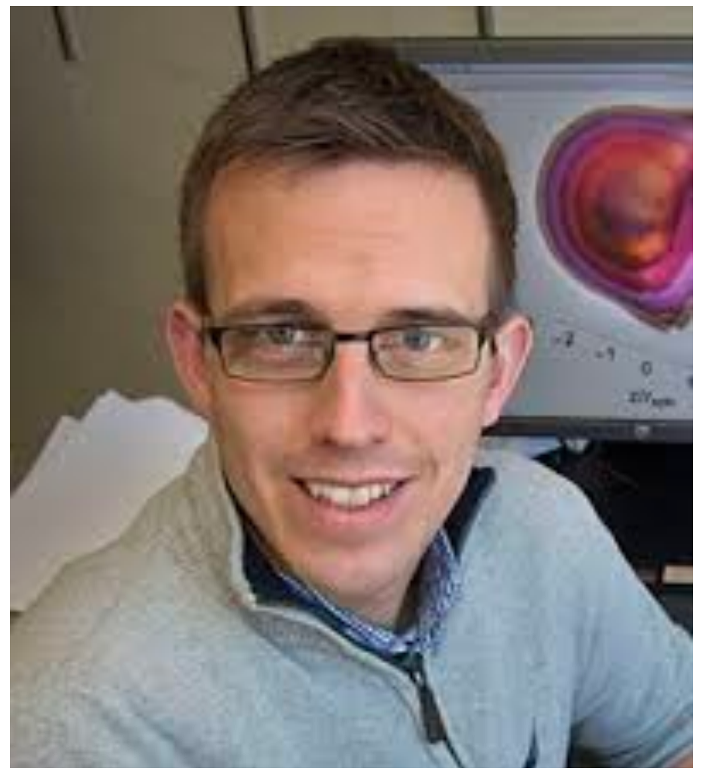
Other ongoing works:

Tensor network calculations for non-Abelian gauge theories

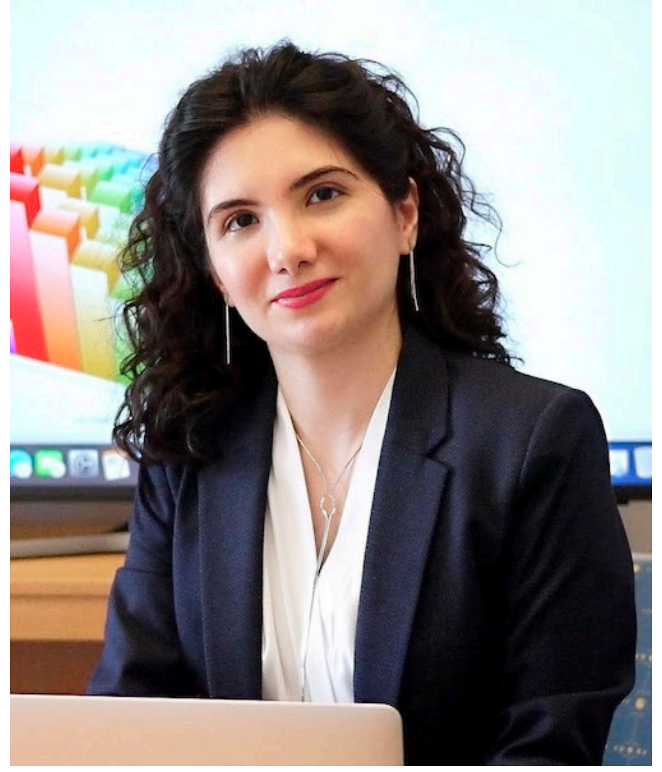
Understanding entanglement structure
for non-Abelian gauge theories

Collaborators:

Aniruddha Bapat



Niklas Mueller



Zohreh Davoudi

The LSH Hamiltonian for (1+1)d SU(3) gauge theory

arXiv:2209.xxxx

Structurally identical to the SU(2) LSH construction

We look forward to demonstrate all such advantages for SU(3)

LSH framework for arbitrary dimension:

Exists for SU(2)

To be developed for SU(3)

The LSH Hamiltonian for (3+1)d SU(3) gauge theory

A concrete step towards quantum simulating QCD

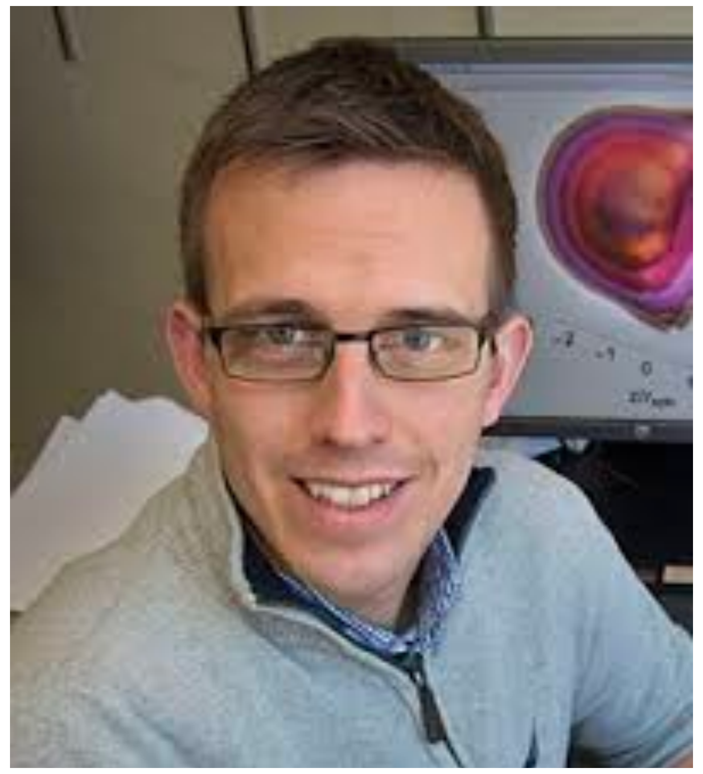

Emil Mathew

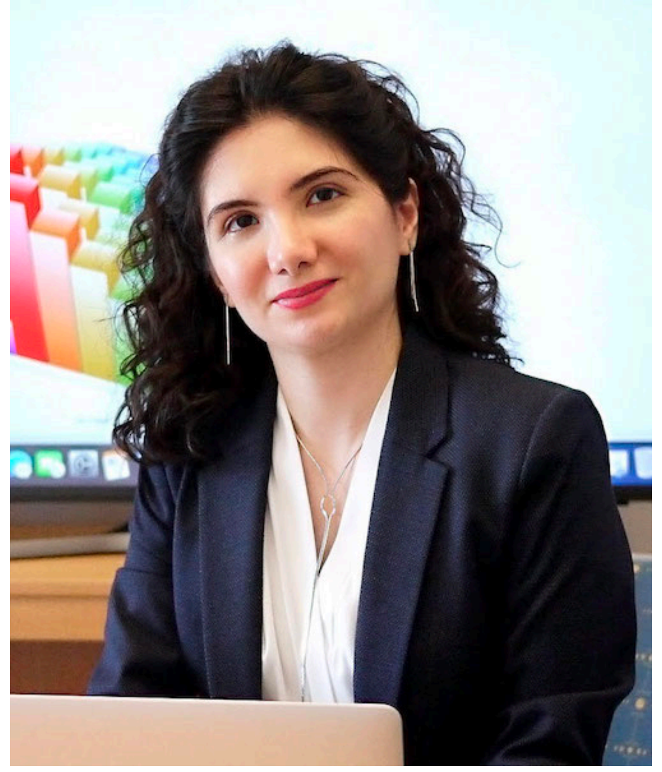

Saurabh Kadam


Jesse Stryker


Aniruddha Bapat


Raka Dasgupta


Niklas Mueller


Zohreh Davoudi

BITS Pilani
PILANI | DUBAI | GOA | HYDERABAD

UNIVERSITY of WASHINGTON

IQuS InQubator for Quantum Simulation

UNIVERSITY OF MARYLAND 1856

BERKELEY LAB
Lawrence Berkeley National Laboratory

Thank You

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Back-up slides

Hamiltonian, describing dynamics of loops, strings and hadrons for SU(2)

$$H^{(\mathrm{LSH})} = H_I^{(\mathrm{LSH})} + H_E^{(\mathrm{LSH})} + H_M^{(\mathrm{LSH})}$$

$$\begin{aligned} H_I^{(\mathrm{LSH})} &= \frac{1}{2a}\sum_n \Bigg\{ \frac{1}{\sqrt{\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x)) + 1}} \\ &\quad \times \left[\hat{S}_o^{++}(x)\hat{S}_i^{+-}(x+1) + \hat{S}_o^{+-}(x)\hat{S}_i^{--}(x+1) \right] \\ &\quad \times \frac{1}{\sqrt{\hat{n}_l(x+1) + \hat{n}_i(x+1)(1-\hat{n}_o(x+1)) + 1}} + \text{h.c.} \Bigg\}, \end{aligned}$$

$$\begin{aligned} H_E^{(\mathrm{LSH})} &= \frac{g^2 a}{2}\sum_n \left[\frac{\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x))}{2} \right. \\ &\quad \left. \times \left(\frac{\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x))}{2} + 1 \right) \right], \end{aligned}$$

$$H_M^{(\mathrm{LSH})} = m\sum_n (-1)^x (\hat{n}_i(x) + \hat{n}_o(x)),$$

$$\begin{aligned} \hat{S}_o^{++} &= \hat{\chi}_o^{+}(\lambda^{+})^{\hat{n}_i}\sqrt{\hat{n}_l + 2 - \hat{n}_i}, \\ \hat{S}_o^{--} &= \hat{\chi}_o^{-}(\lambda^{-})^{\hat{n}_i}\sqrt{\hat{n}_l + 2(1 - \hat{n}_i)}, \\ \hat{S}_o^{+-} &= \hat{\chi}_i^{+}(\lambda^{-})^{1-\hat{n}_o}\sqrt{\hat{n}_l + 2\hat{n}_o}, \\ \hat{S}_o^{-+} &= \hat{\chi}_i^{-}(\lambda^{+})^{1-\hat{n}_o}\sqrt{\hat{n}_l + 1 + \hat{n}_o}, \end{aligned}$$

$$\begin{aligned} \hat{S}_i^{+-} &= \hat{\chi}_o^{-}(\lambda^{+})^{1-\hat{n}_i}\sqrt{\hat{n}_l + 1 + \hat{n}_i}, \\ \hat{S}_i^{-+} &= \hat{\chi}_o^{+}(\lambda^{-})^{1-\hat{n}_i}\sqrt{\hat{n}_l + 2\hat{n}_i}, \\ \hat{S}_i^{--} &= \hat{\chi}_i^{-}(\lambda^{-})^{\hat{n}_o}\sqrt{\hat{n}_l + 2(1 - \hat{n}_o)}, \\ \hat{S}_i^{++} &= \hat{\chi}_i^{+}(\lambda^{+})^{\hat{n}_o}\sqrt{\hat{n}_l + 2 - \hat{n}_o}. \end{aligned}$$

The strong-coupling vacuum of the LSH Hamiltonian is given by

$$\begin{aligned} &n_l(x) = 0, \text{ for all } x, \\ &n_i(x) = 0, \ n_o(x) = 0, \text{ for } x \text{ even}, \\ &n_i(x) = 1, \ n_o(x) = 1, \text{ for } x \text{ odd}. \end{aligned}$$

Hamiltonian, describing dynamics of loops, strings and hadrons: Identical spectrum to KS

Global symmetries of the LSH framework: SU(2)

LSH basis in 1 spatial dimension $|n_l, n_i, n_0\rangle_{(x)} \quad \forall x$

with local constraints

$$n_l + n_o(1-n_i)\Big|_x = n_l + n_i(1-n_o)\Big|_{x+1}$$

The super-selection sectors of the LSH Hamiltonian are defined by:

1. Total fermionic occupation number:

$$Q = \sum_{x=0}^{N-1} [n_i(x) + n_o(x)]$$

   For a N-site lattice, the value of Q can be any integer between $[0, 2N]$.

2. The imbalance between incoming and outgoing strings: relates to the boundary fluxes

$$q = \sum_{x=0}^{N-1} [n_0(x) - n_i(x)]$$

   For a particular Q value, q can take any value from $-Q$ to $+Q$ and defines different disconnected sectors of the larger gauge-invariant LSH Hilbert space.

- **Charge conjugation symmetry:** The particle anti-particle symmetry of the theory identifies (Q, q) sector of the Hamiltonian to the $(Q, -q)$ sector.

Global symmetries of the LSH framework: SU(3)

$$H_I(r,r+1) = \left\{\hat{\chi}_o^\dagger(\hat{\Lambda}_P^+)^{\hat{\nu}_m}\sqrt{1-\frac{\hat{\nu}_m}{\hat{n}_P+2}}\sqrt{1-\frac{\hat{\nu}_i}{\hat{n}_P+\hat{n}_Q+3}}\right\}_r \left\{\sqrt{1+\frac{\hat{\nu}_m}{\hat{n}_P+1}}\sqrt{1+\frac{\hat{\nu}_i}{\hat{n}_P+\hat{n}_Q+2}}\hat{\chi}_o(\hat{\Lambda}_P^+)^{1-\hat{\nu}_m}\right\}_{r+1} + \text{H.c.}$$

$$+\left\{\hat{\chi}_i^\dagger(\hat{\Lambda}_Q^-)^{1-\hat{\nu}_m}\sqrt{1+\frac{\hat{\nu}_m}{\hat{n}_Q+1}}\sqrt{1+\frac{\hat{\nu}_o}{\hat{n}_P+\hat{n}_Q+2}}\right\}_r \left\{\sqrt{1-\frac{\hat{\nu}_m}{\hat{n}_Q+2}}\sqrt{1-\frac{\hat{\nu}_o}{\hat{n}_P+\hat{n}_Q+3}}\hat{\chi}_i(\hat{\Lambda}_Q^-)^{\hat{\nu}_m}\right\}_{r+1} + \text{H.c.}$$

$$+\left\{\hat{\chi}_m^\dagger(\hat{\Lambda}_P^-)^{1-\hat{\nu}_o}(\hat{\Lambda}_Q^+)^{\hat{\nu}_i}\sqrt{1+\frac{\hat{\nu}_o}{\hat{n}_P+1}}\sqrt{1-\frac{\hat{\nu}_i}{\hat{n}_Q+2}}\right\}_r \left\{\sqrt{1-\frac{\hat{\nu}_o}{\hat{n}_P+2}}\sqrt{1+\frac{\hat{\nu}_i}{\hat{n}_Q+1}}\hat{\chi}_m(\hat{\Lambda}_P^-)^{\hat{\nu}_o}(\hat{\Lambda}_Q^+)^{1-\hat{\nu}_i}\right\}_{r+1} + \text{H.c.}$$

The super-selection sectors of the LSH Hamiltonian are defined by:

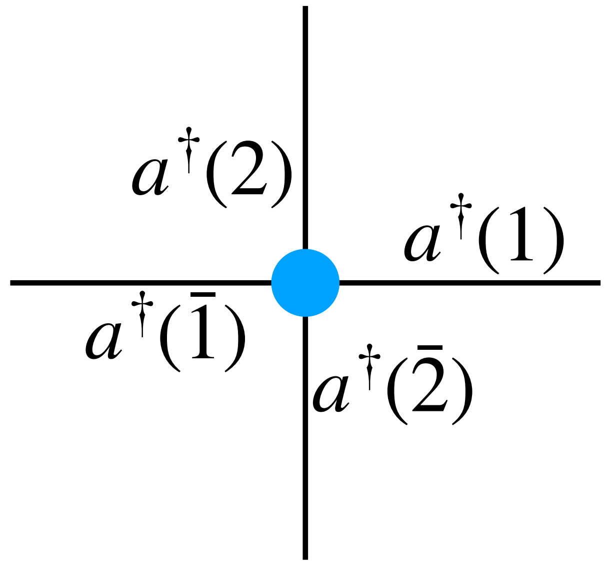
$$\sum_{r=0}^{L-1}\nu_i(r), \quad \sum_{r=0}^{L-1}\nu_m(r), \quad \sum_{r=0}^{L-1}\nu_o(r).$$

Or equivalently by:

$$Q = \sum_{r=0}^{L-1}[\nu_i(r)+\nu_m(r)+\nu_o(r)] \qquad P_{out} = \sum_{r=0}^{L-1}(\nu_m(r)-\nu_i(r)) \quad , \quad Q_{out} = \sum_{r=0}^{L-1}(\nu_o(r)-\nu_m(r))$$

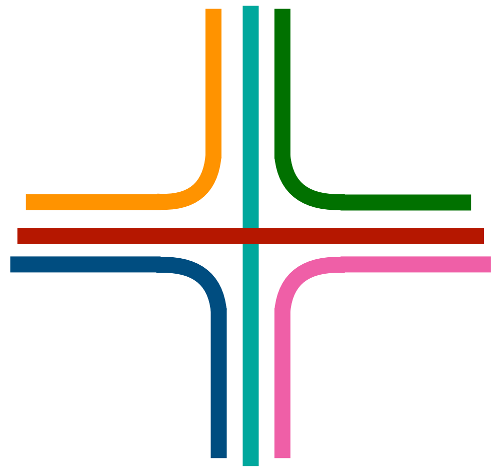
SU(2) LSH framework in $d > 1$

Prepotential Formulation for 2+1 d:



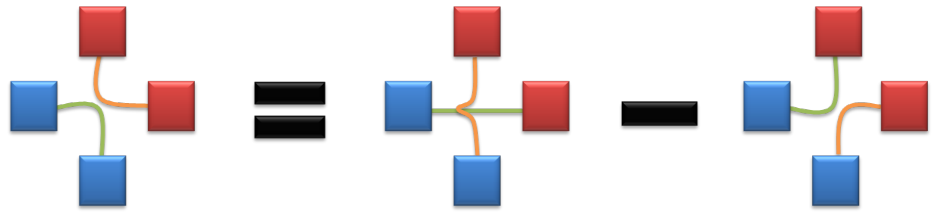
Local Loop Operator: $\mathcal{L}^{++}_{ij} = \epsilon^{\alpha\beta} a^\dagger_\alpha(i) a^\dagger_\beta(j)$

Pictorial representation:



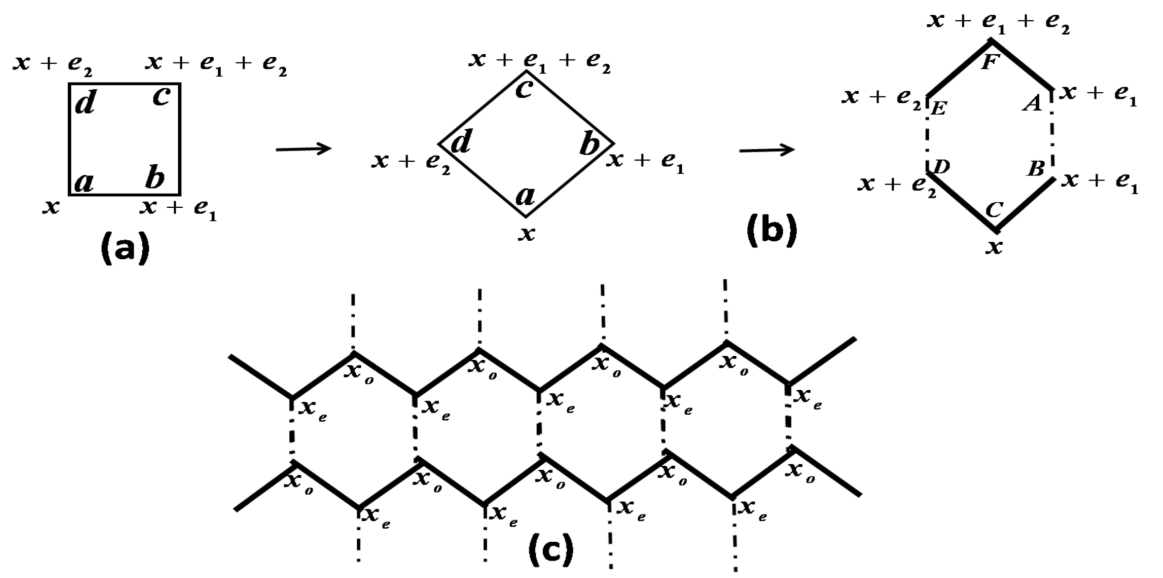
Overcomplete

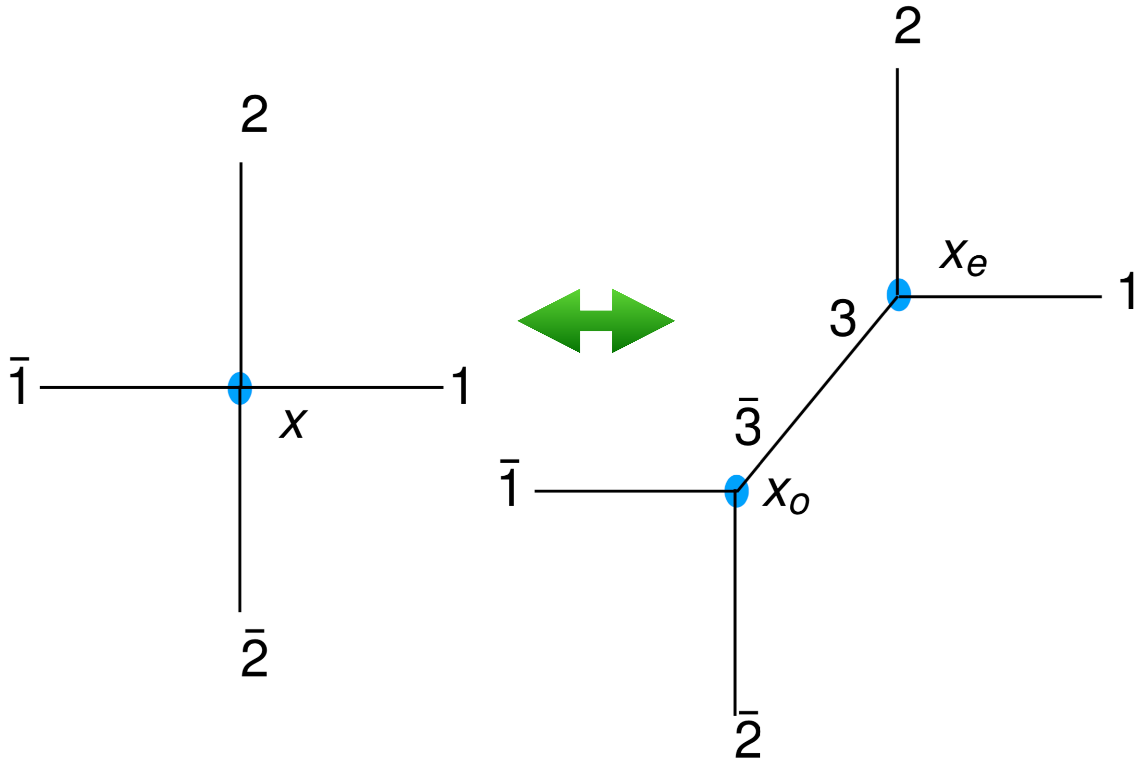
3 physical d.o.f = 6 (local loop quantum numbers in 2d)
- 2(Abelian Gauss' law constraint along 2 link directions)
-1 (**Mandelstam constraint**)

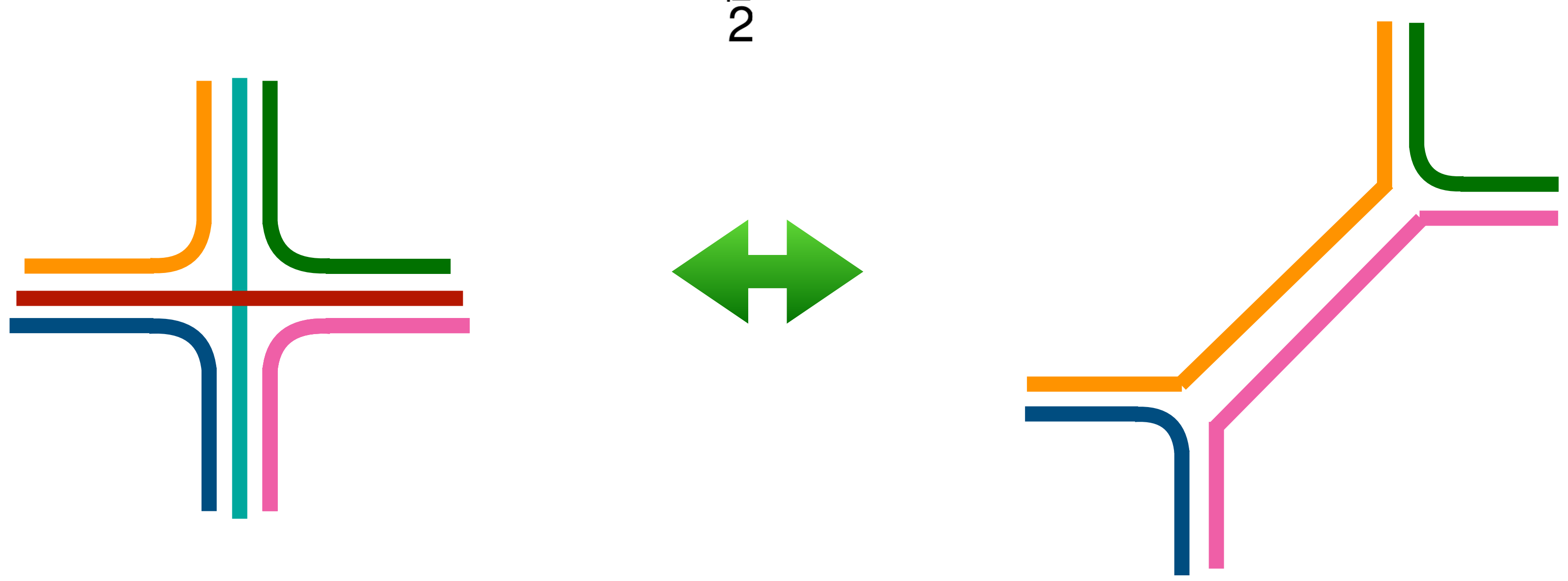


Non-linear constraints, become increasingly complicated with increasing dimension

Way out? Virtual point splitting scheme:



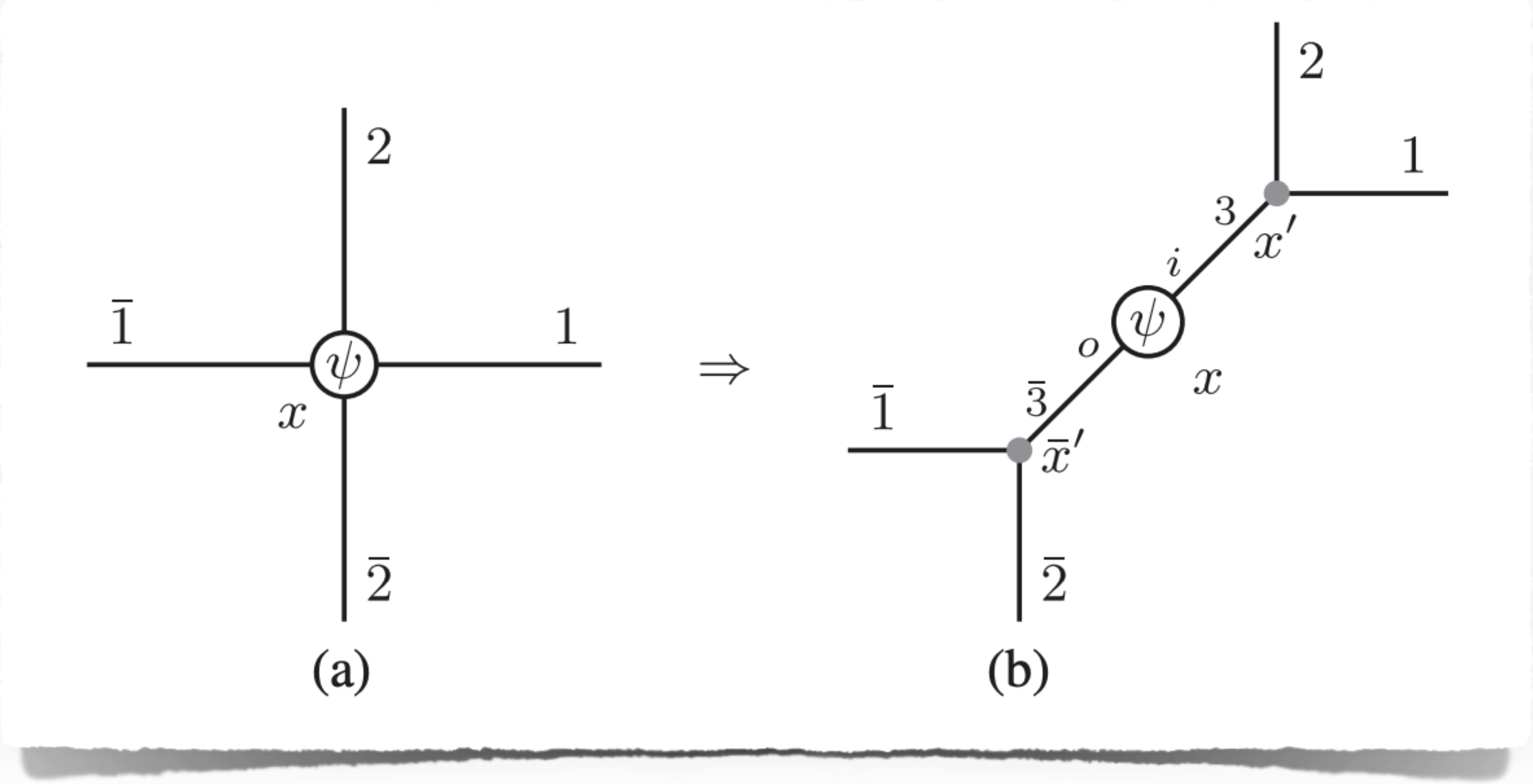


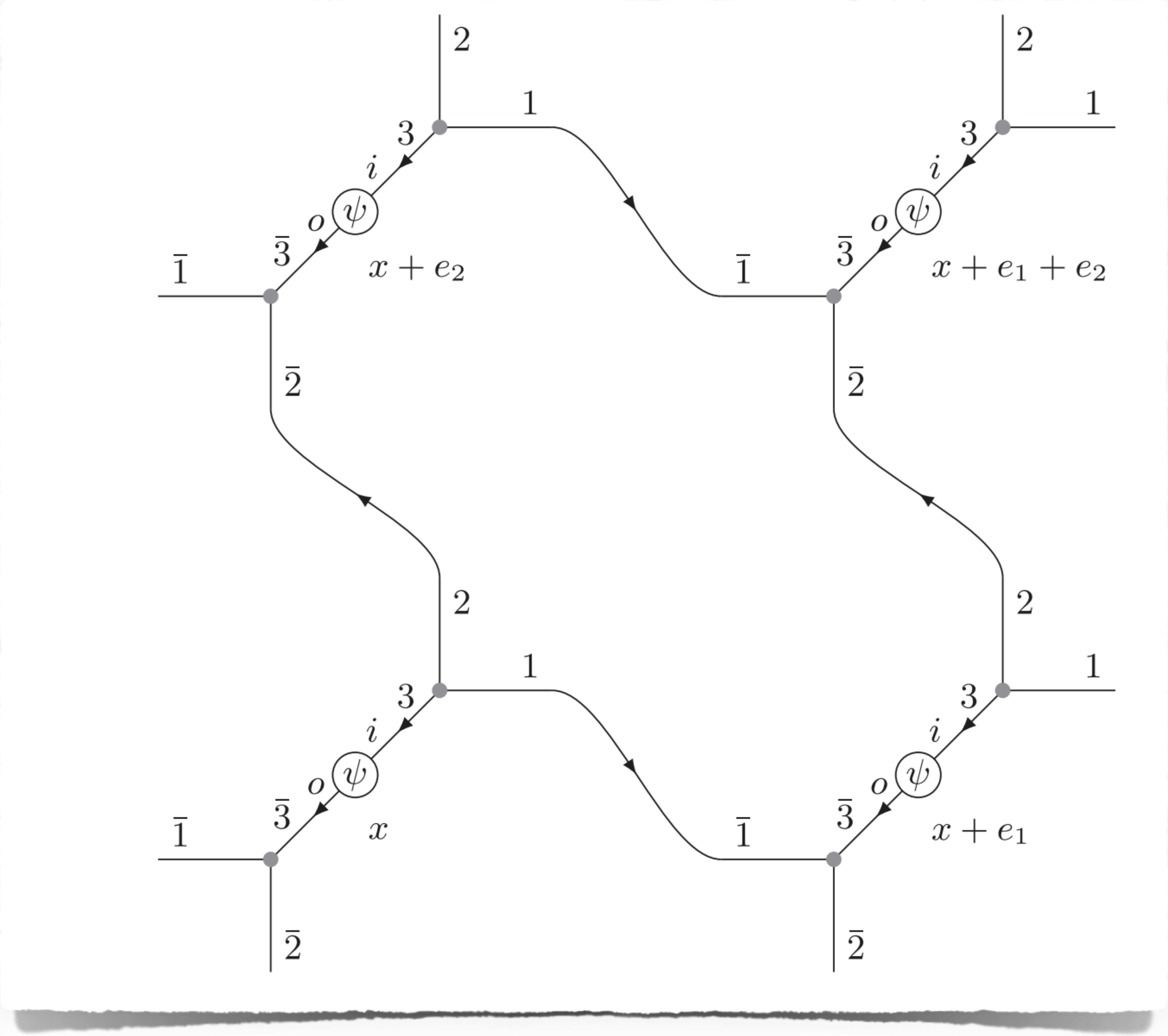


3 physical d.o.f = 2 x 3 (local loop quantum numbers in 2d)
- 3(Abelian Gauss' law constraint)
+ **0** (**Mandelstam constraint**)

Generalized for arbitrary dimension!

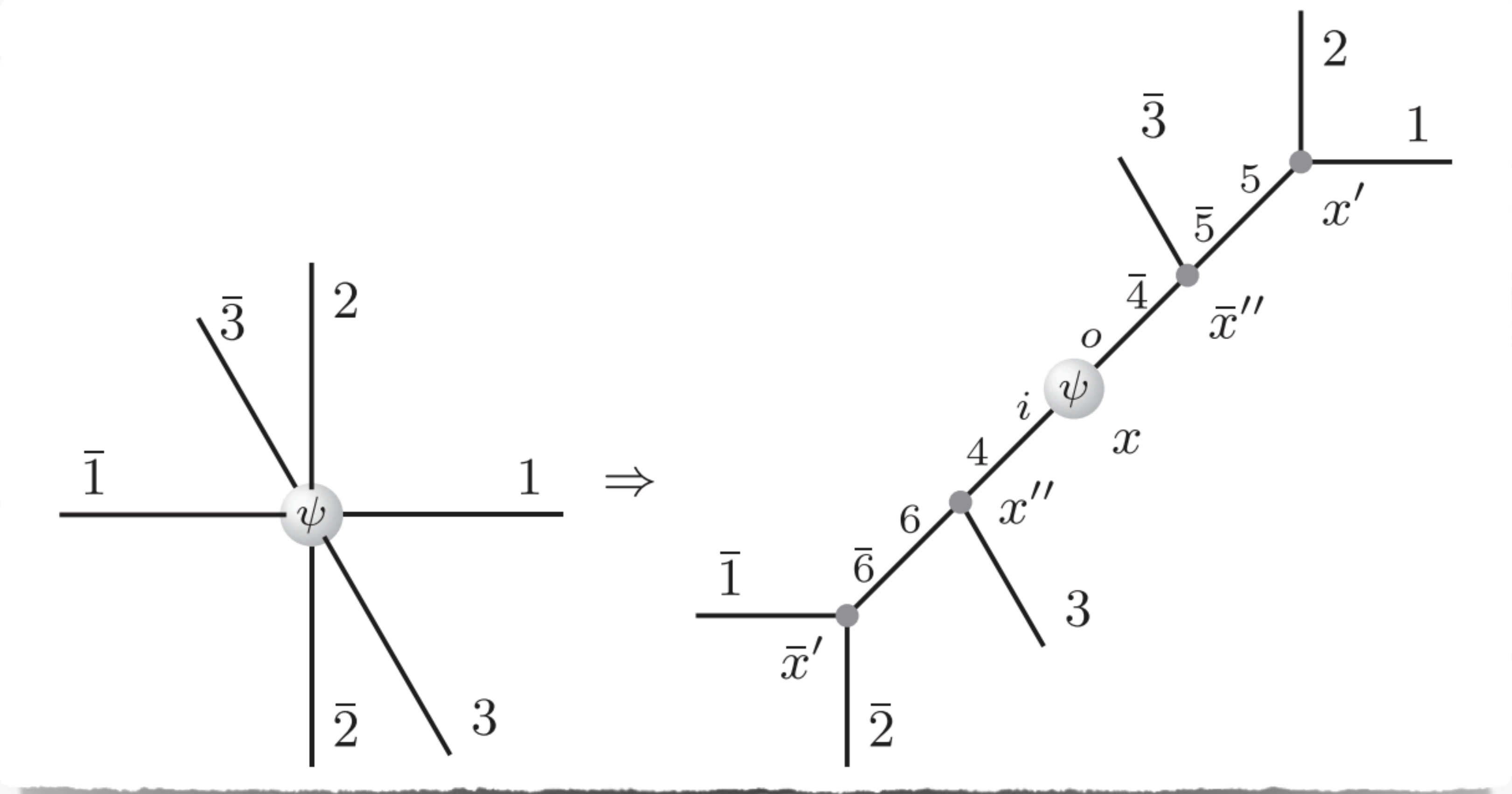
SU(2) LSH Formalism: 2+1 d





Matter-Gauge interactions are same as in 1d

SU(2) LSH Formalism: 3+1 d



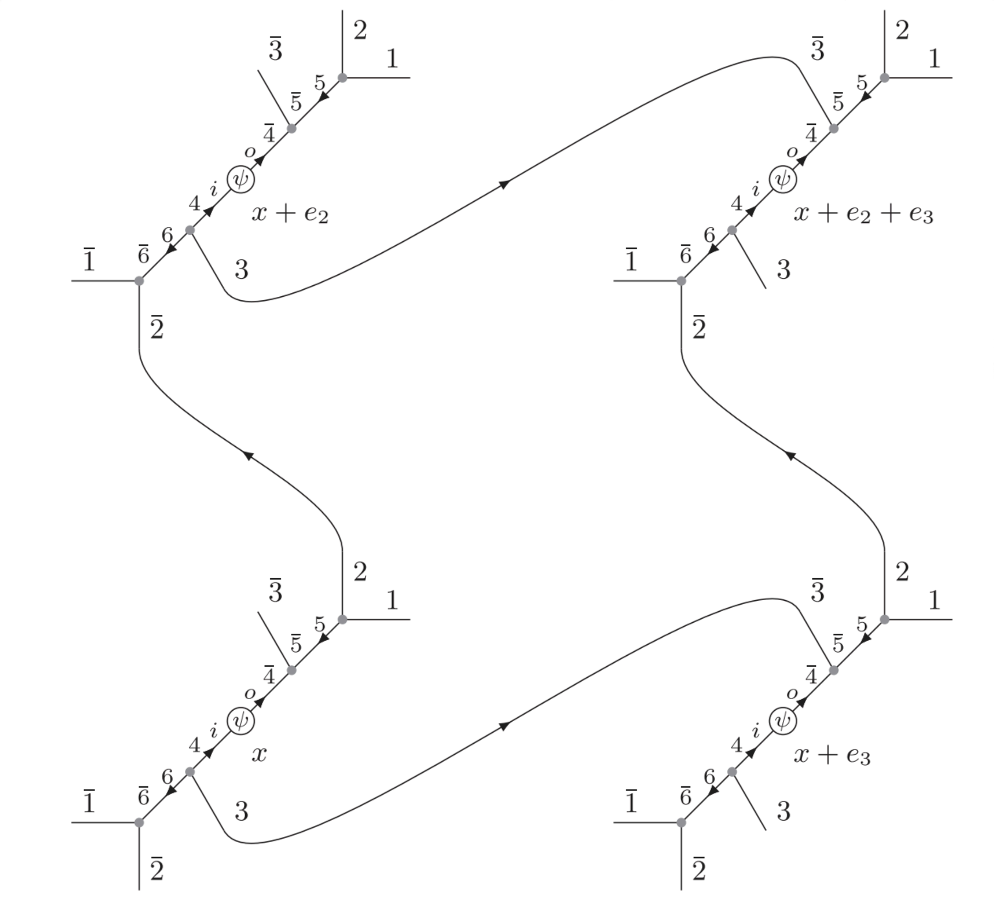


FIG. 8. Connectivity of a *yz*-plaquette in three dimensions.

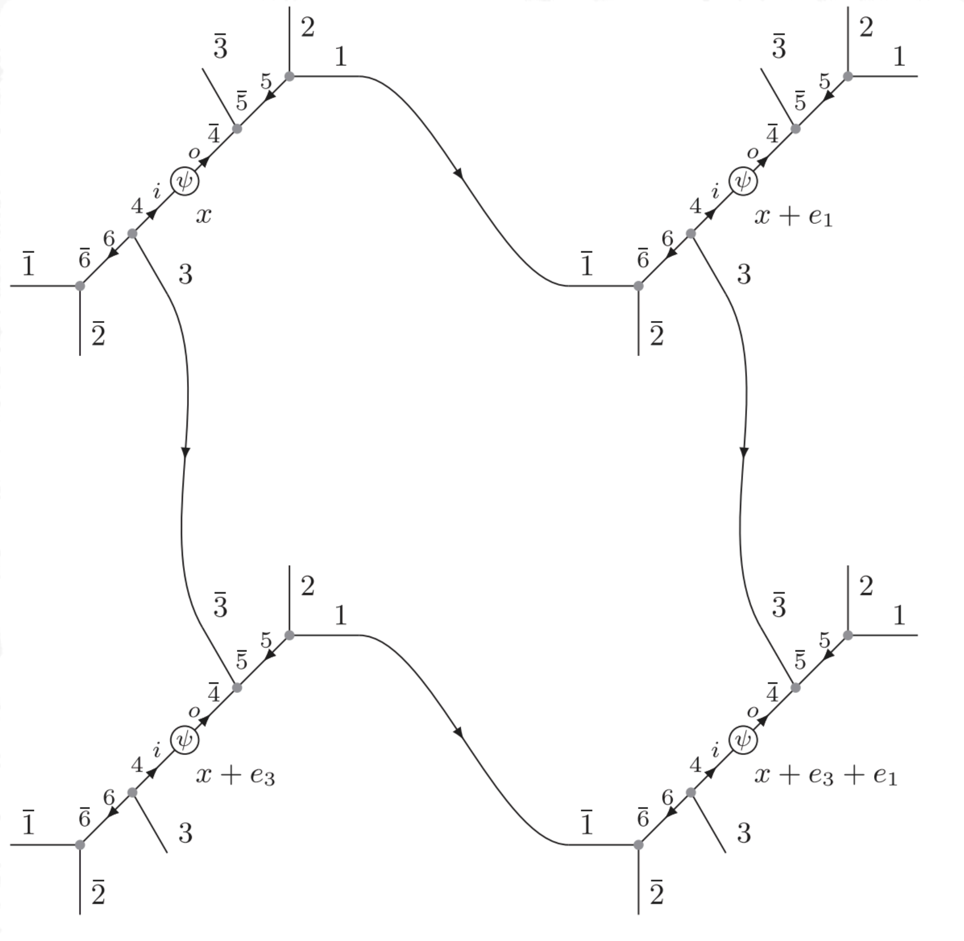


FIG. 9. Connectivity of a *zx*-plaquette in three dimensions.

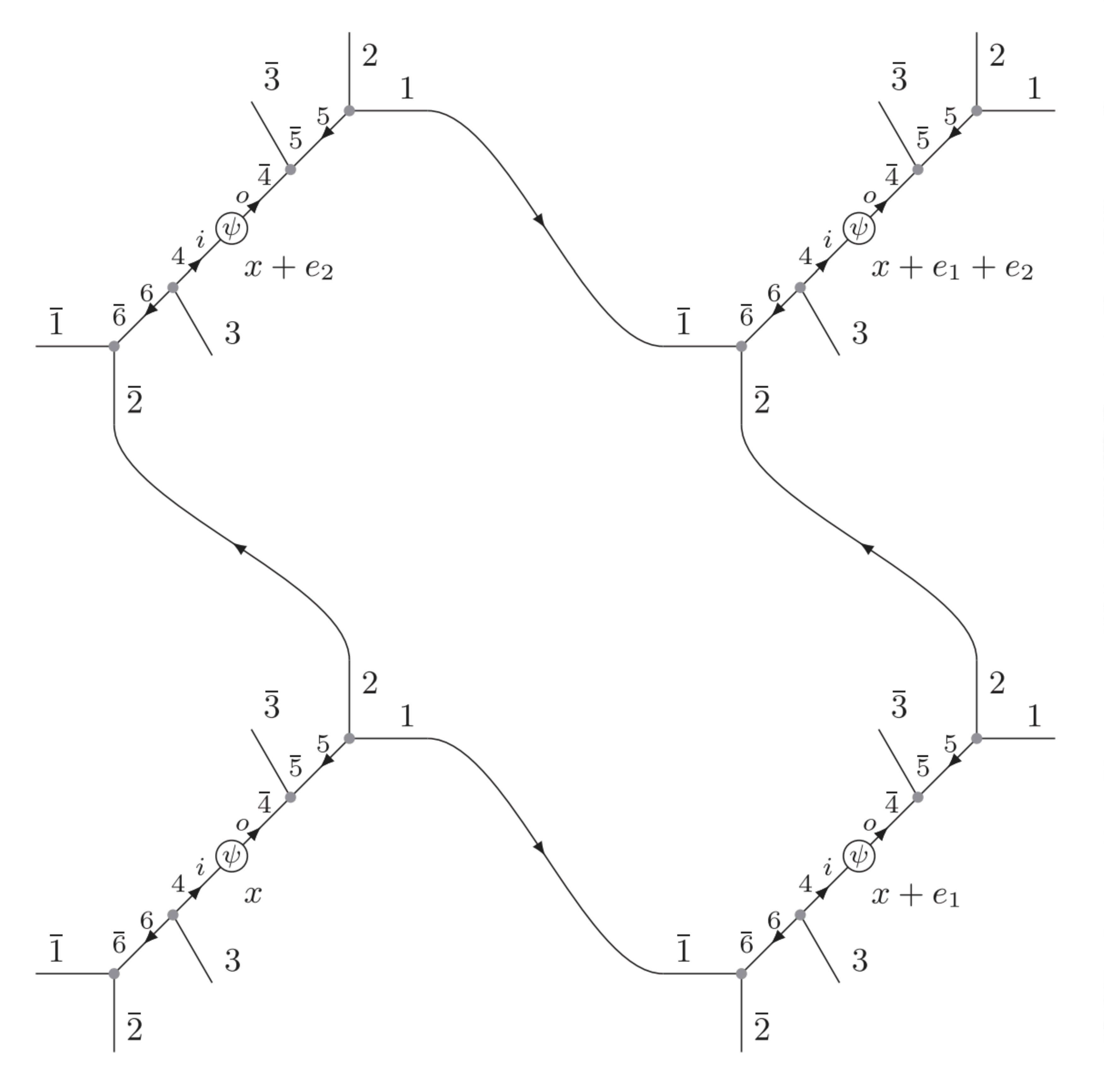


FIG. 7. Connectivity of a *xy*-plaquette in three dimensions.

- Matter-Gauge interactions are same as in 1+1d
- Pure gauge interactions are same as in 2+1d