Extreme Universe
A New Paradigm for Spacetime and Matter from Quantum Information
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GRADUATE SCHOOL OF / FACULTY OF SCIENCE
KYOTO UNIVERSITY

Binary-coupling sparse Sachdev-Ye-Kitaev model

"Nonperturbative and Numerical Approaches to Quantum Gravity, String Theory, and Holography"

ICTS-TIFR, Bengaluru (via online)

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Masaki TEZUKA (Kyoto University)

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  - Onur Oktay, Masanori Hanada (U. Surrey)
  - Enrico Rinaldi, Franco Nori (RIKEN and U. Michigan)
- Collaborators in related works

“Quantum error correction in SYK-like models” in preparation

- Yoshifumi Nakata

“Chaotic-Integrable Transition in the Sachdev-Ye-Kitaev Model” PRL 120, 241603 (2018)

- Antonio M. García-García, Bruno Loureiro, Aurelio Romero-Bermúdez

“Minimal model of many-body localization” PRResearch 3, 013023 (2021)

- Felipe Monteiro, Tobias Micklitz, and Alexander Altland

“Quantum Ergodicity in the Many-Body Localization Problem” PRL 127, 030601 (2021)

- Felipe Monteiro, Alexander Altland, David A. Huse, and Tobias Micklitz

Summary [arXiv:2208.12098]

The binary-coupling sparse SYK model for N Majorana fermions,

$$\hat{H} \propto \sum_{1 \le a < b < c < d \le N} x_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d \,,$$

$$x_{abcd} = \begin{cases} 1 & (\text{probability } p/2) \\ -1 & (\text{probability } p/2) \\ 0 & (\text{probability } 1-p) \end{cases} , \{\hat{\chi}_a, \hat{\chi}_b\} = 2\delta_{ab},$$

obeys random-matrix eigenvalue statistics for $K_{\mathrm{cpl}} = \binom{N}{4} p \gtrsim N$.

Contents

- Introduction: Sachdev-Ye-Kitaev (SYK) model
  - Spectral form factor
- (Gaussian, Binary, Unary)-coupling sparse SYK models
- Summary

Masaki Tezuka, Onur Oktay, Masanori Hanada, Enrico Rinaldi, and Franco Nori, “Binary-coupling sparse SYK: an improved model of quantum chaos and holography”, arXiv:2208.12098

Sachdev-Ye-Kitaev model

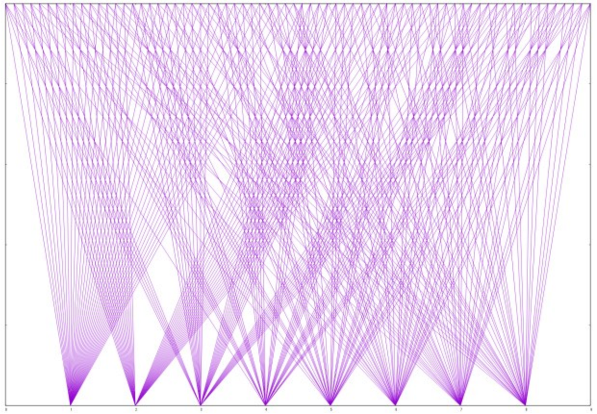
[Kitaev, talks at KITP (2015)]
[Sachdev, PRX (2015)]
[Sachdev and Ye, PRL (1993)]

$$\hat{H}_{\mathrm{SYK}_4} = \sum_{1 \le a < b < c < d \le N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

$\hat{\chi}_{a=1,2,\ldots,N}$: N Majorana fermions ($\{\hat{\chi}_a, \hat{\chi}_b\} = 2\delta_{ab}$)

J_{abcd} : independent Gaussian random couplings ($\overline{J_{abcd}{}^2} = J^2,\ \overline{J_{abcd}} = 0$)

$\binom{N}{4} \cong \frac{N^4}{24}$ terms

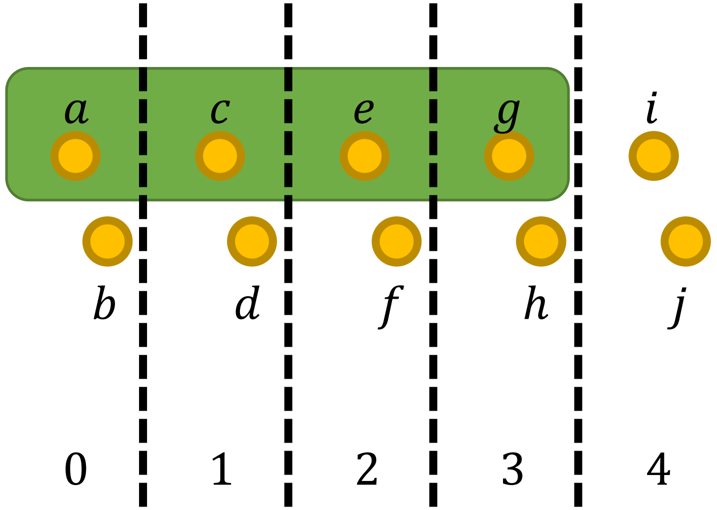


N Majorana fermions

- Solvable in the large-N limit [Maldacena, Shenker, and Stanford, JHEP **1608**(2016)106]
- Maximally chaotic ($\lambda_{\mathrm{Lyapunov}} \xrightarrow{\text{low } T} 2\pi k_{\mathrm{B}} T/\hbar$: chaos bound)
- Correspondence to 1+1d gravity, random matrix

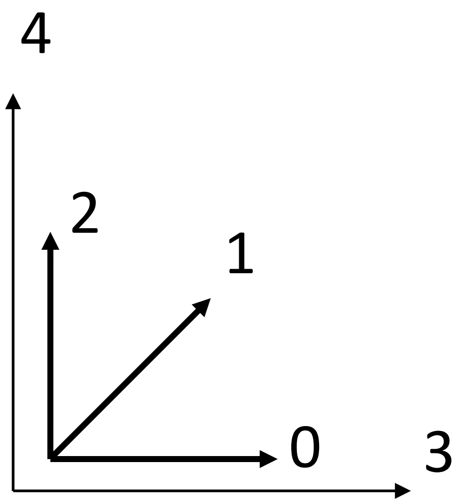
One term of the 10-Majorana fermion $\text{SYK}_{q=4}$

$\chi_a \chi_c \chi_e \chi_g$

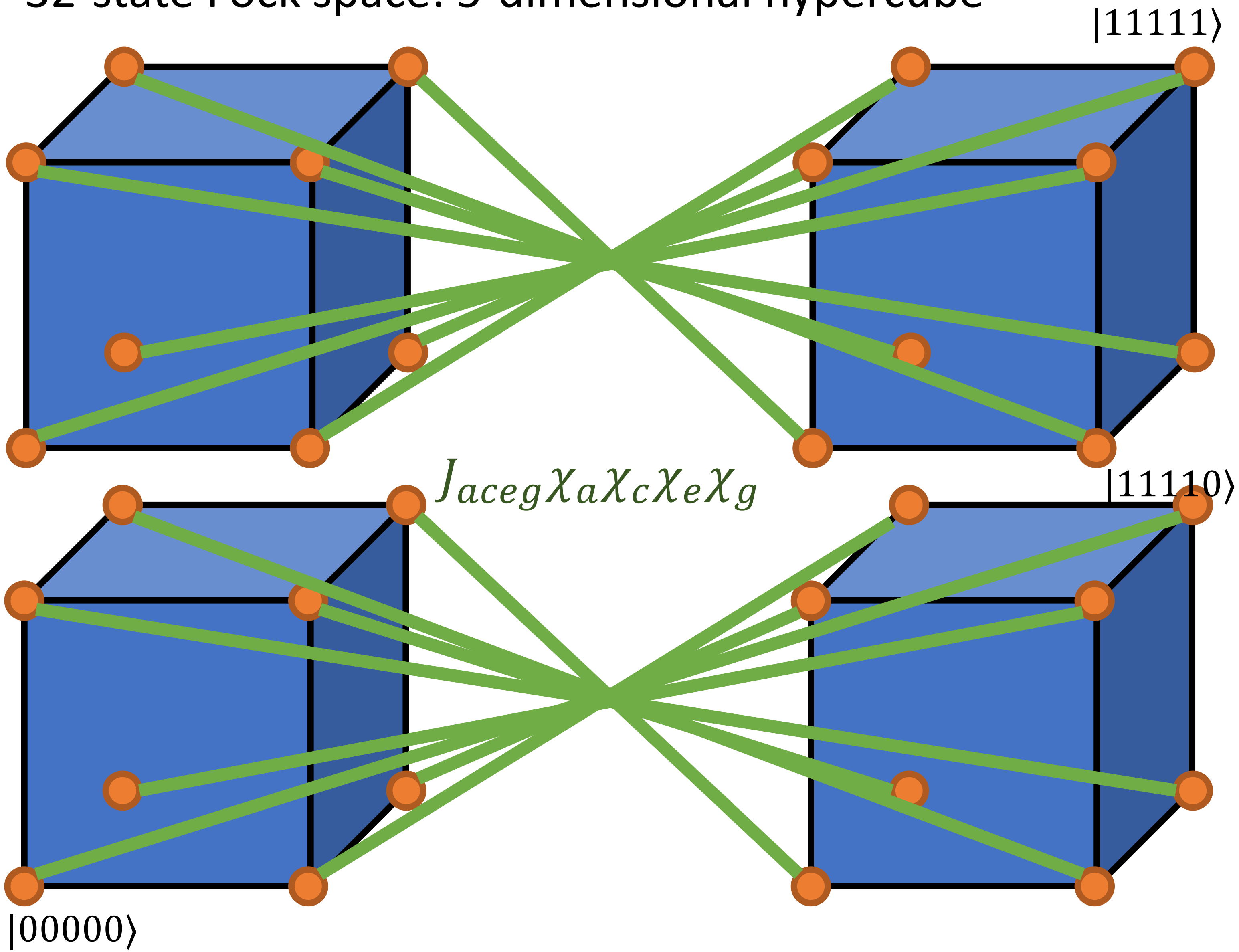


5 qubits

$\binom{10}{4} = 210$ terms



32-state Fock space: 5-dimensional hypercube



Sachdev-Ye-Kitaev model

N Majorana- or Dirac- fermions randomly coupled to each other

[Majorana version]

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a<b<c<d \le N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

[A. Kitaev: talks at KITP (2015)]

[Dirac version]

$$\hat{H} = \frac{1}{(2N)^{3/2}} \sum_{ij;kl} J_{ij;kl} \hat{c}_i{}^{\dagger} \hat{c}_j{}^{\dagger} \hat{c}_k \hat{c}_l$$

[A. Kitaev's talk]

[S. Sachdev: PRX **5**, 041025 (2015)]

cf. SY model [S. Sachdev and J. Ye, 1993]

>1300 citations after 2015

Studied for long time in the nuclear theory context

[French and Wong (1970)][Bohigas and Flores (1971)]

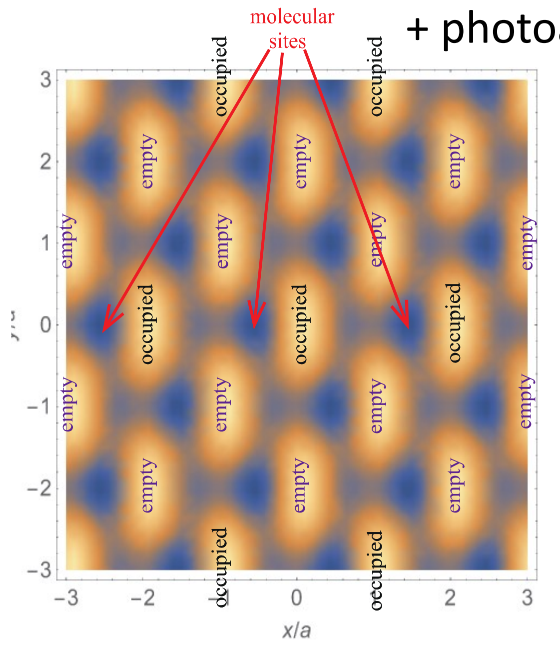
"Two-body Random Ensemble"

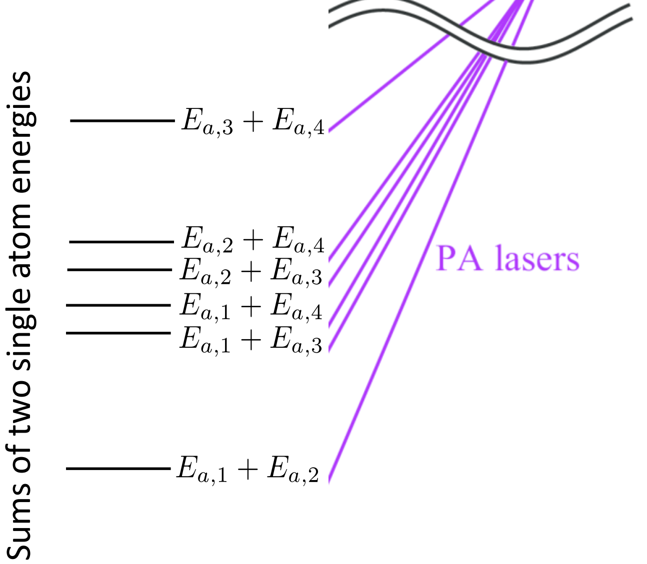
→ Many variants of the model: bosonic, multiflavor, supersymmetric, nonhermitian, …

Proposals for experimental realization

[I. Danshita, M. Hanada, MT: PTEP **2017**, 083I01 (2017)]

Ultracold fermions in optical lattice
+ photoassociation lasers





s: molecular levels

$$\hat{H}_{\rm m} = \sum_{s=1}^{n_s} \left\{ \nu_s \hat{m}_s^\dagger \hat{m}_s + \sum_{i,j} g_{s,ij} \left(\hat{m}_s^\dagger \hat{c}_i \hat{c}_j - \hat{m}_s \hat{c}_i^\dagger \hat{c}_j^\dagger \right) \right\}.$$

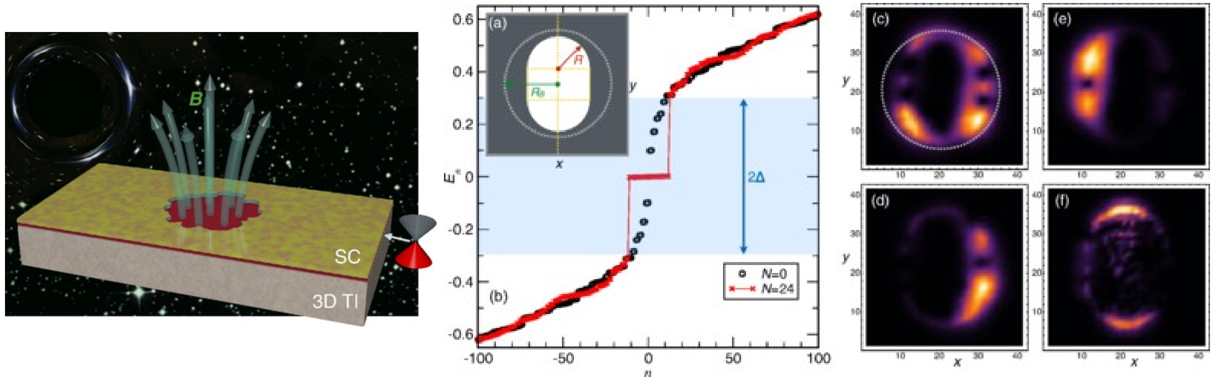
$$|\nu_s| \gg |g_{s,ij}|$$

$$\hat{H}_{\rm eff} = \boxed{\sum_{s,i,j,k,l} \frac{g_{s,ij} g_{s,kl}}{\nu_s}} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l.$$

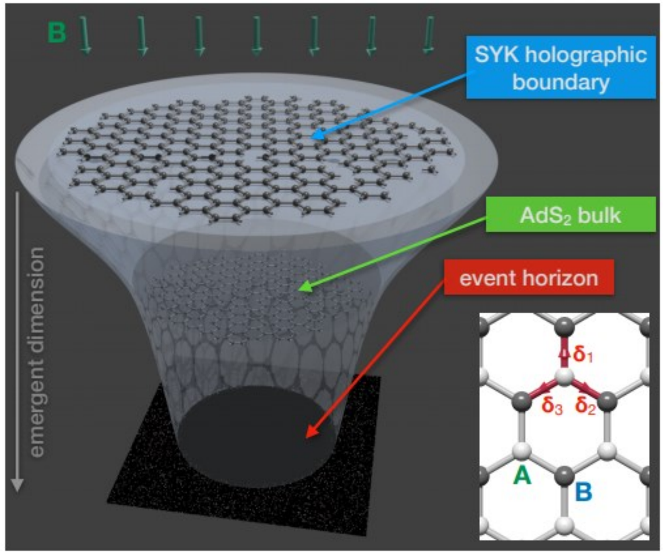
Approximately Gaussian if many levels are used

[D. I. Pikulin and M. Franz, PRX **7**, 031006 (2017)]

N quanta of magnetic flux through a nanoscale hole



[A. Chen, R. Ilan, F. de Juan, D.I. Pikulin, M. Franz, PRL **121**, 036403 (2018)]



Graphene flake with an irregular boundary in magnetic field

NMR experiment for the SYK model

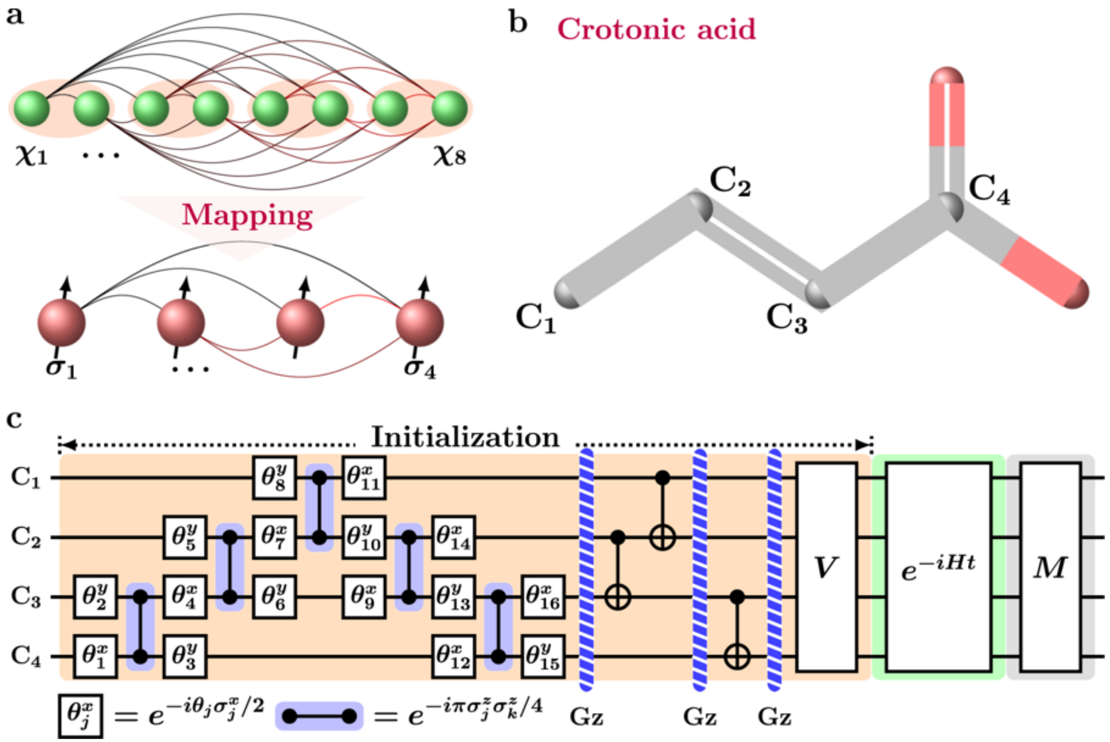
"Quantum simulation of the non-fermi-liquid state of Sachdev-Ye-Kitaev model" Zhihuang Luo, Yi-Zhuang You, Jun Li, Chao-Ming Jian, Dawei Lu, Cenke Xu, Bei Zeng and Raymond Laflamme, npj Quantum Information **5**, 53 (2019)

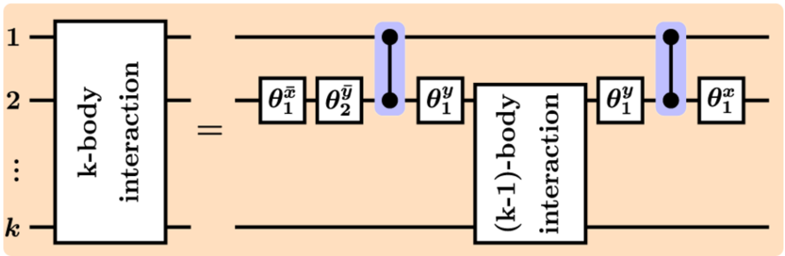
$$H = \frac{J_{ijkl}}{4!}\chi_i\chi_j\chi_k\chi_l + \frac{\mu}{4}C_{ij}C_{kl}\chi_i\chi_j\chi_k\chi_l$$

$$\chi_{2i-1} = \frac{1}{\sqrt{2}}\sigma_x^1\sigma_x^2\cdots\sigma_x^{i-1}\sigma_z^i,\ \chi_{2i} = \frac{1}{\sqrt{2}}\sigma_x^1\sigma_x^2\cdots\sigma_x^{i-1}\sigma_y^i.$$

$$H = \sum_{s=1}^{70} H_s = \sum_{s=1}^{70} a^s_{ijkl}\sigma^1_{\alpha_i}\sigma^2_{\alpha_j}\sigma^3_{\alpha_k}\sigma^4_{\alpha_l}$$

$$e^{-iH\tau} = \left(\prod_{s=1}^{70} e^{-iH_s\tau/n}\right)^n + \sum_{s<s'}\frac{[H_s, H_{s'}]\tau^2}{2n} + O(|a|^3\tau^3/n^2),$$





N: number of fermions

SYK: Solvable in the $N \gg 1$ limit (after sample average $\langle\cdots\rangle_{\{J\}}$)

Non-perturbative Hamiltonian = 0,

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a<b<c<d \le N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

as perturbation

Free two-point function

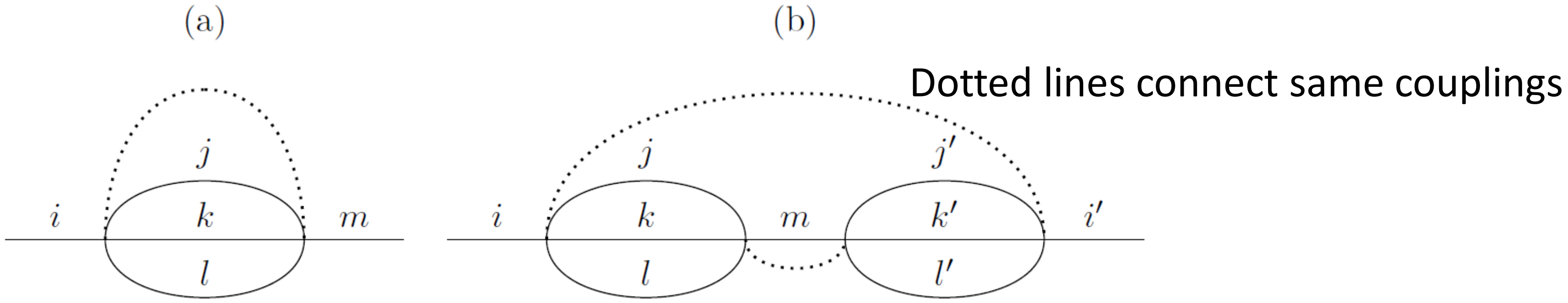
$$G_{0,ij}(t) = -\langle \mathrm{T} \chi_i(t) \chi_j(0) \rangle = -\mathrm{sgn}(t) \delta_{ij}$$

$\langle {J_{abcd}}^2 \rangle_{\{J\}} = J^2$, Gaussian distribution

$\langle J_{abcd} J_{abce} \rangle_{\{J\}} = 0$ if $d \neq e$ ➔ Most diagrams average to zero

Only "melon-type" diagrams survive sample averaging

From Wikimedia Commons (By Aravind Sivaraj (2012))



$O(1)$ melon

$O(N^{-2})$ not melon

Feynman diagrams

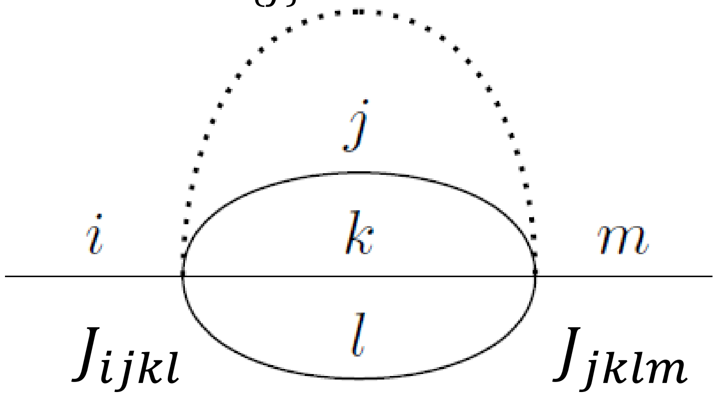
[J. Polchinski and V. Rosenhaus, JHEP **1604** (2016) 001]
[J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)]

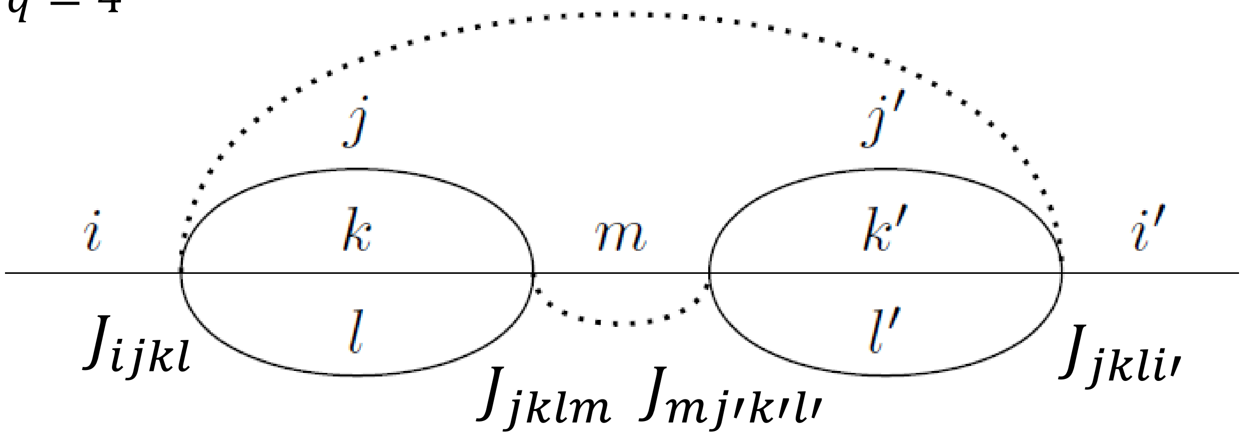
$$\hat{H}_{\mathrm{SYK4}} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1\le a<b<c<d\le N} J_{abcd}\underbrace{\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d}_{q\,=\,4}$$

$$\{\hat{\chi}_a, \hat{\chi}_b\} = 2\delta_{ab}$$

$$\langle J_{abcd}\rangle^2 = J^2 = 1$$

Sample average $\langle\cdots\rangle_{\{J\}}$





$$\sum_{jkl} \langle J_{ijkl} J_{jklm}\rangle_{\{J\}} = \frac{N^3}{3!}\delta_{im}$$

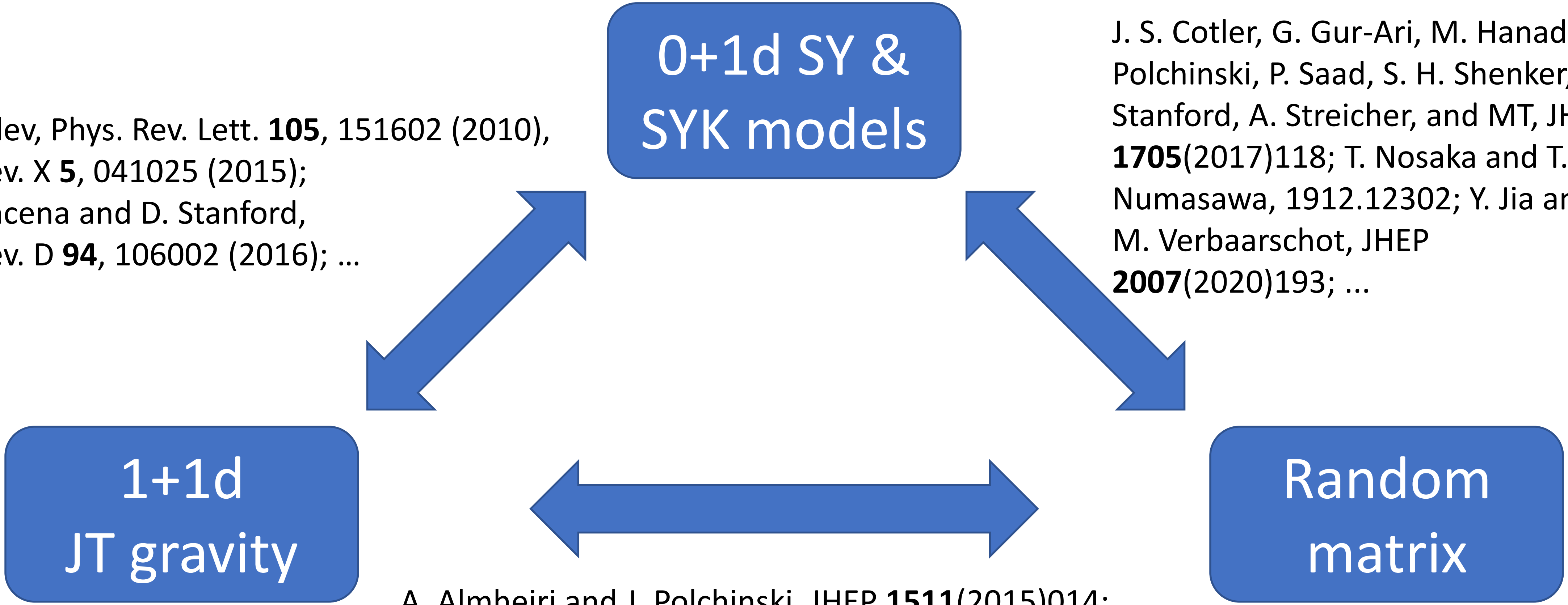
$$\sum_{m\neq i}\ \sum_{jklj'k'l'} \langle J_{ijkl} J_{jklm} J_{m j' k' l'} J_{j'k'l'i'}\rangle_{\{J\}} \propto N^4 \delta_{ii'}$$

➡ $O(N^0)$ contribution

➡ $O(N^{-2})$ contribution

Large-*N*: "Melon diagrams" dominate

Maximally chaotic systems



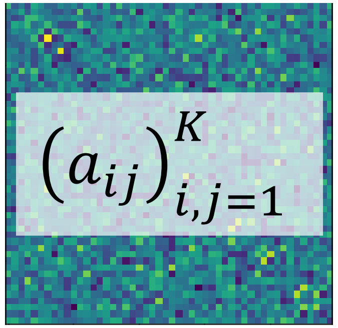
S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010), Phys. Rev. X **5**, 041025 (2015); J. Maldacena and D. Stanford, Phys. Rev. D **94**, 106002 (2016); …

J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and MT, JHEP **1705**(2017)118; T. Nosaka and T. Numasawa, 1912.12302; Y. Jia and J. J. M. Verbaarschot, JHEP **2007**(2020)193; ...

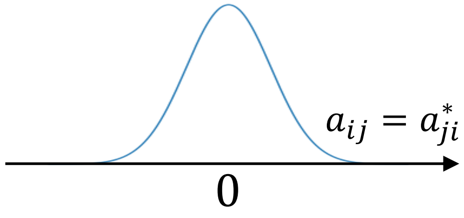
A. Almheiri and J. Polchinski, JHEP **1511**(2015)014;
P. Saad, S. H. Shenker, and D. Stanford, arXiv:1903.11115;
D. Stanford and E. Witten, arXiv:1907.03363; …

Gaussian random matrices

Real ($\beta = 1$): Gaussian Orthogonal Ensemble (GOE)
Complex ($\beta = 2$): G. Unitary E. (GUE)
Quaternion ($\beta = 4$): G. Symplectic E. (GSE)



Gaussian distribution



Density $\propto e^{-\frac{\beta K}{4}\mathrm{Tr}H^2} = \exp\left(-\frac{\beta K}{4}\sum_{i,j}^{K}|a_{ij}|^2\right)$

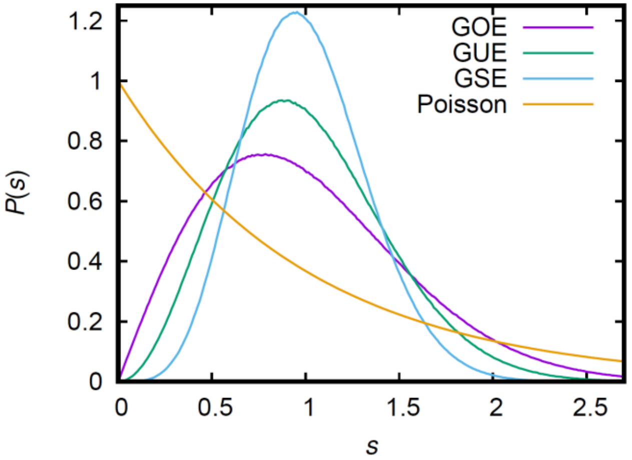
Joint distribution function for energy eigenvalues $\{e_j\}$

Level repulsion

$$p(e_1, e_2, \dots, e_K) \propto \prod_{1\le i<j\le K} |e_i - e_j|^{\beta} \prod_{i=1}^{K} e^{-\beta K e_i^2/4}$$

Distribution of normalized level separation $s_j = \frac{e_{j+1} - e_j}{\Delta(\bar{e})}$

GOE/GUE/GSE: $P(s) \propto s^{\beta}$ at small s, has e^{-s^2} tail



Uncorrelated: $P(s) = e^{-s}$ (Poisson distribution)

Neighboring gap ratio

$$r = \frac{\min(e_{i+1} - e_i\,,\ \ e_{i+2} - e_{i+1})}{\max(e_{i+1} - e_i\,,\ \ e_{i+2} - e_{i+1})}$$

	Uncorrelated	GOE	GUE	GSE
$\langle r \rangle$	2log 2 – 1 = 0.38629…	0.5307(1)	0.5996(1)	0.6744(1)

[Y. Y. Atas *et al.* PRL 2013]

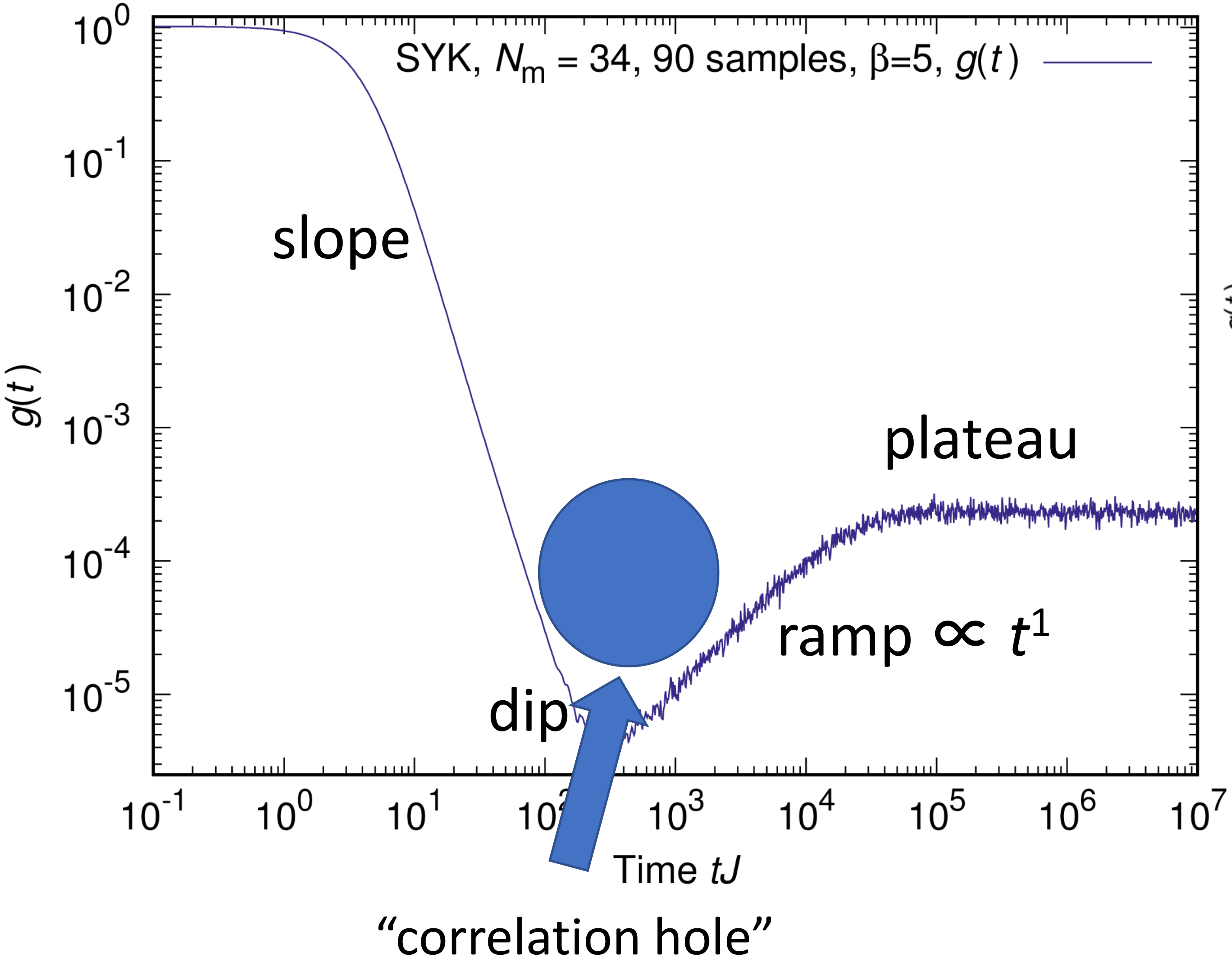
➔ SYK model: level correlation ($P(s), P(r), \langle r \rangle$, etc.) indistinguishable from corresponding Gaussian ensemble

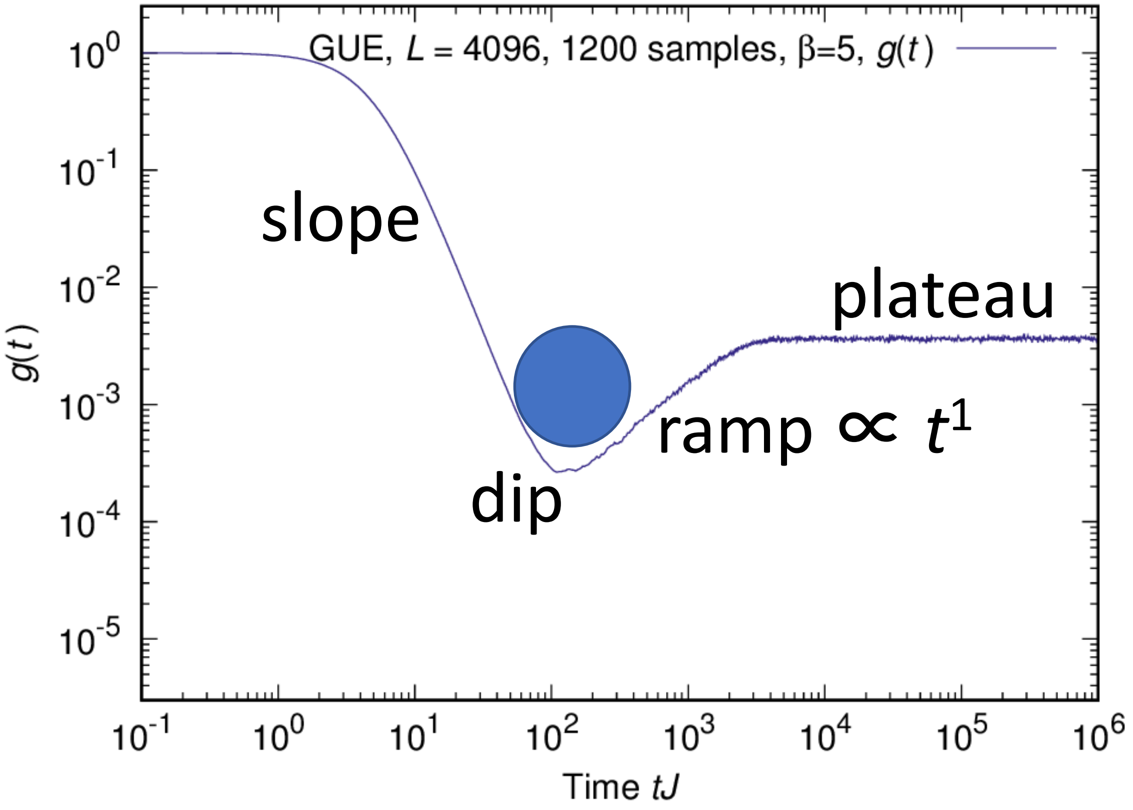
Majorana SYK4 with
$N \equiv 0 \pmod 8$: GOE
$N \equiv 2, 6 \pmod 8$: GUE
$N \equiv 4 \pmod 8$: GSE

[Fidkowski and Kitaev PRB 2010, 2011]
[You, Ludwig, and Xu PRB 2017]

Spectral form factor

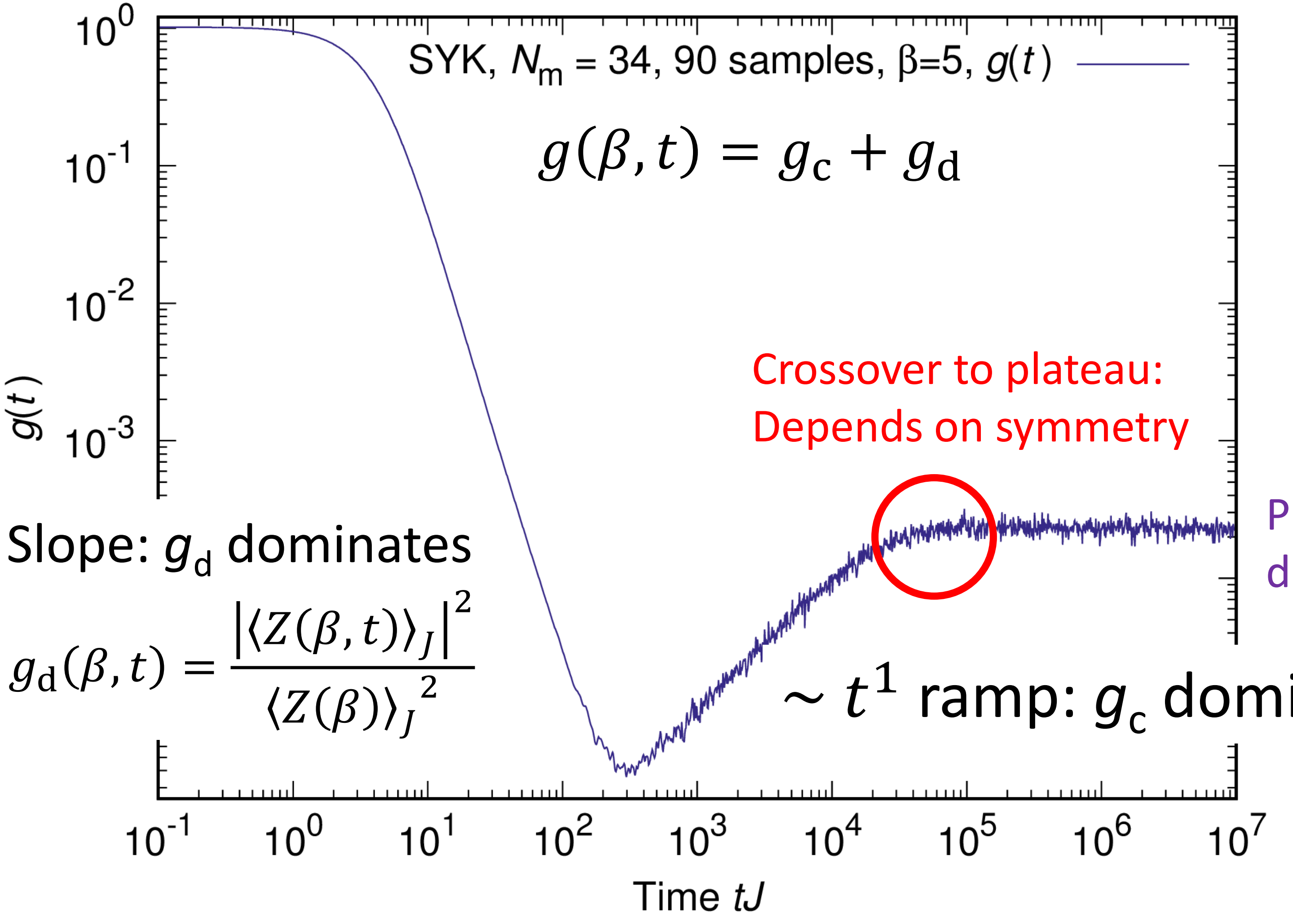
$$g(\beta, t) = \frac{\langle |Z(\beta, t)|^2 \rangle_{\{J\}}}{\langle Z(\beta) \rangle_{\{J\}}{}^2} \qquad \begin{aligned} Z(\beta, t) &= Z(\beta + \mathrm{i}t) \\ &= \mathrm{Tr}\left(\mathrm{e}^{-\beta \hat{H} - \mathrm{i}\hat{H}t}\right) \end{aligned}$$





Slope-dip-ramp-plateau structure

Cotler, Gur-Ari, Hanada, Polchinski, Saad, Shenker, Stanford, Streicher, and MT, JHEP **1705**(2017)118



$$g(\beta,t) = g_\mathrm{c} + g_\mathrm{d}$$

Crossover to plateau:
Depends on symmetry

Slope: g_d dominates

$$g_\mathrm{d}(\beta,t) = \frac{\left|\langle Z(\beta,t)\rangle_J\right|^2}{\langle Z(\beta)\rangle_J^{\,2}}$$

$\sim t^1$ ramp: g_c dominates

$$Z(\beta,t) = \mathrm{Tr}\left(\mathrm{e}^{-\beta\hat{H}-\mathrm{i}\hat{H}t}\right)$$

$$g_\mathrm{c}(\beta,t) = \frac{\langle |Z(\beta,t)|^2\rangle_J - \left|\langle Z(\beta,t)\rangle_J\right|^2}{\langle Z(\beta)\rangle_J^{\,2}}$$

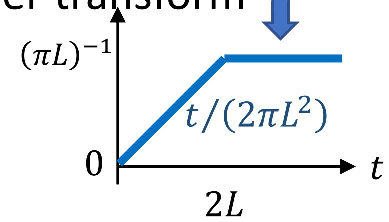
$$\sim \iint d\lambda_1 d\lambda_2 \langle \delta\rho(\lambda_1)\delta\rho(\lambda_2)\rangle e^{\mathrm{i}t(\lambda_1-\lambda_2)}$$

$$\rho(\lambda) = \sum_j \delta(\epsilon_j - \lambda)$$

Plateau height:
determined by degeneracy

$$R(\lambda) = \langle \delta\rho(\lambda_1)\delta\rho(\lambda_1-\lambda)\rangle = \boxed{-\frac{\sin^2 L\lambda}{(\pi L\lambda)^2} + \frac{1}{\pi L}\delta(\lambda)}$$

Fourier transform



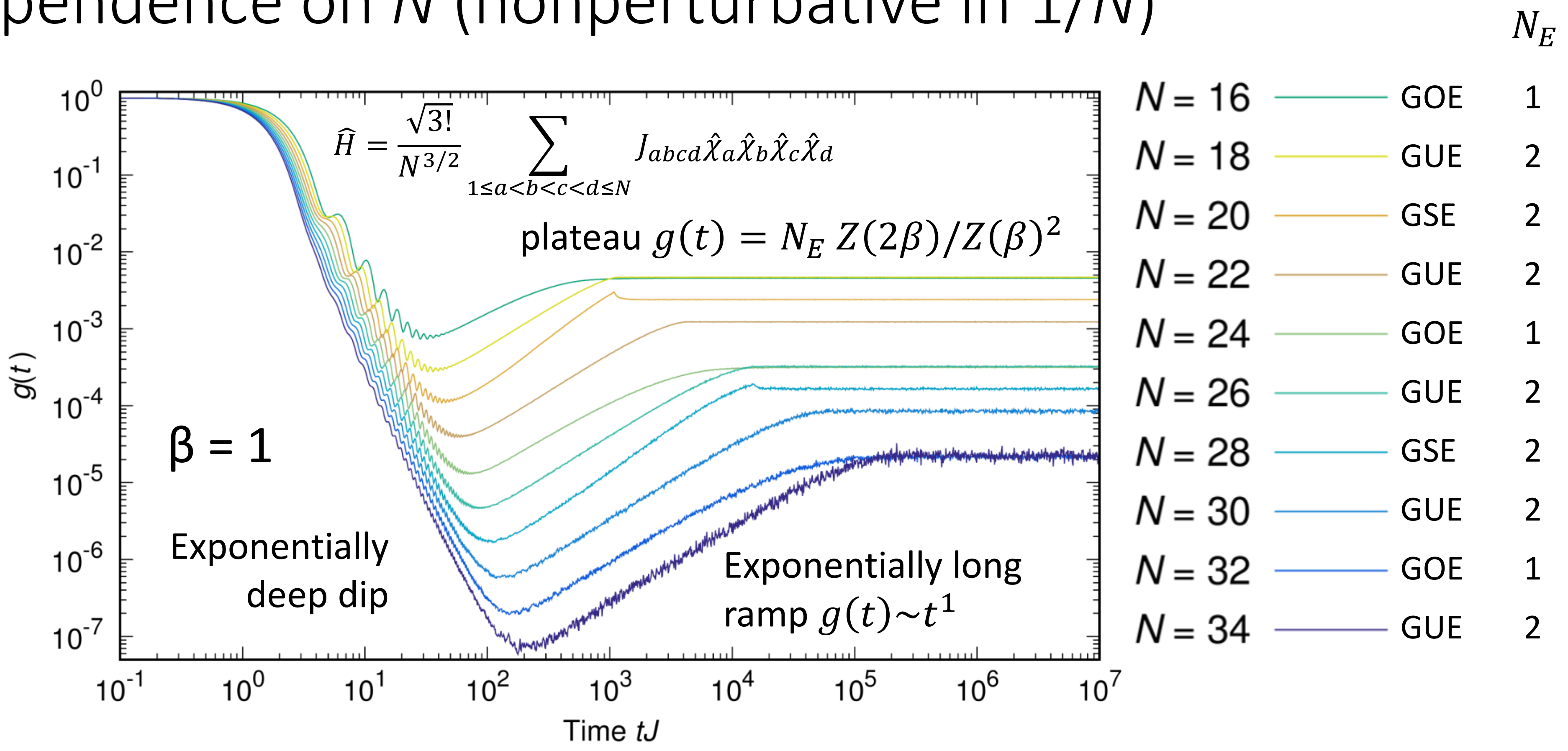
Random matrix theory
(GUE)

Exponentially long $\sim t^1$ ramp

➔ Rigid spectrum of the Sachdev-Ye-Kitaev model

$g(t)$: Dependence on N (nonperturbative in $1/N$)

Cotler et al., JHEP **1705**(2017)118



Classification of SPT order in class BDI: reduced from Z to Z_8 by interaction

[L. Fidkowski and A. Kitaev: PRB **81**, 134509 (2010); PRB **83**, 075103 (2011)]

Many-body level statistics 🡸 corresponding (dense) random matrix ensemble

$N_\chi \pmod 8$	0	1	2	3	4	5	6	7
qdim	1	$\sqrt{2}$	2	$2\sqrt{2}$	2	$2\sqrt{2}$	2	$\sqrt{2}$
lev. stat.	GOE	GOE	GUE	GSE	GSE	GSE	GUE	GOE

[Y.-Z. You, A. W. W. Ludwig, and Cenke Xu, PRB **95**, 115150 (2017)]

Sparse (or pruned) SYK

$$\hat{H} = \sum_{a<b<c<d} x_{abcd} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d \,,\, x_{abcd} = \begin{cases} 1 & \text{(probability } p\text{)} \\ 0 & \text{(probability } 1-p\text{)} \end{cases},\, P(J_{abcd}) = \frac{\exp\left(-\frac{J_{abcd}^2}{2J^2}\right)}{\sqrt{2\pi J^2}}$$

$K_{\text{cpl}} = \binom{N}{4} p$: Number of non-zero x_{abcd}

$K_{\text{cpl}} \sim \mathcal{O}(1) N$ enough for

- Random matrix-like behavior
- Large entropy per fermion at low T !

$$\boxed{p \sim \frac{4!}{N^3} = \mathcal{O}(N^{-3})}$$

- Talk by Brian Swingle at Simons Center (18 September 2019)
- “Sparse Sachdev-Ye-Kitaev model, quantum chaos and gravity duals” A. M. García-García, Y. Jia, D. Rosa, J. J. M. Verbaarschot, Phys. Rev. D **103**, 106002 (2021)
- “A Sparse Model of Quantum Holography” S. Xu, L. Susskind, Y. Su, and B. Swingle, arXiv:2008.02303
- “Spectral Form Factor in Sparse SYK models” E. Cáceres, A. Misobuchi, and A. Raz, JHEP **2208**, 236 (2022)

M. Tezuka, O. Oktay, E. Rinaldi, M. Hanada, and F. Nori, arXiv:2208.12098

Sparse (or pruned) SYK with interaction $= \pm 1$

The product of two Gaussians = Gaussian

The product of two (Gaussians + $(1-p)\delta(x)$) = Gaussian + $(1-p')\delta(xx')$

The product of two ± 1s = ± 1

The product of two $(p\delta(x^2-1)+(1-p)\delta(x))$ = $(p'\delta(x^2-1)+(1-p')\delta(x))$

$$\hat{H} = C_{N,p} \sum_{1\le a<b<c<d\le N} x_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d \,, x_{abcd} = \begin{cases} 1 & (\text{probability } p/2) \\ -1 & (\text{probability } p/2) \\ 0 & (\text{probability } 1-p) \end{cases}$$

Random-matrix statistics for $K_{\text{cpl}} = \binom{N}{4}p \gtrsim N$!

cf. Non-Gaussian disorder average [T. Krajewski, M. Laudonio, R. Pascalie, and A. Tanasa, PRD **99**, 126014 (2019)]

Extra degeneracy for small $K_{\mathrm{cpl}} \lesssim N$

$$\widehat{H} = C_{N,p} \sum_{1 \le a < b < c < d \le N} x_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d \qquad x_{abcd} = \begin{cases} 1 & \text{(probability } p/2) \\ -1 & \text{(probability } p/2) \\ 0 & \text{(probability } 1-p) \end{cases}$$

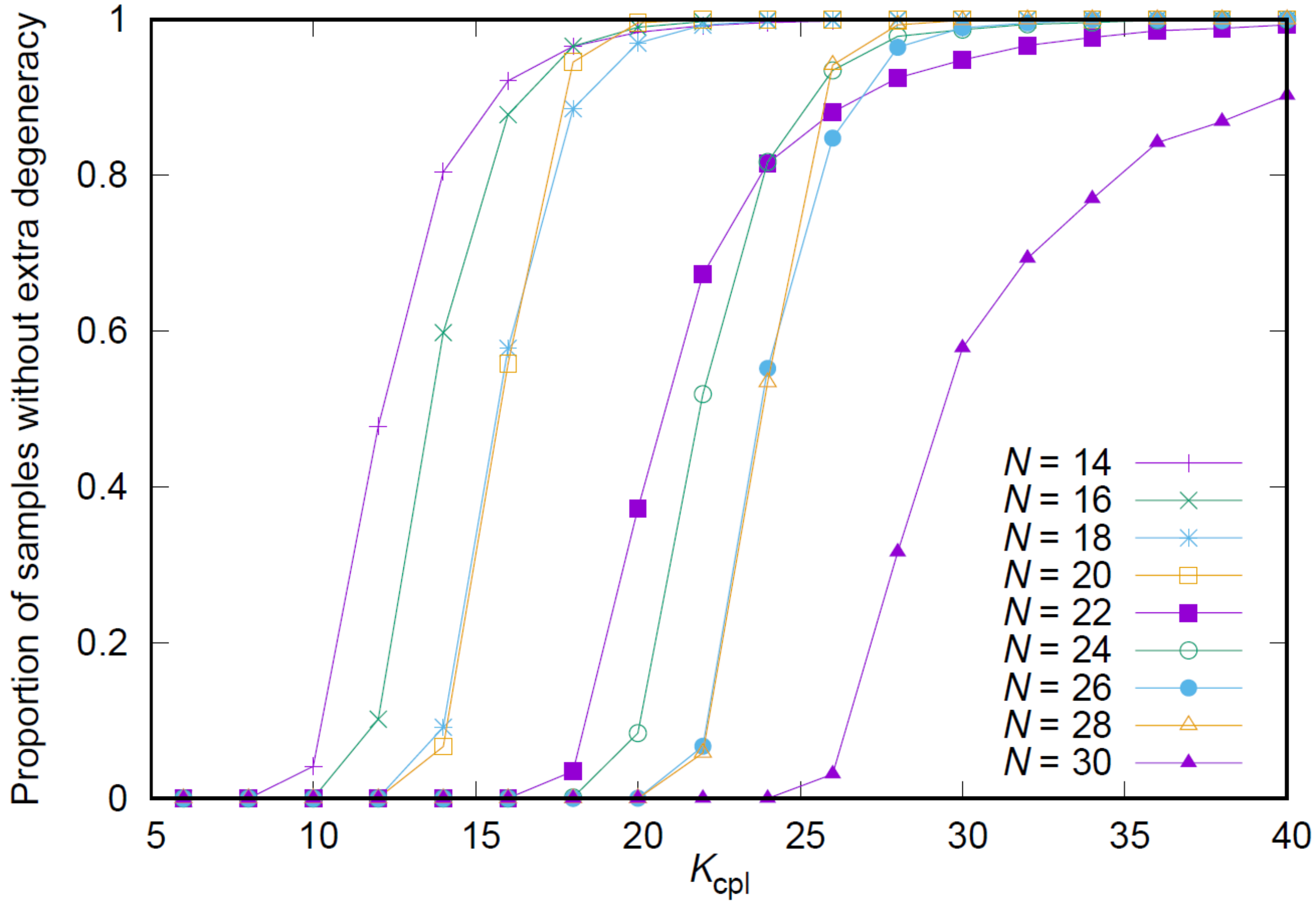
- If only few x_{abcd} are nonzero, some products of $\hat{\chi}_j$ can (anti)commute with the Hamiltonian [A. M. García-García *et al.*, PRD 2021]
- Simple example: if both $\hat{\chi}_{2k}$ and $\hat{\chi}_{2k+1}$ do not appear in $\widehat{H}$
  - The state of the qubit k does not change the energy
  - Twofold extra degeneracy

In the following, we take $C_{N,p} = 1/\sqrt{K_{\mathrm{cpl}}}$ so that the variance of $\{\epsilon_j\}$ is 1:

$$\mathrm{Tr}\widehat{H}^2 = C_{N,p}^2 \sum_{\substack{abcd \\ a'b'c'd'}} x_{abcd} x_{a'b'c'd'} \, \mathrm{Tr} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d \hat{\chi}_{a'} \hat{\chi}_{b'} \hat{\chi}_{c'} \hat{\chi}_{d'} = C_{N,p}^2 K_{\mathrm{cpl}} 2^{\frac{N}{2}} = 2^{\frac{N}{2}}.$$

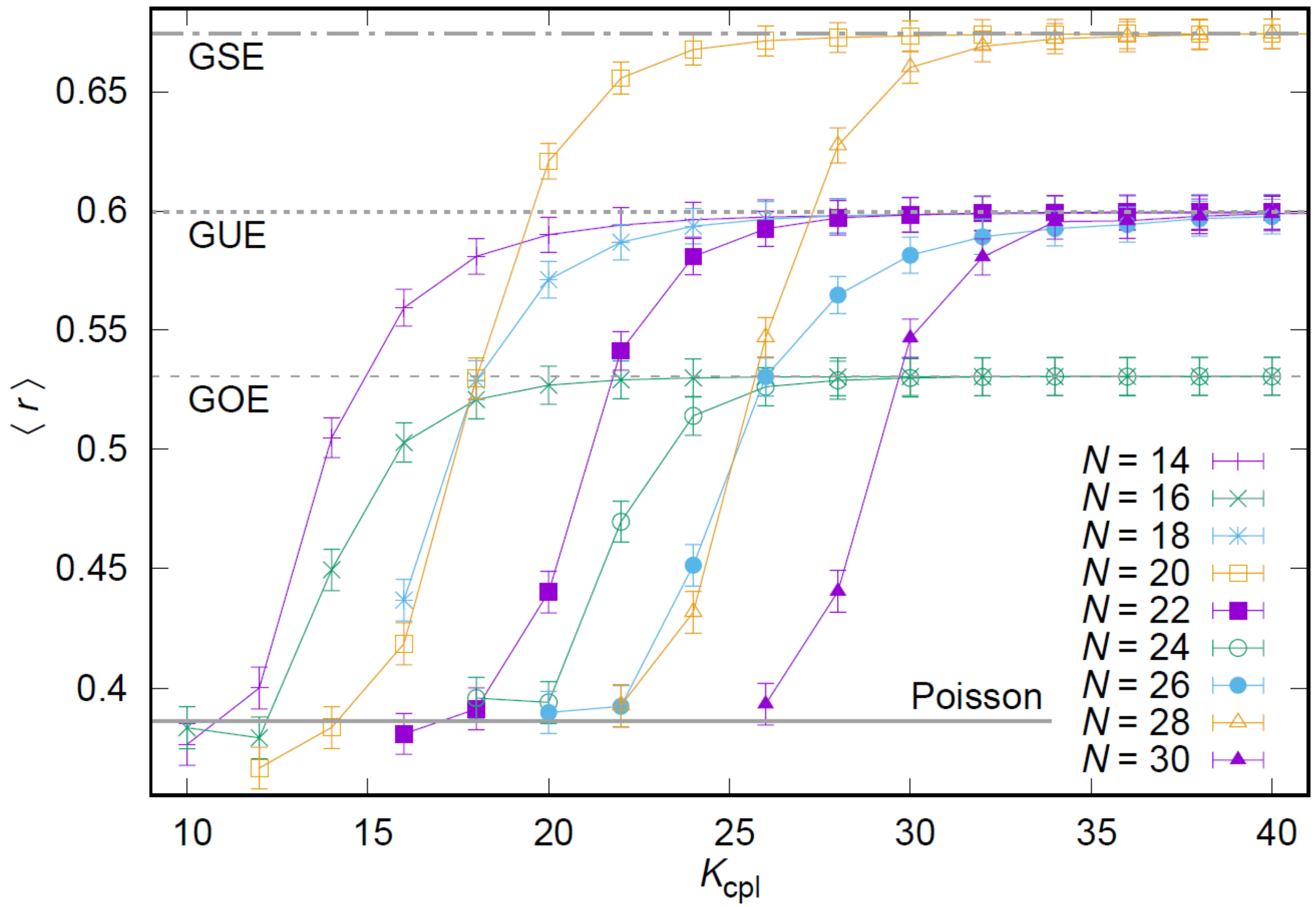
M. Tezuka, O. Oktay, E. Rinaldi, M. Hanada, and F. Nori, arXiv:2208.12098

$K_{\mathrm{cpl}} \gtrsim N$: extra degeneracy disappears



2^{24} eigenvalues ($2^{17} - 2^{9}$ samples)

$\langle r \rangle$ as a function of K_{cpl}: approach RMT value



Neighboring gap ratio

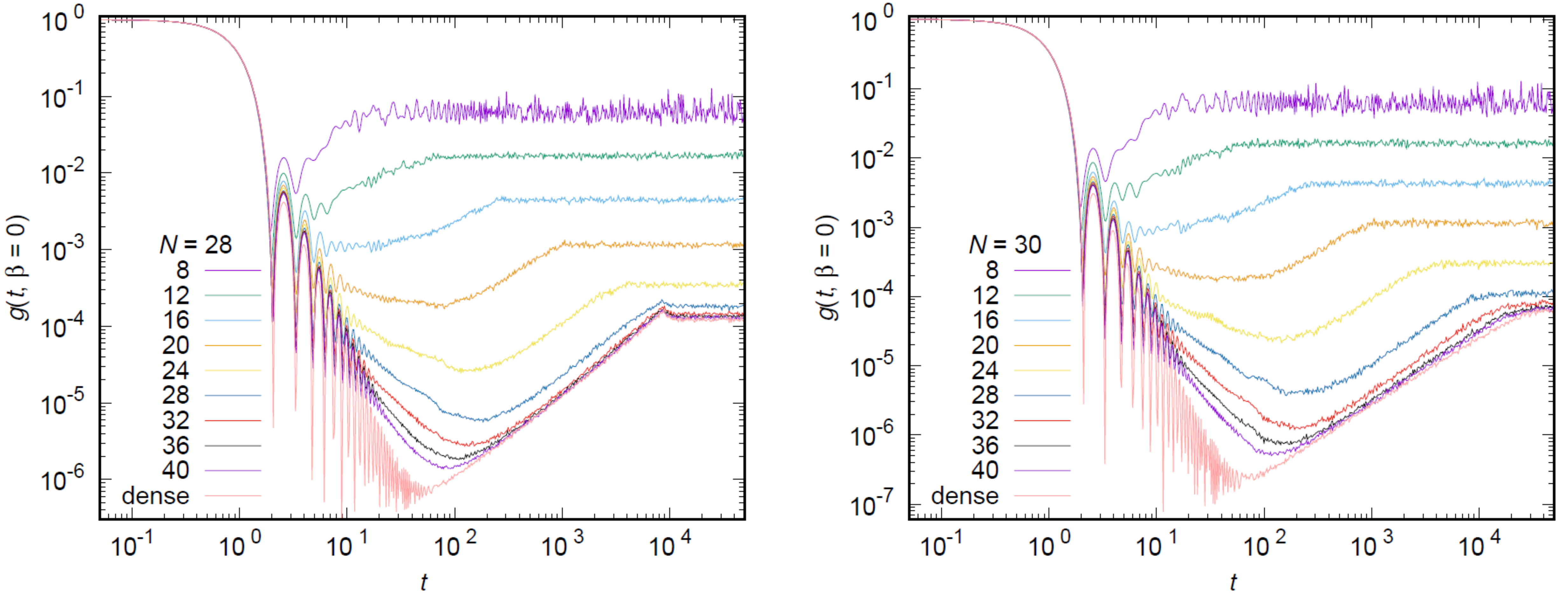
$$r = \frac{\min(e_{i+1} - e_i\,,\ \ e_{i+2} - e_{i+1})}{\max(e_{i+1} - e_i\,,\ \ e_{i+2} - e_{i+1})}$$

	Uncorrelated	GOE	GUE	GSE
$\langle r \rangle$	2log 2 – 1 = 0.38629…	0.5307(1)	0.5996(1)	0.6744(1)

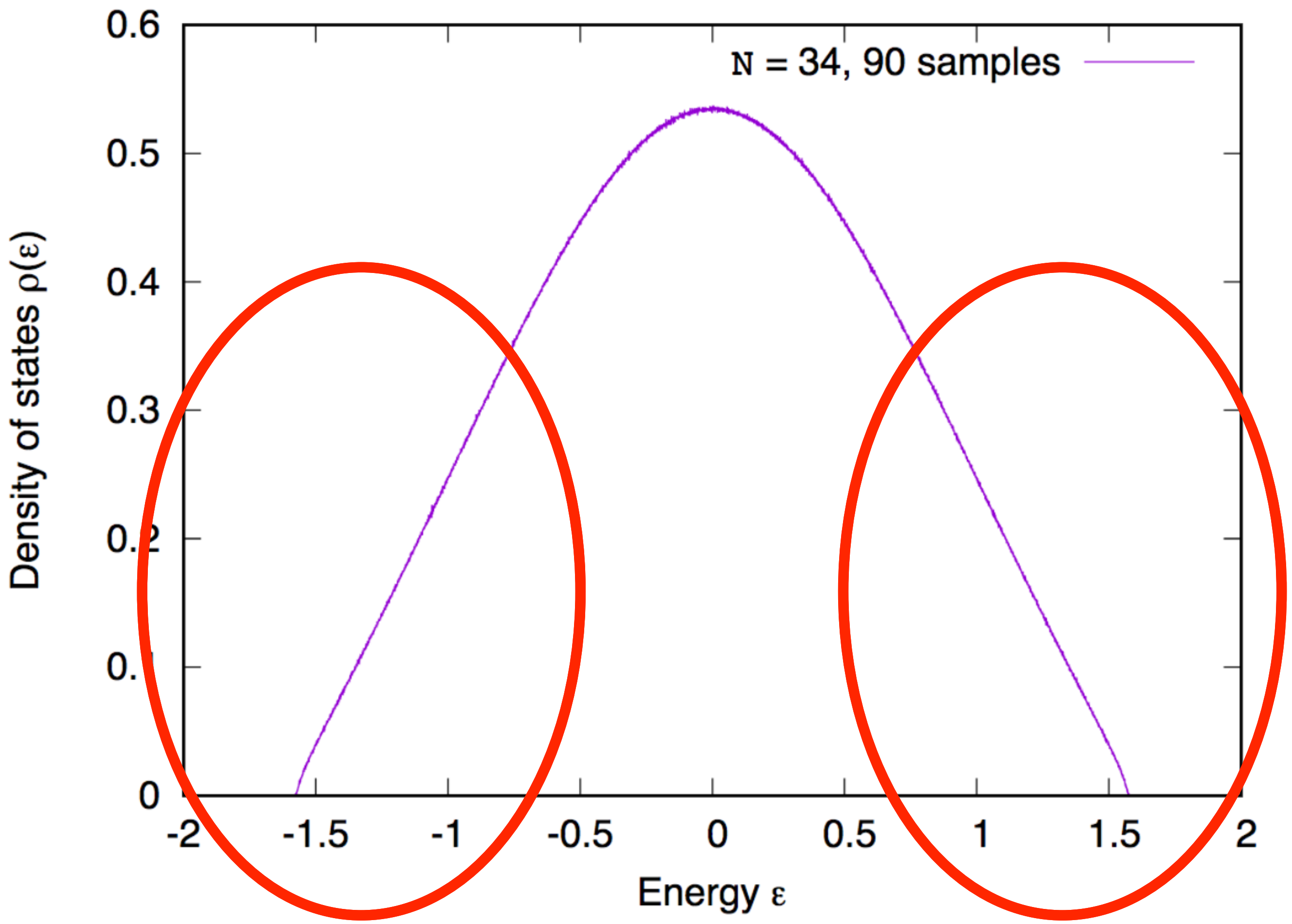
[Y. Y. Atas *et al.* PRL 2013]

Spectral form factor

Clear ramp for $K_{\mathrm{cpl}} \gtrsim N$, coincides with the dense SYK as $N \rightarrow$ large



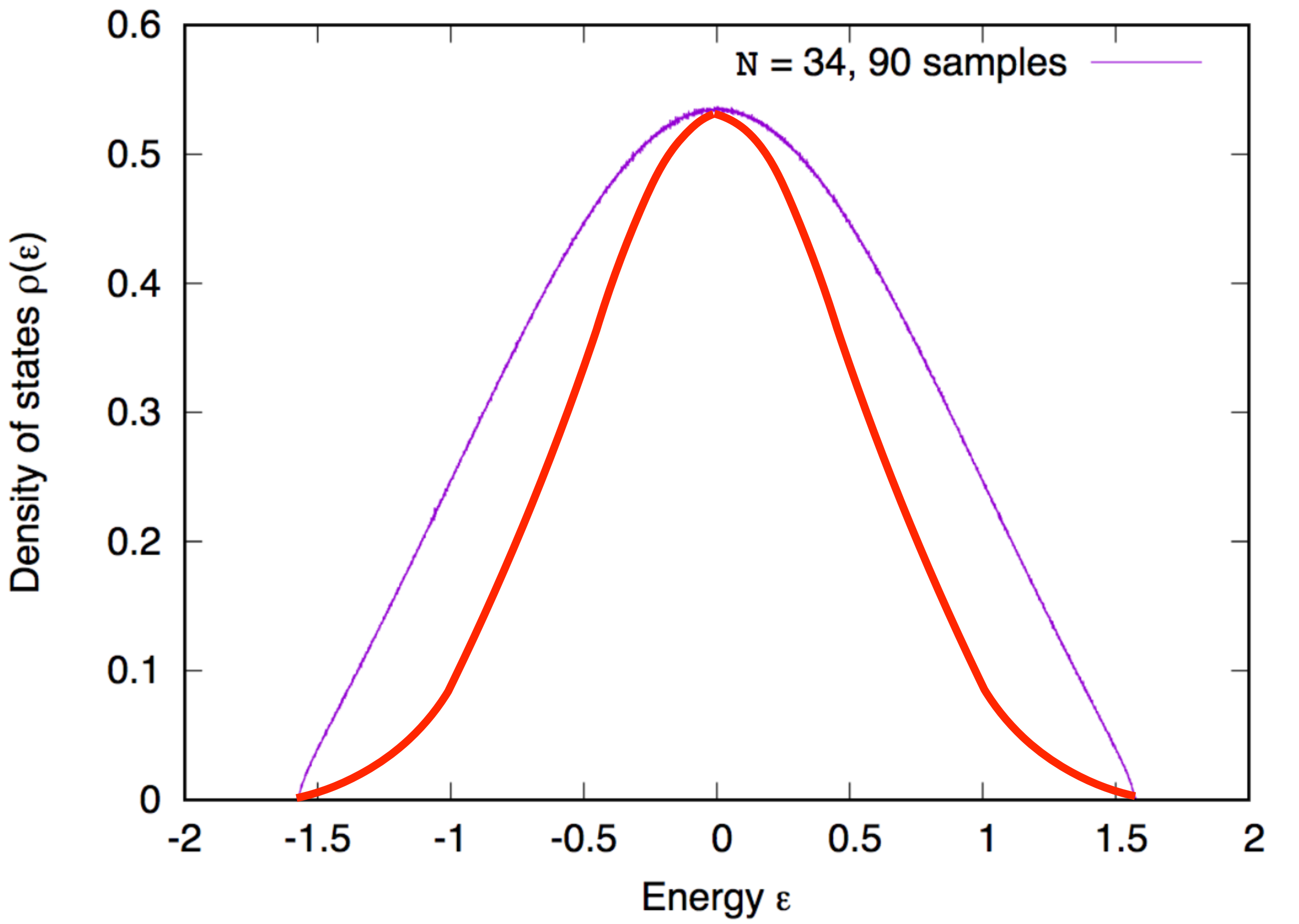
Density of states: SYK



"Slope" depends on the tails

Ghabiryan, Hanada, Shenker, and MT, JHEP **1807**(2018)124

Density of states: SYK

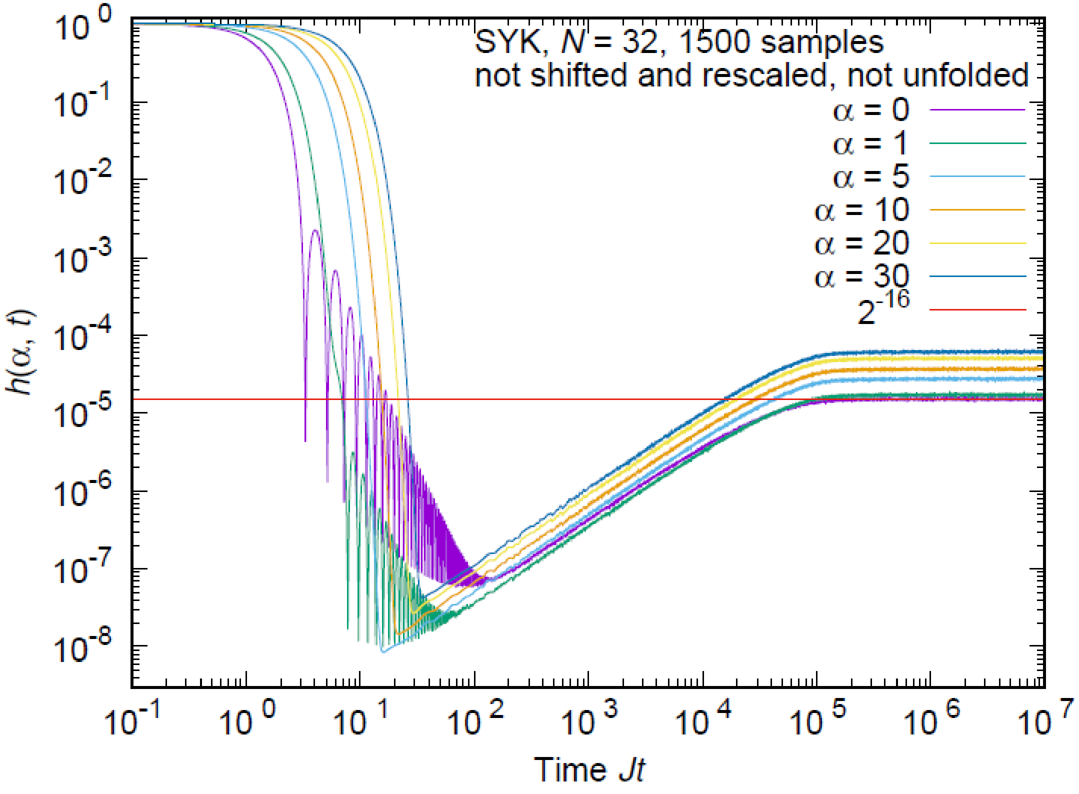


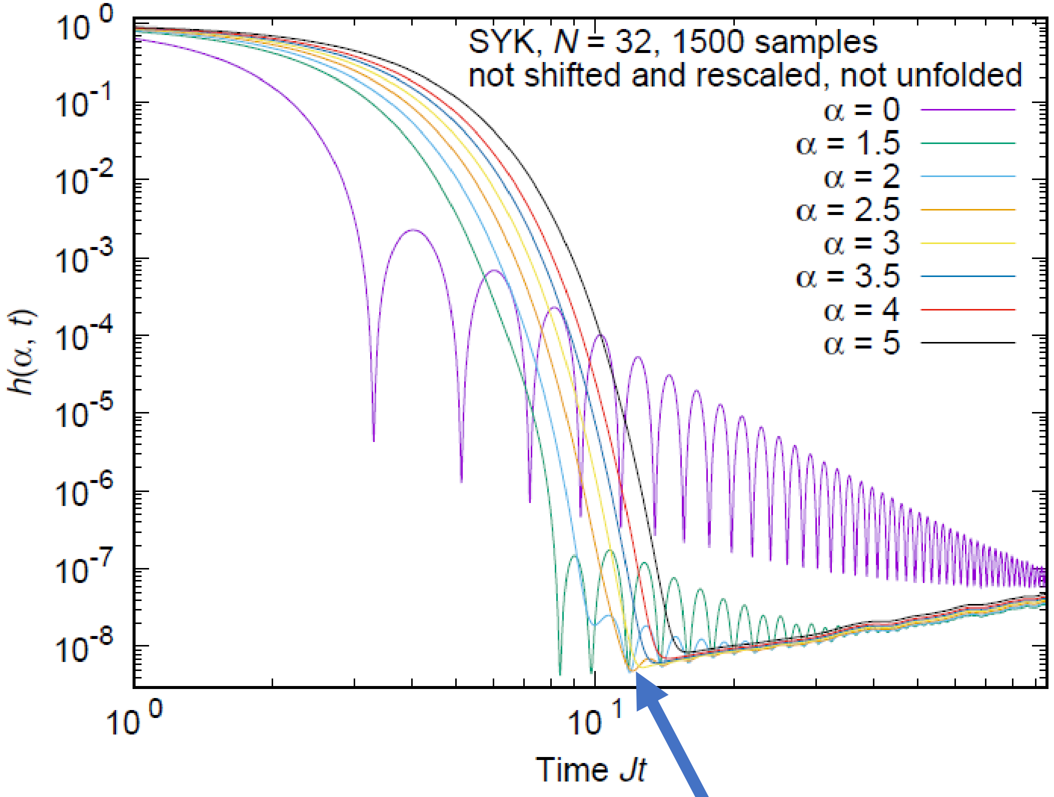
$$Y(\alpha,t)Y^*(\alpha,t)=\sum_{m,n}\boxed{e^{-\alpha(E_n^2+E_m^2)}}e^{+i(E_m-E_n)t}$$

Ghabiryan, Hanada, Shenker, and MT,
JHEP **1807**(2018)124

Modified spectral form factor

$$h(\alpha,t,\beta)=\frac{|Y(\alpha,t,\beta)|^2}{Y(\alpha,0,\beta)^2},\ Y(\alpha,0,\beta)=\sum_j e^{-\alpha\epsilon_j^2-(\beta+it)\epsilon_j}$$





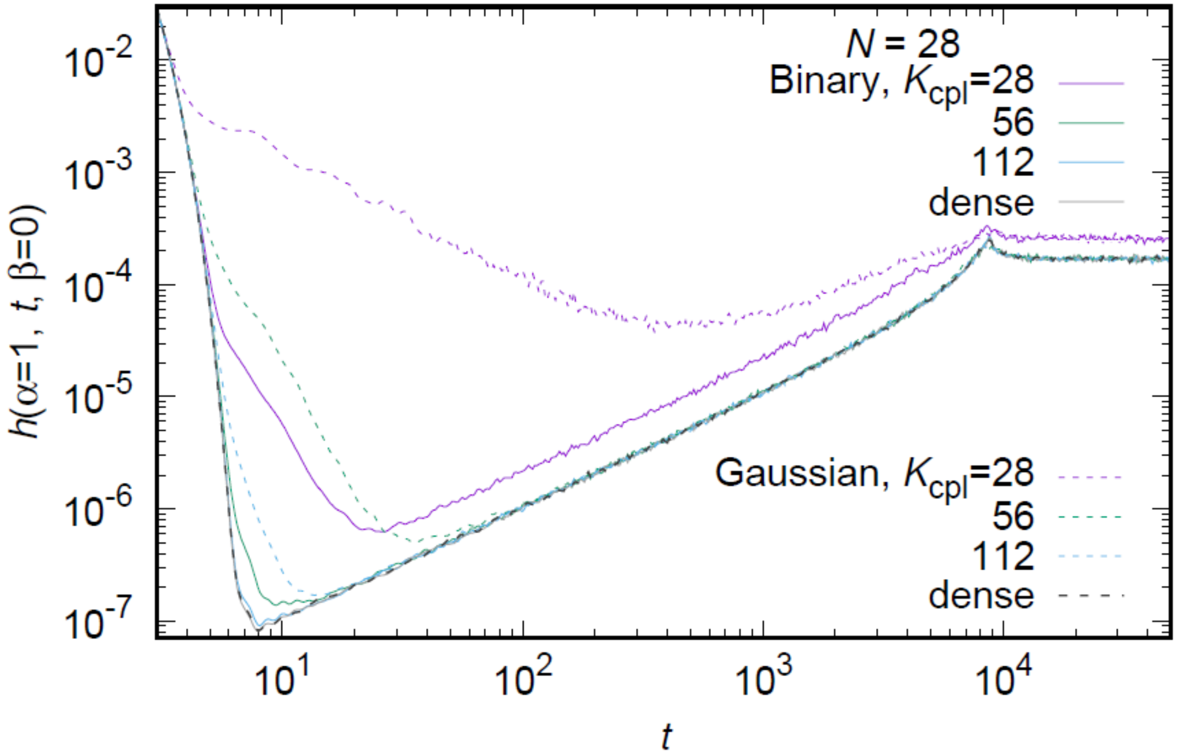
$t_{\min}=12.5$ for $\alpha=2.9$

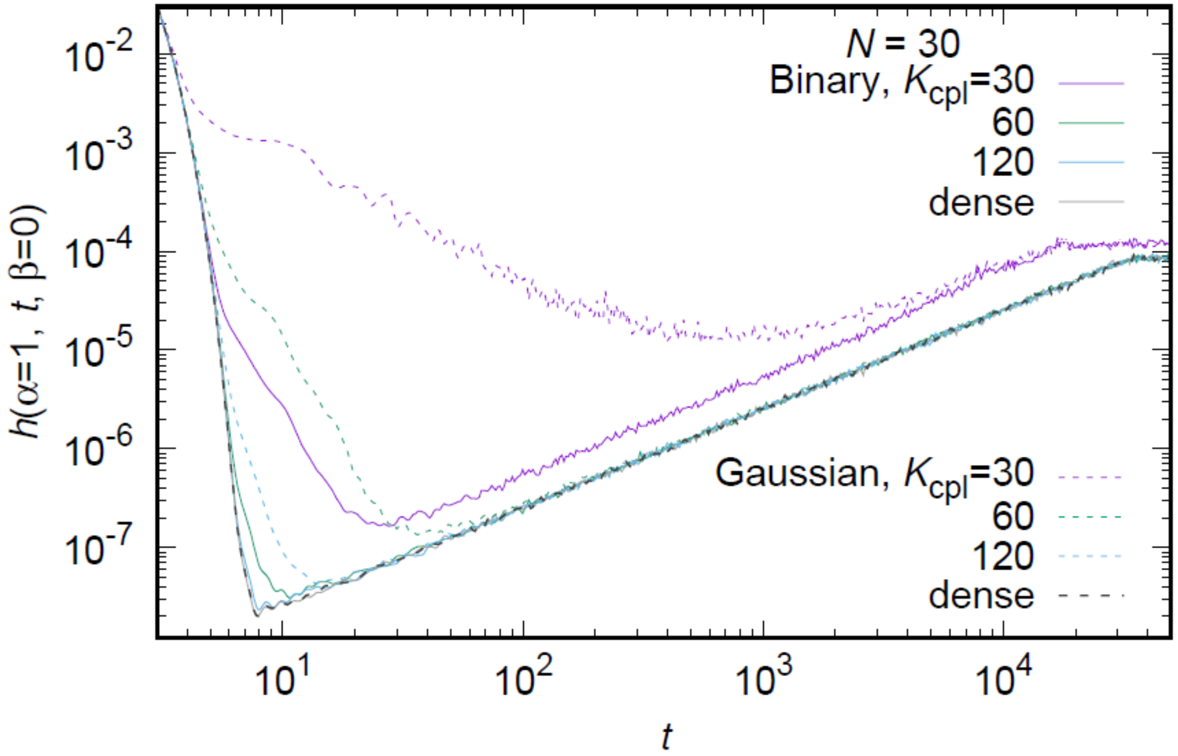
M. Tezuka, O. Oktay, E. Rinaldi, M. Hanada, and F. Nori, arXiv:2208.12098

Modified SFF

$$h(\alpha,t,\beta)=\frac{|Y(\alpha,t,\beta)|^2}{Y(\alpha,0,\beta)^2}, Y(\alpha,0,\beta)=\sum_j e^{-\alpha\epsilon_j^2-(\beta+it)\epsilon_j}$$

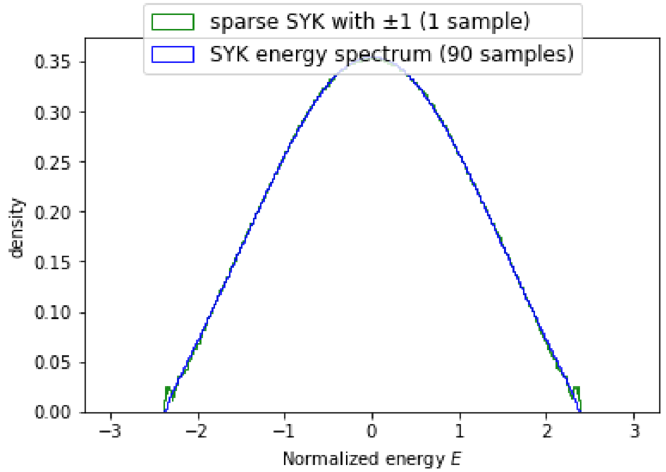
- Binary-coupling sparse model: rigidity of the eigenenergy spectrum $\sim$ Gaussian-coupling model with twice as large K_{cpl}

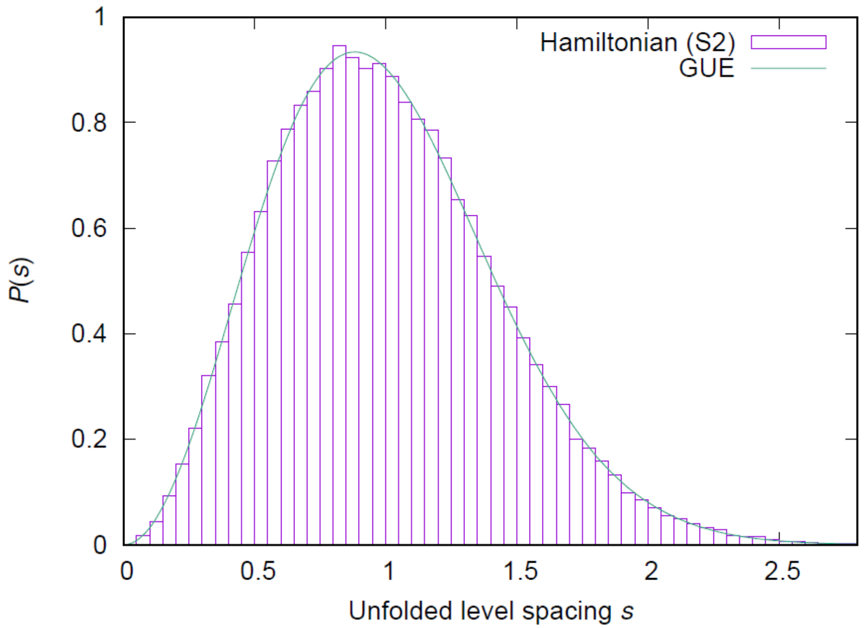


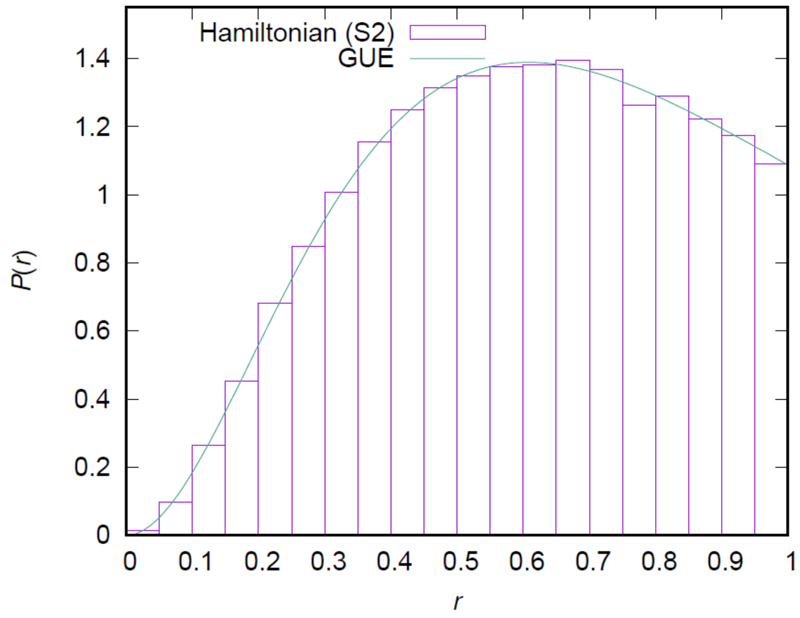


$N = 34, K_{\mathrm{cpl}} = 36, 1$ sample

GUE!







$$
\begin{aligned}
\mathcal{H} = {} & \chi_0\chi_5\chi_{19}\chi_{27} + \chi_0\chi_6\chi_{21}\chi_{23} - \chi_0\chi_9\chi_{14}\chi_{24} - \chi_0\chi_{14}\chi_{18}\chi_{30} - \chi_0\chi_{14}\chi_{20}\chi_{25} - \chi_1\chi_2\chi_{16}\chi_{22} \\
& + \chi_1\chi_{18}\chi_{22}\chi_{23} + \chi_2\chi_4\chi_5\chi_{15} + \chi_2\chi_{13}\chi_{16}\chi_{21} + \chi_2\chi_{14}\chi_{19}\chi_{24} + \chi_2\chi_{20}\chi_{27}\chi_{33} + \chi_2\chi_{22}\chi_{31}\chi_{32} \\
& + \chi_3\chi_4\chi_5\chi_{29} - \chi_3\chi_8\chi_{14}\chi_{28} - \chi_3\chi_8\chi_{29}\chi_{31} + \chi_3\chi_{21}\chi_{26}\chi_{29} - \chi_3\chi_{22}\chi_{25}\chi_{33} + \chi_4\chi_7\chi_{13}\chi_{30} \\
& - \chi_4\chi_9\chi_{14}\chi_{17} - \chi_5\chi_6\chi_{17}\chi_{29} + \chi_5\chi_{12}\chi_{29}\chi_{31} - \chi_5\chi_{13}\chi_{19}\chi_{24} - \chi_5\chi_{14}\chi_{22}\chi_{31} - \chi_5\chi_{17}\chi_{31}\chi_{33} \\
& + \chi_5\chi_{20}\chi_{30}\chi_{31} - \chi_6\chi_{23}\chi_{27}\chi_{29} + \chi_7\chi_{12}\chi_{13}\chi_{18} + \chi_8\chi_{10}\chi_{24}\chi_{28} - \chi_9\chi_{12}\chi_{20}\chi_{33} + \chi_{10}\chi_{11}\chi_{28}\chi_{32} \\
& + \chi_{10}\chi_{21}\chi_{27}\chi_{29} - \chi_{12}\chi_{20}\chi_{22}\chi_{24} + \chi_{14}\chi_{17}\chi_{26}\chi_{27} - \chi_{15}\chi_{24}\chi_{26}\chi_{27} - \chi_{16}\chi_{18}\chi_{23}\chi_{27} - \chi_{18}\chi_{24}\chi_{30}\chi_{32}
\end{aligned}
$$

2^{16} dimensions/parity; dense SYK: 46376 terms ➔ randomly chose $K_{\mathrm{cpl}} = 36$, half +1, half -1

Unary sparse SYK

- $\hat{H} = C_{N,p} \sum_{1 \le a<b<c<d \le N} x_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d \,, x_{abcd} = \begin{cases} 1 & \text{(probability } p) \\ 0 & \text{(probability } 1-p) \end{cases}$
- Reordering Majorana fermions: flips about half of the signs of x_{abcd}
- Similar statistics as binary sparse SYK expected unless p is very large

➔ Numerically checked

Related results on SYK & prospects

A. M. García-García, B. Loureiro, A. Romero-Bermúdez, and M. Tezuka, PRL **120**, 241603 (2018)

- **Chaotic-integrable transition in SYK4+2**: $\hat{H}_{\mathrm{SYK4+2}} = (\cos\theta)\hat{H}_{\mathrm{SYK4}} + (\sin\theta)\hat{H}_{\mathrm{SYK2}}, \delta \equiv \tan\theta$

F. Monteiro, T. Micklitz, M. Tezuka, and A. Altland, PRR **3**, 013023 (2021)

- The transition above: a **many-body localization transition in the Fock space** at $\delta = \delta_{\mathrm{c}} \sim N^2 \log N$
- Localization in the Fock space starts at $\delta \sim \frac{1}{\sqrt{N}}$, inverse participation ratio $\sim \delta^2$ for $1 \lesssim \delta \ll \delta_{\mathrm{c}}$

F. Monteiro, M. Tezuka, A. Altland, D. A. Huse, and T. Micklitz, PRL **127**, 030601 (2021)

- Bipartite entanglement entropy as a function of δ: exhibits a plateau indicating **ergodicity within energy shells**

"Quantum error correction in SYK-like models" with Yoshifumi Nakata (YITP, Kyoto U.) in preparation

- Dynamics: how long does it take for $\{e^{i\hat{H}t}\}$, if ever, to scramble quantum states like Haar random unitaries?
- SYK, binary-coupling sparse SYK ($t \sim \sqrt{N}$), and SYK4+2 (δ suppresses scrambling)
- Spin chains

(Digital or analog) quantum simulations of the binary sparse SYK model

- Smaller number of terms, no Gaussian randomness → more efficient?

… and more!

Summary

- Sachdev-Ye-Kitaev model and Gaussian random matrices
  - $q = 4$: $\mathcal{O}(N^4)$ Gaussian random terms
- Binary sparse SYK: spectral statistics obeys Random Matrix Theory predictions with $\sim N$ terms
  - Unary sparse case: similar statistics
- More efficient than Gaussian sparse SYK
  - Realization in quantum simulators?

<u>Masaki Tezuka</u>, Onur Oktay, Masanori Hanada, Enrico Rinaldi, and Franco Nori, "Binary-coupling sparse SYK: an improved model of quantum chaos and holography", arXiv:2208.12098