QCD in deconfined phase: Heavy quark as probe

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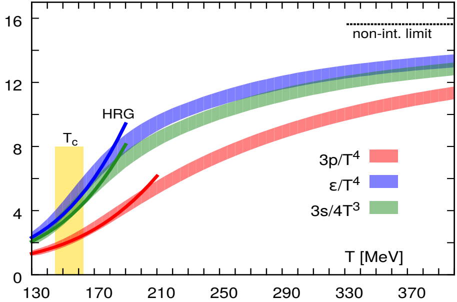
QCD at high temperatures

- At high temperatures, strongly interacting matter exists in a deconfined, chirally symmetric *quark-gluon plasma* phase.
- The energy density rapidly rises to $\sim 85\%$ of a free gas of quarks and gluons.



HotQCD, PRD 90(2014)094503. See also Borsanyi et al., PL B 370(2014)99.

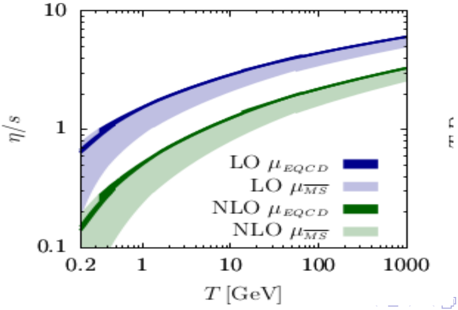
- Insights about strongly coupled gauge theories from AdS/CFT have been invaluable in understanding transport properties of QGP.

Viscosity

- Experimental study of QGP in relativistic heavy ion collision experiments (BNL, LHC) revealed a nearly perfect liquid, with very low shear viscosity $\frac{\eta}{s}$
- But leading order PT suggested large $\eta \sim \frac{T^3}{\alpha_s^2 \log \alpha_s}$
  $\frac{\eta}{s} \sim 1$ for $\alpha_s \sim 0.25$

  Arnold, Moore, Yaffe, JHEP 05 (2003) 051
- Turns out PT is not very well behaved.

  Ghiglieri, Moore & Teaney, JHEP 03 (2018) 179



Viscosity from AdS/CFT

- Insights from AdS/CFT about strongly interacting gauge theories:
  a) Thermodynamic quantities may not be a good marker of the weak coupling nature of a medium
  b) For conformal theories with a gravity dual, $\frac{\eta}{s} = \frac{1}{4\pi}$

  Kovtun, Son, Starinets, PRL 94 (2005) 111601

- Explanation of RHIC data requires $\frac{\eta}{s} \sim \frac{2-3}{4\pi}$

  Luzum & Romatschke, PR C 78 (2008) 034915

- Nonperturbative evaluation of viscosity using lattice QCD is difficult. Requires extraction of the transport peak in the spectral function from Euclidean $T_{12} T_{12}$ correlator.
- Results from gluon plasma: consistent with the low value indicated by RHIC

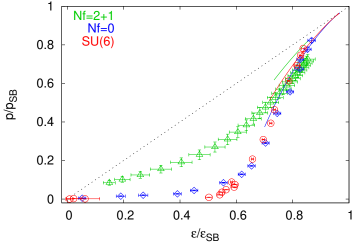
  Meyer, PRD 76 (2007) 101701; Borsanyi et al., PRD 98 (2018) 014512; Astrakhantsev, Braguta & Kotov, JHEP 04 (2017) 101.

Insight

- Similarly, The relaxation time for $N = 4$ SYM is small in the strong coupling limit, significantly smaller than $1/T$. This gives hope for the understanding of rapid thermalization in RHIC.

  See Casaldarrey-Solana, Liu, Mateos, Rajagopal, Wiedemann, *Gauge-String Duality, Hot QCD and Heavy Ion Collisions,* (Cambridge University Press).

- QGP is different in many respects from $N = 4$ SYM: one can only use its results to get qualitative insight into the behavior of a strongly coupled gauge theory.



Datta & Gupta, PRD 82 (2010) 114505; HotQCD (2014)

Heavy quark as probe of the plasma

- Heavy $Q\bar{Q}$ are mostly created in the initial hard scatterings of the heavy ion collision experiments, and then propagate through the plasma.
  $m_b, m_c \gg \Lambda_{QCD}, \; T_{\rm medium}$
- $t_{\rm kin} \sim \frac{m_Q}{T^2}$ large: late/incomplete thermalization?
- The experiments found that moderate momentum charm quarks flow with the medium flow, almost like the light quarks.
- For thermal heavy quark, $M \gg T, \quad p \gtrsim \sqrt{MT}$
- Even in hard collisions momentum transfer $\sim T$:
  Takes $\mathcal{O}(M/T)$ hard collisions to change momentum by $\mathcal{O}(1)$
  Thermal scattering: uncorrelated momentum kicks.
- Langevin equation:

$$\frac{dp_i}{dt} = \xi_i(t) - a(p) p_i, \qquad \langle \xi_i(t)\, \xi_j(t') \rangle = b_{ij}(p)\, \delta(t-t')$$

Svetitsky '88; Mustafa, Pal, Srivastava, PRC '97; Moore & Teaney '05

Diffusion coefficient

- Leads to the Fokker-Planck equation

$$\frac{\partial f_Q(p,t)}{\partial t} = -\frac{\partial}{\partial p_i}\left[p_i\, a(p)\, f_Q(p,t)\right] + \frac{\partial^2}{\partial p_i\,\partial p_j}\left[b_{ij}(p)\, f_Q(p,t)\right]$$

- At small p, the **drag** and **momentum diffusion coefficients**

$$a(p) \to \eta_D, \quad B_{ij}(p) \to \kappa$$

- The thermal distribution is the stable fixed point of FP equation

$$\to \eta_D = \frac{\kappa}{2\,M\,T}$$

- One can calculate κ from the $2 \to 2$ collision processes. Energy loss processes are dominated by $qQ \to qQ$ and $gQ \to gQ$ processes. In leading order,

$$\kappa = \frac{2\alpha^2 T^3}{27\pi}\left[N_c\left(\ln\frac{2T}{m_D} + C\right) + \frac{N_f}{2}\left(\ln\frac{4T}{m_D} + C\right)\right]$$

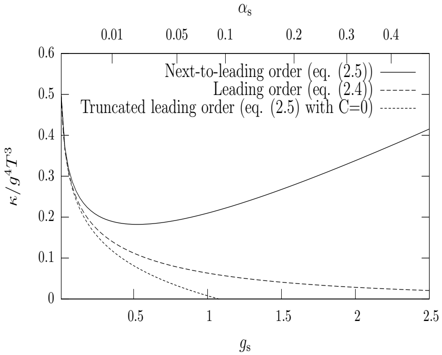
Moore & Teaney '05

κ in kinetic theory

- κ obtained from kinetic theory is too small: explanation of RHIC data requires $\frac{\kappa}{T_c^3} \sim 2.5 - 8$.

  S. Cao, et al., PRC 99 (2019) 054907.

- Weak coupling PT badly behaved: NLO corrections very large.



Caron-Huot & Moore, PRL 100 (2008) 052301

κ for N=4 SYM using AdS/CFT

- κ can be obtained from the force-force correlator:

$$\kappa = \int d^3t \, \langle \xi(t) \, \xi(t') \rangle$$

- In the static limit, the force on the quark is the color electric field gE : one can get κ from the EE correlator.
- Was calculated for strongly coupled $\mathcal{N} = 4$ SYM using AdS/CFT.

$$\kappa_{N=4SYM} = \pi \sqrt{\lambda} \, T^3, \qquad \lambda = g^2 \, N_c$$

J. Casalderrey-Solana & D. Teaney, PRD 74 (2006) 085012.

- Also the drag coefficient η_D was calculated for this theory by taking a quark moving at uniform velocity, and calculating the momentum inflow required. For small velocity,

$$\eta_D = \frac{\pi \, T^2 \sqrt{\lambda}}{2M}$$

C. Herzog, et al., JHEP 07('06)13; S. Gubser, PRD 74 ('06) 126005.

Momentum diffusion coefficient and force-force correlator

- We can estimate κ in QCD from the force-force correlator.
- A field theoretic definition of the force term can be given as $M\frac{dJ^i}{dt}$, where J^i is the Noether current for particle number.
- The force term can be expanded in a series in $1/M$:

$$F^i = M\frac{dJ^i}{dt} = \phi^\dagger \left\{ -gE^i + \frac{[D^i, D^2 + g\sigma\cdot B]}{2M} + \frac{g\,[D_0, \sigma\times E]^i}{4M} + \ldots \right\} \phi$$

- κ can be obtained from the correlator

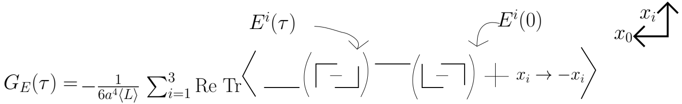
$$\kappa = \lim_{\omega\to 0} \frac{1}{3\chi} \int dt\, e^{i\omega t} \int d^3x \left\langle \frac{1}{2}\left\{ F^i(t,x), F^i(0,0) \right\} \right\rangle$$

S. Caron-Huot, M. Laine & G. Moore, JHEP 04 (2009) 53.
A. Bouttefeux & M. Laine, JHEP 12 (2020) 150

- Nonperturbatively, we can calculate the Matsubara correlator. This can be connected to the above correlator using standard relations.

$m_Q \to \infty$ limit

- In the static limit, we get the electric field correlator.
- It is convenient to use the discretization $gE_i \; = \; [D_i, D_0]$.
- Then we are left with the correlation function



- Extended object: multilevel technique is useful.
- The renormalization factor Z_E is finite, and has been evaluated in perturbation theory.

C. Christensen & M. Laine, PLB 755 (2016) 316.

- One can connect the correlator to the spectral function

$$G_E(\tau) \; = \; \int_0^\infty \frac{d\omega}{\pi} \, \rho_{EE}(\omega) \, \frac{\cosh \, \omega(\tau - 1/2T)}{\sinh \omega/2T}$$

- Then κ can be extracted from the infrared behavior of $\rho_{EE}(\omega)$:

$$\rho_{IR} \; = \; \frac{\kappa \, \omega}{2T}$$

κ_E from lattice

- The EE correlator has been investigated for a gluonic plasma by multiple groups, and κ_E extracted.

Banerjee, Datta, Gavai, Majumdar, PRD 85 (2012) 014510.
Francis, Kaczmarek, Laine, Neuhaus, Ohno, PRD 92 (2015) 116003.
Brambilla, Leino, Petreczky, Vairo, PRD 102 (2020) 074503.
Altenkort, Eller, Kaczmarek, Mazur, Moore, Shu, PRD 103 (2021) 014511.
Brambilla, Leino, Mayer-Steudte, Petreczky, arXiv:2206.02861.
Banerjee, Datta, Gavai, Majumdar, arXiv:2206.15471.

- Besides the scheme outlined above, Gradient flow has been used for both opertor renormalization and signal enhancement.
- Structure of spectral function not complicated.
- Techniques used: modelling of the ultraviolet, Brackus-Gilbert inversion, Bayesian techniques (MEM), ...

$1/m_Q$ correction

The $1/m_Q$ correction has only recently been estimated. Under certain assumptions, (Bouttefeux & Laine, JHEP 12 ('20) 150)

$$\kappa_Q \approx \kappa_E + \frac{2}{3}\langle v^2\rangle\, c_B(\mu)\,\kappa_B\,, \qquad \langle \gamma v^2\rangle = \frac{3T}{M_{\rm kin}}$$

where κ_B is the equivalent of κ_E from the $B-B$ correlator. Note that the BB correlator has an anomalous dimension. This cancels with the scale dependence of c_B, to give a physical result. This becomes equivalent to evaluating $Z_B(\mu = 19.2\,T)$.
We used the clover discretization of the B field, and use the nonperturbative matching to ϕ_{RGI} calculated by ALPHA collaboration.

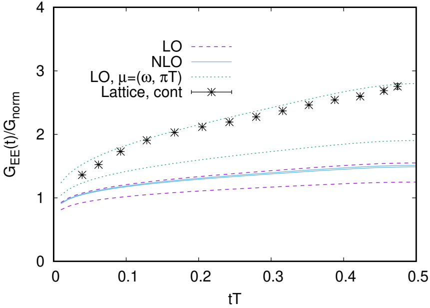
Guazzini, Meyer & Sommer, JHEP 10 (2007) 081.

Then we get to $Z_{\overline{MS}}(\mu = 19.2\,T)$ by integrating the RGE:

$$\mu\frac{d}{d\mu}\left[\frac{\phi_{RGI}}{\phi_{\overline{MS}}(\mu)}\right] = -\gamma(g)\left[\frac{\phi_{RGI}}{\phi_{\overline{MS}}(\mu)}\right]$$

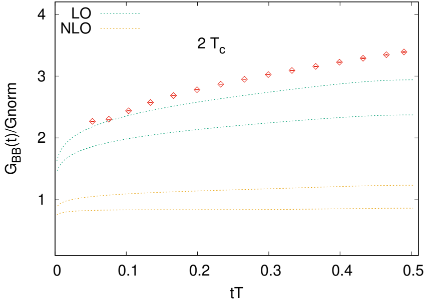
Banerjee, Datta & Laine, JHEP 08 ('22) 128

Perturbative result



EE correlator at 3 T_c vs PT

Banerjee, Datta, Gavai & Majumdar, arXiv:2206.15471



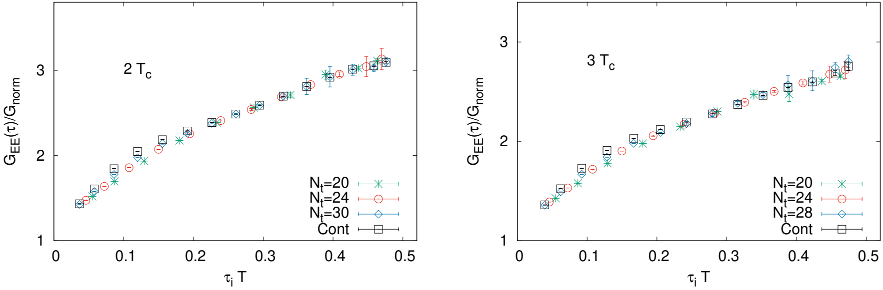
BB correlator at 2 T_c vs PT

Banerjee, Datta & Laine, JHEP 08('22)128 (2204.14075)

Normalized by $G_{\rm norm}$, $G_{LO}(\tau) = g_0^2 \, C_f \, G_{\rm norm}(\tau)$

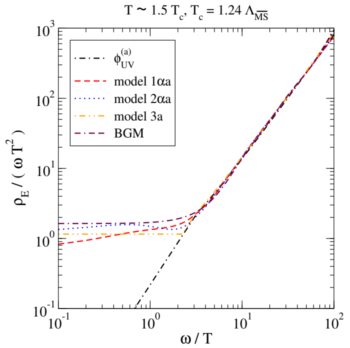
EE correlator

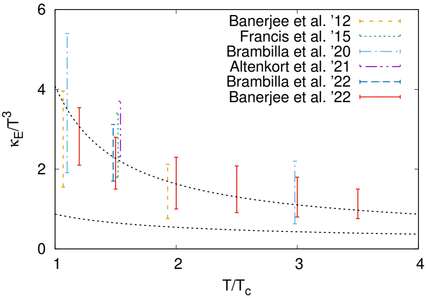
For gluonic plasma, careful continuum limit of the correlator taken and κ_E calculated. Results available from multiple groups.



Banerjee, Datta, Gavai, Majumdar, arXiv:2206.14571

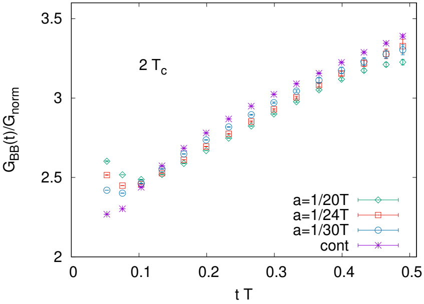
κ_E





Francis, Kaczmarek, Laine, et al., PRD 92 (2015) 116003

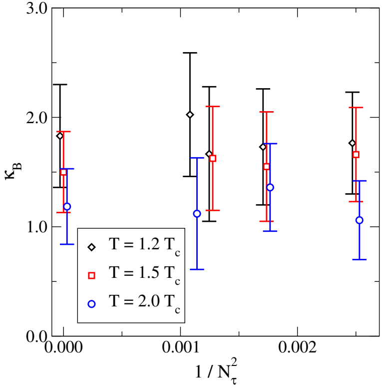
BB correlator from lattice

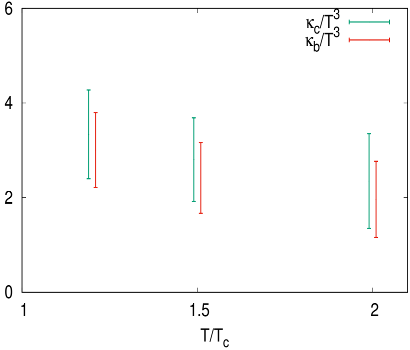


Banerjee, Datta, Laine, JHEP 08 ('22) 128

Unlike the EE correlator, the BB correlator has anomalous dimension, and renormalization is nontrivial.
The renormalization factor we get is large, $Z^2 \approx 3$ for much of our data set.

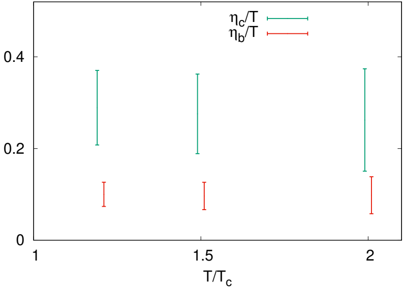
κ_B from BB correlator





Banerjee, Datta, Laine, JHEP08('22)128 (2204.14075)

$\eta_{c,b}$



Indicates $\tau_b \approx 3\tau_c \gtrsim 10fm$
Also $2\pi D_s T_c \sim 2.4 - 5.8$ for charm and $\sim 2.8 - 6.5$ for bottom.

Banerjee, Datta, Laine, JHEP 08 (2022) 128 (2204.14075)

Summary

- The high temperature, deconfined phase of QCD has rich physics, with very nontrivial transport properties.
- Calculating the transport coefficients from QCD is nontrivial. Insights into strongly coupled gauge theories from AdS/CFT have been very helpful.
- One quantity that we have been able to study using numerical lattice techniques is heavy quark momentum diffusion coefficient.
- Results for κ_b and κ_c are available for the gluonic plasma.
- Relaxation time short for charm but larger for the bottom, indicating incomplete thermalization for bottom.
- First studies of full QCD are in progress.

L. Altenkort, Lattice 2022