Collective transport in hardcore run-and-tumble particles

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References

- *Time-dependent properties of run-and-tumble particles: Density relaxation*, **Tanmoy Chakraborty** and PP, Phys. Rev. E **109**, 024124 (2024).
- *Time-dependent properties of run-and-tumble particles. II. Current fluctuations*, **Tanmoy Chakraborty** and PP, arXiv:2309.02896 (2023) (accepted in *Phys. Rev. E*).

Fluctuating hydrodynamics: Equilibrium

- Transport on macroscopic scales is characterized through density- and (other) parameters-dependent transport coefficients.
- Systems having a single conservation law and satisfying detailed balance (near-equilibrium scenario):

$$\frac{\partial \rho(x,\tau)}{\partial \tau} = \frac{\partial}{\partial x}\left[D(\rho)\frac{\partial \rho}{\partial x} + \sqrt{\frac{2\chi(\rho)}{L}}\zeta_x(\tau)\right], \tag{1}$$

$$\langle \zeta_x(\tau)\rangle = 0; \quad \langle \zeta_x(\tau)\zeta_{x'}(\tau')\rangle = \delta(x-x')\delta(\tau-\tau'). \tag{2}$$

Questions:

- What is fluctuating hydrodynamic description for systems violating detailed balance, such as self-propelled particles?
- Do the transport coefficients have singularities?

Previous work

Transport instabilities in interacting self-propelled particles (SPPs)?

- Theory of motility-induced phase separation (MIPS): The bulk-diffusion coefficient vanishes at the "critical point", leading to clustering and phase separation. Widely accepted theory. ....But, ... ??

  Tailleur and Cates PRL (2008) - interacting RTPs;
  Fily and Marchetti et al. PRL (2012); Bialké et al. EPL (2013) - conventional ("strongly-interacting") ABPs.

- MIPS proven, but only when the (drift) rates depend on system size!

  Kourbane-Houssene et al. PRL (2018) - "weakly-interacting" RTPs (vanishing bulk correlations and mean-field hydrodynamics);

- No (reasonably) rigorous theoretical proof yet for *conventional* SPPs !!

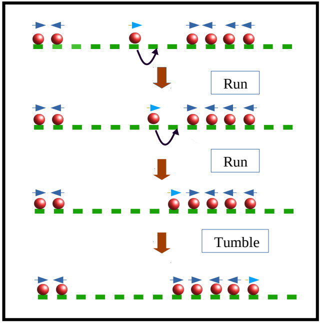
Contd.

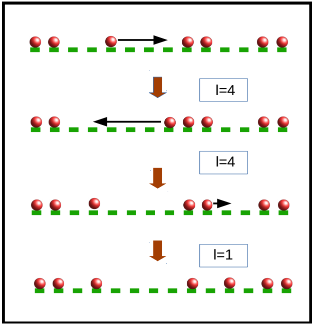
- *GrandPre et al. PRE (2018)* - Current fluctuation in conventional ABPs; non-Gaussian current distribution.
- *Banerjee et al. PRE (2020)* - Current fluctuations in noninteracting run-and-particles (RTPs).
- *Dandekar, Chakraborti, Rajesh PRE (2020)* - a variant of conventional hardcore RTPs; in the leading order of γ.
- *Agranov et al. JSTAT (2021) and SciPost Phys (2023); Jose et al. JSTAT (2023)* - current fluctuation for "weakly-interacting" RTPs.
- *Mukherjee et. al. SciPost Phys. (2023)* - Nonexistence of MIPS in one dimension.

We calculate the transport coefficients for arbitrary density ρ and tumbling rate γ in (strongly-interacting) hardcore RTPs.

- We implement an efficient numerical scheme.
- We develop an analytical closure scheme.

Run-and-Tumble Particles (RTPs) and Long-ranged Lattice Gas (LLG)





Soto, Golestanian, PRE (2014).

Persistence length of RTPs, $l_p = 1/\gamma$.

Hop-length distribution in LLG, $\phi(l) \sim \exp(-l/l_p)$.

Analytic expressions

Transport coefficients for arbitrary ρ and γ in one dimension,

$$D_{LLG}(\rho,\gamma) = -\frac{1}{2}\frac{\partial}{\partial\rho}\left[\rho\sum_{l=1}^{\infty}\phi(l)\left(\sum_{g=1}^{l-1} gP(g) + l\sum_{g=l}^{\infty}(g-l+1)P(g)\right)\right], \quad (3)$$

$$\chi_{LLG}(\rho,\gamma) = \frac{\rho}{2}\sum_{l=1}^{\infty}\phi(l)\left[\sum_{g=1}^{l-1} g^2 P(g) + l^2\sum_{g=l}^{\infty} P(g)\right]. \quad (4)$$

$P(g) \equiv P(g|\rho,\gamma)$ - steady-state gap distribution function.

Main results

- In the limit $\rho, \gamma \ll 1$ and $\psi = \rho/\gamma$ finite:

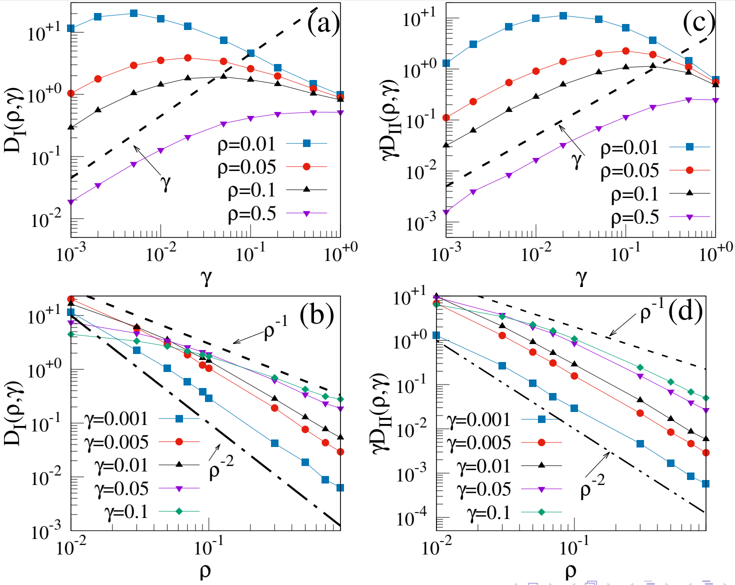
$$\mathcal{D}(\rho,\gamma) \equiv \mathcal{D}^{(0)} \mathcal{F}\left(\frac{\rho}{\gamma}\right), \tag{5}$$

$$\chi(\rho,\gamma) \equiv \chi^{(0)} \mathcal{H}\left(\frac{\rho}{\gamma}\right). \tag{6}$$

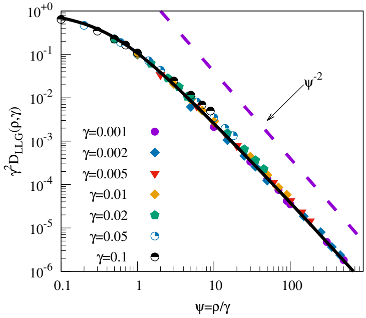
- In the limit $T, L \gg 1$ and DT/L^2 finite:

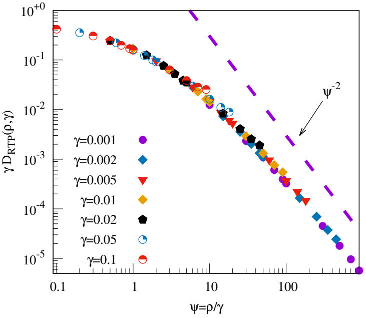
$$\frac{\mathcal{D}(\rho,\gamma)}{2\chi(\rho,\gamma)} \langle Q_X^2(T) \rangle \equiv L \mathcal{W}\left(\frac{DT}{L^2}\right). \tag{7}$$

Bulk-diffusion coefficient



Scaling collapse of $D(\rho, \gamma)$

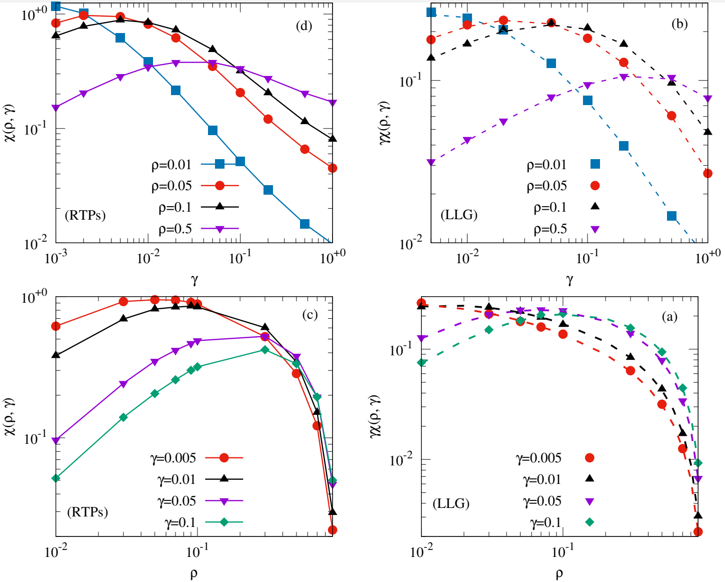




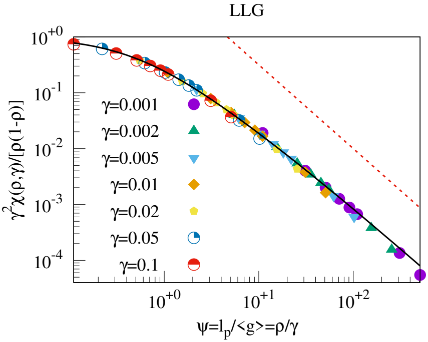
$$D(\rho, \gamma) = D^{(0)} \mathcal{F}(\psi), \quad \mathcal{F}_{LLG}(\psi) = \frac{(2+\psi)}{2(\psi + \sqrt{1+\psi})^3}$$

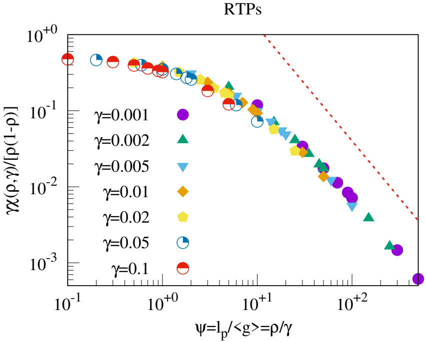
Chakraborty and PP, PRE (2024).

Collective particle mobility



Scaling Collapse of $\chi(\rho, \gamma)$

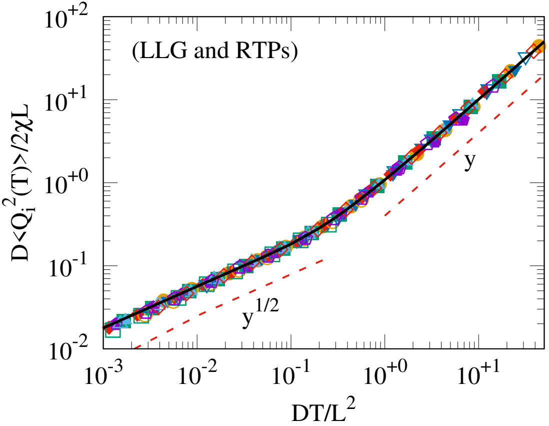




$$\chi_{LLG}(\rho,\gamma) = \frac{\rho(1-\rho)}{\gamma^2}\mathcal{H}_{LLG}(\psi), \quad \mathcal{H}_{LLG}(\psi) = \frac{\sqrt{1+\psi}}{(\psi+\sqrt{1+\psi})^2}.$$

Chakraborty and PP, PRE (2024).

Bond-current fluctuation



$$\mathcal{W}(y) \simeq y + \left(\frac{y}{\pi}\right)^{1/2} \mathrm{erfc}\left(2\pi\sqrt{y}\right) + \frac{1 - \exp(-4\pi^2 y)}{4\pi^2}. \tag{8}$$

Conclusions

- The *bulk-diffusion coefficient* and the particle *mobility* for arbitrary density and tumbling rates in two models of hardcore RTPs.
- A scaling law in the low density and strong-persistence limit.
- No diffusive instability in the system.
- The temporal growth of current fluctuation obeys a universal scaling law.
- The above scaling functions calculated analytically for a variant of hardcore RTPs.

THANK YOU!