Quantum Field Theory on Causal Sets

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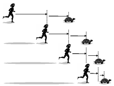
Women at the Intersection of Mathematics and Theoretical Physics, ICTS, December 31, 2025

Outline

1. Background
2. Free quantum Field theory on Causal sets
3. A diagrammatic expansion for in-in correlators

Fundamental nature of spacetime

Zeno Paradox : 

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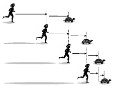
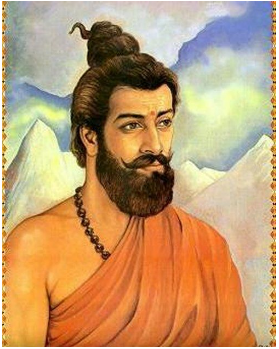
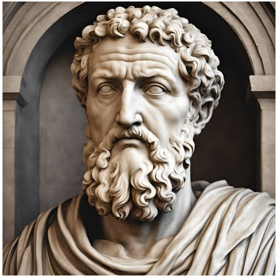
Zeno Paradox : 



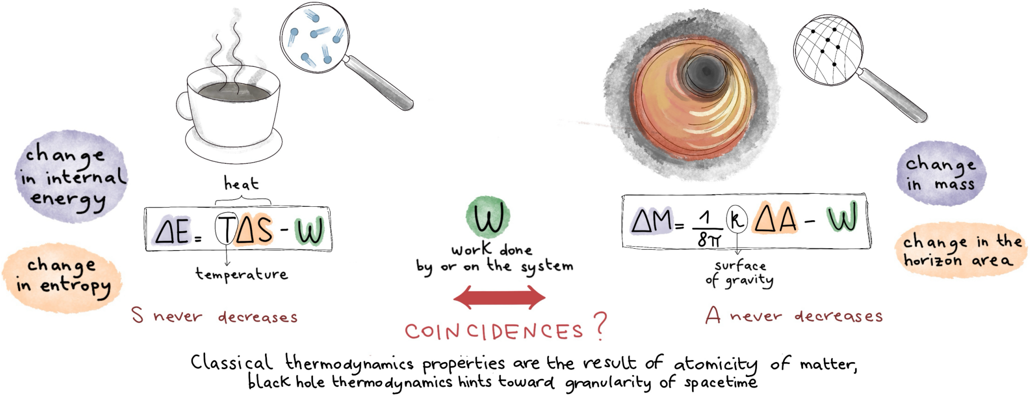
Kanada - Vaisheshika school, 6^{th} to 2^{nd} BC

Leucippus (Democritus and the pre-Socratics, 450 BC)



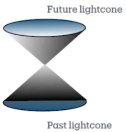
Lucretius (100 BC) (Epicurus (250 BC) and the Hellenic School)

Thermodynamics

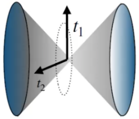


Causal Sets

A causal set is defined as a locally finite partially ordered set.
Take a pair $(\mathcal{C}, \prec)$ where $\mathcal{C}$ is a set with a partial order relation $\prec$ which satisfies:



Poset associated with $(-,+,+,+,\ldots)$



$(-,-,+,+)$ has no associated poset

- $\forall\ a, b \in \mathcal{C},\ \ a \prec b \prec a \Rightarrow a = b$ **Acyclicity**
- $\forall\ a, b, c \in \mathcal{C},\ \ a \prec b \prec c \Rightarrow a \prec c$ **Transitivity**

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- $\forall\ a, c \in \mathcal{C}, |\ [a, c]\ | < \infty$, where the set $[a, c] := \{b \in \mathcal{C}|\ a \prec b \prec c\}$ is a causal interval and $|X|$ is the cardinality of a set X. **Locally finiteness**

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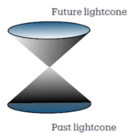
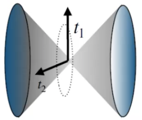


Poset associated with $(-,+,+,+,\dots)$



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MOTTO:

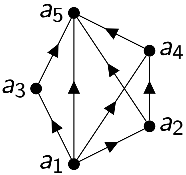
"Order + Number $\sim$ Lorentzian Geometry"

Hawking, King, and McCarthy 1976, Malament 1977

Representation

$$\mathcal{C} = \{a_1, a_2, a_3, a_4, a_5\}$$

$$a_1 \prec a_2, a_1 \prec a_3, a_1 \prec a_4, a_1 \prec a_5,$$
$$a_2 \prec a_4, a_2 \prec a_5,$$
$$a_3 \prec a_5, a_4 \prec a_5,$$



$$C = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad L = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

A **labelling** is an injective map: $\mathcal{C} \to \mathbb{N}$, analogue of a choice of coordinate system in the continuum.

Discrete/Continuum Correspondance

A causal set $(\mathcal{C}, \prec)$ is well-approximated by a continuum geometry $(\mathcal{M}, g)$ of dimension d if there exists a *faithful embedding* of $\mathcal{C}$ in $\mathcal{M}$, that is a map $f : \mathcal{C} \to \mathcal{M}$ such that,

(*i*) $q \prec p \iff f(q) \in J^-(f(p))$, where $J^-(x)$ denotes the causal past of $x \in \mathcal{M}$,

(*ii*) the points $f(\mathcal{C})$ are distributed in $\mathcal{M}$ according to a uniform distribution in $\mathcal{M}$ in agreement with the volume measure on $\mathcal{M}$

(*iii*) the discreteness length $l = \rho^{-d}$ is small compared to any curvature length scale.

Discrete/Continuum Correspondance

We must ensure covariance, since the goal is to be able to recover the approximate covariant spacetime geometry:
A regular lattice picks a preferred frame.

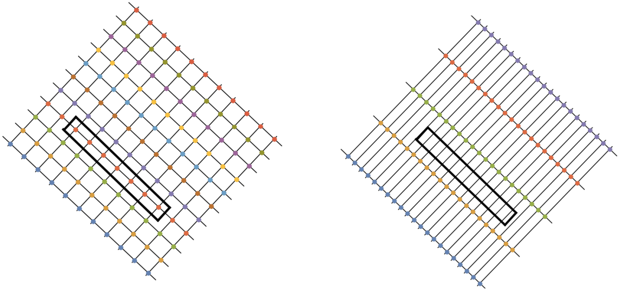


Figure: The lattice on the left looks "regular" in a fixed frame but transforms into the "stretched" lattice on the right under a boost. The highlighted Alexandrov interval has n = 7 lattice points in the lattice in the left but is empty after a boost. From S.Surya arxiv.1903.11544

Sprinkling: generating causal sets from continuum

Select points in (M, g) uniformly at random $\rightarrow \langle n \rangle = \rho V$ via a Poisson distribution and impose a partial ordering via the induced spacetime causality relation.

$$P(|\mathcal{C} \cap V| = n) = \frac{(\rho V)^n e^{-\rho V}}{n!}, \tag{1}$$

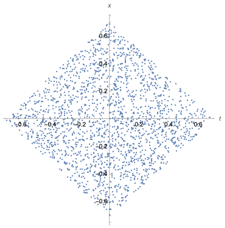


Figure: CS with 1000 elements approximated by a portion of $1+1$ Minkowski

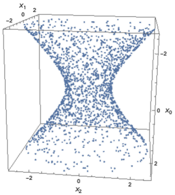


Figure: CS with 1000 elements approximated by dS_2

This process is Lorentz invariant! only uses the invariant volume measure.
Dowker, Henson and Sorkin, gr-qc/0311055, Bombelli, Henson and Sorkin, gr-qc/0605006

Uniqueness of the continuum approximation

The Hauptvermutung - Fundamental Conjecture

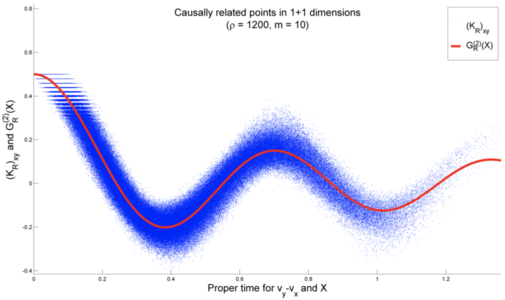
$\mathcal{C}$ can be faithfully embedded at density ρ into two distinct spacetimes, $(\mathcal{M}, g)$ and $(\mathcal{M}', g')$ iff they are approximately isometric.

By an approximate isometry , $(\mathcal{M}, g) \sim (\mathcal{M}', g')$ at density $\rho \implies$ differ only at scales smaller than ρ

Free QFT on Causal sets

- Non-Local discrete structure $\implies$ No tangent space $\implies$ No equation of motion.
- Start with the retarded propagators Δ^R_{xy}: hops and stops model at each element of the trajectory. Johnston. Particle propagators on discrete spacetime, arXiv:0806.3083

$$\Delta^R_{(2D)} := \frac{1}{2}C(\mathbb{I} + \frac{1}{2}\frac{m^2}{\rho}C)^{-1} \tag{2}$$



Free QFT on Causal sets

- Non-Local discrete structure $\implies$ No tangent space $\implies$ No equation of motion.
- Start with the retarded propagators Δ^R_{xy}. Associate a field operator $\phi(x)$ to each $x \in \mathcal{C}$ and impose the Peierls bracket,

$$[\phi(x), \phi(y)] = i\Delta_{xy} = i(\Delta^R_{xy} - \Delta^R_{yx}). \quad (3)$$

This is the Pauli-Jordan function Note: $[\phi(x), \phi(y)] = 0$ if $x \natural y$.

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- $i\Delta$ is **skew-symmetric** and **Hermitian** $\implies$ even rank (2s) and real positive and negative pairs of non-zero eigenvalues

$$i\Delta_{xy} v_y^{(\lambda)} = \lambda v_x^{(\lambda)} \qquad i\Delta_{xy} \bar{v}_y^{(\lambda)} = -\lambda \bar{v}_x^{(\lambda)}. \qquad i\Delta_{xy} w_y^k = 0, \tag{4}$$

where $\lambda > 0$, $(\lambda) = 1, \ldots, s; k = 1, \ldots, |\mathcal{C}| - 2s$, with bars denoting complex conjugation. **FINITELY MANY MODES!**

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- The Pauli-Jordan function can be written as

$$i\Delta_{xy} = \sum_{\lambda>0} \lambda v_x^{(\lambda)} \bar{v}_y^{(\lambda)} - \overline{\sum_{\lambda>0} \lambda v_x^{(\lambda)} \bar{v}_y^{(\lambda)}} \tag{5}$$

SJ-vacuum

- The eigenvectors of $i\Delta$ can be used to define a Gaussian vacuum state $|0\rangle$ by requiring that,

$$W(x, y) \equiv \langle 0|\phi(x)\phi(y)|0\rangle = Pos(i\Delta) = \sum_{\lambda>0} \lambda v_x^{(\lambda)} \bar{v}_y^{(\lambda)} \tag{6}$$

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- Fock representation: for each $\lambda > 0$, introduce ladder operators $(a_\lambda, a_\lambda^\dagger)$ and impose the commutation relations. Fields can be expanded as

$$\phi(x) = \sum_{\lambda>0} \sqrt{\lambda}\left(v_x^\lambda a_\lambda + \bar{v}_x^\lambda a_\lambda^\dagger\right), \tag{7}$$

  As in the continuum, $\phi(x)$ is a 'solution' if it is not in the kernel of $i\Delta$
  $\rightarrow w^k \cdot \phi = 0 \quad \forall k$ where $i\Delta w^k = 0$

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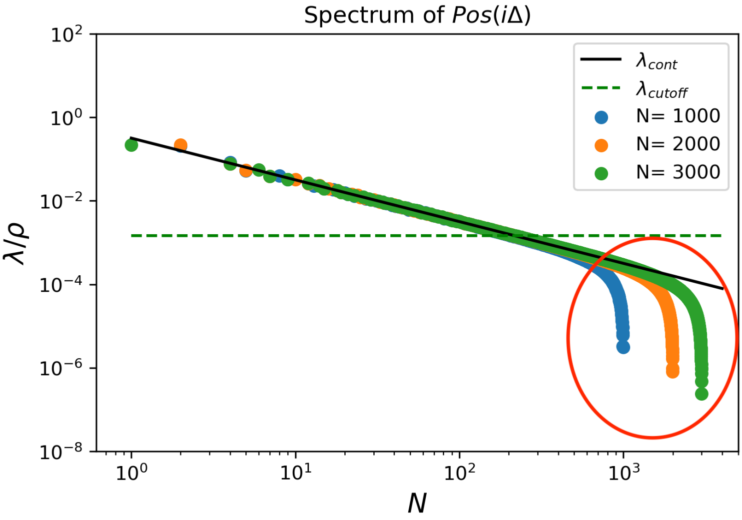
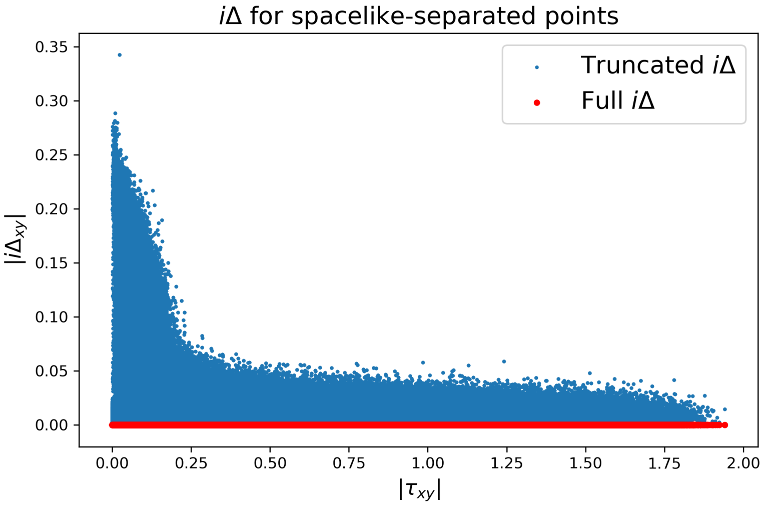
NOTE: SJ vacuum is UNIQUE on curved backgrounds as well! Afshordi et al., A Ground State for the Causal Diamond in 2 Dimensions, arXiv:1205.1296, A. Mathur, S. Surya, Sorkin-Johnston vacuum for a massive scalar field in the 2D causal diamond hep-th/1906.07952 , S. Surya, Nomaan X, Y. K. Yazdi, Studies on the SJ Vacuum in de Sitter Spacetime, gr-qc/ 1812.10228

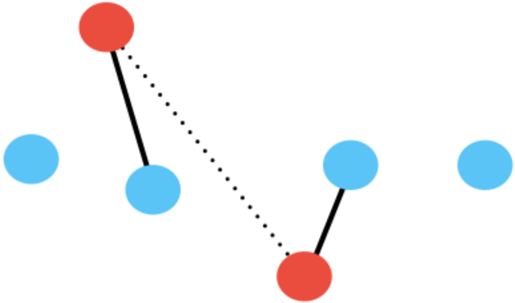
Massless Scalar Field on 1+1 Causal Diamond



Violation of causality as the spectrum is truncated λ_{cutoff}



Initial Value Problem



An antichain is not a Cauchy surface. It fails to encode the causal relation represented by the dashed line.

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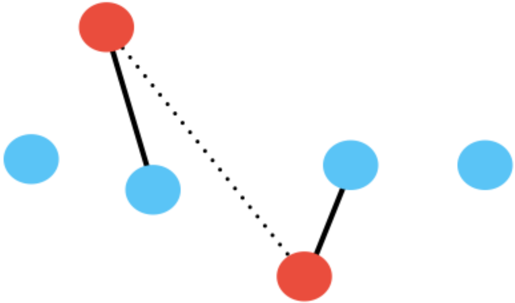

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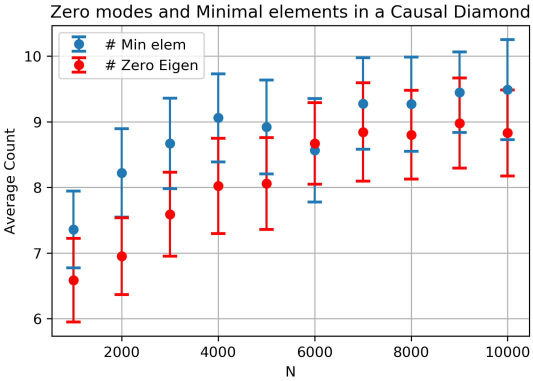
What is the number of physical modes that propagate?



Specifying the field on the minimal elements alone is insufficient for solving the initial value problem!

Causal Ordering

Define the causal ordering operator C whose action on a product of two fields is,

$$C[\phi(x)\phi(y)] = \begin{cases} \phi(x)\phi(y) & \text{if } x \succ y \\ \phi(y)\phi(x) & \text{if } x \prec y, \end{cases} \tag{8}$$

- For a spacelike pair of points $x \natural y$: $C[\phi(x)\phi(y)] = \phi(x)\phi(y) = \phi(y)\phi(x)$.
- In a labelled causal set: ordering a product of operators by decreasing label from left to right = causal ordering, *e.g.* $\phi(4)\phi(4)\phi(2)\phi(1)$

Causal ordering is the causal set analogue of the time ordering of the continuum.
Define the Feynman propagator as

$$\Delta^F_{xy} = \langle C[\phi(x)\phi(y)] \rangle. \tag{9}$$

Key Features

- Finitely many modes $\sim$ non-local collection of harmonic oscillators
- Manifestly causal formulation
- The SJ-vacuum is UNIQUE on any curved background!
- The SJ vacuum is NON-Hadamard
- The path integral formulation $\rightarrow$ In-In formalism. Decoherence functional
  Sorkin. Scalar field theory on a causal set in histories form, arXiv:1107.0698
- The free theory is gaussian $\rightarrow$ can define any n-point function by Wick's theorem.

The Heisenberg field in the continuum

In the continuum, the Heisenberg field $\phi^H(t, \mathbf{x})$ is related to the interaction picture field $\phi(t, \mathbf{x})$ via,

$$\phi^H(t, \mathbf{x}) = U^\dagger(t, t_0)\phi(t, \mathbf{x})U(t, t_0). \tag{10}$$

where,

$$U(t, t_0) = T\left[e^{-i\int_{t_0}^{t} H(t)dt}\right] \quad (t \geq t_0) \tag{11}$$

is the time-evolution operator and where H is the interacting Hamiltonian in the interaction picture.

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Recipe:

- Replace the time integral by a sum over causal set points
- Replace the time-ordering T with the causal ordering C. Under the action of C, all field commutators vanish and we can express the exponential of a sum as a product of exponentials.

Evolution Operators in Causal Set

Define the following family of evolution operators,

$$V_x = C\left[\prod_{z\prec x} e^{-i\mathcal{H}(z)}\right], \quad U_{x,y} = C\left[\prod_{y\leq z<x} e^{-i\mathcal{H}(z)}\right] \text{ for } x>y, \quad U_x = \begin{cases} 1 \\ C\left[\prod_{z<x} e^{-i\mathcal{H}(z)}\right] \end{cases}$$

and note that they satisfy the following relations,

$$V_x^\dagger V_x = U_x^\dagger U_x = 1, \qquad V_x^\dagger \mathcal{O}(x) V_x = U_x^\dagger \mathcal{O}(x) U_x \text{ for any local operator } \mathcal{O}(x). \qquad (12)$$

V_x is a **covariant operator**, relying only on the partial order $\prec$ (physical operator).
$U_{x,y}$ and U_x are **label-dependent operators**, relying on the total order $<$ of the labelling (gauge-dependent operators)
EA, Dowker, Nasiri, Zalel, Phys.Rev.D 109 (2024) 10, 106014
Dable-Heath, Fewster, Rejzner, Woods,Phys.Rev.D 101 (2020) 065013

Example : $\mathcal{H} = \frac{g}{3!}\phi^3$

Consider 1 interaction point such as $z \prec x$ (totally ordered),

$$\langle \phi_x^H \rangle = \langle e^{\frac{ig}{3!}\phi_z^3} \phi_x e^{-\frac{ig}{3!}\phi_z^3} \rangle$$
$$= \langle \phi_x \rangle - \frac{ig}{3!}\langle [\phi_x, \phi_z^3] \rangle + \frac{1}{2}\left(-\frac{ig}{3!}\right)^2 \langle [[\phi_x, \phi_z^3], \phi_z^3] \rangle = \frac{g}{2}\Delta^R_{x,z}\langle \phi_z^2 \rangle$$

given $\langle \phi_x \rangle = 0$, $[\phi_x, \phi_z] = i\Delta_{x,z} = i\Delta^R_{x,z}$

The perturbative series terminates at $\mathcal{O}(g)$ since $[\phi_z, \phi_z] = 0$, giving an exact answer!

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\begin{aligned}
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Can we derive a diagrammatic expansion of $\phi^H(x)$ and expectation values for an arbitrary interaction region?

The diagrammatic expansion for the Heisenberg field

- Associate a vertex with each of the points $x, z_1 \dots, z_n$.
  x is the external vertex and z_i are the internal vertices.
- The number of half-legs meeting at each vertex is equal to the number of fields at the associated point.
- Connect the half-legs in all possible ways to form directed edges with the following properties:
  (i) every internal vertex is connected to the external vertex by at least one directed path
  (ii) every directed edge is of the form $z_i \to x$ or $z_i \to z_j$ with $i > j \to$ the factors of Δ_{x,z_i} and Δ_{z_i,z_j}.

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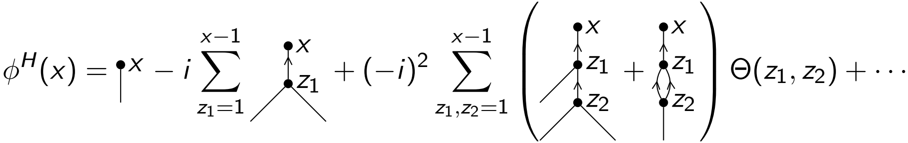
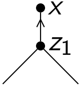
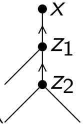
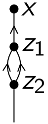


$$= \phi(x) + \frac{g}{2} \sum_{z_1}^{x-1} \Delta^R_{x,z_1} \phi(z_1)^2 + \frac{g^2}{2} \sum_{z_1,z_2=1}^{x-1} \Delta^R_{x,z_1} \Delta^R_{z_1,z_2} \left(\phi(z_1)\phi(z_2)^2 - i\Delta^R_{z_1,z_2}\phi(z_2)\right) \Theta(z_1, z_2) + \cdots ,$$

where in the second line we used the relation,

$$\Delta_{z_i z_j} \Theta(z_1, \ldots, z_n) = \Delta^R_{z_i z_j} \Theta(z_1, \ldots, z_n)$$

Example : $\mathcal{H} = \frac{g}{3!}\phi^3$

$\phi^H(x) =$  $- i \sum_{z_1=1}^{x-1}$  $+ (-i)^2 \sum_{z_1,z_2=1}^{x-1} \Big($  $+$  $\Big) \Theta(z_1, z_2) + \cdots$

$$= \phi(x) + \frac{g}{2}\sum_{z_1}^{x-1}\Delta^R_{x,z_1}\phi(z_1)^2 + \frac{g^2}{2}\sum_{z_1,z_2=1}^{x-1}\Delta^R_{x,z_1}\Delta^R_{z_1,z_2}\left(\phi(z_1)\phi(z_2)^2 - i\Delta^R_{z_1,z_2}\phi(z_2)\right)\Theta(z_1,z_2) + \cdots,$$

where in the second line we used the relation,

$$\Delta_{z_i z_j}\Theta(z_1,\ldots,z_n) = \Delta^R_{z_i z_j}\Theta(z_1,\ldots,z_n)$$

The field expansion terminates at a finite order in the interaction coupling. This order increases with the order of the interaction Hamiltonian and with the number of points to the past of x which are contained in the interaction region.

Properties of the field algebras

- *Causality.* Heisenberg fields at spacelike separated points commute,

$$[\phi^H(x), \phi^H(y)] = 0 \text{ for all } x, y \in \mathcal{C} \text{ with } x \natural y. \tag{13}$$

- *Polynomial property.* The Heisenberg field $\phi^H(x)$ can be written as,

$$\phi^H(x) = \phi(x) + Q_x(\phi(y); y \prec x), \tag{14}$$

  where Q_x is a finite order polynomial in the interaction picture fields in the past of x. Inverting this relationship,

$$\phi(x) = \phi^H(x) + P_x(\phi^H(y); y \prec x), \tag{15}$$

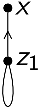
  where P_x is a finite order polynomial in the Heisenberg fields in the past of x.
- *Observable algebras.* Given a causet $\mathcal{C}$ and a subcauset $\mathcal{R} \subseteq \mathcal{C}$, we write $\mathfrak{A}^H_{\mathcal{R}}$ and $\mathfrak{A}_{\mathcal{R}}$ denote the algebras generated by $\{\phi^H(x)\}_{x\in\mathcal{R}}$ and $\{\phi(x)\}_{x\in\mathcal{R}}$, respectively. Then $\mathfrak{A}^H_{\mathcal{R}} = \mathfrak{A}_{\mathcal{R}}$ if and only if $\mathcal{R}$ is a stem in $\mathcal{C}$.

Expectation value of a field $\langle \phi^H(x) \rangle$

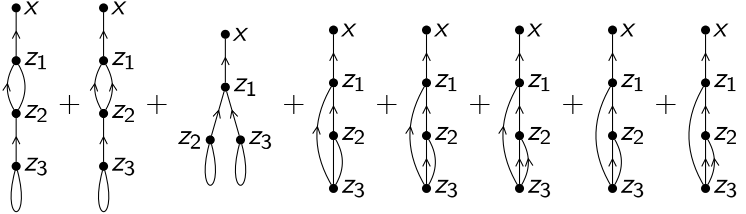
- We can apply Wick's theorem to the expansion of expanded $\phi^H(x)$ to obtain $\langle \phi^H(x) \rangle$.
- Diagrammatically, this corresponds to taking each diagram in the expansion of $\phi^H(x)$ and joining its half-legs to create undirected edges in all possible ways. Each undirected edge is a Feynman propagator, and each diagram is weighted by an additional Wick factor.
- Assuming $\langle \phi(x) \rangle = 0$, only the diagrams in which all half-legs are contracted contribute to the expectation value $\langle \phi^H(x) \rangle$.

Example: $\mathcal{H} = \frac{g}{3!}\phi^3$

$$\langle \phi^H(x) \rangle = -i \sum_{z_1=1}^{x-1}$$



$$+(-i)^3 \sum_{z_1,z_2,z_3=1}^{x-1} \Big(\quad \Big) \Theta(z_1, z_2, z_3)$$



$$+ \cdots \tag{16}$$

Unlabelled Diagrams

- Consider some labelled diagram G_n with n internal vertices which appears in the expansion of $\langle \phi^H(x) \rangle$ and permute the labels of its internal vertices to produce another diagram G'_n.
- For each isomorphism-class $[G_n]$ choose some representative G_n, and for each diagram $G'_n \in [G_n]$ apply the necessary coordinate transformation $(z_1 \ldots z_n) \to (z_1 \ldots z_n)$ to bring G'_n into the labelling of G_n.

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- Collecting the identical diagrams and taking into account the action of each coordinate transformation on Θ, we obtain,

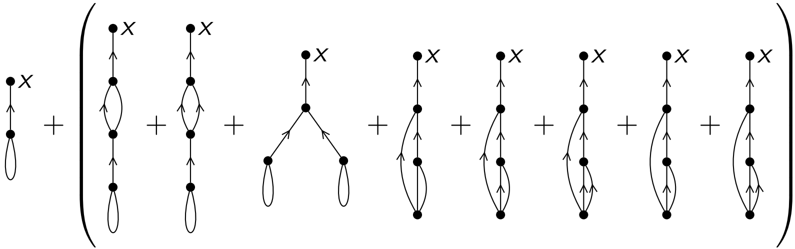
$$\langle \phi^H(x) \rangle = \sum_{n=0}^{\infty} \sum_{[G_n]} (-i)^n \sum_{z_1 \cdots z_n = 1}^{x_1 - 1} \frac{G_n}{|Aut(G_n)|} \sum_{\pi} \Theta(\pi), \tag{17}$$

  where $[G_n]$ denotes an unlablled diagram, G_n denotes a labelled diagram representative of $[G_n]$, $Aut(G_n)$ is the group of automorphisms of G_n which keep the x vertex fixed, $\pi = z_{i_1}, \dots, z_{i_n}$ is a permutation of $z_1, \dots, z_n$.
- $\sum_\pi \Theta(\pi) = 1$, the expansion simplifies to a sum over unlabelled diagrams $[G_n]$.

Example: $\mathcal{H} = \frac{g}{3!}\phi^3$

Our example can be rewritten in terms of $[G_1]$ and $[G_3]$ as

$$\langle \phi^H(x) \rangle =$$



$$+ \cdots \quad (18)$$

Key Features

- The field expansion **terminates at a finite order** in the interaction coupling.
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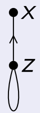
Key Features

- The field expansion **terminates at a finite order** in the interaction coupling.
- **MANIFESTLY CAUSAL**: Each internal vertex is connected to at least one external vertex by at least one directed path.
- What is the continuum approximation of Interacting QFT on CS? does it recover the QFT on the continuum?



Causal set:

$$g_{cs} \sum_z W_{zz}\, i\Delta^R_{xz}$$

Continuum:

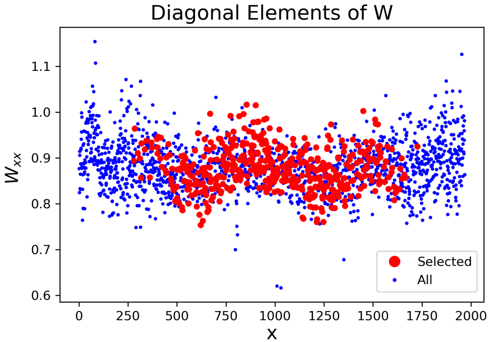
$$\frac{g_{\text{cont}}\, \rho}{2} \int_{-L}^{L} dv_z \int_{-L}^{L} du_z\, J_0(m\tau_{xz})\, \Delta^F_{\text{cont}}(0)$$

$$= g_{cs}\, \frac{1}{m^2} \Big(1 - J_0(2\sqrt{2}\, mL)\Big)\, \Delta^F_{\text{cont}}(0)$$

How $Diag(W)$ **and** $\Delta^F_{cont}(0)$ **are related?**

Propagator in the coincidence limit

Diag(W) is finite for finite $\rho \to$ Covariant regularization scheme. Work in progress with S. Surya, K. Rejzner

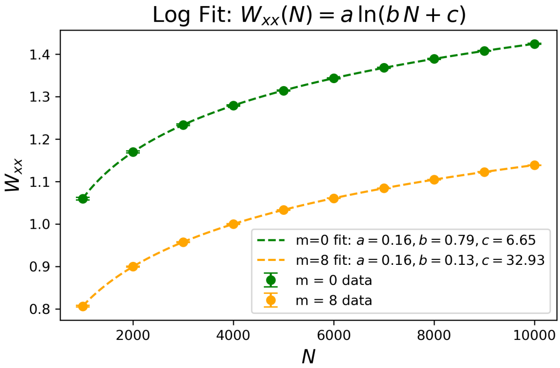


Afshordi et al, A Ground State for the Causal Diamond in 2 Dimensions, arXiv:1207.7101,

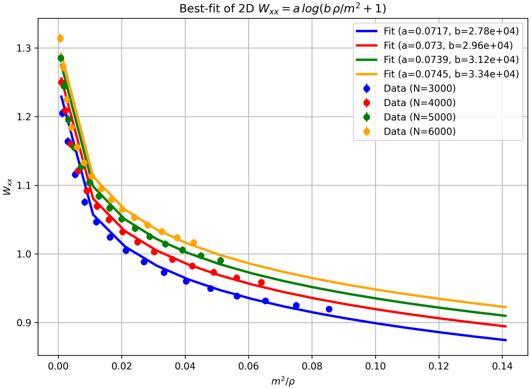
Mathur, Surya, Sorkin-Johnston vacuum for a massive scalar field in the 2D causal diamond, arXiv:1906.07952

Propagator in the coincidence limit

$$W^{cont}(0) = \int_{-\infty}^{\infty} \frac{dp_0}{4\pi^2} dp_1 \frac{-i}{-p_0^2 + p_1^2 + m^2}$$
$$= \int_0^{2\pi} \frac{d\theta}{4\pi^2} \int_0^{\Lambda} dpp \frac{1}{p^2 + m^2}$$
$$= \frac{1}{4\pi} Ln\left(\frac{\Lambda_{cont}^2 + m^2}{m^2}\right)$$



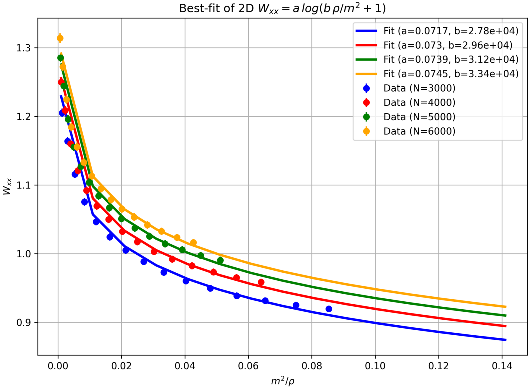
Propagator in the coincidence limit

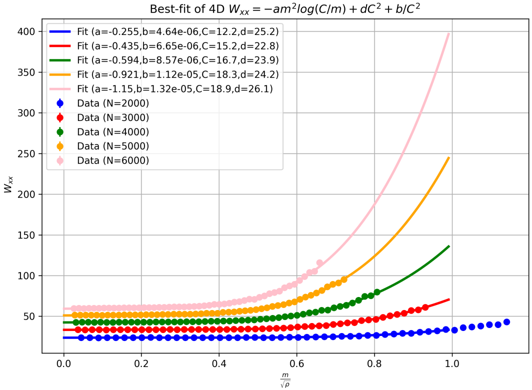


$$\Lambda^2_{cont} = b\rho \rightarrow \Lambda^{-1}_{cont} \ll l_p^2 \text{ for } b = 10^4$$

(?????)

Propagator in the coincidence limit





$$\Lambda_{cont}^2 = b\rho \rightarrow \Lambda_{cont}^{-1} \ll l_p^2 \text{ for } b = 10^4$$
(?????)

Outlook: The discrete cosmological collider

- Can we compute cosmological correlators on a causal set background? Yes!
- A new tool for cosmological collider physics, can produce predictions to compare against cosmological data to test for spacetime discreteness. E.g. Non- linearities during the inflationary era: M. Musso, A new diagrammatic representation for correlation functions in the in-in formalism, arXiv:hep-th/0611258

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- Can offer a novel regularization of the continuum, since there are no UV divergences on a causal set.

  Open Problems aka new year's resolutions:
  - Wetterich equation - renormalization group flow E. D'Angelo, Nicolò Drago, N. Pinamonti, K. Rejzner, An algebraic QFT approach to the Wetterich equation on Lorentzian manifolds, arXiv:2202.07580
  - Initial Value Problem - No Cauchy Hypersurface, What are the spacetime configurations for which is enough to specify the field values on the minimal elements?
  - Particle Production - Non locality of SJ vacuum vs Sandwich Spacetime set up and Bogoliubov transformations ...

Summary

- Causal Set Theory is an approach to quantum gravity in which spacetime is fundamentally discrete.
- It's a tool for new discoveries of non- local and Lorentz-invariant physics.
- New developments are enabling us to make concrete predictions, including for cosmological collider physics.

THANKS FOR LISTENING!


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ANOTHER DANCE AROUND
THE SUN