Quantum Trajectories as Unravellings

How does the nature of the trajectory depend on the detector, and does it really?

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CENTRE FOR QUANTUM COMPUTATION & COMMUNICATION TECHNOLOGY
AUSTRALIAN RESEARCH COUNCIL CENTRE OF EXCELLENCE

Outline

1. A Brief History of Quantum Trajectory Theory
   - Quantum Jumps 1913-1993
   - Not Just Quantum Jumps!
   - The Dynamics of Knowledge
2. Unravellings and EPR-Steering
   - Are quantum trajectories detector-dependent?
   - Back to the Future: EPR, 1935
   - Applying EPR-Steering to Atomic Fluorescence Experiments
   - Can we do better?
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   - Contrived Questions for Future Lectures / Work

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Bohr+Einstein: Quantum Jumps (1913-29)

The passing of the systems between different stationary states ... cannot be treated [using] ordinary mechanics ... [and] is followed by the emission of a homogeneous radiation, for which [$h\nu = \Delta E$]. (Bohr, 1913.)

[T]he theory ... leaves the moment and direction of the elementary processes to 'chance'. (Einstein, 1917.)

- The emission, and the jumps, were envisaged by Bohr and Einstein as **objective** microscopic physical events.
- Even in the New Quantum Theory it seemed quantum jumps remained, to the exasperation of Schrödinger:

If I had known we were going to go on having all this damned quantum-jumping, I would never have got involved in the subject. (Schrödinger, 1929.)

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The Long Night (1930–1985)

- The NQT enabled physicists to calculate a spontaneous emission rate γ from microscopic physics (Wigner–Weisskopf, 1930).
- In the 1960s physicists upgraded from rate equations to quantum optical master equation derived using the Born-Markov approximation *e.g.*,

$$\dot{\rho} = \mathcal{L}\rho \equiv -i[\hat{H}, \rho] + \gamma\mathcal{D}[\hat{\sigma}_-]\rho,$$

  - $\hat{\sigma}_- = |g\rangle\langle e|$ is an atomic lowering operator,
  - γ is the sponaneous emission rate (Einstein A coefficient),
  - $\mathcal{D}[\hat{c}]\rho \equiv \hat{c}\rho\hat{c}^\dagger - \frac{1}{2}\left\{\hat{c}^\dagger\hat{c}, \rho\right\}$
  - $\hat{H}$ is the Hamiltonian in the Interaction Frame,
  - *e.g.*, in *resonance fluorescence* (driving on-resonance) $\hat{H}_\Omega = \frac{\Omega}{2}\hat{\sigma}_x$.
- Now *most* physicists did not even talk about jumps. They just “shut up and calculated” photocurrent correlation functions.

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The Modern Understanding (c.1993-)

- The master equation is derived by ignoring (tracing over) the bath.
- It is not always appropriate to ignore the bath — often it can be measured, yielding information about the system.
- *If* the Born-Markov approximation is a good one *then* the bath can be measured repeatedly, on a time scale which is short compared to the interesting system evolution, *without invalidating the master equation*.
- This is called monitoring the system. If the monitoring is perfect, then this produces a *pure* conditioned system state $|\psi_c(t)\rangle$.
- We say the stochastic evolution for $|\psi_c(t)\rangle$ *unravels* the ME:

$$\mathrm{E}[|\psi_c(t)\rangle\langle\psi_c(t)|] = \rho(t) = \exp(\mathcal{L}t)|\psi(0)\rangle\langle\psi(0)|.$$

- If $\psi \in \mathbb{C}^D$, and D is very large, $\rho \in \mathbb{C}^{D\times D}$ may be too big to store. Then using a unravelling can be helpful numerically to calculate a running ensemble average: $\mathrm{E}_N[\langle\psi_c(t)|\hat{A}|\psi_c(t)\rangle] \approx \mathrm{Tr}\left[\rho(t)\hat{A}\right]$.

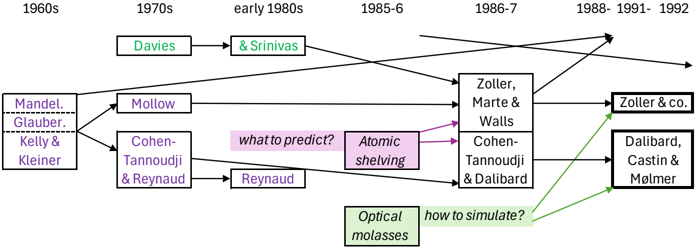
What happened 1986–1992?

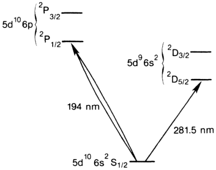
Formalisation of Quantum Jumps within Quantum Measurement Theory

Quantum Optics Theory: Photon Counting & Resonance Fluorescence

Single trapped ion expt.

Laser cooling experiments





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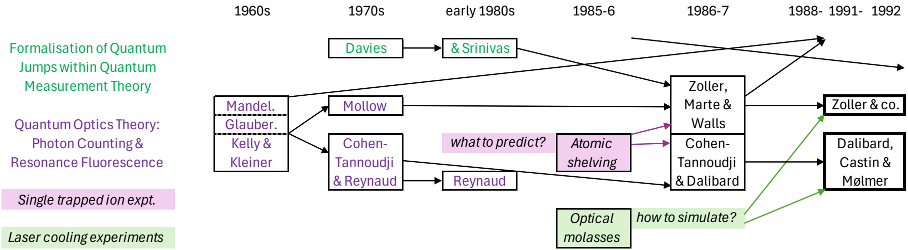

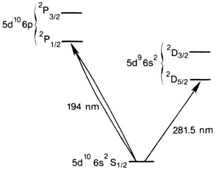

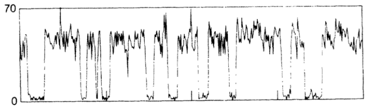

photodetections per 10 ms.
Wineland and co., PRL 1986.

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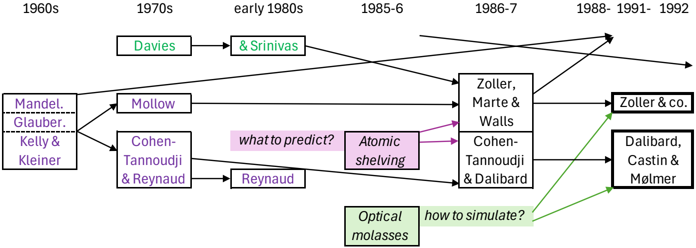

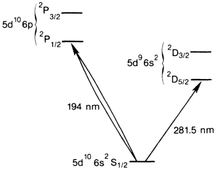

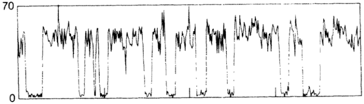

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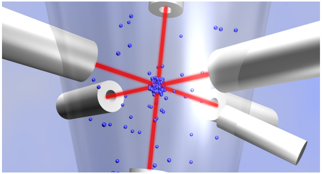

Contrast 1913 and 1993

- The emission, and the jumps, were envisaged by Bohr and Einstein as **objective** microscopic physical events.
- The jump occurs when a photon is **emitted**.
- In modern quantum jump theory, **perfect** monitoring of the bath produces a *pure* conditioned system state $|\psi_c(t)\rangle$.
- We say the stochastic evolution for $|\psi_c(t)\rangle$ *unravels* the ME:

$$\mathrm{E}[|\psi_c(t)\rangle\langle\psi_c(t)|] = \rho(t) = \exp(\mathcal{L}t)|\psi(0)\rangle\langle\psi(0)|.$$

- The jump occurs when a photon is **detected** (or even: when a "photo-detection" happens)

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Detection, or Emission — Who Cares?

- If there were only one way to detect a field, no-one should care.
- But there isn't. For an atom (or any Markovian system) the average system dynamics $\dot{\rho} = \mathcal{L}\rho = \mathcal{D}[\hat{c}]\rho - i[\hat{H}, \rho]$ is unchanged by any processing of the system output fields prior to detection.

- e.g. we can add a local oscillator field β.
- Mathematically, this amounts to $\hat{c} \to \hat{c} + \beta$, $\hat{H} \to \hat{H} - \frac{i}{2}(\beta^* \hat{c} - \beta \hat{c}^\dagger)$.
- In the limit $|\beta| \to \infty$, $\arg \beta = \Phi$, this is called *homodyne detection*.

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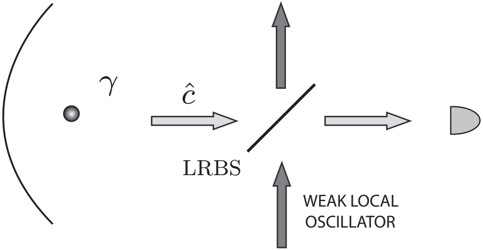

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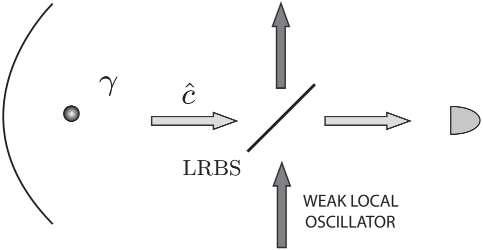

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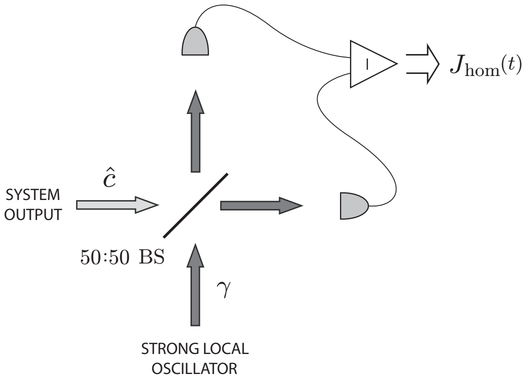

How this came about

Quantum squeezing expts.

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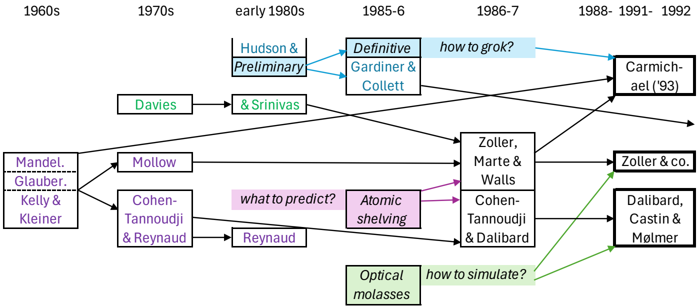

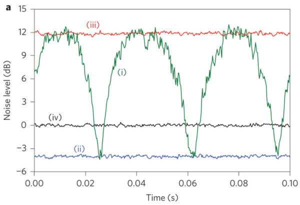

Different Stochastic Schrödinger Equations

- Master equation is fixed: $\dot{\rho} = \mathcal{L}\rho = -i[\hat{H}, \rho] + \mathcal{D}[\hat{c}]\rho$,
  where $\mathcal{D}[\hat{c}]\rho \equiv \hat{c}\rho\hat{c}^\dagger - \frac{1}{2}\left\{\hat{c}^\dagger\hat{c}, \rho\right\}$

- Quantum jump unravelling SSE [DalCasMøl92,DumZolRit92,GarParZol92]

$$d|\psi_\text{c}\rangle = \left[dN\left(\frac{\hat{c}}{\sqrt{\langle\hat{c}^\dagger\hat{c}\rangle_\text{c}}} - 1\right) - dt\left(i\hat{H} + \tfrac{1}{2}\hat{c}^\dagger\hat{c} - \tfrac{1}{2}\langle\hat{c}^\dagger\hat{c}\rangle_\text{c}\right)\right]|\psi_\text{c}\rangle,$$

  with $J_\text{direct}(t) = dN(t)/dt$, where $dN(t) \in \{0, 1\}$ is a count increment of mean $\text{E}[dN(t)] = \langle\psi_\text{c}(t)|\hat{c}^\dagger\hat{c}|\psi_\text{c}(t)\rangle$.

- Quantum diffusion (homodyne) unravelling SSE [Car93]

$$d|\psi_\text{c}\rangle = \left[dW(t)\left(\hat{c}_\Phi - \langle\hat{x}_\Phi\rangle_\text{c}\right) - dt\left(i\hat{H} + \tfrac{1}{2}\hat{c}^\dagger\hat{c} - 2\langle\hat{x}_\Phi\rangle_\text{c}\hat{c}_\Phi + \langle\hat{x}_\Phi\rangle_\text{c}^2\right)\right]|\psi_\text{c}\rangle$$

  with $J_\text{hom}(t) = 2\langle\hat{x}_\Phi\rangle_\text{c}dt + dW$, where $\hat{c}_\Phi = e^{-i\Phi}\hat{c}$, $2\hat{x}_\Phi = \hat{c}_\Phi + (\hat{c}_\Phi)^\dagger$, and $dW(t)$ is a Wiener increment satisfying $\text{E}[dW] = 0$, $\text{E}[dW^2] = dt$.

Illustration of these different unravellings

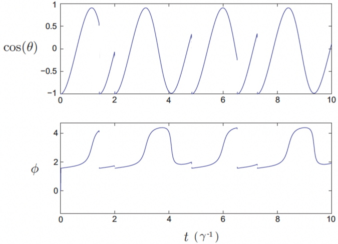

Fig. 4.4 A typical trajectory for the conditioned state of an atom under direct detection in terms of the Bloch angles ϕ and $\cos\theta$. The driving and detuning are $\Omega = 3$ and $\Delta = 0.5$, in units of the decay rate γ.

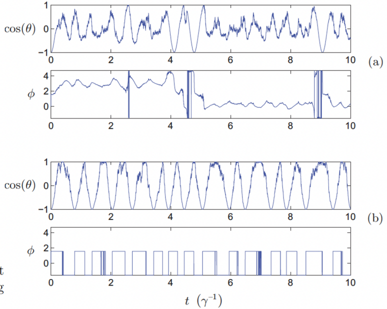

Fig. 4.6 Segment of a trajectory of duration $10\gamma^{-1}$ of an atomic state on the Bloch sphere under homodyne detection. The phase Φ of the local oscillator relative to the driving field is 0 in (a) and $\pi/2$ in (b). The driving and detuning are $\Omega = 3$ and $\Delta = 0$.

Wiseman and Milburn, *Quantum Measurement and Control*, Cambridge 2010

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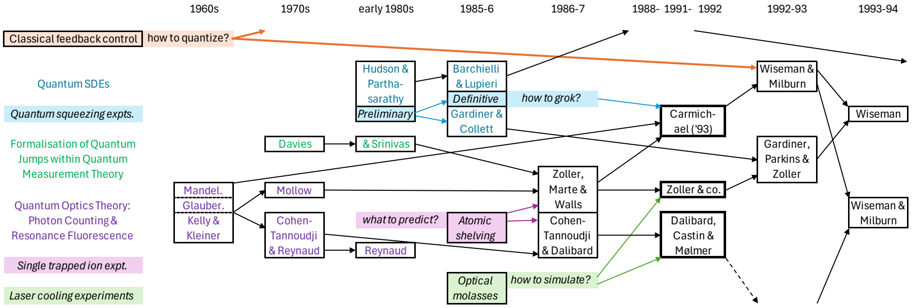
My entry into “history”



- 1991: my Honours (4th year undergrad) thesis with Gerard Milburn on *attempting* to describe quantum feedback, among other things.
- January 1992: at a Summer School at ANU, Howard Carmichael gave an unscheduled lunchtime lecture on quantum trajectories.
- I immediately — at least that’s how my memory flatters me ☺ — recognized that this was the tool I needed to do quantum feedback *right*.

Taking quantum trajectories seriously

- Real detection is not perfect!
  - Real systems "leak" — not all quantum information in the output fields makes it into detectors — and detectors are inefficient [WisMil93Jan].
  - The input (and therefore output) fields themselves may have thermal noise, or more general white noise [WisMil94].
  - Other detector imperfections: dark counts, finite bandwidth [WarWis03].
- $\Longrightarrow$ the actual conditioned state will be *mixed*, $\rho_{\rm c}(t)$, and its evolution described by a Stochastic Master Equation* (SME) [WisMil93Jan]
- Unlike an SSE, a SME is not† useful for simulating ME averages.
- But it *is* useful for describing feedback control, *e.g.*, that generated by

$$\hat{H}_{\rm fb}(t) = \hat{Z} \int_0^\infty h(s)\, J(t-s)\, ds$$

  for generic (not analytically solvable) quantum systems.
- Also, this $\rho_{\rm c}(t)$ formulation can actually make the jump and diffusion unravellings look less different, mathematically ...

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The usual formulation of jump and diffusive SME

- The (usual) jump SME:

$$d\rho = \mathcal{L}\rho\, dt + \left(\frac{\hat{c}\rho\hat{c}^\dagger}{\mathrm{Tr}[\hat{c}\rho\hat{c}^\dagger]} - \rho \right) \left(dN - \eta \mathrm{Tr}[\hat{c}\rho\hat{c}^\dagger]\, dt \right)$$

$$\text{with} \begin{cases} \mathbb{P}[dN = 0] = 1 - \mathbb{P}[dN = 1] \\ \mathbb{P}[dN = 1] = \eta \mathrm{Tr}[\hat{c}\rho\hat{c}^\dagger]\, dt \end{cases}$$

- The (usual) diffusive SME:

$$d\rho = \mathcal{L}\rho\, dt + \sqrt{\eta} \left(\hat{c}\rho + \rho\hat{c}^\dagger - \mathrm{Tr}[(\hat{c} + \hat{c}^\dagger)\rho]\rho \right) dW$$

$$\text{with } dY = \sqrt{\eta}\mathrm{Tr}[(\hat{c} + \hat{c}^\dagger)\rho]\, dt + dW$$

Can we unify the two formulations?

- The main intuition came from the **unified structure** of the correlation functions formula for $I_t = \mathrm{d}N_t/\mathrm{d}t$ or $I_t = \mathrm{d}Y_t/\mathrm{d}t$:

$$\mathbb{E}[I_{t_1} \dots I_{t_n}] = \mathrm{Tr}\left[\mathcal{M}e^{(t_n - t_{n-1})\mathcal{L}} \dots \mathcal{M}e^{t_1\mathcal{L}}\rho_0\right]$$

for $t_1 < \cdots < t_n$ and $\mathcal{L}_t = \mathcal{L}$

$$\text{with} \begin{cases} \mathcal{M}\rho = \theta\rho + \eta\hat{c}\rho\hat{c}^\dagger & \text{for the jump SME} \\ \mathcal{M}\rho = \sqrt{\eta}(\hat{c}\rho + \rho\hat{c}^\dagger) & \text{for the diffusive SME} \end{cases}$$

Proof in e.g. *Pierre Guilmin, Pierre Rouchon and Antoine Tilloy. "Correlation functions for realistic continuous quantum measurement." IFAC-PapersOnLine 56.2 (2023).*

- This will be our **guiding light**: we want to preserve this structure. Writing $\mathrm{d}R_t = \mathrm{d}N_t$ or $\mathrm{d}R_t = \mathrm{d}Y_t$, we have

$$\mathbb{E}[\mathrm{d}R_t/\mathrm{d}t] = \mathrm{Tr}[\mathcal{M}e^{t\mathcal{L}}\rho_0] \implies \mathbb{E}[\mathrm{d}R_t] = \mathrm{Tr}[\mathcal{M}\rho_t]\,\mathrm{d}t$$

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Derivation and history

- **Part one** | January 2024 (with Antoine Tilloy and Pierre Rouchon)
  - Use the jump SME with the dark-count rate θ.
  - Replace $\mathrm{d}W$ by the signal $\mathrm{d}Y$ in the diffusive SME.
  - Use the superoperator $\mathcal{M}$ whenever possible.
  - Get stuck with the normalisation.
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  - Ask Howard's help.
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The new unified formulation

$$\mathrm{d}\rho = \left[\mathcal{L}\,\mathrm{d}t + \frac{(\mathrm{d}R - \mathbb{E}[\mathrm{d}R])\,(\mathrm{d}\mathcal{M} - \mathbb{E}[\mathrm{d}R])}{\sigma^2_{\mathrm{d}R}}\right]\rho$$

$$\text{with} \begin{cases} \mathcal{M}\rho = \theta\rho + \eta\hat{c}\rho\hat{c}^\dagger & \text{for the jump SME} \\ \mathcal{M}\rho = \sqrt{\eta}(\hat{c}\rho + \rho\hat{c}^\dagger) & \text{for the diffusive SME} \end{cases}$$

and $\mathbb{E}[\mathrm{d}R] = \mathrm{Tr}[\mathrm{d}\mathcal{M}\rho] = \mathrm{Tr}[\mathcal{M}\rho]\,\mathrm{d}t$ (with $\mathrm{d}\mathcal{M} = \mathcal{M}\,\mathrm{d}t$)

Why is it interesting?

$$\mathrm{d}\rho = \left[\mathcal{L}\,\mathrm{d}t + \frac{(\mathrm{d}R - \mathbb{E}[\mathrm{d}R])\,(\mathrm{d}\mathcal{M} - \mathbb{E}[\mathrm{d}R])}{\sigma^2_{\mathrm{d}R}}\right]\rho$$

with $\mathbb{E}[\mathrm{d}R] = \mathrm{Tr}[\mathrm{d}\mathcal{M}\rho]$

- **Unification.** The two seemingly different SMEs are **the same**!
- **Unconditionned evolution.** The first term $(\mathrm{d}R - \mathbb{E}[\mathrm{d}R])$ guarantees that we recover **Lindblad** on average for $\bar{\rho}_t = \mathbb{E}[\rho_t] \to \mathrm{d}\bar{\rho}/\mathrm{d}t = \mathcal{L}\bar{\rho}$.
- **Trace preservation.** The second term $(\mathrm{d}\mathcal{M} - \mathbb{E}[\mathrm{d}R])\rho = (\mathrm{d}\mathcal{M}\rho - \mathrm{Tr}[\mathrm{d}\mathcal{M}\rho]\rho)$ is traceless and guarantees that the trace of the state is **preserved** at all times: $\mathrm{Tr}[\rho_t] = 1$.
- **Signal normalisation.** The denominator $\sigma^2_{\mathrm{d}R}$ accounts for the **gauge freedom** in choosing the signal unit (invariance under $\mathrm{d}R \to \alpha\,\mathrm{d}R$).
- **Measurement backaction interpretation**: the more the observer is **surprised**, the stronger the backaction.

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The usual jump-diffusive SME

$$\mathrm{d}\rho = \mathcal{L}\rho\,\mathrm{d}t + \sum_{k\in S_{\text{diffusive}}} \sqrt{\eta_k}\left(\hat{c}_k\rho + \rho\hat{c}_k^\dagger - \mathrm{Tr}[(\hat{c}_k + \hat{c}_k^\dagger)\rho]\rho\right)\mathrm{d}W_k$$

$$+ \sum_{k\in S_{\text{jump}}}\left(\frac{\theta_k\rho + \eta_k\hat{c}_k\rho\hat{c}_k^\dagger}{\theta_k + \eta_k\mathrm{Tr}[\hat{c}_k\rho\hat{c}_k^\dagger]} - \rho\right)\left(\mathrm{d}N_k - (\theta_k + \eta_k\mathrm{Tr}[\hat{c}_k\rho\hat{c}_k^\dagger])\,\mathrm{d}t\right)$$

with for all $k \in S_{\text{jump}}$, $\begin{cases} \mathbb{P}[\mathrm{d}N_k = 0] = 1 - \mathbb{P}[\mathrm{d}N_k = 1] \\ \mathbb{P}[\mathrm{d}N_k = 1] = (\theta_k + \eta_k\mathrm{Tr}[\hat{c}_k\rho\hat{c}_k^\dagger])\,\mathrm{d}t \end{cases}$

and for all $k \in S_{\text{diffusive}}$, $\mathrm{d}Y_k = \sqrt{\eta_k}\mathrm{Tr}[(\hat{c}_k + \hat{c}_k^\dagger)\rho]\,\mathrm{d}t + \mathrm{d}W_k$

The unified jump-diffusive SME

$$d\rho = \left[\mathcal{L}\, dt + \sum_k \frac{(dR_k - \mathbb{E}[dR_k])\,(d\mathcal{M}_k - \mathbb{E}[dR_k])}{\sigma^2_{dR_k}} \right] \rho$$

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$$\text{and for all } k \in S_{\text{diffusive}}, \begin{cases} dR_k = \text{Tr}[d\mathcal{M}_k \rho] + dW_k \\ \mathcal{M}_k \rho = \sqrt{\eta}(\hat{c}_k \rho + \rho \hat{c}_k^\dagger) \end{cases}$$

Back to our guiding light

The correlation functions for the signals $\{I_k = \mathrm{d}R_k/\mathrm{d}t\}_{k \,\in\, S_{\text{jump}} \,\cup\, S_{\text{diffusive}}}$ are:

$$\mathbb{E}[I_{k_1,t_1} \ldots I_{k_n,t_n}] = \mathrm{Tr}\left[\mathcal{M}_{k_n} e^{(t_n - t_{n-1})\mathcal{L}} \ldots \mathcal{M}_{k_1} e^{t_1 \mathcal{L}} \rho_0\right]$$

for $t_1 < \cdots < t_n$ and $\mathcal{L}_t = \mathcal{L}$

What's next for this formula

- **Pedagogical purpose.** Four axes when you learn about quantum trajectories: jump vs. diffusive, SSE vs. SME, linear vs. non-linear, discrete-time vs. continuous-time.
- **Unified proofs.** For example for the correlation functions formula?
- **From a unified formulation to a complete characterisation.** Does this form characterise all possible unravellings?
  - Only allowed stochastic process that give a Markovian ρ: Poisson and Wiener (Lévy-Itô decomposition).
  - Additional constraints on the measurement backaction (form of $\mathcal{M}$):
    - CP map: $\rho + \mathrm{d}\rho \geq 0$.
    - For perfect detection ($\theta = 0, \eta = 0$), we want an initial pure state to remain pure at all times $\mathrm{Tr}[\rho_t^2] = 1$.

$$\mathrm{d}\rho = \left[\mathcal{L}\,\mathrm{d}t + \frac{(\mathrm{d}R - \mathbb{E}[\mathrm{d}R])\,(\mathrm{d}\mathcal{M} - \mathbb{E}[\mathrm{d}R])}{\sigma^2_{\mathrm{d}R}}\right]\rho$$

- **Why do we care?** For example, to build new models of the world!

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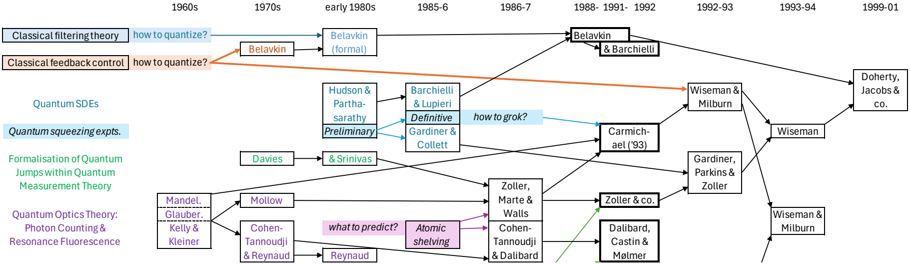
For the pros, here is the general linear SME

For any **ostensible distribution** ϱ (possibly time-dependent), the non-unit state $\tilde{\rho}$ satisfies:

$$\mathrm{d}\tilde{\rho}_{:\varrho} = \left[\mathcal{L}\,\mathrm{d}t + \frac{(\mathrm{d}R - \mathbb{E}_\varrho[\mathrm{d}R])\,(\mathrm{d}\mathcal{M} - \mathbb{E}_\varrho[\mathrm{d}R])}{\sigma^2_{\mathrm{d}R,\varrho}} \right] \tilde{\rho}$$

$$\text{with } \mathbb{E}_\varrho[\mathrm{d}R] = \mathrm{Tr}[\mathrm{d}\mathcal{M}\varrho]$$

A proper appreciation of $\rho_c(t)$

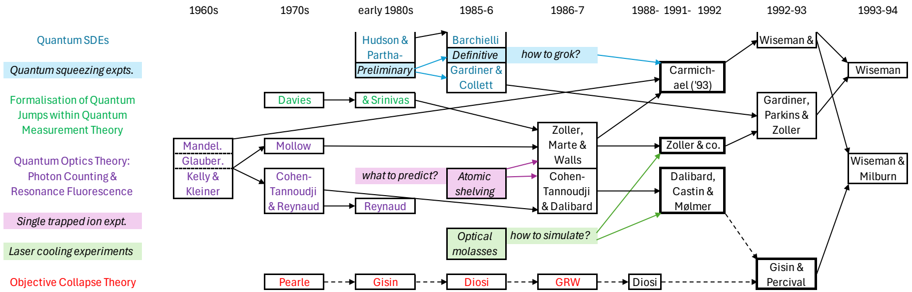


- More than being useful for *modelling* general feedback control, $\rho_c(t)$ is the object that (*if* you can calculate it in real time) is all you need to determine the optimal control $\mathbf{u}(t)$ to apply at time t provided the control objective is to maximize a function of the form $\mathrm{E}[\int dt \langle \hat{h}(\mathbf{u}(t), t)\rangle]$, with $\hat{h}$ a system operator.
- This was shown (very) formally by Belavkin in 1983(?).
- It was not appreciated in physics until Doherty and Jacobs and co. independently made the connection for linear systems, and then 'we' started to understand the generality of work by Belavkin and co.

Outline

1. A Brief History of Quantum Trajectory Theory
   - Quantum Jumps 1913-1993
   - Not Just Quantum Jumps!
   - The Dynamics of Knowledge
2. Unravellings and EPR-Steering
   - Are quantum trajectories detector-dependent?
   - Back to the Future: EPR, 1935
   - Applying EPR-Steering to Atomic Fluorescence Experiments
   - Can we do better?
3. Conclusion
   - Summary
   - Contrived Questions for Future Lectures / Work

Quantum state confusion?



- GP's model for Quantum State Diffusion with $dV^2 = 0$, $|dV|^2 = dt$,

$$d|\psi_c\rangle = \left[dV(t)\,(\hat{c} - \langle\hat{c}\rangle_c) - dt\left(i\hat{H} + \tfrac{1}{2}\hat{c}^\dagger\hat{c} - 2\langle\hat{c}^\dagger\rangle_c\hat{c} + |\langle\hat{c}\rangle_c|^2 \right) \right] |\psi_c\rangle,$$

  is invariant under transformations that leave the ME invariant.

- [WisMil93March] showed that this is the *heterodyne detection* SSE.

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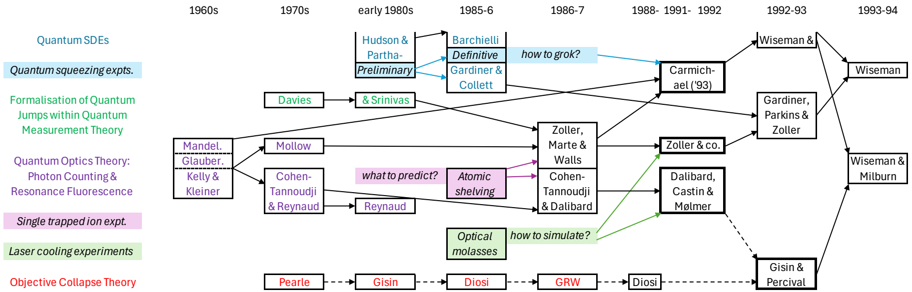

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Are quantum trajectories detector-dependent?

- Yes!
- But this was not obvious to people even in 1989:

  What role does photoelectric detection actually play in the return of the atom to its ground state after each photon emission? … We argue … that photoelectric detection does not cause atomic state reduction. Projection of the atom into its ground state is caused by the dissipative nature of the atomic dynamics … with complete indifference to the presence or absence of an observer. Photoelectric detection merely monitors emitted (realized) photons.

- Nor to Gisin and Percival in 1992:

  QSD [Quantum state diffusion] is a model for the motion of an [individual] quantum system in interaction with its environment. … [The quantum jump model] provides a different insight [into the behaviour of individual systems], and it remains to be seen which, if any, is preferable.

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Can we derive **realistic** *experimental tests that would rule out* **all** *OPSDMs, including objective quantum jumps, and QSD?*

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Einstein, Podolsky & Rosen, 1935; Schrödinger, 1935

EPR introduce a general pure state held by (say) Alice and Bob:

$$|\Psi\rangle = \sum_{n=1}^{\infty} c_n |u_n\rangle|\psi_n\rangle = \sum_{s=1}^{\infty} d_s |v_s\rangle|\varphi_s\rangle. \quad (1)$$

If Alice measures in the $\{|u_n\rangle\}$ (resp. $\{|v_s\rangle\}$) basis, she would instantly collapse Bob's system into one of the states $|\psi_n\rangle$ (resp. $|\varphi_s\rangle$):

> *[A]s a consequence of two different measurements performed upon the first system, the* [distant] *second system may be left in states with two different* [types of] *wavefunctions.*

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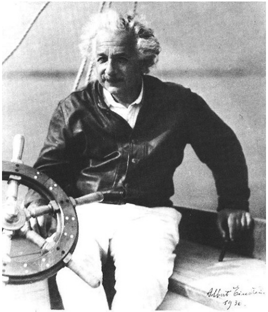

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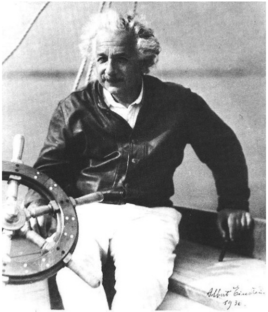

Formalizing EPR-Steering, 2007

- HMW, Jones & Doherty (PRL, 2007) formalized and generalized EPR-steering: to demonstrate EPR-steering is to demonstrate that a **Local Hidden State** assumption for Bob cannot hold.
- The LHS assumption is that Bob has a local hidden state π_ξ (hidden to him, but perhaps known to Alice) with probability $\wp_\xi$.
- No assumptions at all are made about Alice, except that, being distant, she cannot alter Bob's state.
- That is, different measurements for Alice can only mean different processing of her potential information (ξ).
- In analogy with Bell inequalities, one can construct EPR-steering inequalities (bipartite correlation functions), the violation of which demonstrates the failure of the LHS assumption.

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EPR-steering Inequality for a Qubit

- If Bob's system is a qubit, then for any π, $\langle\hat{\sigma}_x\rangle^2 + \langle\hat{\sigma}_y\rangle^2 + \langle\hat{\sigma}_z\rangle^2 \leq 1$.
- Say that Alice can perform two different measurements A_1 and A_2.
- Then under the LHS assumption it follows that (for example),

$$\mathrm{E}^{A_1}\left\{\left(\langle\hat{\sigma}_x\rangle_j^{A_1}\right)^2\right\} + \mathrm{E}^{A_2}\left\{\left(\langle\hat{\sigma}_y\rangle_j^{A_2}\right)^2 + \left(\langle\hat{\sigma}_z\rangle_j^{A_2}\right)^2\right\} \leq 1.$$

where j (Alice's "result") is the index for the ensemble, so

$$\text{e.g. } \mathrm{E}^{A_1}\left\{\left(\langle\hat{\sigma}_x\rangle_j^{A_1}\right)^2\right\} \equiv \sum_j \wp_j^{A_1}\left(\mathrm{Tr}\left[\rho_j^{A_1}\hat{\sigma}_x\right]\right)^2$$

is an average property of Bob's state conditioned on Alice's "result" j.
- If this inequality is violated, that demonstrates EPR-steering.

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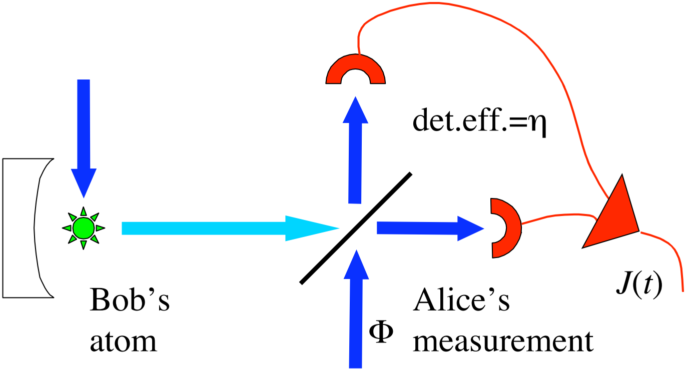
EPR-steering for a continuously monitored system

- If Bob's atom evolved according to an objective pure-state dynamical model (OPSDM) then at all times t it would be in some pure state π_ξ, and Alice's best knowledge would be if she knew ξ.
- We can disprove every OPSDM if Alice can implement two different monitoring schemes on the atom's fluorescence, A_1 and A_2, which allow her to violate an EPR-steering inequality.
- Wait until **steady-state**, when entanglement has built up.
- Thus to test the EPR-steering inequality Bob should:

1. Randomly choose $\alpha = 1$ or 2, and tell Alice to implement A_1 or A_2.
2. Randomly choose the time t ($\gg$ the system relaxation time) and measure $\hat{\sigma}_x$ or $\hat{\sigma}_y$ or $\hat{\sigma}_z$ at this time.
3. Ask Alice which state (from a set $\{\rho_j^{A_\alpha}\}$ nominated earlier by her) pertained to his atom at time t.
4. Store his data in different files for different α and j.

What types of monitoring schemes?

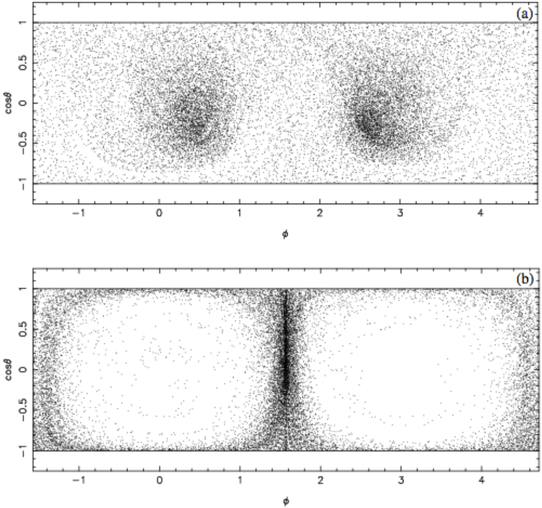
- Presently, the best efficiency is with homodyne measurement.
- This uses a strong local oscillator with a **choice of phase** Φ.



- The index j defining the state $\rho_j^{A\alpha}$ will depend on the complete photocurrent record $J^\alpha(s)$ for $0 \le s < t$.

Homodyne x versus Homodyne y

In the strong driving limit ($\hat{H} = \frac{\Omega}{2}\hat{\sigma}_x; \Omega \gtrsim \gamma$) these two monitorings with $\eta = 1$ should give distinctly different atomic-state trajectories: [WisMil93March]



- A_1: homo-x ($\Phi = 0$).

  ρ_c *tends* to localize at longitude $\phi = 0$ or $\phi = \pi$, near the states: $\langle \hat{\sigma}_x \rangle = \pm 1$.

- A_2: homo-y ($\Phi = \frac{\pi}{2}$).

  ρ_c is confined to the $\hat{\sigma}_x = 0$ great circle ($\phi = \pm\pi/2$) where $\langle \hat{\sigma}_y \rangle^2 + \langle \hat{\sigma}_z \rangle^2 = 1$.

Applying the Steering Inequality[1]

Recall: LHS $\implies$ $\mathrm{E}^{A_1}\left\{\left(\langle\hat{\sigma}_x\rangle_j^{A_1}\right)^2\right\} + \mathrm{E}^{A_2}\left\{\left(\langle\hat{\sigma}_y\rangle_j^{A_2}\right)^2 + \left(\langle\hat{\sigma}_z\rangle_j^{A_2}\right)^2\right\} \leq 1.$

- The above behaviours of the two-level atom under unravellings A_1 (homo-x) and A_2 (homo-y) suggest this is a good inequality to try to violate.
- As a function of η (assumed the same for A_1 and A_2), we find that $\eta > 73\%$ suffices.

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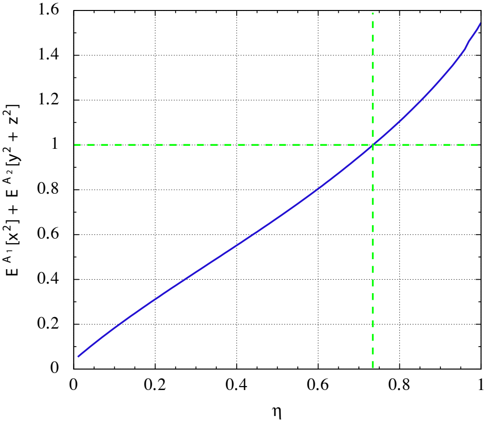

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No-go theorem[2]

- Can we do better (i.e. a lower threshold efficiency), e.g. by considering more than two homodyne schemes?
- Consider an *arbitrary* master equation for an *arbitrary* system,

$$\dot{\rho} = -i[\hat{H}, \rho] + \sum_{\ell=1}^{L} \mathcal{D}[\hat{c}_\ell]\rho$$

with an *arbitrary* number of different diffusive unravellings.

- Say the efficiency with which each channel (ℓ) can be monitored is η_ℓ.
- If $\forall \ell, \eta_\ell < 0.5$, then **one cannot show detector-dependent quantum dynamics**, no matter what EPR-steering inequality one uses.

[2] S. Daryanoosh & H. M. Wiseman, New J. Physics **16**, 063028 (2014).

Go no further, no-go theorem

"Quantum jumps are more quantum than quantum diffusion"[3]

- A different master equation

$$\mathcal{L} = \delta\mathcal{D}[\hat{\sigma}_-] + \epsilon\mathcal{D}[\hat{\sigma}_+]$$

- It has infinitely many **adaptive** unravellings giving two-state ensembles (for $\eta = 1$).
- Applying a large number of these types of schemes (with $\eta < 1$) can demonstrate EPR-steering with η as low as 37%.

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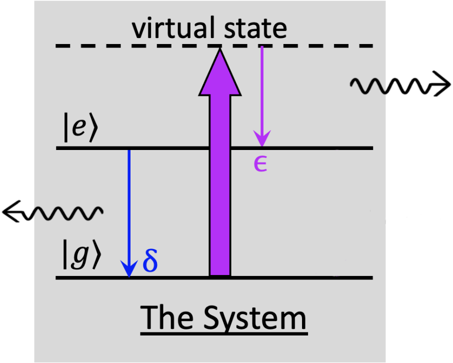

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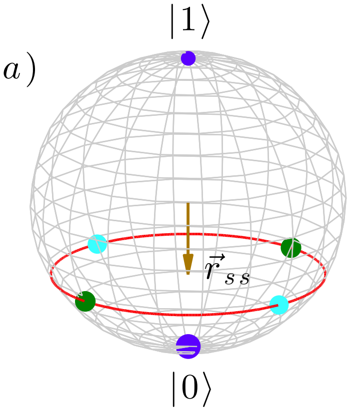

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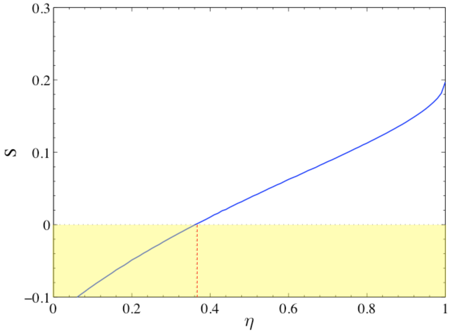
$$S \equiv \frac{1}{n}\sum_{j=1}^{n} \mathrm{E}^{\varphi_j}\left[\left|\left\langle \hat{\sigma}_{\varphi_j}\right\rangle\right|\right] - f(n)\mathrm{E}^{z}\left[\sqrt{1-\langle\hat{\sigma}_z\rangle^2}\right]$$



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- From 1993 it has been generally recognized that OPSDMs are false: different *distant* detection schemes lead to different *unravellings* of the master equation into pure conditioned state trajectories.
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- We have proposed an experiment that could rule out all OPSDMs by being able to implement two different homodyne measurements on a 2LA, to violate an EPR-steering inequality.
- The required efficiency (collection and detection) is only 73%.
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Contrived Questions for Future Lectures / Work

- Will experimentalists try to disprove OPSDMs? Hopefully.
- Will I explain more about adaptive unravellings? Yes, tomorrow.
- Given that classical filtering theory could be "quantized" to get quantum trajectories, what about classical *smoothing* theory? See my talk 4th Feb.
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