Josephson-Current Signatures of Unpaired Floquet Majorana Bound States

Rekha Kumari, Babak Serdjeh (IUB), Arijit Kundu (IITK)

Phys. Rev. Lett. 113, 196601 (2024)

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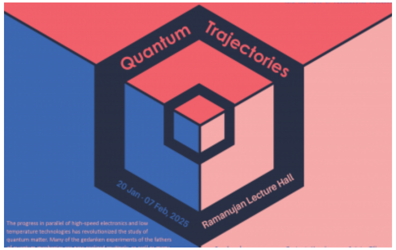

Quantum Trajectories,

ICTS Bengaluru,

2025

Motivation

Topological quantum computation based on chiral Majorana fermions

Biao Lian[a,b,1], Xiao-Qi Sun[b,c,1], Abolhassan Vaezi[b,c], Xiao-Liang Qi[b,c,d], and Shou-Cheng Zhang[b,c,2]

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Contributed by Shou-Cheng Zhang, September 5, 2018 (sent for review June 11, 2018; reviewed by Eduardo Fradkin, Naoto Nagaosa, and Fuchun Zhang)

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REVIEW ARTICLE **OPEN**

Majorana zero modes and topological quantum computation

Sankar Das Sarma[1,2], Michael Freedman[2] and Chetan Nayak[2,3]

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PHYSICAL REVIEW B **88**, 035121 (2013)

Flux-controlled quantum computation with Majorana fermions

T. Hyart,[1] B. van Heck,[1] I. C. Fulga,[1] M. Burrello,[1] A. R. Akhmerov,[2] and C. W. J. Beenakker[1]

[1]*Instituut-Lorentz, Universiteit Leiden, P. O. Box 9506, 2300 RA Leiden, The Netherlands*

[2]*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*

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REVIEW ARTICLE OPEN

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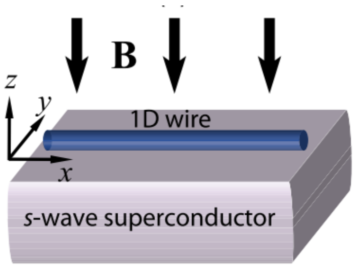
Majorana fermions in a tunable semiconductor device

Jason Alicea
Department of Physics, California Institute of Technology, Pasadena, California 91125, USA
(Received 16 December 2009; published 15 March 2010)

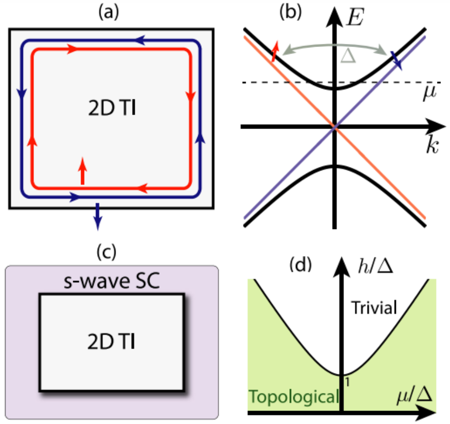
New directions in the pursuit of Majorana fermions in solid state systems

To cite this article: Jason Alicea 2012 *Rep. Prog. Phys.* **75** 076501



Basic architecture required to stabilize a topological superconducting state in a 1D spin–orbit-coupled wire.

Two-dimensional topological insulator + s-wave superconductivity + Zeeman field -> The effective low-energy Hamiltonian is equivalent to the Kitaev system.



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PRL **111,** 047002 (2013) PHYSICAL REVIEW LETTERS week ending 26 JULY 2013

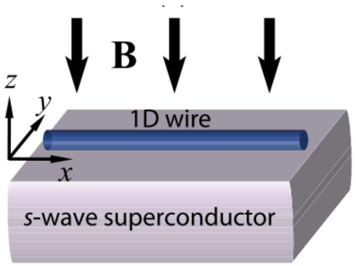
Floquet Majorana Fermions for Topological Qubits in Superconducting Devices and Cold-Atom Systems

Dong E. Liu,[1,2,*] Alex Levchenko,[2] and Harold U. Baranger[1]
[1]*Department of Physics, Duke University, Box 90305, Durham, North Carolina 27708-0305, USA*
[2]*Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA*
(Received 6 November 2012; published 22 July 2013)

Floquet Topological Superfluid and Majorana Zero Modes in Two-Dimensional Periodically Driven Fermi Systems

Xiaosen Yang, Beibing Huang & Zhengling Wang

Scientific Reports **8**, Article number: 2243 (2018) | Cite this article



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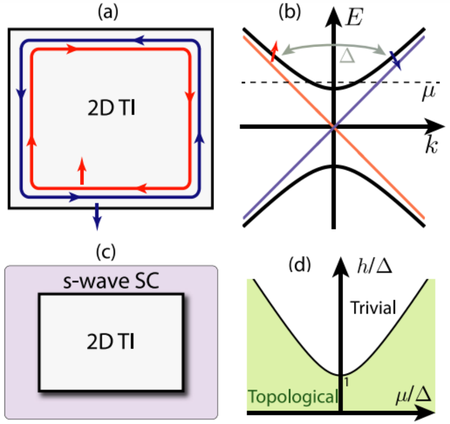

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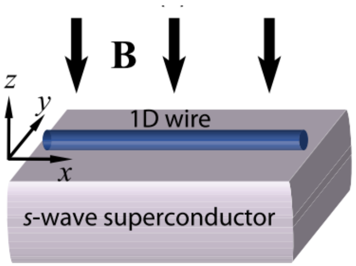

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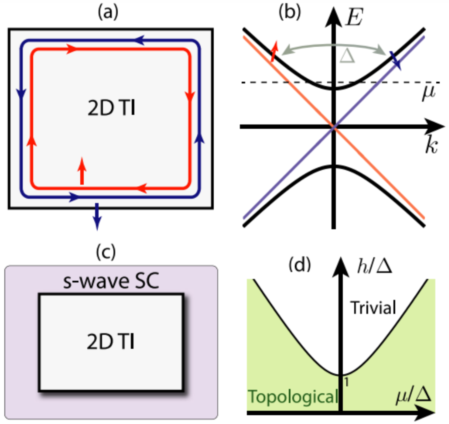

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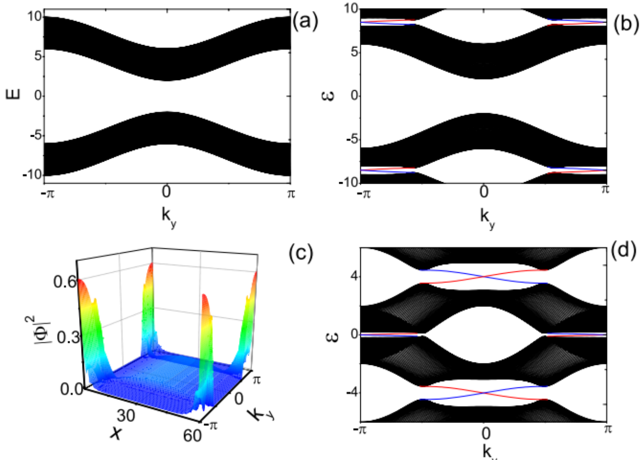


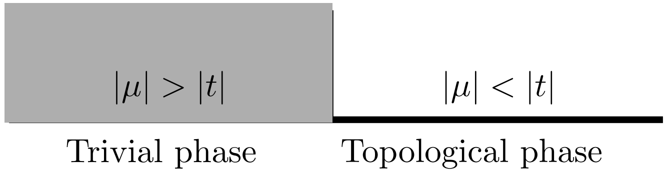
The edge states of a Floquet topological superfluid: P-wave superconductor in the presence of an external Floquet field.

Majorana Fermions in Kiteav model (static case)

- The Kitaev model describes a one-dimensional spinless p-wave superconductor. The Hamiltonian in particle basis is given by:

$$H = \mu \sum_{i}^{N} c_i^\dagger c_i - \frac{t}{2} \sum_{i=0}^{N-1} (c_{i+1}^\dagger c_i + c_i^\dagger c_{i+1}) - \frac{\Delta}{2} \sum_{i=0}^{N-1} (c_{i+1} c_i + c_i^\dagger c_{i+1}^\dagger)$$

Here t, μ, $\Delta$are hopping parameter, onsite chemical potential, and superconducting gap.



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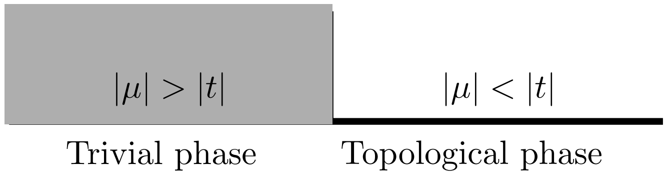
- Majorana basis transformation: In parameter space the system can have two distinct phases

$$H = -t\frac{i}{2} \sum_{i=0}^{N-1} \gamma_{2,i}\gamma_{1,i+1} + \frac{i}{2}\mu \sum_{i=0}^{N} \gamma_{1,i}\gamma_{2,i}$$



$$c_i^\dagger = \frac{1}{2}(\gamma_{1,i} + i\gamma_{2,i})$$

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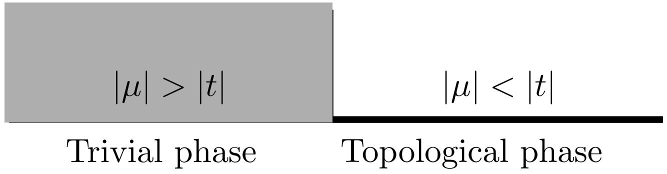

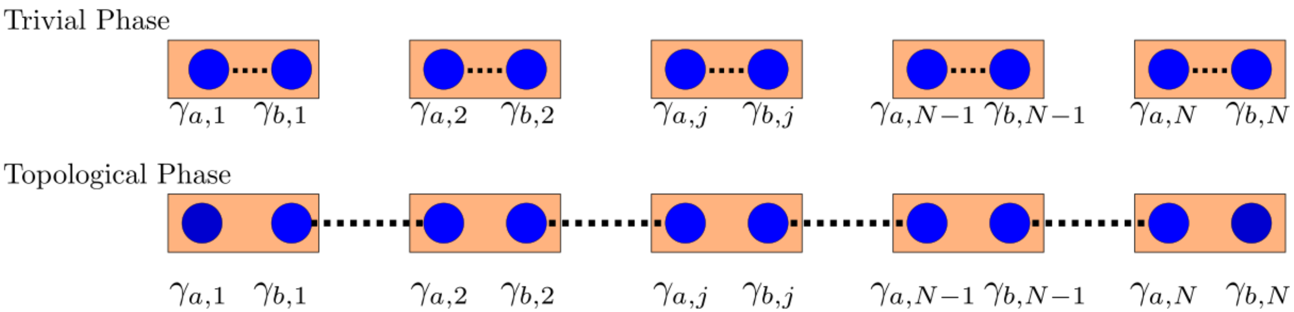

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Floquet Majorana Fermions : in periodically driven systems

- Floquet systems :

$$\mathbf{H}(t+\mathcal{T}) = \mathbf{H}(t) \qquad\qquad i\hbar\frac{d}{dt}|\psi(t)\rangle = (\mathbf{H}(t) - i\mathbf{\Sigma})|\psi(t)\rangle$$

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- Floquet theory confirms that for such a time-periodic Hamiltonian, there exists a complete set of solutions given as :

Floquet-states $\psi_\alpha(t) = e^{i\epsilon_\alpha t} u_\alpha(t)$ $\qquad \epsilon = i\hbar\frac{\log \mathbf{U}(\mathcal{T},0)}{\mathcal{T}}$

Quasi-energy $-\pi/T \leq \epsilon_\alpha \leq \pi/T$

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- Periodically driven superconductor : We can have two kinds of Majorana Fermions, one in the middle and one at the edge of the Floquet zone of quasi-energies.

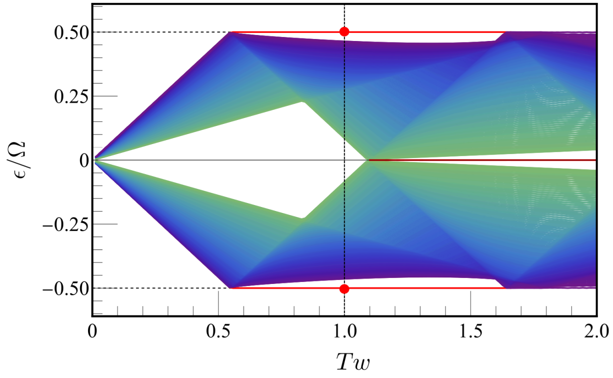


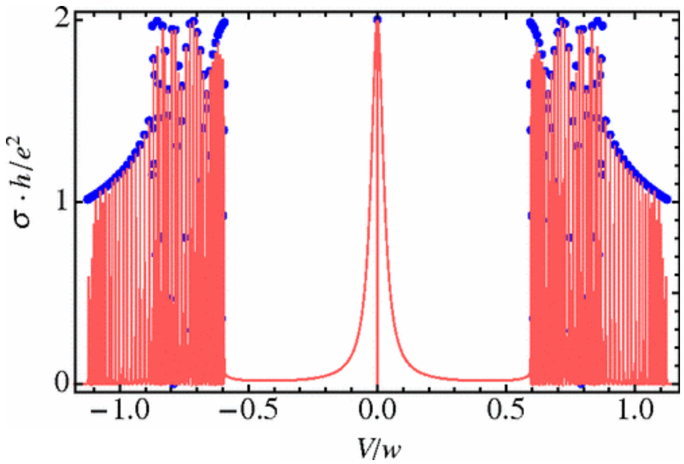
FIG: Quasi energy spectrum of a periodically driven Kiteav chain showing the emergence of Floquet Majorana modes

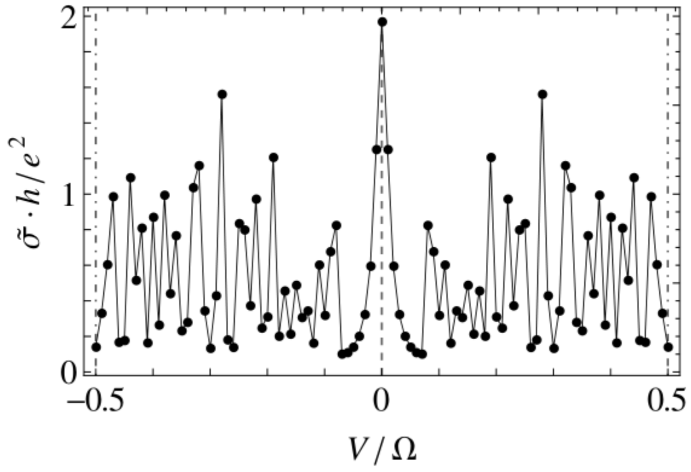
Phys. Rev. Lett. 113, 196601 (2024)

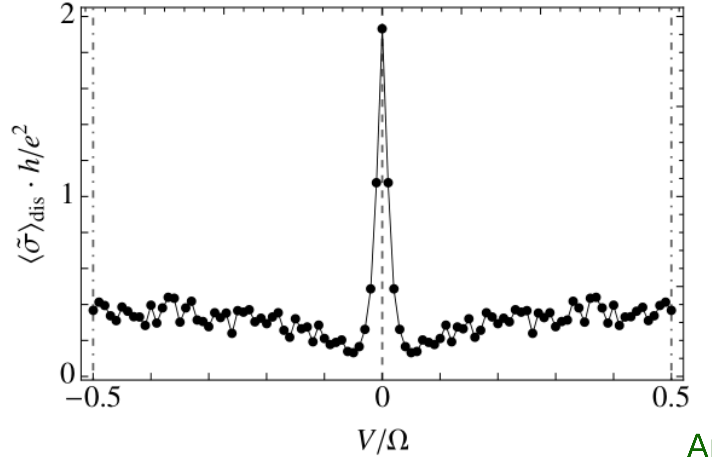
Transport signatures of Majorana Fermions

- Tunneling signatures of Majorana modes
- Summed conductance is quantized : Essentially takes into account emission and absorption of integer quantas of energy in multiplication of the frequency of the drive.

$$\lim_{\mu\to 0,\pm\Omega/2}\sum_n \ \sigma(\mu+n\Omega)=\tilde{\sigma}(0,\pm\Omega/2)=2e^2/h$$







Arijit Kundu, Babak Seradjeh (IUB), Phys. Rev. Lett. 111, 136402 (2013)

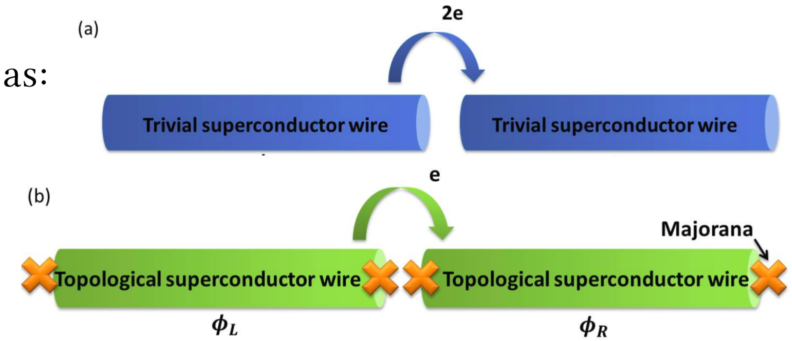
Transport signatures of Majorana Fermions

- Josephson effect : When two superconductors are placed in proximity and connected by a weak link, a current flows across the weak link without any voltage difference. This current is called the Josephson current, and the device is known as a Josephson junction.

- This effect is primarily described by the Josephson equations, given as:

$$I(t) = I_0 \sin(\phi) \qquad \frac{\partial \phi}{\partial t} = \frac{2eV}{\hbar}$$

- $\phi = \phi_R - \phi_L$, is the phase difference across the junction.



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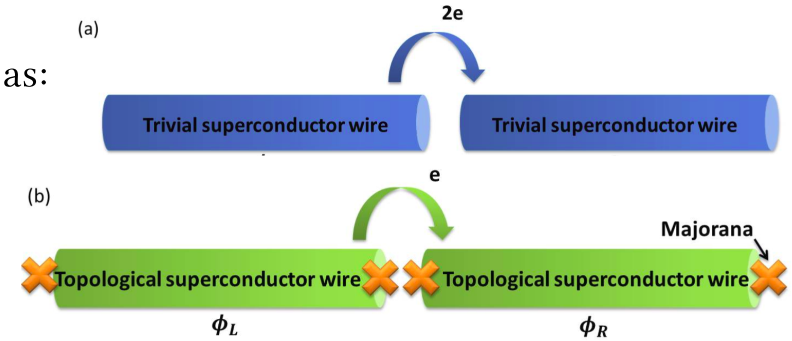

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- In case of topological superconductors current phase relations becomes 4pi periodic.

Effective low energy Hamiltonian if two Majorana's are at zero energy

$$H_{eff} = -t_{cpl}\left(\hat{n}_1 - \frac{1}{2}\right)\cos\frac{\delta\phi}{2}$$

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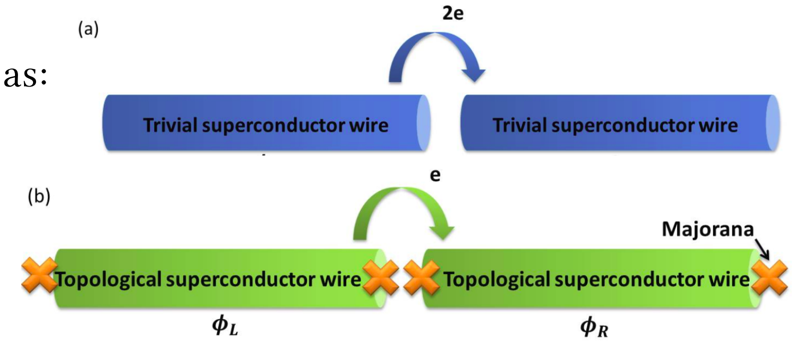

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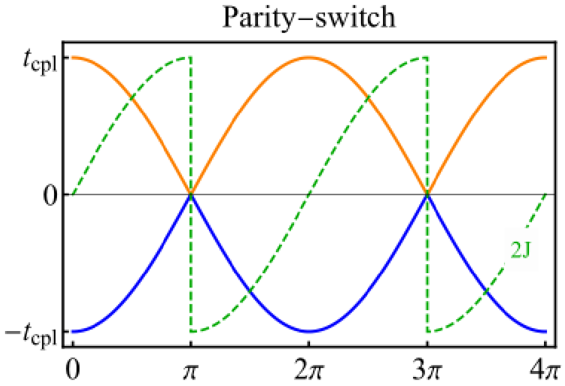

Role of system size

- Effective low energy Hamiltonian if two Majorana's are at ±δ (vanishes exponentially as their separation increases).

$$E_{eff} \approx \left(\left(t_{cpl} \cos \frac{\delta\phi}{2} \right)^2 + \delta^2 \right)^{1/2}$$

$$J = \frac{2e}{\hbar} \left(\frac{t_{cpl}^2}{2} \sin \delta\phi \right) \left(\left(t_{cpl} \cos \frac{\delta\phi}{2} \right)^2 + \delta^2 \right)^{-1/2}$$

- In steady state (JJ coupled to the reservoir):

$$\mathrm{J}_{\mathrm{stat}} \approx f(E_{\mathrm{eff}}, \mu) \frac{\partial E_{\mathrm{eff}}}{\partial \phi} = \Delta n \frac{\partial E_-}{\partial \phi}$$

$$\Delta n = f(E_+, \mu) - f(E_-, \mu) = -1$$

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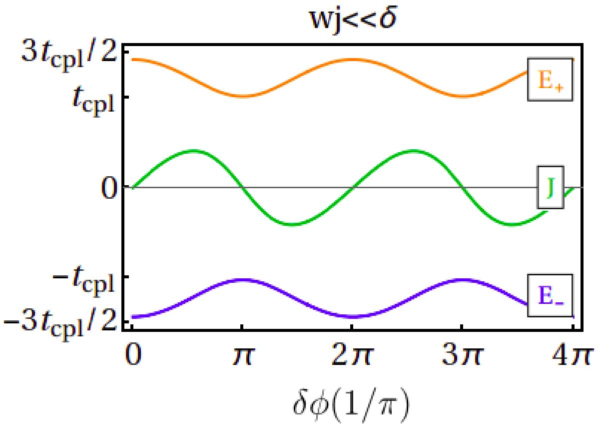

$E_{\text{eff.}} \propto \cos(\phi)$

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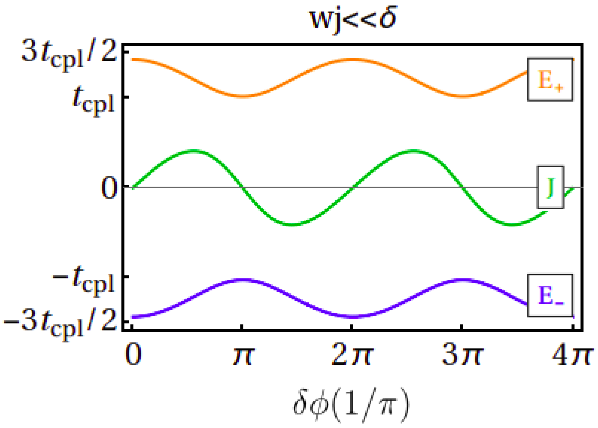

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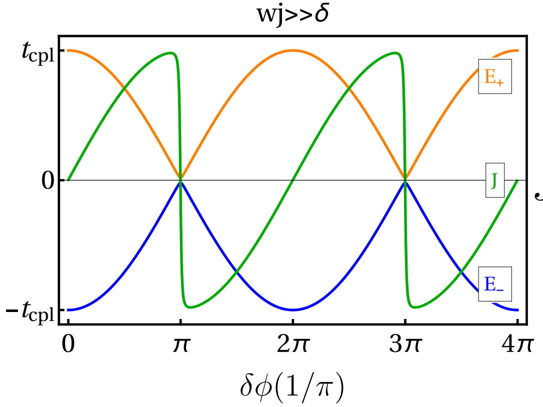

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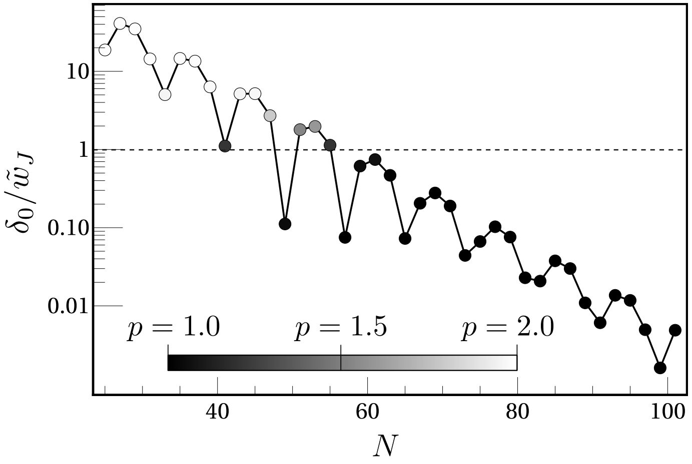
$$J \propto t_{\text{cpl}}^p$$



Also true for driven case.

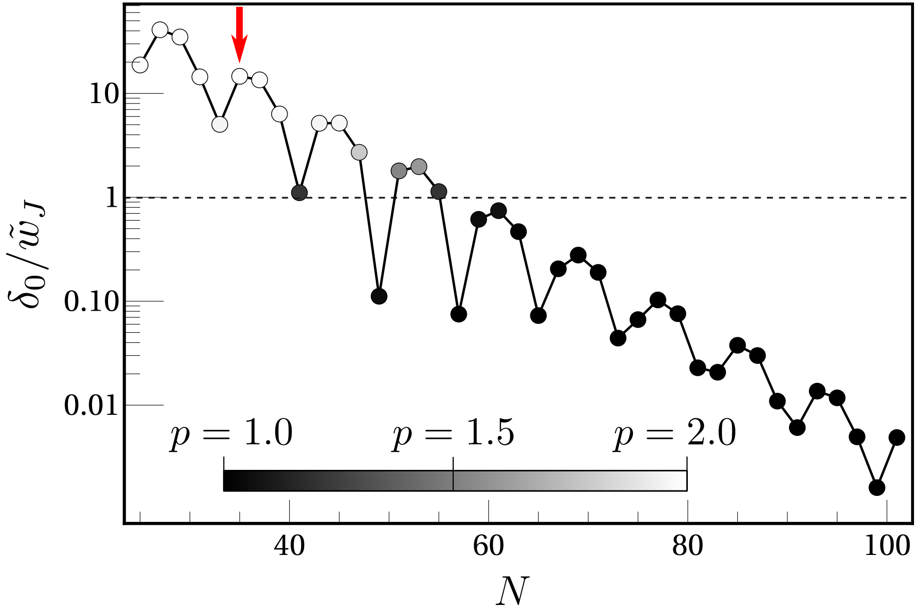
The localization of FMFs at the edge depends on the system size.

The quasienergy gaps are exponential in system-size.

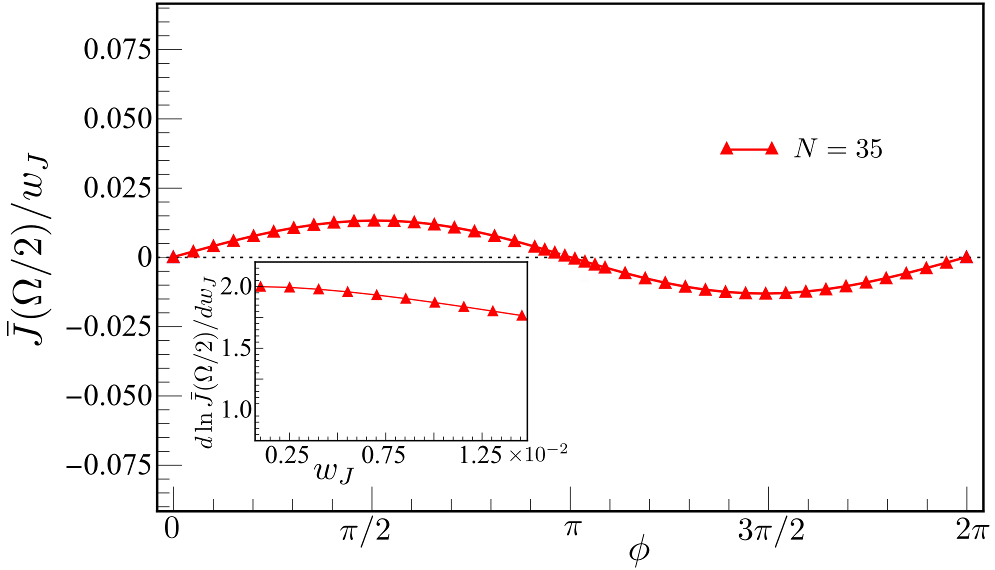
Josephson current can be computed using the Floquet non-equilibrium Green's function method.

$$H(t) = H_{SC}(t)^L + H_T + H_{SC}(t)^R + \sum_\lambda H_\lambda + H_{SC-\lambda}$$

Josephson current signatures of FMF's



Floquet NEGF



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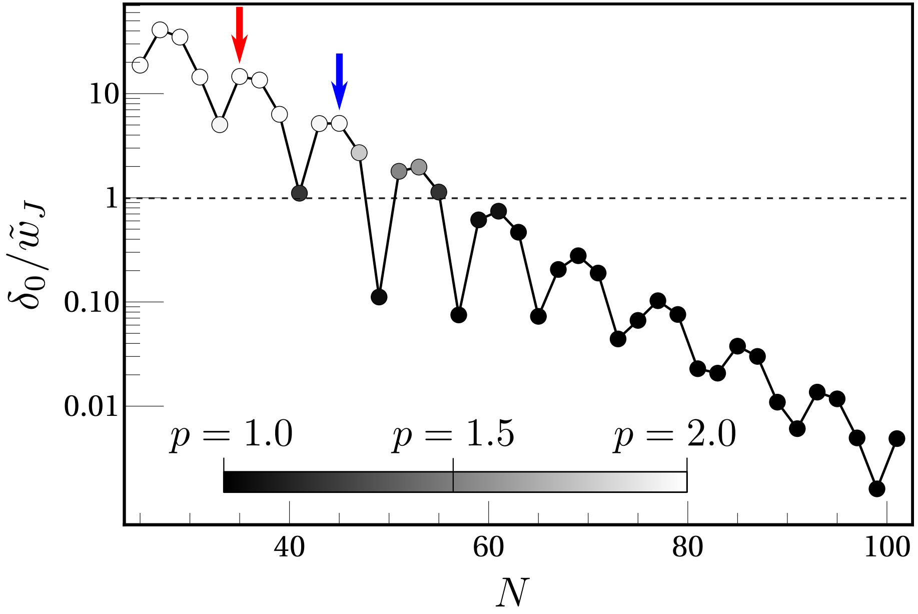

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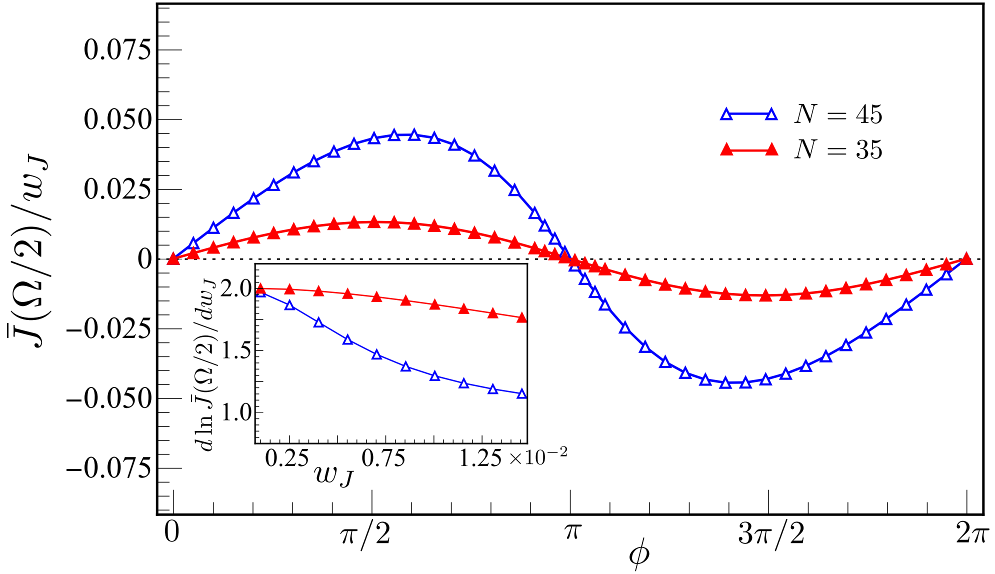

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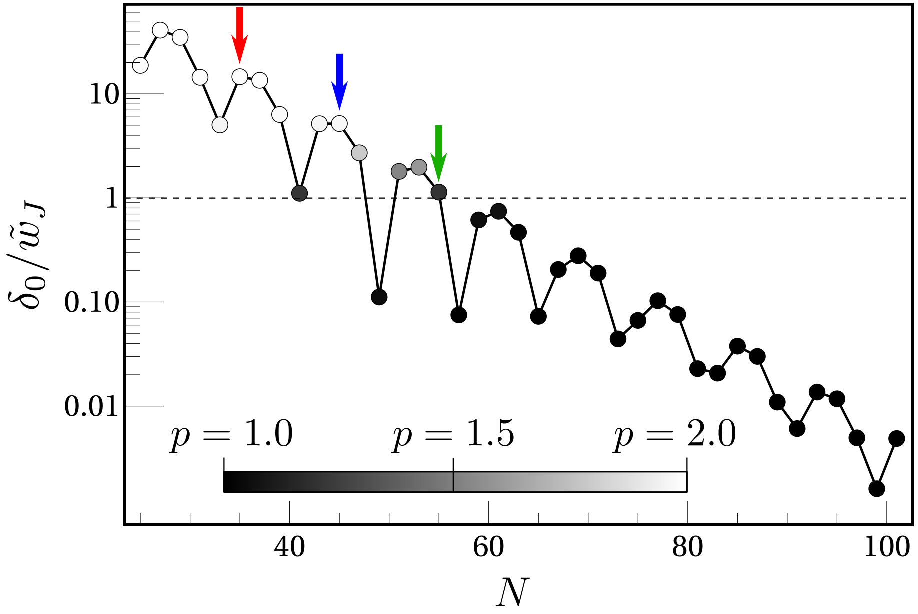
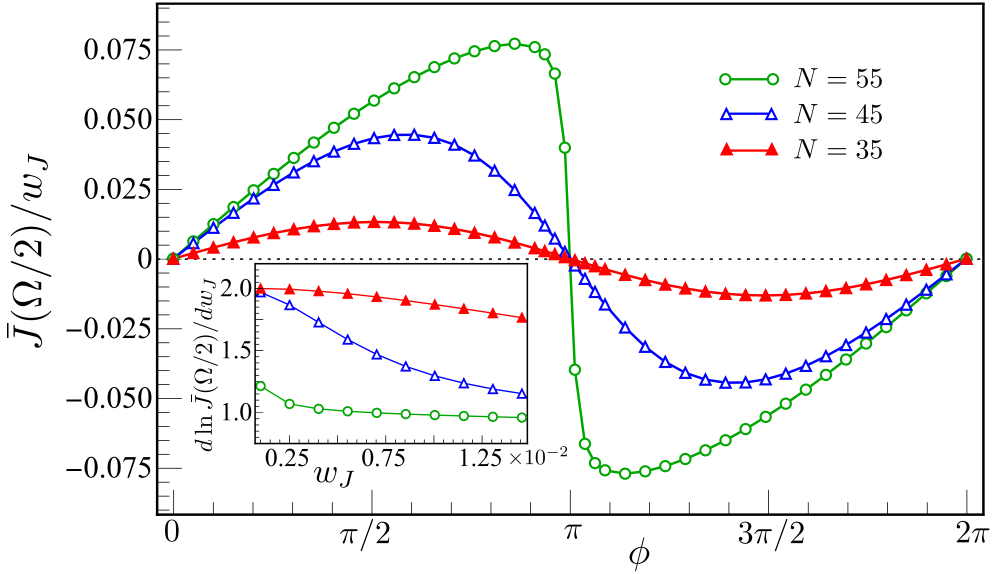


Floquet NEGF



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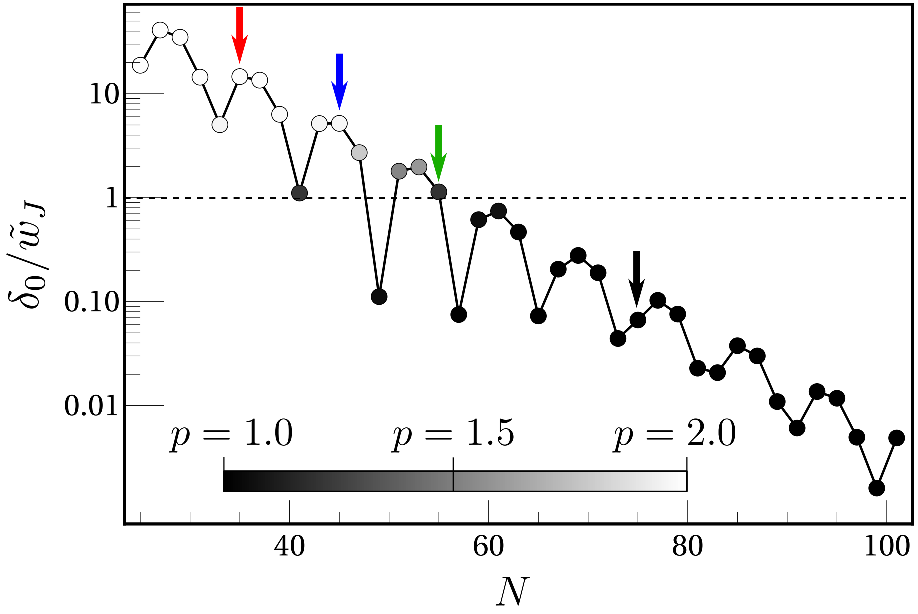

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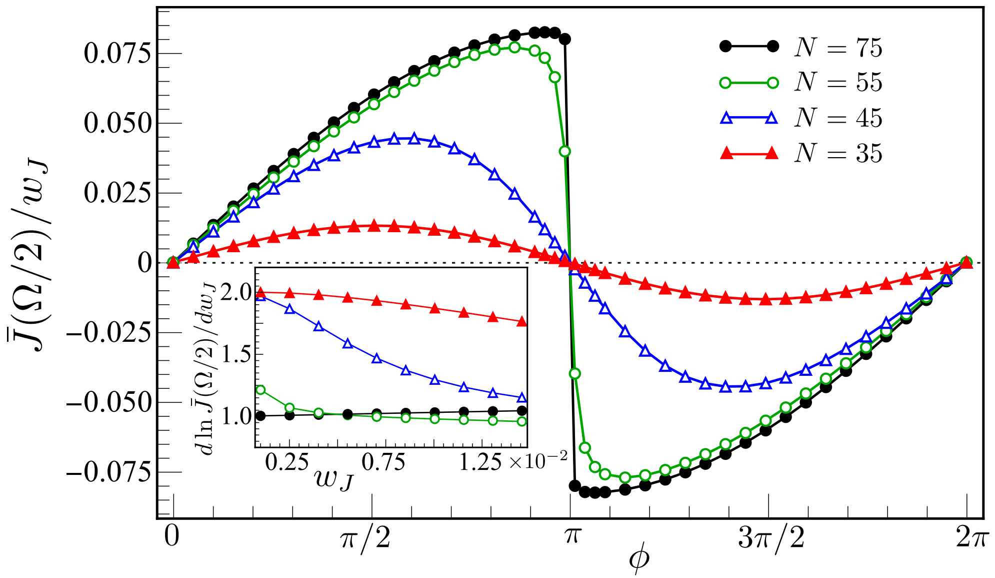

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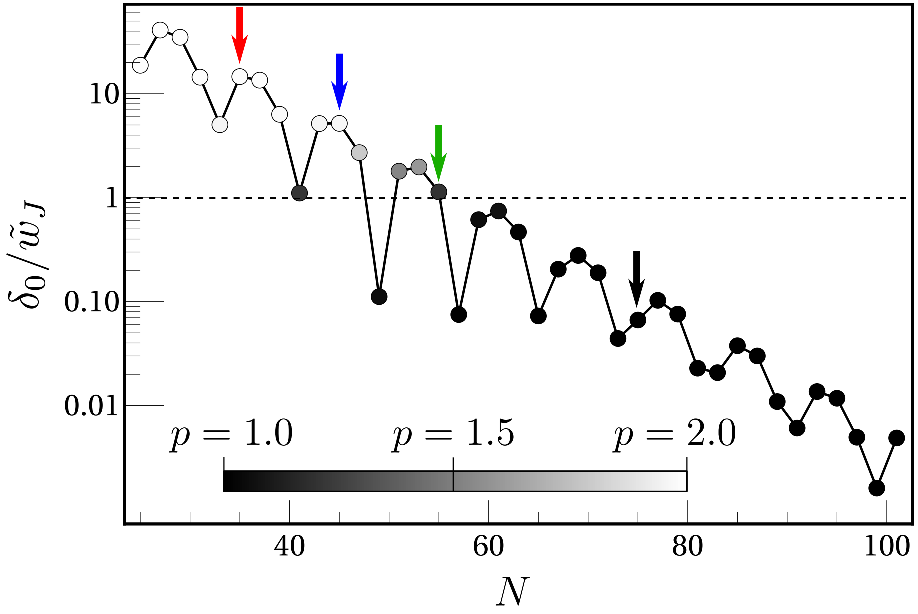

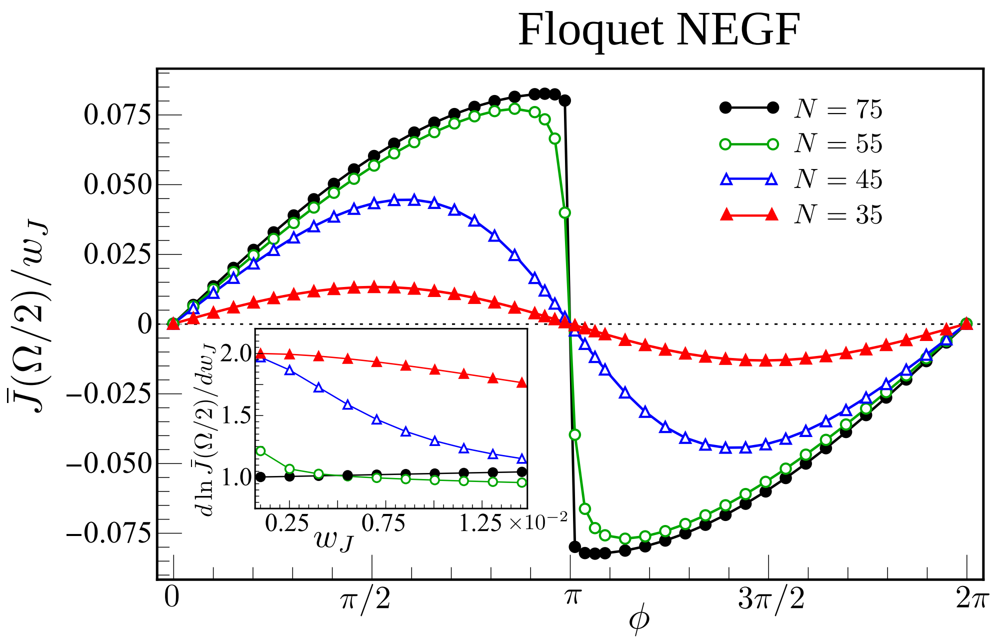

- **Steep jump** of the current, a reminiscent of static MF mediated Josephson junction.
- The jump is steeper and the current becomes **linear in tunneling** with increasing system size

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Occupation of Floquet states

- In static case Josephson current can also be written as:

$$\pm E_{\text{eff}} \propto \pm \left(w_J^2 \cos^2(\phi/2) + \delta^2 \right)^{1/2}$$

$$J_{\text{stat}} \approx f(E_{\text{eff}}) \frac{\partial E_{\text{eff}}}{\partial \phi} \propto w_J^2 \left(w_J^2 \cos^2(\phi/2) + \delta^2 \right)^{-1/2} \sin \phi$$

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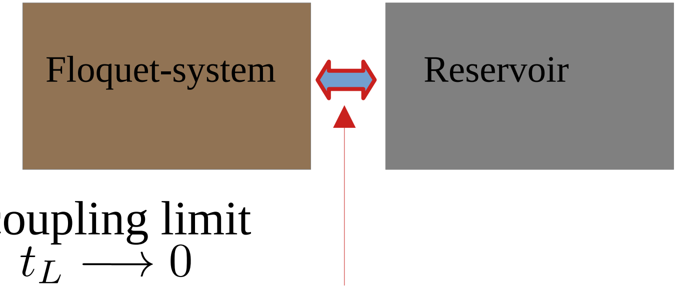
- Density matrix in the basis of Floquet states :

$$\hat{\rho}(t) = \sum n_{\alpha\beta}(t) |u_\alpha(t)\rangle\langle u_\beta(t)|$$

$$n_{\alpha\beta}(t) = \left\langle \Psi_\alpha^\dagger(t)\Psi_\beta(t) \right\rangle_{\text{Lead avg.}}$$



Weak-coupling limit
$t_L \longrightarrow 0$

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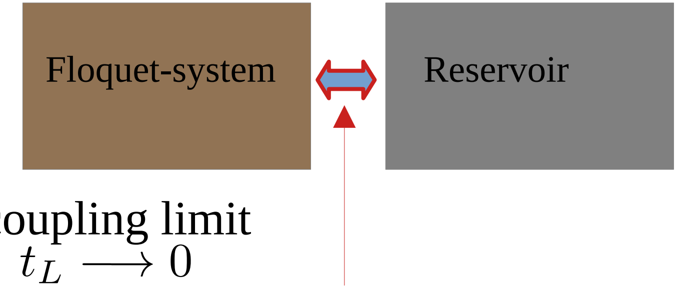

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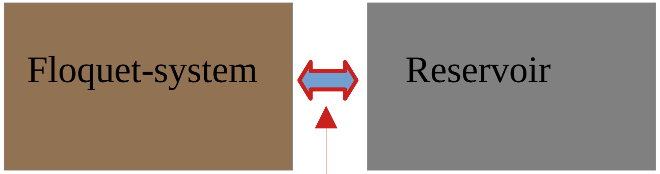

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$$n_\alpha = \sum_{k \in \mathbb{Z}} f_r(\epsilon_\alpha + k\Omega - \mu_r) \langle u_\alpha^{(k)} | u_\alpha^{(k)} \rangle.$$

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Occupation of Floquet states

- Time averged Josephson current:

$$\bar{J}(\mu_r) = \langle\langle \hat{J}_{\mathcal{S}} \rangle\rangle = \frac{1}{T}\int_0^T dt \langle \hat{J}_{\mathcal{S}} \rangle(\mu_r, t) = \sum_\alpha n_\alpha(\mu_r)\partial_\phi \epsilon_\alpha,$$

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  - The summed occupation difference :

$$\nu_0^F = 1$$

$$\nu_\pi^F = -1$$

  - The summed Josephson current

$$\bar{J}_F(\mu_b) \equiv \sum_k \bar{J}(\mu_b + k\Omega) \approx \nu_b^{\mathrm{F}} \frac{\partial \delta_J}{\partial \phi}$$

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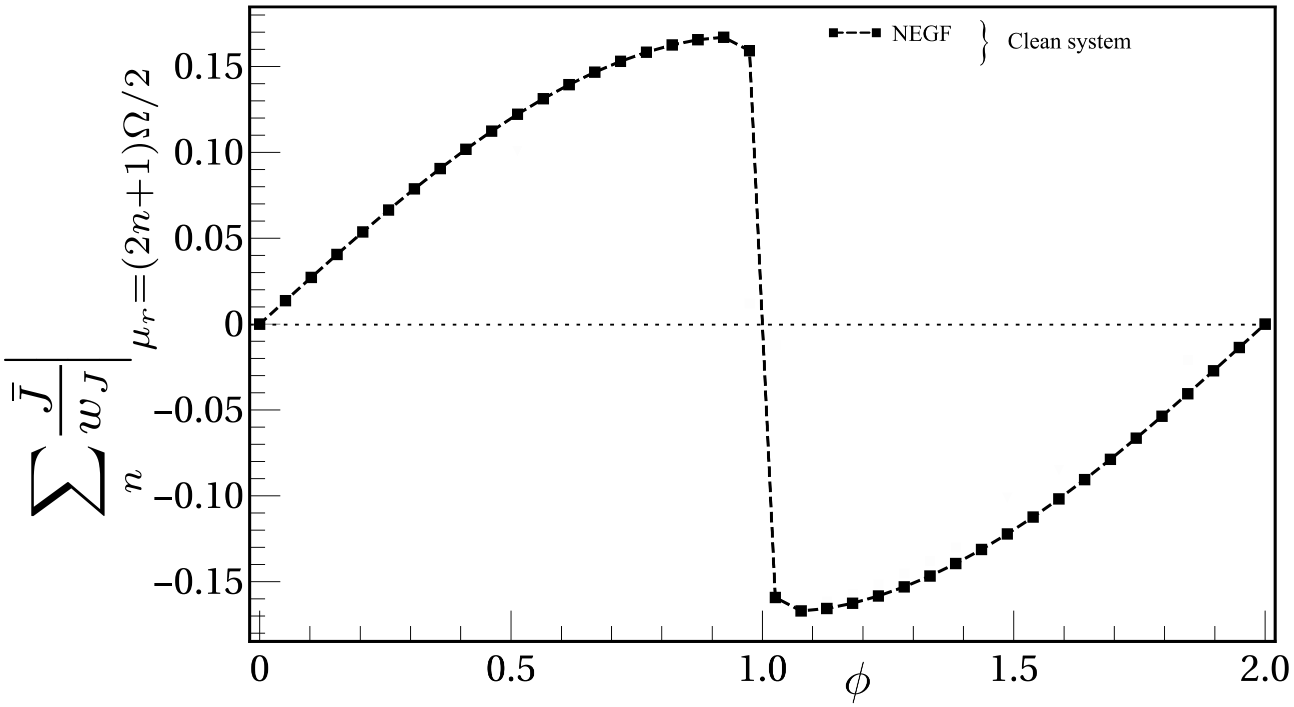
- Similarity to the static-system's current : With relevant energy-scales are dictated by details of the driving (and assisted tunneling)`

$$\bar{J}_F \approx \frac{\partial \epsilon_F}{\partial \phi} \propto \tilde{w}_J^2 \left(\tilde{w}_J^2 \cos^2(\phi/2) + \delta^2\right)^{-1/2} \sin\phi$$

Sum-rule : robust against small (static) disorder



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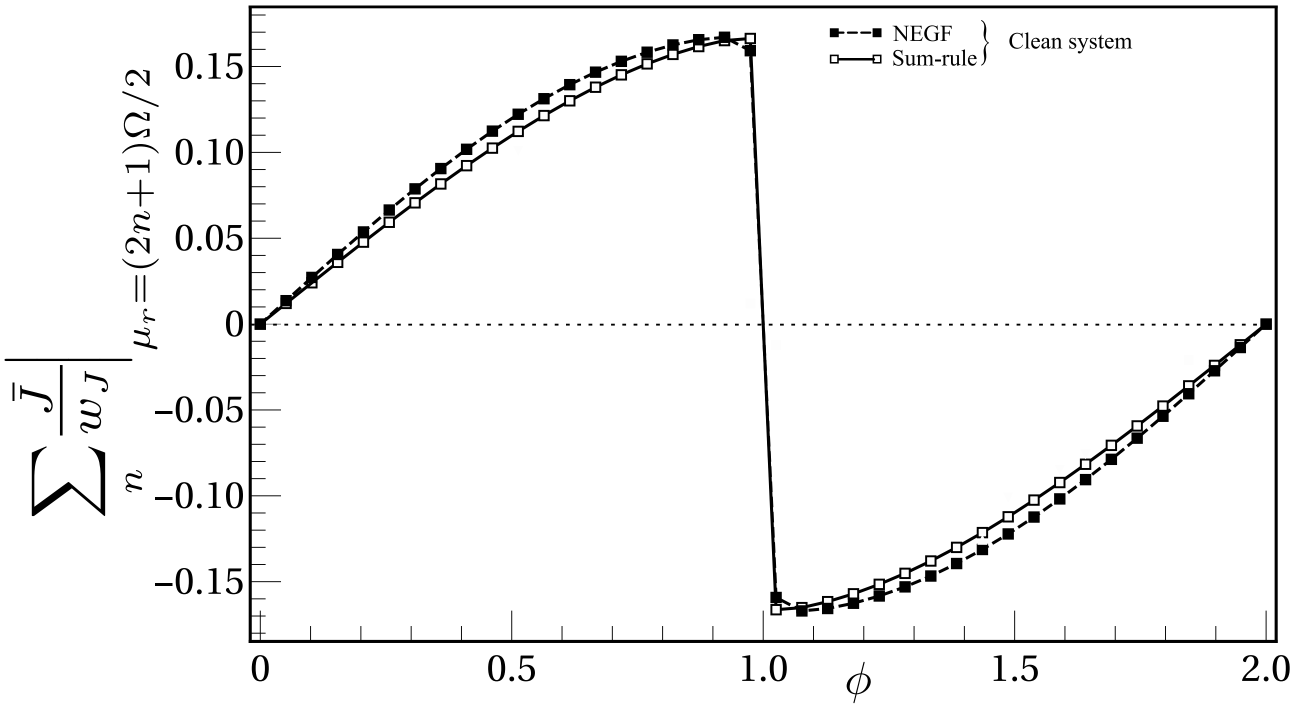

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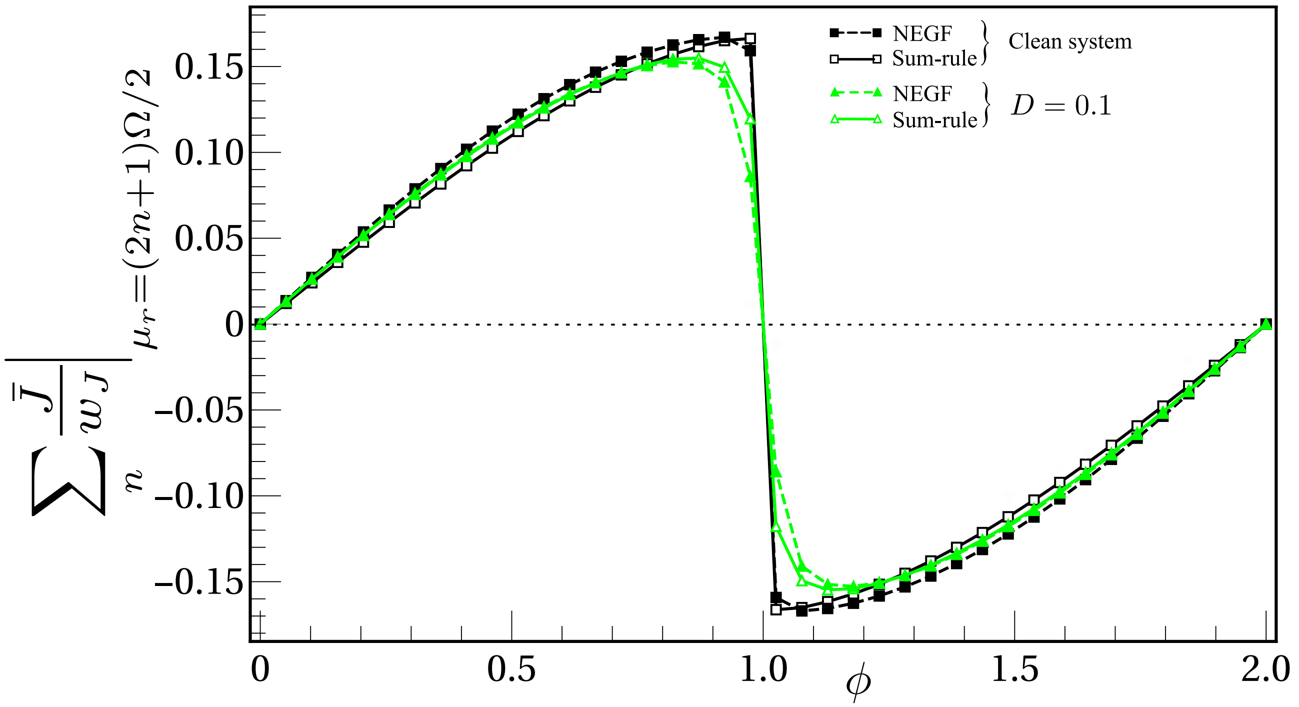

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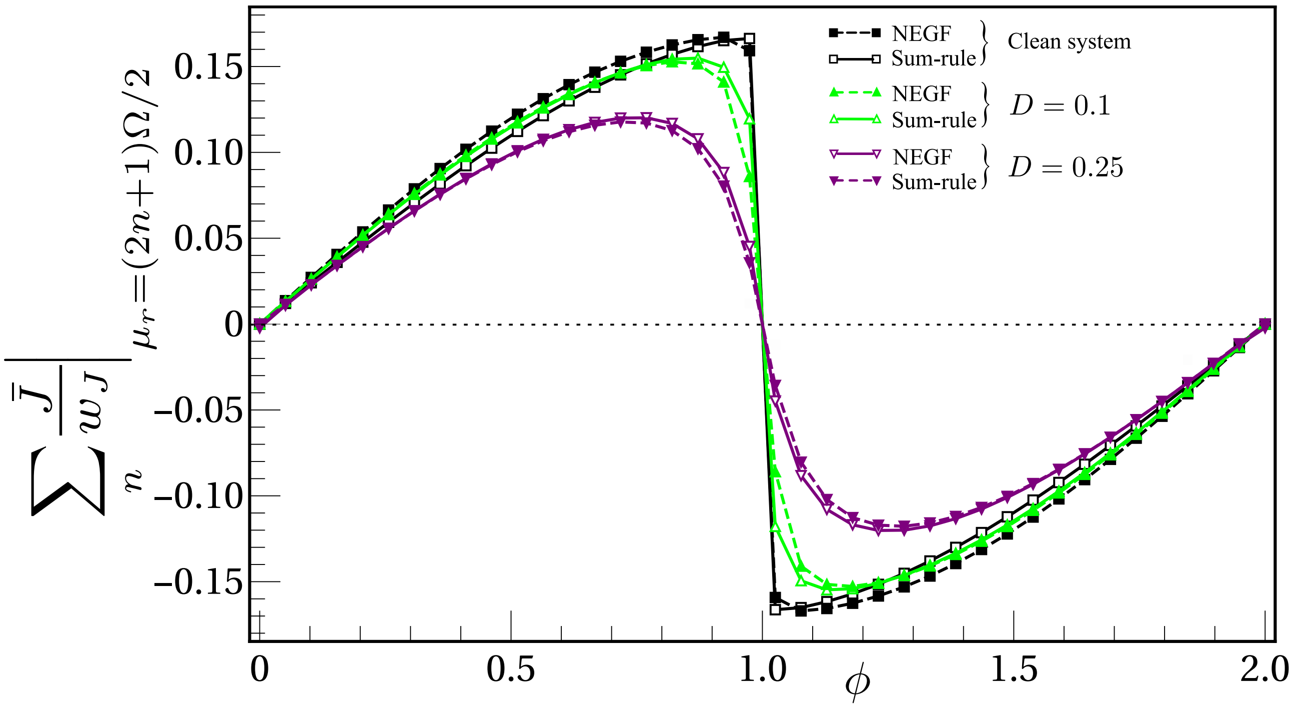

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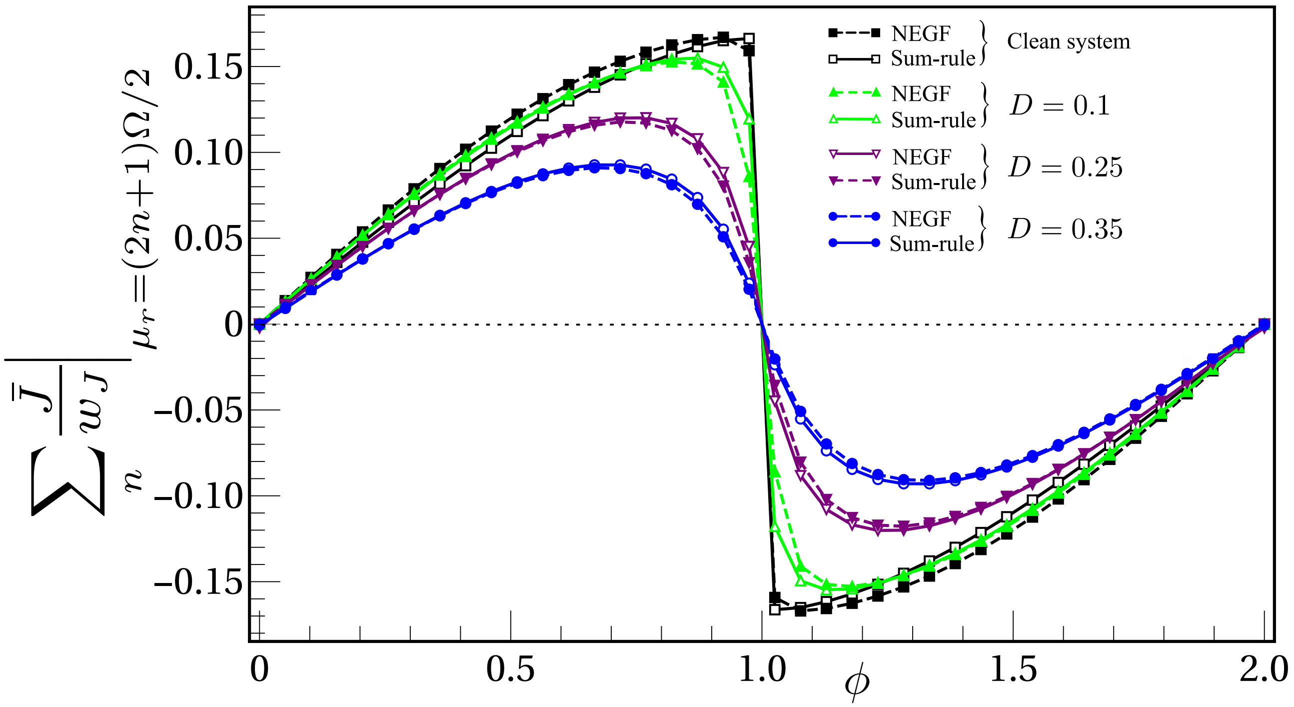

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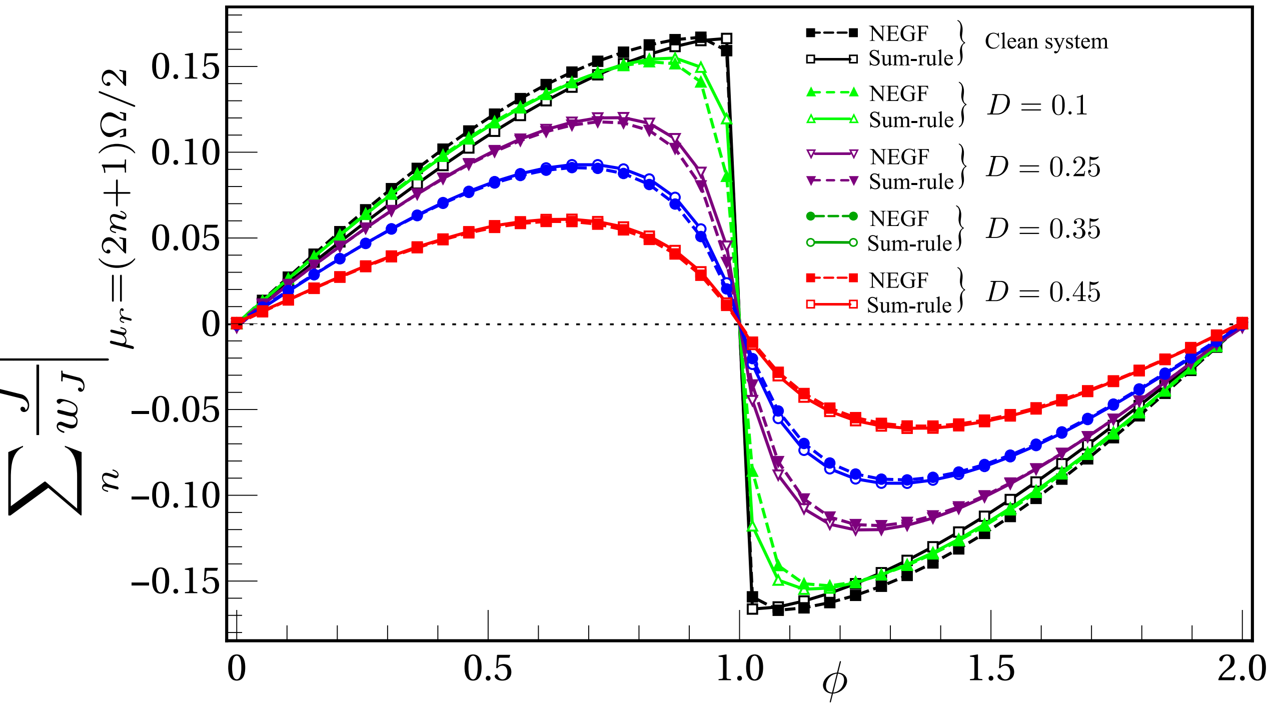

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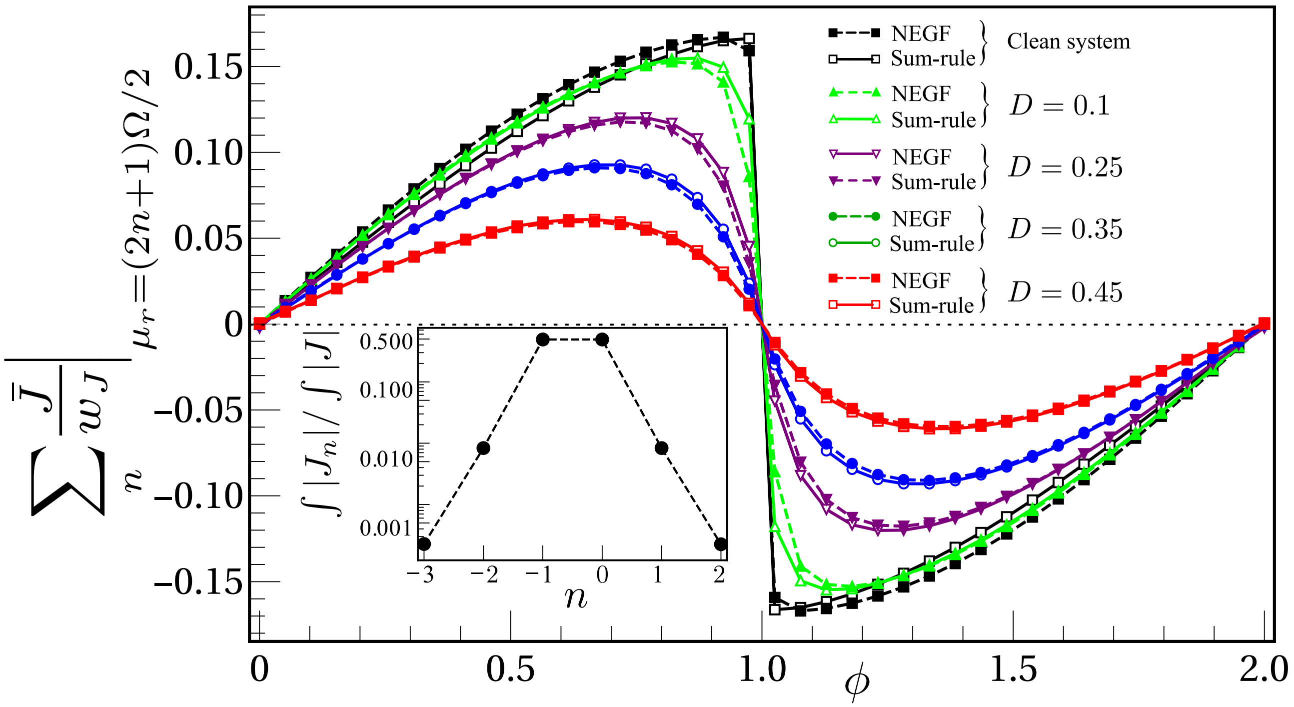

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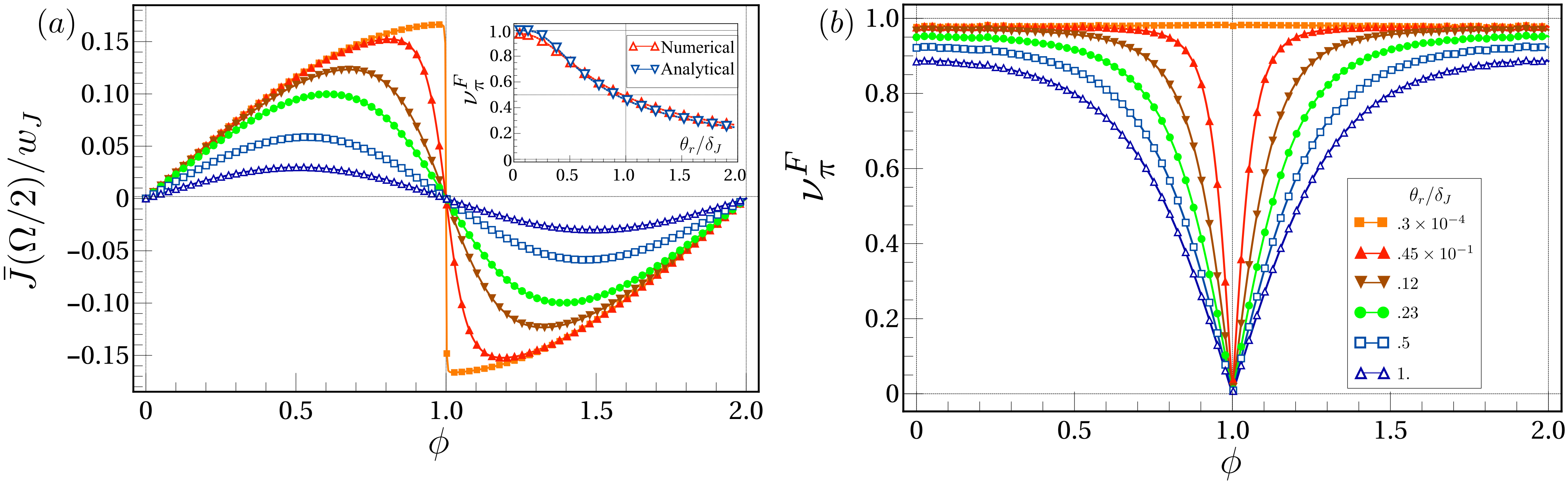
Sum-rule in current: Temperature dependence



$$\nu_\pi^F = \frac{2\delta_J}{\Omega} - \tanh\left(\frac{\delta_J}{2\theta_r}\right)$$

For more information - Phys. Rev. Lett. 113, 196601 (2024)

Conclusion

- Similar to the static case, Josephson current phase relation show 4pi-periodicity for the case of FMF's.

- We have presented the sum rules for the current and FMF's occupations.

- We have also given the simplified current expressions in terms of quasi-energy and occupations of Floquet states.

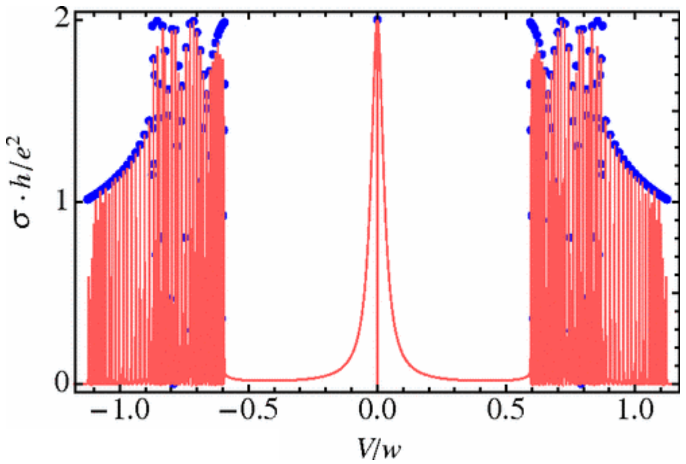
- Results are robust for weak disorder and small temperature case.

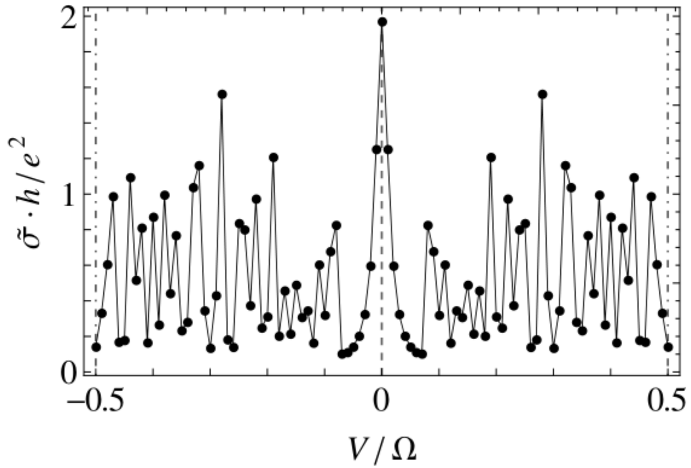
Thank-you

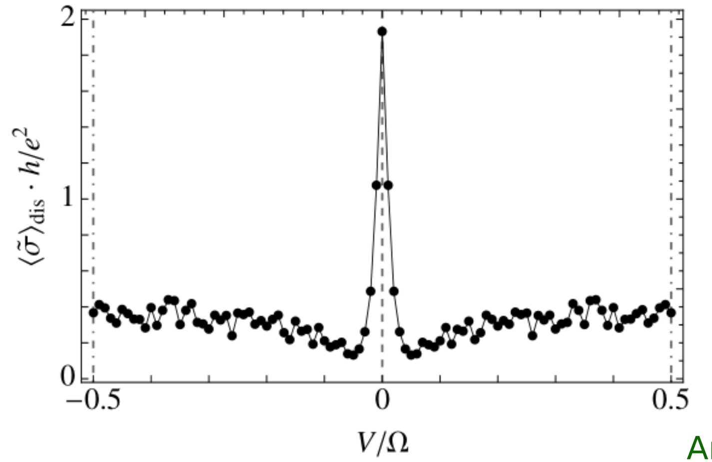
Transport signatures of Majorana Fermions

- Tunneling signatures of Majorana modes
- Summed conductance is quantized : Essentially takes into account emission and absorption of integer quantas of energy in multiplication of the frequency of the drive.

$$\lim_{\mu\to 0,\pm\Omega/2} \sum_n \ \sigma(\mu + n\Omega) = \tilde{\sigma}(0, \pm\Omega/2) = 2e^2/h$$







Arijit Kundu, Babak Seradjeh (IUB), Phys. Rev. Lett. 111, 136402 (2013)

I. FLOQUET THEORY

Let's consider a quantum system with its Hamiltonian being a periodic function in time, such that $\mathbf{H}(t+\mathcal{T}) = \mathbf{H}(t)$, where $\mathcal{T}$ is the period of the perturbation. The symmetry of the Hamiltonian under discrete time translations, $t \to t + \mathcal{T}$, enables the use of the Floquet formalism. The Schrödinger equation of this periodically driven system is

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = (\mathbf{H}(t) - i\mathbf{\Sigma})|\psi(t)\rangle \ , \tag{1}$$

where $\mathbf{\Sigma}$ results from environmental degrees of freedom of the reservoir (making the time evolution non-unitary). Here, $H(t)$ contains two terms: $H(t) = H_0 + V(t)$, where H_0 is the unperturbed Hamiltonian and $V(t)$ is the time-periodic perturbation.

1. **Floquet theory** confirms that for such a time-periodic Hamiltonian there exists a complete set of solutions $\{|\psi_\alpha(t)\rangle\}$ of equation (1) of the form

$$|\psi_\alpha(t)\rangle = e^{-(i\epsilon_\alpha/\hbar+\gamma_\alpha)t}|u_\alpha(t)\rangle \ , \qquad |u_\alpha(t)\rangle = |u_\alpha(t+\mathcal{T})\rangle \ . \tag{2}$$

$|u_\alpha(t)\rangle$ are called *Floquet modes* obeying equation (2), and ϵ_α are quasi-energies (with width γ_α) (reference : section A1 of [1]).

7. By inserting the ansatz Eq. (2) into Eq. (1), one easily verifies that the Floquet states fulfill the eigenvalue equation:

$$\left(\mathbf{H}(t) - i\mathbf{\Sigma} - i\hbar\frac{d}{dt}\right)|u_\alpha(t)\rangle = (\epsilon_\alpha - i\hbar\gamma_\alpha)|u_\alpha(t)\rangle \tag{11}$$

The above equation can be written as:

$$(\mathbf{H}_{\text{eff}} - i\mathbf{\Sigma})\,|u_\alpha(t)\rangle = (\epsilon_\alpha - i\hbar\gamma_\alpha)|u_\alpha(t)\rangle \tag{12}$$

Here we have defined $\mathbf{H}_{\text{eff}} = \left(\mathbf{H}(t) - i\hbar\frac{d}{dt}\right)$.

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8. As, in the presence of $\mathbf{\Sigma}$, the time evolution is not unitary, its eigensystem may not be chosen as orthogonal. Taking the conjugate of Eq. (11),

$$(\mathbf{H}_{\text{eff}} + i\mathbf{\Sigma})\,|u_\alpha^+(t)\rangle = (\epsilon_\alpha + i\hbar\gamma_\alpha)|u_\alpha^+(t)\rangle \ , \tag{13}$$

and assuming the completeness of the eigenstates of $\mathbf{U}(\mathcal{T},0)$, the Floquet states form a bi-orthonormal system with these adjoint modes

$$\langle u_\alpha^+(t)|u_\beta(t)\rangle = \delta_{\alpha\beta} \qquad \text{and} \qquad \sum_\alpha |u_\alpha^+(t)\rangle\langle u_\alpha(t)| = \mathbf{I} \ . \tag{14}$$

9. As the Floquet states are time periodic, it is convenient to introduce the composite Hilbert space made up of the Hilbert space of square integrable functions on configuration space and the time space of time periodic functions of period $2\pi/\mathcal{T}$. By taking the Fourier transform of the Floquet modes, (reference: section A2 of [1])

$$|u_\alpha(t)\rangle = \sum_{n=-\infty}^{\infty} e^{-in\Omega t}|u_\alpha^{(n)}\rangle \ . \tag{15}$$

And similarly, for the Hamiltonian, if $\mathbf{H}_{\text{tot}} = (\mathbf{H} - i\mathbf{\Sigma})$ is time periodic, then

$$\mathbf{H}_{\text{tot}}(t) = \sum_{n=-\infty}^{\infty} e^{-in\Omega t}\mathbf{H}_{\text{tot}}^{(n)} \ . \tag{16}$$

In this discrete momentum space, the Hamiltonian equation becomes by substituting Eq. (15) and (16) in Eq. (11) gives:

$$\sum_{l=-\infty}^{\infty} \mathbf{H}_{\text{tot}}^{(k-l)}|u_\alpha^{(l)}\rangle = (\epsilon_\alpha - i\hbar\gamma_\alpha + k\Omega)|u_\alpha^{(k)}\rangle \ . \tag{17}$$

$$\sum_{l=-\infty}^{\infty} \left(\mathbf{H}_{\text{tot}}^{(k-l)} - \delta_{k,l}l\Omega\right)|u_\alpha^{(l)}\rangle = (\epsilon_\alpha - i\hbar\gamma_\alpha)|u_\alpha^{(k)}\rangle \ . \tag{18}$$

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Floquet-extended zone picture

The above equation can be written in matrix format as:

$$
\begin{array}{c}
\vdots \\ . \\ k=-2 \\ k=-1 \\ k=0 \\ k=1 \\ k=2 \\ \\
\end{array}
\begin{pmatrix}
 & l=-2 & l=-1 & l=0 & l=1 & l=2 \\
\ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \mathbf{H}^0+2\Omega & \mathbf{H}^{-1} & \mathbf{H}^{-2} & . & \vdots \\
\vdots & \mathbf{H}^1 & \mathbf{H}^0+\Omega & \mathbf{H}^{-1} & \mathbf{H}^{-2} & \vdots \\
\vdots & \mathbf{H}^2 & \mathbf{H}^1 & \mathbf{H}^0 & \mathbf{H}^{-1} & \mathbf{H}^{-2} \\
\vdots & \vdots & \mathbf{H}^2 & \mathbf{H}^1 & \mathbf{H}^0-\Omega & \mathbf{H}^{-1} \\
\vdots & \vdots & . & \mathbf{H}^2 & \mathbf{H}^1 & \mathbf{H}^0-2\Omega \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\begin{pmatrix}
\vdots \\ |u_\alpha^{(-2)}\rangle \\ |u_\alpha^{(-1)}\rangle \\ |u_\alpha^{(0)}\rangle \\ |u_\alpha^{(1)}\rangle \\ |u_\alpha^{(2)}\rangle \\ \vdots
\end{pmatrix}
= (\epsilon_\alpha - i\gamma_\alpha)
\begin{pmatrix}
\vdots \\ |u_\alpha^{(-2)}\rangle \\ |u_\alpha^{(-1)}\rangle \\ |u_\alpha^{(0)}\rangle \\ |u_\alpha^{(1)}\rangle \\ |u_\alpha^{(2)}\rangle \\ \vdots
\end{pmatrix}
\qquad (19)
$$

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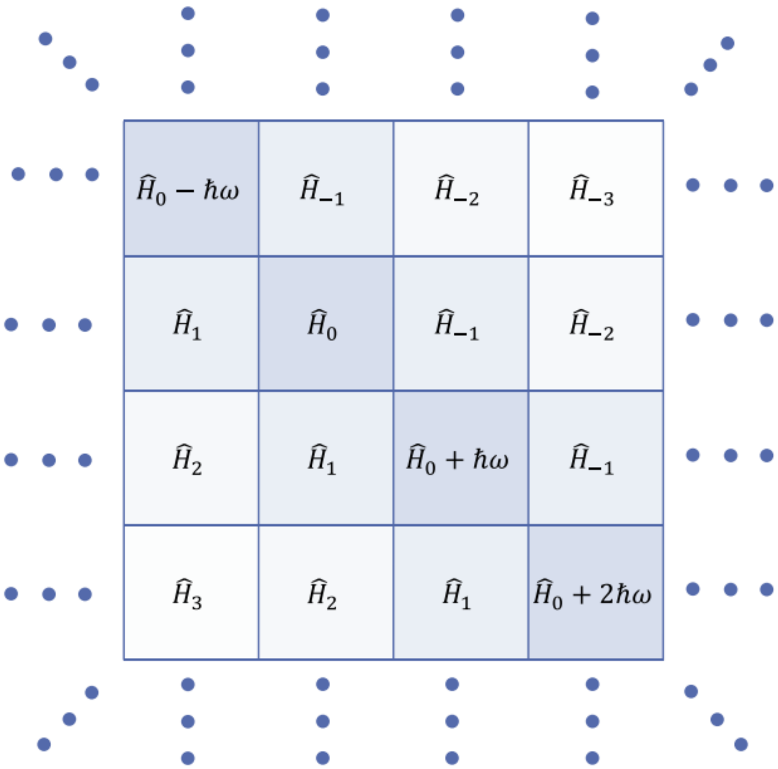

Figure 1. Block structure of the quasienergy operator $\bar{Q}$ with respect to the 'photon' index m. Each block corresponds to an operator $\hat{Q}_{m'm} = \hat{H}_{m'-m} + \delta_{m'm} m\hbar\omega$ acting in the full state space $\mathcal{H}$. The diagonal blocks $\hat{H}_0 + m\hbar\omega$ can be interpreted to act in the subspace of relative 'photon' number m and the off-diagonal blocks $\hat{H}_{m'-m}$, which obey $\hat{H}_{m'-m} = \hat{H}^{\dagger}_{m-m'}$, describe $(m' - m)$-'photon' processes.

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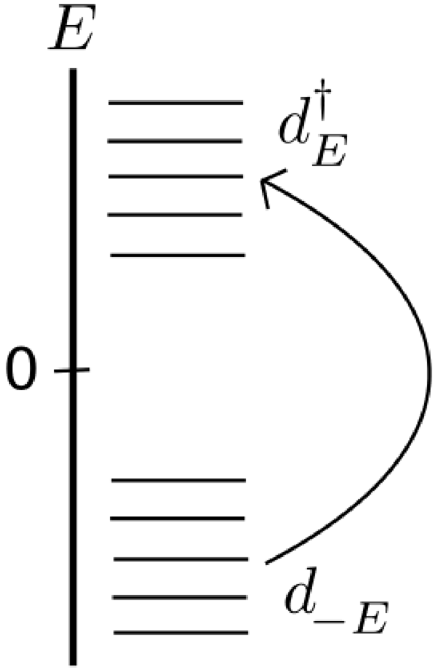
- More understandings - Occupations and Sum-rules for FMF.

Majorana Fermions (MFs)



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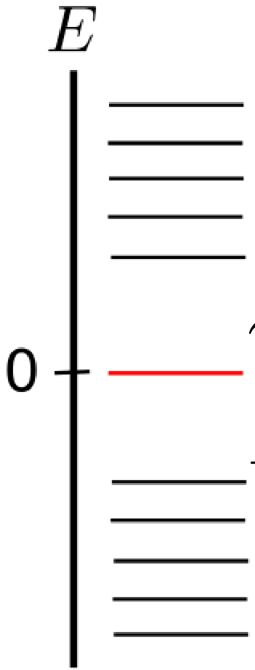

$$\gamma^{\dagger}(E) = \gamma(-E)$$

$$E = 0$$

- MFs are particle-hole symmetry-related states of positive and negative energy excitations (in superconductors).
- A Majorana state is an equal mixture of particle and hole-like excitations and, thus, a topologically protected zero-energy mode of superconductor.
- 1D topological superconductors host such Majorana-like states as edge modes.

Floquet Majoranas (FMFs)

- Periodically driven system :

$$H(t+T) = H(t)$$

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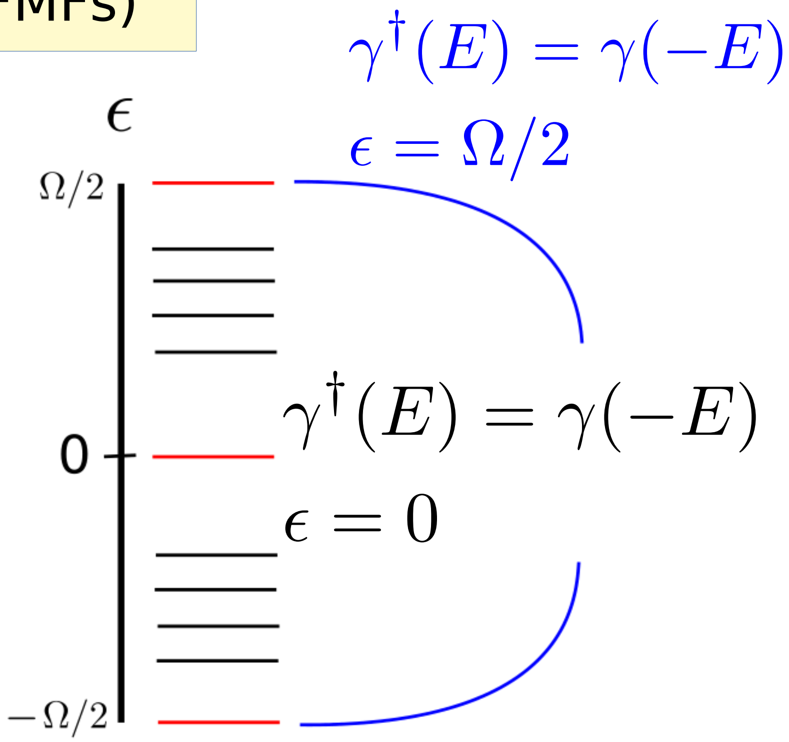

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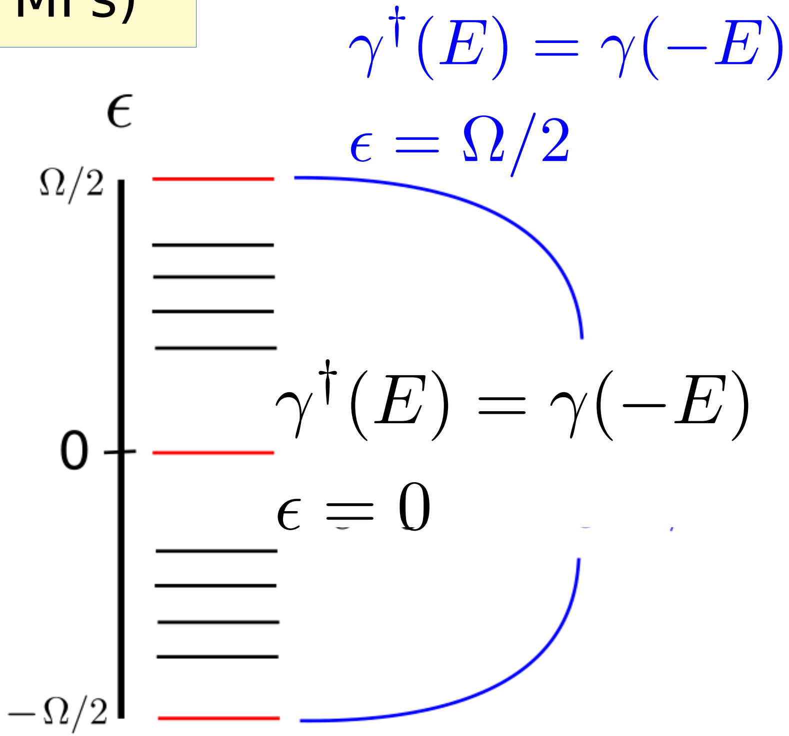

- We can have two kinds of Majorana, like steady-states, one in the middle and one at the edge of the Floquet zone of quasi-energies.
    - Zero and pi-FMF.

MF in Kiteav chain

- Kiteav chain :

$$H_p = -\frac{t}{2}\sum_{j=0}^{N-1}\left(c_{j+1}^{\dagger}c_j + c_j^{\dagger}c_{j+1}\right) - \frac{\Delta}{2}\sum_{j=0}^{N-1}\left(c_j c_{j+1} + c_{j+1}^{\dagger}c_j^{\dagger}\right) - \mu\sum_{j=1}^{N} c_j^{\dagger}c_j$$

MF in Kiteav chain

- Kiteav chain :

$$H_p = -\frac{t}{2}\sum_{j=0}^{N-1}\left(c_{j+1}^{\dagger}c_j + c_j^{\dagger}c_{j+1}\right) - \frac{\Delta}{2}\sum_{j=0}^{N-1}\left(c_j c_{j+1} + c_{j+1}^{\dagger}c_j^{\dagger}\right) - \mu\sum_{j=1}^{N}c_j^{\dagger}c_j$$

- Kiteav chain model in Majorana basis:

$$H_p = -\frac{i}{2}\Delta\sum_{j=0}^{N-1}\gamma_{b,j}\gamma_{a,j+1} + \frac{i}{2}\mu\sum_{j=1}^{N}\gamma_{b,j}\gamma_{a,j}$$

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- Majorana basis :

$$c_j = \frac{1}{2}(\gamma_{b,j} + i\gamma_{a,j}) \qquad \gamma_{a,j} = \gamma_{a,j}^\dagger$$

$$c_j^\dagger = \frac{1}{2}(\gamma_{b,j} - i\gamma_{a,j}) \qquad \{\gamma_{\alpha,i}, \gamma_{\beta,j}\} = 2\delta_{\alpha\beta}\delta_{ij}$$

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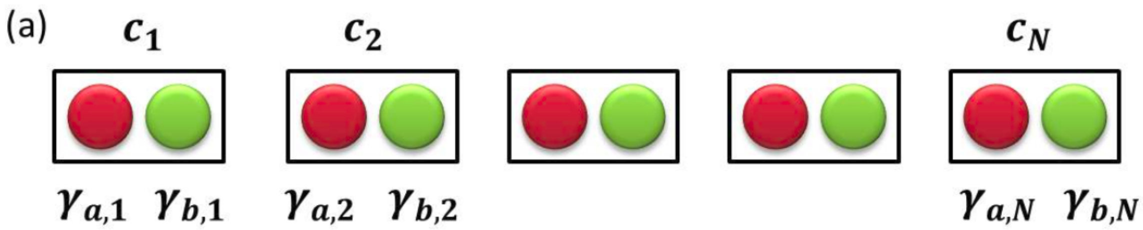

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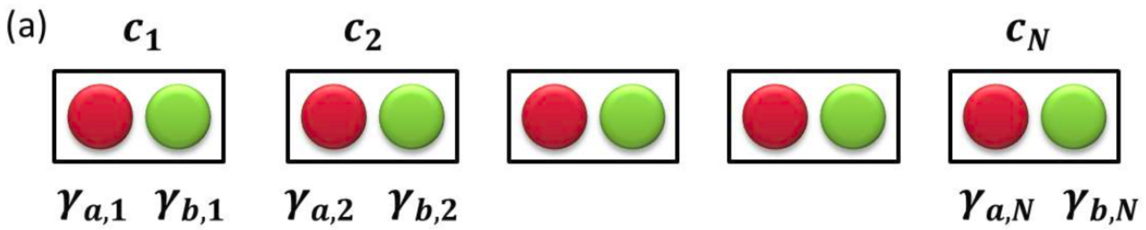

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https://www.semanticscholar.org/paper/Signatures-of-Topological-Superconductors-Lee

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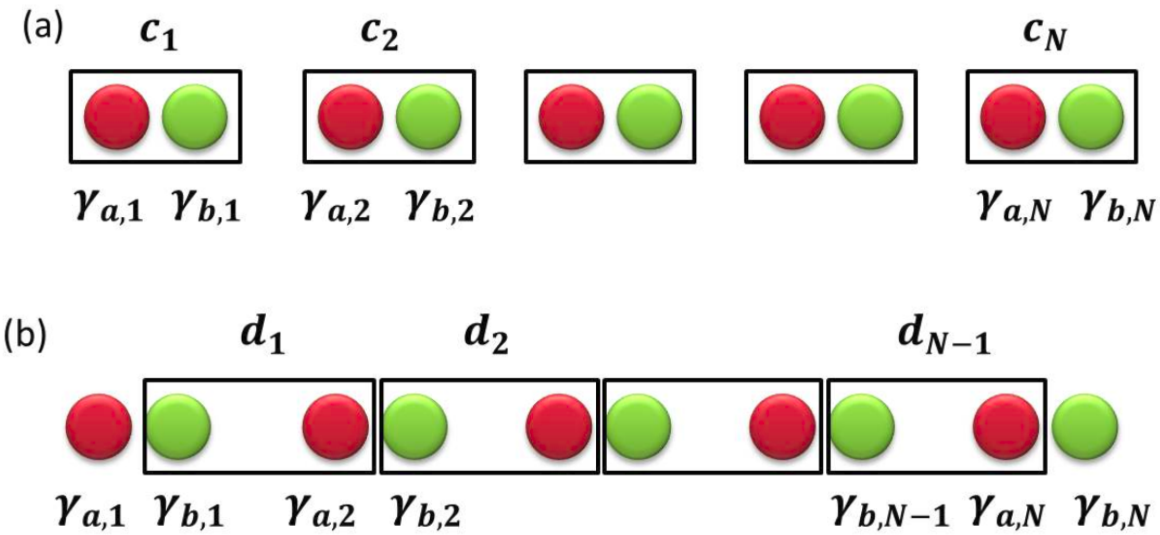

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$$H_p = \left[\Delta \sum_{j=0}^{N-1} \left(d_j^\dagger d_j - \frac{1}{2}\right)\right] + 0\left(d_N^\dagger d_N - \frac{1}{2}\right)$$

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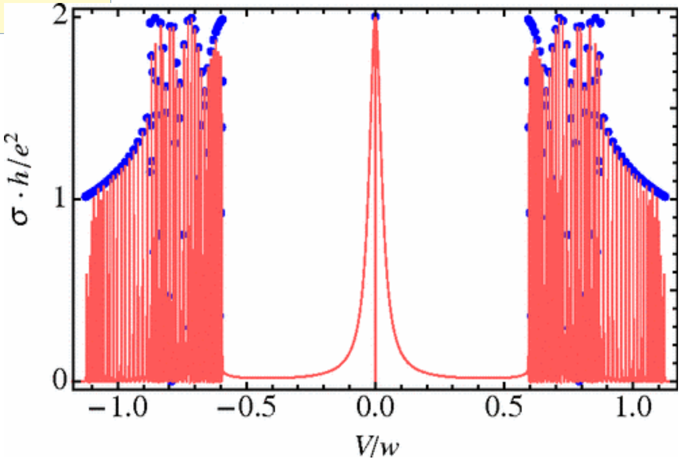
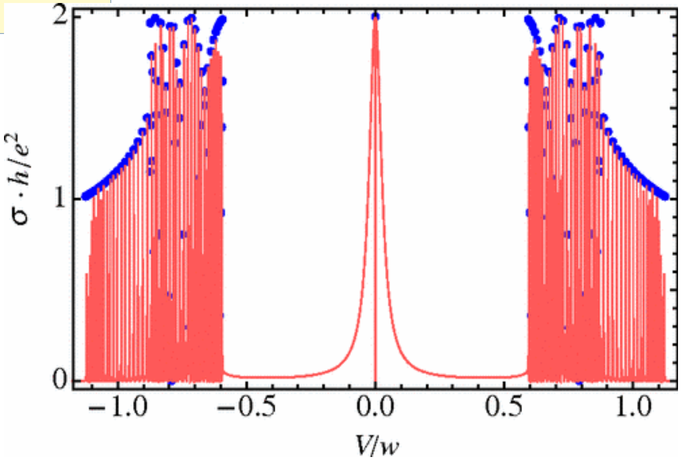
Quantized zero-bias conductance

- Two terminal transport signature of FMF :



Arijit Kundu, Babak Seradjeh (IUB), Phys. Rev. Lett. 111, 136402 (2013)

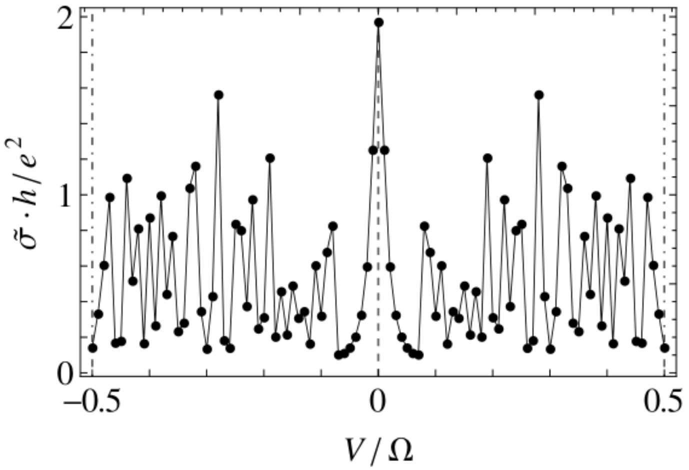
Quantized zero-bias conductance

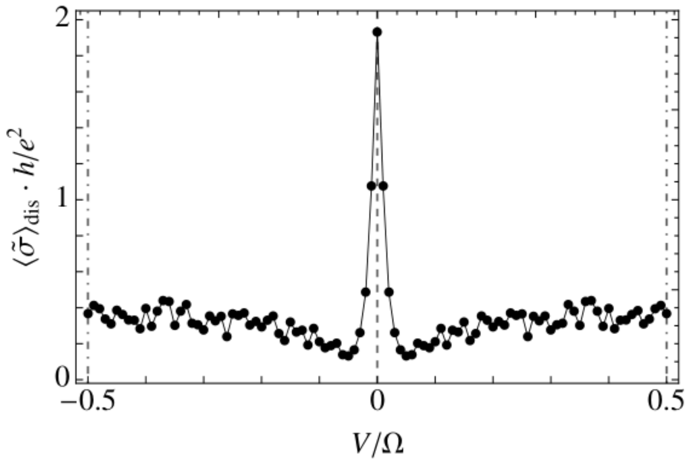


- Two terminal transport signature of FMF :

- Summed conductance is quantized : Essentially takes into account emission and absorption of integer quantas of energy in multiplication of the frequency by the stead-states.

$$\lim_{\mu\to 0,\pm\Omega/2}\sum_n \ \sigma(\mu+n\Omega)=\tilde{\sigma}(0,\pm\Omega/2)=2e^2/h$$





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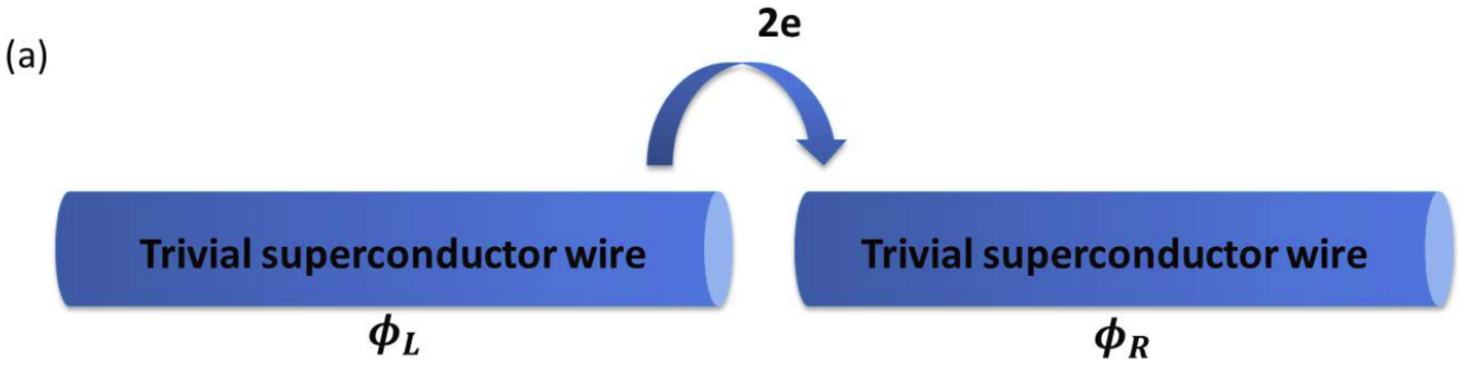
Josephson current signatures



(a) A Josephson junction built by trivial superconductors. Cooper pair tunneling leads the 2π current phase relation.

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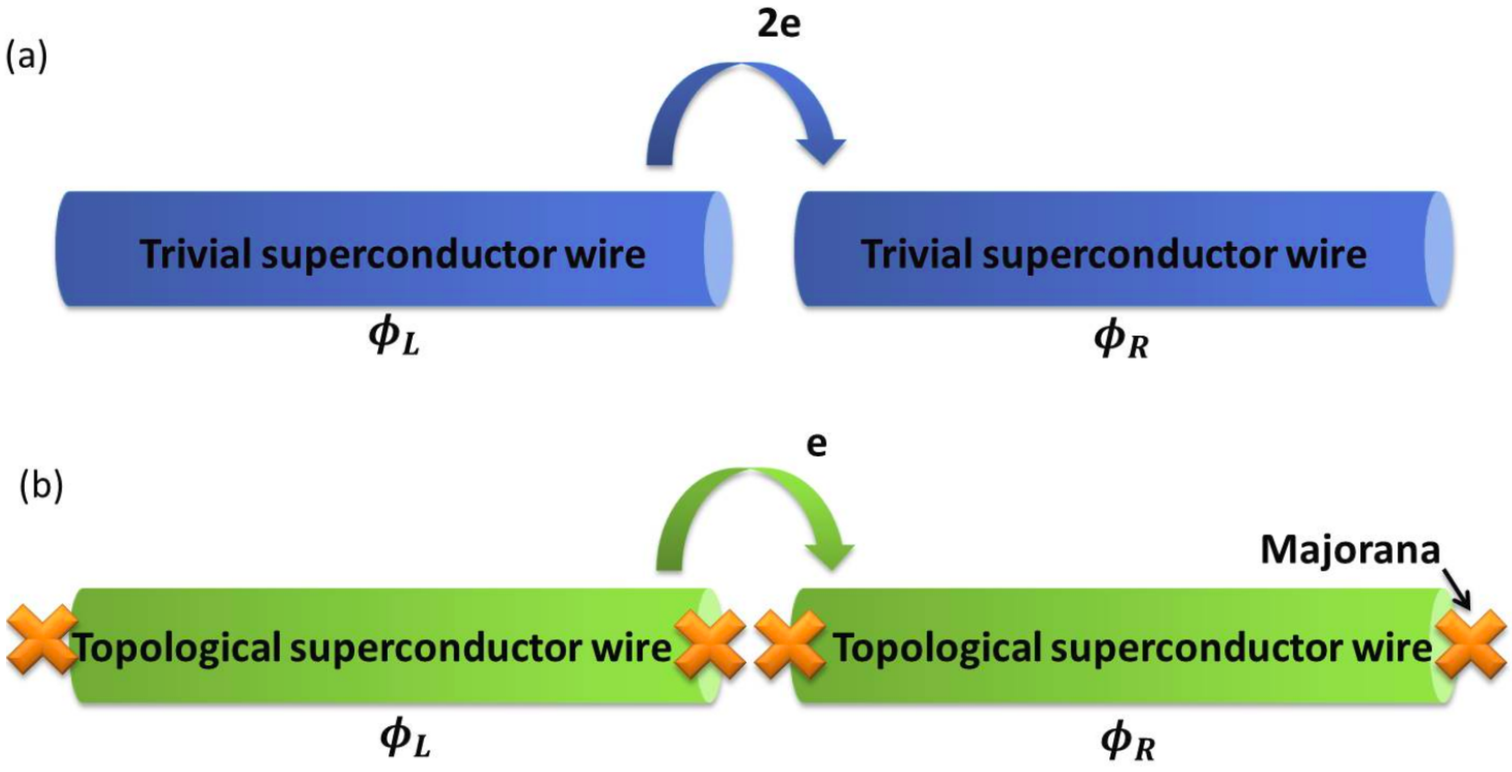


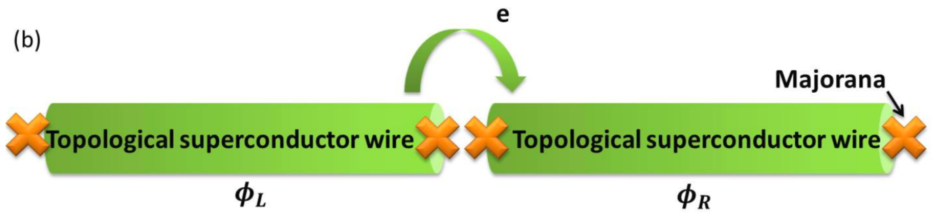
(a) A Josephson junction built by trivial superconductors. Cooper pair tunneling leads the 2π current phase relation.

(b) A Josephson junction built by spinless p-wave superconducting wires.Two Majorana modes form a single electron state in the Josephson junction, which allows single electrons to tunnel through the junction

https://www.semanticscholar.org/paper/Signatures-of-Topological-Superconductors-Lee

Josephson current signatures

$$H_\alpha = -\frac{\Delta}{2}\sum_{x=0}^{N-1}\left[c^\dagger_{\alpha,x+1}c_{\alpha,x} + c^\dagger_{\alpha,x}c_{\alpha,x+1} + e^{i\phi_\alpha}c_{\alpha,x}c_{\alpha,x+1} + e^{-i\phi_\alpha}c^\dagger_{\alpha,x+1}c^\dagger_{\alpha,x}\right]$$



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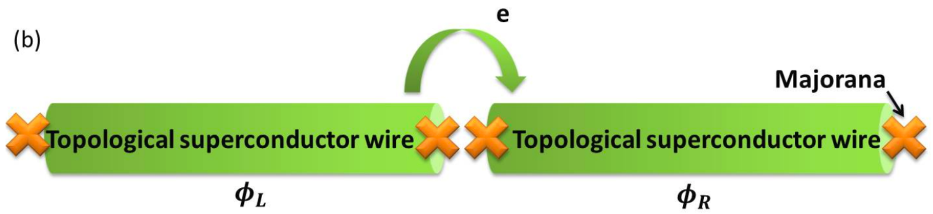
$$H_t = w_J\left(c^\dagger_{R,N}c_{L,1} + h.c.\right)$$



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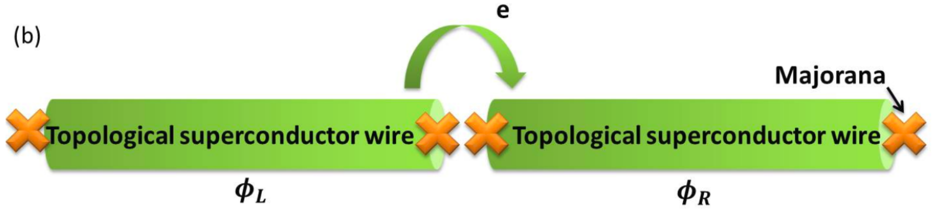

$$H_{eff.} \approx -\frac{i}{2}w_J\gamma_{bN}\gamma_{a1}\cos\left(\frac{\phi}{2}\right) = -w_J\cos\left(\frac{\phi}{2}\right)\left(d^\dagger d - \frac{1}{2}\right)$$

‘d’ - is the annihilation operator of the single fermionic states that appears at the Josephson junction

https://www.semanticscholar.org/paper/Signatures-of-Topological-Superconductors-Lee

Transport mediated by Majoranas : Role of system size

- In the static case, the energy of edge modes at the junction is $\pm\delta$ (vanishes exponentially as their separation increases).

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$$\pm E_{\text{eff}} \propto \pm \left(w_J^2 \cos^2(\phi/2) + \delta^2 \right)^{1/2}$$

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$$\pm E_{\text{eff}} \propto \pm \left(w_J^2 \cos^2(\phi/2) + \delta^2\right)^{1/2}$$

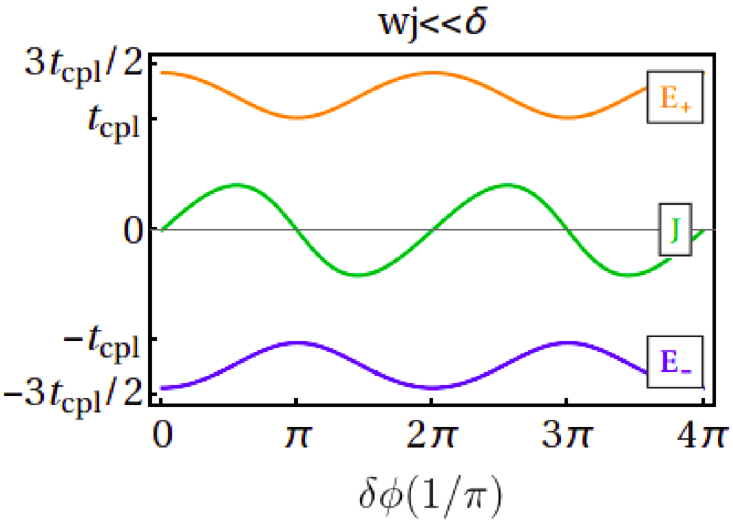
- At zero temperature, the current carried by these states is,

$$J_{\text{stat}} \approx f(E_{\text{eff}}) \frac{\partial E_{\text{eff}}}{\partial \phi} \propto w_J^2 \left(w_J^2 \cos^2(\phi/2) + \delta^2\right)^{-1/2} \sin\phi$$

$w_J \ll \delta$

$$E_{\text{eff.}} \propto \cos(\phi)$$

$$J_{\text{stat.}} \propto w_J^2 \sin(\phi)$$



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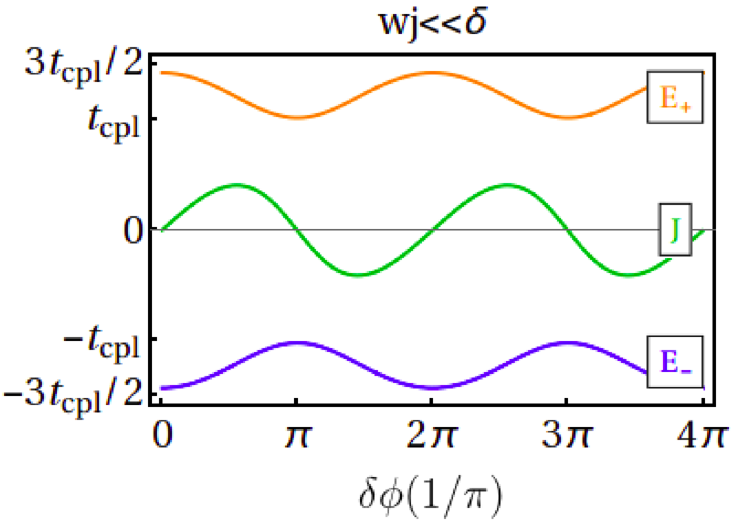

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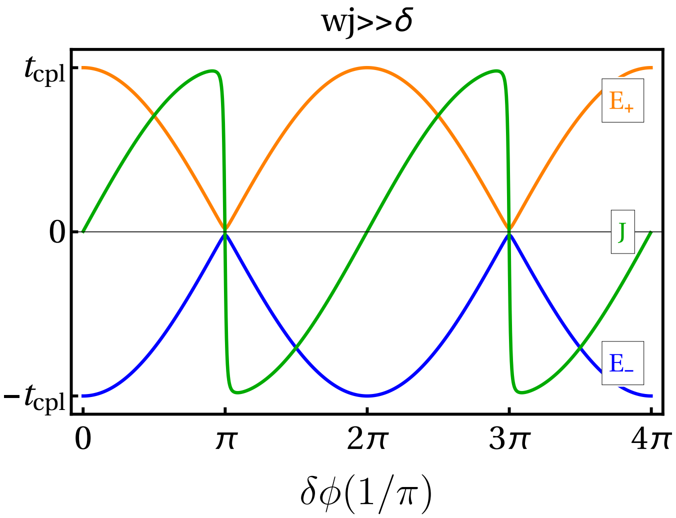

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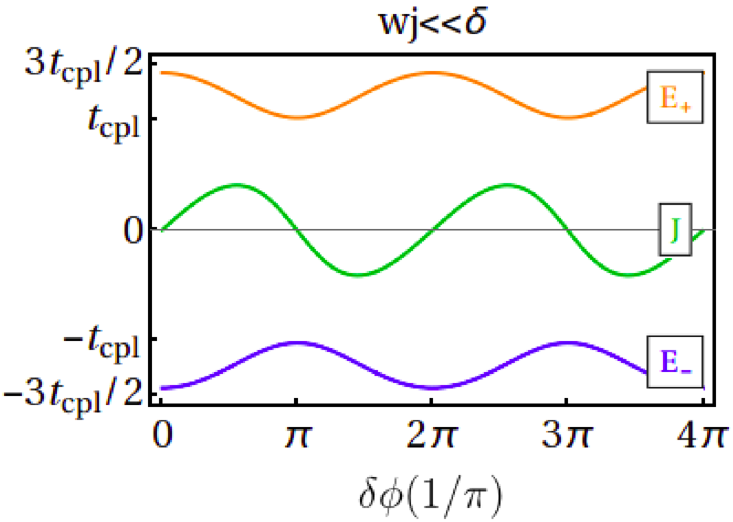
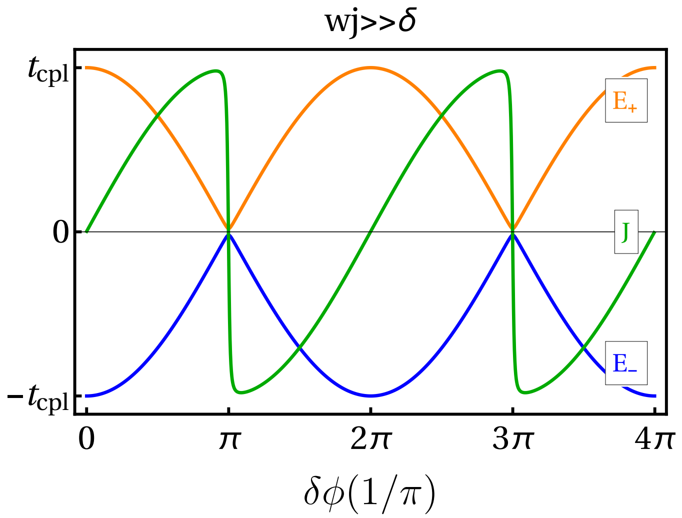
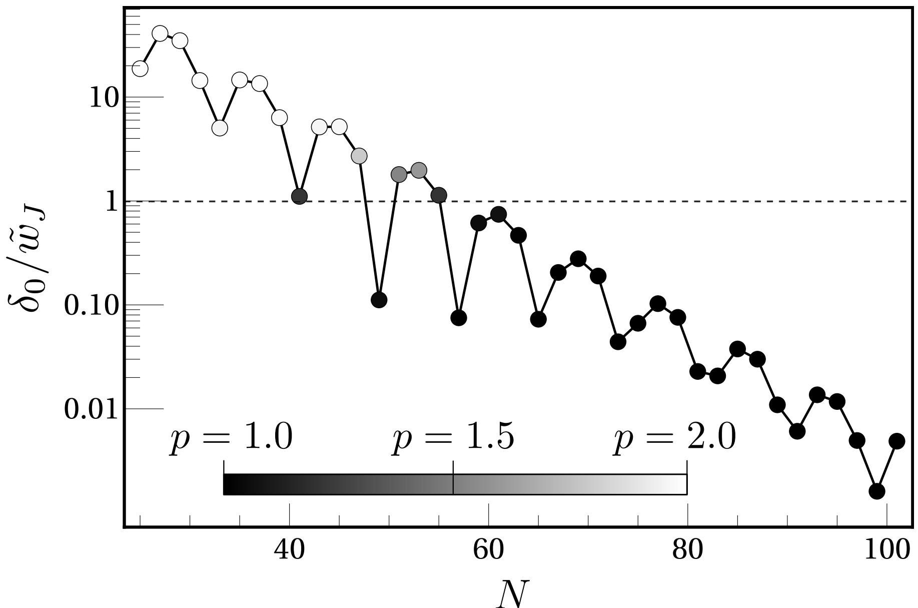




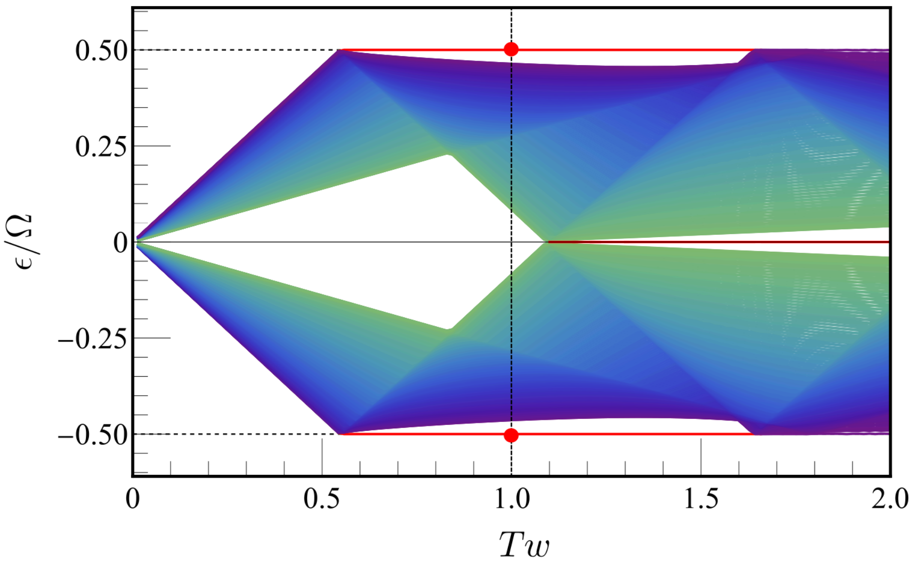
FMF in driven Kitaev model

The localization of FMFs at the edge depends on the system size.

The quasienergy gaps are exponential in system-size.

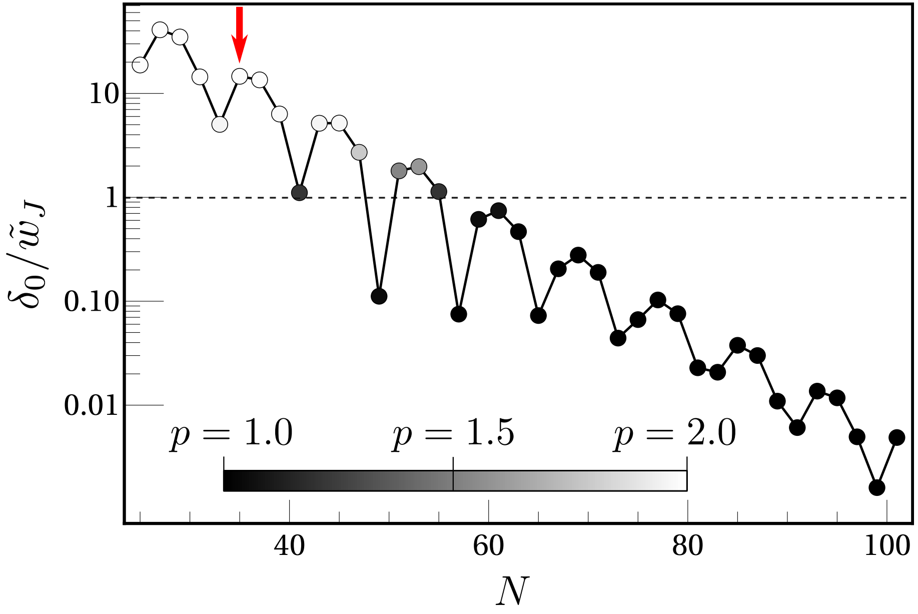


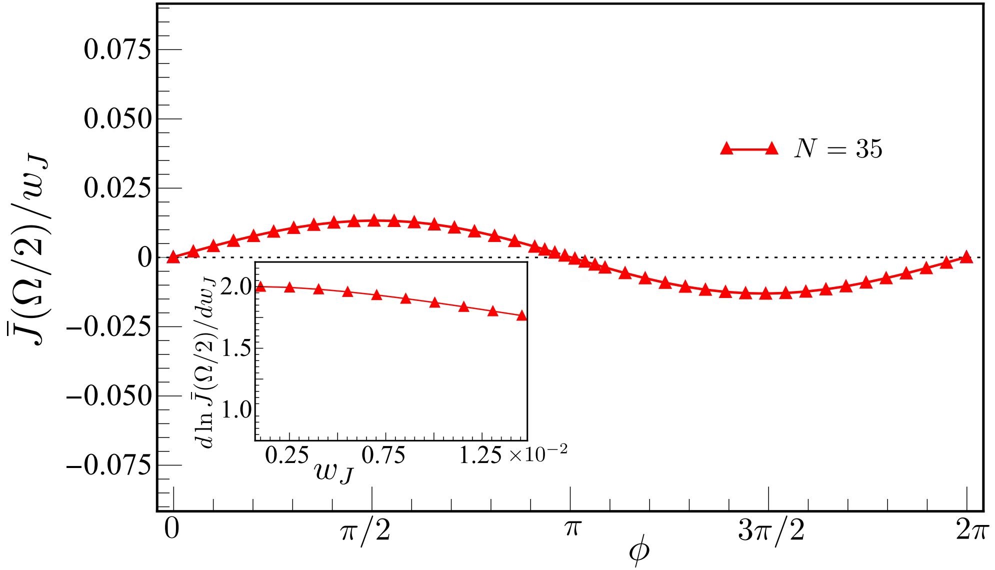
$$H(t+T) = H(t)$$



- System-size controls the signatures of FMF's.

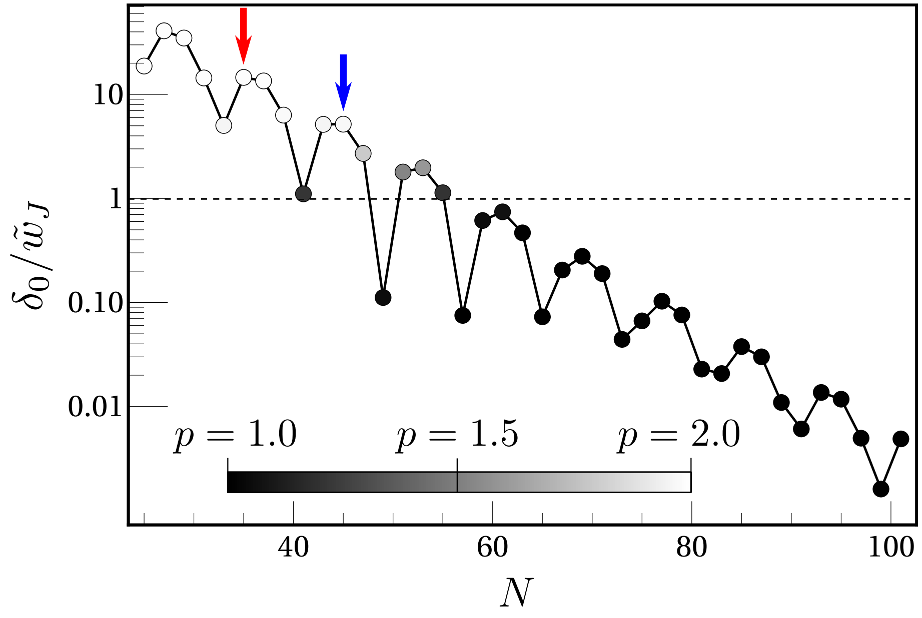
FMF in driven Kitaev model

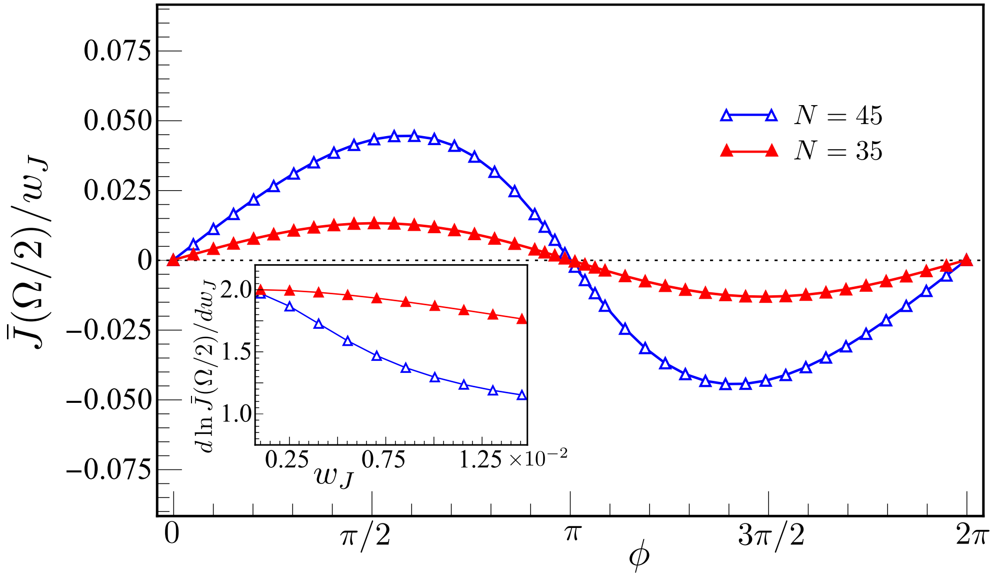




arXiv:2301.07707

FMF in driven Kitaev model





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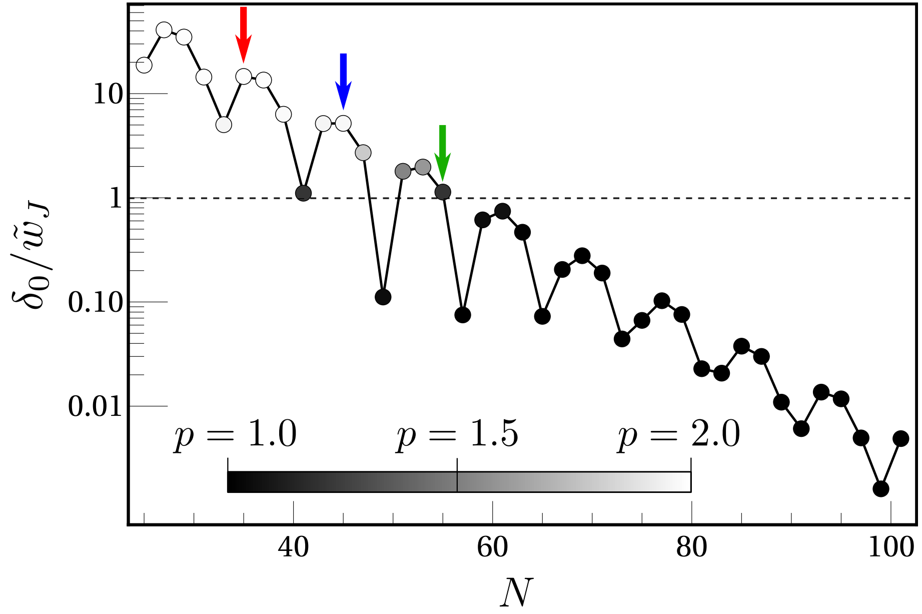
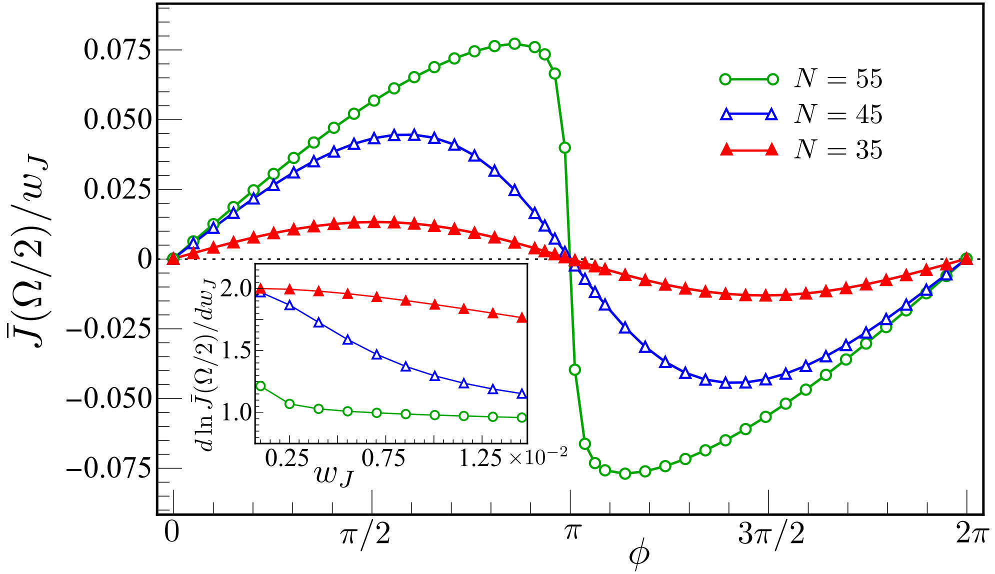
FMF in driven Kitaev model





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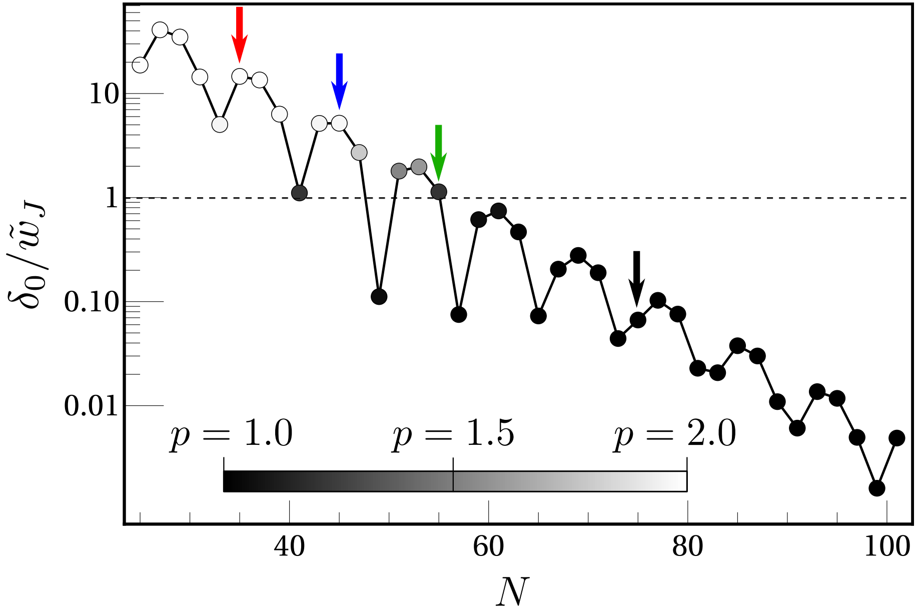

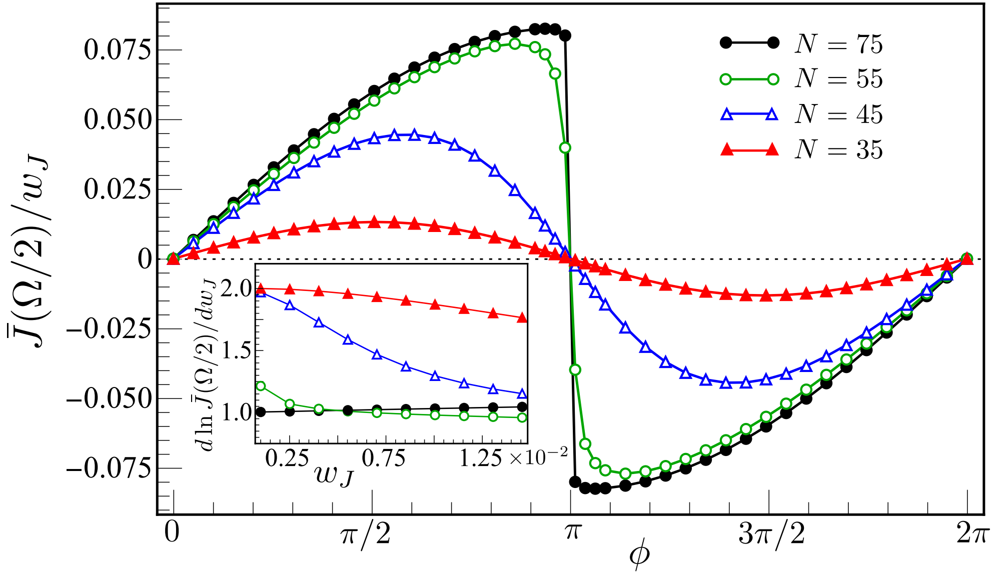

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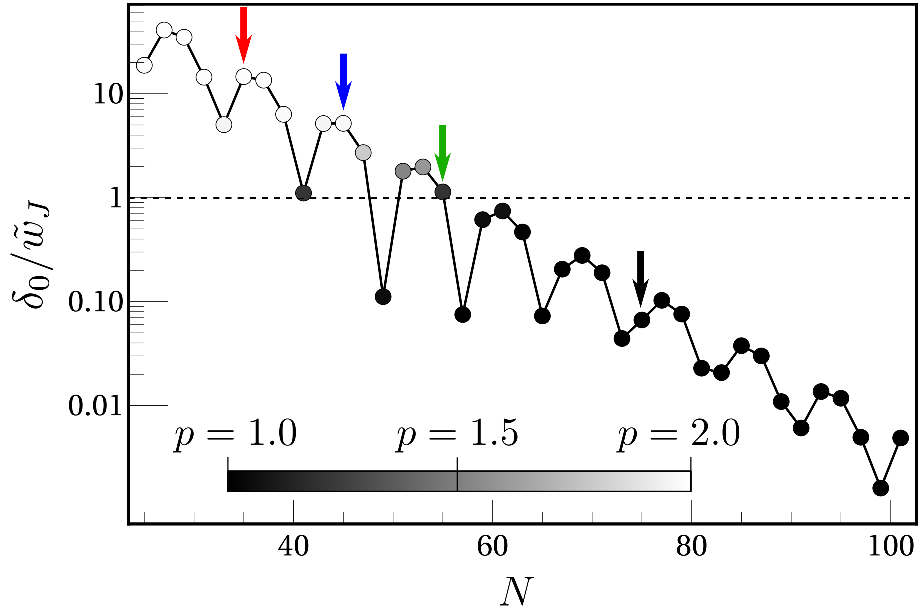
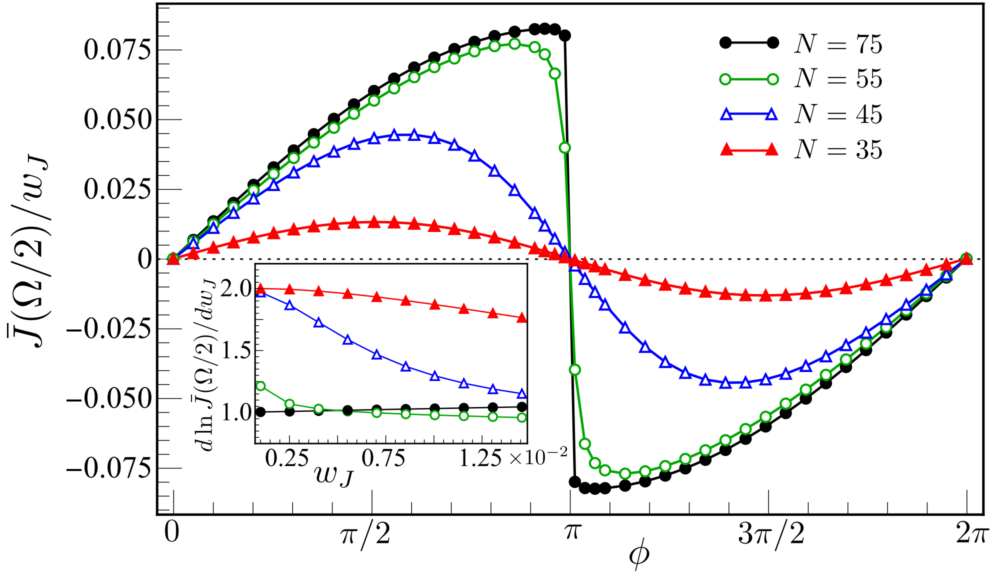




- **Steep jump** of the current, a reminiscent of static MF mediated Josephson junction.

- The jump is steeper and the current becomes **linear in tunneling** with increasing system size

arXiv:2301.07707

Steady-state occupation of Floquet states

- Can these signatures also be understood in terms of steady state occupations? Similar to static systems?

$$\mathrm{J}_{\mathrm{stat}} \approx f(E_{\mathrm{eff}})\frac{\partial E_{\mathrm{eff}}}{\partial \phi} = \Delta n \frac{\partial E_{-}}{\partial \phi}$$

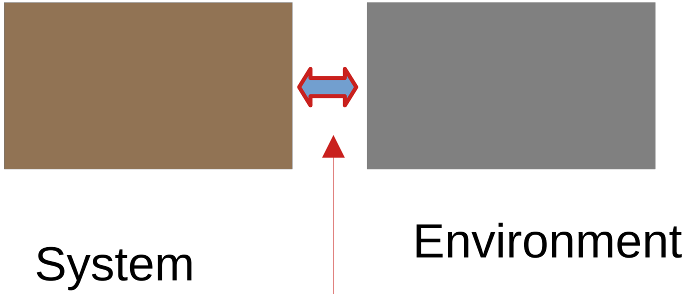
- Josephson current in terms of Quasi-energy derivatives and occupations?

- Occupation differences? $\Delta n = f(E_{+}) - f(E_{-}) = -1$

- What is different in driven case?

arXiv:2301.07707

Steady-state occupation of Floquet states



Weak-coupling limit

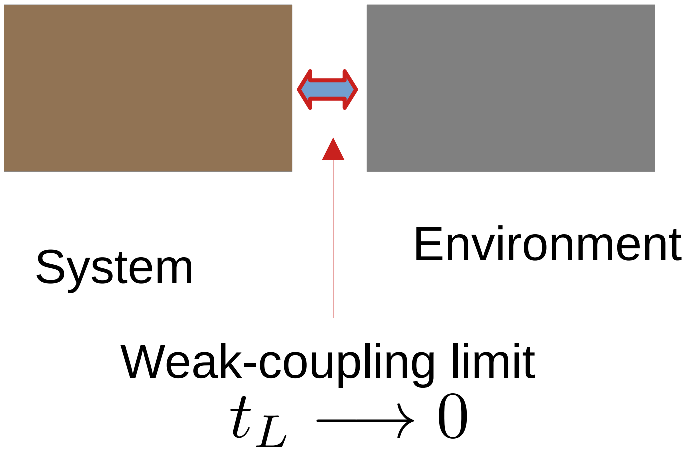
$$t_L \longrightarrow 0$$

arXiv:2301.07707

Steady-state occupation of Floquet states

- Steady-state density matrix in the basis of Floquet states

$$\hat{\rho}(t) = \sum n_{\alpha\beta}(t)|u_\alpha(t)\rangle\langle u_\beta(t)|$$



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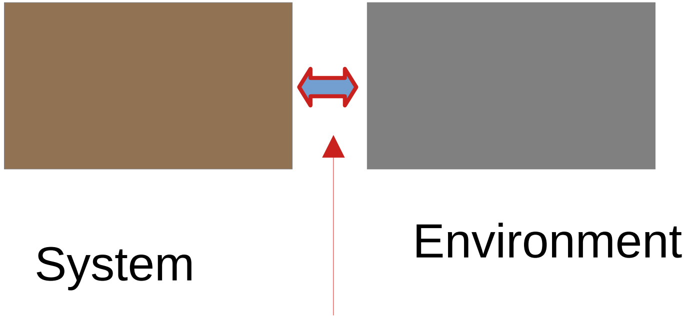
Where

$$n_{\alpha\beta}(t) = \frac{1}{T}\int_0^T \left\langle \Psi^\dagger_\alpha(t)\Psi_\beta(t)\right\rangle_{\text{Lead avg.}}$$

Can be computed using a ‘Floquet Green’s function technique’, in the flat-band limit.



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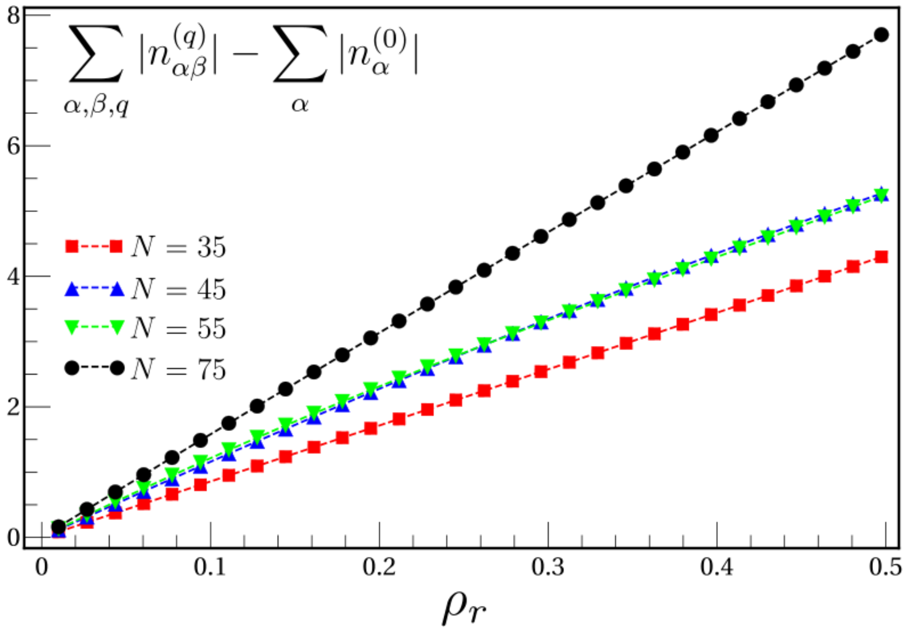
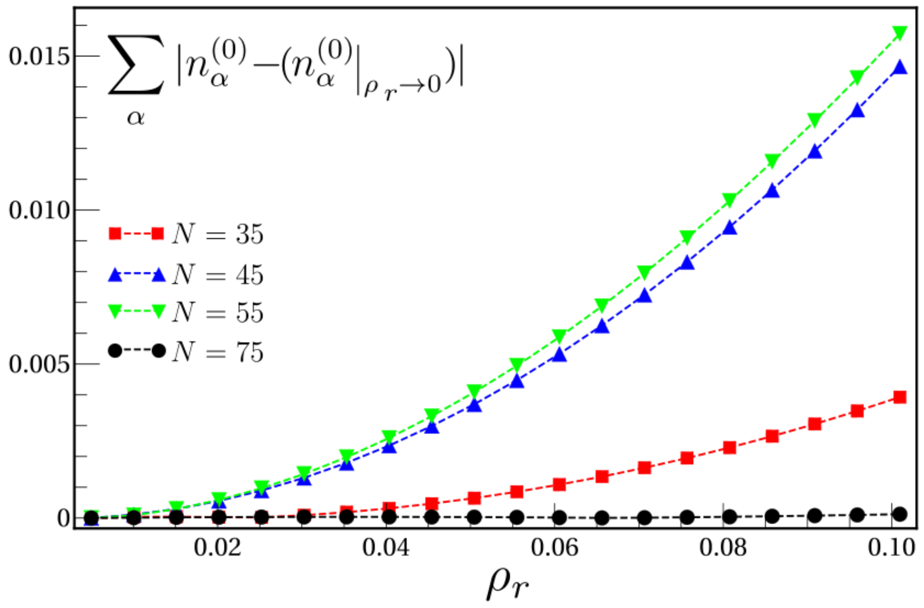


- If we subtract the diagonal, zero frequency terms, then the rest goes to zero linearly with the coupling to the environment.

Steady-state occupation of Floquet states



- If we subtract the diagonal, zero frequency terms, then the rest goes to zero linearly with the coupling to the environment.
- In the limit when the environmental coupling is the smallest energy-scale, one can derive an expression of the steady-state occupation.
- The difference from the analytical value of such value differs at finite environmental coupling only at quadratic manner.

Steady-state occupation of Floquet states

$$n_{\alpha\beta}^{(q)} = \sum_{\lambda k} \int_{-\infty}^{\infty} \frac{\langle u_{\beta}^{+\ (k)} | \mathbb{V}^{\lambda} | u_{\alpha}^{+\ (k+q)} \rangle f^{\lambda}(\omega, \mu^{\lambda}, \beta^{\lambda}) d\omega}{(\omega - \epsilon_{\alpha}^{(k+q)} - i\gamma_{\alpha})(\omega - \epsilon_{\beta}^{(k)} + i\gamma_{\beta})}$$

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$$f(\omega, \mu^{\lambda}, \beta^{\lambda}) = \frac{1}{e^{\beta^{\lambda}(\omega - \mu^{\lambda})}}$$

$$= -\frac{1}{\beta^{\lambda}} \sum_{n \in I} \frac{1}{(\omega - \mu^{\lambda}) - \frac{(2n+1)i\pi}{\beta^{\lambda}}} + \frac{1}{2}$$

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- Flat band limit :
- Reservoir connected at every site:
- Weak coupling limit:

$$f(\omega, \mu^\lambda, \beta^\lambda) = \frac{1}{e^{\beta^\lambda(\omega - \mu^\lambda)}}$$

$$= -\frac{1}{\beta^\lambda} \sum_{n \in I} \frac{1}{(\omega - \mu^\lambda) - \frac{(2n+1)i\pi}{\beta^\lambda}} + \frac{1}{2}$$

Steady-state occupation of Floquet states

$$n_{\alpha\beta}^{(q)} = \sum_{\lambda k} \int_{-\infty}^{\infty} \frac{\langle u_\beta^{+\,(k)} | \mathbb{V}^\lambda | u_\alpha^{+\,(k+q)} \rangle f^\lambda(\omega, \mu^\lambda, \beta^\lambda) d\omega}{(\omega - \epsilon_\alpha^{(k+q)} - i\gamma_\alpha)(\omega - \epsilon_\beta^{(k)} + i\gamma_\beta)}$$

- Flat band limit :
- Reservoir connected at every site:
- Weak coupling limit:

$$f(\omega, \mu^\lambda, \beta^\lambda) = \frac{1}{e^{\beta^\lambda(\omega - \mu^\lambda)}}$$

$$= -\frac{1}{\beta^\lambda} \sum_{n \in I} \frac{1}{(\omega - \mu^\lambda) - \frac{(2n+1)i\pi}{\beta^\lambda}} + \frac{1}{2}$$

Occupation of Floquet states:

$$n_\alpha = \sum_{k \in \mathbb{Z}} f_r(\epsilon_\alpha + k\Omega - \mu_r) \langle u_\alpha^{(k)} | u_\alpha^{(k)} \rangle$$

Steady-state occupation of Floquet states

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- Incorporates relaxation due to particle tunneling to and forth the reservoir.

Steady-state occupation of Floquet states

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- Josephson current : In terms of quasi-energy and occupation

$$\langle \hat{J}_{\mathcal{S}} \rangle (\mu_r, t) = \sum_\alpha n_\alpha(\mu_r) \langle u_\alpha(t) | \partial_\phi H_{\mathcal{S}} | u_\alpha(t) \rangle.$$

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$$\frac{1}{T}\int_0^T dt\langle u_\alpha(t)|(\partial_\phi H_{\mathcal{S}})|u_\alpha(t)\rangle$$

$$= \frac{1}{T}\int_0^T \langle u_\alpha(t)|\partial_\phi\left(H_{\mathcal{S}}|u_\alpha(t)\rangle\right) - \langle u_\alpha(t)|H_{\mathcal{S}}|\partial_\phi u_\alpha(t)\rangle \quad = \partial_\phi \epsilon_\alpha.$$

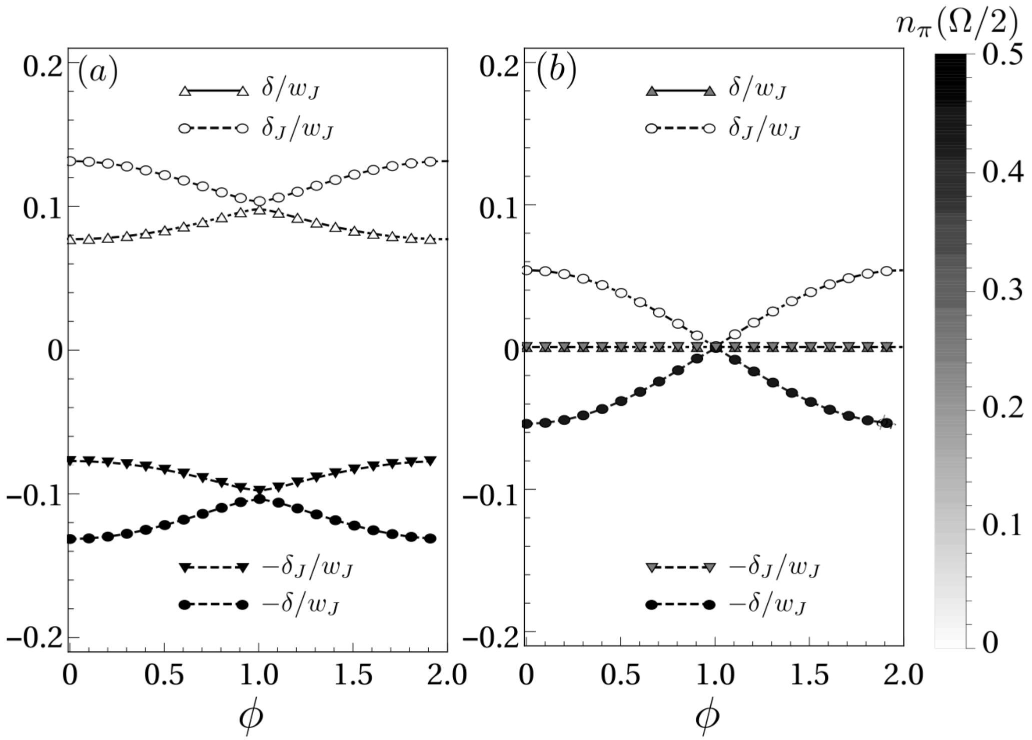
Josephson current

$$\bar{J}(\mu_r) = \langle\langle \hat{J}_{\mathcal{S}} \rangle\rangle = \frac{1}{T}\int_0^T dt \langle \hat{J}_{\mathcal{S}} \rangle(\mu_r, t) = \sum_\alpha n_\alpha(\mu_r)\partial_\phi \epsilon_\alpha$$

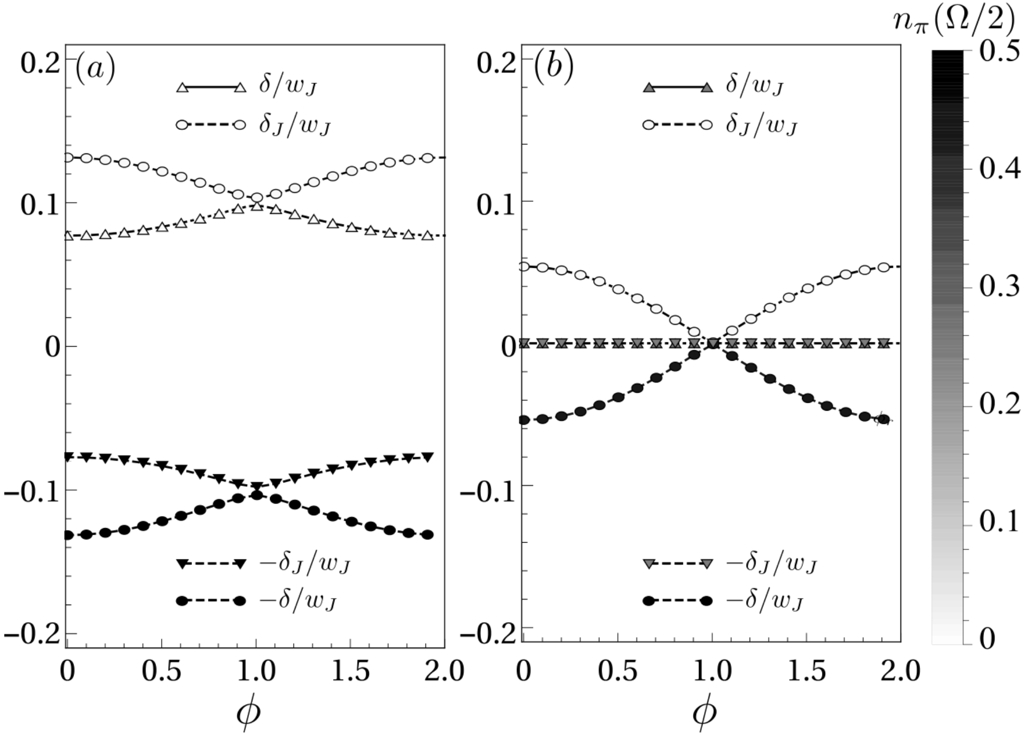
$$n_\alpha = \sum_{k\in\mathbb{Z}} f_r(\epsilon_\alpha + k\Omega - \mu_r) \langle u_\alpha^{(k)} | u_\alpha^{(k)} \rangle$$

If the chemical potential of the lead is integer multiples of FMF quasienergies, then most of theContribution to the current is from the FMF states themselves.

A jump at ϕ = π

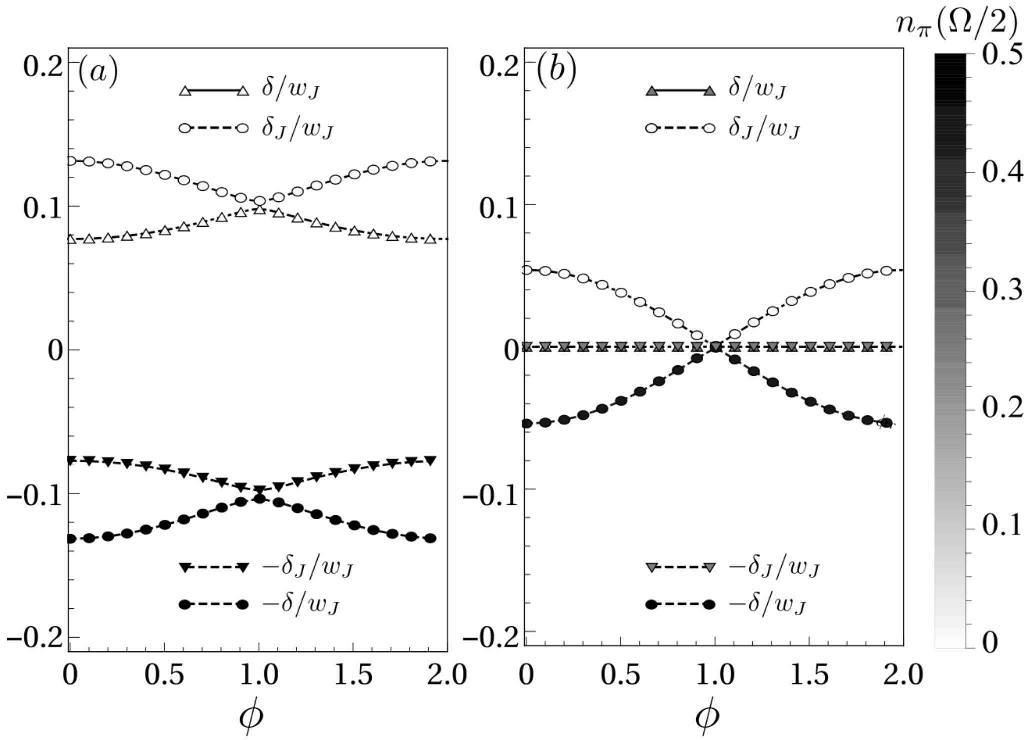

(a)
δ/w_J
δ_J/w_J
−δ_J/w_J
−δ/w_J
(b)
δ/w_J
δ_J/w_J
−δ_J/w_J
−δ/w_J
n_π(Ω/2)
0.5
0.4
0.3
0.2
0.1
0
0.2
0.1
0
−0.1
−0.2
0
0.5
1.0
1.5
2.0
ϕ

A jump at $\phi = \pi$



- For a small system size, the FMF at the junction (*J*) never interchange occupation
  at $\phi = \pi$.

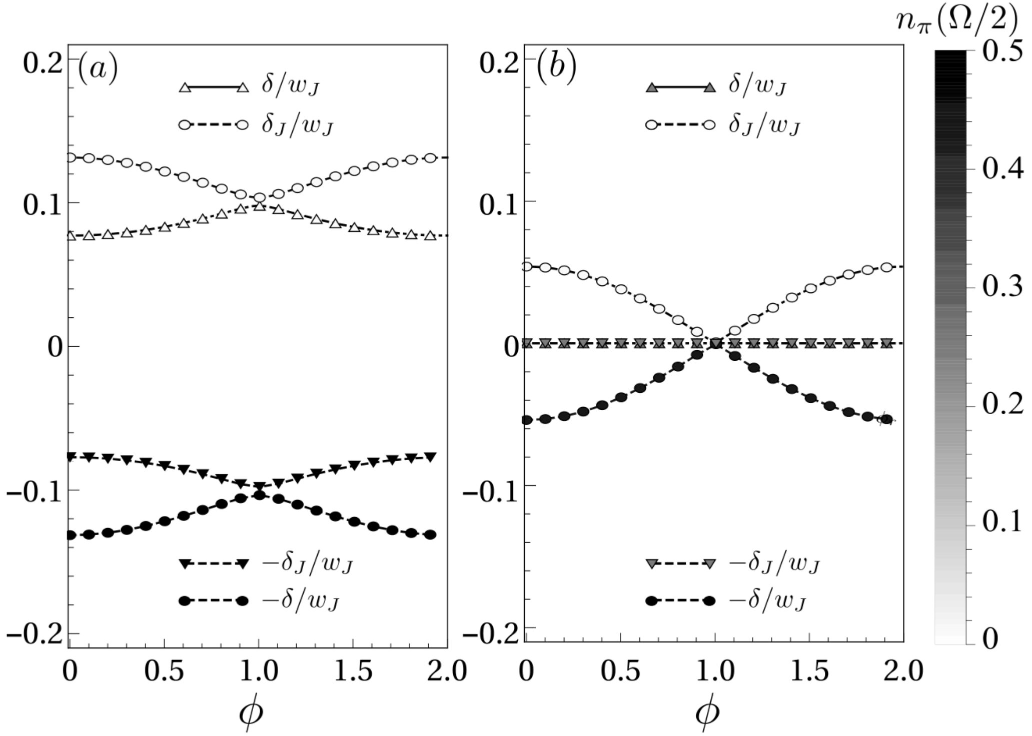
A jump at ϕ = π



- For a small system size, the FMF at the junction (J) never interchange occupation
  at ϕ = π.

- For a larger system size, we observe that the π-FMFs at the junction with quasienegies $\pm\Omega/2 \mp \delta_J$ exchange their occupation probability at ϕ = π and this results in a jump of the Josephson current.

A jump at ϕ = π



- For a small system size, the FMF at the junction (J) never interchange occupation at ϕ = π.
- For a larger system size, we observe that the π-FMFs at the junction with quasienegies ±Ω/2 ∓ δ_J exchange their occupation probability at ϕ = π and this results in a jump of the Josephson current.
- How much the difference will be, that is determined by Fourier components of the Floquets states of the FMFs.

Summed occupation difference

$$\bar{J}(\mu_r) = \sum_{\epsilon_\alpha<0} \left(n_{\epsilon_\alpha}(\mu_r) \frac{\partial \epsilon_\alpha}{\partial \phi} + n_{\epsilon_{\bar{\alpha}}}(\mu_r) \frac{\partial \epsilon_{\bar{\alpha}}}{\partial \phi} \right) = \sum_{\epsilon_\alpha<0} \left[n_{\epsilon_\alpha}(\mu_r) - n_{\epsilon_{\bar{\alpha}}}(\mu_r) \right] \frac{\partial \epsilon_\alpha}{\partial \phi}$$

$$\equiv \sum_{\epsilon_\alpha<0} \nu_\alpha(\mu_r) \frac{\partial \epsilon_\alpha}{\partial \phi}.$$

- Thus the current has a some of only negative quasienergy states.

Summed occupation difference

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- Thus the current has a some of only negative quasienergy states.
- Lets look at `summed occupation difference’:

$$\nu_\alpha^F = \lim_{N \to \infty} \sum_{k=-\infty}^{N} \left(n_{\epsilon_\alpha}(\mu_r + k\Omega) - n_{\epsilon_{\bar{\alpha}}}(\mu_r + k\Omega) \right)$$

Summed occupation difference

- The `summed occupation difference' has a rather interesting form

$$\nu_\alpha^F = \frac{2\epsilon_\alpha}{\Omega} - \frac{1}{\pi}\int_0^T dt \langle u_\alpha(t)|H(t)|u_\alpha(t)\rangle + \lim_{N\to\infty}\sum_{k=-\infty}^{N}\left[f_r(\epsilon-\mu_r-k\Omega) - f_r(-\epsilon-\mu_r-k\Omega)\right]$$

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$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}$$

An `winding’ in time phase

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An `winding' in time phase

Goes to zero for FMF states!

arXiv:2301.07707

Summed occupation difference: FMF cases

$$
\begin{aligned}
\nu_0^F &= -\frac{2\delta_J}{\Omega} + \lim_{N\to\infty} \sum_{k=-\infty}^{N} \left[f_r(-\delta_J - k\Omega) - f_r(\delta_J - k\Omega) \right] \\
&= -\frac{2\delta_J}{\Omega} + \left[f_r(-\delta_J) - f_r(\delta_J) \right] \\
&= -\frac{2\delta_J}{\Omega} + \tanh\left(\frac{\delta_J}{2\theta_r} \right) \approx 1
\end{aligned}
$$

Summed occupation difference: FMF cases

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$$= -\frac{2\delta_J}{\Omega} + \left[f_r(-\delta_J) - f_r(\delta_J) \right]$$

$$= -\frac{2\delta_J}{\Omega} + \tanh\left(\frac{\delta_J}{2\theta_r}\right) \approx 1$$

$$\nu_\pi^F = \frac{-\Omega + 2\delta_J}{\Omega} + \lim_{N\to\infty} \sum_{k=-\infty}^{N-m} \left[f_r(\epsilon - \mu_r - k\Omega) - f_r(-\epsilon - \mu_r - (k+1)\Omega) \right] + 1$$

$$= \frac{2\delta_J}{\Omega} + \left[f_r(\delta_J) - f_r(-\delta_J) \right]$$

$$= \frac{2\delta_J}{\Omega} - \tanh\left(\frac{\delta_J}{2\theta_r}\right) \approx -1$$

Sum-rule: Josephson current

- The `summed Josephson current'

$$\bar{J}_F(\mu_b) \equiv \sum_k \bar{J}(\mu_b + k\Omega) \approx \nu_b^{\mathrm{F}} \frac{\partial \delta_J}{\partial \phi}$$

Sum-rule: Josephson current

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$$\bar{J}_F(\mu_b) \equiv \sum_k \bar{J}(\mu_b + k\Omega) \approx \nu_b^{\mathrm{F}} \frac{\partial \delta_J}{\partial \phi}$$

- Similarity to the static-system's current

$$J_{\mathrm{stat}} \approx \frac{\partial E_{\mathrm{eff}}}{\partial \phi} \propto w_J^2 \left(w_J^2 \cos^2(\phi/2) + \delta^2 \right)^{-1/2} \sin \phi$$

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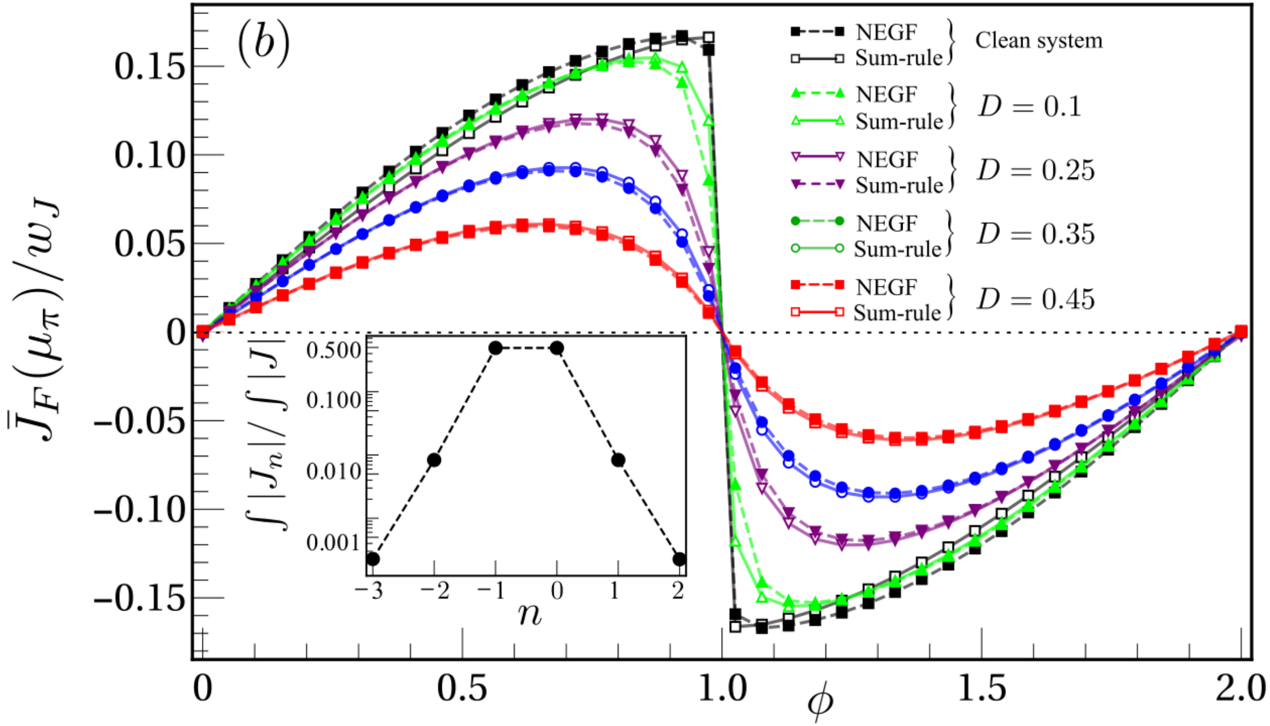
- With relevant energy-scales are dictated by details of the driving (and assisted tunneling)`

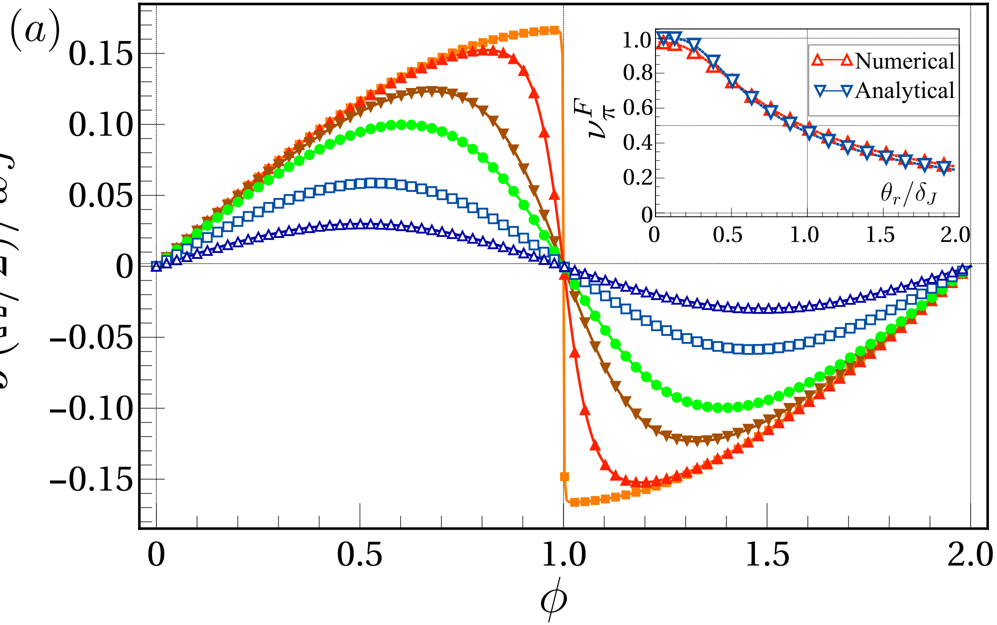
$$\bar{J}_F \approx \frac{\partial \epsilon_F}{\partial \phi} \propto \tilde{w}_J^2 \left(\tilde{w}_J^2 \cos^2(\phi/2) + \delta^2 \right)^{-1/2} \sin\phi$$

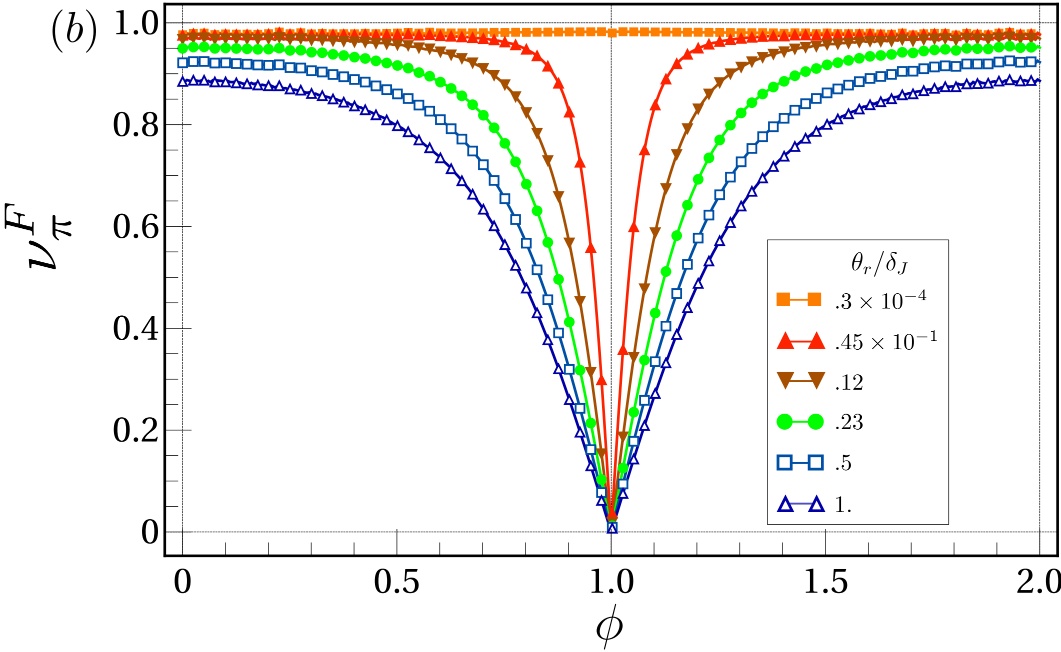
Sum-rule in current: robust against small (static) disorder



For more information - arXiv:2301.07707

Sum-rule in current: Temeperature dependence





For more information - arXiv:2301.07707

Summary

- When floquet systems are weakly coupled to the fermionic bath, the sum of Fermi functions gives the occupation probabilities of steady states.

- This leads to an exciting form of a summed occupation difference for particle-hole pairs of excitations.

- Leading to a ‘sum-rule’ expression of Josephson current, reminiscent of static MF mediated Josephson current, and especially recovering the jump at phase difference $\phi = \pi$.

- These signatures are robust again weak disorder and small temperature.

For more information - arXiv:2301.07707

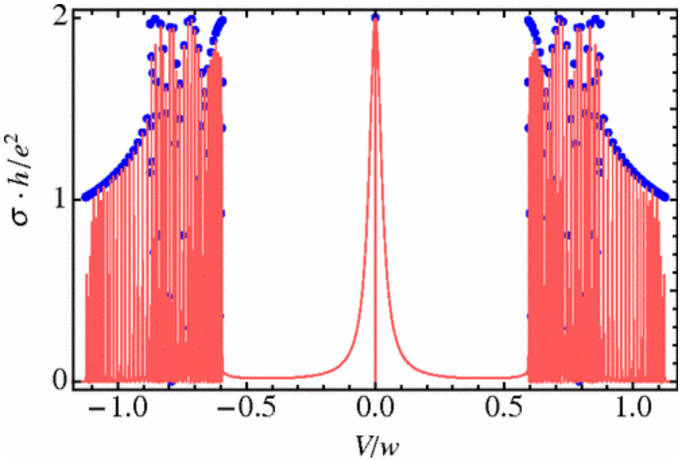
Thank You

Thank You

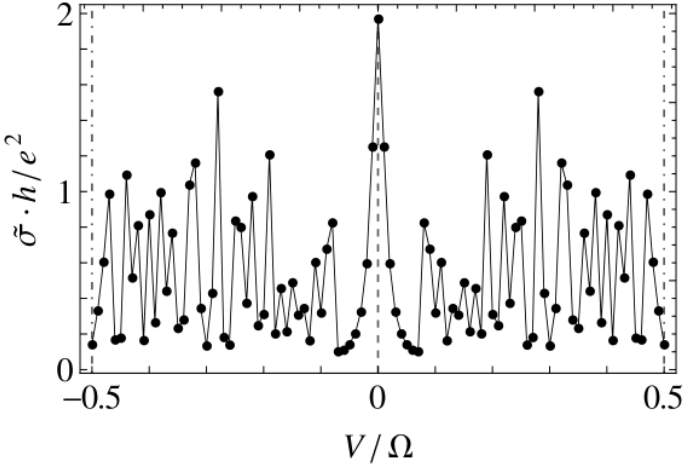
Quantized zero-bias conductance

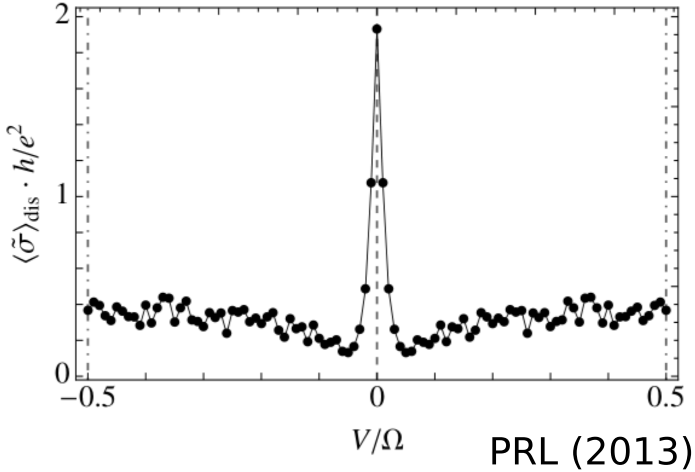
- Two terminal transport signature of FMF :

- Summed conductance is quantized : Essentially takes into account emission and absorption of integer quantas of energy in multiplication of the frequency by the stead-states.



$$\lim_{\mu\to 0,\pm\Omega/2}\sum_n \ \sigma(\mu+n\Omega)=\tilde{\sigma}(0,\pm\Omega/2)=2e^2/h$$





PRL (2013), PRB(R) (2020)

Steady-state occupation of Floquet states

$$\Gamma_{k\alpha}^{n} = \frac{2\pi}{\hbar}\sum_{\ell}|\langle\phi_{k\alpha}^{n}|H_{\text{tun}}|\ell\rangle|^{2}\delta(\mathcal{E}_{k\alpha} + n\hbar\Omega - E_{\ell}).$$

$$I_{k\alpha}^{\text{tun}} = \sum_{n}\Gamma_{k\alpha}^{n}[\bar{F}_{k\alpha}D(\mathcal{E}_{k\alpha}^{n}) - F_{k\alpha}\bar{D}(\mathcal{E}_{k\alpha}^{n})].$$

$$\mathcal{E}_{k\alpha}^{n} \equiv \mathcal{E}_{k\alpha} + n\hbar\Omega$$

$$\dot{F}_{k\alpha} = I_{k\alpha}^{\text{tun}}(F_{k\alpha})$$

$$\tilde{F}_{k\alpha} = \frac{\sum_{n}\Gamma_{k\alpha}^{n}D(\mathcal{E}_{k\alpha}^{n})}{\sum_{n}\Gamma_{k\alpha}^{n}}$$

PRX 5, 041050 (2015)

Steady-state occupation of Floquet states

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$$\frac{1}{T}\int_0^T dt \langle u_\alpha(t)|(\partial_\phi H_{\mathcal{S}})|u_\alpha(t)\rangle$$

$$= \frac{1}{T}\int_0^T \langle u_\alpha(t)|\partial_\phi \left(H_{\mathcal{S}}|u_\alpha(t)\rangle\right) - \langle u_\alpha(t)|H_{\mathcal{S}}|\partial_\phi u_\alpha(t)\rangle$$

$$= -\frac{1}{\beta^\lambda} \sum_{n \in I} \frac{1}{(\omega - \mu^\lambda) - \frac{(2n+1)i\pi}{\beta^\lambda}} + \frac{1}{2}$$

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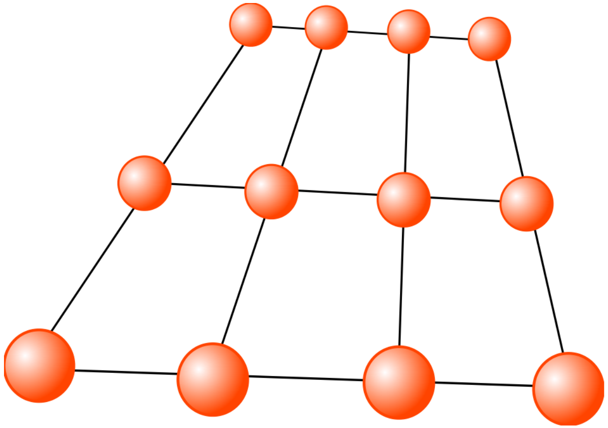
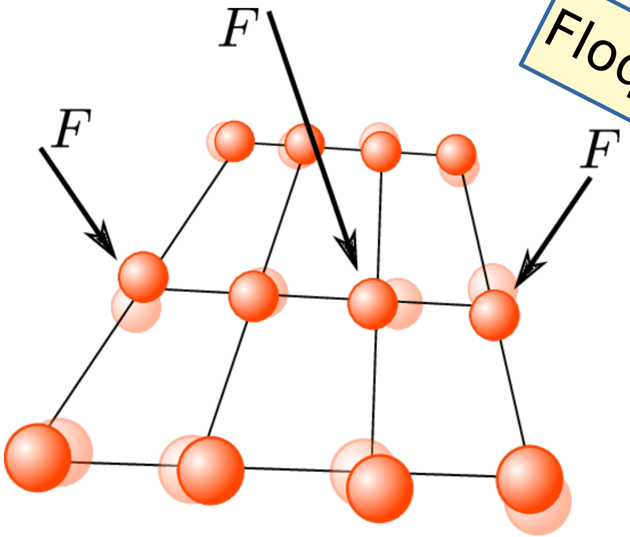
Floquet systems





$$\psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}\phi_{\vec{k}}(\vec{r})$$

$$\psi_{\alpha}(t) = e^{i\epsilon_{\alpha}t}u_{\alpha}(t)$$

Bloch Momentum

$$-\pi/a \leq k \leq \pi/a$$

Quasi-energy

$$-\pi/T \leq \epsilon_{\alpha} \leq \pi/T$$