RG flow is Lindbladian evolution

w. S. Goldman, R. Leigh 24xx.xxxxx

based on

Goldman, NL, Leigh 2301.09669
Furuya, NL, Moosa, Oseph 2112.05099
Furuya, NL, Oseph 2012.14001

Results: A fully non-pert. framework for RG that

1) Incorporates & generalizes continuous tensor networks

2) Provides net of approx. local QEC codes

3) Manifestly erases information, has a zoo of RG monotones

RG flow eq: Lindblad master eq.

In fact, high-energy people already know it

Exact RG of Polchinski applied to density matrices

Motivation: Why is this interesting?

1) Unifies diff. approaches to RG
   - Tensor Network RG
   - Momentum shell integration
   - Holographic RG
2) Generalizes cont. Tensor networks to interacting regime
3) Generalizes connection between holographic RG, radial direction & QEC to general QFT
4) Makes the info. loss along the RG a RG monotone maybe. (new C-theorem?)

Outline

1) Review of approaches to RG on a lattice

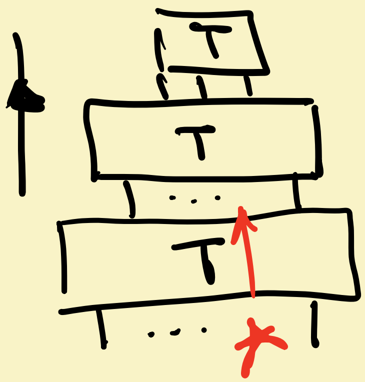
Problems in the continuum limit

2) RG is a net of approx. local QEC codes

3) ERG for density matrices is given by Markov master eq. (Lindbladian)

1) Approaches to RG on lattice

Real-space state RG: Kadanoff's spin-blocking



Properties:

- isometric
- Multi-layer

subalgebra of observables at each level (scale)

$$A_1 \subset A_2 \subset A_3 \subset \cdots \subset A_\infty$$

local UV errors become exponentially weak in the IR:

1st connection to error correction:

Net of Approx. Local classical EC code

local UV errors: noise   IR data is protected

1111 ... 111

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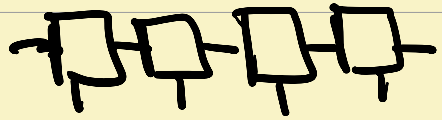
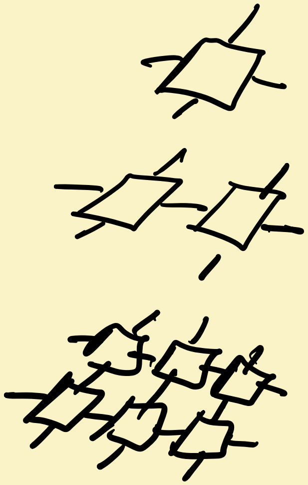


2. DMRG (1d Quantum)

White

Partition function RG : effective action

3. Coarse-graining partition function



Levin, Nave

Partition function of classical 2d model

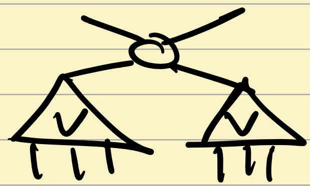
or Path-integral of 1d quantum system

(SVD)

Problems of 1 & 2 & 3

- Remaining short range correl.
- do not capture near criticality physics

Near Criticality: emergent discrete geom.

1. MERA 

   Vidal

2. Tensor network renormalization (TNRG)

   Vidal, Evenbly

TNR yields MERA on the boundary

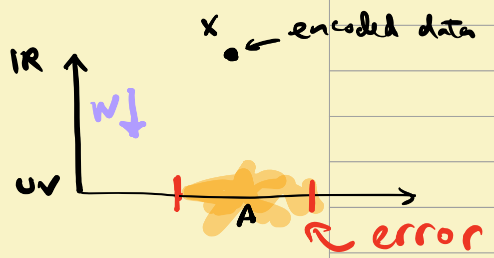
→ MERA

TNR → Euclidean path integral

Approx. local QEC in MERA

Kim, Kastoryano

$\frac{l_{UV}}{l_{IR}} = e^{s}$ $\quad W : \mathcal{H}_{IR} \to \mathcal{H}_{UV}$ $\quad$ isometries : $\begin{cases} \text{disentanglers} \\ \text{coarsegrainings} \end{cases}$



cMERA

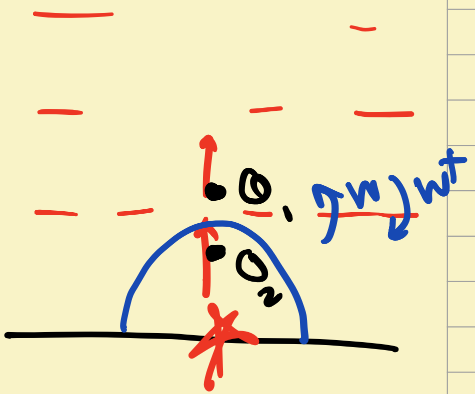
$R(O_{UV}) = W^\dagger O_{UV} W$ $\quad$ scaling map

$R(O_\Delta) = e^{-s\Delta} O_\Delta$ → conf. primary

dimensions: Δ → conf. dimension

RG flow of local density matrices is a superoperator whose eigenoperators are conf. primaries

Holographic RG Flow



- RG flow is radial evolution

de Boer, Verlinde, Verlinde

building on RT formula

- QEC in holography

Almheiri, Dong, Harlow

A net of QEC codes protected against UV noise (erasure)

Knill-Laflamme eq.

$P O_2 P \simeq P$ approx. Knill-Laflamme

$W^\dagger W = P$

Continuum limit

Near cont. phase transitions

correl. length $\to \infty$ cont. limit

Haegeman, et. al. — cMERA: unitary tensor network

Evenbly, Vidal — CTNR :

$$\phi_\Lambda(x) = \int u_\Lambda(x-y)\phi(y)$$

* both free & including even pert. interaction proved hard

→ still proposal: * Tadashi & friends: motivated by holography

$\text{CTNR} \not\to \text{cMERA}$

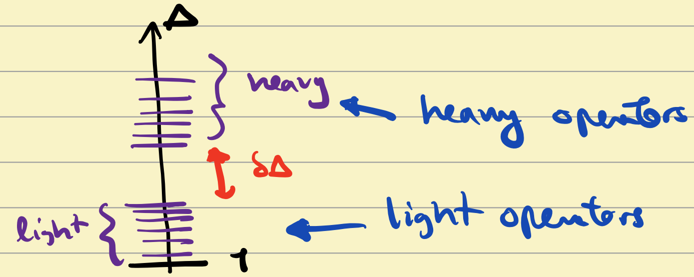
Vidal, Evenbly

2) cMERA is a net of approx. local QEC codes

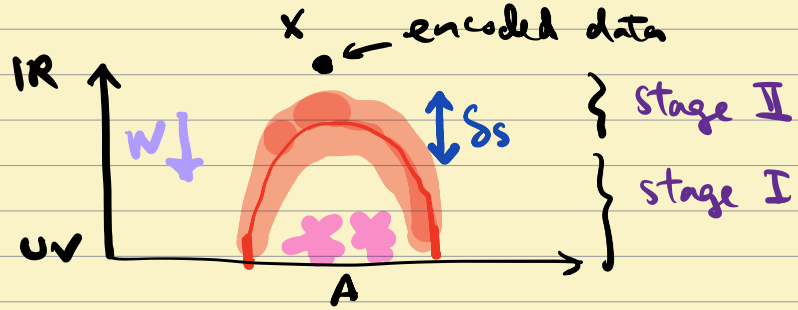
(2112.05099)

error:

$$\int_{g\in A} O_H(g)\, f(g)$$



A gap in the spectrum of conf. dim.



Stage I: support of operators shrink exponentially fast

Stage II: norm of the heavy operator falls off exponentially fast with the gap

State vs. Partition function RG

$\rho(\mu;\Lambda)$

State RG : real space $P(\mu;\Lambda)$

$\mu = e^{-s}\Lambda$ $\quad \partial_s \rho_s = \cdots$ $\quad \partial_s P_s = \cdots$

Partition function RG :

effective $\quad \partial_s Z[\mu;\Lambda]$

$\log Z = S_{\mathrm{eff}}$ quantum

free energy classical

Path-integrals with boundaries give states

3) Exact RG: nonpert. RG in cont. limit

Polchinski

$$S = -\frac{1}{2}\int (\partial_\mu \phi)\, K^{-1}\left(\frac{-\partial^2}{\Lambda^2}\right)(\partial^\mu \phi) + S_{int} + \int J\phi$$

smooth cut-off

$$K^{-1}\left(\frac{p^2}{\Lambda^2}\right)\phi(-p)\,\phi(p)$$

for $p/\Lambda \gg 1$ $\quad S_{free} \gg 1$

$$e^{-S_{free}} \to 0$$

$$0 = \Lambda \partial_\Lambda Z = \int D\phi \left(\Lambda \partial_\Lambda S_0 + \Lambda \partial_\Lambda S_I\right)$$

$G^{-1} = K^{-1}\partial^2$

$$\Lambda \partial_\Lambda S_0 = -\frac{1}{2} \frac{\delta S_0}{\delta \phi} \cdot \Lambda \partial_\Lambda G \frac{\delta S_0}{\delta \phi}$$

Schwinger-Dyson $\quad \mathcal{O} \frac{\delta S_0}{\delta \phi} = \frac{\delta \mathcal{O}}{\delta \phi} - \mathcal{O} \frac{\delta S_I}{\delta \phi}$

$$\Lambda \partial_\Lambda S_I = -\Lambda \partial_\Lambda S_0$$

$$= \frac{1}{2} \frac{\delta S_I}{\delta \phi} \Lambda \partial_\Lambda G \frac{\delta S_I}{\delta \phi} - \frac{1}{2} \mathrm{tr}\left(\frac{\delta^2 S_I}{\delta \phi \delta \phi} \Lambda \partial_\Lambda G \right)$$

β-Functions

$$D_\Lambda^2 = K^{-1}\left(z^2 \vec{\nabla}^2 / \Lambda^2\right)\left(-\partial_t^2 + \vec{\nabla}^2\right)$$

$$\lim_{s\to\infty} K(s) = 0 \qquad \lim_{s\to 0} K(s) = 1$$

$$S[\phi] = -\frac{1}{2 z^{d-2}}\left(\int_{M} \phi \cdot D^2 \cdot \phi - \int_{\partial\Sigma} \phi \cdot K^{-1} \partial_+ \phi\right)$$

step 1 : $\Lambda \to e^{-s}\Lambda$

step 2 : (sym) : $z \to e^{s} z$

$\Lambda \to e^{s}\Lambda$

$\phi \to e^{s\left(\frac{d-2}{2}\right)}\phi$

$$\langle \varphi \,|\, \Omega[\Lambda, z]\rangle = \langle e^{s\left(\frac{d-2}{2}\right)}\varphi \,|\, \Omega[\Lambda e^{s}, e^{s} z]\rangle$$

ERG for wave-functionals

Fliss, Leigh, Parrikar

ERG

acts as unitary flow on the boundary Hilbert space

Goldman, NL, Leigh 2301.09669

diff. regul. for ∂_t & $\vec{\nabla}$

$$\tilde{D}^2 = -K_0^{-1}\partial_t^2 + K_s^{-1}\vec{\nabla}^2 \qquad \omega = \sqrt{\frac{K_0}{K_s}}\,p$$

Magic

CMERA Zou, Ganahl, Vidal

under the RG two steps

$$\partial_s \langle \varphi | \Omega[J] \rangle = \langle \varphi | \left(-\mathrm{tr}_M \left(\tilde{\beta}[J] \frac{\delta}{\delta J} + i\hat{K} + i\hat{L} \right) | \Omega[J] \rangle \right.$$

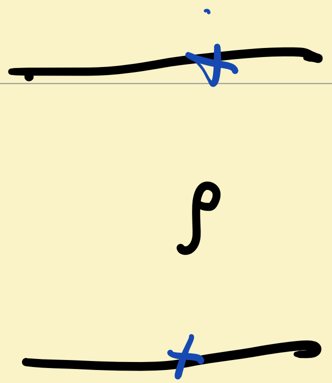
sources

$$\tilde{\beta}[J] = J \cdot \Lambda \partial_\Lambda (\vec{D}^{-2}) \cdot J$$

$$\hat{K} = \frac{1}{4} \int_{\vec{x}\,\vec{r}} g_2(\vec{r}) \left(\hat{\varphi}(\vec{x}) \hat{\pi}(\vec{x}+\vec{r}) + h.c \right)$$

$g_2 = \Lambda \partial_\Lambda \log K$

$$\hat{L} = \frac{d-2}{4} \int_{\vec{x}} \left(\hat{\varphi}(\vec{x}) \hat{\pi}(\vec{x}) + h.c \right)$$



$$\frac{d}{ds}\rho = i\,[\hat{K}+\hat{L},\rho] + D(\rho)$$

$$D(\rho) \sim \varphi\rho\varphi \quad \pi\rho\pi \quad \varphi\rho\pi$$
$$\pi\rho\varphi \; - \underbrace{\{\varphi^2,\rho\}}_{\text{normalization}} - \{\pi^2,\rho\}$$

$$a(\vec{p};\Lambda) = \sqrt{\frac{g(\vec{p};\Lambda)}{2}}\,\hat{\varphi}(\vec{p}) + i\sqrt{\frac{1}{2g(\vec{p};\Lambda)}}\,\hat{\pi}(\vec{p})$$

effective at scale Λ

$$\gamma_\pm = \coth\left(\frac{\beta\omega}{2}\right) \pm 1$$

$$\Delta_\Sigma(\vec{p}) = \Lambda \partial_\Lambda \log K(p^2/\Lambda^2)$$

$$\mathcal{D}_\beta(\rho) = \int_{\vec{p}} \Delta_\Sigma(\vec{p}) \Big(\gamma_+ \big(a\rho a^\dagger - \tfrac{1}{2}\{a^\dagger a, \rho\}\big) + \gamma_- \big(a^\dagger \rho a - \tfrac{1}{2}\{a a^\dagger, \rho\}\big) \Big)$$

emission   absorption

- Comments:

  γ_+, γ_- : quantum detailed balance

- Lindblad eq (scale-dep.)

- Markovian; memoriless

dissipative : any dist. measure is a monotone

Conclusion:

ERG flow of density matrices is given by Lindblad master eq.

this unifies varies approach to RG, connects to quantum Error Correction codes & makes the ERG monotones manifest.