Building the Trigonometric Tables

Exploration 1

The year is 2024 BCE. The world's mightiest king has commanded the world's greatest architect to build a monument that will last to eternity. The world's greatest sailors want to explore beyond the limits of known world, by navigating through vast open oceans. The world's greatest astronomers want to want to measure the size and distance of heavenly bodies. They have all come to you—the world's greatest mathematician of the era.

They all have a single request: **create the world's first trigonometric table**. How will you accomplish this challenge?

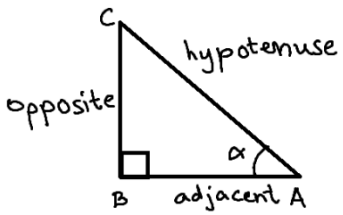
Rules:

1. You must build the trigonometric tables (sine table, cosine table, tangent table) from first principles. You cannot use math or machines that have not been invented yet.
2. The devices available to you are a ruler, compass, protractor. Since you are a great mathematician, you have assistants to help you with simple calculations.
   (In other words, you can use a calculator to perform the four basic arithmetic operations and extract square roots)
3. However, you do not have access to any oracle. (In other words, no Google, no software programs, no coding, no internet for help)
4. If you use a formula or method, you must derive it from first principles. **You must justify your results with logical reasoning!**

This sheet provides challenges at different levels. Choose the level which you feel is apt for you. There are many ways to solve the problems. Any method is good if you can justify it.

I am an absolute beginner

The basic trigonometric ratios of sine, cosine and tangent as defined as follows. ΔABC is a right triangle. For any given angle α, the values of sin α, cos α, and tan α are defined as shown in the figure.



$\sin \alpha = \frac{\text{opp}}{\text{hyp}}$

$\cos \alpha = \frac{\text{adj}}{\text{hyp}}$

$\tan \alpha = \frac{\text{opp}}{\text{adj}} = \text{slope}$

1. Find the sine, cosine, and tangent of the following angles

a) 45° b) 30° c) 60°

2. If you know the value of sin x, how will you find the values of cos x and tan x?
3. If you know the value of sin x, how will you find the value of sin (90 - x)?
4. For which other angles can you find the trigonometric ratios?

I am familiar with trigonometry

1. If you know the value of sin x, how will you find the value of

a) Sin $2x$ b) sin $(x/2)$ c) sin $3x$ d) sin $(x/3)$

2. If you know the value of sin x and sin y, how will you find the value of

a) Sin $(x + y)$ b) sin $(x - y)$

Similarly, find a way to get cosine and tangent values of new angles from known values. Use these results to build the trigonometric tables.

I am a smarty-pants

1. Find sin 18°, sin 36°, sin 54°, sin 72°.
2. Build the most accurate trigonometric table you can.
3. How will you find the sine of *any* angle?