Building the Trigonometric Tables

Exploration 2

We will continue our quest to build trigonometric tables from the scratch. Along the way, we will learn the methods of Aryabhata and Bhaskara.

This sheet builds on the problems from the previous sheet of Exploration 1. The problems from the previous sheet are retained for the benefit of newcomers. In case you haven't solved the challenges from last sheet, you can have another go at it.

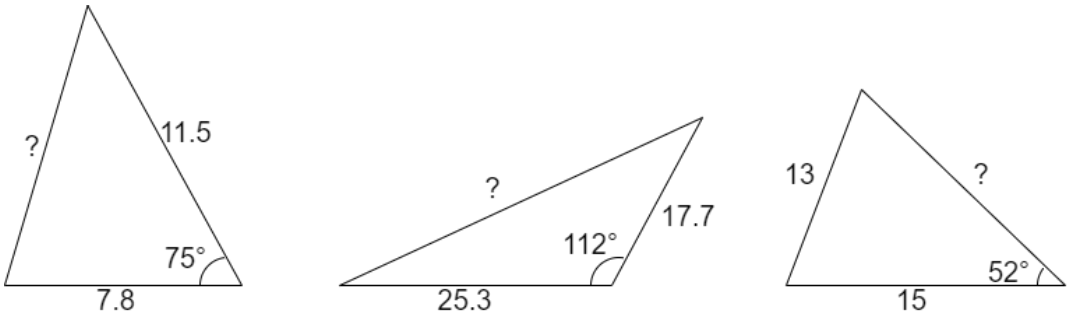
Rules:

1. You must build the trigonometric tables (sine table, cosine table, tangent table) from first principles.
2. The devices available to you are a ruler, compass, protractor. You can use a calculator, but only to perform the four basic arithmetic operations and extract square roots.
3. No Google, no software programs, no coding, no internet for help.
4. If you use a formula or method, you must derive it from first principles. **You must justify your results with logical reasoning!**

This sheet provides challenges at different levels. Choose the level which you feel is apt for you. There are many ways to solve the problems. Any method is good if you can justify it.

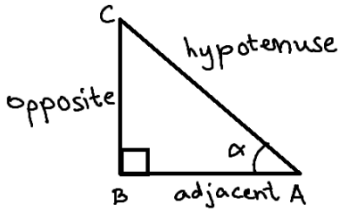
I am an absolute beginner

1. Consider a triangle with sides 29, 43, 56 units. Will it be an acute-angled, obtuse-angled, or right-angled triangle?
2. Find the unknown side in the triangles below.



In case you cannot find the exact length of the side, you can estimate a range.

The basic trigonometric ratios of sine, cosine and tangent as defined as follows. ΔABC is a right triangle. For any given angle α, the values of sin α, cos α, and tan α are defined as shown in the figure.



$$\sin\alpha = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\alpha = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\alpha = \frac{\text{opp}}{\text{adj}} = \text{slope}$$

1. Find the sine, cosine, and tangent of the following angles

a) 45° b) 30° c) 60°

2. If you know the value of sin x, how will you find the values of cos x and tan x?
3. If you know the value of sin x, how will you find the value of sin (90 - x)?
4. For which other angles can you find the trigonometric ratios?

I am familiar with trigonometry

1. If you know the value of sin x, how will you find the value of

a) Sin $2x$ b) sin $(x/2)$ c) sin $3x$ d) sin $(x/3)$

2. If you know the value of sin x and sin y, how will you find the value of

a) Sin $(x + y)$ b) sin $(x - y)$

Similarly, find a way to get cosine and tangent values of new angles from known values. Use these results to build the trigonometric tables.

3. Can you come up with an expression for the following in terms of sine or cosine of $(x + y)$ or $(x - y)$?

a) sin x + sin y b) sin x – sin y c) cos x + cos y d) cos x – cos y

4. Suppose you know that sin 3° = 0.0523, can you find

a) sin 48° b) sin 27°

5. How will you use the result from Q2 & Q3 to build a sine table in increments of 3°?

Angle	Sine	Cosine	Tangent
3°			
6°			
…			
90°			

I am a smarty-pants

1. Find sin 18°, sin 36°, sin 54°, sin 72°.
2. Build the trigonometric table with increments of 3.75°. This is the first known trigonometric table, computed by Aryabhata.

Angle	Sine	Cosine	Tangent
3.75°			
7.5°			
11.25°			
…			
90°			

3. Bhaskara subsequently came up with this formula for computing sines.

$$\sin x\ (in\ degrees) = \frac{4x\ (180° - x)}{40500 - x(180° - x)}.$$

a) Verify if this formula is correct.

b) Give a proof of correctness.