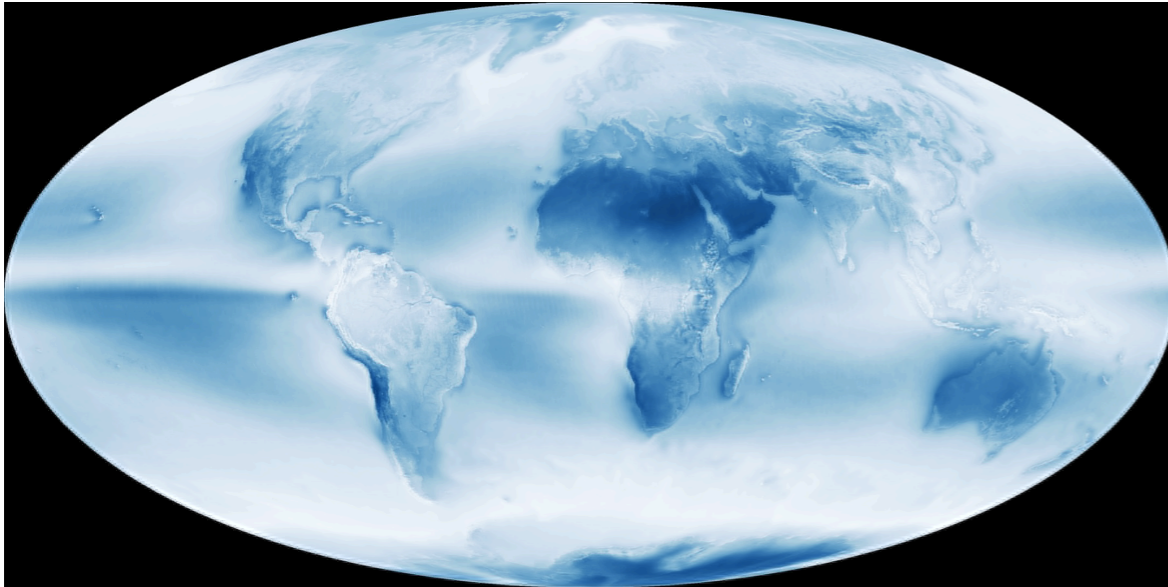
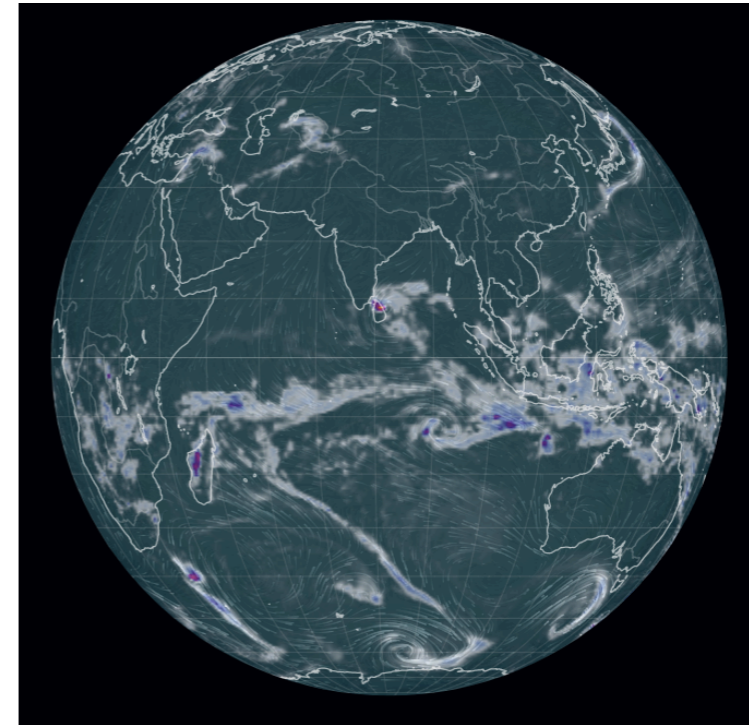


Most clouds don't rain



https://eoimages.gsfc.nasa.gov/images/imagerecords/85000/85843/globalcldfr_amo_200207-201504_lrg.jpg



https://earth.nullschool.net/#current/wind/surface/level/overlay=precip_3hr/orthographic

How turbulent should a cloud be?

Rama Govindarajan
ICTS

$Re = ?$



$\sim 1km$

$\sim 1km$

$1mm$



$$N \sim 10^9 m^{-3}$$

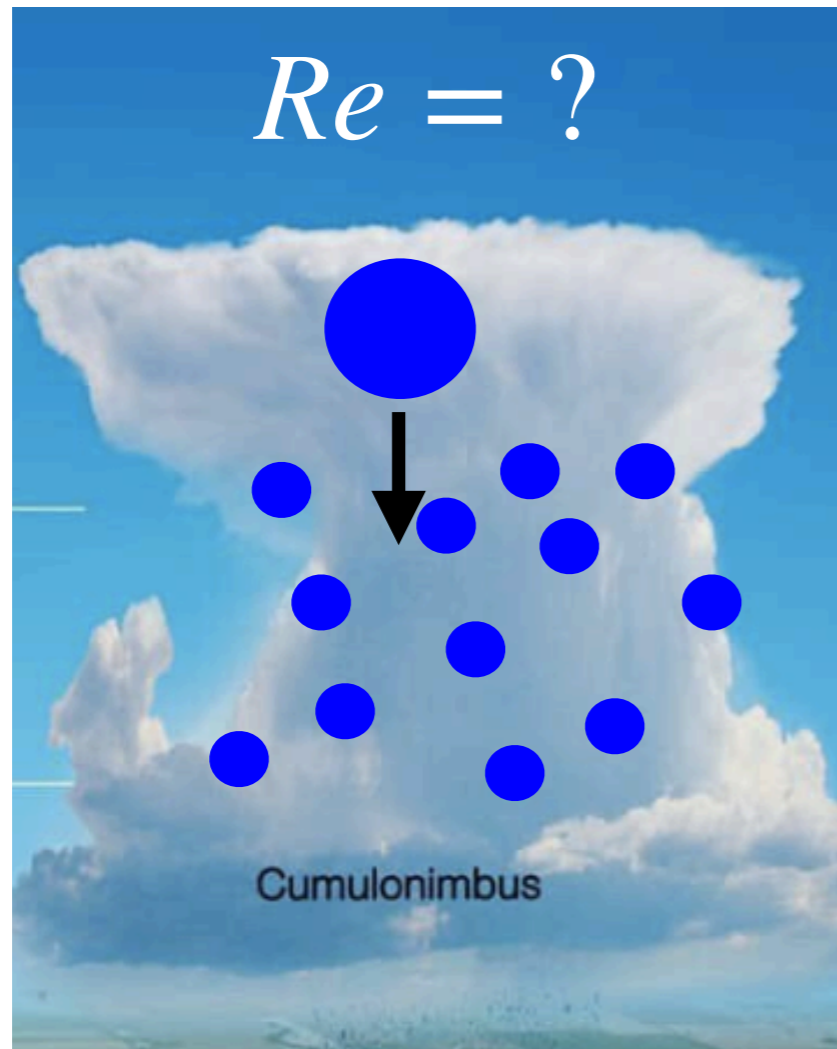
$$\text{Condensation} \implies a_0 \sim 10 \mu m$$

$$q_L \sim 4 \cdot 10^{-3} m^3/m^2$$

$$t_{evap} = \frac{\mathcal{C}}{2} \rho_w a^2 \quad t_{fall} = \frac{H}{v_p}$$

$$\chi \equiv \frac{t_{evap}}{t_{fall}} > 1 \implies a > 70 \mu m$$

Need $\sim 10^6$ $1 mm$ drops per unit area



$$\frac{1}{2}c_D\rho_a v_p^2 \pi a^2 = \frac{4}{3}\pi a^3 \rho_w g$$

$$\frac{d}{dt} \left[\frac{4}{3}\pi a^3 \right] = (v_p - v_{p0})(a + a_0)^2 N \epsilon \frac{4}{3}\pi^2 a_0^3$$

Some updraft helps

A 1mm drop starts out as a $100\mu m$ one

Rain initiation needs enough “lucky” drops in a cloud

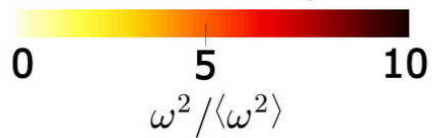
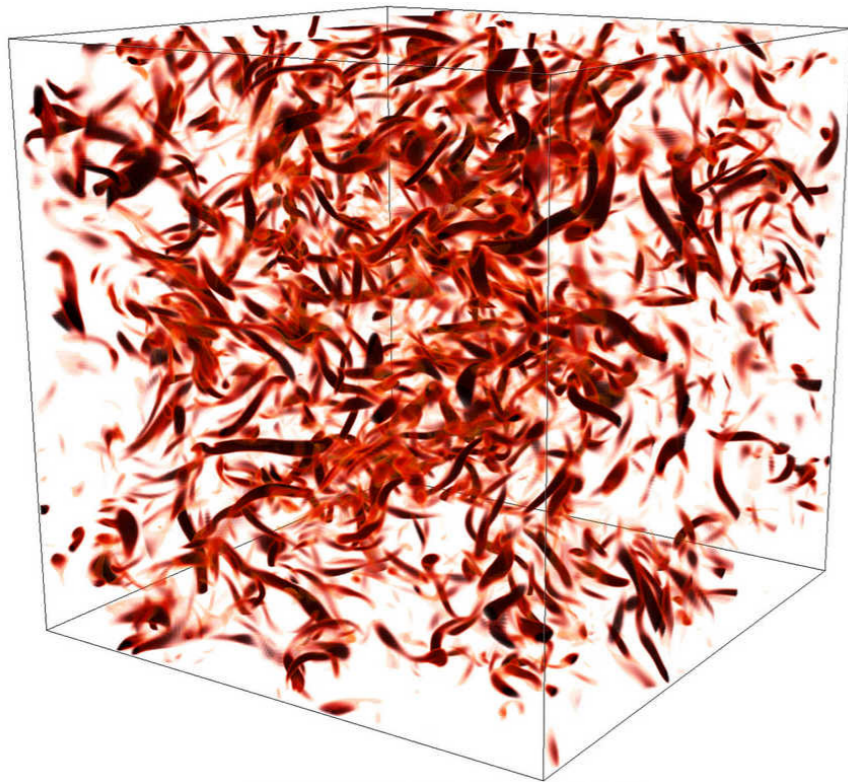
See e.g. Konstantin & Shaw, Bull. Am. Met. Soc., 2005

$10^6/1000m$ out of 10^{12}
 “Lucky” (one in a million) drop:

Now lets go to the tiniest

Lamb-Oseen vortices

Intense vorticity:
tubular structures



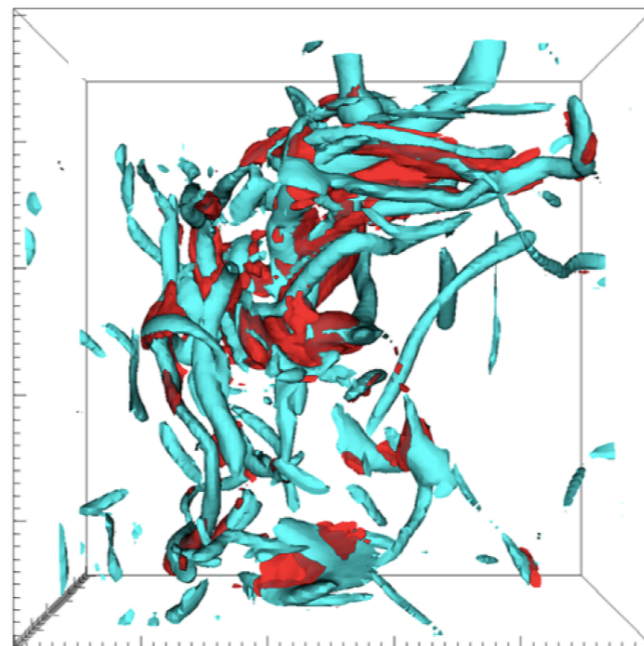
Simulation of Jason Picardo

Bhuharia & Pumir,
Phys. Rev. Lett. 2022

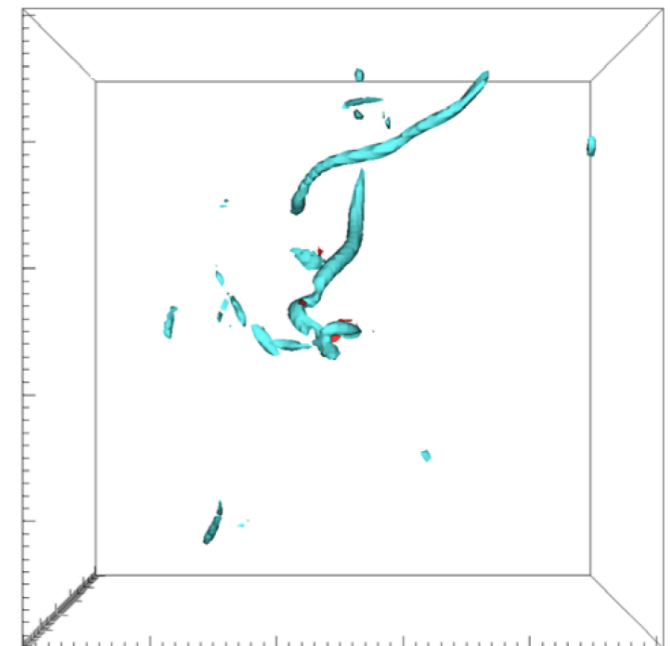
→ R_λ



$R_\lambda = 1300$



$\Omega \tau_K^2 = 500$



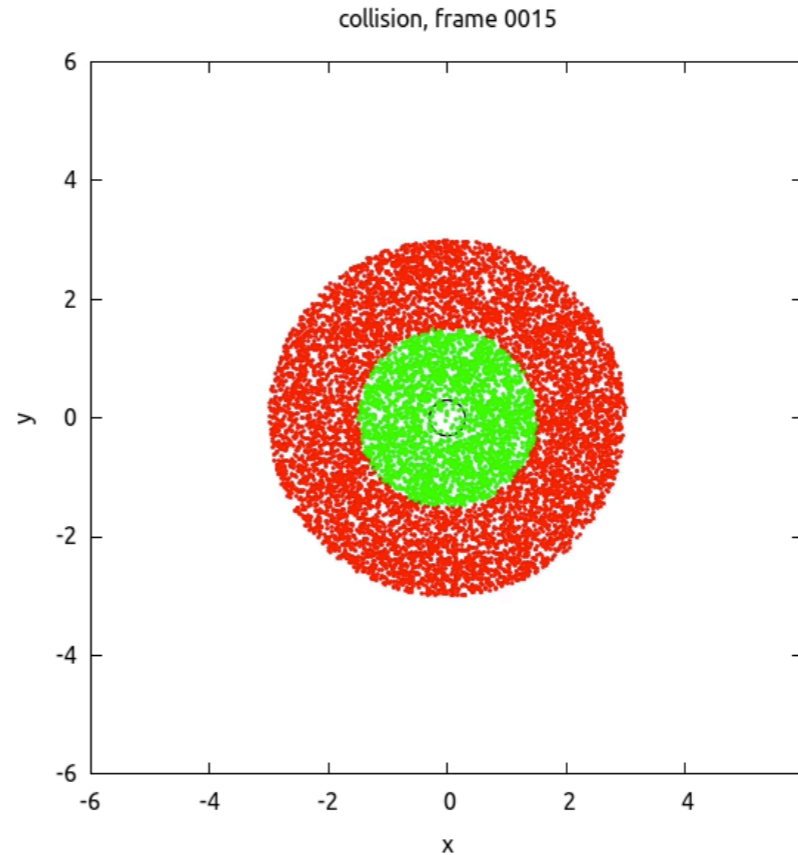
$\Omega \tau_K^2 = 4000$

Dynamics near a single vortex

Simplest form of Maxey-Riley equation

$$\dot{z} = v_p$$

$$\dot{v}_p = -\frac{1}{\tau}(v_p - u)$$



$$r_c \sim (\Gamma\tau)^{1/2}$$

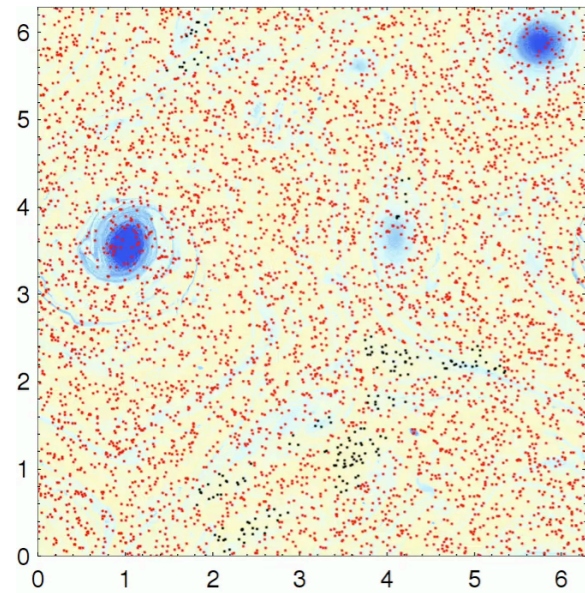
Deepu, Ravichandran & RG, Phys. Rev. Fluids 2017

$$U_v = \frac{\Gamma}{r_v} [1 - e]$$

$$\left[\frac{r_c}{r_v} \right]^2 = \frac{\tau}{T_v} = St$$

We need $St > 1$

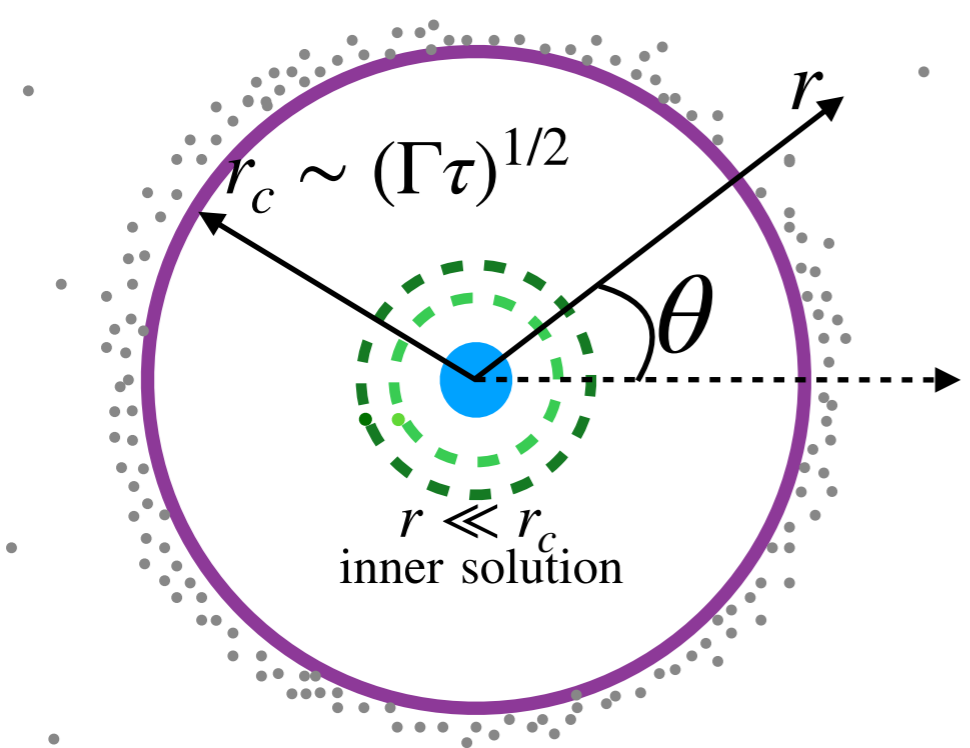
$f_{void} \propto$ number of lucky drops



Simulation of Jason Picardo

Standard arguments not working

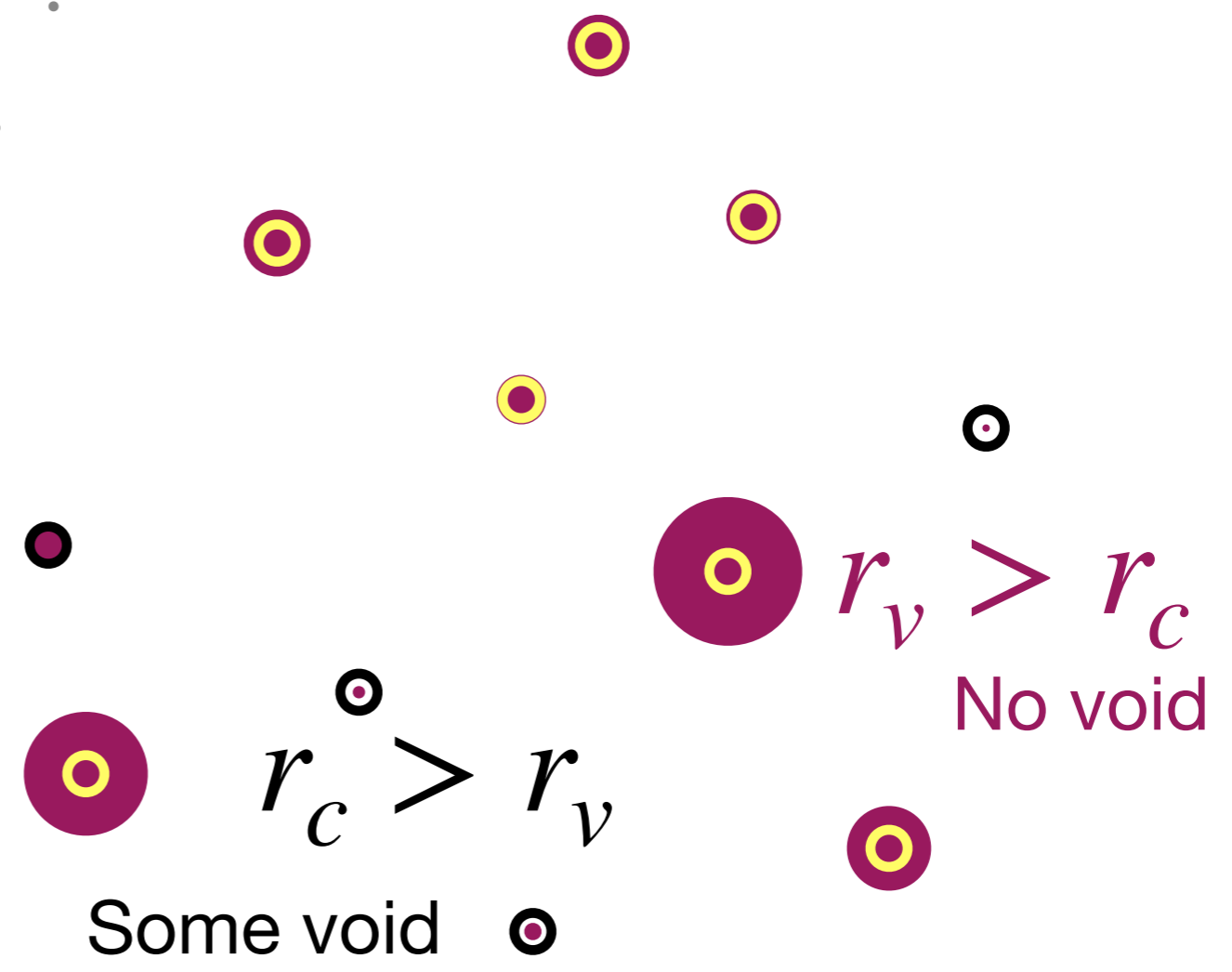
Calculation of void fraction in turbulence



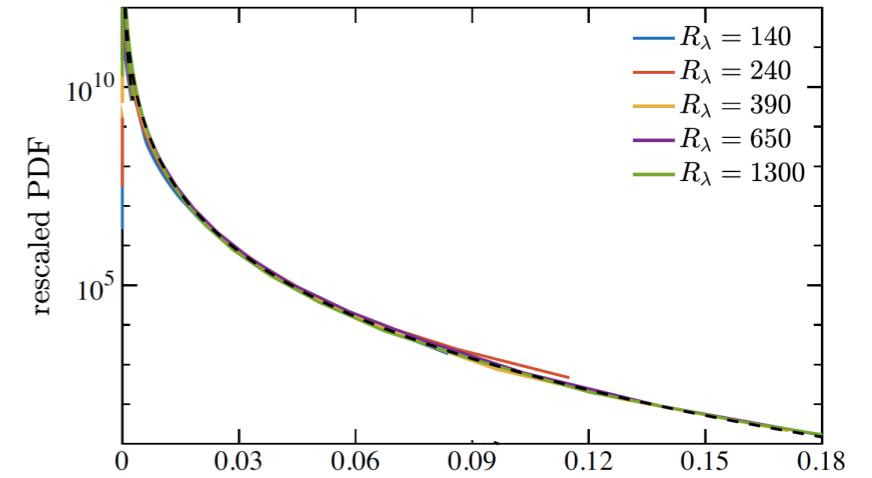
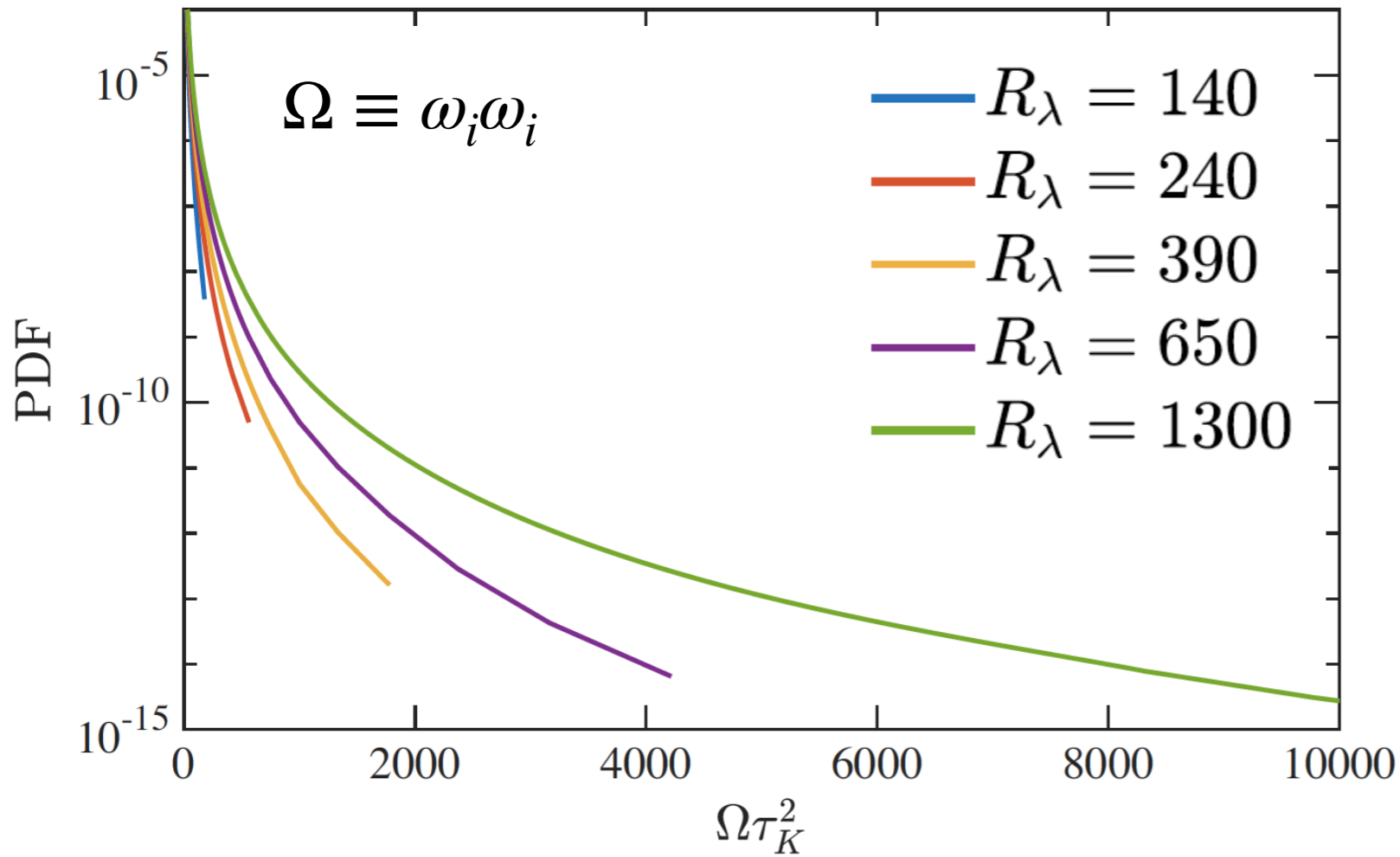
$$\Omega \equiv \omega_i \omega_i, \quad x \equiv \Omega t_K^2$$

$$\Omega_{min} = \frac{1}{\tau^2}$$

$$f_{void} = \int_{x_{min}}^{\infty} p(x) \pi r_c^2 dx$$



Need $f_{void} > 10^{-6}$



$$p(x) \sim \exp(-bx^c)$$

Bhuaria & Pumir, Phys. Rev. Lett. 2022

$$\frac{r_v}{\eta_K} \simeq (\Omega t_K^2)^{-\gamma/4}$$

R_λ	t_K	η_K	$x_{min} = \Omega_{min} t_K^2$	f_{void}
650	0.1s	10^{-3} m	2500	$\sim 10^{-10}$
1300	0.027s	5×10^{-4} m	200	$\sim 10^{-4}$
10^4	10^{-2} s	3×10^{-4} m	25	$O(1)$

Summary

Single vortex caustics could be important
A threshold Reynolds number for rain formation is indicated



*Ref: Ravichandran & Govindarajan,
Phys. Rev. Fluids 2022*