A Tale of Twisting Columns

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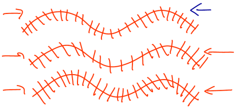
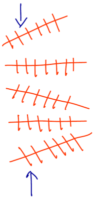
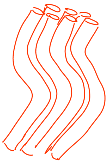
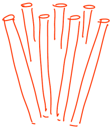
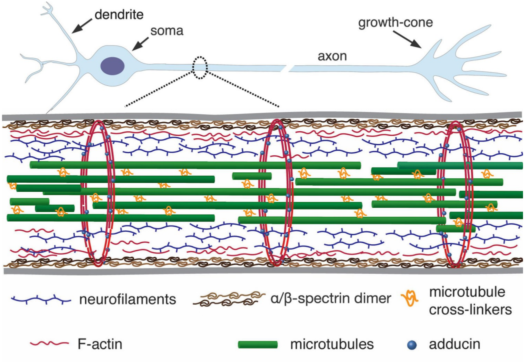
Active systems with translational order

- Active matter: collections of self-driven particles
- Particles carry intrinsic stresses – e.g. swimmers
- Orientationally ordered states: dramatic instabilities
- Translational elasticity resists active instability
- Liquid and solid directions: the best of both worlds

Active systems with translational order

- Motor-microtubule bundle as active chiral columnar phase?



Dubey et al., eLife 2020

Columnar phase

- 2D solid, fluid-like along column direction $\hat{z}$, displacement $\mathbf{u}_\perp$

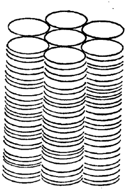


Figure: Chandrasekhar, Sadashiva & Suresh, Pramana 1977

- Statics at equilibrium: 2D Lamé elasticity + column bending

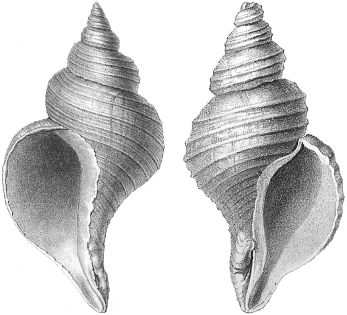
$$F = \int \left[\frac{\lambda}{2} (\mathrm{Tr}\mathbf{E})^2 + \mu \mathrm{Tr}(\mathbf{E}^2) + \frac{\kappa}{2} \nabla^2 u_k \nabla^2 u_k \right]$$

where the strain tensor **E** has components

$$E_{ij} = \frac{1}{2} \left(\partial_i u_j + \partial_j u_i - \partial_k u_i \partial_k u_j \right)$$

Chiral active matter

- Chirality: can't superpose on mirror image



- chirality + activity + broken trans inv: odd and odder elasticity

chiral active solid: Scheibner et al. NPhys 2020; active cholesteric: Kole et al. PRL 2021

Framework: active model H*

- Pseudoscalar ψ coupled to velocity field
- Columnar phase = two-dimensional modulation of ψ

$$\psi = \psi_0 + \sum_j \psi_j e^{i\mathbf{q}_j \cdot (\mathbf{x} - \mathbf{u}_\perp)}$$

$\mathbf{q}_j \in$ 2D reciprocal lattice; displacement field $\mathbf{u}_\perp$

- 3D chiral active stress $\zeta_c \boldsymbol{\nabla}\psi \times \boldsymbol{\nabla}\boldsymbol{\nabla}\psi$
- achiral active stress $\zeta \boldsymbol{\nabla}\psi \boldsymbol{\nabla}\psi$

Active hydrodynamics: *apolar* chiral columnar phase

- z and $-z$ equivalent
- Displacement field kinematics

$$\partial_t \mathbf{u}_\perp = \mathbf{v}_\perp$$

- Stokes hydrodynamics for velocity field $\mathbf{v}$

$$\eta \nabla^2 \mathbf{v} = \nabla P - (\lambda + \zeta)\nabla_\perp^2 \mathbf{u}_\perp - (\mu + \zeta)\nabla_\perp \nabla_\perp \cdot \mathbf{u}_\perp - \zeta \partial_z^2 \mathbf{u}_\perp$$
$$-\zeta \partial_z (\nabla_\perp \cdot \mathbf{u}_\perp)\hat{z} - \zeta_c \nabla^2 \partial_z \boldsymbol{\epsilon} \cdot \mathbf{u}_\perp + \zeta_c \nabla^2 \nabla_\perp \times \mathbf{u}_\perp$$

- Achiral activity: ζ breaks time reversal symmetry (TRS)
  Chiral activity: ζ_c breaks TRS & inversion symmetry

Spontaneous chiral-symmetry breaking

- Extensile active *achiral* stress $\rightarrow$ helical instability of columns
- cf. Helfrich-Hurault

Threshold activity $\zeta_{th} = B\frac{\pi\ell}{L}$ for $L \times L \times H$ system

$\ell = \sqrt{\kappa/B}$; 2D bulk modulus $B = \lambda + \mu$

- Selected wave vector of helical modulation:
  $u = u_0(\cos qz\mathbf{e_x} + \sin qz\mathbf{e_y})$

$$q^2 = \frac{\pi}{\ell L};$$ Handedness selected spontaneously

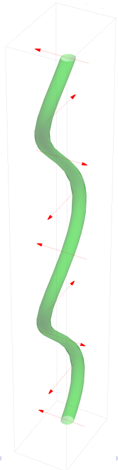
Column curvature sources flow

Spontaneous chiral flows

- Chiral active stress (ζ_c) contributes to

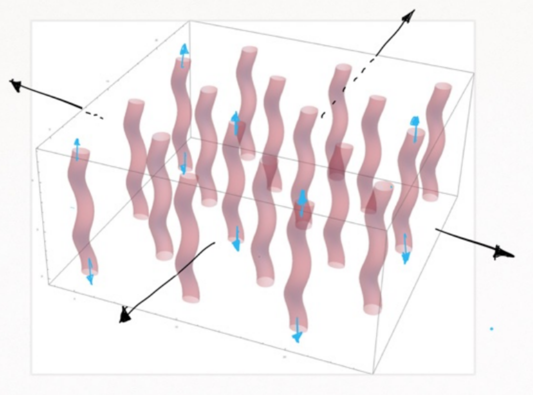
$$\eta \nabla^2 \mathbf{v} = \zeta_c \nabla^2 (-\partial_z \boldsymbol{\epsilon} \cdot \mathbf{u}_\perp + \nabla_\perp \times \mathbf{u}_\perp)$$

$z \to -z$ invariant: $\boldsymbol{\epsilon}$ and ∂_z both odd
Forcing perpendicular to column tilt
and along local rotation



Mapping: External imposed stress and active stress

- Extensile active stress along $\hat{\mathbf{z}} \leftrightarrow$ in-plane dilational external stress



- Correspondence $\rightarrow$ quantitative measurement and control of activity.

Active hydrodynamics: *polar* chiral columnar phases

- z and $-z$ inequivalent; natural 2D antisym tensor ϵ_{ijz}

$$\partial_t \mathbf{u}_\perp = \mathbf{v}_\perp$$

$$\eta\nabla^2\mathbf{v} = \nabla P - (\lambda + \zeta)\nabla_\perp^2\mathbf{u}_\perp - (\mu + \zeta)\nabla_\perp\nabla_\perp\cdot\mathbf{u}_\perp - \zeta\partial_z^2\mathbf{u}_\perp - \zeta\partial_z(\nabla_\perp\cdot\mathbf{u}_\perp)\hat{z} - \zeta_c\nabla^2\boldsymbol{\epsilon}\cdot\mathbf{u}_\perp + \zeta_c\partial_z\nabla_\perp\cdot\boldsymbol{\epsilon}\cdot\mathbf{u}_\perp\hat{z}$$

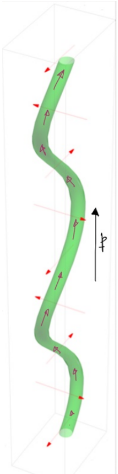
- Active chiral effects[1] realise 2D odd elasticity
- In-plane velocity field due to odd elastic stress (schematically):

$$\mathbf{v}_\perp = \zeta_c\boldsymbol{\epsilon}\cdot\mathbf{u}_\perp$$

[1] which enter at same order as equilibrium elasticity

Flows

- Archimedean screw: Helices post the HH instability → spinning about the ideal column axis → material transport along the polar axis.



Conclusion

- Surprisingly rich active hydrodynamics of columnar phases
- Spontaneous chiral-symmetry breaking, helical instability, odd flows
- The active-imposed connection
  - generic for active translationally-ordered systems
  - allows estimate and control of active stress
- Polar columns
  - projected 2D elasticity is odd
  - viscous Archimedean screw
- Future directions: flows, defects