Inflationary Gravitational Waves as a Probe of post-inflationary History of Universe
Hearing BSM via GWs @ ICTS (Bengaluru)

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Accessing New Physics from the Early Universe

→ **Early Universe is Nature's ultimate HEP laboratory!**

→ Fundamental Observables:

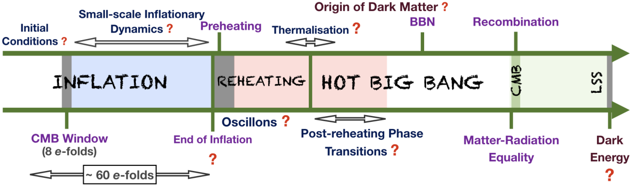
1. Primordial Relics: **PBHs**, Solitons (e.g. Oscillons), **Heavy Nuclei**
2. Primordial Signals: **Stochastic GWs**, **Photons (CMB)**, **Neutrinos**

$$\langle\, \hat{\zeta}(t,\vec{x})\, \hat{\zeta}(t,\vec{x})\, \hat{\zeta}(t,\vec{x}) \ldots \rangle_{\rm ini} \longrightarrow \langle\, \boldsymbol{\mathcal{O}}(t,\vec{x})\, \boldsymbol{\mathcal{O}}(t,\vec{x})\, \boldsymbol{\mathcal{O}}(t,\vec{x}) \ldots \rangle_{\rm late}$$

→ **Probing Beyond-Standard-Model Physics & Cosmology** between

A) **Hubble-exit of smallest CMB scales $\boldsymbol{k \geq k_{\rm CMB}}$** during inflation

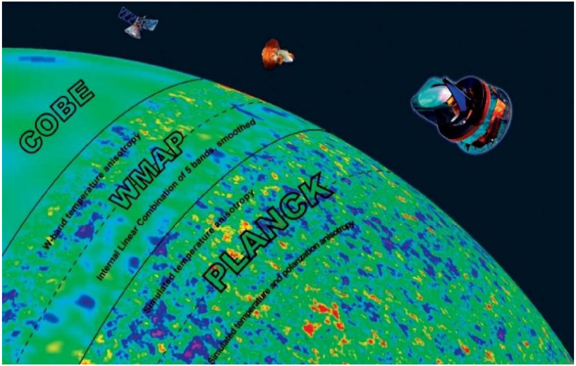
B) **Until the onset of Big Bang Nucleosynthesis (BBN)**



Best Probes of the Early Universe

Cosmic Microwave Background (CMB) Radiation

+ BBN + LSS + BAO



1. **What is the origin of the constituents of the plasma?**
2. **What is the origin of the fluctuations in the plasma?**

COSMIC INFLATIONARY SCENARIO

Single-field Slow-roll Inflation (Successful Framework!)

$$\textbf{System} = \textbf{Gravity}\ (g_{\mu\nu}) + \textbf{Scalar Field}\ (\phi)$$

$$S[g_{\mu\nu},\phi] = \int \mathrm{d}^4x\,\sqrt{-g}\,\left(\frac{m_p^2}{2}\,R - \frac{1}{2}\,\partial_\mu\phi\,\partial_\nu\phi\,g^{\mu\nu} - V(\phi) + ...\right)$$

$$\mathrm{d}s^2 \;=\; -\beta^2(t)\,\mathrm{d}t^2 \;+\; a^2(t)\,\left[\left(e^{-2\boldsymbol{\Psi}(t,\vec{x})}\,\delta_{ij} + 2\,\boldsymbol{h_{ij}}(t,\vec{x})\right)\,\mathrm{d}x^i\mathrm{d}x^j\right]$$

In particular, **two light fields** ($\boldsymbol{m \ll H}$) are guaranteed to exist –

1) Comoving Curvature Perturbations: $-\boldsymbol{\zeta}(t,\vec{x}) = \boldsymbol{\Psi} + \frac{1}{\sqrt{2\epsilon_H}}\frac{\boldsymbol{\delta\phi}}{m_p}$

2) Transverse & traceless Tensor Perturbations: $\boldsymbol{h_{ij}}(t,\vec{x})$

Slow-roll regime: $\boldsymbol{\epsilon_H},\ |\boldsymbol{\eta_H}| \ll \mathbf{1}$ with

$$\boldsymbol{\epsilon_H = -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{3m_p^2H^2}}\,;\quad \boldsymbol{\eta_H = \epsilon_H - \frac{1}{2}\frac{\mathrm{d}\ln\epsilon_H}{\mathrm{d}N} = -\frac{\ddot{\phi}}{H\dot{\phi}}}$$

Power-spectra: Linear Perturbation Theory

Slow-roll Primordial power-spectrum on **large scales** –

(CMB pivot scale $\boldsymbol{k_* = 0.05\ \mathrm{Mpc}^{-1}}$)

Scalar power spectrum

$$\mathcal{P}_\zeta(k) = \frac{1}{8\pi^2}\left(\frac{H}{m_p}\right)^2 \frac{1}{\epsilon_H} = A_S\left(\frac{k}{k_*}\right)^{n_S-1}$$

Tensor power spectrum

$$\mathcal{P}_\mathcal{T}(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2 = A_\mathcal{T}\left(\frac{k}{k_*}\right)^{n_\mathcal{T}}$$

Scalar spectral index

$$\boxed{n_S - 1 = 2\eta_H - 4\,\epsilon_H} \ll 1$$

Tensor spectral index

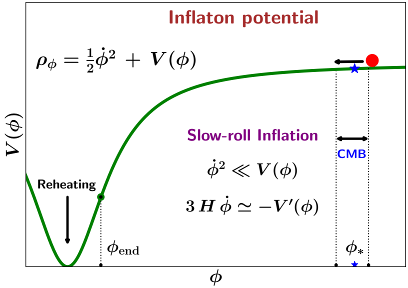
$$\boxed{n_\mathcal{T} = -2\,\epsilon_H} \ll 1$$

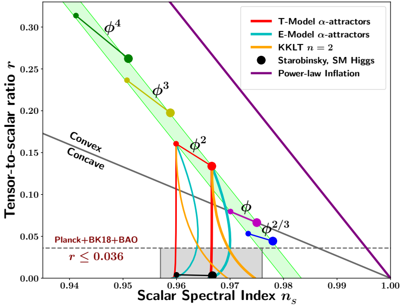
Tensor-to-scalar ratio: $$\boxed{r = \frac{A_\mathcal{T}}{A_S} = 16\,\epsilon_H} \ll 1$$

$\Rightarrow$ **Tiny fluctuations** that are **nearly scale-invariant**

Observational Constraints

$$A_s = 2.1 \times 10^{-9}\ ; \quad n_s - 1 \in [-0.043, -0.024]\ ; \quad r \leq 0.036$$





Latest CMB Data [**BICEP/Keck + Planck**] $\Rightarrow$ On CMB scales:

Single-field slow-roll paradigm of Inflation &

Asymptotically-flat concave potentials!

**Planck(2018); **BICEP/Keck(2021); **SSM & Sahni(2022), **Bhatt, SSM *et.al.*(2022)

Cosmic Inflation: Targets for 2025-2045

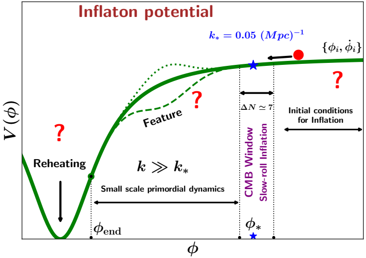
Theoretically

Origin of Inflaton field?

Single or Multi-field?

Primordial Interactions?

Cosmological Collider Signals?



Phenomenologically

Primordial GWs? ✓

Inflaton Decay & Reheating?

Small-scale Inflationary Dynamics?

Primordial Non-Gaussianity?

Observationally

B-Mode Polarization

GW Observatories ✓

PNG from CMB, LSS

PNG from 21 cm

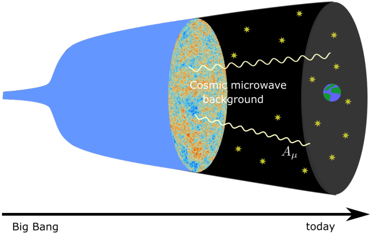
GWs: Messengers from the Early Universe

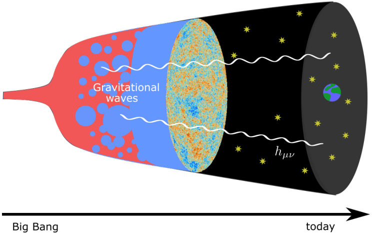
→ Probing **Energy Scale of Inflation** via **CMB B-Mode Missions**

→ Early Universe is expected to host **a number of distinct epochs**

→ Probing these unknown epochs *via* **Gravitational Waves**

→ Physics is encoded in the **amplitude** & **tilt** of the GW spectrum





EM Waves from Recombination **GWs from very early Universe**

Tensor Vacuum Fluctuations during Inflation

Tensor Action:

$$S^{(2)}[h_{ij}] = \frac{1}{2}\int \mathrm{d}\tau\, \mathrm{d}^3\vec{x} \left(\frac{a m_p}{2}\right)^2 \left[\left(h'_{ij}\right)^2 - (\vec{\nabla} h_{ij})^2\right]$$

Polarization $h_{ij} = \frac{1}{\sqrt{2}} \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} = \epsilon^+_{ij}\, h_+ + \epsilon^\times_{ij}\, h_\times$

$$\Rightarrow S^{(2)}[h_+,\, h_\times] = \frac{1}{2}\int \mathrm{d}\tau\ \mathrm{d}^3\vec{x} \left(\frac{a m_p}{2}\right)^2 \sum_{\lambda=+,\times} \left[(h_\lambda{}')^2 - (\partial_l h_\lambda)^2\right]$$

In terms of canonical **Starobinsky variable** $v_\lambda = \left(\frac{a m_p}{2}\right) h_\lambda$

Tensor Action becomes

$$S^{(2)}[v_+,\, v_\times] = \frac{1}{2}\int \mathrm{d}\tau\, \mathrm{d}^3\vec{x} \sum_{\lambda=+,\times} \left[(v_\lambda{}')^2 - (\partial_i v_\lambda)^2 + \frac{a''}{a} v_\lambda{}^2\right]$$

****Starobinsky (1979) Pioneering work on dS GWs**

Tensor Power Spectrum at the end of Inflation

$$S^{(2)}[\boldsymbol{v_+},\, \boldsymbol{v_\times}] = \frac{1}{2}\int \mathrm{d}\tau\, \mathrm{d}^3\vec{x} \sum_{\lambda=+,\times} \left[(\boldsymbol{v_\lambda}')^2 - (\boldsymbol{\partial_i v_\lambda})^2 - \left(-\frac{\boldsymbol{a''}}{\boldsymbol{a}}\right) \boldsymbol{v_\lambda}^2 \right]$$

$\equiv$ **2 decoupled massive scalar fields in Minkowski spacetime**

With a **tachyonic** $\boldsymbol{m_{\text{eff}}^2 = -\frac{a''}{a} \simeq -2(aH)^2}$

Power spectrum definition

$$\boldsymbol{\mathcal{P}_{\mathcal{T}}(k) = \frac{k^3}{2\pi^2}\left[|h_+|^2 + |h_\times|^2\right] = \frac{k^3}{2\pi^2}\left(\frac{2}{am_p}\right)^2 \left[|v_+|^2 + |v_\times|^2\right]}$$

which **at the end of inflation on super-Hubble scales** $\boldsymbol{k \ll aH}$

$$\boldsymbol{\mathcal{P}_{\mathcal{T}}(k)\Big|_{k\ll aH} = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2} \simeq \boldsymbol{A_{\mathcal{T}}\left(\frac{k}{k_*}\right)^{n_{\mathcal{T}}}} \; ; \quad \boldsymbol{n_{\mathcal{T}} = -2\epsilon_H = -\frac{r}{8}}$$

→ **Remains frozen/constant on outside Hubble radius**

→ **Initial conditions for tensor propagation post inflation**

Post-inflationary Evolution of Tensor Modes

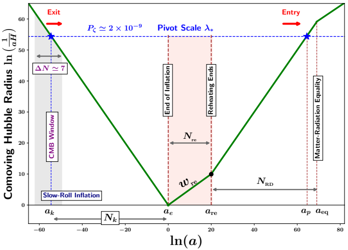
Inflationary Output $\longrightarrow$ **Reheating** $\longrightarrow$ **Hot Big Bang Input**

Inflationary Tensor Modes $\longrightarrow$ **Primordial GWs**

Inflationary Scalar Modes $\longrightarrow$ **CMB Acoustic Waves** $\longrightarrow$ **LSS**

Tensor mode functions satisfy

$$h_k''^{\lambda} + 2\left(\frac{a'}{a}\right) h_k'^{\lambda} + k^2\, h_k^{\lambda} = 0$$



For **post-inflationary epoch** with **EoS** w

$$\frac{a'}{a} = a_i H_i \left[1 + \frac{a_i H_i\,(\tau - \tau_i)\,(1+3w)}{2}\right]^{-1}$$

General solution

$$h_k^{\lambda}(y) = \frac{1}{(\alpha\, y)^{\alpha - \frac{1}{2}}}\left[A_k\, J_{(\alpha-\frac{1}{2})}(\alpha\, y) + B_k\, J_{-(\alpha-\frac{1}{2})}(\alpha\, y)\right] \qquad \alpha = \frac{2}{(1+3w)}$$

**SSM Lecture Notes [arXiv:2403.10606]

Unknown Coeffs. $\{A_k,\, B_k\}$; $\quad y = \frac{k}{aH}$

Spectral Energy Density of Primordial GWs

Definition: $$\boldsymbol{\Omega_{\rm GW}(\tau,k) = \frac{1}{\rho_c}\frac{{\rm d}\rho_{\rm GW}}{{\rm d}\ln k}}$$

$$\boldsymbol{\rho_{\rm GW}}(\tau) \equiv \langle 0\,|\hat{\rho}_{\rm GW}(\tau,\vec{x})|\,0\rangle = \frac{m_p^2}{8a^2(\tau)}\int \mathbf{d}\ln \boldsymbol{k}\;\boldsymbol{\frac{k^3}{\pi^2}\left[\left|h_k'^{\lambda}(\tau)\right|^2 + k^2\left|h_k^{\lambda}(\tau)\right|^2\right]}$$

$$\hat{\rho}_{\rm GW}(\tau,\vec{x}) = -\hat{T}^0_{\ 0}(\tau,\vec{x}) = \frac{m_p^2}{8a^2(\tau)}\left[(\hat{h}'_{ij}(\tau,\vec{x}))^2 + (\vec{\nabla}\hat{h}_{ij}(\tau,\vec{x}))^2\right]$$

$$\hat{h}_{ij}(\tau,\vec{x}) = \sum_{\lambda=+,\times}\int\frac{{\rm d}^3\vec{k}}{(2\pi)^3}\,\epsilon^{\lambda}_{ij}\,\left[h^{\lambda}_k(\tau)\,\hat{a}_{\vec{k}}\,e^{i\vec{k}\cdot\vec{x}} + \left(h^{\lambda}_k(\tau)\right)^*\,\hat{a}^{\dagger}_{\vec{k}}\,e^{-i\vec{k}\cdot\vec{x}}\right]$$

GW Spectral energy density (for $\boldsymbol{k \gg aH}$)

$$\boldsymbol{\Omega_{\rm GW}(\tau,k) = \frac{k^2}{12\,a^2(\tau)H^2(\tau)}\,\mathcal{P}_T(\tau,k)}$$

With $\boldsymbol{\mathcal{P}_T(\tau,k) = \frac{k^3}{2\pi^2}\left(|h_k^{+}(\tau)|^2 + |h_k^{\times}(\tau)|^2\right)}$

Inflationary GW Spectral Energy Density

RD Epoch $$\Omega^{\rm RD}_{\rm GW}(f) \simeq \left(\frac{r\,A_S}{24}\right)\,\Omega_{0r}\left(\frac{f}{f_*}\right)^{n_{\mathcal{T}}}$$ $f_{\rm eq} < f < f_{\rm re}$

Reheating $$\Omega^{\rm re}_{\rm GW}(f) \simeq \Omega^{\rm RD}_{\rm GW}(f)\left(\frac{f}{f_{\rm re}}\right)^{2\left(\frac{w_{\rm re}-1/3}{w_{\rm re}+1/3}\right)}$$ $f_{\rm re} < f < f_{\rm end}$

Primordial (inflationary) tensor tilt

$$0 < |n_{\mathcal{T}}| \leq 0.005$$

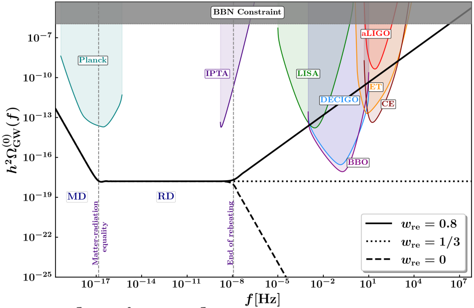
GW Spectral Tilt

$$n_{\rm GW}(w) = n_{\mathcal{T}} + 2\left(\frac{w_{\rm re} - 1/3}{w_{\rm re} + 1/3}\right)$$

→ **Red-tilted** for $w_{\rm re} < \frac{1}{3}$

→ **Flat-spectrum** for $w_{\rm re} = \frac{1}{3}$

→ **Blue-tilted** for $w_{\rm re} > \frac{1}{3}$ ⇒ **Stiff-matter dominated**



Range of $\{w_{\rm re}, N_{\rm re}\}$ leading to detectable GWs in aLIGO, LISA

Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

Ability of LIGO and LISA to probe the equation of state of the early Universe

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Observationally Favorable: $\{$ **High** r, **Stiff(er)** $w_{\rm re}$, **Low** $T_{\rm re}$ $\}$

General Reheating Dynamics

1. **Non-perturbative inflaton decay:** **(Early stages)**

   e. g. In $\mathcal{L}_{\rm int} \supset -\frac{1}{2}g^2\varphi^2\chi^2$, for **Broad Band resonance**,

$$q = \frac{g^2}{4}\left(\frac{\phi_0}{m}\right)^2 \geq 1 \;\Rightarrow\; g^2 \geq 4\left(\frac{m}{m_p}\right)\left(\frac{m_p}{\phi_0}\right)$$

   For $m \simeq 10^{-5}\, m_p$ and $\phi_0 \simeq 0.2\, m_p$, we get $\boxed{g^2 \geq 10^{-8}}$

2. **Perturbative inflaton decay:** **(certainly at Late times)**

$$\varphi\varphi \longrightarrow \chi\chi \;\; \text{for} \;\; g^2 < 10^{-8}\,; \quad \varphi \longrightarrow \bar{\psi}\psi \;\; \text{for} \;\; h \lesssim 10^{-2}$$

3. **Coherent Oscillations:**

   For $h,\, g \sim 0$, the inflaton condensate oscillates for a long time

$$\boxed{\phi(t) = \phi_0(t)\, \cos(mt)\,; \quad \langle w_\phi \rangle \simeq 0 \;\Rightarrow\; \rho_\phi \propto a^{-3}}$$

$\Rightarrow$ universe remains in a **condensate-dominated phase** ×

Oscillons & Transients formation

GWs as a Probe of Unknown Reheating History

→ **Reheating Dynamics is Complex!**

→ **Early Universe might exhibit Phase Transitions**

→ **Likely involving multiple epochs with $\{w_1, w_2, ..., w_n\}$**

Inflationary Gravitational Waves as a probe of the unknown post-inflationary primordial Universe

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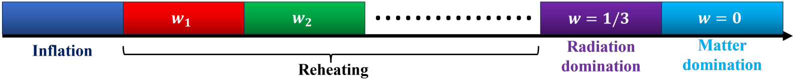
****Soman, SSM *et. al.* [arXiv:2407.07956] (soon to appear on PRD)**

Multiple EoS Parameters during Reheating

Multiple sharp transitions in EoS during reheating

Model Agnostic Approach



Assumptions

- Instantaneous Transitions in EoS

$$w_{\rm re} = w_1 + (w_2 - w_1)\,\Theta\,(\tau - \tau_1) + (w_3 - w_2)\,\Theta\,(\tau - \tau_2) + \ldots$$

$\Rightarrow$ Mode functions $h_k^\lambda(\tau)$ via Israel junction matching

- Large scale inflationary GWs to be detectable *via* CMB B-mode in the upcoming decade, *i.e.* $r \geq 0.001$

Q. Parameter space leading to GW detection via GW observatories

**Soman, SSM, Shafi & Basak [arXiv:2407.07956] (2024)

Determining Coefficients $\{A_k, B_k\}$ in $h_k^\lambda(\tau)$

- Inflationary output as initial conditions for tensor modes $\Rightarrow$ For any post-inflationary epoch **at Hubble-entry** $\boldsymbol{\tau < \tau_k}$

$$\boxed{h_k^\lambda(\tau_k) = h_{k,\,\mathrm{inf}}^\lambda \; ; \quad h_k'^\lambda(\tau_k) = 0}$$

- Apply **Israel Junction matching** conditions at transition

$$h_{k,\,\mathrm{Before}}^\lambda\left(\tau_1^-\right) = h_{k,\,\mathrm{After}}^\lambda\left(\tau_1^+\right) \qquad \text{(Continuity)}$$

$$h_{k,\,\mathrm{Before}}'^\lambda\left(\tau_1^-\right) = h_{k,\,\mathrm{After}}'^\lambda\left(\tau_1^+\right) \qquad \text{(Differentiability)}$$

Standard Cosmological transitions:

Inflation $\longrightarrow$ **Reheating** $\longrightarrow$ **RD** $\longrightarrow$ **MD**

$\Rightarrow \; w = -1 \;\longrightarrow\; w = w_{\mathrm{re}} \;\longrightarrow\; w = 1/3 \;\longrightarrow\; w = 0$

**Sahni (1990), **Giovanni(1990s), **Figueroa & Tanin (2019)

Solved Expressions of Coefficients $\{A_k, B_k\}$

$$\boldsymbol{A_{k,m+1}} = \frac{(\alpha_{m+1} y_m)^{\left(\alpha_{m+1}-\frac{1}{2}\right)}}{(\alpha_m y_m)^{\left(\alpha_m-\frac{1}{2}\right)}} \left[\frac{(g_2 f_3 + g_4 f_1)\,\boldsymbol{A_{k,m}} + (f_2 f_3 - f_4 f_1)\,\boldsymbol{B_{k,m}}}{f_1 g_3 + g_1 f_3}\right]$$

$$\boldsymbol{B_{k,m+1}} = \frac{(\alpha_{m+1} y_m)^{\left(\alpha_{m+1}-\frac{1}{2}\right)}}{(\alpha_m y_m)^{\left(\alpha_m-\frac{1}{2}\right)}} \left[\frac{(g_2 g_3 - g_4 g_1)\,\boldsymbol{A_{k,m}} + (f_2 g_3 + f_4 g_1)\,\boldsymbol{B_{k,m}}}{f_1 g_3 + g_1 f_3}\right]$$

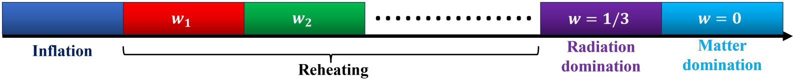
With
$$g_1 = J_{\left(\alpha_{m+1}-\frac{1}{2}\right)}(\alpha_{m+1}\, y_m)\,, \quad f_1 = J_{-\left(\alpha_{m+1}-\frac{1}{2}\right)}(\alpha_{m+1}\, y_m)$$
$$g_2 = J_{\left(\alpha_m-\frac{1}{2}\right)}(\alpha_m\, y_m)\,, f_2 = J_{-\left(\alpha_m-\frac{1}{2}\right)}(\alpha_m\, y_m)$$
$$g_3 = J_{\left(\alpha_{m+1}+\frac{1}{2}\right)}(\alpha_{m+1}\, y_m)\,, \quad f_3 = J_{-\left(\alpha_{m+1}+\frac{1}{2}\right)}(\alpha_{m+1}\, y_m)$$
$$g_4 = J_{\left(\alpha_m+\frac{1}{2}\right)}(\alpha_m\, y_m)\,, \quad f_4 = J_{-\left(\alpha_m+\frac{1}{2}\right)}(\alpha_m\, y_m)$$

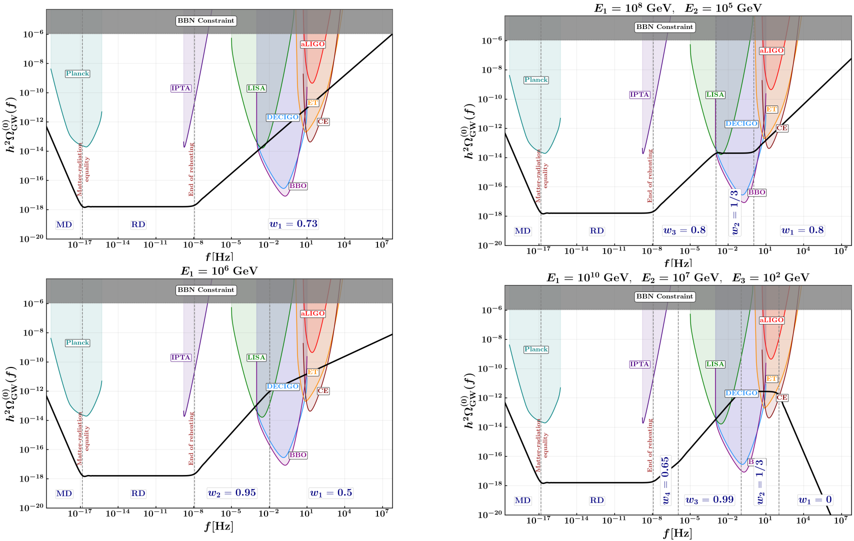
$$\boldsymbol{h^{\lambda}_{k,n}}(y) = \frac{1}{(\alpha_n\, y)^{\alpha_n-\frac{1}{2}}} \left[\boldsymbol{A_{k,n}\, J_{\left(\alpha_n-\frac{1}{2}\right)}}(\alpha_n\, y) + \boldsymbol{B_{k,n}\, J_{-\left(\alpha_n-\frac{1}{2}\right)}}(\alpha_n\, y)\right]$$

Compute Ω_{GW}

**Soman, SSM, Shafi & Basak (2024)

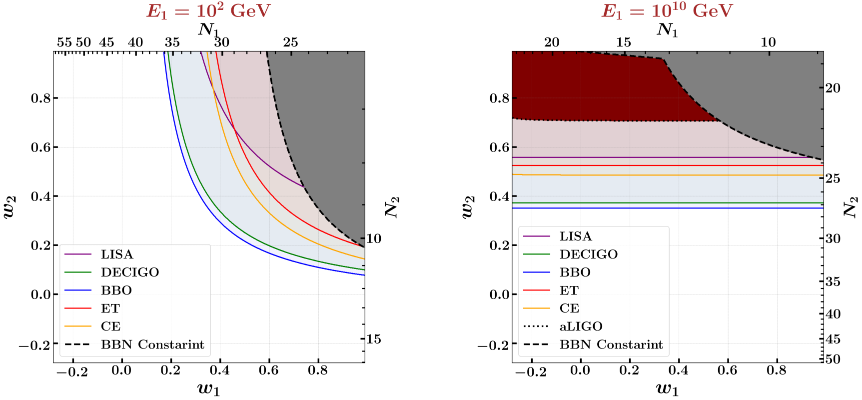
Multiple EoS Parameters during Reheating





Parameter space for a single transition $w_1 \to w_2$

Low reheating temperature $E_{\rm re} = 10\,\mathrm{MeV}$



→ **Detectable parameter space**: Colour–Shaded regions

→ **BBN Constraints**: Dark-Grey regions

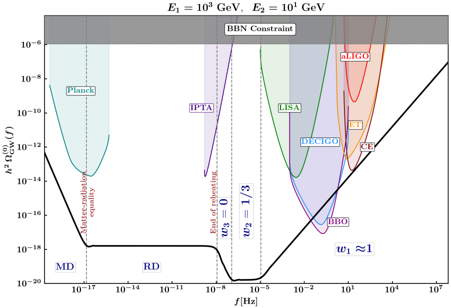
→ **LIGO Constraints**: Deep-Maroon regions

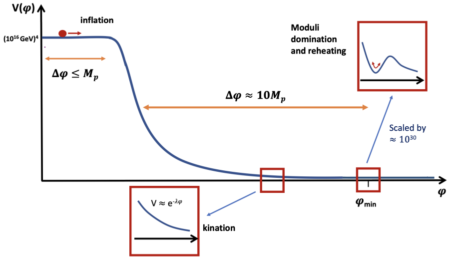
****Details in GitHub Repository**

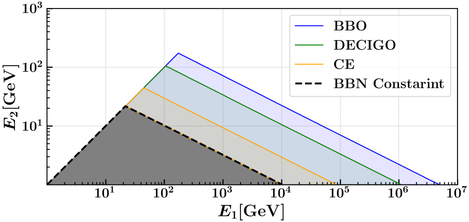
Application to a String Theory inspired Scenario

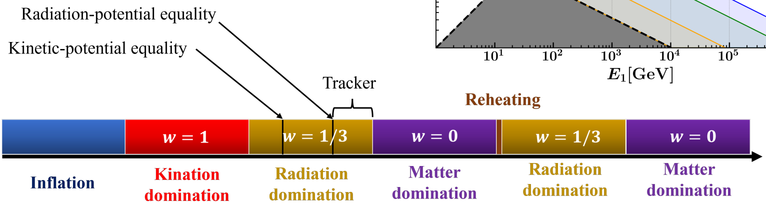
**Apers, Conlon, Copeland *et.al.* [2401.04064]

String Theory & The First-half of the Universe



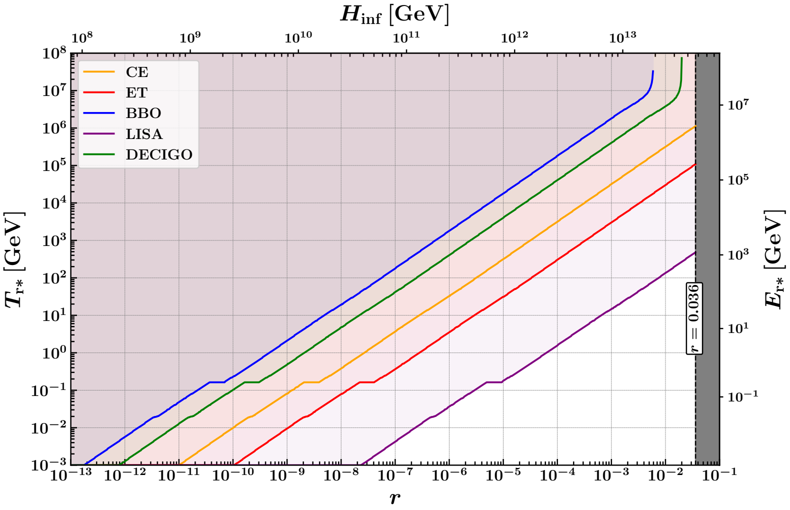






**Soman, SSM, Shafi & Basak [2407.07956]

Effect of low tensor-to-scalar ratio



Only the non-shaded region will be detectable!

Upcoming & future work on Inflationary GWs

- **Application to concrete phenomenological scenarios**
- **Smooth (instead of instantaneous) transitions** of EoS parameters
- **Scalar-induced (2^{nd}-order) GWs for multiple EoS**
- Breaking degeneracies between various stochastic GW signals (future goal)

GitHub Link

github.com/athul104/Spectral_Energy_Density_FO_GWs

Extra Slides

What happened to other fields during inflation?

- Observations favour '**single-field slow-roll**' inflation.
- '**Cold inflationary paradigm**:' $\Rightarrow\ \mathcal{L}_\chi\,,\ \mathcal{L}_\psi \ll \mathcal{L}_\varphi$

  and Negligible coupling to external fields $\boldsymbol{g^2,\, h\ \ll\ 1}$

$$S[\varphi,\chi,\psi] = -\int \mathrm{d}^4x\sqrt{-g}\ \Big[\ \frac{1}{2}\,\partial_\mu\varphi\,\partial^\mu\varphi + V(\varphi) + \cancelto{0}{\frac{1}{2}\partial_\mu\chi\partial^\mu\chi} + \cancelto{0}{\frac{1}{2}m_{0\chi}^2\chi^2}$$

$$+\ \cancelto{0}{\bar{\psi}\,(i\gamma^\mu\partial_\mu + m_{0\psi})\,\psi}$$

$$+\ \frac{1}{2}\cancelto{0}{\boldsymbol{g^2\,\varphi^2\,\chi^2}} + \cancelto{0}{\boldsymbol{h\,\psi\bar{\psi}\varphi}} + \ldots\Big]$$

$\Rightarrow$ **particle production during inflation can be neglected**.

- **Effects of the small coupling?**
  1. **Primordial Non-Gaussianity**: inflaton interactions.
  2. Decay of the inflaton field: **Reheating the universe**.

Universe Reheating after Inflation

Reheating may proceed in two distinct regimes –

1. **Perturbative Decay:**
   - Relevant primarily for **fermionic decay** $\varphi \longrightarrow \bar{\psi}\psi$
   - When bosonic couplings are extremely weak $g^2 \ll 10^{-8}$
   - **Slow** and **in-efficient**
   - Does not incorporate the presence of **coherently oscillating inflaton condensate** background

2. **Non-perturbative Decay:**
   - When bosonic couplings are high enough $g^2 \gtrsim 10^{-8}$
   - Particle production in presence of oscillating inflaton field $\Rightarrow$ Parametric resonance **- Collective phenomenon**
   - Fast, explosive, efficient, **highly non-thermal**

Reheating *via* Non-perturbative decay

→ Reheating dynamics is complicated, non-perturbative physics

→ **Particle production in presence of oscillating inflaton condensate**

→ Dynamics can be broadly divided into three distinct phases :

1. **Preheating** (**linear parametric resonance**)
2. **Backreaction** (**quenching of resonant particle production**)
3. **Scattering & thermalization** (**perturbative decay, turbulence**)

****Kofman, Linde, Starobinsky(1994-97); **Shtanov, Traschen, Brandenberger(1995)**

Equations of Reheating Dynamics

System : **Inflaton φ $\longrightarrow$ massless offspring χ** ; $m \gg m_{0\chi}$

Described by the **action**

$$S[\varphi,\chi] = -\int d^4x\,\sqrt{-g}\left[\frac{1}{2}\,\partial_\mu\varphi\partial^\mu\varphi + V(\varphi) + \frac{1}{2}\,\partial_\mu\chi\partial^\mu\chi + \mathcal{I}(\varphi,\chi)\right]$$

With interaction $\mathcal{I}(\phi,\,\chi) = \frac{1}{2}g^2\varphi^2\chi^2$

The corresponding **coupled classical field equations** are

$$\ddot{\varphi} - \frac{\nabla^2}{a^2}\varphi + 3H\dot{\varphi} + V_{,\varphi} + \mathcal{I}_{,\varphi} = 0$$

$$\ddot{\chi} - \frac{\nabla^2}{a^2}\chi + 3H\dot{\chi} + \mathcal{I}_{,\chi} = 0$$

with **Hubble parameter**

$$H^2 = \frac{1}{3\,m_p^2}\left[\frac{1}{2}\,\dot{\varphi}^2 + \frac{1}{2}\,\frac{\vec{\nabla}\varphi}{a}.\frac{\vec{\nabla}\varphi}{a} + V(\varphi) + \frac{1}{2}\,\dot{\chi}^2 + \frac{1}{2}\,\frac{\vec{\nabla}\chi}{a}.\frac{\vec{\nabla}\chi}{a} + \mathcal{I}(\varphi,\,\chi)\right].$$

Present-epoch Frequency of GWs

$$\boldsymbol{f_k = 7.43 \times 10^{-8} \left(\frac{g_{s,0}}{g_{s,T_k}}\right)^{1/3} \left(\frac{g_{*,T_k}}{90}\right)^{1/2} \left(\frac{T_k}{\mathrm{GeV}}\right) \mathrm{Hz}}$$

$$\Rightarrow f_k = 1.03 \times 10^{-8} \left(\frac{g_{s,0}}{g_{s,T_k}}\right)^{1/3} g_{*,T_k}^{1/4} \left(\frac{E_k}{\mathrm{GeV}}\right) \mathrm{Hz}$$

$\boldsymbol{T_k}$: **Hubble-entry epoch temperature**

Cosmic Events	**Energy scale $\boldsymbol{E_k}$**	**Frequency $\boldsymbol{f_k}$ (Hz)**
M-R Equality	~ 1 eV	1.4×10^{-17}
CMB pivot-scale entry	~ 5 eV	7.2×10^{-17}
Onset of BBN	~ 1.4 MeV	1.8×10^{-11}
QCD Phase Transition	~ 320 MeV	3.7×10^{-9}
Electro-Weak SB	~ 240 GeV	2.7×10^{-6}
Baryogenesis	$\gtrsim 10$ TeV	$\gtrsim 1.4 \times 10^{-4}$

GW Observatories : Primordial HEP Detectors