Recovering redshifted 21-cm power spectrum : residual gain effects in Interferometric Observations

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Radio Cosmology and Continuum Observations in the SKA Era: A Synergic View

Bias and variance in redshifted 21-cm signal

$$\langle \mathcal{V}(\vec{U},\nu_1)\mathcal{V}(-\vec{U},\nu_2)\rangle \sim C_\ell(\Delta\nu) + \mathcal{B}_{C_\ell} \qquad \sigma^2_{C_\ell} = \sigma^2_T + \sigma^2_E$$

$$n_{13} = 2n_1 + n_3 \qquad n_{12} = 2/N_B + n_2 \qquad \Sigma^\delta_2 = \sigma^2_{\delta R} + \sigma^2_{\delta I}$$

$$\mathcal{B}_{C_\ell} = \left[(n_{13}\chi + n_{12})\Sigma^\delta_2 + (n_{13} + n_{12})\Sigma^b_2 \xi\right] \frac{C_\ell}{N_d}$$

$$\sigma^2_E = 2\frac{\mathcal{B}^2_{C_\ell}}{N_G} + 2\left[\Sigma^\delta_2 + \Sigma^b_2\right]\frac{N_2 C_\ell}{N_B N^2_d} + 4\left[\Sigma^\delta_2 + \Sigma^b_2\right]^2 \frac{C^2_\ell}{N_G N^2_d}$$

From Prasun's talk …

$$C_\ell(\Delta\nu) = C_\ell(\Delta\nu)\,|_{FG} + C_\ell(\Delta\nu)\,|_{HI} \sim C_\ell(\Delta\nu)\,|_{FG}$$

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(In communication)

uGMRT Band 3 observation: MAPS at $\Delta\nu = 0$

SKA1-Low Prediction: MAPS at $\Delta\nu \neq 0$

Observation Summary (uGMRT)

Foreground Model

$$C_\ell |_F = A \left(\frac{1200}{\ell} \right)^{\beta} \quad \mathrm{mK}^2$$

$$A = 33.4 \pm 3.4 \quad \mathrm{mK}^2$$

$$\beta = 3.1 \pm 0.3$$

Project code	32_120
Observation date	May 05, 06, 07 of 2017
	June 27 of 2017
Bandwidth	200 MHz
Frequency range	300-500 MHz
Channels	8192
Integration time	2s
Correlations	RR RL LR LL
Total on-source time	13 h (ELAIS N1)
Working antennas	28
Flux Calibrator	
Source	3C286
Flux Density	23 Jy
Source	3C48
Flux Density	42 Jy
Phase Calibrator	
Source	J1549+506
Flux Density	0.3 Jy
Target Field	
Source	ELAIS N1

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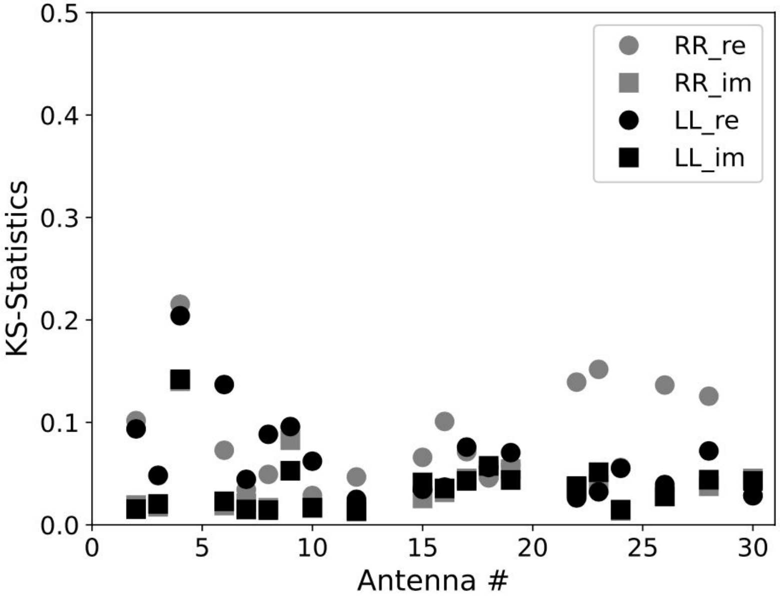
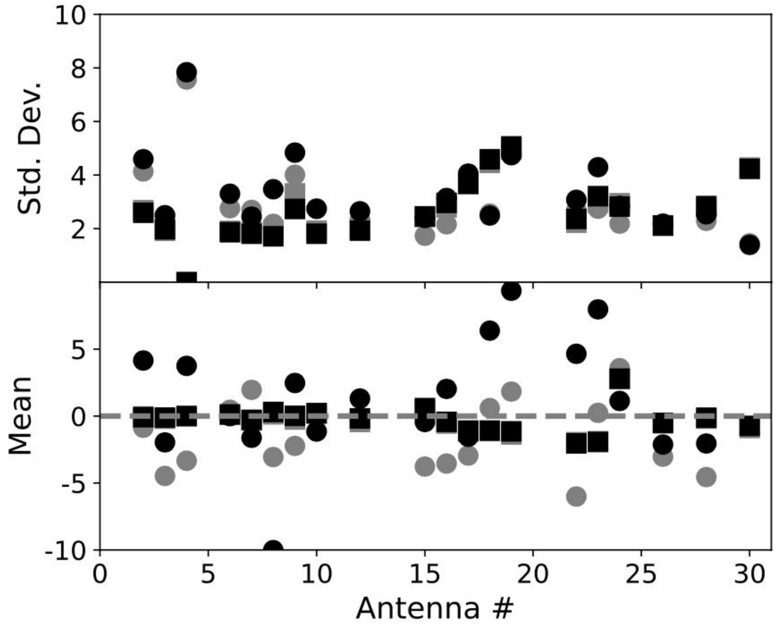
Gain Characteristics

KS Statistics



Standard Deviation of residual gain of corresponding antennae



- # 1, 13, 20, 21, 25, 27 & 29 are flagged as stdv is more than 8%

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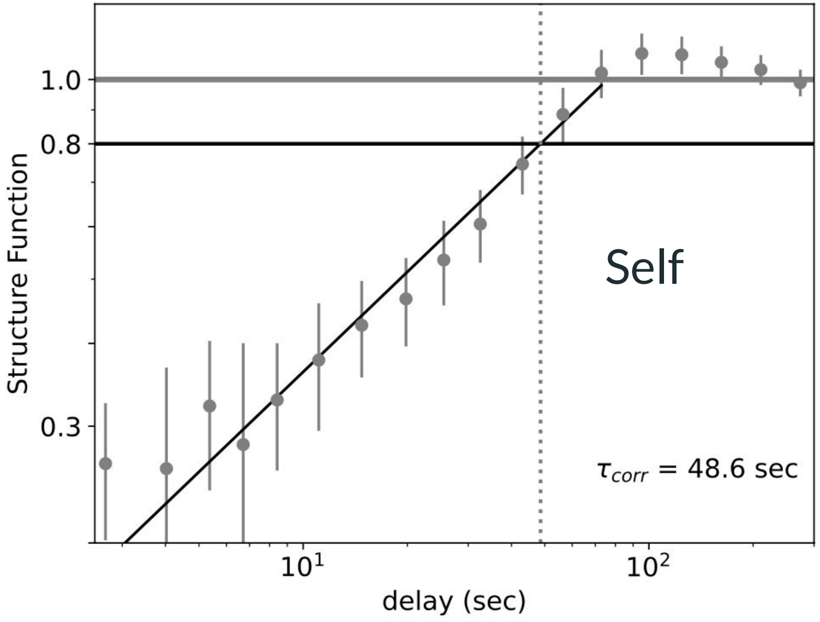
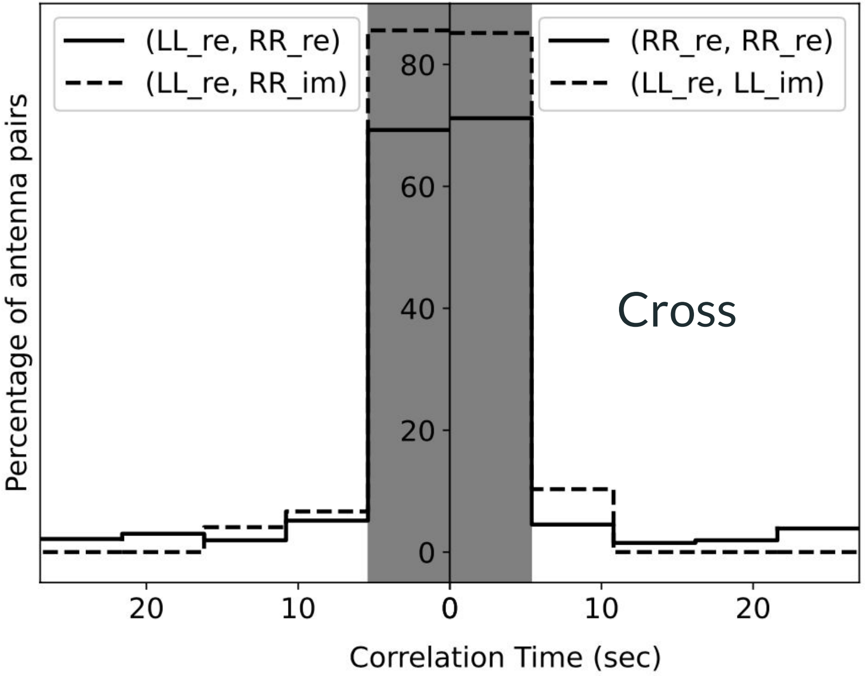
Gain Characteristics : Time correlation effect

$$S_2(\tau)_{XY} = \langle [\delta_X(t) - \delta_Y(t+\tau)]^2 \rangle / (\sigma_X \sigma_Y)$$



Integration time
2.68 sec



- Normalized self structure fn residual gain error for LL_im of #16 antenna

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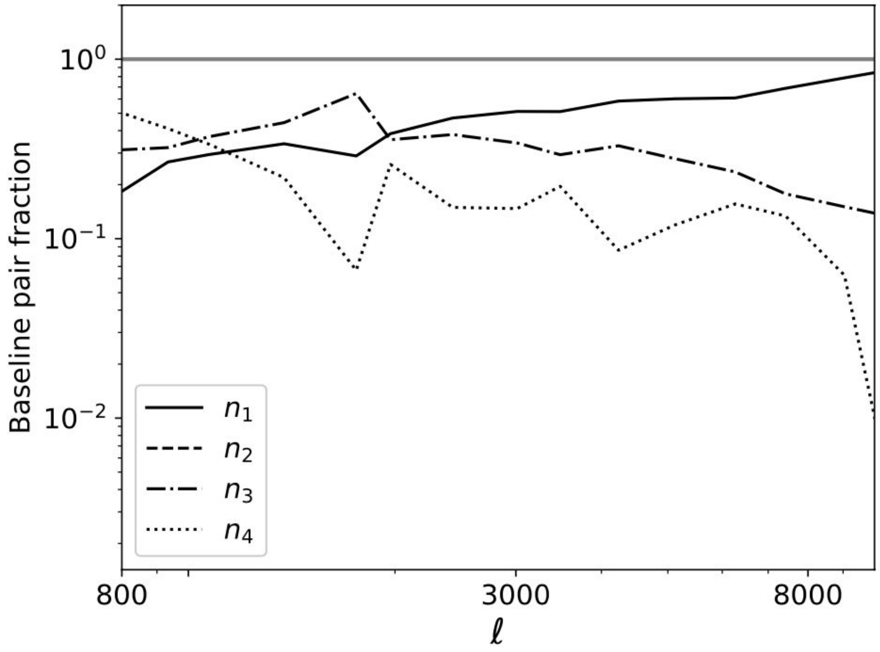
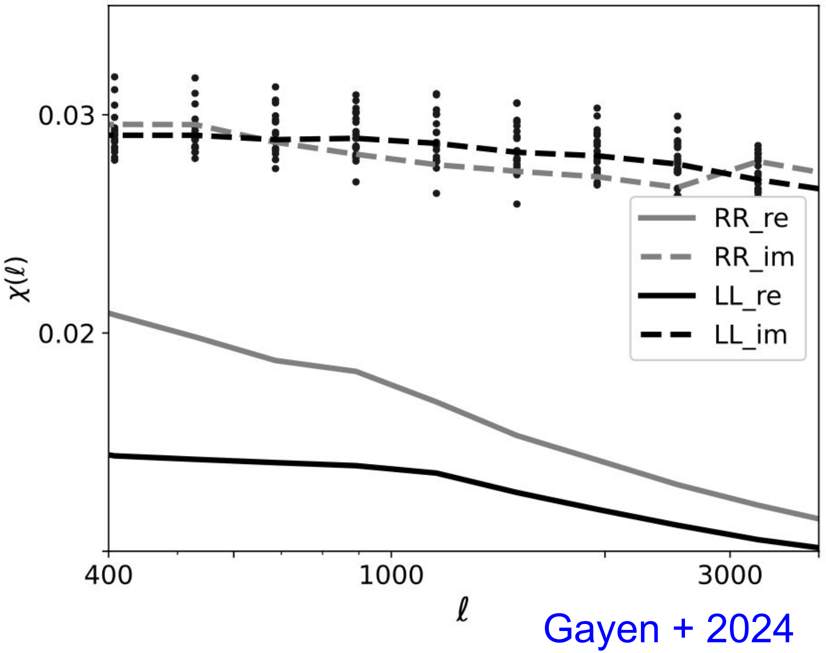
Baseline pair fractions

- Baseline-pair fractions for 3.5 hours of observation with uGMRT [6th May, 2017]

$$\chi(\ell) = \frac{1}{T_D^2} \int_{\Delta\tau}^{T_D} (T_D - \tau)\xi(\tau) d\tau$$





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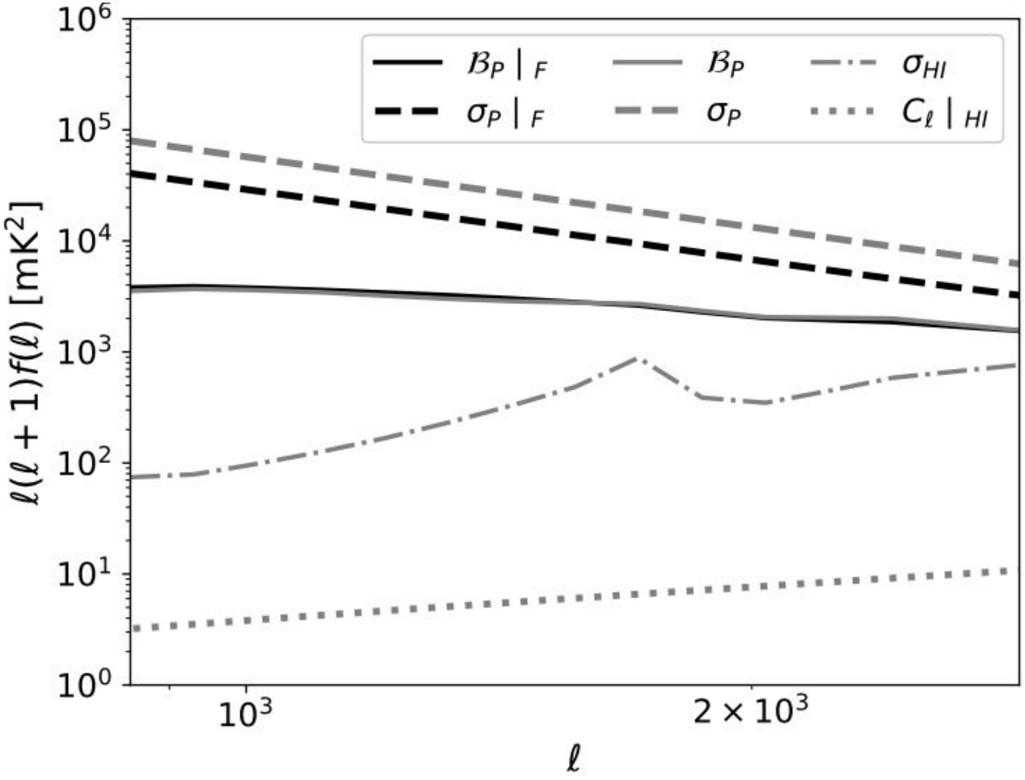
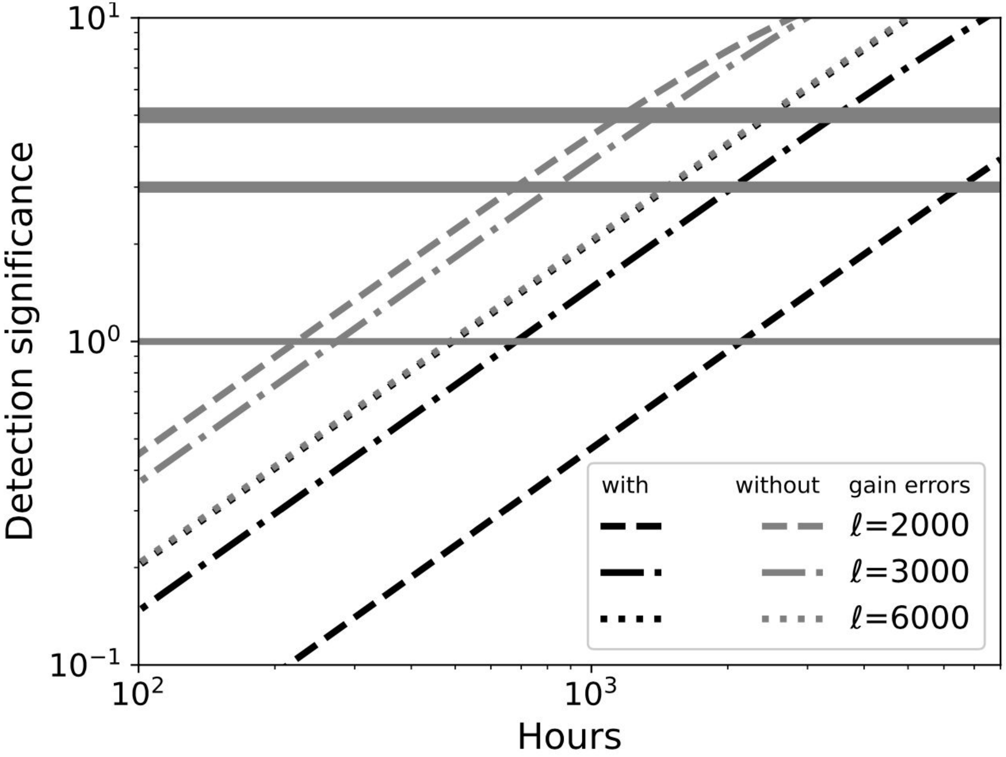
Results: Effect in PS ; Bias and variance [06 th May]



- $\mathcal{B}_{C_\ell}$ and σ_{C_ℓ} > σ_{T}
- Addition flagging of several antennae improve the uncertainty in PS coming from residual gain errors.

Results : Predictions



- For lower value of l (shorter baseline) residual gain error effect is significant, it takes longer time to detect signal when we consider residual gain effect.

- For larger baseline uncertainty in PS mostly coming from thermal noise

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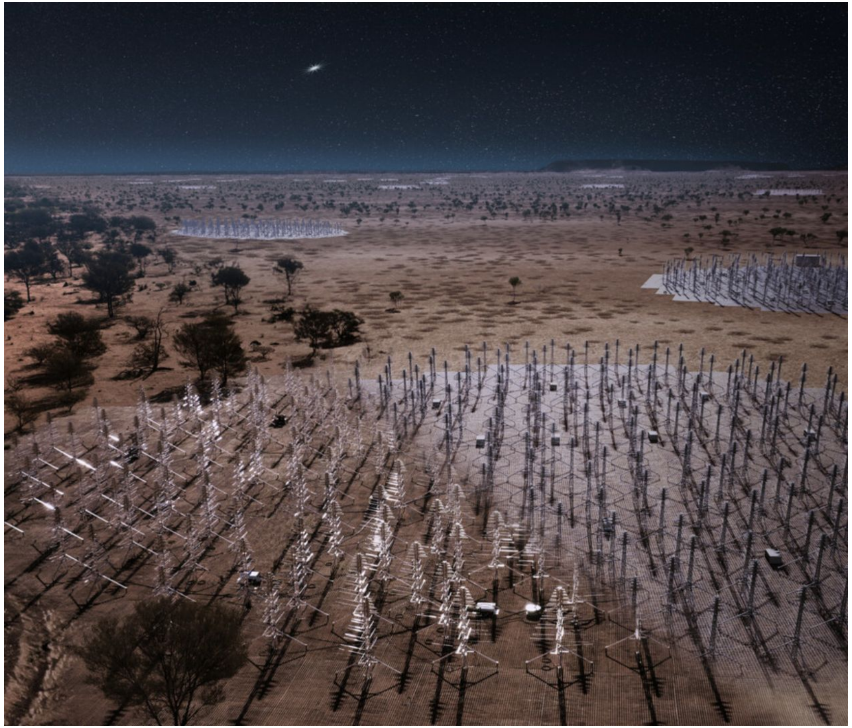
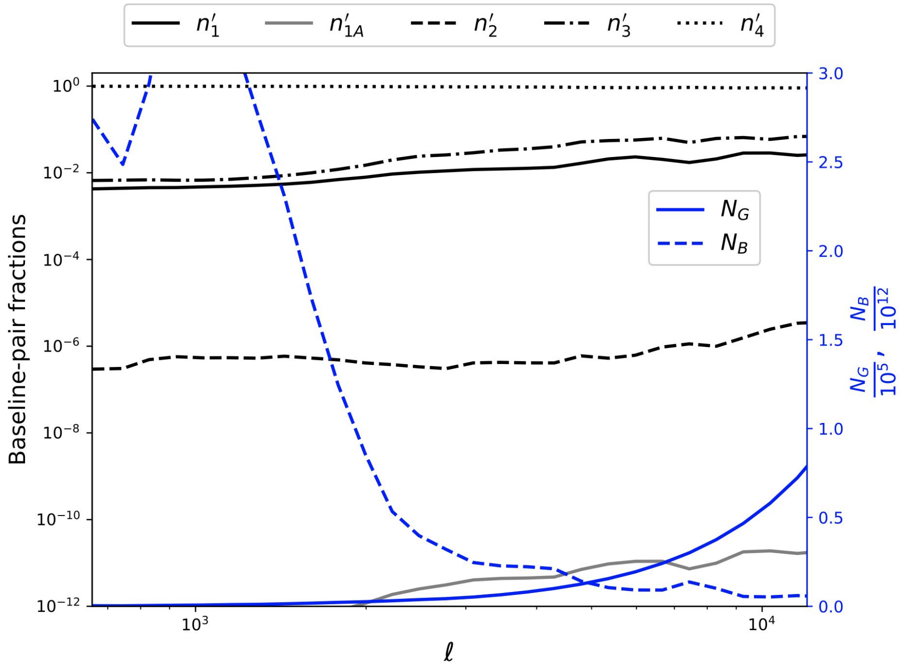
"Observation" (SKA1-Low AA4 configuration)



Bandwidth	50 MHz
Channels	1024
Integration time	1 sec
Total on-source time	8 hrs
Working antennae	512

Baseline pair fraction

- Baseline-pair fractions with angular multi-pole for 8 hours of observation with SKA1-Low



Type 1: $\langle \tilde{\mathscr{V}}_{AB}(t,\nu_1)\tilde{\mathscr{V}}^*_{AB}(t',\nu_2)\rangle$

Type 1A: $\langle \tilde{\mathscr{V}}_{AB}(t,\nu_1)\tilde{\mathscr{V}}^*_{AB}(t,\nu_2)\rangle$

Type 2: $\langle \tilde{\mathscr{V}}_{AB}(t,\nu_1)\tilde{\mathscr{V}}^*_{AC}(t,\nu_2)\rangle$

Type 3: $\langle \tilde{\mathscr{V}}_{AB}(t,\nu_1)\tilde{\mathscr{V}}^*_{AC}(t',\nu_2)\rangle$

Type 4: $\langle \tilde{\mathscr{V}}_{AB}(t,\nu_1)\tilde{\mathscr{V}}^*_{CD}(t',\nu_2)\rangle$

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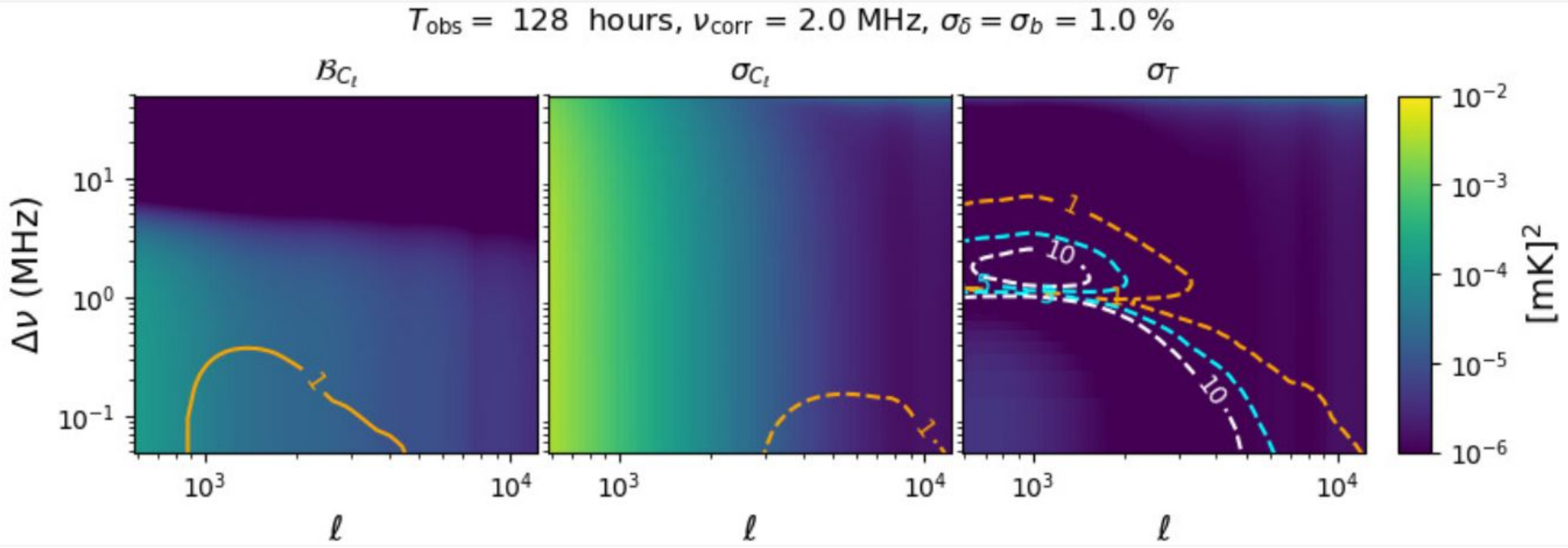
Results

- Solid contours $= \mathcal{R}_B = \frac{C_{\ell_{HI}}}{\mathcal{B}_{C_\ell}}$
- Dashed contours $= \mathcal{R}_\sigma = \frac{C_{\ell_{HI}}}{\sigma_{C_\ell}}$
- 1 (orange), 5 (cyan) and 10 (white)



- In absence of residual gain and bandpass errors, 128 hrs observation with SKA1-Low adequate to detect HI MAPS.

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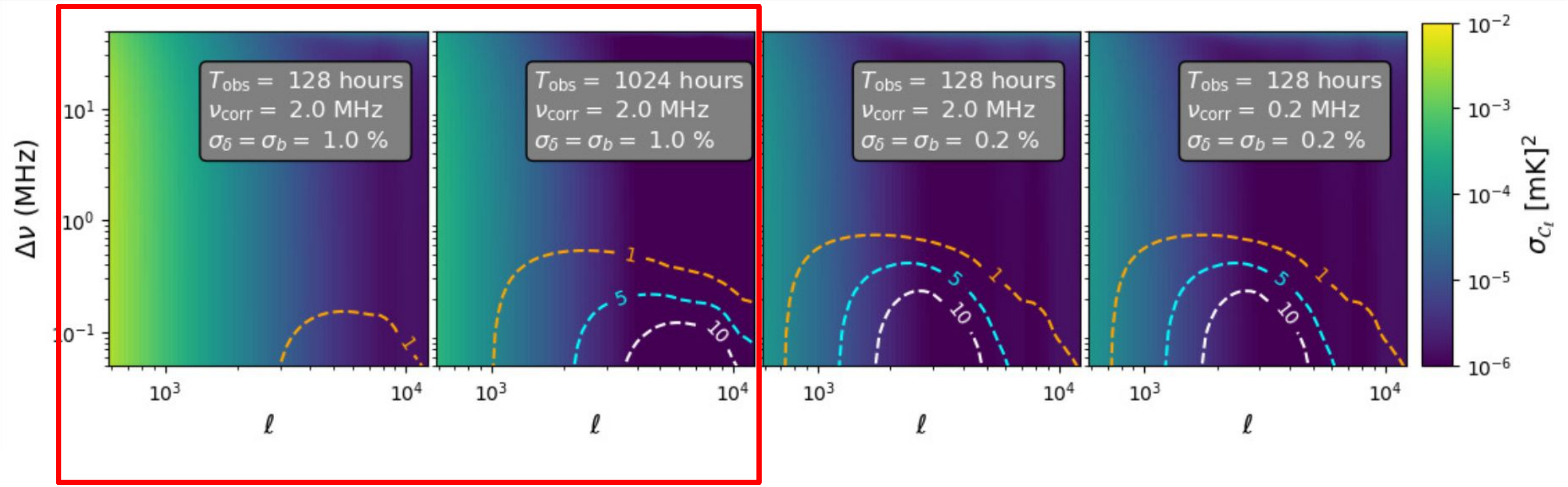
(In communication)

Results



- Increasing the observation time, keeping other parameters fixed, increase the detection significance significantly .

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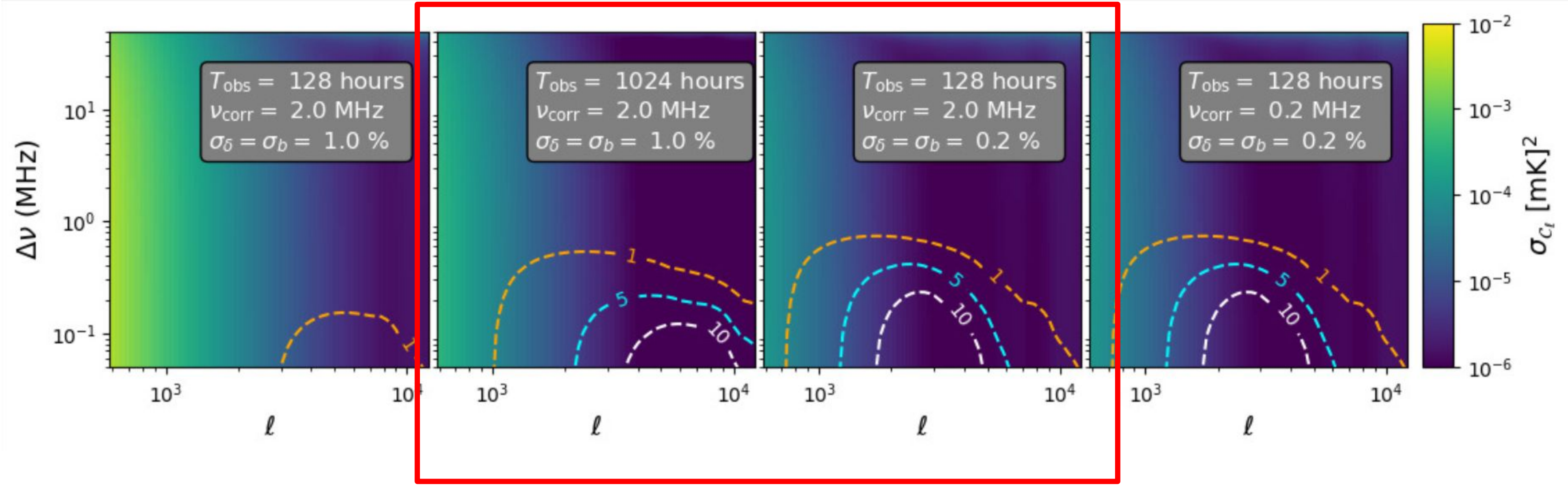
Results



- With 5 times better calibration accuracy, a significant detection possible even with only 128 hrs of observation.

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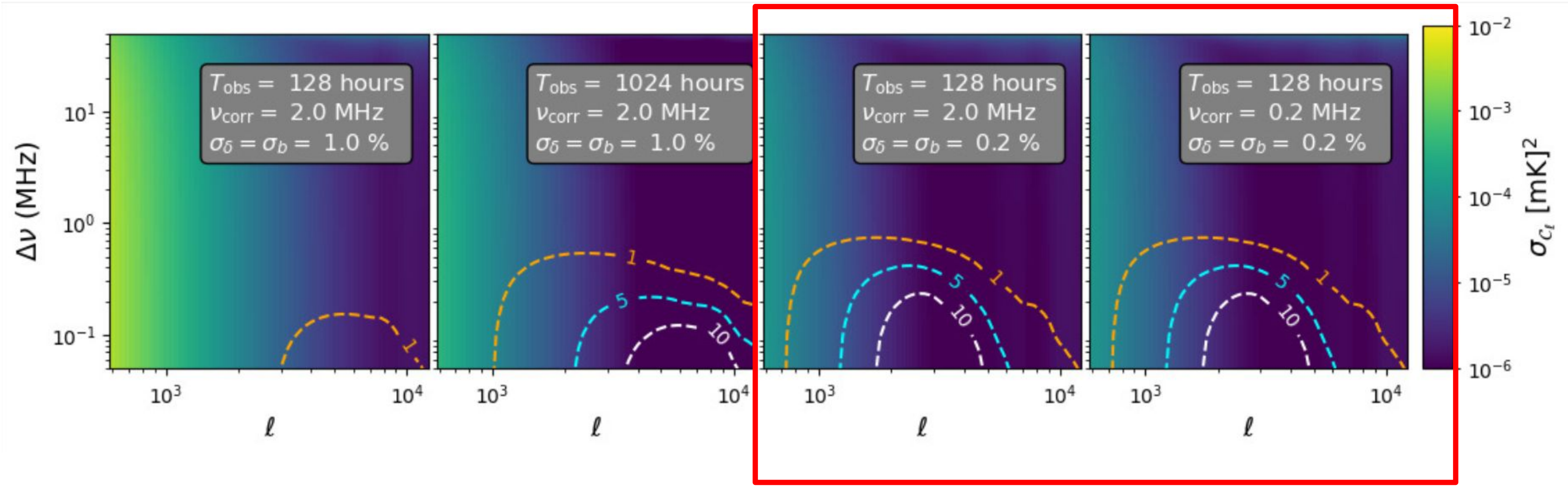
(In communication)

Results



- Change in bandpass-correlation by a factor of 10 does not change the detection significance.

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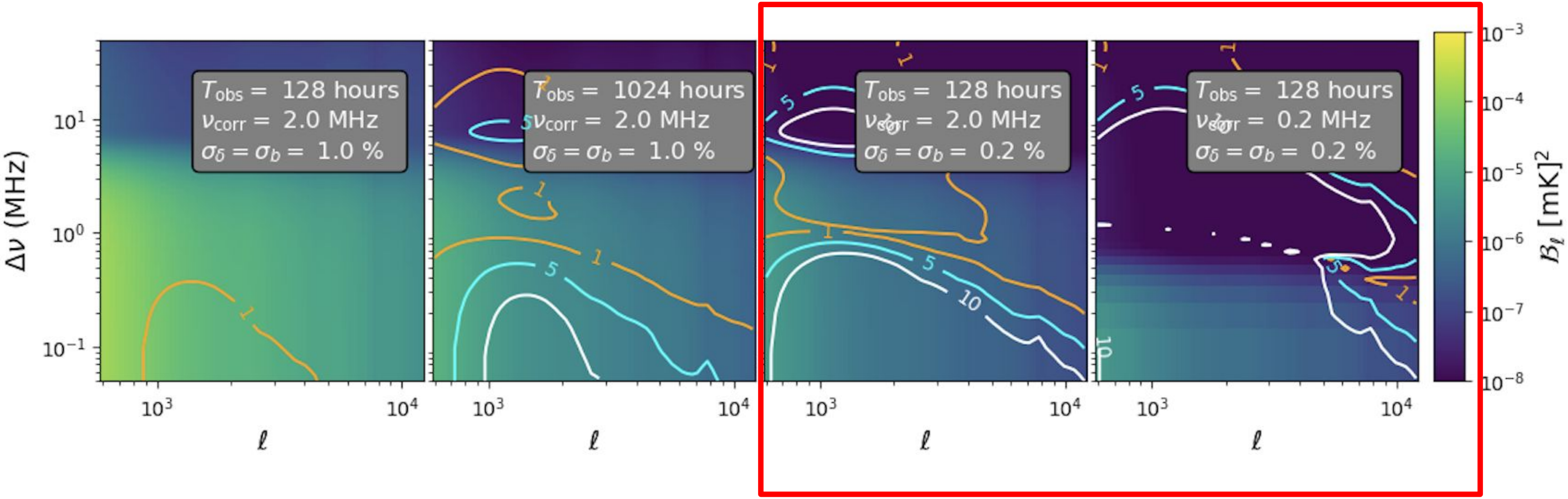
(In communication)

Results



- Change in bandpass-correlation by a factor of 10 does increase the detection significance significantly in case of bias as a part of bias is depends on frequency.

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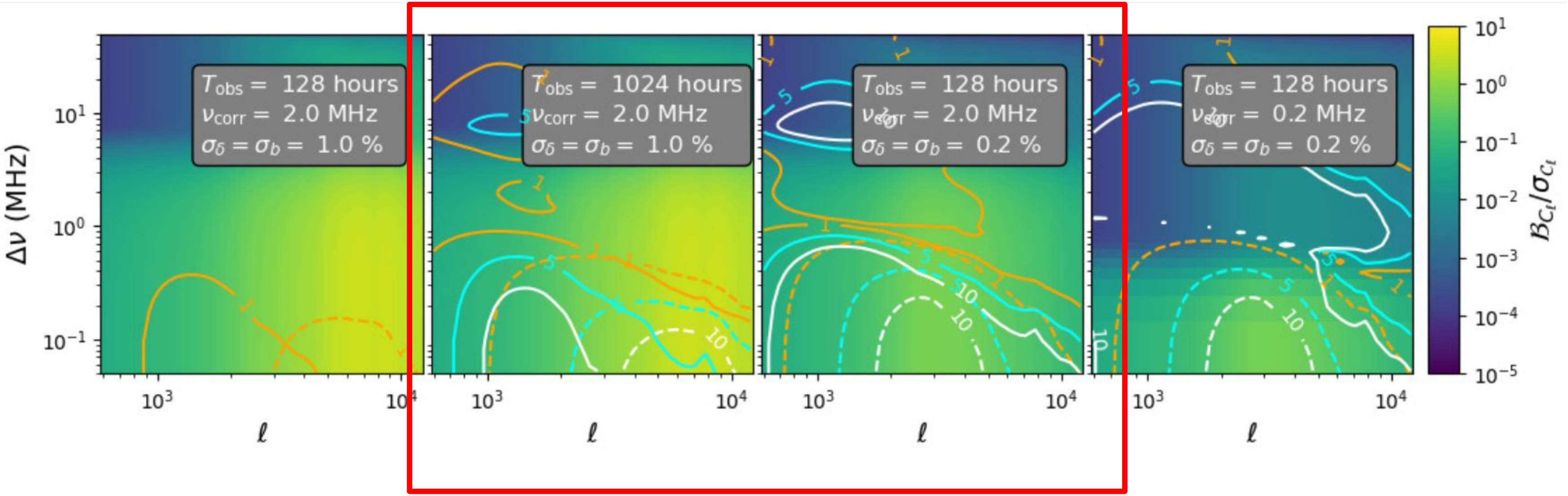
(In communication)

Results



- Even with large observation time, 1024 hrs a statistically significant detection of MAPS is difficult as intersection area of 5 sigma contours for bias and variance is limited.

- Improving the calibration accuracy does result in significant detection even only with 128 hrs of observation.

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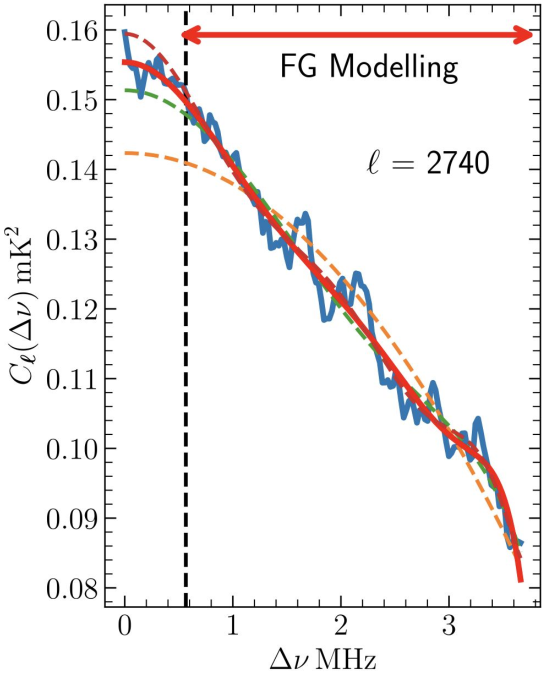
(In communication)

Results



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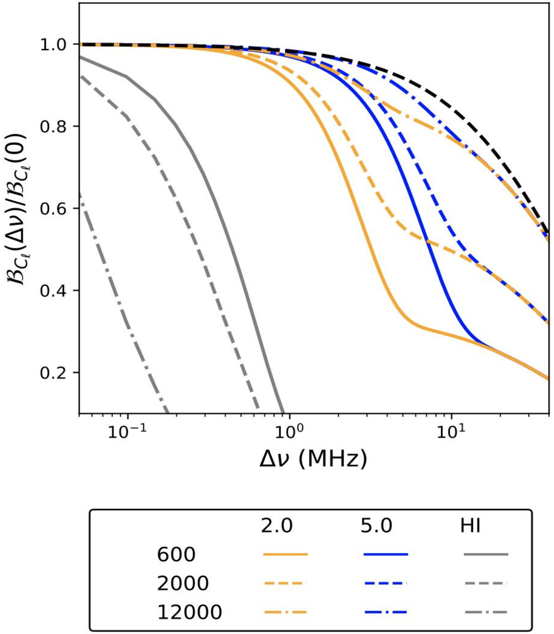

- We have to consider bandpass gain as it correlates a certain frequency range higher than HI MAPS.
- The frequency dependency in 1st term of the Bias originate from that in the foreground MAPS so this part can be mitigated.

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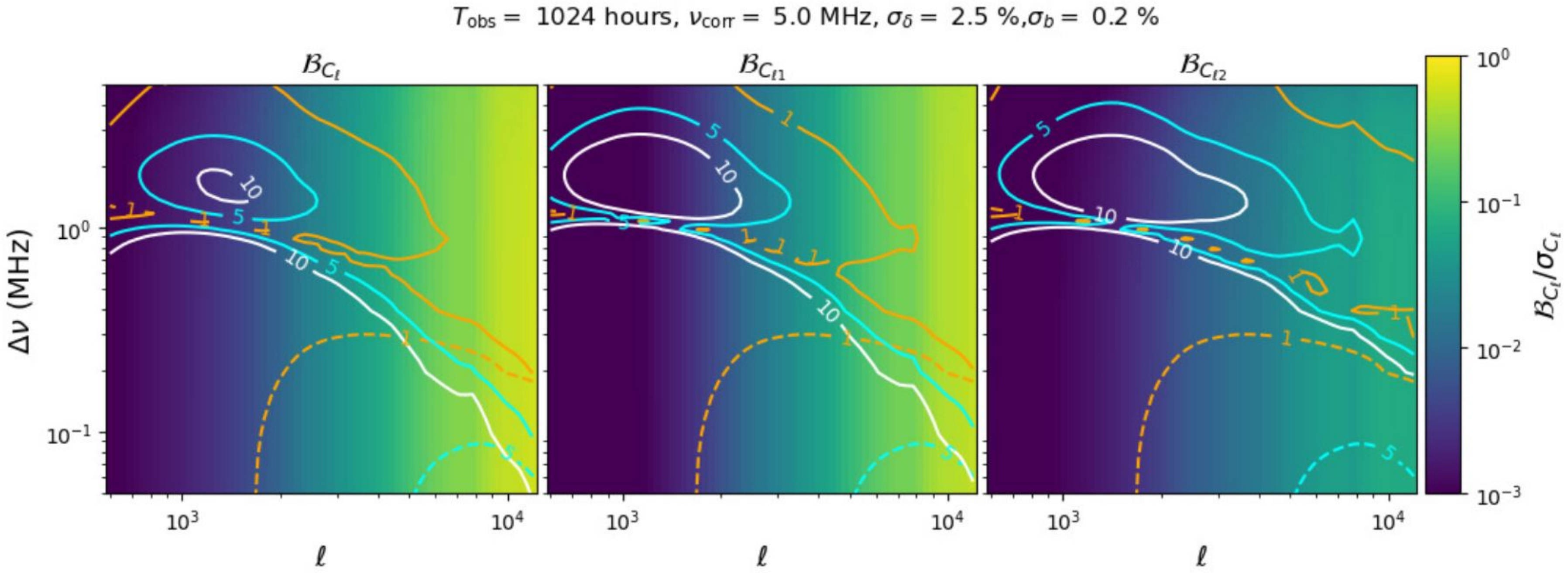
(In communication)

Results



- If 2nd term of Bias is subdominant, there is a possibility of unbiased detection , even in the presence of significant time dependent residual gain errors.

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(In communication)

Conclusion

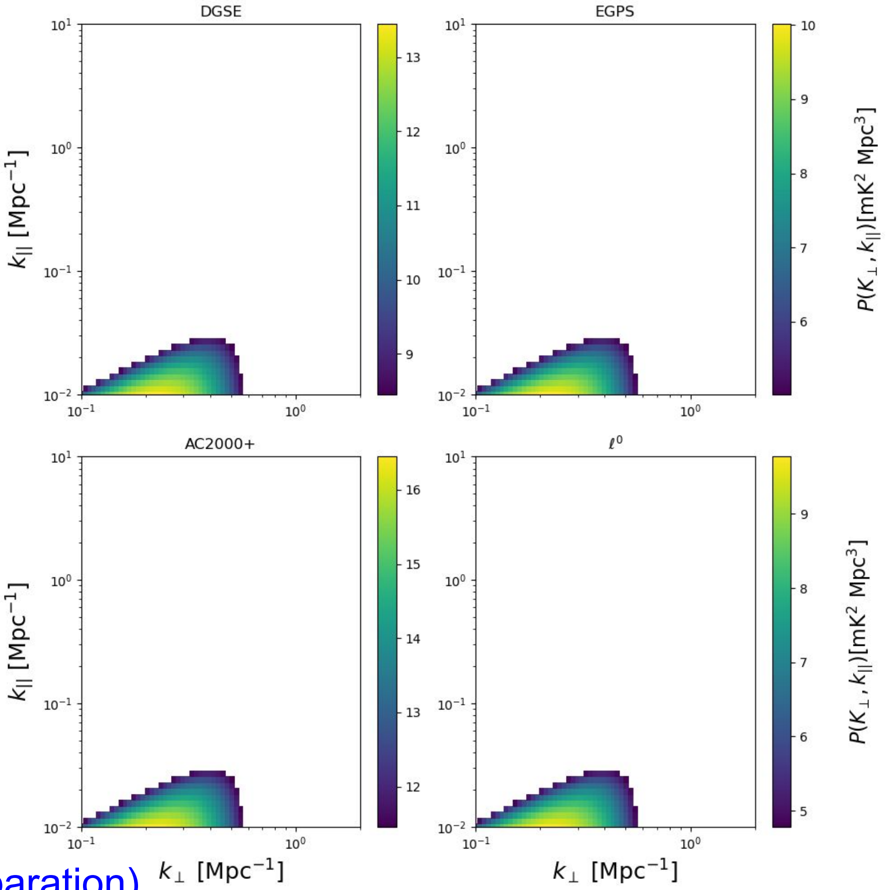
- In the absence of residual gain and bandpass errors, 128 hrs observation with SKA1-Low adequate to detect HI MAPS.
- Increasing the observation time to 1024 hours, keeping other parameters fixed, increase the detection significance significantly .
- With better calibration accuracy (1%->0.2%), significant detection is possible with only 128 hrs of observation.
- In significant part of the parameter space the measurement can be biased.
- The frequency dependency in 2nd term of the Bias originate from that in the bandpass and hence frequency correlation in bandpass need to be minimized.

Mathematical formulation for Foreground wedge

$$P(k_\perp, k_\parallel) \approx \pi\theta_0^2 \left(\frac{\Delta B}{\Delta T}\right)^2 \sqrt{\frac{\pi}{\pi\theta_0 u/\nu_0}} \int \mathrm{d}u_1 \int \mathrm{d}\eta B\left(\pi\theta_0(u-u_1), \frac{\tau-\eta}{\theta_0 u/\nu_0}\right) C(2\pi u_1, \eta; \nu_0),$$

$$B\left(\pi\theta_0(u-u_1), \frac{\tau-\eta}{\theta_0 u/\nu_0}\right) = A\left(\pi\theta_0(u-u_1)\right) A\left(\frac{\tau-\eta}{\theta_0 u/\nu_0}\right) \exp\left(-i2\pi\frac{u-u_1}{u/\nu_0}(\tau-\eta)\right)$$

$$u = \frac{k_\perp r_c}{2\pi}, \quad \tau = \frac{k_\parallel r_c'}{2\pi}.$$



Gayen + (In preparation)