Neutrinos and their interactions with matter

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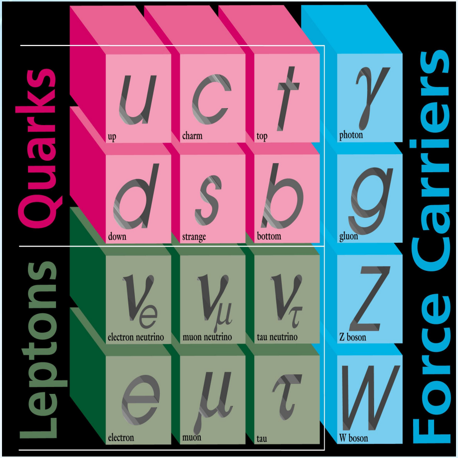
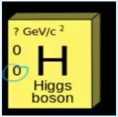
Outline

- Introduction
- $e^- - \mu^-$ scattering
- $\nu_e e^- \to \nu_e e^-$ scattering
- Quasielastic scattering
- Meson production from nucleon
- Deep Inelastic Scattering
- Quark-Hadron Duality
- Neutrino-Nucleus Interaction
- Quasielastic scattering
- Conclusion

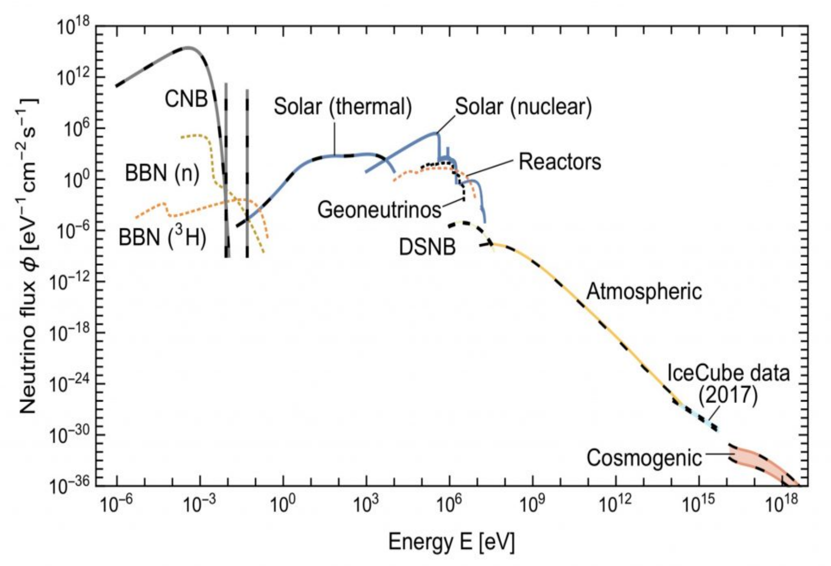
Fundamental particles





Mass $\sim$ 125 GeV

Neutrino flux



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- Innovations in particle detectors developed for neutrino experiments can be applied to medical imaging or national security.

Why study ν_l interactions?

Good understanding of neutrino interactions is important for:

- neutrino detection, energy reconstruction, neutrino flux calibration
- determination of backgrounds
- reduction of systematic errors
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- Near detectors help to reduce systematic errors, still there are limitations:

ND vs FD: These detectors are exposed to the

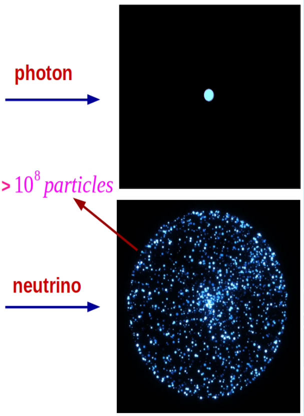
- different fluxes with different flavor composition
- different geometry, acceptance and targets

Interaction of neutrinos

number density of photon = 450/cm^3

number density of neutrino = 330/cm^3

Interaction of neutrinos



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Fundamental interactions and mediating quanta

Interaction	Mediating particle	Cross section (cm^2)	Range	Typical coupling
Strong	gluon	10^{-26}	10^{-15} m	1
Electromagnetic	photon	$10^{-30} - 10^{-32}$	∞	10^{-2}
Weak	$W^{\pm}$	$10^{-38} - 10^{-40}$	10^{-18} m	10^{-6}
	Z^0			

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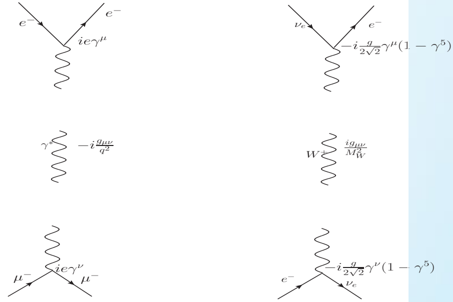
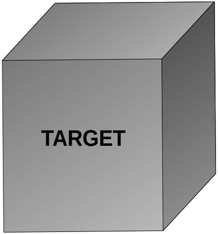
$e^- \mu^- \to e^- \mu^-$ $\qquad$ $\nu_e e^- \to \nu_e e^-$



$$e = \sqrt{4\pi\alpha} = \sqrt{\frac{4\pi}{137}}; \qquad \frac{g^2}{8M_W^2} \sim 10^{-5} GeV^{-2}.$$

Cross section



Water density= 1 g/cc

Iron density= 7.8 g/cc

Lead density= 11.3 g/cc

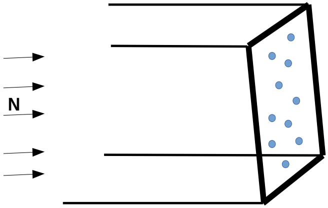
If M is the target mass in kg then

$$N_{nucleons} = M\,(kg)\,(10^3\ kg\ g^{-1})\,N_A$$

Number of nucleons in a gm of matter is N_A.

If the targets are nuclei of mass number A:

$$N_{nuclei} = M\,(kg)\,(10^3\ kg\ g^{-1})\,N_A / A\,(mol\ g^{-1})$$



$$dN \propto N, \qquad dN \propto \rho\, dx$$

Due to reduction in flux:

$$dN = -\sigma N \rho\, dx$$

$$\frac{dN}{N} = -\sigma \rho\, dx$$

$$\ln \frac{N_f}{N_i} = -\sigma \rho x$$

$$N_f = N_i \exp(-\sigma \rho x)$$

Neutrino mean free path

$$Z = 10,\, N = 8,\, A = 18$$

$$\text{1 mole of } H_2O = 18 \text{ gm} \quad = \quad 6.023 \times 10^{23} \text{ molecules of water}$$

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Trillions of neutrinos pass through us without a single interaction

Neutrino interactions with point particles

Possible reactions are:

$$\nu_e + e^- \quad \to \quad \nu_e + e^-; \quad \text{posssible via both CC and NC}$$
$$\nu_l + e^- \quad \to \quad \nu_l + e^-; \quad l = \mu, \tau; \quad \text{posssible only via NC}$$

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Scattering cross section for $\nu_\mu e^- \to \nu_e \mu^-$:

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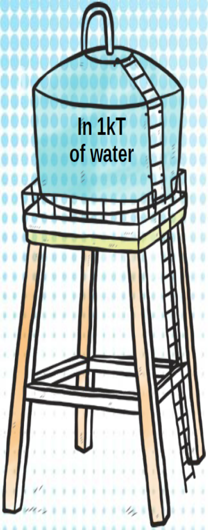
- **Hence gigantic nuclear targets are being used.**

Neutrino events



For 1 GeV Neutrino energy:

$\nu - N$ QE scattering process
$\sigma \propto 10^{-38}\ cm^2$

Volume of a cubic Detector of side 10 m=1000 m^3=10^9 cm^3

10^9 cm^3 hold 6.023 x 10^{32} nucleons

$$Hourly\ event = \sigma_{\nu N} \times \varphi \times t \times D$$

$$H.E. = 10^{-38}\ cm^2 \times 100\ \nu / cm^2 / sec \times 3600\ sec \times 6.023 \times 10^{32} \sim 2$$

Detector response true to reconstructed energy

cross section flux selection efficiency

$$N_{ND}(E_\nu) \equiv \int \sigma(E_\nu) \times \phi_1(E_\nu) \times \varepsilon_1(E_\nu) \times D_1(E_\nu, E_\nu^{recons}) dE_\nu$$

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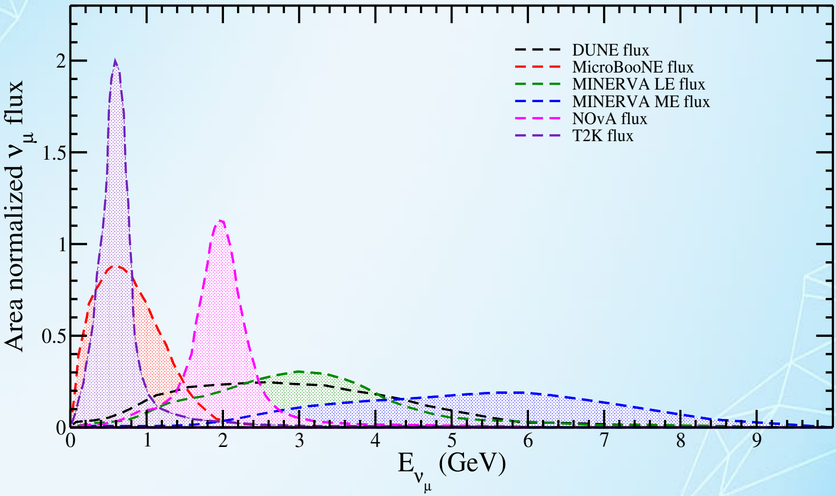
Detector response true to reconstructed energy

cross section flux selection efficiency Oscillation probability

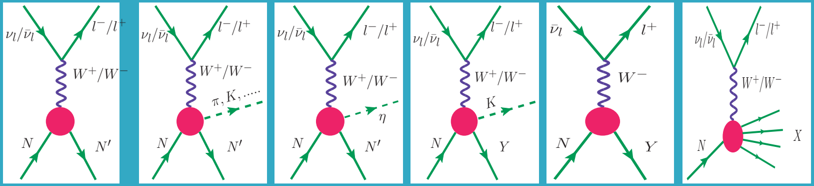
$$N_{FD}(E_\nu) \equiv \int \sigma(E_\nu) \times \phi_2(E_\nu) \times \varepsilon_2(E_\nu) \times D_2(E_\nu, E_\nu^{recons}) \times P_{\nu_\alpha \to \nu_\beta}(E_\nu) dE_\nu$$

Accelerator neutrino experiments

The ongoing accelerator experiments like NOvA, MINERvA and T2K, and the upcoming DUNE experiment have (anti)neutrino peak energy in the few GeV energy region.



Various neutrino interaction processes



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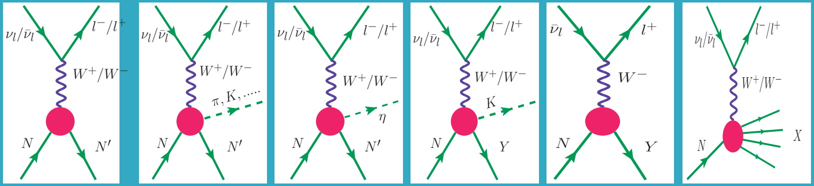
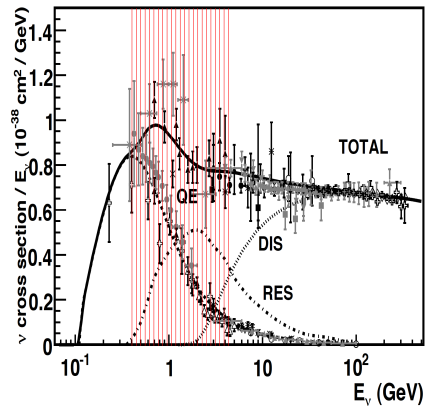


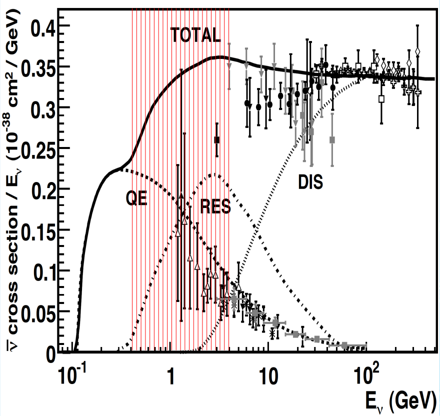
$$
\begin{aligned}
\nu_l/\bar{\nu}_l + N &\to l^{\mp} + N' \\
\nu_l/\bar{\nu}_l + N &\to l^{\mp} + N' + \pi \\
\bar{\nu}_l + N &\to l^{+} + Y \\
\nu_l/\bar{\nu}_l + N &\to l^{\mp} + \eta + N' \\
\nu_l/\bar{\nu}_l + N &\to l^{\mp} + X
\end{aligned}
$$

Neutrino cross section

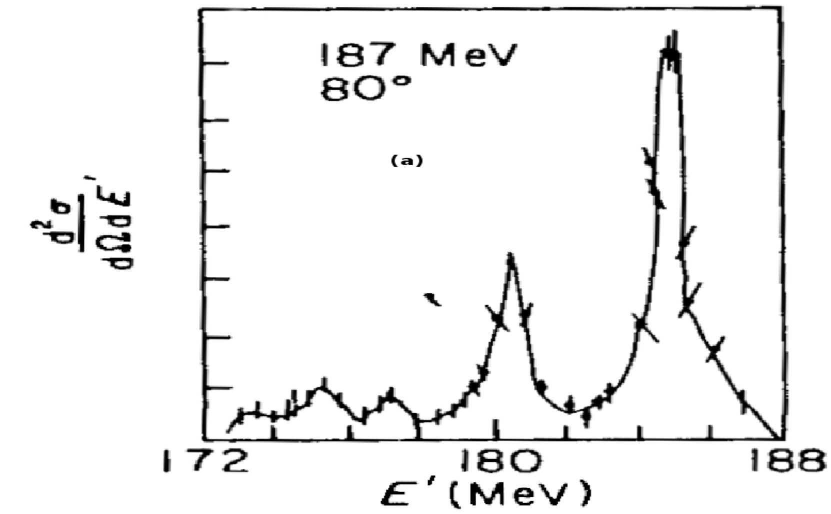
Neutrino



Anti-neutrino



$e - ^{12}C$ scattering $d^2\sigma$ vs E'



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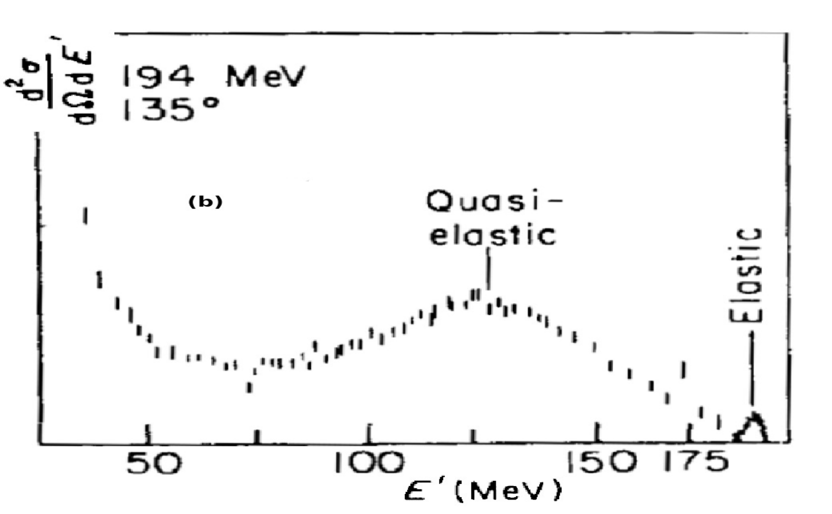
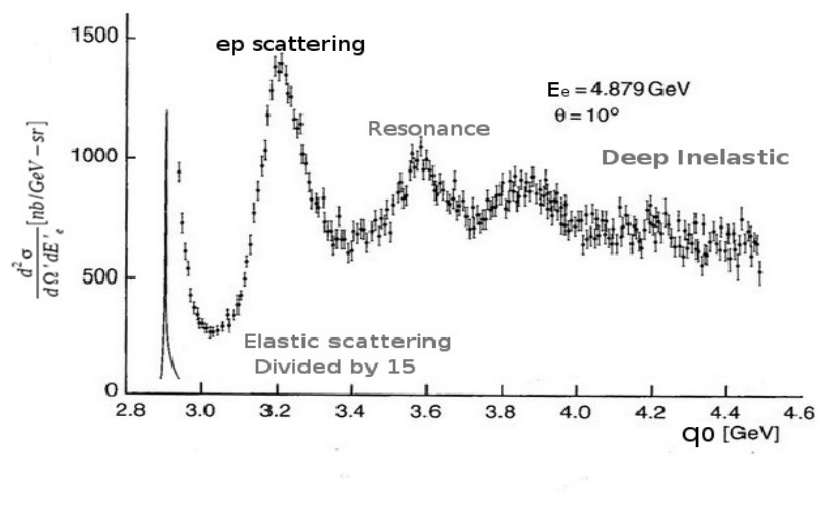
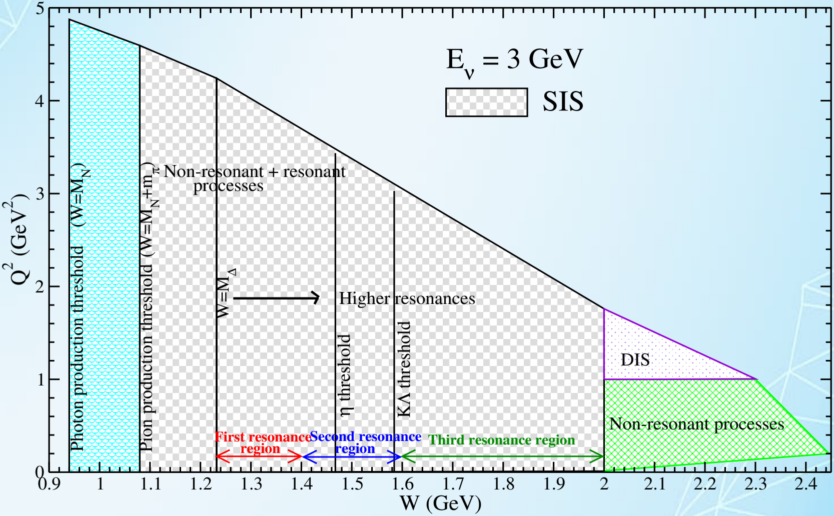


$e - p$ scattering $d^2\sigma$ vs q_0



Q^2 vs W region

- Q^2 is the square of the four momentum transferred to the hadronic system
- $W = \sqrt{s}$ is the invariant hadronic mass



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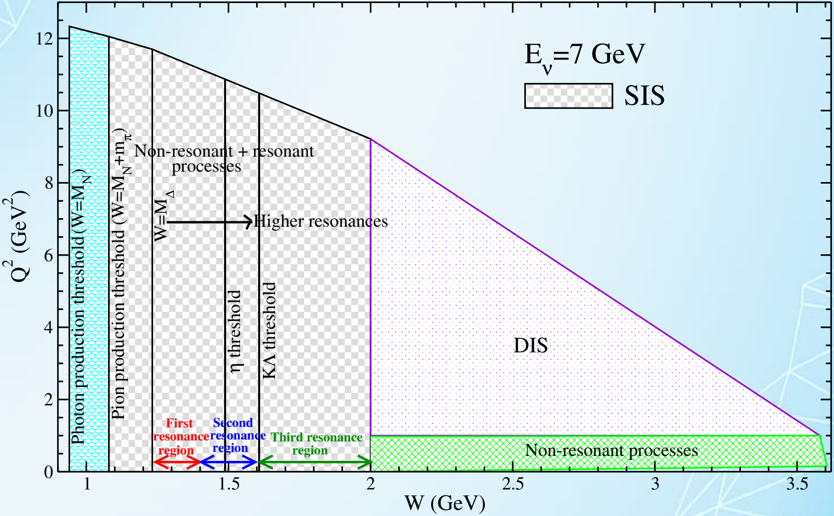

Weinberg-Salam interaction Lagrangian

$$L_I = -e\left[\frac{1}{2\sqrt{2}\sin\theta_W}\left(j_\mu^{CC} W^{\mu+} + h.c.\right) + \frac{1}{2\sin\theta_W\cos\theta_W} j_\mu^{NC} Z^\mu + j_\mu^{EM} A^\mu\right],$$

where $W_\mu^\pm$, Z_μ and A_μ are the charged, neutral and electromagnetic gauge fields and

$$j_\mu^{CC} = \sum_{l=e,\mu,\tau} \overline{\Psi}_l \gamma_\mu (1-\gamma^5)\psi_{\nu_l}$$

$$j_\mu^{NC} = \sum_{l=e,\mu,\tau} \left[\overline{\Psi}_l \gamma_\mu (g_V^l - g_A^l \gamma^5)\psi_l + \overline{\Psi}_{\nu_l} \gamma_\mu (g_V^{\nu_l} - g_A^{\nu_l}\gamma^5)\psi_{\nu_l}\right]$$

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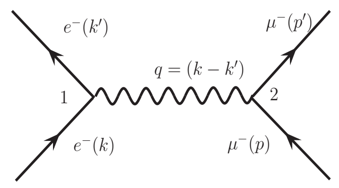
$$j_\mu^{EM} = \sum_{l=e,\mu,\tau}\overline{\psi}_l\gamma_\mu\psi_l,$$

- $\sin\theta_W = \frac{e}{g}$
- $g_V^l = -\frac{1}{2} + 2\sin^2\theta_W$
- $g_A^l = -\frac{1}{2}$
- $g_V^{\nu_l} = \frac{1}{2}$
- $g_A^{\nu_l} = \frac{1}{2}$
- $e = \sqrt{4\pi\alpha}$
- $sin^2\theta_W \simeq 0.23$
- $\alpha = \frac{1}{137}$
- $\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}$

$e^- - \mu^-$ scattering



The transition amplitude $\mathscr{M}$ is given as:

$$-i\mathscr{M} = \text{current at vertex 1} \times \text{propagator} \times \text{current at vertex 2}$$

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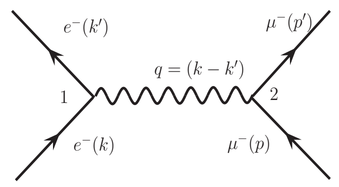

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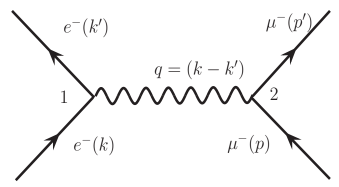

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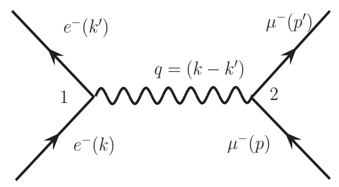

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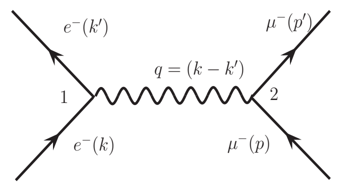

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$$\overline{\sum}\sum |\mathscr{M}|^2 = \frac{e^4}{q^4}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot Tr\left[(\not{k}'+m)\gamma^\mu(\not{k}+m)\gamma^\nu\right] \times Tr\left[(\not{p}'+M)\gamma_\mu(\not{p}+M)\gamma_\nu\right]$$

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The general expression for the differential scattering cross section is given by

$$d\sigma = \frac{1}{4\sqrt{(p\cdot k)^2 - m^2M^2}} \overline{\sum}\sum |\mathscr{M}|^2 \frac{d^3k'}{(2\pi)^3 2E'} \cdot \frac{d^3p'}{(2\pi)^3 2E'_\mu} \times (2\pi)^4 \delta^4(p + k - p' - k')$$

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- $\frac{1}{4\sqrt{(p\cdot k)^2 - m^2M^2}} \Longrightarrow$ relative flux factor
- $\delta^4(p + k - p' - k') \Longrightarrow$ ensures energy-momentum conservation
- $\frac{d^3p'}{(2\pi)^3 2E'} \Longrightarrow$ available phase space for final particles
- $\bar{\sum}\sum |\mathcal{M}|^2 \Longrightarrow$ matrix element square averaged over initial particles spin and summed over final particles spin

Using

$$\int \frac{d\mathbf{p}'}{2E'_p}\delta^4(p'+k'-p-k) = \frac{1}{2M}\delta^0\left(\nu + \frac{q^2}{2M}\right)$$

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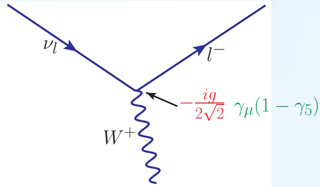
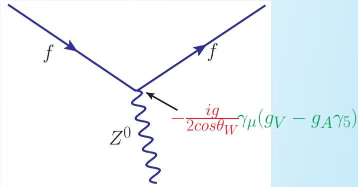
Lagrangian for the weak interaction At each vertex, the strength and nature of the interaction is taken.

CC



NC



$$\mathscr{L}_I = -i\frac{g}{2\sqrt{2}}\overline{\psi}(k')\gamma^\mu(1-\gamma_5)W_\mu\psi(k)$$

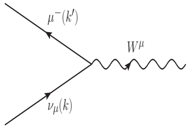
$$\mathscr{L}_I = -i\frac{g}{2cos\theta_W}\overline{\psi}(k')\gamma^\mu(g_V - g_A\gamma_5)Z_\mu\psi(k)$$

Inverse muon decay

$$\nu_\mu(\vec{k}, E_{\nu_\mu}) + e^-(\vec{p}, E_e) \longrightarrow \mu^-(\vec{k}', E_\mu) + \nu_e(\vec{p}\,', E_{\nu_e}).$$

The interaction Lagrangian for the leptons interacting with a W^+ field is given by:

$$\mathscr{L}_I = \frac{-g}{2\sqrt{2}} \bar{\psi}(\vec{k}')\gamma_\mu(1-\gamma_5)\psi(\vec{k})W^{+\mu}.$$



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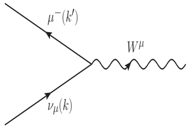
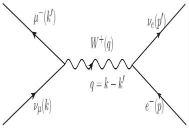


Transition matrix element:

$$-i\mathscr{M}_{CC} = \bar{u}(\vec{k}')\left[\frac{-ig}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)\right]u(\vec{k})\left(\frac{-ig_{\mu\nu}}{M_W^2}\right) \times \bar{u}(\vec{p}\,')\left[\frac{-ig}{2\sqrt{2}}\gamma^\nu(1-\gamma_5)\right]u(\vec{p})$$



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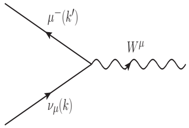

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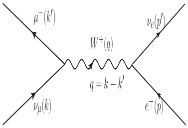


Differential cross section in the Lab frame:

$$\left.\frac{d\sigma}{d\Omega}\right|_{Lab} = \frac{1}{\pi^2 m_e} G_F^2 \frac{E_\mu^2}{E_{\nu_\mu}}\left[E_{\nu_\mu}E_\mu - E_{\nu_\mu}|\vec{k}'|\cos\theta + m_e E_\mu - m_\mu^2\right]$$

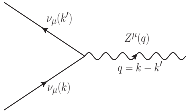
$\nu_\mu + e^- \to \nu_\mu + e^-$ scattering

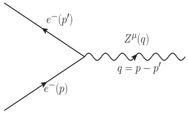
For $\nu\nu Z$ interaction:

$$\mathscr{L}_I^{neutrino} = \frac{-g}{2cos\theta_W}\bar{\psi}(\vec{k}')\gamma_\mu(g_V^\nu - g_A^\nu\gamma^5)\psi(\vec{k})Z^\mu.$$

At electron vertex:

$$\mathscr{L}_I^{electron} = \frac{-g}{2cos\theta_W}\bar{\psi}(\vec{p}')\gamma_\mu(g_V^e - g_A^e\gamma^5)\psi(\vec{p})Z^\mu.$$





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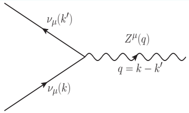

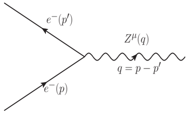

The differential cross section in the Lab frame:

$$\begin{aligned}\left.\frac{d\sigma}{d\Omega}\right|_{Lab} &= \frac{1}{4\pi^2 m_e}G_F^2\left(\frac{E_e'^2}{E_\nu}\right)\Big[(g_V^e + g_A^e)^2(E_\nu E_e' - E_\nu|\vec{p}'|\cos\theta + m_e E_e' - m_e^2)\\ &+ (g_V^e - g_A^e)^2(E_\nu + m_e - E_e')(E_e' - |\vec{p}'|\cos\theta)\\ &- m_e\{(g_V^e)^2 - (g_A^e)^2\}(m_e - E_e' + |\vec{p}'|\cos\theta)\Big].\end{aligned}$$

Invariant matrix element square:

$$\overline{\sum_i}\sum_f |\mathscr{M}|^2 = \frac{1}{2}\left(\frac{4G_F^2}{2}\right) L_{\mu\nu}^{neutrino} L^{\mu\nu}_{electron}$$

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Leptonic tensors are given by

$$L_{\mu\nu}^{neutrino} = 2\left[k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k\cdot k' - i\epsilon_{\sigma\mu\rho\nu} k^\rho k'^\sigma\right]$$

$$L^{\mu\nu}_{electron} = 4[\{(g_V^e)^2 + (g_A^e)^2\}(p'^\mu p^\nu - p'\cdot p g_{\mu\nu} + p'^\nu p^\mu) + 2p'_\lambda p_\theta i\epsilon^{\lambda\mu\theta\nu} g_V^e g_A^e + m_e^2 g^{\mu\nu}\{(g_V^e)^2 - (g_A^e)^2\}].$$

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The differential cross section in the Lab frame is obtained as:

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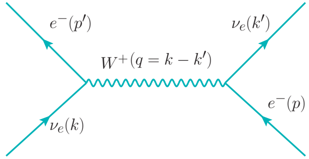
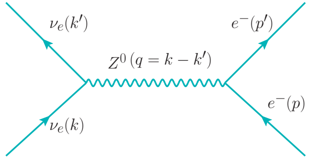
$\nu_e e^- \to \nu_e e^-$ scattering

It is mediated via both neutral and charged current.

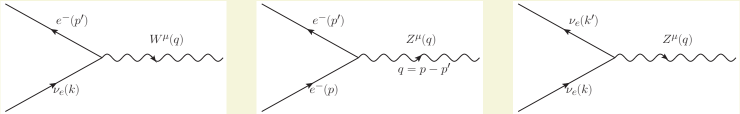


Charged current(CC)



Neutral current(NC)

$\nu_e + e^- \to \nu_e + e^-$ scattering



The Lagrangian for the charged current W^+ boson exchange:

$$\mathscr{L}_{\nu_e e^- W^+} = \frac{-g}{2\sqrt{2}} \bar{\psi}(\vec{p}\,')\gamma_\mu(g_V^e - g_A^e \gamma_5)\psi(\vec{k})W^{+\mu}$$

The Lagrangians for the neutral current Z^0 exchange:

$$L^{\nu\nu Z} = \frac{-g}{2cos\theta_W} \bar{\psi}(\vec{k}')\gamma_\mu(g_V^\nu - g_A^\nu \gamma^5)\psi(\vec{k})Z^\mu,$$

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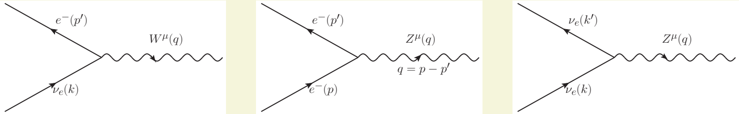

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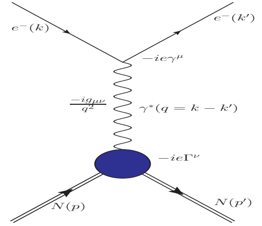
Differential cross section in the Lab frame:($g'_{V,A} = g_{V,A} + 1$)

$$\left.\frac{d\sigma}{d\Omega}\right|_{Lab} = \frac{1}{4\pi^2 m_e} G_F^2 \left(\frac{E_e'^2}{E_\nu}\right) \left[(g'_V + g'_A)^2(E_\nu E'_e - E_\nu|\vec{p}\,'|\cos\theta + m_e E'_e - m_e^2)\right.$$
$$\left. +(g'_V - g'_A)^2(E_\nu + m_e - E'_e)(E'_e - |\vec{p}\,'|\cos\theta) - m_e(g_V'^2 - g_A'^2)(m_e - E'_e + |\vec{p}\,'|\cos\theta)\right].$$

$e^- - p$ elastic scattering process



Invariant amplitude is written

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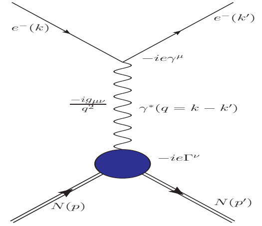

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Four Vectors

(p^μ, p'^μ) **or** (p^μ, q^μ) **or** (q^μ, P^μ) *or...*

Bilinear covariants

1 (*scalar*)

γ^μ (*vector*)

γ^5 (*pseudo scalar*)

$\gamma^\mu\gamma^5$ (*axial vector*)

$\sigma^{\mu\nu}$ (*tensor*)

Γ^μ is written in terms of p, p', q and γ-matrices:

$$\begin{aligned}\Gamma^\mu &= A(Q^2)\gamma^\mu + B(Q^2)(p'-p)^\mu + C(Q^2)(p'+p)^\mu \\ &+ D(Q^2)i\sigma^{\mu\nu}(p'-p)_\nu + E(Q^2)i\sigma^{\mu\nu}(p'+p)_\nu\end{aligned}$$

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$$\Gamma^\mu \quad = \quad [F_1(q^2) + F_2(q^2)]\,\gamma^\mu - \frac{P^\mu}{2M}\, F_2(q^2)$$

where

$$P^\mu = (p+p')^\mu \quad ; \quad q_\nu = (p'-p)_\nu .$$

- Hadronic current is given by

$$j^\mu = \bar{u}(p') \left\{ [F_1(q^2) + F_2(q^2)]\, \gamma^\mu - \frac{P^\mu}{2M} F_2(q^2) \right\} u(p)$$

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$$\begin{aligned} L_{\mu\nu} &= \sum l_\mu \, (l_\nu)^\dagger \\ L_{\mu\nu} &= \frac{1}{2} 4[k_\mu k'_\nu + k'_\mu k_\nu - (k \cdot k' - m^2) g_{\mu\nu}] \end{aligned}$$

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- The hadronic tensor is obtained as

$$\begin{aligned} J^{\mu\nu} &= \sum j^\mu\, (j^\nu)^\dagger \\ \Rightarrow J^{\mu\nu} &= \frac{1}{2} Tr \left[(\not{p}' + M) \left\{ \gamma^\mu (F_1(q^2) + F_2(q^2)) - P^\mu \frac{F_2(q^2)}{2M} \right\} (\not{p} + M) \right. \\ & \left. \left\{ \gamma^\nu (F_1(q^2) + F_2(q^2)) - P^\nu \frac{F_2(q^2)}{2M} \right\} \right] \end{aligned}$$

The differential scattering cross section is given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha^2}{4E^2 sin^4\left(\frac{\theta}{2}\right)}\right)\frac{E'}{E}\left\{\left(F_1^2(q^2) - \frac{q^2}{4M^2}F_2^2(q^2)\right)cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2M^2}\left(F_1(q^2)+F_2(q^2)\right)^2 sin^2\left(\frac{\theta}{2}\right)\right\}$$

This is known as Rosenbluth formula.

Parametrisation of Form factors

- $F_1(q^2)$ and $F_2(q^2)$ are the Dirac and Pauli form factors which are given in terms of the Sachs Form factors $G_E(q^2)$ and $G_M(q^2)$.

$$F_1^{p,n}(q^2) = \frac{G_E^{p,n}(q^2) - \frac{q^2}{4M^2} G_M^{p,n}(q^2)}{\left(1 - \frac{q^2}{4M^2}\right)}$$

$$F_2^{p,n}(q^2) = \frac{G_M^{p,n}(q^2) - G_E^{p,n}(q^2)}{\left(1 - \frac{q^2}{4M^2}\right)}$$

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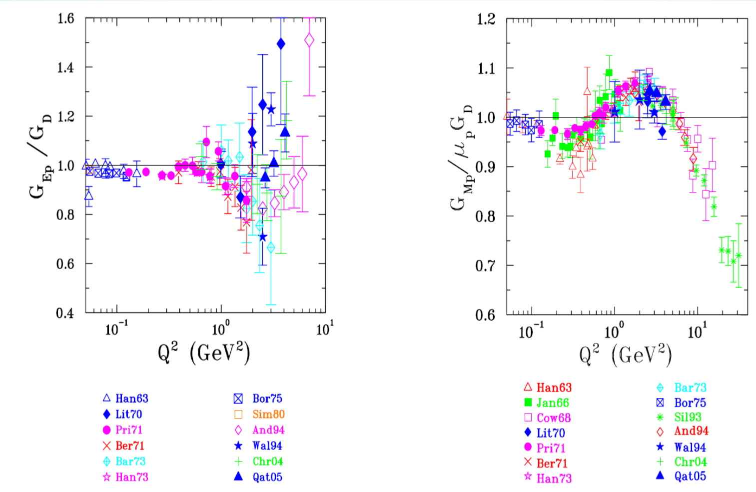
- The dipole form of electromagnetic Sachs Form factors is given by

$$\begin{aligned} G_E^p(q^2) &= \frac{1}{(1 - q^2/M_V^2)^2} = G_D(q^2) \\ G_M^p(q^2) &= (1 + \mu_p) G_E^p(q^2) \\ G_M^n(q^2) &= \mu_n G_E^p(q^2) \\ G_E^n(q^2) &= (\frac{q^2}{4M^2}) \mu_n G_E^p(q^2) \xi_n \\ \xi_n &= \frac{1}{(1 - \lambda_n \frac{q^2}{4M^2})} \end{aligned}$$

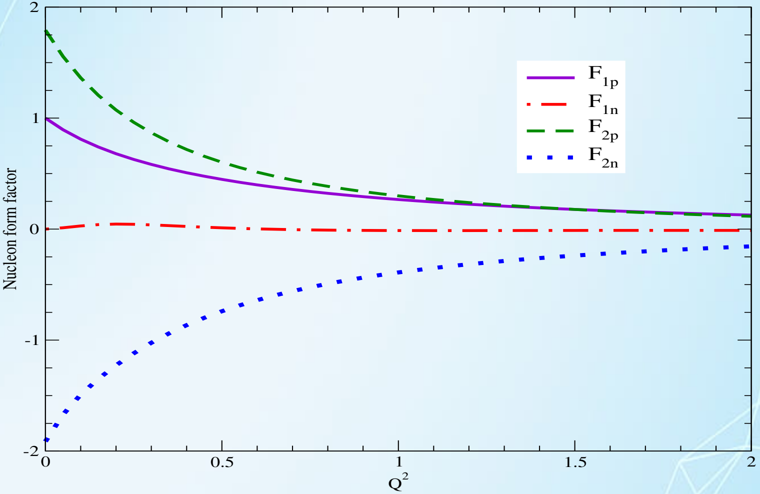
Parametrisation of Form factors

- $\mu_p = 1.7927\mu_N$
- $\mu_n = -1.913\mu_N$
- $M_V = 0.84\ GeV$
- $\lambda_n = 5.6$
- Other parameterizations in recent years:
  - Gari-Krüempelmann
  - Kelly
  - Alberico et al.
  - BBA03, BBBA05, BBBA07 etc.

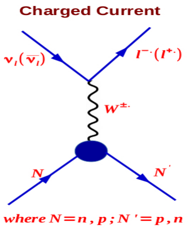
Dipole Sach's Form Factors



Nucleon Form Factors



Quasielastic and Elastic $\nu-$scattering on Nucleons



Quasielastic

$$|\Delta S| = 0$$

$$\nu_l + n \longrightarrow l^- + p$$
$$\bar{\nu}_l + p \longrightarrow l^+ + n$$

$$|\Delta S| = 1,\ \Delta S = \Delta Q$$

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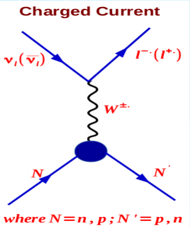

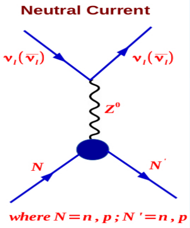

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Quasielastic hyperon production

$$\nu_l + n \longrightarrow l^- + \Sigma^+; \Delta Q = -\Delta S$$

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$$\nu_l/\bar{\nu}_l(k) + N(p) \longrightarrow l^{\pm}(k') + N(p')$$

$$\mathcal{M} = \frac{G_f \cos\theta_c}{\sqrt{2}} \bar{u}(k')\gamma_\mu(1 \pm \gamma_5)u(k) \;\; \bar{u}(p')\left[V^\mu(p',p) - A^\mu(p',p)\right]u(p)$$

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Isospin properties of the weak hadronic current

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- ★ The isospin group of transformations is generated by the three 2×2 Pauli matrices τ_i $(i = 1 - 3)$, which along with the vector currents constitute the isovector part of the hadronic current.

Isospin properties of the weak hadronic current– contd.

★ By defining the isospin raising and lowering operators $\tau^{\pm} = \frac{\tau_1 \pm i\tau_2}{2}$, we can write

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★ Similarly, for the electromagnetic vector current, the hadronic current is given by

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$$V^{em}_\mu(p,n) = \left[\gamma_\mu F_1^{p,n}(Q^2) + i\sigma_{\mu\nu} \frac{q^\nu}{(2M)} F_2^{p,n}(Q^2)\right], \quad (5)$$

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★ $q = p' - p$ with $Q^2 = -q^2$. $F_1^{p,n}(Q^2)$ and $F_2^{p,n}(Q^2)$ are, respectively, the Dirac and Pauli form factors of the nucleon.

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- ★ In fact, a stronger hypothesis of the isotriplet of the vector currents was proposed, which goes beyond the hypothesis of CVC and predicts the form factors $f_{1,2}(Q^2)$ describing the matrix elements of the weak vector current in terms of the electromagnetic form factors of hadrons.

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★ According to the isotriplet hypothesis, the weak vector currents V_μ^+, V_μ^- and the isovector part of the electromagnetic current V_μ^{em} are assumed to form an isotriplet under the isospin symmetry such that f_1 and f_2 are given in terms of the isovector electromagnetic form factors i.e.

$$\begin{aligned} f_1(Q^2) &= F_1^V(Q^2) = F_1^p(Q^2) - F_1^n(Q^2), \\ f_2(Q^2) &= F_2^V(Q^2) = F_2^p(Q^2) - F_2^n(Q^2). \end{aligned} \qquad (6)$$

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- ★ The CVC hypothesis, i.e. $\partial_\mu V^\mu(x) = 0$ implies $f_3(Q^2) = 0$.

PCAC hypothesis

★ In contrast to the vector current which is conserved, the axial-vector current is not conserved. For example, consider the matrix element of the axial-vector current between one pion state and vacuum which enters in the πl_2 decay of pion i.e. $\langle 0|A^\mu(x)|\pi^-\rangle = i f_\pi q^\mu e^{-iq\cdot x}$, where q is the four momentum of the pion.

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- ★ Therefore, the axial-vector current is not conserved. However, since the pion is the lightest hadron, we can work in the limit of $m_\pi \to 0$, and say that the axial-vector current is conserved in the limit

$$\lim_{m_\pi \longrightarrow 0} \partial_\mu A^\mu(x) = 0,$$

which is termed as the partial conservation of axial-vector current (PCAC).

Symmetry properties

- T invariance $\Rightarrow$ form factors are real
- CVC $\Rightarrow f_3(Q^2) = 0$
- G invariance $\Rightarrow f_3(Q^2) = 0$ and $g_2(Q^2) = 0$
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- ★ G violation $\Rightarrow g_2(Q^2) \neq 0$
- ★ T invariance $\Rightarrow$ Real values of $g_2(Q^2)$
- ★ T violation $\Rightarrow$ Imaginary values of $g_2(Q^2)$

Axial vector FF

- Axial vector form factors are parameterized in dipole form as:

$$g_1(Q^2) = \frac{g_A(0)}{\left(1+\frac{Q^2}{M_A^2}\right)^2}\,; \qquad g_3(Q^2) = \frac{2M^2 g_1(Q^2)}{m_\pi^2 + Q^2}$$

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- $g_1(0) = 1.267$ from β decay and $M_A = 1.026$ GeV

Axial vector FF

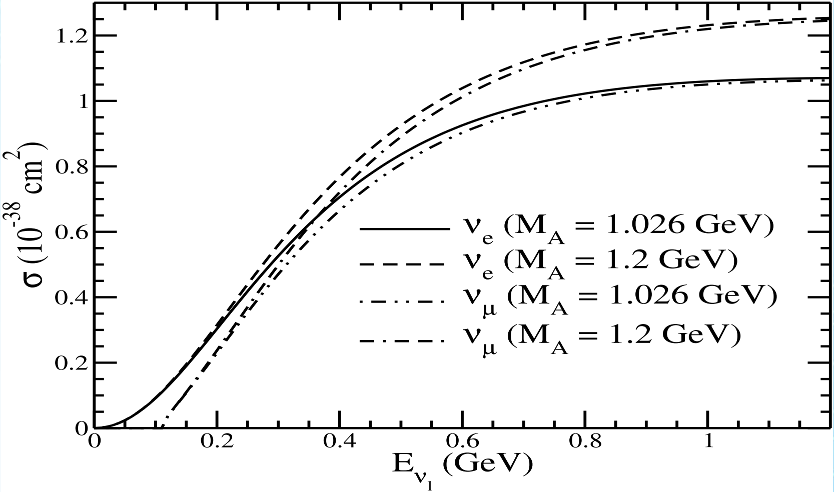
- Axial vector form factors are parameterized in dipole form as:

$$g_1(Q^2) = \frac{g_A(0)}{\left(1 + \frac{Q^2}{M_A^2}\right)^2} \; ; \qquad g_3(Q^2) = \frac{2M^2 g_1(Q^2)}{m_\pi^2 + Q^2}$$

- $g_1(0) = 1.267$ from β decay and $M_A = 1.026$ GeV
- To be determined from experimental data in total cross section and angular distributions of leptons in QE scattering from nucleus and nucleons.

Experiment	M_A (GeV)	Experiment	M_A (GeV)
MINERνA	0.99	SciBooNE	1.21±0.22
NOMAD	1.05±0.02±0.06	K2K-SciBar	1.144±0.077
MiniBooNE	1.23±0.20	K2K-SciFi	1.20±0.12
MINOS	1.19$(Q^2 > 0)$ 1.26$(Q^2 > 0.3GeV^2)$	World Average	1.026± 0.021 1.014±0.014

Cross section as a function of neutrino energy



CC and NC induced meson production channels

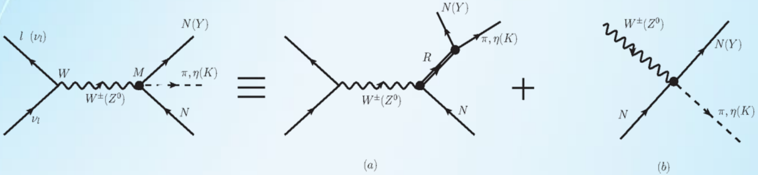
CC reactions

$$
\begin{aligned}
\nu_l(\bar{\nu}_l) + N &\rightarrow l^{\mp} + N' + \pi \\
\nu_l(\bar{\nu}_l) + N &\rightarrow l^{\mp} + N' + n\pi \\
\nu_l(\bar{\nu}_l) + N &\rightarrow l^{\mp} + N' + K(\bar{K}) \\
\nu_l(\bar{\nu}_l) + N &\rightarrow l^{\mp} + N' + \eta \\
\nu_l(\bar{\nu}_l) + N &\rightarrow l^{\mp} + Y + K \\
\bar{\nu}_l + N &\rightarrow l^{+} + Y + \pi \\
&\vdots
\end{aligned}
$$

NC reactions

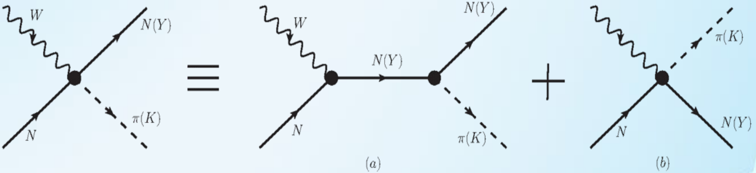
$$
\begin{aligned}
\nu_l(\bar{\nu}_l) + N &\rightarrow \nu_l(\bar{\nu}_l) + N' + \pi \\
\nu_l(\bar{\nu}_l) + N &\rightarrow \nu_l(\bar{\nu}_l) + N' + n\pi \\
\nu_l(\bar{\nu}_l) + N &\rightarrow \nu_l(\bar{\nu}_l) + N + \eta \\
\nu_l(\bar{\nu}_l) + N &\rightarrow \nu_l(\bar{\nu}_l) + Y + K \\
&\vdots
\end{aligned}
$$

Generic Feynman diagrams For the Meson Production



Resonance excitation

Born terms



Born terms

Pole diagrams

Contact term

Nucleon and Delta resonances: PDG

First resonance region

Resonances R_{IJ}	πN branching ratio (%)	ηN branching ratio (%)	$\pi\pi N$ branching ratio (%)	$K\Lambda$ branching ratio (%)	$K\Sigma$ branching ratio (%)
P_{33}(1232)	100	–	–	–	–
P_{11}(1440)	55 – 75	< 1	17 – 50	–	–
D_{13}(1520)	55 – 65	0.07 – 0.09	25 – 35	–	–
S_{11}(1535)	32 – 52	30 – 55	3 – 14	–	–
S_{31}(1620)	25 – 35	–	55 – 80	–	–
S_{11}(1650)	50 – 70	15 – 35	8 – 36	5 – 15	–
D_{15}(1675)	38 – 42	< 1	25 – 45	–	–
F_{15}(1680)	60 – 70	< 1	20 – 40	–	–
D_{33}(1700)	10 – 20	–	10 – 55	–	–
D_{13}(1700)	–	60 – 90	–	–	
P_{11}(1710)	5 – 20	10 – 50	–	5 – 25	–
P_{13}(1720)	8 – 14	1 – 5	50 – 90	4 – 5	–
S_{11}(1895)	2 – 18	15 – 40	–	13 – 23	6 – 20
P_{13}(1900)	1 – 20	2 – 14	40 – 80	2 – 20	3 – 7
F_{35}(1905)	9 – 15	–	–	–	–

Nucleon and Delta resonances: PDG

Second resonance region

Resonances R_{IJ}	πN branching ratio (%)	ηN branching ratio (%)	$\pi\pi N$ branching ratio (%)	$K\Lambda$ branching ratio (%)	$K\Sigma$ branching ratio (%)
$P_{33}(1232)$	100	–	–	–	–
$P_{11}(1440)$	$55-75$	<1	$17-50$	–	–
$D_{13}(1520)$	$55-65$	$0.07-0.09$	$25-35$	–	–
$S_{11}(1535)$	$32-52$	$30-55$	$3-14$	–	–
$S_{31}(1620)$	$25-35$	–	$55-80$	–	–
$S_{11}(1650)$	$50-70$	$15-35$	$8-36$	$5-15$	–
$D_{15}(1675)$	$38-42$	<1	$25-45$	–	–
$F_{15}(1680)$	$60-70$	<1	$20-40$	–	–
$D_{33}(1700)$	$10-20$	–	$10-55$	–	–
$D_{13}(1700)$	–	$60-90$	–	–	
$P_{11}(1710)$	$5-20$	$10-50$	–	$5-25$	–
$P_{13}(1720)$	$8-14$	$1-5$	$50-90$	$4-5$	–
$S_{11}(1895)$	$2-18$	$15-40$	–	$13-23$	$6-20$
$P_{13}(1900)$	$1-20$	$2-14$	$40-80$	$2-20$	$3-7$
$F_{35}(1905)$	$9-15$	–	–	–	–

Nucleon and Delta resonances: PDG

Third and higher resonance regions

Resonances R_{IJ}	πN branching ratio (%)	ηN branching ratio (%)	$\pi\pi N$ branching ratio (%)	$K\Lambda$ branching ratio (%)	$K\Sigma$ branching ratio (%)
P_{33}(1232)	100	–	–	–	–
P_{11}(1440)	55 – 75	< 1	17 – 50	–	–
D_{13}(1520)	55 – 65	0.07 – 0.09	25 – 35	–	–
S_{11}(1535)	32 – 52	30 – 55	3 – 14	–	–
S_{31}(1620)	25 – 35	–	55 – 80	–	–
S_{11}(1650)	50 – 70	15 – 35	8 – 36	5 – 15	–
D_{15}(1675)	38 – 42	< 1	25 – 45	–	–
F_{15}(1680)	60 – 70	< 1	20 – 40	–	–
D_{33}(1700)	10 – 20	–	10 – 55	–	–
D_{13}(1700)	–	60 – 90	–	–	
P_{11}(1710)	5 – 20	10 – 50	–	5 – 25	–
P_{13}(1720)	8 – 14	1 – 5	50 – 90	4 – 5	–
S_{11}(1895)	2 – 18	15 – 40	–	13 – 23	6 – 20
P_{13}(1900)	1 – 20	2 – 14	40 – 80	2 – 20	3 – 7
F_{35}(1905)	9 – 15	–	–	–	–

Spin $\frac{1}{2}$ resonance

$$j^{\mu}_{\frac{1}{2}} = \bar{u}(p')\Gamma^{\mu}_{\frac{1}{2}} u(p)$$

Adjoint Dirac Spinor

N-$R_{\frac{1}{2}}$ transition vertex

Dirac Spinor

Spin $\frac{1}{2}$ resonance

$$j^{\mu}_{\frac{1}{2}} = \bar{u}(p')\Gamma^{\mu}_{\frac{1}{2}} u(p)$$

Adjoint Dirac Spinor — **N-$R_{\frac{1}{2}}$ transition vertex** — **Dirac Spinor**

Transition vertex

- **Positive parity state**

$$\Gamma^{\mu}_{\frac{1}{2}^+} = V^{\mu}_{\frac{1}{2}} - A^{\mu}_{\frac{1}{2}}$$

- **Negative parity state**

$$\Gamma^{\mu}_{\frac{1}{2}^-} = \left[V^{\mu}_{\frac{1}{2}} - A^{\mu}_{\frac{1}{2}}\right]\gamma_5$$

Spin $\frac{1}{2}$ resonance

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Adjoint Dirac Spinor — **N-$R_{\frac{1}{2}}$ transition vertex** — **Dirac Spinor**

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$$\Gamma^\mu_{\frac{1}{2}^-} = \left[V^\mu_{\frac{1}{2}} - A^\mu_{\frac{1}{2}}\right]\gamma_5$$

$$V^\mu_{\frac{1}{2}} = \left[\frac{f_1(Q^2)}{(2M)^2}\left(Q^2\gamma^\mu + \not{q}q^\mu\right) + \frac{f_2(Q^2)}{2M} i\sigma^{\mu\alpha}q_\alpha\right]\gamma_5 \qquad A^\mu_{\frac{1}{2}} = g_1(Q^2)\gamma^\mu + \frac{g_3(Q^2)}{M}q^\mu$$

$N - R_{\frac{1}{2}}$ transition vector form factors

★ **Isospin symmetry relates weak vector form factors with electromagnetic form factors**

$$f^V_{1,2}(Q^2) = F^{R+}_{1,2}(Q^2) - F^{R0}_{1,2}(Q^2).$$

$N - R_{\frac{1}{2}}$ transition vector form factors

★ **Isospin symmetry relates weak vector form factors with electromagnetic form factors**

$$f^V_{1,2}(Q^2) = F^{R+}_{1,2}(Q^2) - F^{R0}_{1,2}(Q^2).$$

★ **EM form factors are derived from the helicity amplitudes extracted from the real and/or virtual photon scattering experiments**

$$A^{p,n}_{\frac{1}{2}} = \sqrt{\frac{2\pi\alpha}{M}\frac{(M_R \mp M)^2 + Q^2}{M_R^2 - M^2}}\left[\frac{Q^2}{4M^2}F_1^{R^+,R^0} + \frac{M_R \pm M}{2M}F_2^{R^+,R^0}\right]$$

$$S^{p,n}_{\frac{1}{2}} = \mp\sqrt{\frac{\pi\alpha}{M}\frac{(M \pm M_R)^2 + Q^2}{M_R^2 - M^2}}\frac{(M_R \mp M)^2 + Q^2}{4M_R M}\left[\frac{M_R \pm M}{2M}F_1^{R^+,R^0} - F_2^{R^+,R^0}\right]$$

$N - R_{\frac{1}{2}}$ transition form factors

- Experimentally, the information regarding the axial vector form factors is scarce
- PCAC and PDDAC relates $g_1(0)$ with $g_{RN\pi}$
- Generalized GT relation gives g_3 in terms of g_1
- $g_{RN\pi}$ is obtained using partial decay width of the $R \to N\pi$

Spin $\frac{3}{2}$ resonances

$$J_\mu^{\frac{3}{2}} = \bar{\psi}^\nu(p')\Gamma_{\nu\mu}^{\frac{3}{2}} u(p)$$

Rarita-Schwinger Spinor **N-$R_{\frac{3}{2}}$ transition vertex** **Dirac Spinor**

Transition vertex

- **Positive Parity states:**

$$\Gamma_{\nu\mu}^{\frac{3}{2}^+} = \left[V_{\nu\mu}^{\frac{3}{2}} - A_{\nu\mu}^{\frac{3}{2}} \right] \gamma_5$$

- **Negative Parity states**

$$\Gamma_{\nu\mu}^{\frac{3}{2}^-} = V_{\nu\mu}^{\frac{3}{2}} - A_{\nu\mu}^{\frac{3}{2}}$$

$N - R_{\frac{3}{2}}$ transition form factors

$$V^{\frac{3}{2}}_{\nu\mu} = \left[\frac{C_3^V}{M}(g_{\mu\nu}\not{q} - q_\nu\gamma_\mu) + \frac{C_4^V}{M^2}(g_{\mu\nu}q\cdot p' - q_\nu p'_\mu) + \frac{C_5^V}{M^2}(g_{\mu\nu}q\cdot p - q_\nu p_\mu) + g_{\mu\nu}C_6^V \right]$$

- **CVC** $\Rightarrow C_6^V = 0$; **Isospin symmetry relates C_i^V (i=3–5) in terms of EM form factors**

$N - R_{\frac{3}{2}}$ transition form factors

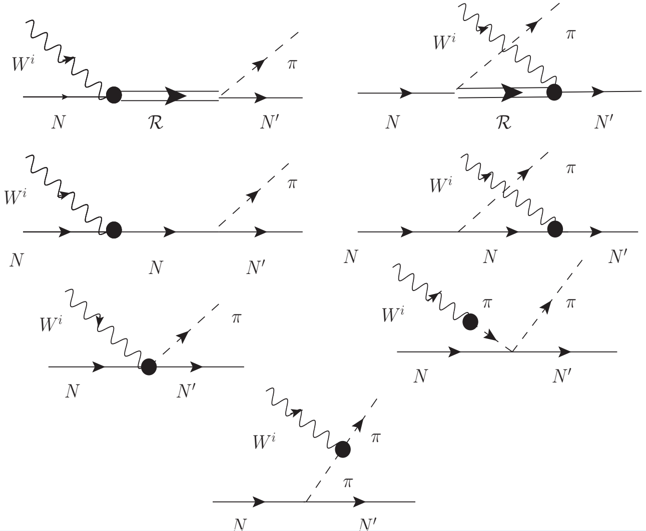
$$V^{\frac{3}{2}}_{\nu\mu} = \left[\frac{C_3^V}{M}(g_{\mu\nu}\not{q} - q_\nu \gamma_\mu) + \frac{C_4^V}{M^2}(g_{\mu\nu} q\cdot p' - q_\nu p'_\mu) + \frac{C_5^V}{M^2}(g_{\mu\nu} q\cdot p - q_\nu p_\mu) + g_{\mu\nu} C_6^V \right]$$

- **CVC $\Rightarrow C_6^V = 0$; Isospin symmetry relates C_i^V (i=3–5) in terms of EM form factors**

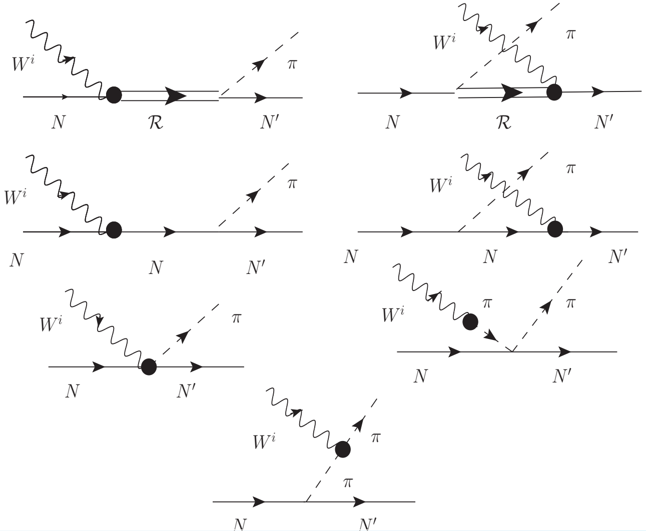
$$A^{\frac{3}{2}}_{\nu\mu} = -\left[\frac{C_3^A}{M}(g_{\mu\nu}\not{q} - q_\nu \gamma_\mu) + \frac{C_4^A}{M^2}(g_{\mu\nu} q\cdot p' - q_\nu p'_\mu) + C_5^A g_{\mu\nu} + \frac{C_6^A}{M^2} q_\nu q_\mu \right] \gamma_5$$

- **Information about the axial vector form factors is poorly known**
- **For Δ, Adler's model guided by SU(6) quark model is adopted, i.e., $C_3^A = 0$ and $C_4^A = -\frac{C_5^A}{4}$**
- **For higher resonances, $C_{3,4}^A$ are taken as zero**
- **PCAC and generalized GT relation relates C_6^A to C_5^A**
- **$C_5^A(0)$ is obtained in terms of $g_{RN\pi}$**

Single pion production: Feynman diagrams



Single pion production: Feynman diagrams



Resonances considered

- $P_{33}(1232)$
- $P_{11}(1440)$
- $S_{11}(1535)$
- $D_{13}(1520)$
- $S_{31}(1620)$
- $S_{11}(1650)$
- $D_{33}(1700)$
- $P_{13}(1720)$

Hadronic current for Born terms: 1π production

$$
\begin{aligned}
j^\mu\big|_{NP} &= a\,\mathscr{A}^{NP}\bar{u}(\vec{p}\,')\not{p}_\pi\gamma_5\frac{\not{p}+\not{q}+M}{(p+q)^2-M^2}\left[V_N^\mu(Q^2)-A_N^\mu(Q^2)\right]u(\vec{p}),\\
j^\mu\big|_{CP} &= a\,\mathscr{A}^{CP}\bar{u}(\vec{p}\,')\left[V_N^\mu(Q^2)-A_N^\mu(Q^2)\right]\frac{\not{p}'-\not{q}+M}{(p'-q)^2-M^2}\not{p}_\pi\gamma_5 u(\vec{p}),\\
j^\mu\big|_{CT} &= a\,\mathscr{A}^{CT}\bar{u}(\vec{p}\,')\gamma^\mu\left(g_A f_{CT}^V(Q^2)\gamma_5 - f_\rho\left((q-p_\pi)^2\right)\right)u(\vec{p}),\\
j^\mu\big|_{PP} &= a\,\mathscr{A}^{PP} f_\rho\left((q-p_\pi)^2\right)\frac{q^\mu}{m_\pi^2+Q^2}\bar{u}(\vec{p}\,')\,\not{q}\,u(\vec{p}),\\
j^\mu\big|_{PF} &= a\,\mathscr{A}^{PF} f_{PF}(Q^2)\frac{(2p_\pi-q)^\mu}{(p_\pi-q)^2-m_\pi^2}2M\bar{u}(\vec{p}\,')\gamma_5 u(\vec{p}),
\end{aligned}
$$

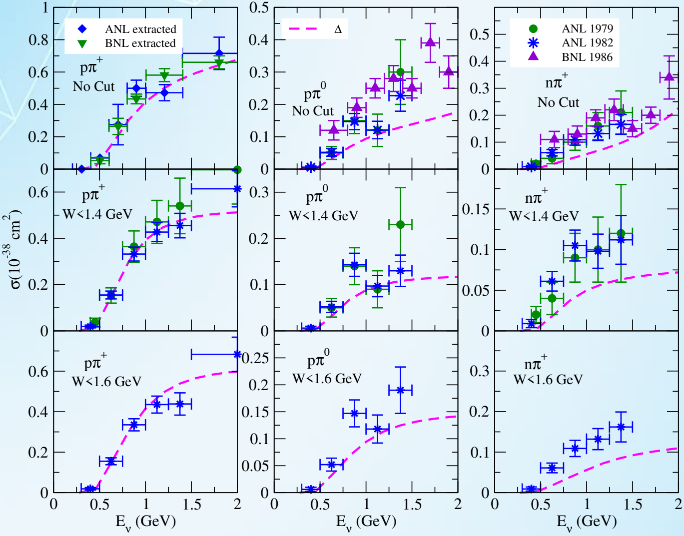
Hadronic current for Born terms: 1π production

$$
\begin{aligned}
j^\mu\big|_{NP} &= a\,\mathscr{A}^{NP}\bar{u}(\vec{p}\,')\not{p}_\pi\gamma_5\frac{\not{p}+\not{q}+M}{(p+q)^2-M^2}\left[V_N^\mu(Q^2)-A_N^\mu(Q^2)\right]u(\vec{p}),\\
j^\mu\big|_{CP} &= a\,\mathscr{A}^{CP}\bar{u}(\vec{p}\,')\left[V_N^\mu(Q^2)-A_N^\mu(Q^2)\right]\frac{\not{p}'-\not{q}+M}{(p'-q)^2-M^2}\not{p}_\pi\gamma_5 u(\vec{p}),\\
j^\mu\big|_{CT} &= a\,\mathscr{A}^{CT}\bar{u}(\vec{p}\,')\gamma^\mu\left(g_A f_{CT}^V(Q^2)\gamma_5 - f_\rho\left((q-p_\pi)^2\right)\right)u(\vec{p}),\\
j^\mu\big|_{PP} &= a\,\mathscr{A}^{PP}f_\rho\left((q-p_\pi)^2\right)\frac{q^\mu}{m_\pi^2+Q^2}\bar{u}(\vec{p}\,')\,\not{q}\,u(\vec{p}),\\
j^\mu\big|_{PF} &= a\,\mathscr{A}^{PF}f_{PF}(Q^2)\frac{(2p_\pi-q)^\mu}{(p_\pi-q)^2-m_\pi^2}2M\bar{u}(\vec{p}\,')\gamma_5 u(\vec{p}),
\end{aligned}
$$

$$V_N^\mu(Q^2) = f_1(Q^2)\gamma^\mu + f_2(Q^2)i\sigma^{\mu\nu}\frac{q_\nu}{2M}$$

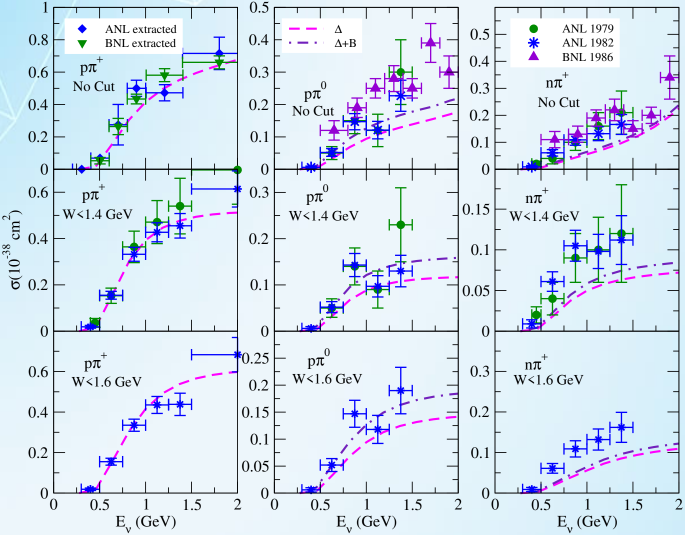
$$A_N^\mu(Q^2) = \left(g_1(Q^2)\gamma^\mu + g_3(Q^2)\frac{q^\mu}{M}\right)\gamma^5$$

σ for CC neutrino induced pion production processes



MSA, AF, SKS, Prog.Part.Nucl.Phys. 129 (2023) 104019.

σ for CC neutrino induced pion production processes



MSA, AF, SKS, Prog.Part.Nucl.Phys. 129 (2023) 104019.

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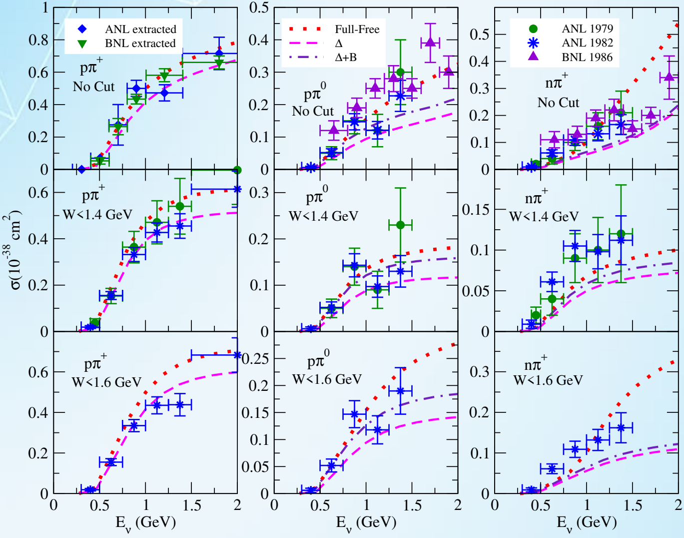

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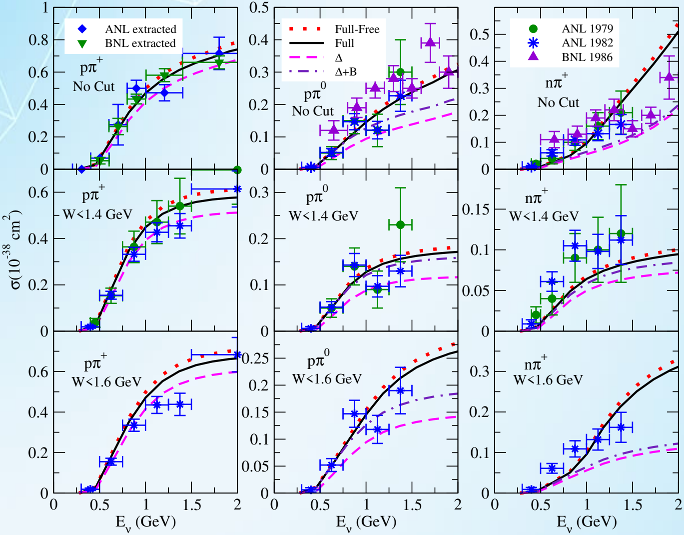
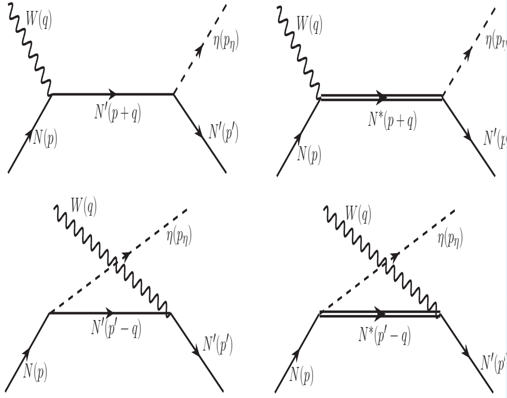


MSA, AF, SKS, Prog.Part.Nucl.Phys. 129 (2023) 104019.

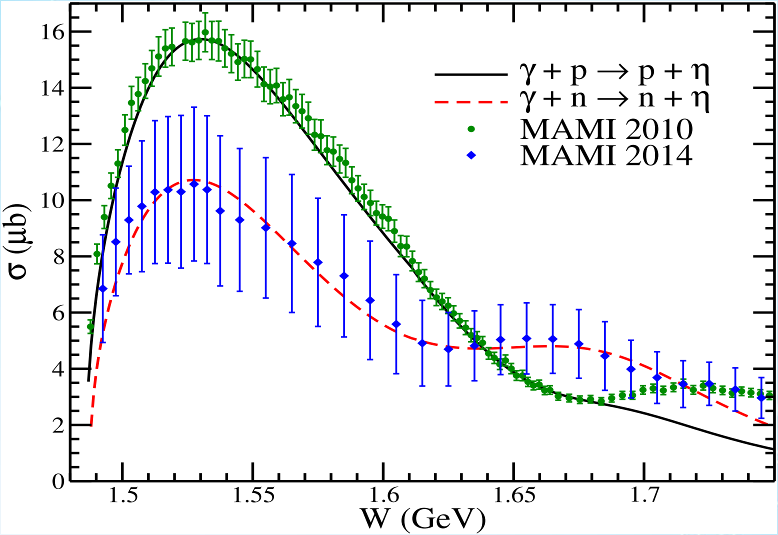
Eta production: Feynman diagrams



Resonances considered

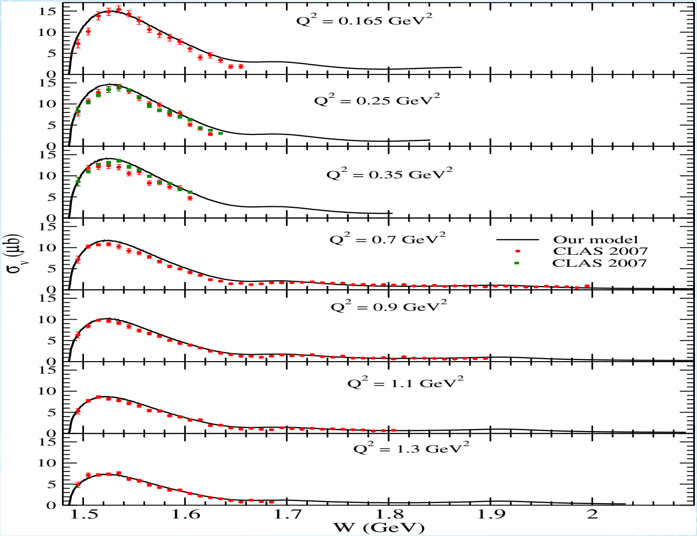
- $S_{11}(1535)$
- $S_{11}(1650)$
- $P_{11}(1710)$

σ for eta photoproduction processes



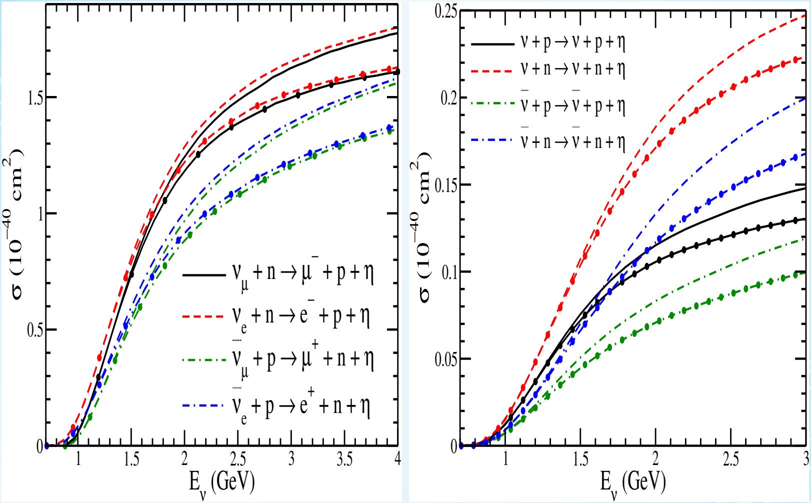
AF, MSA, SKS, Phys. Rev. D 108 (2023), 053009

σ for eta electroproduction processes



AF, MSA, SKS, Phys. Rev. D 108 (2023), 053009

σ for CC and NC induced eta production processes

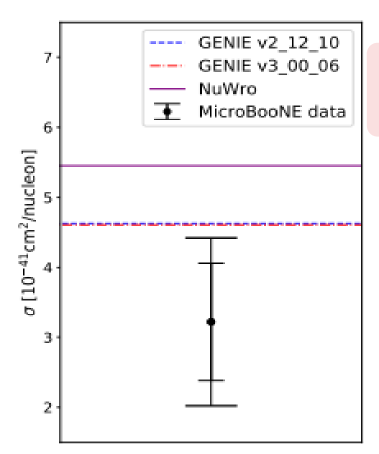


AF, MSA, SKS, Phys. Rev. D 108 (2023), 053009

Lines with solid dots show $S_{11}(1535)$ contribution only

MicroBooNE η production result

$$\langle\sigma\rangle = (3.22 \pm 0.84 \pm 0.86) \times 10^{-41}\ \text{cm}^2/\text{nucleon}$$



- $\langle\sigma\rangle_{\text{free}} = 1.87 \times 10^{-41}$ cm^2/nucleon
- $\langle\sigma\rangle_{^{40}\text{Ar}} = 1.78 \times 10^{-41}$ cm^2/nucleon

Phys. Rev. Lett. 132, 151801 (2024)

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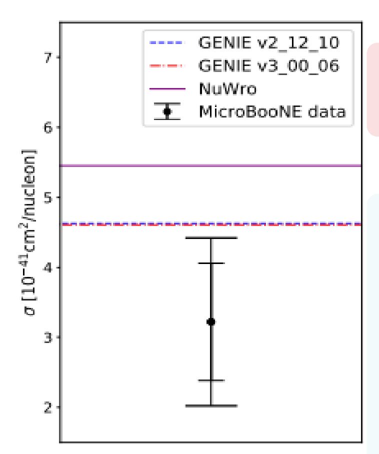

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- GENIE $v2\_12\_10$: 4.63×10^{-41} cm^2/nucleon
- GENIE $v3\_00\_06G18\_10a\_02\_11a$: 4.61×10^{-41} cm^2/nucleon
- NuWro 19.02.1: 5.45×10^{-41} cm^2/nucleon
- NEUT v5.4.0: 11.9×10^{-41} cm^2/nucleon

Phys. Rev. Lett. 132, 151801 (2024)

Deep-inelastic Scattering Region

★ When an (anti)neutrino interacts with a nucleon via the exchange of intermediate vector boson, at sufficiently large four momentum transfer squared (Q^2), the nucleon breaks up, completely loses its identity and produces a jet of hadrons, mainly mesons, in the final state along with the corresponding lepton.

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★ As Q^2 grows it becomes sufficient for interactions to take place with a single quark, which is the true or rigorous deep inelastic scattering (DIS) perturbative QCD region.

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★ These higher-Q^2 interactions are used in determining the parton distribution functions within the nucleon. DIS cross sections are expressed in terms of the nucleon structure functions, which are in turn described in terms of the parton distribution functions (PDFs).

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- ★ As Q^2 grows it becomes sufficient for interactions to take place with a single quark, which is the true or rigorous deep inelastic scattering (DIS) perturbative QCD region.
- ★ These higher-Q^2 interactions are used in determining the parton distribution functions within the nucleon. DIS cross sections are expressed in terms of the nucleon structure functions, which are in turn described in terms of the parton distribution functions (PDFs).
- ★ The inclusion of W dependence in the definition of DIS enters qualitatively to avoid the region dominated by the resonance excitations and quantitatively through the second DIS requirement, determining the probability of finding a quark on which to scatter with the chosen Q^2 restriction.

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- ★ A cut of W $<$ 2 GeV that isolates a region dominated by resonance production, which starts with the main contribution of the Δ resonance (W=1.232 GeV) and multiple smaller higher-W resonances. The nonresonant meson production and nonperturbative multi-quark meson production from the SIS region intermix with the resonant meson production and there is no possible way to separate, experimentally, the meson produced by these processes.

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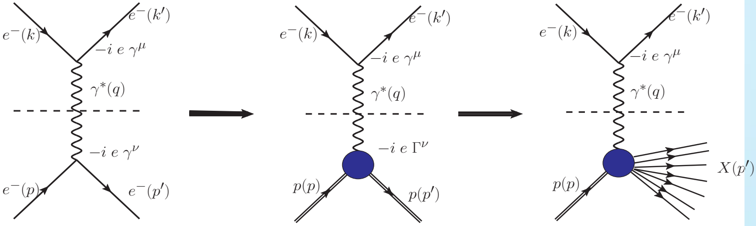
★ Therefore, in practice, it is difficult to experimentally have a well-defined SIS region, which separates from the true DIS region (the region where perturbative QCD goes to nonperturbative QCD) or an SIS region which separates from the resonance region (nonperturbative QCD to resonant meson production).

$e^- - p$ deep inelastic scattering



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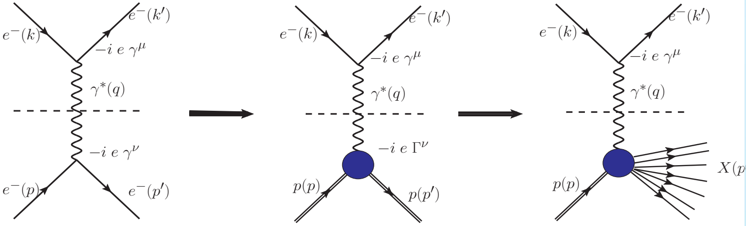


For the two body exclusive process $1+2 \to 3+4+...+n$ the differential cross section is

$$\sigma = \frac{1}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \int |\mathscr{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - ...) \times$$

$$\Pi_{j=3}^{n} 2\pi \ \ \delta(p_j^2 - m_j^2) \ \ \theta(p_j^0) \frac{d^4 p_j}{(2\pi)^4}$$

Dynamics of the scattering is contained in

$$|\mathscr{M}|^2 = \frac{\alpha^2}{q^4} \frac{E'}{E} L_{\mu\nu} W^{\mu\nu}$$

We write a general parameterization of the hadronic tensor

$$W^{\mu\nu} = -g_{\mu\nu} W_1 + \frac{p_\mu p_\nu}{M^2} W_2 - i\varepsilon_{\mu\nu\lambda\sigma} \frac{p^\lambda q^\sigma}{2M^2} W_3 + \frac{q_\mu q_\nu}{M^2} W_4 + \frac{(p_\mu q_\nu + p_\nu q_\mu)}{2M^2} W_5 + \frac{i(p_\mu q_\nu - p_\nu q_\mu)}{2M^2} W_6$$

- By contraction of hadronic tensor with $L_{\mu\nu}$, the terms with W_3 and W_6 become zero

$$L_{\mu\nu}\left(i\varepsilon_{\mu\nu\lambda\sigma} \frac{p^\lambda q^\sigma}{2M^2}\right) \to 0$$

$$L_{\mu\nu}\left(\frac{i(p_\mu q_\nu - p_\nu q_\mu)}{2M^2}\right) \to 0$$

Applying CVC: $q_\mu W^{\mu\nu} = 0$ which leads to

$$W_4 = \frac{-2p\cdot q}{q^2} W_2$$

$$W_5 = \frac{M^2}{q^2} W_1 + \left(\frac{p\cdot q}{q^2}\right)^2 W_2$$

Therefore, hadronic tensor can be written as:

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1 + \left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2} q^\nu\right)\frac{W_2}{M^2}$$

W_1 and W_2 can be the functions of any two Lorentz-invariant scalars like:

- q^2
- $\nu = \frac{p.q}{M}$
- $x = \frac{-q^2}{2p.q}$
- $y = \frac{p.q}{p.k}$

$$L_{\mu\nu}W^{\mu\nu} = 4W_1(k.k') + \frac{2W_2}{M^2}[2(p.k)(p.k') - M^2(k.k')]$$

In the Lab frame, we have

$$k.k' = 2EE' \sin\frac{\theta}{2}, \qquad p.k = EM, \qquad p.k' = E'M$$

Therefore, $L_{\mu\nu}W^{\mu\nu}$ is obtained as:

$$L_{\mu\nu}W^{\mu\nu} = 4EE'\left[\cos^2\frac{\theta}{2}W_2(\nu,q^2) + \sin^2\frac{\theta}{2}2W_1(\nu,q^2)\right]$$

The differential scattering cross section in the energy and angle of the scattered electron for $ep \longrightarrow eX$ is

$$\frac{d\sigma}{dE'd\Omega} = \frac{\alpha^2}{4E^2 sin^4(\frac{\theta}{2})}\left[W_2(\nu,Q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,Q^2)\sin^2\frac{\theta}{2}\right]$$

- Dimensionless SF

$$\begin{aligned} MW_1(\nu, Q^2) &= F_1(x) \\ \nu W_2(\nu, Q^2) &= F_2(x) \end{aligned}$$

- In terms of PDFs

$$\begin{aligned} F_2(x) &= \sum_i e_i^2 \, x \, (q_i(x) + \bar{q}_i(x)) \\ F_{2p}(x) &= x \left[\frac{4}{9}(u(x) + \bar{u}(x)) \; + \; \frac{1}{9}(d(x) + \bar{d}(x)) \right. \\ &\left. + \frac{1}{9}(s(x) + \bar{s}(x)) + \frac{4}{9}(c(x) + \bar{c}(x)) \right] \\ F_{2n}(x) &= x \left[\frac{4}{9}(d(x) + \bar{d}(x)) \; + \; \frac{1}{9}(u(x) + \bar{u}(x)) \right. \\ &\left. + \frac{1}{9}(s(x) + \bar{s}(x)) + \frac{4}{9}(c(x) + \bar{c}(x)) \right] \end{aligned}$$

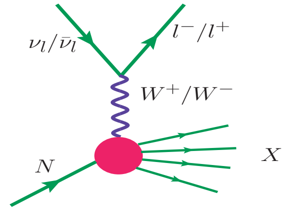
- Therefore,

$$\frac{d\sigma}{dE' d\Omega} = \frac{4 \, \alpha^2 \, E'^2 \, \cos^2 \frac{\theta}{2}}{q^4 M \nu} \left[2 \, \nu \, F_1(x) \, tan^2 \frac{\theta}{2} + M \, F_2(x) \right]$$

$\nu - N$ deep inelastic scattering

- The charged current $\nu_l(\bar{\nu}_l) - N$ DIS is given by

$$\nu_l/\bar{\nu}_l(k) + N(p) \to l^-/l^+(k') + X(p')$$



- For the weak interaction the hadronic tensor

$$W^N_{\mu\nu} = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) W_1 + \frac{1}{M_N^2} \times \left(p_\mu - \frac{p.q}{q^2} q_\mu\right)\left(p_\nu - \frac{p.q}{q^2} q_\nu\right) W_2 - \frac{i}{2M^2}\epsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma W_3$$

contains an additional term arising due to the parity violation.

- Dimensionless SF

$$
\begin{aligned}
M_N W_{1N}(\nu, Q^2) &= F_{1N}(x), \\
\nu W_{2N}(\nu, Q^2) &= F_{2N}(x), \\
\nu W_{3N}(\nu, Q^2) &= F_{3N}(x).
\end{aligned}
$$

- In terms of PDFs

$$
\begin{aligned}
F_2^{\nu}(x) &= \sum_i x\,(q_i(x) + \bar{q}_i(x)) \\
x\,F_3^{\nu}(x) &= \sum_i x\,(q_i(x) - \bar{q}_i(x)) \\
F_2^{\nu p}(x) &= 2x[d(x) + s(x) + \bar{u}(x) + \bar{c}(x)], \\
F_2^{\bar{\nu} p}(x) &= 2x[u(x) + c(x) + \bar{d}(x) + \bar{s}(x)], \\
xF_3^{\nu p}(x) &= 2x[d(x) + s(x) - \bar{u}(x) - \bar{c}(x)], \\
xF_3^{\bar{\nu} p}(x) &= 2x[u(x) + c(x) - \bar{d}(x) - \bar{s}(x)],
\end{aligned}
$$

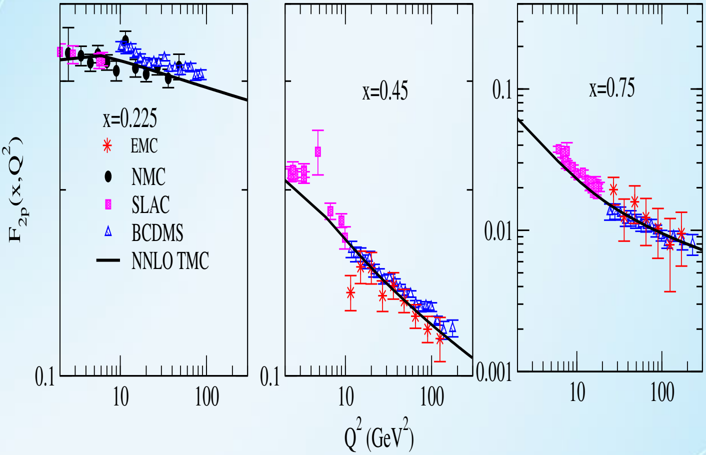
The differential scattering cross section is given by

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{G_F^2\, E'^2\, cos^2\left(\frac{\theta}{2}\right)}{2\,\pi^2\, M\, \nu}\left(\frac{M_W^2}{M_W^2+Q^2}\right)^2\left[2\nu\, tan^2\left(\frac{\theta}{2}\right)\, F_1(x) + M\, F_2(x) \pm (E+E')\, tan^2\left(\frac{\theta}{2}\right)\, F_3(x)\right]$$

The term corresponding to $F_3(x)$ will have

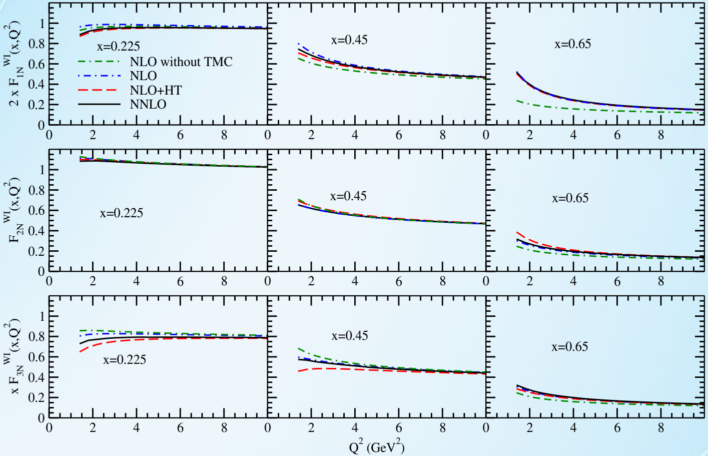
- positive sign for neutrino
- negative sign for antineutrino

EM structure function



Phys. Rev. D 99, 093011 (2019).

Weak structure function



Phys. Rev. D 101, 033001 (2020).

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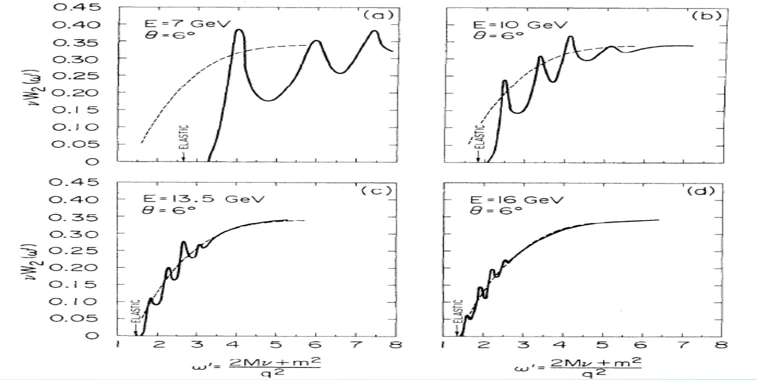
- ★ The question arises "whether there exists a region where both processes apply simultaneously" i.e. whether a parton based description can "on the average" reproduce data in the kinematic region of hadronic resonances.
- ★ To understand this transition region, the phenomenon of Quark-Hadron duality comes into play; this duality basically connects the inclusive production cross sections in the two regions.

Bloom and Gilman defined duality by comparing the structure functions obtained from inclusive electron-nucleon scattering with resonance production.



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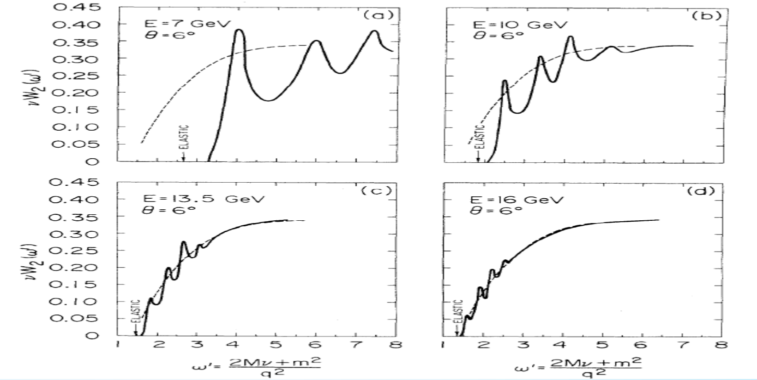

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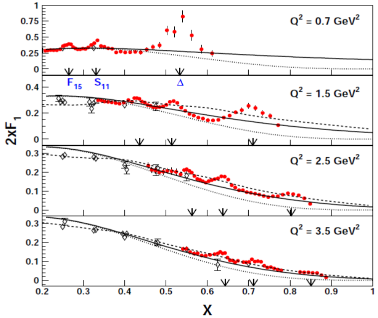
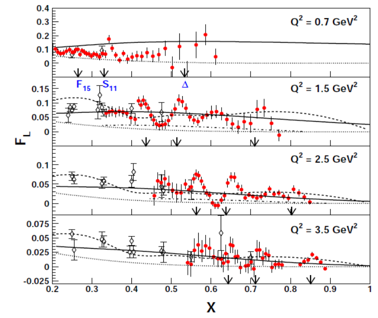
★ This seems to be valid in each resonance region individually (local duality) as well as in the entire resonance region (global duality), when the structure functions are summed over higher resonances.

Electromagnetic structure functions





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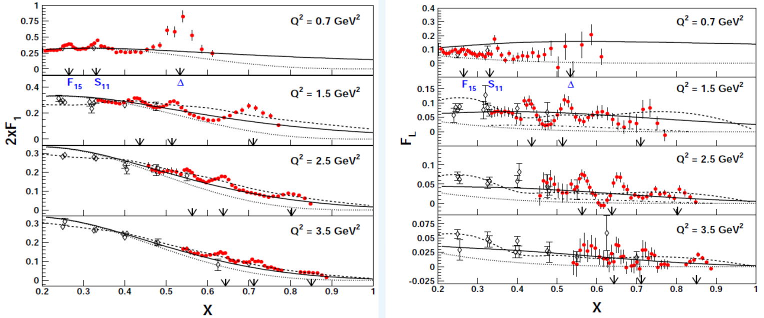

- The mass peak regions move to the large x values with increasing Q^2.
- Peak positions are somewhat different for longitudinal and transverse structure functions.
- Above $Q^2 \geq 1\ GeV^2$, the mass peaks are relatively more prominent for F_L than $2xF_1$, signifies their W dependence.

Liang et al., Phys.Rev.C 105 (2022) 6, 065205.

$\nu_l/\bar{\nu}_l - p$ scattering – QH-Duality

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- **In this case the resonance structure functions are significantly larger than the LT functions, $F_i^{\nu p(\mathrm{res})} > F_i^{\nu p(\mathrm{LT})}$, and quark-hadron duality is clearly violated for a proton target.**

$\nu_l/\bar{\nu}_l - n$ scattering – QH-Duality

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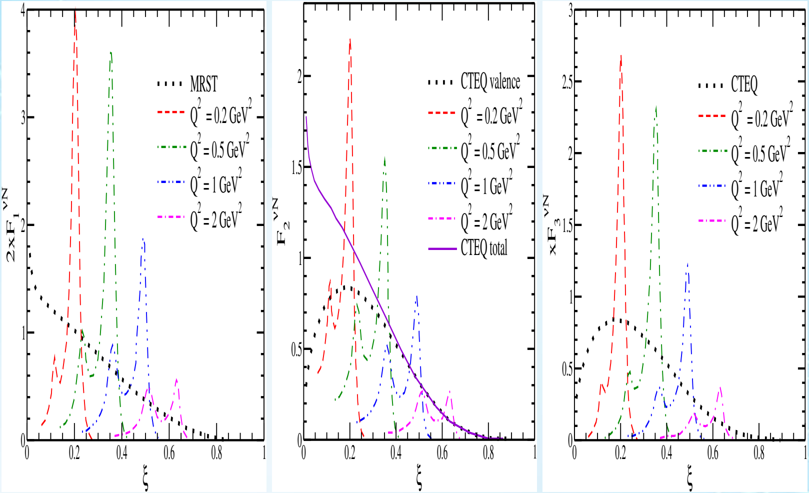
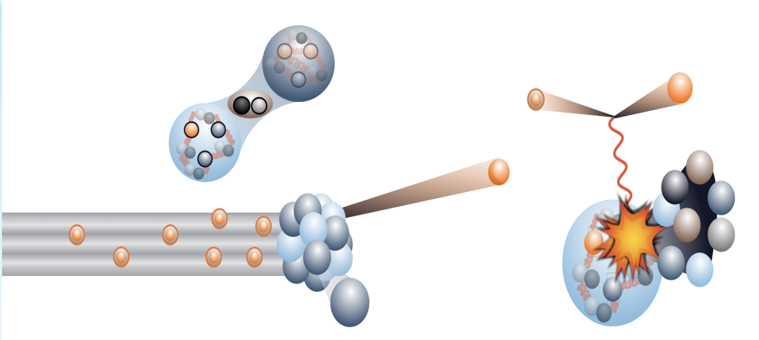
- $F_i^{\nu n(\mathrm{res})} < F_i^{\nu n(\mathrm{LT})}$**, so that quark-hadron duality does not hold for this case either.**
- **In neutrino scattering, duality for the average of proton and neutron structure functions holds with better accuracy than electron scattering.**

QH duality in neutrino scattering



Lalakulich et al., Phys. Rev. C 75, 015202 (2007)

(Anti)neutrino interaction with nuclear target



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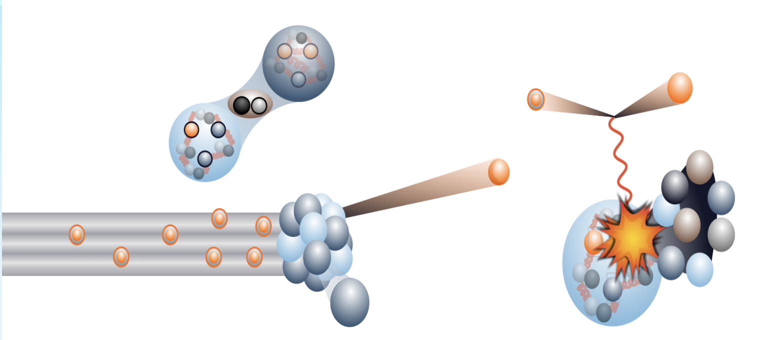
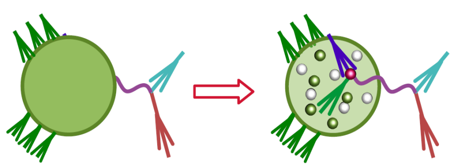


When interaction takes place with a bound nucleon "NME" come into play.

Impulse Approximation

At relevant kinematics, the dominant process of neutrino-nucleus interaction is scattering off a single nucleon, with the remaining nucleons acting as a spectator system.

This description is valid when the momentum transfer $|\vec{q}|$ is high enough ($|\vec{q}| > 200$ MeV).



Nuclear medium effects

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Fermi Gas Model and Spectral Functions:

Include effects of Fermi motion and binding energy.

Pauli Blocking:

Nucleons are fermions and obey Fermi-Dirac statistics which allows only two nucleons per energy level. Scatterings which would take the nucleon to a new state already occupied by other nucleons are not allowed.

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Final State Effects:

Any hadrons we produce in the interaction now have to travel through the nucleus before we have any chance of detecting them. Along the way they can interact with other.

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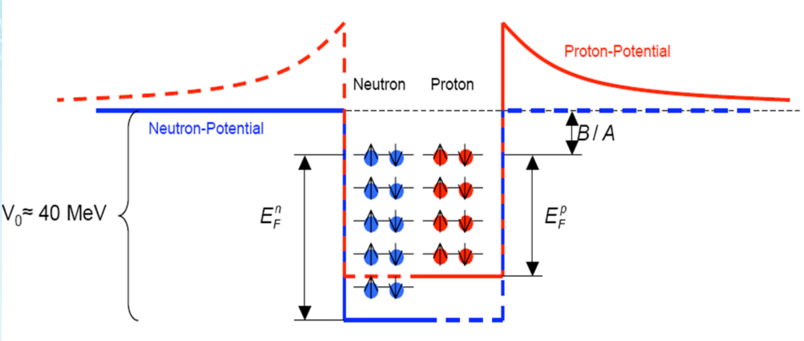
  In such a gas at T=0K (nucleus in its ground state), all the energy levels up to a maximum, known as Fermi energy E_F are occupied by the particles.
- In other words at temperature T = 0, the lowest states will be filled up to a maximum momentum, called the Fermi momentum p_F, the maximum possible momentum of the ground state.


Proton-Potential
Neutron
Proton
Neutron-Potential
B / A
V0≈ 40 MeV
E_F^n
E_F^p

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- The potential that every nucleon feels is a superposition of the potentials of the other nucleons.
- The neutron potential well is deeper that the proton well because of the missing Coulomb repulsion. The model assumes common Fermi energy for the protons and neutrons in stable nuclei, otherwise $p \to n$ decay would happen spontaneously. This implies that there are more neutrons states available and therefore $N > Z$ for heavier nuclei.

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- These may lead to an interaction of W/Z with additional degrees of freedom in nuclei, which may be present due to nucleon interactions
- Moreover after the interaction, new particles may be produced which are subsequently absorbed in the nucleus, leaving only leptons leading to QE like events

Theory of QE $\nu-$ Nucleus scattering

Nuclear calculations are generally done in Nucleon only Impulse Approximation(NOIA). The following nuclear effects are taken into account:

- Pauli Blocking of Nucleons
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Beyond Impulse Approximation:

- Short range and long range correlations
- Meson Exchange currents
- Initial state interactions, spectral functions
- Final state interactions(FSI) of nucleons and pions in nuclear medium

QE neutrino-nucleus scattering from nuclear targets are studied in the entire energy region of ν-energy.

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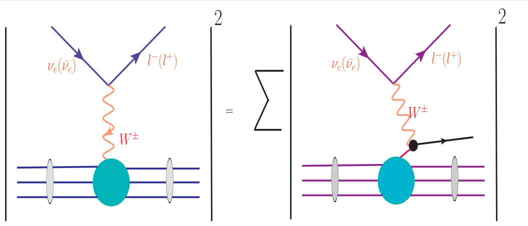
- In the low energy scattering few nuclear states are excited. Calculations are done using the specified final states and a sum over all the final states are performed.
- Simplest calculations are done using shell model (with its various extensions like RPA, CRPA, QRPA) for describing the initial and final state of nucleus.

  In Impulse Approximation, it is assumed that the cross section is given as incoherent sum of scattering from individual nucleons.



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- Fermi Gas Model(with various versions)
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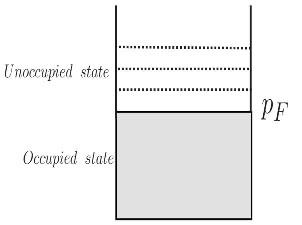
In this model it is assured that the nucleons in a nucleus (or nuclear matter) occupy one nucleon per unit cell in phase space so that the total number of nucleons N is given by

$$N = 2V \int_0^{p_F} \frac{d^3\vec{p}}{(2\pi)^3}$$

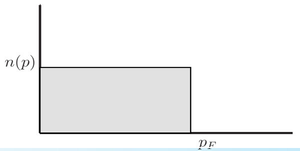
where a factor of two to account spin degree of freedom. All states upto a maximum momentum p_F $(p < p_F)$ are filled. The momentum states higher than $\vec{p} > \vec{p}_F$ are unoccupied.



Fermi Gas Model (with various versions)

The occupation number $n(\vec{p})$ is defined as:

$$\begin{aligned} n(\vec{p}) &= 1, \vec{p} < \vec{p}_F \\ &= 0, \vec{p} > \vec{p}_F \\ \Longrightarrow \rho &= \frac{N}{V} = \frac{p_F^3}{3\pi^2} \\ \therefore p_F &= (3\pi^2 \rho)^{\frac{1}{3}} \end{aligned}$$



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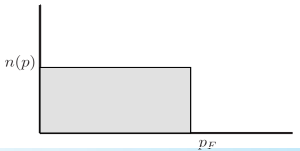

Protons and neutrons are supposed to have different Fermi sphere so

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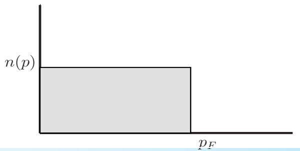

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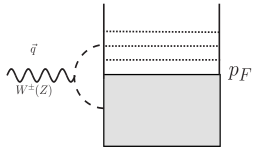
Under a weak interaction induced by $\nu/\bar{\nu}$ a nucleon is excited from an occupied state to an unoccupied state i.e.



Creating a hole in the Fermi sea and a particle above the sea. This is known as 1p1h excitation, with the condition that:

- initial momentum: $\vec{p} < \vec{p}_F^{\,i}$
- final momentum: $|\vec{p} + \vec{q}| > \vec{p}_F^{\,f}$

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For free nucleon

$$\frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE_l'} = \frac{M^2}{E_n E_p}\frac{|\vec{k}'|}{|\vec{k}\,|}\frac{G^2}{4\pi^2}L_{\mu\nu}J^{\mu\nu}\delta(q_0+E_n-E_p)$$

where $J^{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[(\not{p}'+M)\Gamma^\mu(\not{p}+M)\tilde{\Gamma}^\nu\right]$

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Inside the nucleus

$$\left.\frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE_l'}\right|_{Nucleus} = \frac{G^2}{4\pi^2}\int \frac{M^2}{E_n E_p} 2d\vec{p} \frac{1}{(2\pi)^3} n_n(\vec{p})(1 - n(|\,\vec{p}+\vec{q}\,|)) \frac{|\vec{k}'|}{|\vec{k}\,|} \times \delta(q_0 + E_n - E_p) L_{\mu\nu} J^{\mu\nu}$$

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$$\frac{M^2}{E_n E_p} J_{\mu\nu} \delta(q_0 + E_n - E_p) \longrightarrow \int f(q,p) J_{\mu\nu}(p) \frac{d^3 p}{(2\pi)^3}$$

$$f(q,p) = n(\mid \vec{p} \mid)(1 - n(\mid \vec{p} + \vec{q} \mid) \frac{M^2}{E_n E_p} \delta(q_0 + E_n - E_p)$$

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- $J_{\mu\nu}$ involves terms like $g_{\mu\nu}, q_\mu q_\nu, p_\mu p_\nu$ and $p_\mu p_\nu$.
  Now $\int f(q,p) J_{\mu\nu}(p) \frac{d^3p}{(2\pi)^3}$ can be evaluated explicitly.

These are the main features of **Smith and Moniz** **RFG** model.

Inside the nucleus, neutrino interacts with a bound nucleon.

Local Density Approximation

Cross section is evaluated as a function of local Fermi momentum($p_F(r)$). The free ν-N cross section is folded over the density of the nucleon in the nucleus and integrated over the size of whole nucleus.

Differential scattering cross section

$$\left(\frac{d\sigma}{dE_l d\Omega_l}\right)_{\nu A} = \int d\vec{r}\, \rho_n(r) \left(\frac{d\sigma}{dE_l d\Omega_l}\right)_{\nu N}$$

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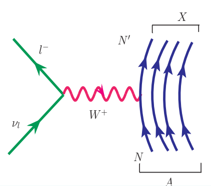
$$N = 2V \int_0^{p_F(r)} \frac{d^3\vec{p}}{(2\pi)^3}$$

$$\sigma(E_l, |\vec{k}'|) = \int 2d\vec{r}\,d\vec{p}\,\frac{1}{(2\pi)^3} n_n(\vec{p},\vec{r})\,\sigma_0(E_l, |\vec{k}'|)$$

where $\sigma_0(E_l, |\vec{k}'|)$ is the free ν-N cross section.

Theory of QE ν–Nucleus scattering

Inclusive CCQE Process



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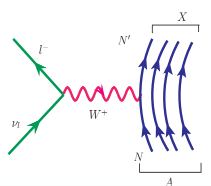

Nuclear medium effects

- Fermi motion & binding energy
- Pauli blocking
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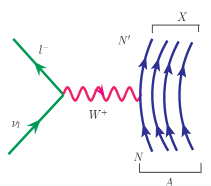
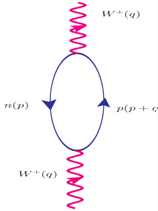


1p-1h Excitation



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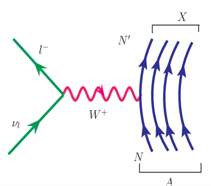

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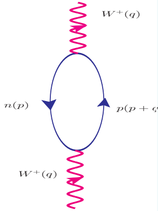
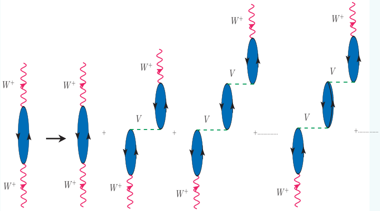


RPA



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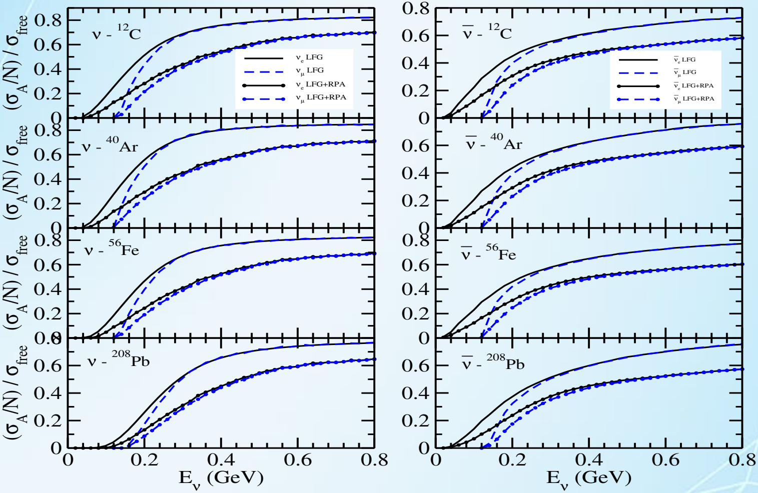
- The lepton energy(and angles) of these QE like events correspond to the scattering from strongly correlated nucleons(not quasi-free) or mesons in flight or nucleons in excited state for which the above kinematics does not hold.
- One needs theoretical inputs for description of QE like events to make correction for those events in determination of E_ν.

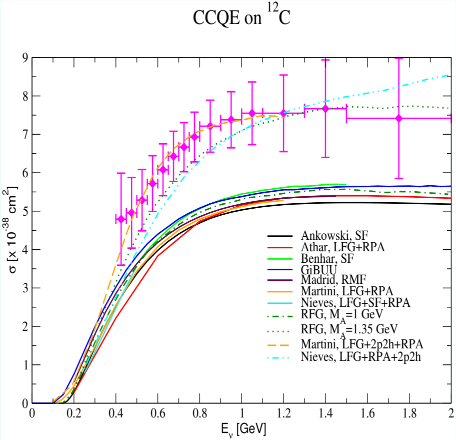
σ vs. E_ν for ^{12}C, ^{40}Ar, ^{56}Fe and ^{208}Pb targets



F. Akbar et al., IJMPE 24 (2015) 1550079.

Multinucleon correlation effect

Cross Section



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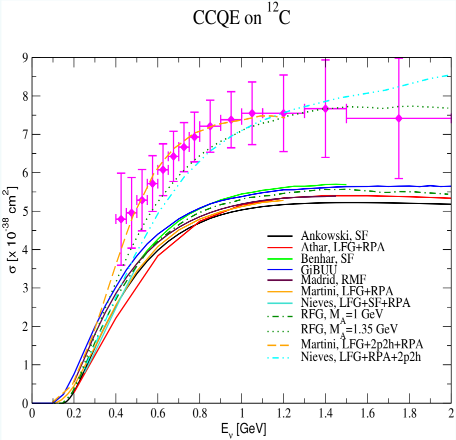
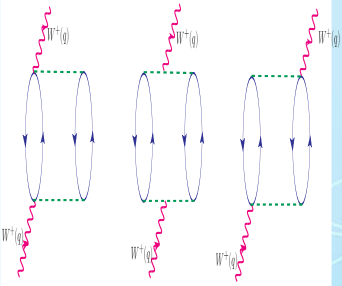


2p-2h Excitations



Marco Martini
Juan Nieves

Production of pions inside the nucleus

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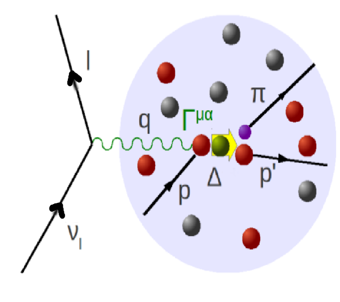


Modification in the width $\tilde{\Gamma}$

$$\frac{\tilde{\Gamma}}{2} \to \frac{\tilde{\Gamma}}{2} - Im\Sigma_\Delta$$

and in mass M_Δ of the Δ resonance

$$M_\Delta \to M_\Delta + Re\Sigma_\Delta$$

To evaluate Δ self energy

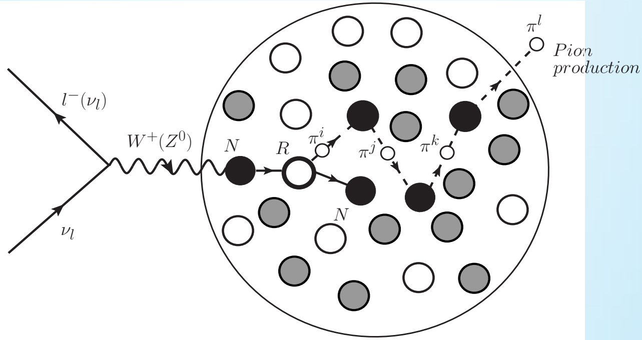
- **Many body expansion in terms of ph and Δh excitations and spin-isospin induced interaction**

Imaginary part of Δ self energy accounts for

- **Quasielastic corrections($WN \to N\pi$)**
- **Two body absorption($WNN \to NN$) and**
- **Three body absorption($WNNN \to NNN$)**

FSI of produced pions: elastic and QE scattering

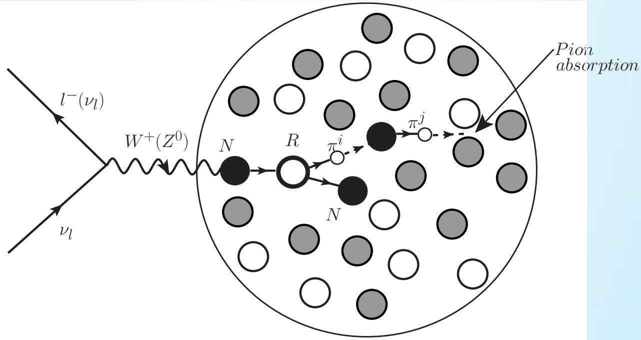
$$\pi^+ + n \to \pi^0 + p; \qquad \pi^- + p \to \pi^0 + n; \qquad \pi^i + N \to \pi^i + N$$



The Physics of Neutrino Interactions (CUP) 2020

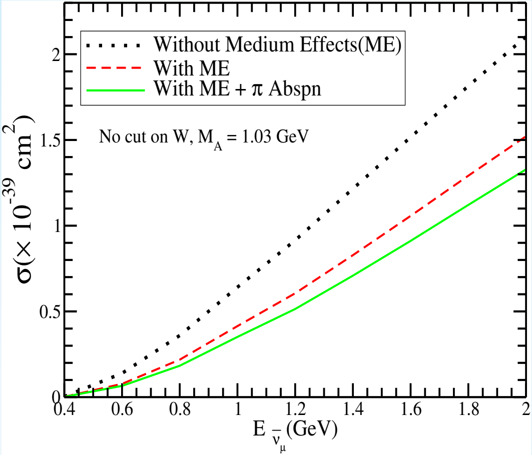
FSI of produced pions: absorption and QE like events

$$\pi^+ + n \to p; \qquad \pi^- + p \to n; \qquad \pi^0 + N \to N'$$



The Physics of Neutrino Interactions (CUP) 2020

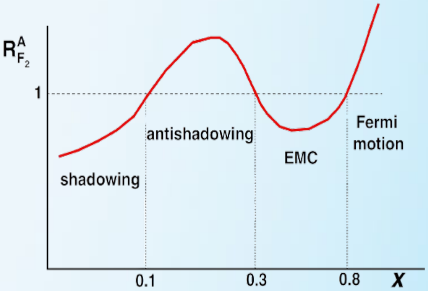
NMEs in Δ production & it's subsequent decay



Nuclear medium effects in DIS

Differential cross section for $\nu_l/\bar{\nu}_l$ induced DIS process off nuclear target:

$$\frac{d^2\sigma_A}{dxdy} = \frac{G_F^2 y}{16\pi}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 L^{\alpha\beta} W^A_{\alpha\beta},$$



The nuclear hadronic tensor $(W^A_{\alpha\beta})$ is written in terms of $F_{iA}(x,Q^2);\ i = 1-5$ as:

$$W^A_{\alpha\beta} = -g_{\alpha\beta}F_{1A}(x,Q^2) + \frac{p_\alpha p_\beta}{p\cdot q}F_{2A}(x,Q^2) - \frac{i}{2p\cdot q}\epsilon_{\alpha\beta\rho\sigma}p^\rho q^\sigma F_{3A}(x,Q^2) + \frac{q_\alpha q_\beta}{p\cdot q}F_{4A}(x,Q^2) + (p_\alpha q_\beta + p_\beta q_\alpha)F_{5A}(x,Q^2).$$

$\nu_l - A$ DIS process: Theoretical model

NME which have been recently considered:

- **Fermi motion, binding energy and nucleon correlations through spectral function.**

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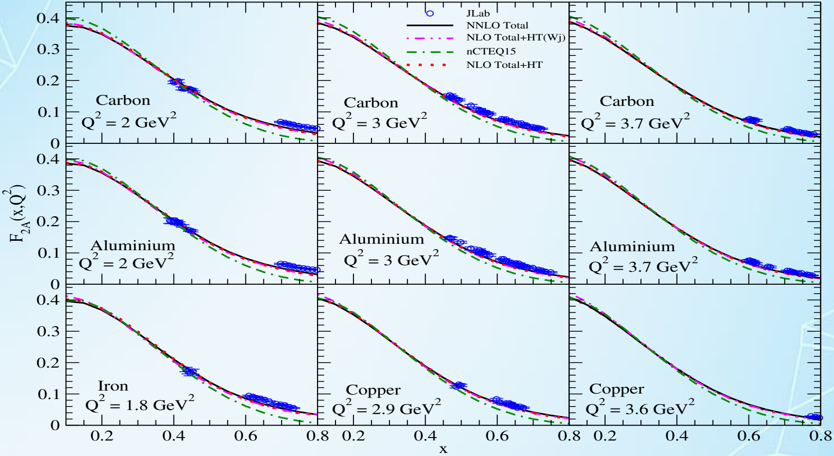
- Shadowing and antishadowing effects.

EM nuclear structure functions



F. Zaidi, et al., Phys. Rev. D 99, (2019) 093011.

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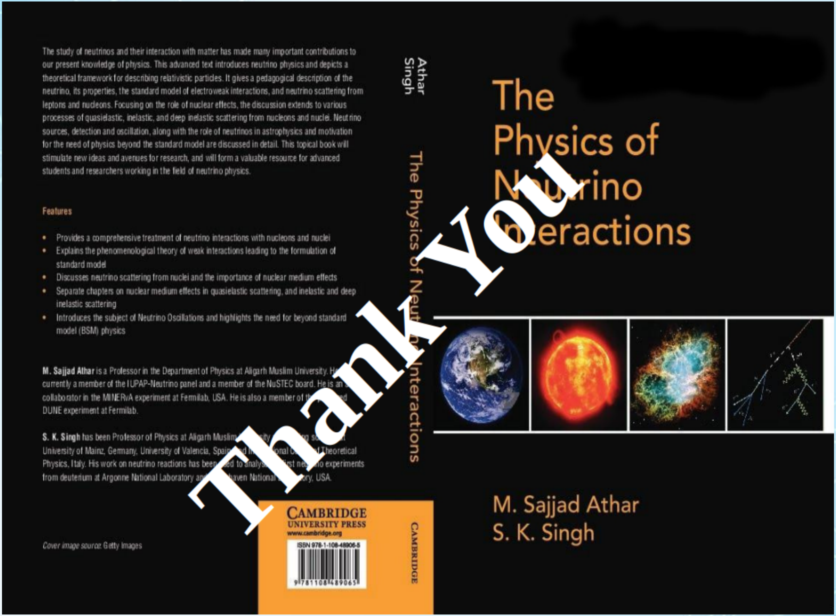
- ★ Theoretical as well as Experimental efforts to understand medium modification in a wide region of x and Q^2 are required.


The study of neutrinos and their interaction with matter has made many important contributions to our present knowledge of physics. This advanced text introduces neutrino physics and depicts a theoretical framework for describing relativistic particles. It gives a pedagogical description of the neutrino, its properties, the standard model of electroweak interactions, and neutrino scattering from leptons and nucleons. Focusing on the role of nuclear effects, the discussion extends to various processes of quasielastic, inelastic, and deep inelastic scattering from nucleons and nuclei. Neutrino sources, detection and oscillation, along with the role of neutrinos in astrophysics and motivation for the need of physics beyond the standard model are discussed in detail. This topical book will stimulate new ideas and avenues for research, and will form a valuable resource for advanced students and researchers working in the field of neutrino physics.
Features
• Provides a comprehensive treatment of neutrino interactions with nucleons and nuclei
• Explains the phenomenological theory of weak interactions leading to the formulation of standard model
• Discusses neutrino scattering from nuclei and the importance of nuclear medium effects
• Separate chapters on nuclear medium effects in quasielastic scattering, and inelastic and deep inelastic scattering
• Introduces the subject of Neutrino Oscillations and highlights the need for beyond standard model (BSM) physics
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Thank You