Quantum meets Topology: Quantum Hall Effect
ICTS, January 2025, 100 years

Perhaps, the most fundamental constant in quantum mechanics ***(CGS units, NOT SI) :***

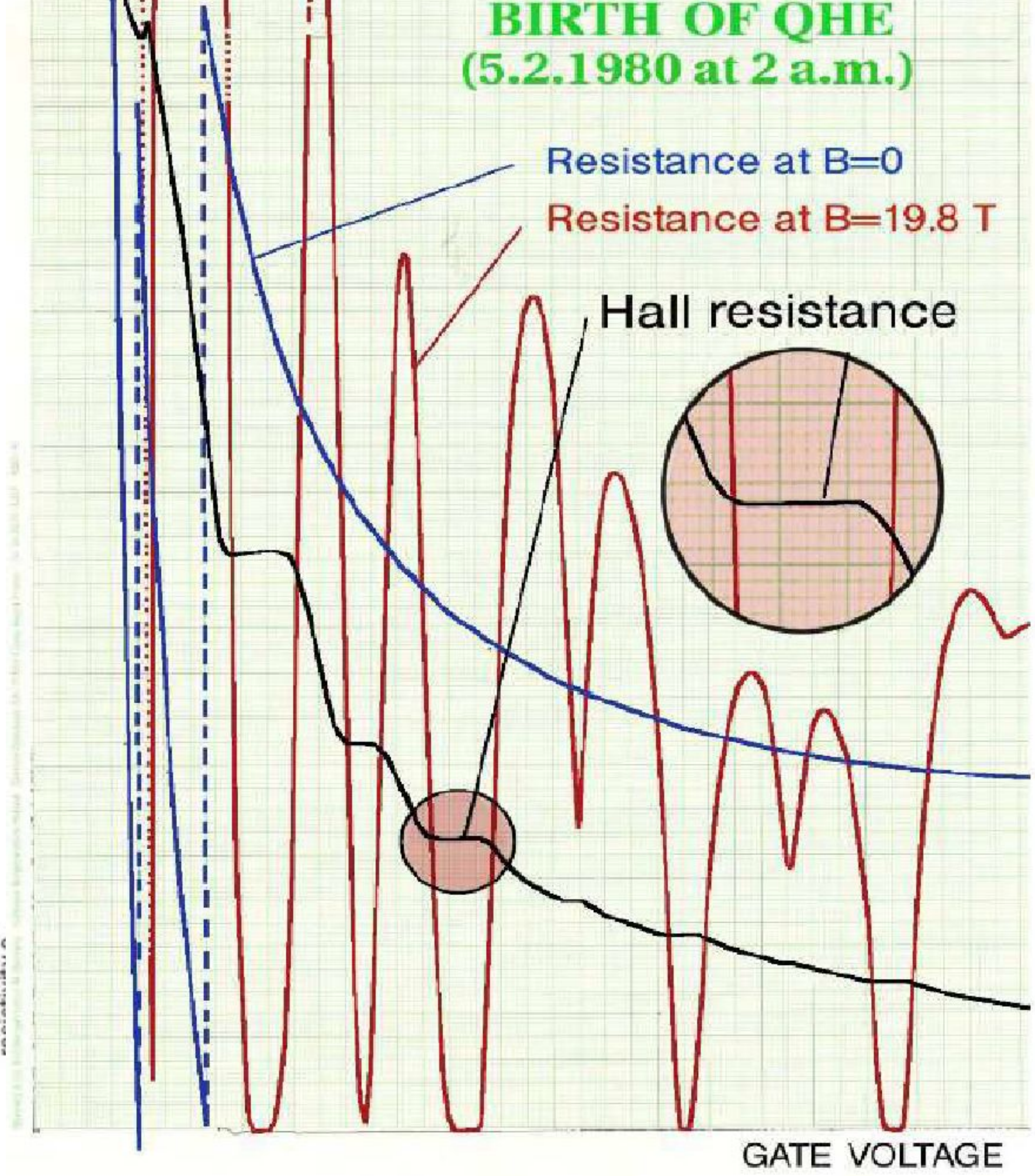
Fine structure constant $\alpha = e^2/\hbar c$ ~ 1/137 (0.007 297 352 5693)

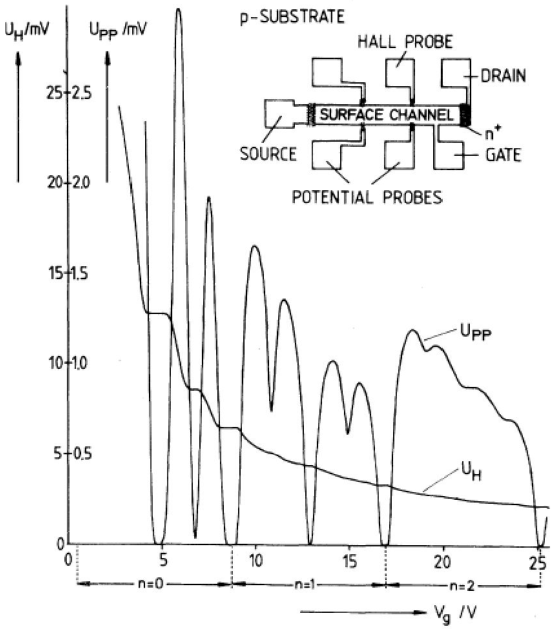
But e/ ħ/c (also m/kg/s) are now precisely defined metrologically! (Vacuum permittivity!)

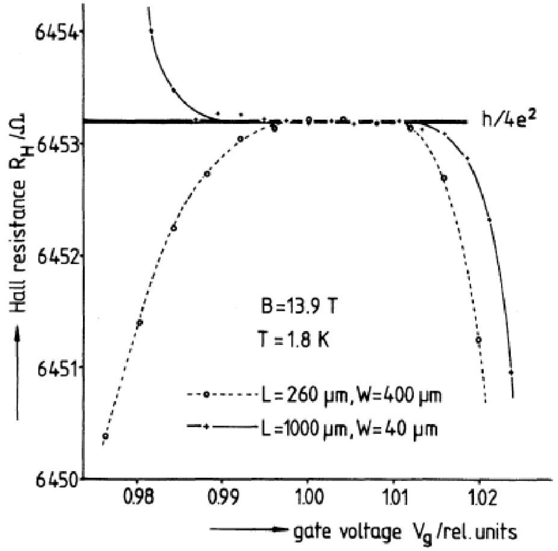
It is protected as an invariant by topology although conductance is a macroscopic property of 10^{23} electrons

Indeed electrical conductance has the dimension of velocity

This invariant 'velocity' $\alpha c/ 2\pi$~1/26k-ohm is quantum Hall conductance (1980)







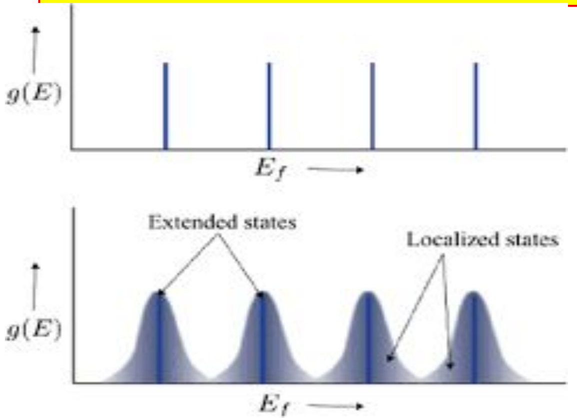
R_K = 25812.80745 Ω = h/e^2 DEFINED as of 2020

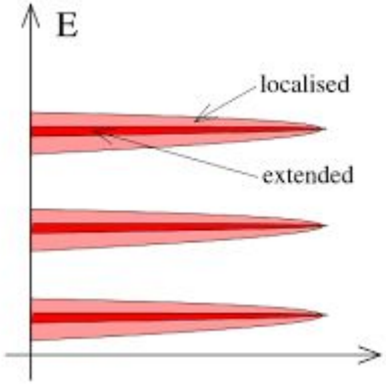
α is measured accurately to 10 decimal places (g-2)

Vacuum permittivity is known to 10 decimals also

c= 299,792,458 m/s ALL CONSISTENT m,kg,s defined

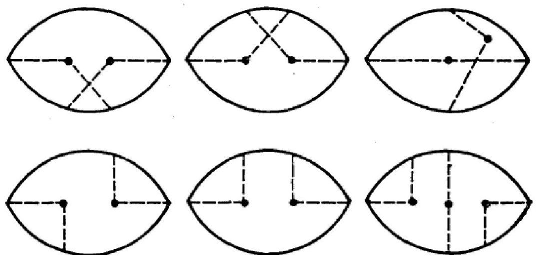
Ironically, 1980 PRL did not determine α It defined





case of repulsive scatterers.* This means that electrons which fully occupy impurity bands do not contribute to the Hall current, while those which occupy the main Landau level give rise to the same Hall current as that obtained when all i.e. $1/2\pi l^2$ electrons of the Landau level move freely. 1975

the effects of higher Born scattering does not vanish



Presence of individual delta-function quenched impurities does not affect the Hall conductivity– isolated bound states form, but the remaining free electrons carry extra current exactly compensating for this

Φ

In a ring geometry pierced by a flux, a change by a flux quantum must

ge”), and hence the only possibility is

nsfer between the edges: $\rho_{xy} = h/(ne^2)$

ge invariance, adiabaticity:

IQHE is a topological phase protected by a gap and characterized by the Chern number invariant– the first and the prototypical example of SPT in physics (also the only decisive one)

$$C = \frac{1}{2\pi}\int_{T^2} d^2\theta\, \mathcal{F}_{xy}$$

The First ‘Chern Number’
Number of filled Landau levels

$$\frac{\partial}{\partial\theta_x}\langle\psi_0|\frac{\partial\psi_0}{\partial\theta_y}\rangle\Big] \qquad \mathcal{A}_i(\Phi) = -i\langle\psi_0|\frac{\partial}{\partial\theta_i}|\psi_0\rangle$$

n for an insulator with no degrees of freedom

$$I_{\text{eff}} = kCS = \frac{\ }{4\pi}\int_{M_3} d^3x\,\epsilon^{ijk} A_i\partial_j A_k$$

$\sigma_{xy} = ke^2/h$. Quantization since k is integer k=C

Disorder essential

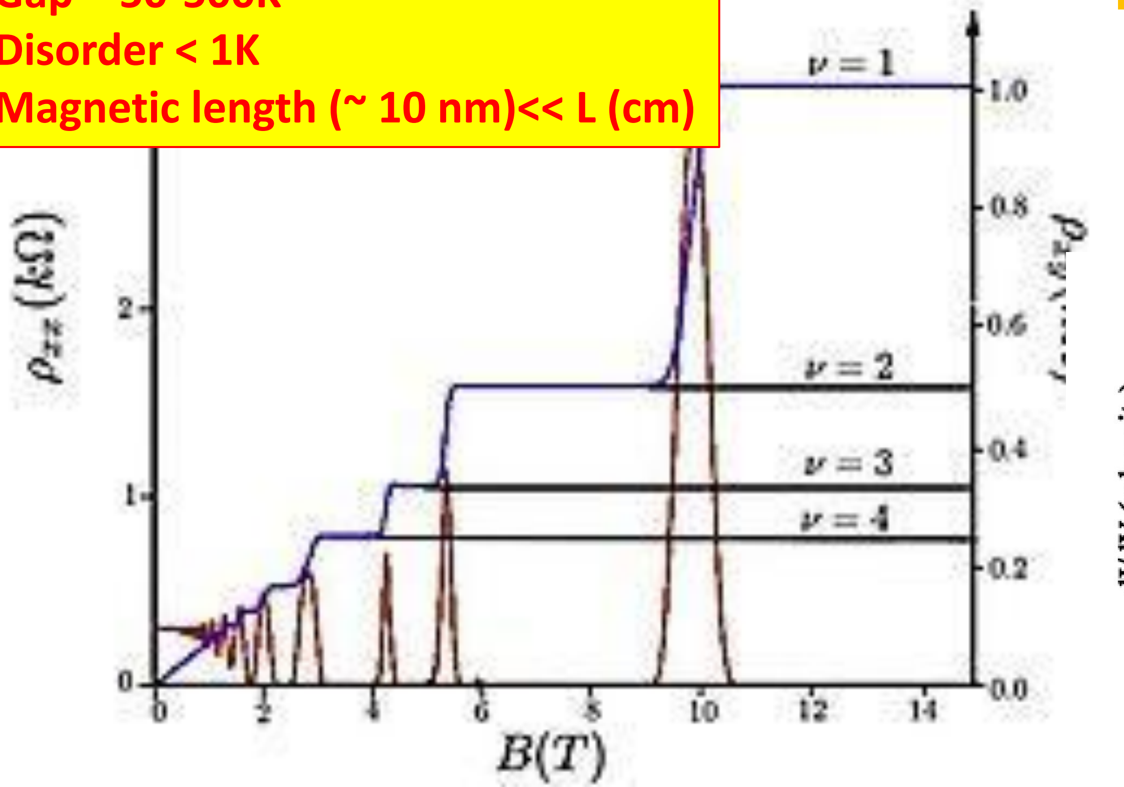
These T=0 topological theories establish QHE as Chern-invariant but are useless experimentally

CS theory asserts that a T=0 quantization is guaranteed for infinite systems, but says NOTHING about its experimental observability at finite temperature/disorder-- **THE GAP IS THE TOPOLOGICAL PROTECTION**

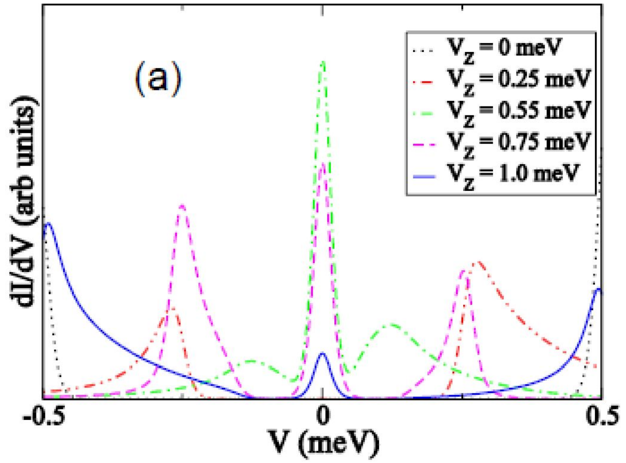
So, the theory 'explains' quantization after the fact, but does not predict it because it provides no technique for calculating corrections!

Also, spectral gap is not sufficient, one must have a transport gap for the chemical potential to move, which necessitates having some disorder

Gap ~ 50-500K
Disorder < 1K
Magnetic length (~ 10 nm)<< L (cm)



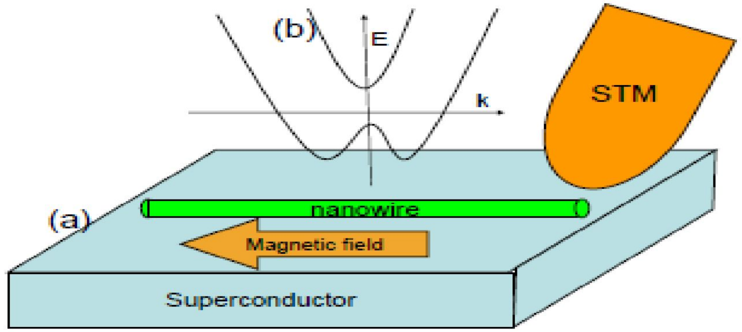
Robust quantization and topology not observed yet in spite of huge research efforts

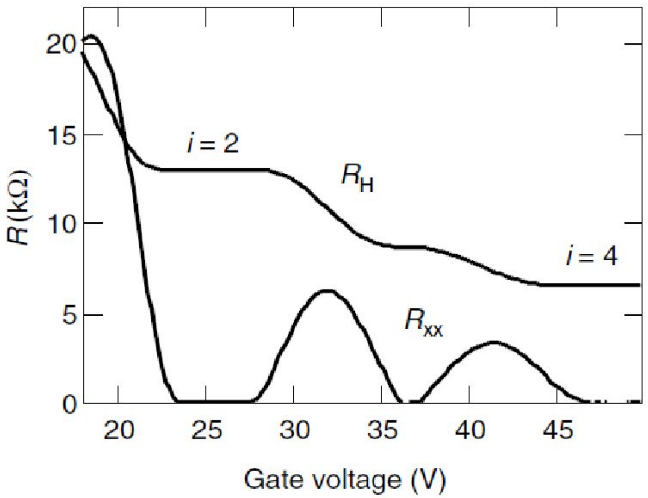


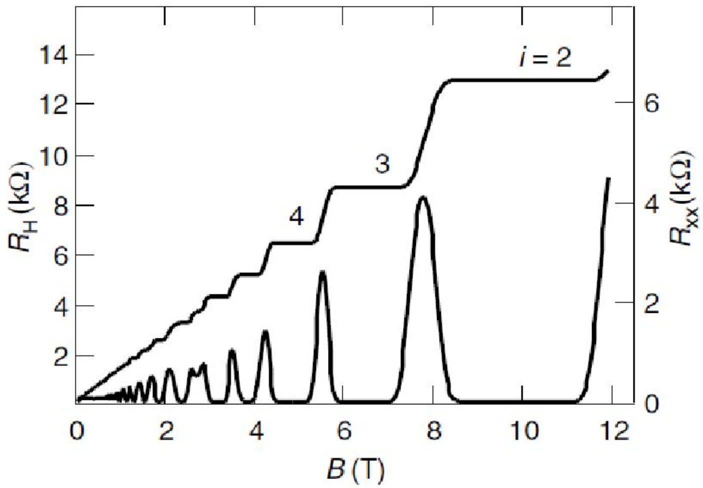
TQC

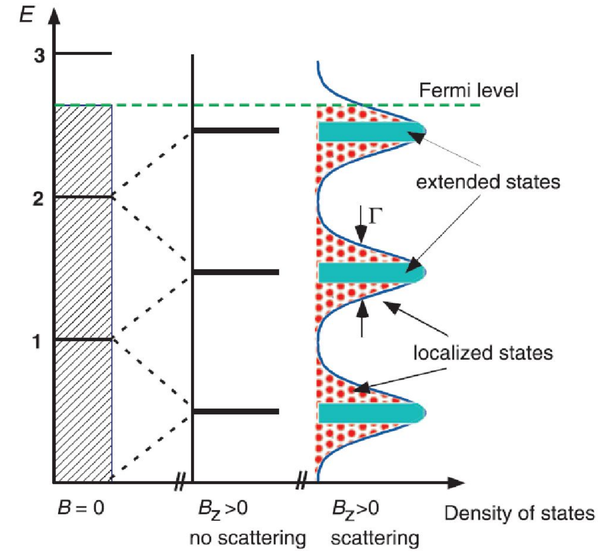
$(SU)_2$ Anyons ; MZMs
Topo invariant: pfaffian

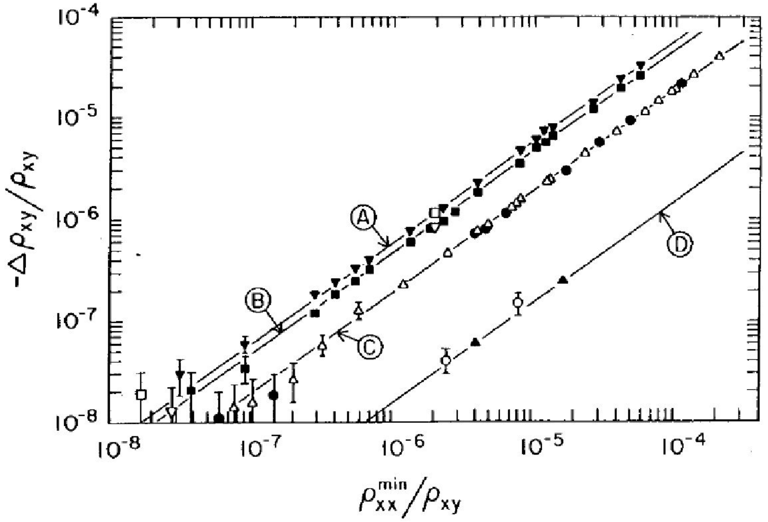
Gap ~ 100-500 mK
Disorder~ 1-20K
Coherence length>>L

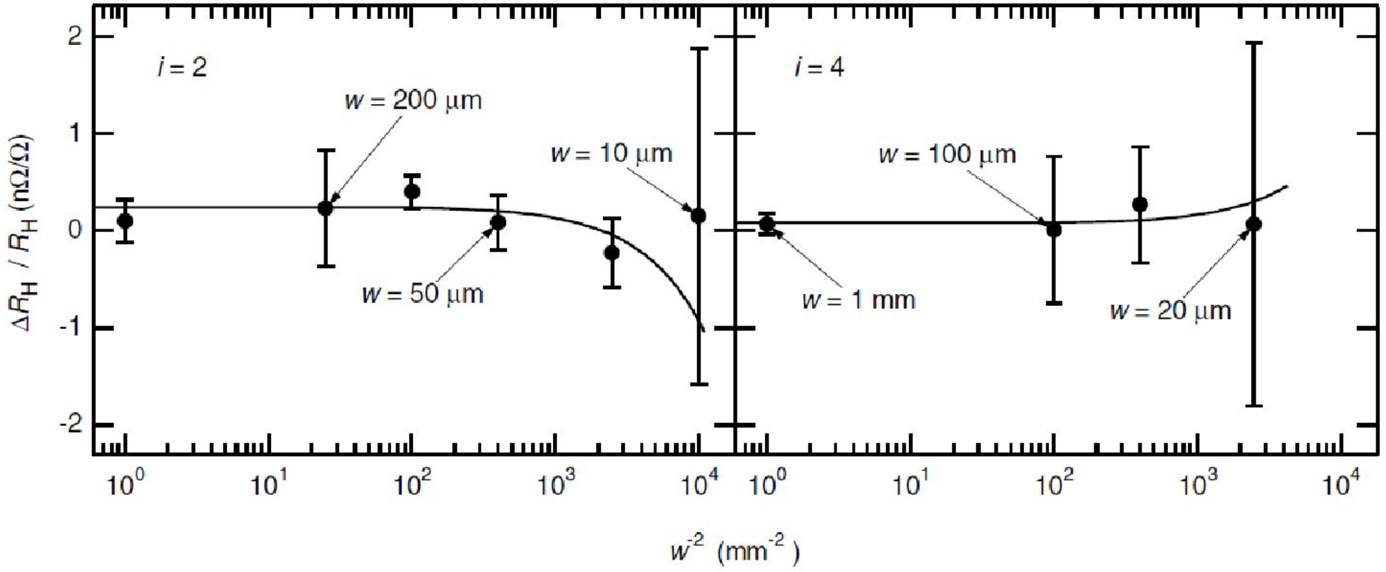










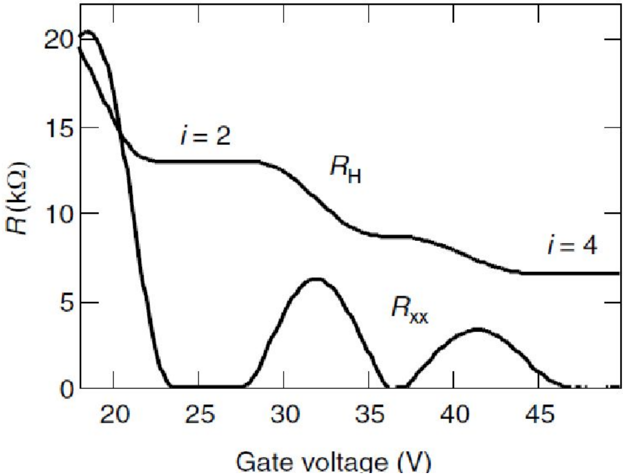


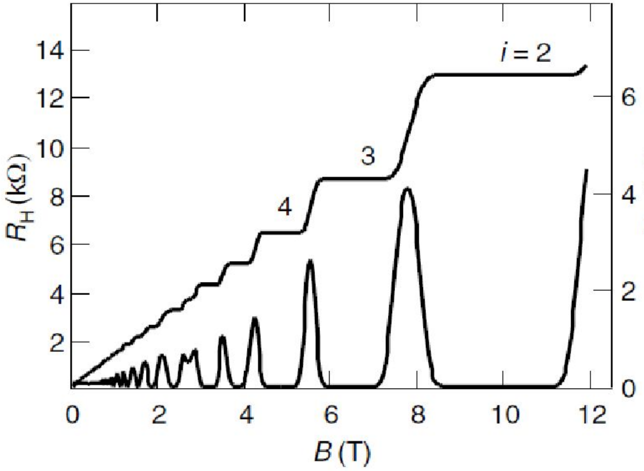
T-dependence? Disorder dependence? Cyclotron energy dependence? Size dependence? LL, μ?

Reconciling B=0 Anderson localization with the existence of IQHE?

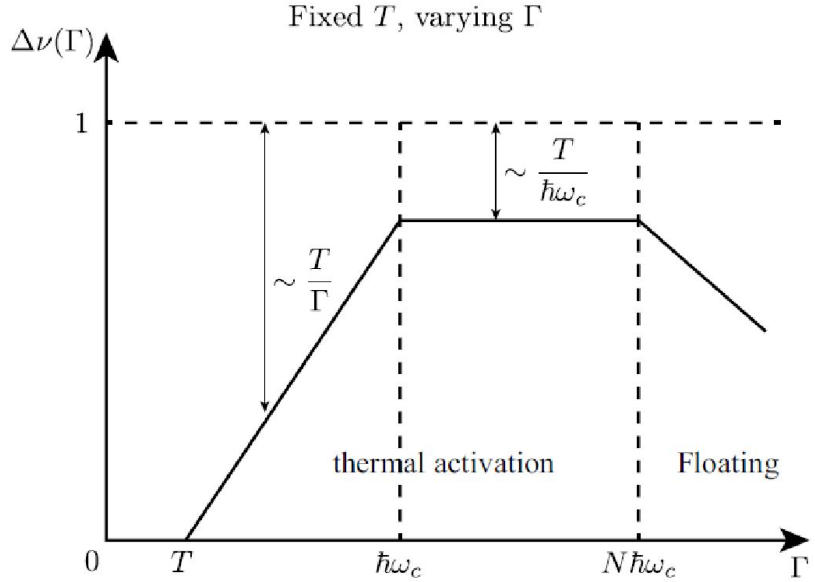
None of these relevant questions can be addressed within the universal CS theories!

How does the QH plateau width depend on T, Γ, and ν?
Increase/Decrease/Constant/No idea ? (3/3/3/1)
Can strong disorder destroy IQHE though a quantum phase transition at any ν ?
How is the B=0 limit of orthogonal class reached in IQHE?

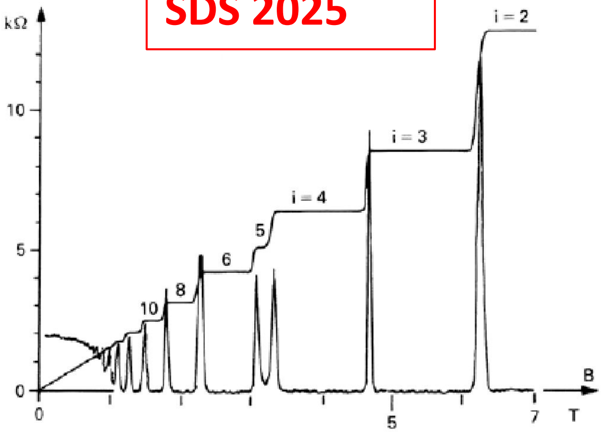




Sit on a plateau: Increase disorder, temperature
What happens to the plateau width?
What happens when the disorder is very strong?
The problem is nontrivial even if it is single particle physics
Topology, localization, Γ, ω_c, μ, T, L : many scales



Yi-Thomas, Huang, Sau, SDS 2025



$$H = -\sum_{\langle ij \rangle} t\, e^{i a_{ij}}\, c_i^\dagger c_j + \text{h.c.} + \sum_i V_i c_i^\dagger c_i$$

Deceptively simple Hamiltonian with chirality/topology hidden as constraints
QHE is a nonperturbative topological effect
H can be diagonalized directly
A percolation network model appropriate

Exact T=0 finite size calculations for the two lowest Landau levels

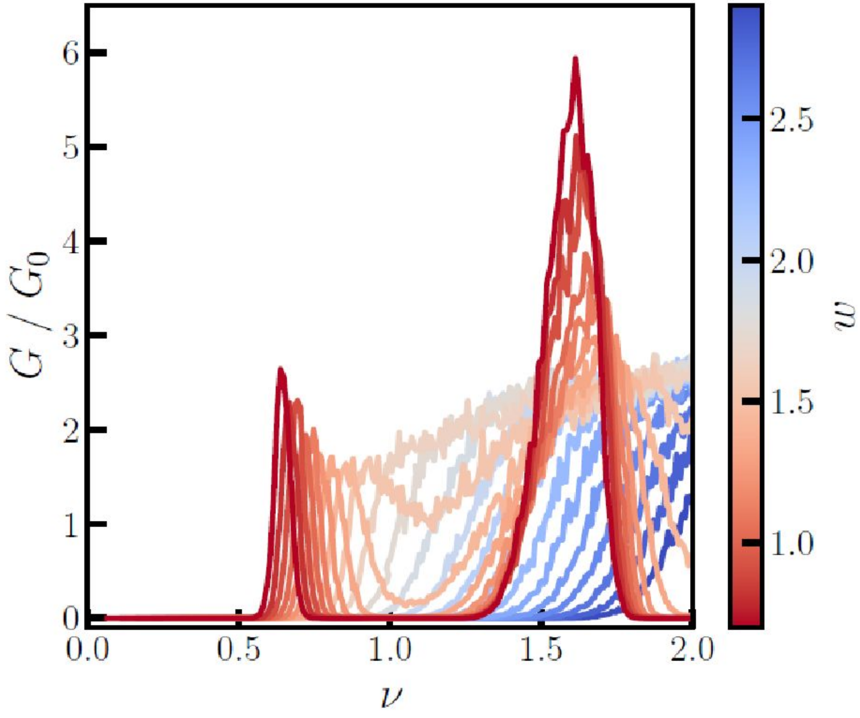
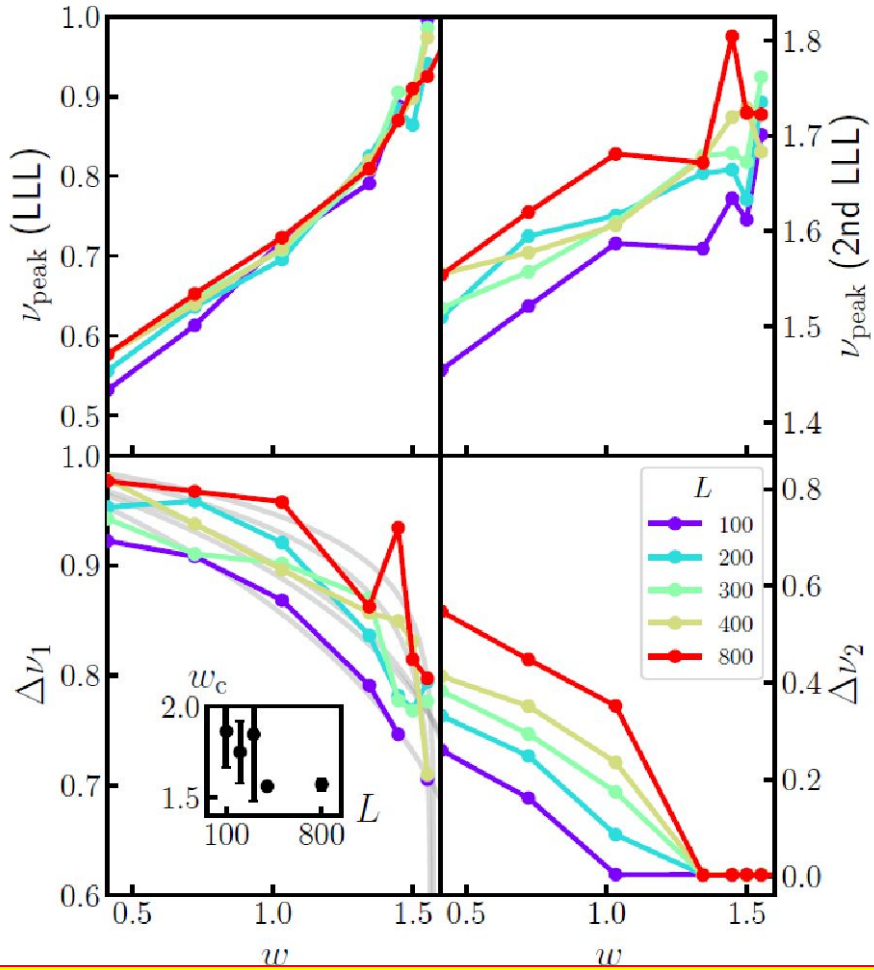
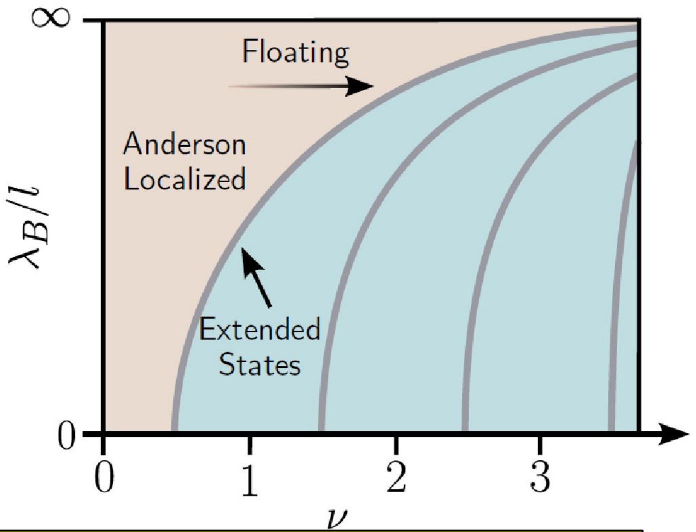


FIG. 2. Longitudinal conductance G (scaled by $G_0 = e^2/l$ of the first two Landau levels as a function of filling ν wit increasing disorder strength w. $L = 800$ and $\lambda_B = 4$.





For B=0 or very strong disorder, Γ/ω_c >>1, no QHE: Floating

T=0: The QHE plateau decreases continuously with increasing disorder with the LLL extended state moving up in energy with increasing disorder: Strong disorder (>2ω_c) 'destroys' QHE

Conductance peak associated with the extended state at LL center shows scaling

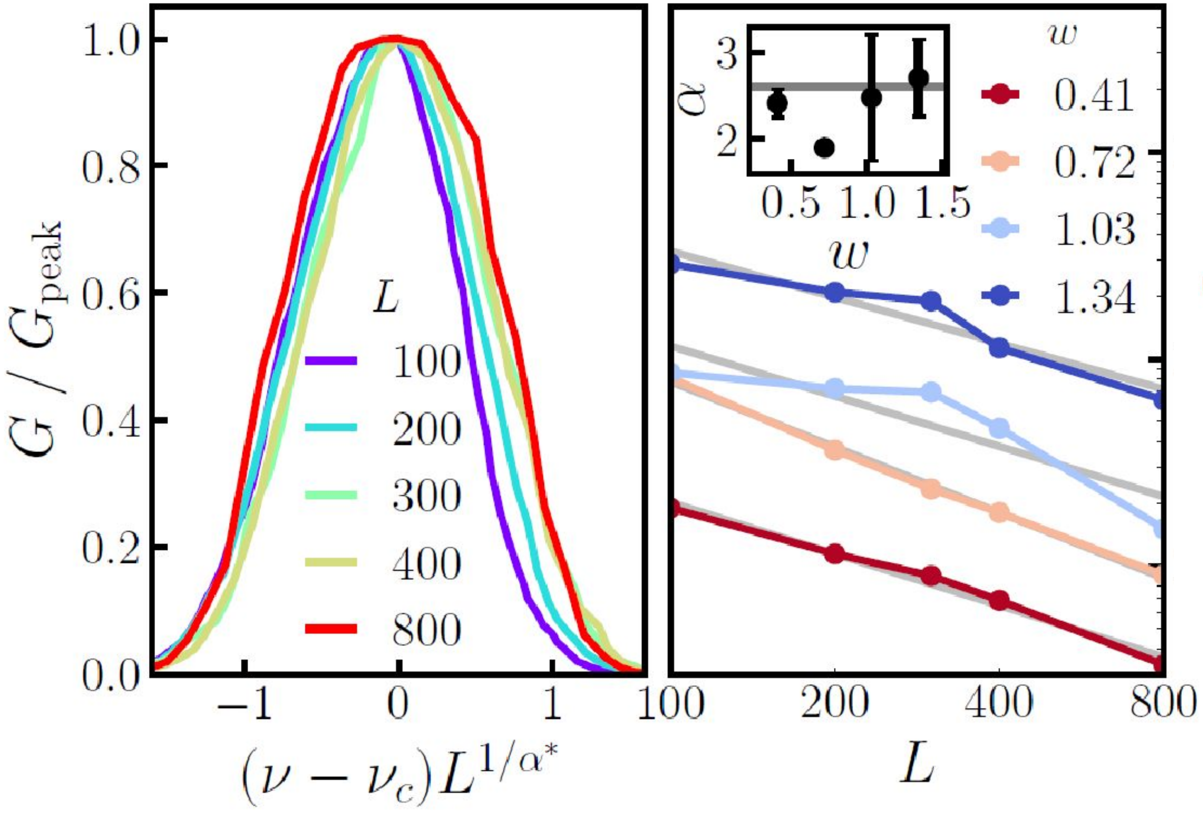
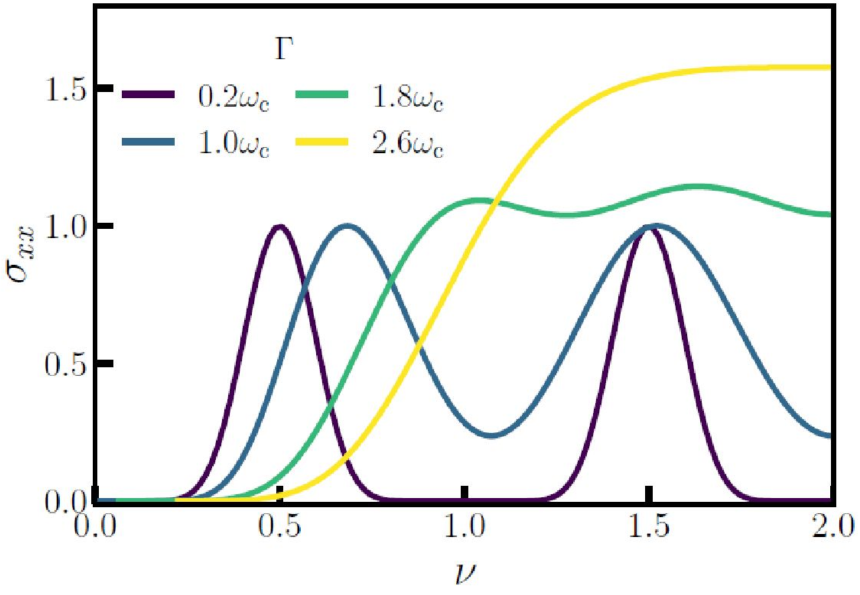
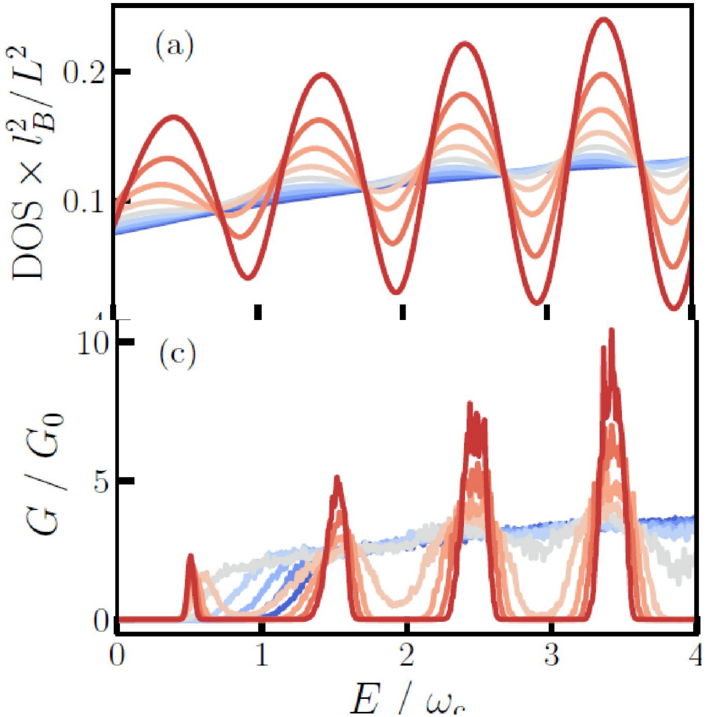
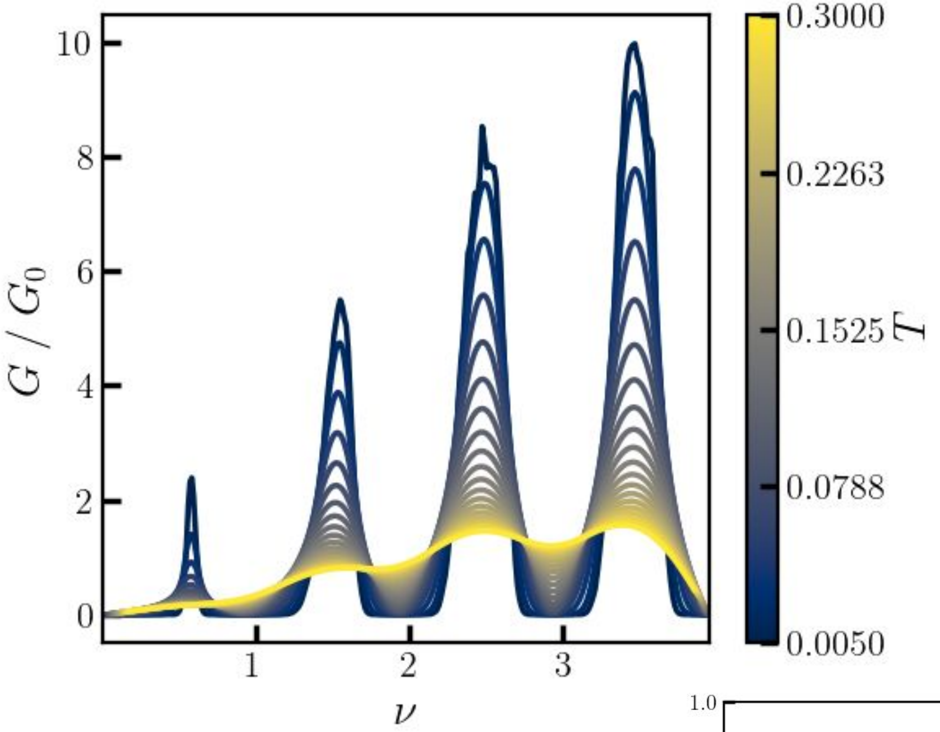


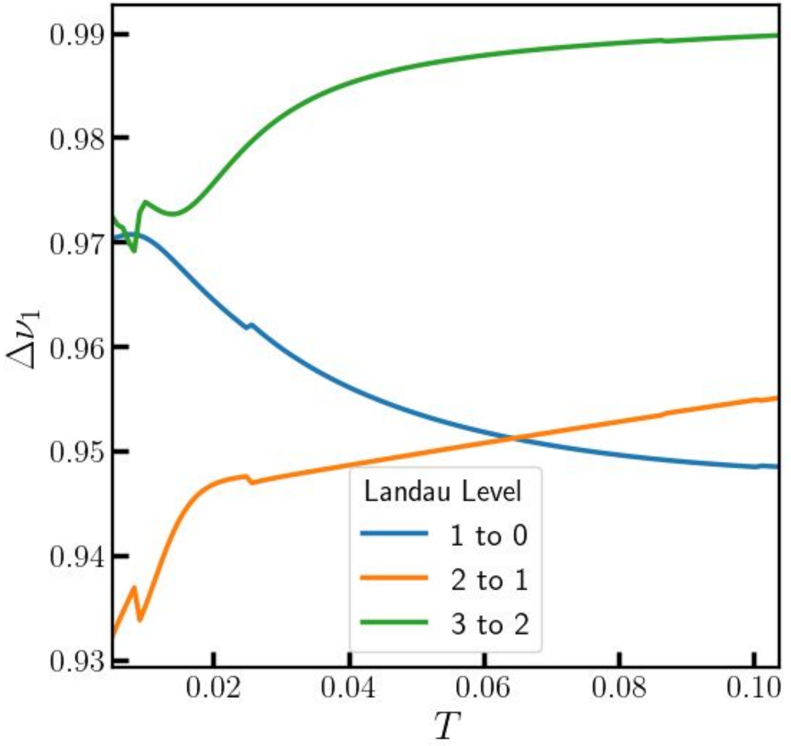
FIG. 5. (Left) The scaling of the conductance peak demonstrating approximated collapse for the $w = 0.41$ disorder case. We use the exponent $\alpha^* = 2.609$ from Ref. 35. (Right) The width σ^2 of the unscaled Gaussian calculated by fitting $\log G$ by $(\nu - \nu_c)^2$. Resulting scaling exponents are given in the inset compared with $\alpha = \alpha^*$, denoted by the horizontal line.

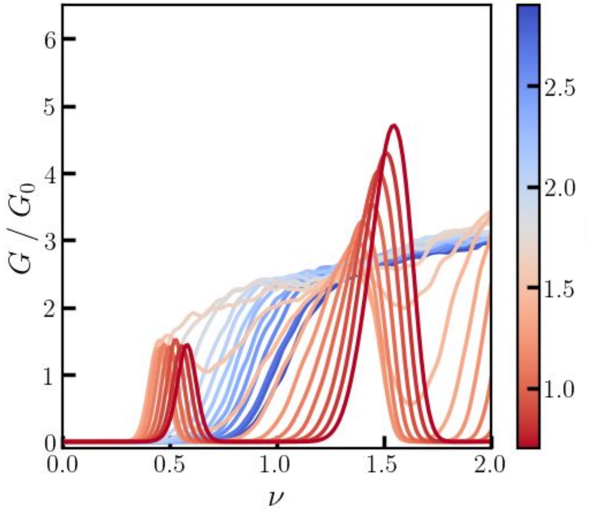


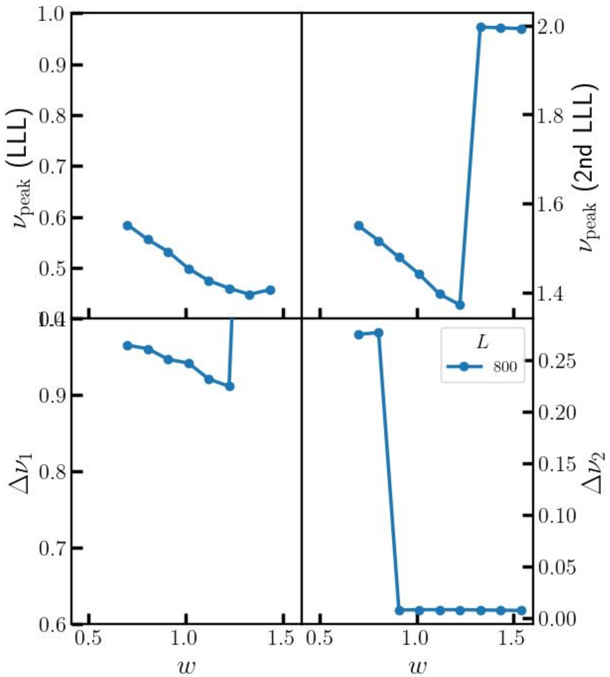




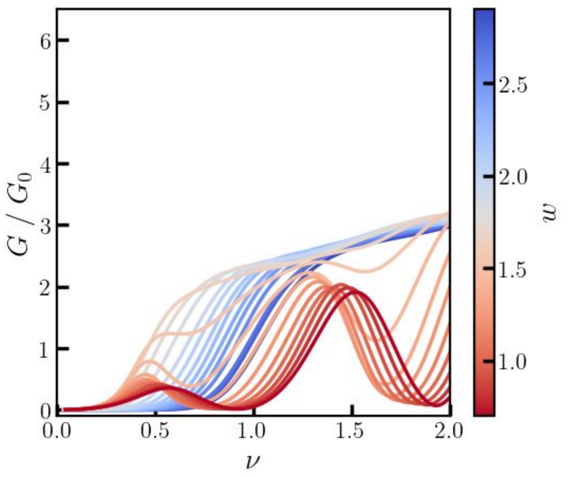
w=0.7, L=800
Increasing T typically shrinks a plateau except for competing with disorder broadening of the DOS

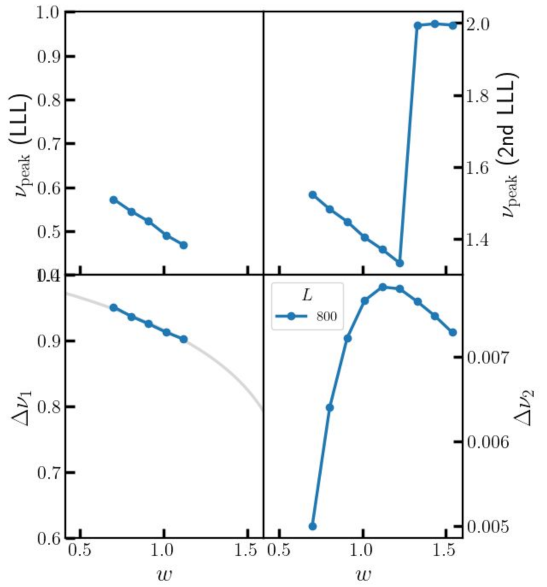






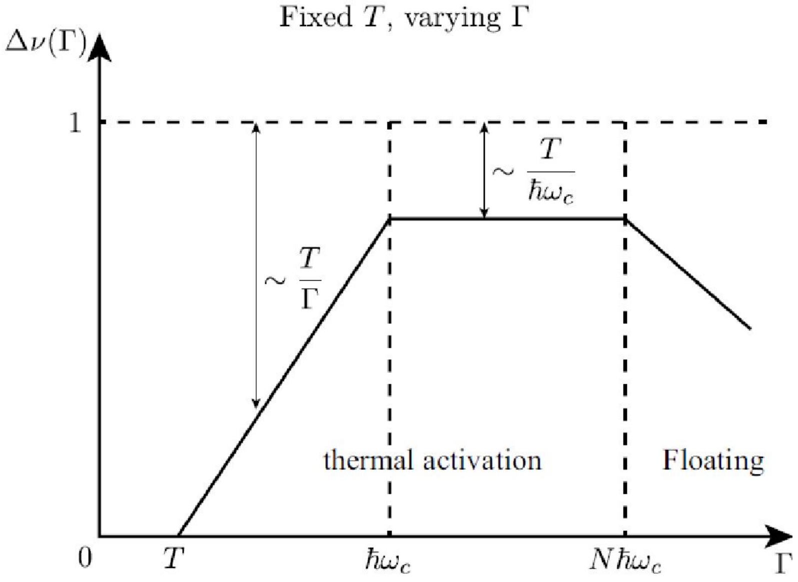
T=0.02 L=800

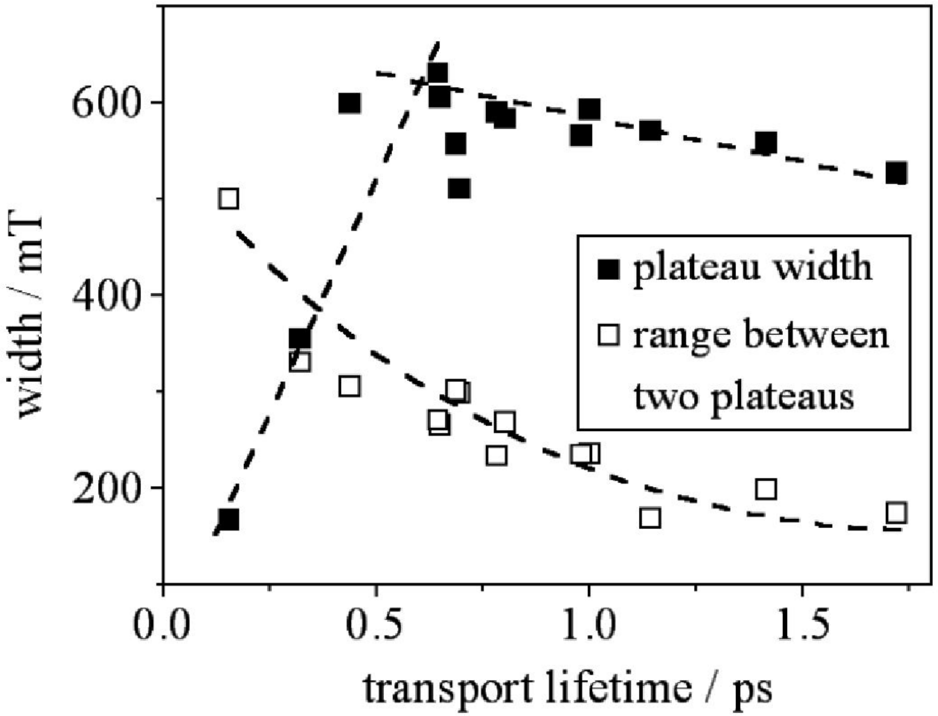




T=0.1 L=800

Semiclassical percolation network theory for IQHE including both disorder and temperature: The main effect of finite T is activation of carriers into extended states



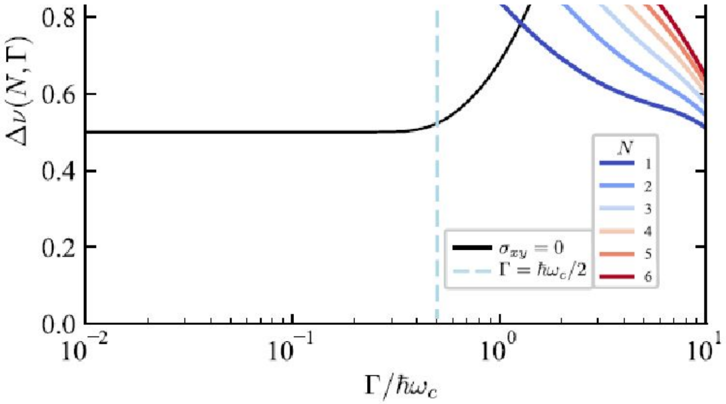


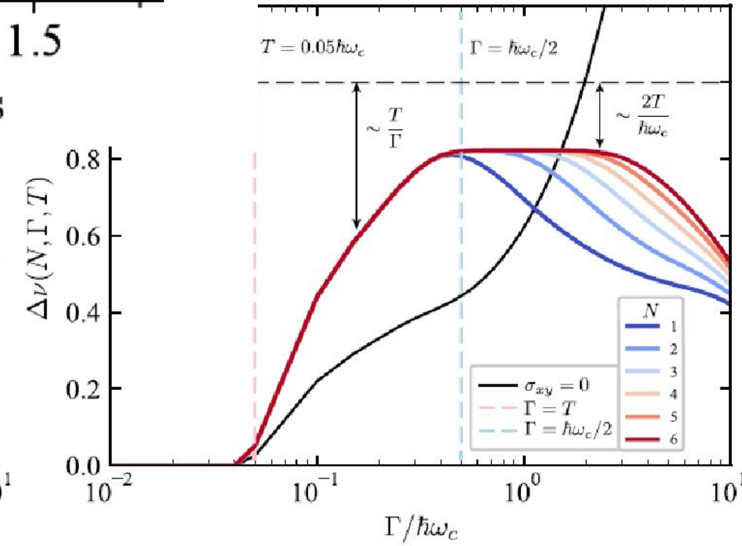
$\Delta\nu_p = 1$, if T>Γ,ω_c

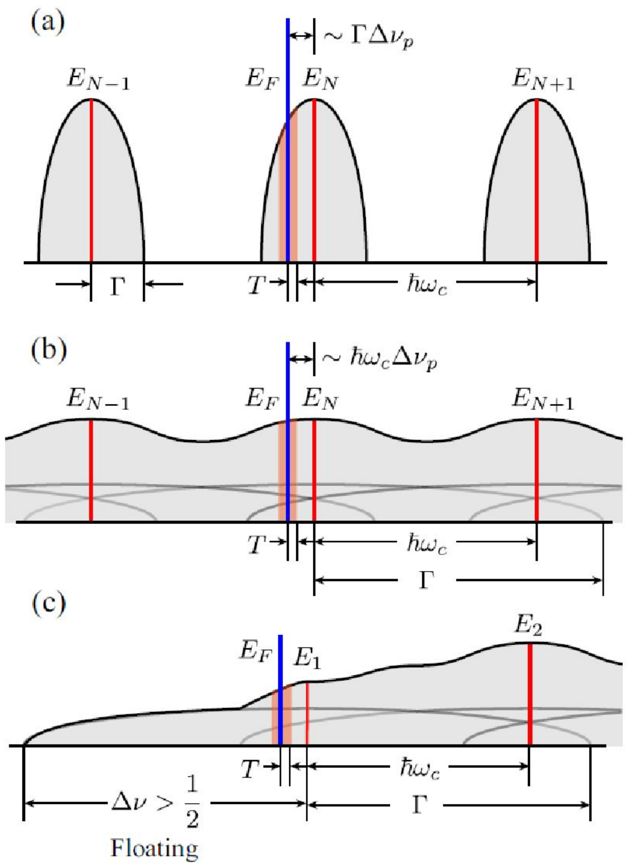
0: Only slow shrinkage starting
th v=0 moving upward
nite T: Nonmonotonic

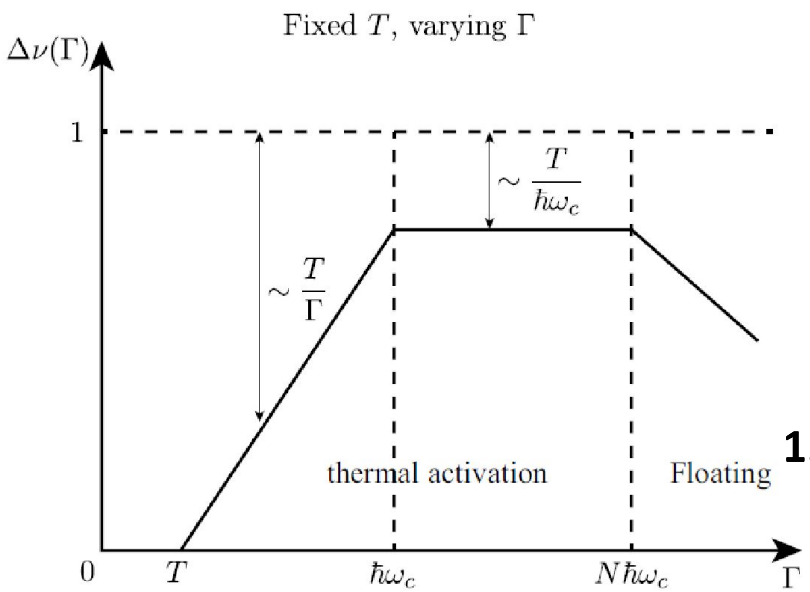
$< \Gamma$

Depending on Γ,ω_c and T, E_F the QH plateau may grow or shrink with increasing disorder and there is slow floating with increasing Γ/ω_c

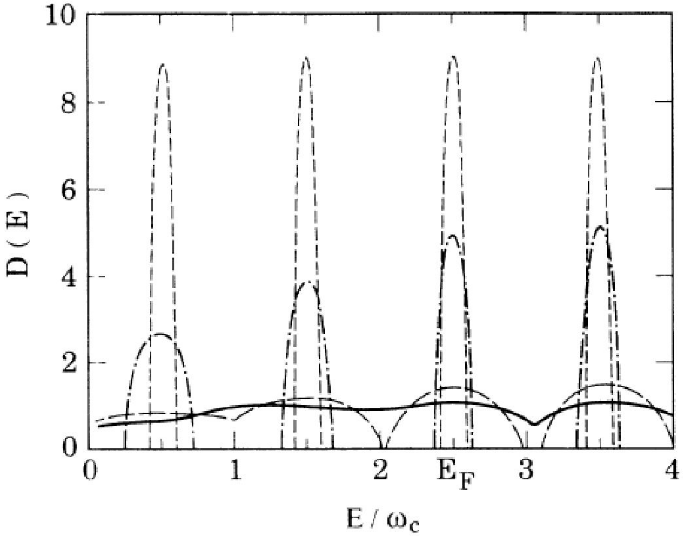


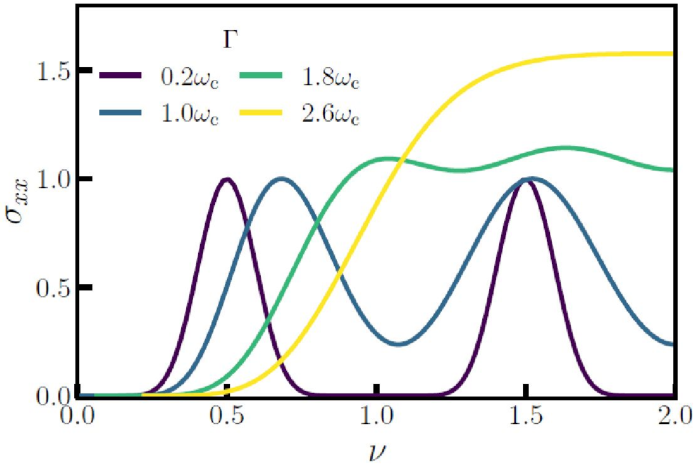


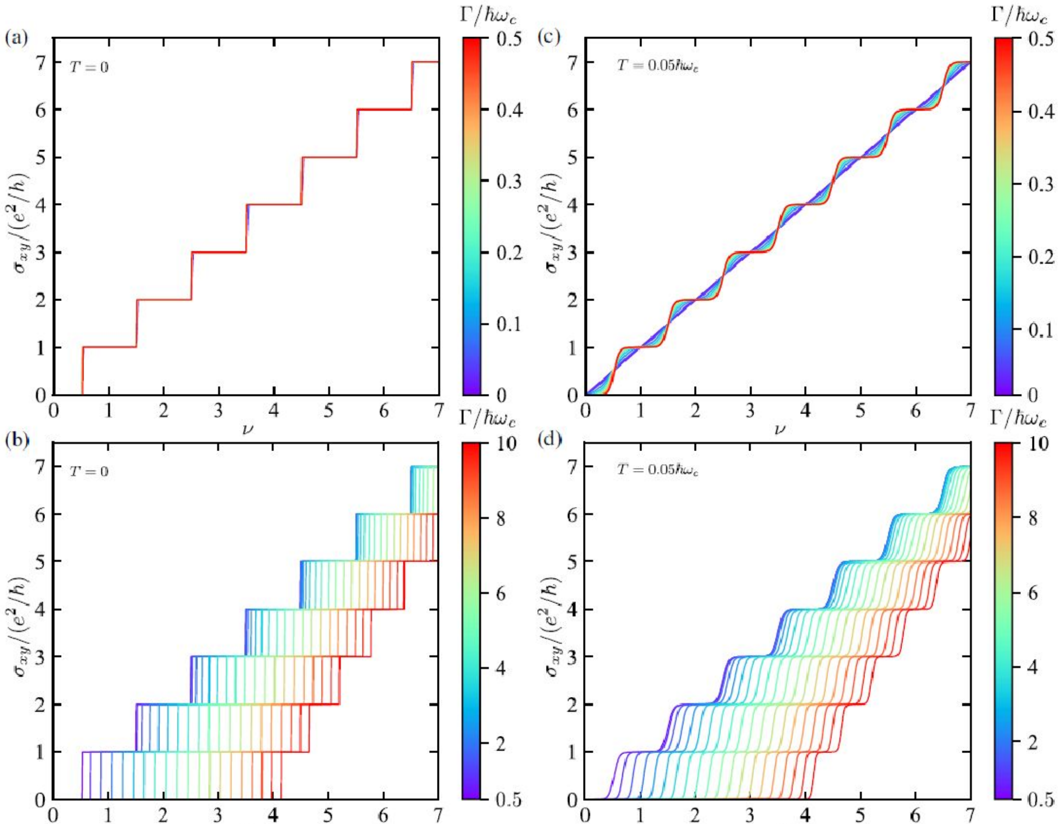


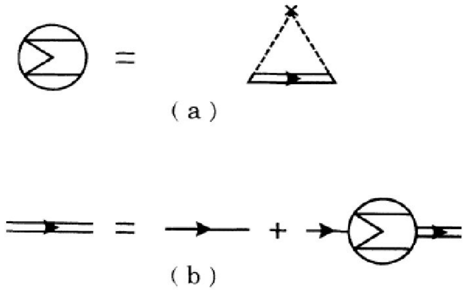


1.2K







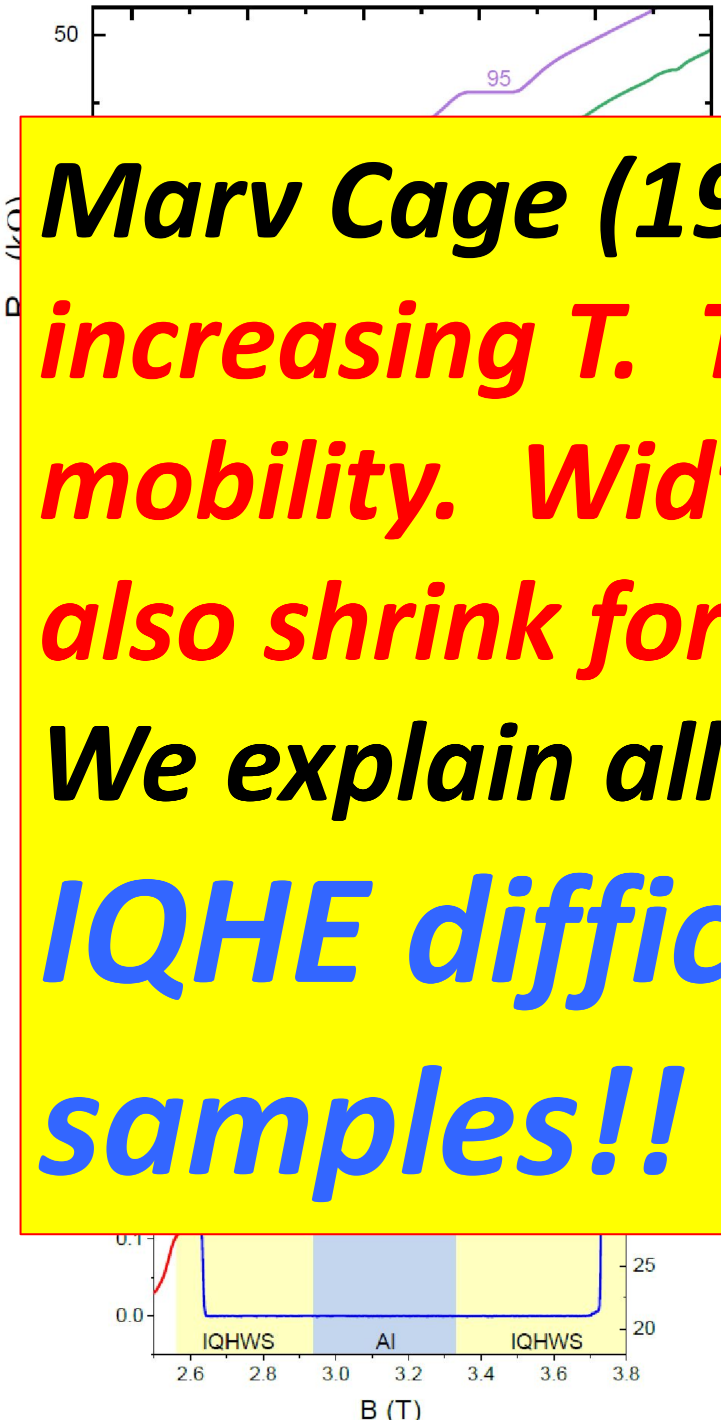


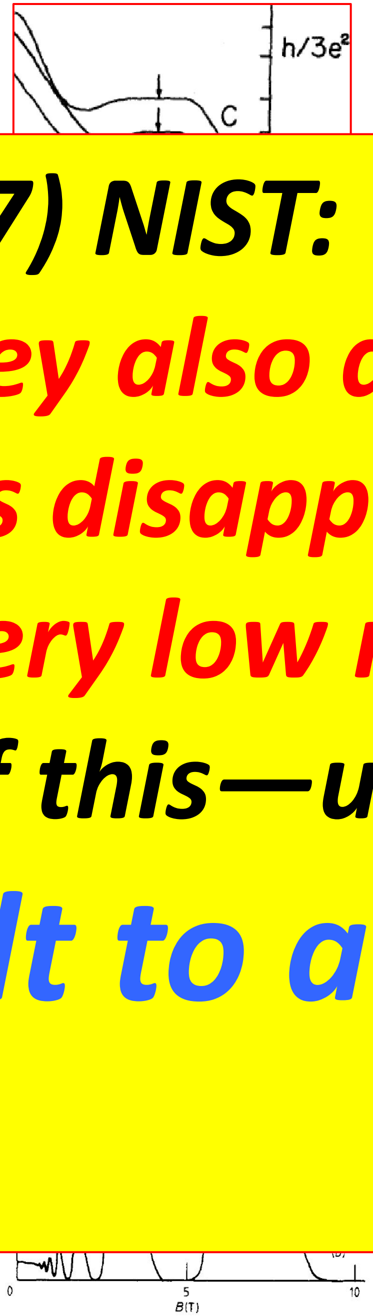
IQHE difficult in high-quality samples!!

2024 GaAs

26 million mobility

Γ ~ 5mK





GaAs 1981

Marv Cage (1987) NIST: "Widths decrease with increasing T. They also decrease with increasing mobility. Widths disappear at high mobilties, and also shrink for very low mobilities."

We explain all of this—universal CS theory cannot

IQHE difficult to achieve in better samples!!

Fig. 4. Plateau width (ΔV_g) of ρ_{xy} in gate voltage scans (V_g) normalized to inter-plateau distances (V_g^0) as a function of reciprocal mobility

Figure 4. Resistivity ρ_{xx} as a function of the magnetic field at $T = 1.5$ K for two different heterostructures with approximately the same carrier density but different mobilities: (a), $\mu = 10^6\ cm^2 V^{-1} s^{-1}$; (b), $\mu = 10^5\ cm^2 V^{-1} s^{-1}$. The plateau $\rho_{xx} = 0$ around $B = 5$ T is better

von Klitzing

Si 1989

Can IQHE/FQHE occur for B=0 and/or in lattice systems?
Yes: All we need are 2D flat band, topology
Spontaneous breaking of time reversal invariance
Chern insulators and FCI; QAHE/FQAHE

Selected for a Viewpoint in *Physics*

PRL **106**, 236803 (2011) PHYSICAL REVIEW LETTERS week ending 10 JUNE 2011

Active huge current subject with many papers in 2023-25 in experiment and theory

Nearly Flatbands with Nontrivial Topology

Kai Sun,[1] Zhengcheng Gu,[2] Hosho Katsura,[3] and S. Das Sarma

In the presence interparticle interactions, these models lead to Abelian and non-Abelian fractionalChern insulators.
We test with hardcore bosons at 1/3 filling, and
a fractional quantum Hall state is observed.

Chern number arising from the band Berry curvature

RAPID C

PHYSICAL REVIEW B **86**, 241112(R) (2012)

Topological flat band models with arbitrary Chern numbers

Shuo Yang,[1] Zheng-Cheng Gu,[2] Kai Sun,[1] and S. Das Sarma[1]

As the strength of disorder increases, the FCI/FQH phase turns into a compressible Fermi liquid and then into a topologically trivial insulator.

Topology and Quantum Mechanics will continue their synergy, becoming a single subject in physics, and will also lead to the first practical fully quantum machine: Topological Quantum Computer (QHE is the just the beginning of this journey)