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Brownian non-Gaussian diffusion

Background:Brownian diffusion

Brownian diffusion

- In 1905 Einstein gave the theory of Brownian motion.
- Two main characteristics of Brownian diffusion are:
  - Position probability distribution (PDF) is Gaussian.
  - Mean square displacement (MSD) varies linear in time.

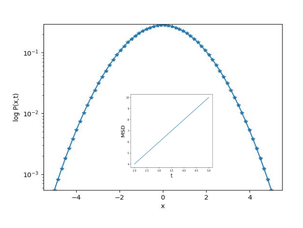
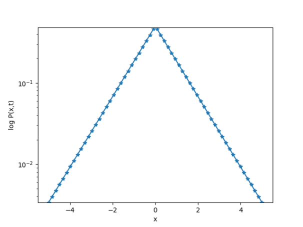


Figure: Gaussian PDF and MSD vary linear in time.

More than Brownian diffusion

- But Brownian diffusion is not the end of the story.
  - It is found in some experiments that position distribution can show non-Gaussian (like below figure). But the MSD vary linear in time. This is known as Brownian non-Gaussian diffusion.



About the talk

- Origin of Brownian non-Gaussian diffusion
- Origin of Brownian yet non-Gaussian diffusion from the polymerization process
  - **Baldovin, Fulvio, Enzo Orlandini, and Flavio Seno. "Polymerization induces non-Gaussian diffusion." Frontiers in Physics 7 (2019): 124.**
  - **Polymers critical point originates Brownian non-Gaussian diffusion. Sankaran Nampoothiri, Enzo Orlandini, Flavio Seno and Fulvio Baldovin, Physical Review E 104.6 (2021): L062501.**
  - **Brownian non-Gaussian polymer diffusion and queing theory in the mean-field limit. Sankaran Nampoothiri, Enzo Orlandini, Flavio Seno and Fulvio Baldovin, New Journal of Physics (2022).**
- Survival characteristics of Brownian non-Gaussian diffusion in the presence of absorbing boundaries.
  - **Being heterogeneous is advantageous: Extreme Brownian non-Gaussian searches. Vittoria Sposini, Sankaran Nampoothiri ,Aleksei Chechkin, Enzo Orlandini,Flavio Seno and Fulvio Baldovin (Physical review letters 132, 117101 (2024)).**
  - **Being heterogeneous is disadvantageous: Brownian non-Gaussian searches. Vittoria Sposini, Sankaran Nampoothiri, Aleksei Chechkin, Enzo Orlandini,Flavio Seno and Fulvio Baldovin (Phys. Rev. E 109, 034120 (2024))**

Central idea to understand the Brownian non-Gaussian diffusion

Brownian non-Gaussian diffusion theoretical modeling

- **Central idea: Subordination (idea of superposition of different diffusivities)**

Superposition of statistics (SS) [Beck, Cohen Phys. A (2003), Wang et al. Nat. Mat. (2012)] Diffusing diffusivities (DD) [Chubynsky, Slater, Phys. Rev. Lett. (2014) Chechkin et al., PRX (2017); Jain, Sebastian, J. Chem. Sci. (2017); Tyagi, Cherayil, J. Phys. Chem. B (2017); Miyaguchi, PRE (2017); Sposini et al., New J. Phys. (2018), J. Phys. A (2018)]

Idea to understand origin of Brownian non-Gaussian diffusion

Suppose diffusion coefficient D is stochastic. Then, what is the position distribution and MSD?

- Suppose 'D' time-dependent variable:
  with $s \equiv \int_0^t \mathrm{d}t' \ D(t')$, we have the solution for position distribution $P(x,t)$ at time 't': $P(x,t) = \frac{1}{\sqrt{4\pi s}} e^{\frac{-x^2}{4s}}$.
- Now suppose D is stochastic. Since D is stochastic s is also stochastic. The distribution becomes then:
  $P(x,t) =< \frac{1}{\sqrt{4\pi s}} e^{\frac{-x^2}{4s}} >_{all\ s}$
  $P(x,t) = \int_0^\infty \mathrm{d}s \, \frac{\mathrm{e}^{-\frac{x^2}{4s}}}{(4\pi\, s)^{1/2}} P(s,t)$ where $P(s,t)$ is the probability distribution for the process s.

$P(x,t)$ **is weighted sum of Gaussian.**

Consequence of weighted sum of Gaussian

- Weighted sum of Gaussian $\longrightarrow$ non-Gaussian PDF with MSD vary linear in time.

Summary of understanding

- Stochastic $D \longrightarrow$ non-Gaussian PDF with MSD vary linear in time.

- Narrow D distribution $\longrightarrow$ Narrow distribution of $P(s,t) \longrightarrow$ Gaussian.
- Broad D distribution $\longrightarrow$ Broad distribution of $P(s,t) \longrightarrow$ non-Gaussian.

Brownian non-Gaussian from polymerization

non-Gaussian diffusion from polymerization

- The number of subunits n vary stochastically during polymerization. Here, monomer (subunit) addition/deletion take place stochastically.
- It is known in polymer physics that the center of mass (CM) of the polymer chain with n subunits diffuses with diffusion coefficient $D = D_0/n$ where D_0 is diffusion coefficient specific of the considered subunit.
- **Time-dependent stochastic $n \longrightarrow$ time-dependent stochastic D $\longrightarrow$ non-Gaussian PDF for the polymer CM.**
- **When can the above process give a noticeable form of non-Gaussian PDF?**
- **It is possible when monomer distribution is broad.**

Brownian non-Gaussian from polymerization

- Broad *n* distribution possible near critical point (critical point seperate dilute polymer phase from that of dense phase) $\longrightarrow$ noticeable non-Gaussian PDF.

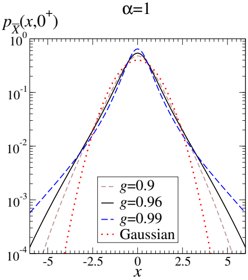


Figure: PDF of polymer C.M for different g (g is measure of how close to critical point).

Figure: Initial non-Gaussian PDF.

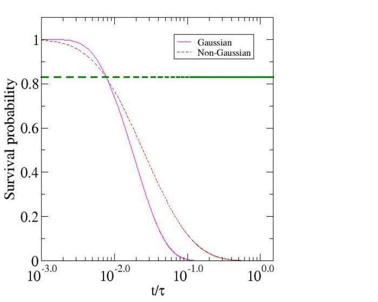
- We can see noticeable form of non-Gaussian PDF near critical point.

Consequences of non-Gaussian PDF on the survival

Survival of Brownian non-Gaussian diffusion

- **Consider the diffusive process (with non-Gaussian PDF) in a domain with absorbing boundaries. Does the survival increase or decrease due to non-Gaussian PDF?**

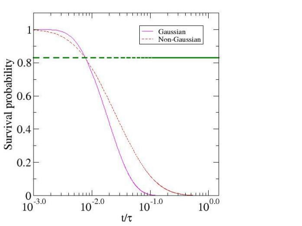
$$\mathcal{S}(t)_{nonGau} = <\mathcal{S}(s)>,\ \mathcal{S}(t)_{Gau} = \mathcal{S}(<s>),\ \mathcal{S}(s|x_0) = \frac{4}{\pi}\sum_{k=0}^{\infty} \sin\left(\frac{(2k+1)\pi}{L}x_0\right) \frac{e^{\frac{-(2k+1)^2\pi^2 s}{2L^2}}}{2k+1}$$



- **Survival is more for non-Gaussian tracers.**

Application side:Consequences of non-Gaussian PDF on the survival

Survival of Brownian non-Gaussian diffusion



- **Non-Gaussian diffusion is efficient for situations where a few tracers are needed to reach the boundary fast to activate some function. This is because of the initial survival drop for non-Gaussian PDF.**
- **Gaussian diffusion is efficient for situations where the majority of the tracers are required to reach the target.**

Consequences of non-Gaussian PDF on the survival

- **Why there is an initial survival drop for non-Gaussian diffusion? Why non-Gaussian diffusion has more survival?**

Some understanding of survival of Brownian non-Gaussian diffusion

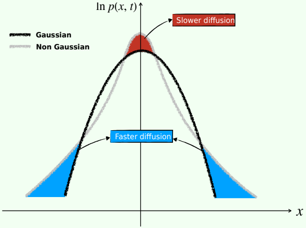


Figure: **Comparison between Gaussian and non-Gaussian PDF (from subordination process).The tail of non-Gaussian PDF decay slow leading to an excess of probability in the tails (faster diffusion). There is also an excess probability in the central part (slower diffusion) in the non-Gaussian PDF.**

Consequence of tail and centre effect?

- **The tail effect (faster diffusion) of the non-Gaussian PDF dominate in the initial dynamics $\longrightarrow$ initial survival drop.**
- **The central part (slower diffusion) dominant in the dynamics after some time $\longrightarrow$ slower decay of survival probability. This lead to more survival.**

Summary of the talk

Message 1

- The monomer distribution broadens near polymer critical point leading to a noticeable form of initial non-Gaussian PDF for the polymer C.M.

Message 2

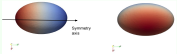
- The non-Gaussian PDF can have interesting consequences on the survival in an absorbing medium.
  - non-Gaussian diffusion is efficient for situations where a few tracers are needed to reach the boundary.
  - Gaussian diffusion is efficient for the situations where majority of tracers are required to reach the boundary.

Other research interests

Reaction-diffusion patterns refer to spatial patterns that emerge due to the interplay of reaction and diffusion of chemicals.

Reaction-diffusion (RD) patterns on curved surfaces

- **Sankaran Nampoothiri. "Preferential localization of a single spot in reaction-diffusion systems on non-spherical surfaces." Soft Matter 19.10 (2023): 1977-1986.**



Patterns on axi-symmetric surfaces

Surface shape plays an important role in determining pattern alignment.

- Importance: Patterns on cell surface can be modelled using RD.

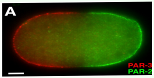


Figure: Single spot-like pattern of PAR protein on C.elegan cell surface. Red (green) represents maxima (minima). Ref:Nance, Jeremy, and Jennifer A. Zallen. "Elaborating polarity: PAR proteins and the cytoskeleton." Development 138.5 (2011): 799-809.

The reaction-diffusion (RD) mechanisms play an important role in modeling protein patterns, which is crucial in cell division on cell surfaces. But, the cell surfaces are curved, so the study may throw new insights into the role of cell surface morphology in controlling the division plane.Understand the role of the geometry of the cell surface in guiding the protein patterns on cell surface.

Acknowledgements

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- I would like to thank Fulvio, Enzo and Flavio for various interesting discussions and support.
- I also thank Vittoria and Aleksei for many useful discussions about the survival of non-Gaussian diffusion.


Thank you!