Optimal Speed of Quantum Control in Open Quantum Systems

Sarfraj Fency (PhD student)

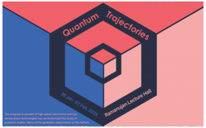
Collaborator: Dr. Riddhi Chatterjee (IISc Bangalore)

Supervisor: Prof. Rangeet Bhattacharyya

IISER KOLKATA

Indian Institute of Science Education and Research Kolkata, India

7th February, 2025



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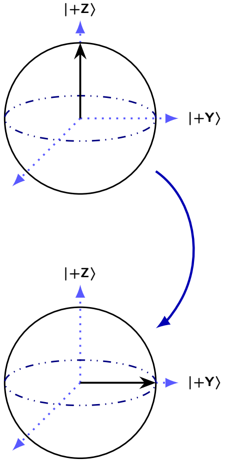
- Quantum technologies require **high precision** and **fast manipulation** of quantum systems.



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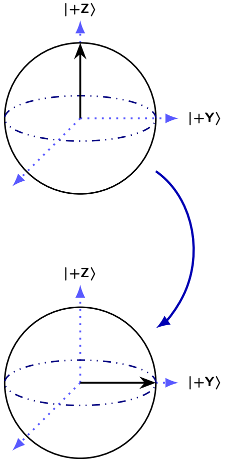

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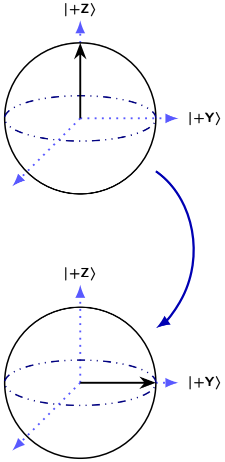

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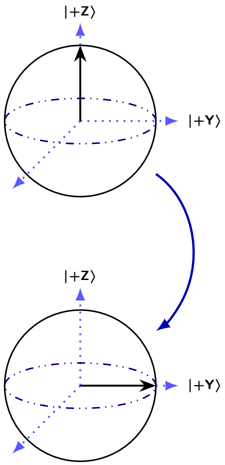
- Strong external drives, necessary for fast operations, introduce **drive-induced dissipation** (DID).

Chakrabarti *et al.*, EPL 121, 57002 (2018)

Chakrabarti *et al.*, Phys. Rev. A 97 6 (2018)

Chanda *et al.*, PRA 101, 042326 (2020)



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- Can we achieve **fast** and **high-fidelity** quantum operations in open quantum systems while accounting for these dissipation?
- Can we design **optimal control strategies** to mitigate these challenges?

The Model

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- The thermal bath is modeled as a two-level system:

$$\mathcal{H}_{\mathrm{SL}} = \omega_{\mathrm{SL}} \left(\sigma_+ L_- + \sigma_- L_+\right)$$

Quantum master Equation

- The standard quantum master equations $\Longrightarrow$ environmental dissipation

Bloch Equation, Phys. Rev. 89, 728 (1953)

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- Several theoretical papers have shown the existence and implication of DID

Chanda *et al.*, PRA 104, 022436 (2021) Saha *et al.*, PRA 107, 022206 (2023)

Chatterjee *et al.*, The European Physical Journal D 78, 44 (2024) Das *et al.*, PRA 110, 062211 (2024)

and more....

Fluctuation Regulated Quantum Master Equation

$$\dot{\rho}_s = -\, i\, \mathrm{Tr_L}[H_{\text{eff}},\ \rho]^{\text{sec}} - \int_0^{\infty} d\tau\, e^{-\frac{\tau}{\tau_c}}\, \mathrm{Tr_L}[H_{\text{eff}}(t),\ [H_{\text{eff}}(t-\tau),\ \rho]]^{\text{sec}}$$

Chakrabarti *et al.*, Phys. Rev. A 97 6 (2018)

$H_{\text{eff}} = H_{\text{dr}} + H_{\text{SL}}$, $\tau_c = \frac{2}{\kappa^2}$,

ρ = total density matrix, ρ_s = system's density matrix

$\mathrm{Tr_L}$ = partial trace taken over the bath degrees of freedom

"sec" = secular approximation where only the slow oscillating terms are retained

Cohen-Tannoudji, Atom-photon interactions: basic processes and applications

The dynamical equation

The entire dynamical equation is scaled with ω_{SL}

$$\dot{\rho}_s' = \boxed{-i\left(\alpha'(t')\,[\sigma_+,\ \rho_s']e^{-i\Delta_-' t'} + h.c.\right)} + \boxed{\chi\left(P_1\,\mathcal{D}[\sigma_+] + P_2\,\mathcal{D}[\sigma_-]\right)}$$

$$+ \boxed{2|\alpha'(t')|^2\beta\left(\mathcal{D}[\sigma_+] + \mathcal{D}[\sigma_-]\right) - \alpha_1'(t')\beta_2\left(\sigma_+\rho_s'\sigma_+\ e^{-2i\Delta_-' t'} + h.c.\right)}$$

$\alpha(t) = \frac{u_1(t) - iu_2(t)}{4}$, $t' = \omega_{\rm SL} t$, $\beta = \beta_1 + \beta_2,\ \beta_1 = J[\Delta_+'],\ \beta_2 = J[\Delta_-']$,

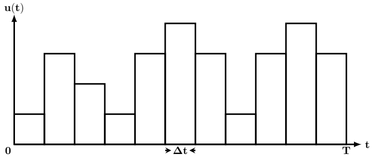
$\alpha'(t') = \frac{\alpha(t')}{\omega_{\rm SL}}$, $\Delta_- = \omega - \Omega =$ frequency of the co-rotating frame, $\Delta_-' = \frac{\Delta_-}{\omega_{\rm SL}}$,

$\alpha_1'(t') = \alpha'(t')^2 + \alpha'^*(t')^2$, $\mathcal{D}[\mathcal{O}] = \mathcal{O}\rho_s'\mathcal{O}^\dagger - \frac{1}{2}\{\mathcal{O}^\dagger\mathcal{O}, \rho_s'\}$,

$\chi = \omega_{\rm SL}\tau_c =$ environmental correlation time

The GRAPE algorithm

- GRAPE = GRadient Ascent Pulse Engineering



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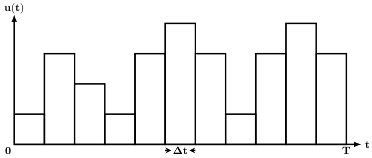

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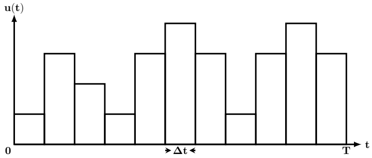

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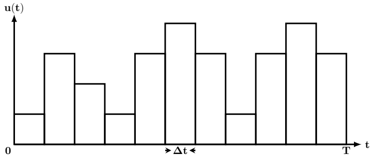

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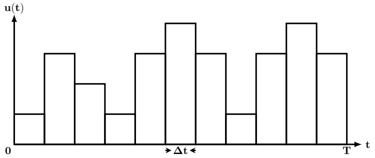

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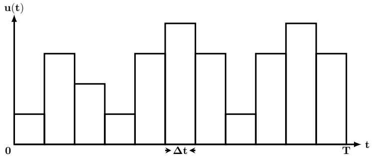

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- The Hamiltonian at j^{th} time step is:

$$H(j) = H_{\circ} + \sum_{k=1}^{m} \mathrm{u}_k(j) H_k$$

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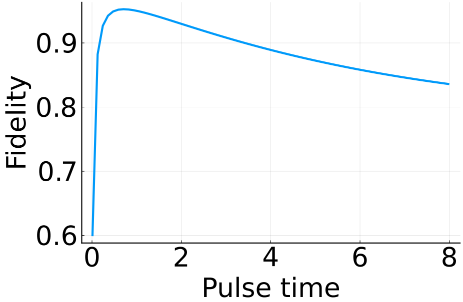
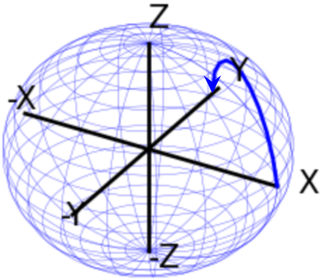
  - It gives us an **optimal pulse profile** for the given number of steps

The results: Pulse Time vs Fidelity





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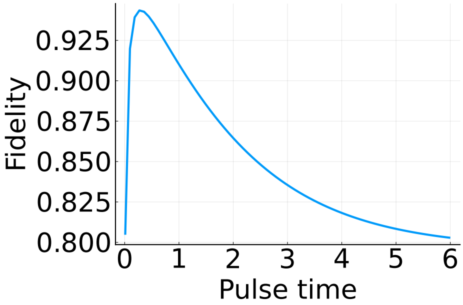

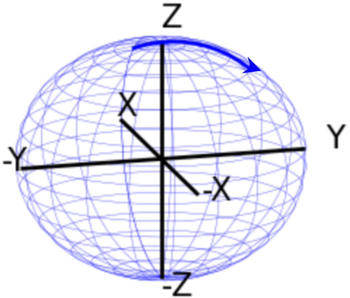

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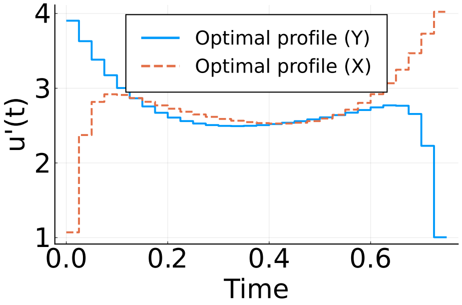

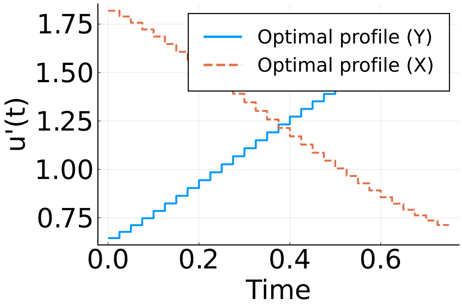

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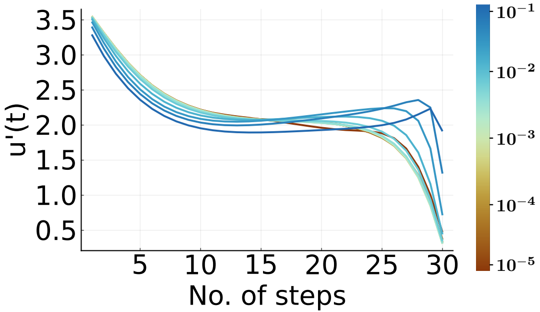

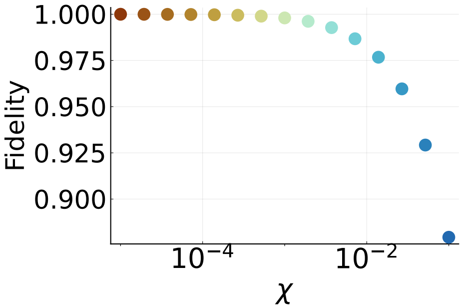

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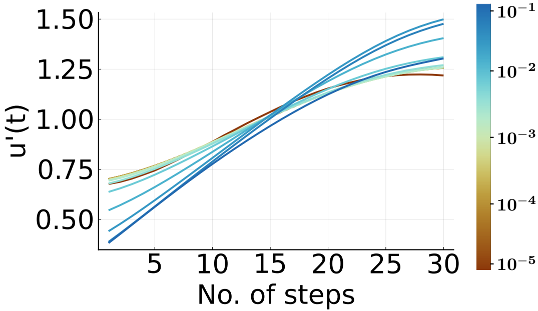

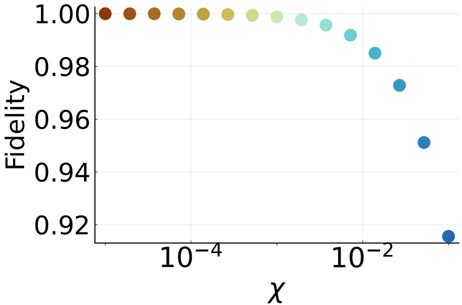

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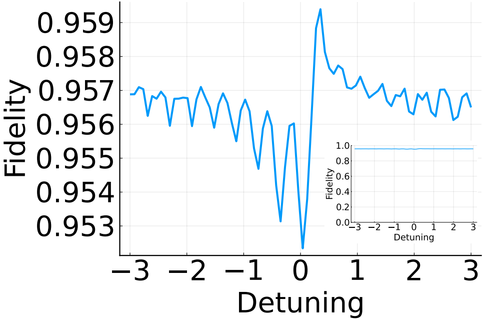

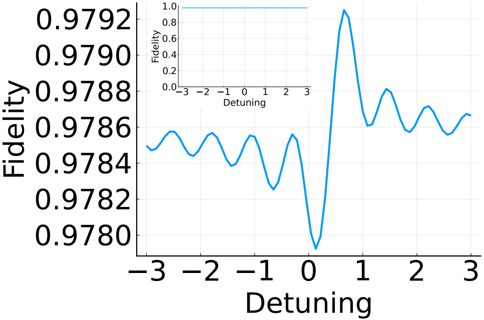

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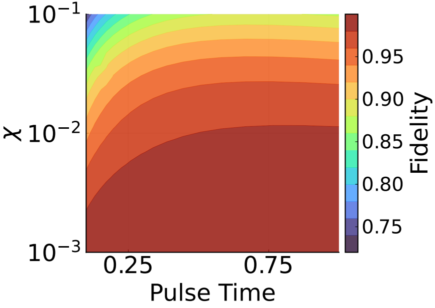

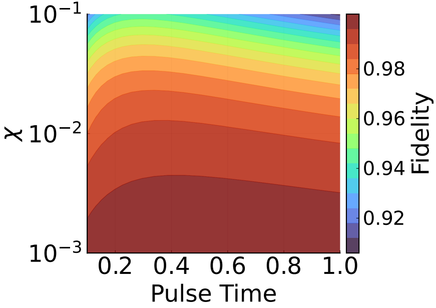

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Thank You

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Manuscript under preparation

FRQME

- The total Hamiltonian in the lab frame is given as follows:

$$\mathcal{H}(t) = \mathcal{H}_{\mathsf{S}}^{\circ} + \mathcal{H}_{\mathsf{L}}^{\circ} + \mathcal{H}_{\mathsf{SL}} + \mathcal{H}_{\mathsf{S}}(t) + \mathcal{H}_{\mathsf{L}}(t)$$

  where $\mathcal{H}_{\mathsf{S}}^{\circ}$ = static Hamiltonian of the quantum system
  $\mathcal{H}_{\mathsf{L}}^{\circ}$ = static Hamiltonian of local-environment
  $\mathcal{H}_{\mathsf{SL}}$ = Coupling between system and local environment
  $\mathcal{H}_{\mathsf{S}}(t)$ = external drive applied on the quantum system.

- The total Hamiltonian of the system in interaction picture of $\mathcal{H}_{\mathsf{S}}^{\circ} + \mathcal{H}_{\mathsf{L}}^{\circ}$ takes the following form:

$$H = H_{\mathsf{S}} + H_{\mathsf{L}} + H_{\mathsf{SL}}$$

Chakrabarti *et al.*, Phys. Rev. A 97 6 (2018)

FRQME

- The thermal noise from the environment were chosen to be diagonal in the eigen basis of the static Hamiltonian of the environment

$$\mathcal{H}_L = \Sigma_j f_j |\phi_j\rangle\langle\phi_j|$$

here f_j is assumed to be Gaussian, δ correlated stochastic variable with zero mean and standard deviation κ.

FRQME

- Finite propagator $\implies$ regulator from the thermal fluctuations (Schrödinger equation)
- The propagator = system Hamiltonian (infinitesimal) + thermal fluctuations (finite)
- Time scale separation: $\tau_c \ll \Delta t \ll \tau_s$
- Born approximation: $\rho(t) = \rho_{\mathrm{s}}(t) \otimes \rho_{\mathrm{L}}^{\mathrm{eq}}$
- Secular approximation: $|\omega_r|\Delta t \ll 1$

Final form of FRQME

- The final form of FRQME:

$$\dot{\rho}_s = -\, i\, \mathrm{Tr}_{\mathrm{L}}[H_{\mathrm{eff}},\ \rho]^{\mathrm{sec}} - \int_0^{\infty} d\tau\ e^{-\frac{\tau}{\tau_c}}\ \mathrm{Tr}_{\mathrm{L}}[H_{\mathrm{eff}}(t),\ [H_{\mathrm{eff}}(t-\tau),\ \rho]]^{\mathrm{sec}}$$

here, $\tau_c = \frac{2}{\kappa^2}$, $\rho =$ total density matrix, $\rho_s =$ system's density matrix
"sec" $=$ secular approximation

U. Haeberlen, High Resolution NMR in solids selective averaging (Elsevier, 2012)

$\mathrm{Tr}_{\mathrm{L}} \implies$ partial trace taken over the bath degrees of freedom

FRQME

- The effective Hamiltonian: $H_{\mathrm{eff}} = H_{\mathrm{S}} + H_{\mathrm{SL}}$.
- The bath is assumed to be isotropic in nature and hence

$$\mathrm{Tr}_{\mathrm{L}}[H_{\mathrm{SL}},\ \rho] = 0$$

- Second term of FRQME, predicts the presence of Drive induced Dissipation (DID)