Lecture 1: Emergent symmetries, Luttinger's theorem and 't Hooft anomalies in metals

Yesterday: illustrate for 2d Fermi. Liquids (and in 1d Luttinger liquids)

Lecture 2: Electrical transport in clean metals: general constraints and some routes to non-fermi liquid transport

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Plan

First explore the generalization of Luttinger's theorem to more general compressible quantum phases of matter which are not necessarily Fermi liquids.

Then explore how the result affects physical properties, in particular electrical transport.

General constraints on clean compressible metals

Else, Thorngren, TS 21;
Else, TS, 21

Low energy theory must(*) have

(i) an emergent continuous internal symmetry group G_{IR} => emergent conserved quantities O

(ii) these symmetries have a specific `t Hooft anomaly

(With some further restrictions, we can prove that, for d > 1, G_{IR} is in fact a large continuous symmetry - bigger than any compact Lie group).

(*) Rigorous proof in 1d, and with some further assumptions for d > 1; `physicist' proof in general.

Constraints as a Generalized Luttinger Theorem

Else, Thorngren, TS, 2020, Else, TS 2021

Low energy theory must have

(i) an emergent continuous internal symmetry group G_{IR} => emergent conserved quantities O

(ii) these symmetries have a specific 't Hooft anomaly

Fermi liquids satisfy this by way of

(1) infinite dimensional emergent symmetry

(2) anomaly of this emergent symmetry <=> Luttinger's theorem

Beyond Fermi liquids: a simple possibility

Emergent ***internal symmetry*** of a class of non-fermi liquid metals (with G_{UV} = U(1) x lattice translations):

- Infinite dimensional emergent continuous symmetry (same as Fermi liquid or some variant thereof)

``Ersatz Fermi Liquids"

Almost all examples of non-fermi liquid models in literature fall in this class.

Other more exotic structure of emergent symmetries may be possible but the class above is already very rich and poorly understood.

Ersatz Fermi liquids

Even if G_{IR} and the anomaly are the same as in the fermi liquid, detailed (universal) dynamical properties can be very different.

Only `kinematic' properties will be the same.

Infinite dimensional continuous symmetry => infinite number of emergent conserved quantities.

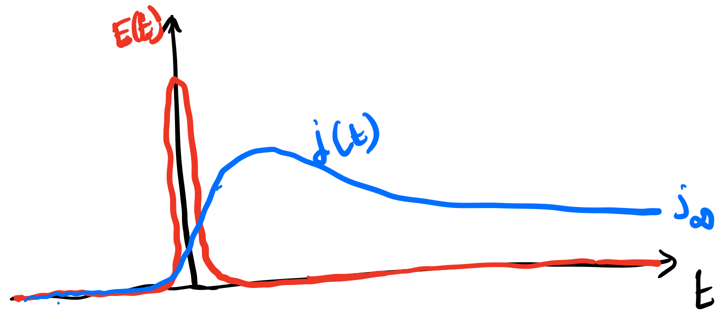
Very strong implications for transport and other dynamical properties in such non-fermi liquid metals.

Electrical transport in clean metals: general constraints and some routes to non-fermi liquid transport

Transport and conservation laws

Electrical resistivity requires mechanism for electric current to decay.

If an electric current can mix with a conserved quantity, then the dc conductivity will generally be infinite.



If $j_\infty \neq 0$, then $Re\ \sigma(\omega) \propto \delta(\omega)$

(homework: prove this!)

Familiar in Fermi liquids with conserved momentum

Theories with conserved momentum density $\overrightarrow{P}$

``Momentum bottleneck"

Consider equilibrium ``Hamiltonian" density $\mathscr{H}(v) = \mathscr{H} - \overrightarrow{v} \cdot \overrightarrow{P}$

Interpret $\overrightarrow{v}$ as ``velocity".

The electrical current density $\langle \overrightarrow{J} \rangle = \chi_{JP} \overrightarrow{v} \equiv Q \overrightarrow{v}$

(Q = charge density)

The average momentum $\langle P \rangle = \chi_{PP} \overrightarrow{v} \equiv M \overrightarrow{v}$

Uniform electric field: $\dfrac{d\langle \overrightarrow{P} \rangle}{dt} = Q\overrightarrow{E} \quad => \dfrac{d\langle \overrightarrow{J} \rangle}{dt} = \dfrac{Q^2}{M}\overrightarrow{E} = \dfrac{\chi_{JP}^2}{\chi_{PP}}\overrightarrow{E}$

Conductivity $Re\ \sigma(\omega) = \dfrac{\pi \chi_{JP}^2}{\chi_{PP}} \delta(\omega) \quad (+ \quad Re\ \sigma_{inc}(\omega))$

Electrical resistivity in metals

Can a clean metal on a lattice have an electrical resistivity?

Can also consider the opposite question.

Can a clean metal support a dissipationless current (i.e zero resistivity)?

Bloch's (other) theorem (Bohm 1949): In equilibrium, the electrical current is zero.

How then to support a dissipationless current?

A modern proof of Bloch's theorem

Watanabe 2019

Grand canonical ensemble $\rho = (1/Z)e^{-\beta(H-\mu Q)}$ satisfies

$$\langle V^\dagger K V\rangle \geq \langle K\rangle \text{ where } K = H - \mu Q$$

(V = unitary)

Apply to a `slow' U(1) rotation $V = e^{-i\int dx \lambda(x) n(x)}$ ($n(x)$ = charge density)

Then $\langle V^\dagger K V\rangle = \langle K\rangle + \int dx \langle j(x)\rangle \frac{d\lambda}{dx} + \ldots.$

(j(x) = current)

Can show that the inequality then implies $\langle j(x)\rangle = 0$ which is Bloch's theorem

A loophole to Bloch's theorem

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Suppose that in addition to the charge Q, there is another commuting conserved quantity O (i.e [Q, O] = 0 = [H, O]).

Then equilibrium is described by a generalized Gibbs ensemble

$$\rho = (1/Z)e^{-\beta(H-\mu Q-hO)}$$

Re-running the Bloch theorem proof we need [n(x), O].

As [Q,O] = 0, this vanishes when integrated over all space.

Suppose $[n(x), O] = -i\dfrac{d\Sigma}{dx}$, then can show that $\langle j\rangle = h\displaystyle\int \frac{dx}{L}\langle\Sigma\rangle$

This gives an equillibrium current so long as $\langle\Sigma\rangle$ is non-zero.

A loophole to Bloch's theorem (cont'd)

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With $[n(x), O] = -i\dfrac{d\Sigma}{dx}$, when can $\langle\Sigma\rangle$ be non-zero?

If the symmetry generator O = $\sum_i O_i$, then we must have Σ = 0 (to satisfy [Q, O] = 0)

=> O must be a ``non-onsite symmetry''

Examples:

(i) Continuous translation symmetry with O = P (the momentum), $\Sigma = n$ (and j = nv)

(ii) if the symmetry generated by O has a mixed anomaly with the global U(1)

(eg, as in the chiral anomaly of the massless Dirac fermion in 1+1-D)

Then $\Sigma = \dfrac{m}{2\pi}$ where m is a quantized anomaly coefficient.

Anomaly and the cross-susceptibility

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For the equilibrium state described by a generalized Gibbs ensemble

$$\rho = (1/Z)e^{-\beta(H-\mu Q-hO)}$$

the electric current $\langle j \rangle = h \int \frac{dx}{L} \langle \Sigma \rangle = h \left(\frac{m}{2\pi} \right)$

where m characterizes the mixed anomaly.

We could also write $\langle j \rangle = \chi_{JO} h$ with χ_{JO} a thermodynamic cross-susceptibility.

Thus this cross-susceptibility is determined entirely by the mixed anomaly.

Recall the generalized Luttinger constraint on clean compressible metals

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Low energy theory must(*) have

(i) an emergent continuous symmetry group G_{IR} => emergent conserved quantities O

(ii) these symmetries have an anomaly

Emergent conservation laws violated by external EM fields

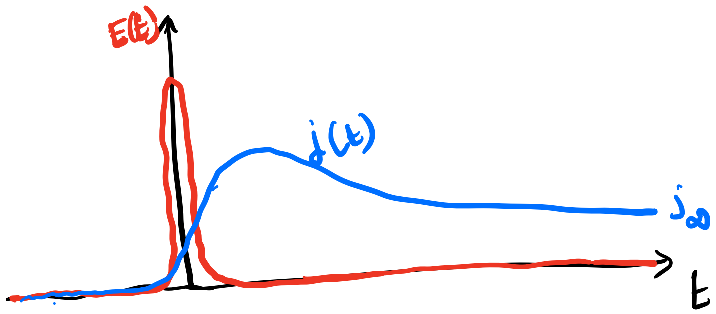
$$dO/dt = (m/2\pi)E = \chi_{JO}E$$

with m an integer (a many body topological invariant).

Anomaly guarantees that current overlaps with O.

(*) Rigorous proof in 1d, and with some further assumptions for d > 1; `physicist' proof in general.

Dissipationless current



Perturb the system with an electric field pulse at t = 0: $E(t) = \mathscr{E}_0 \delta(t)$

The anomaly equation $dO/dt = \chi_{JO} E$ implies that at late times O changes by $\delta O = \chi_{JO} \mathscr{E}$,

and correspondingly the conjugate thermodynamic variable h by

$\delta h = \delta O / \chi_{OO} = \chi_{JO} / \chi_{OO} \mathscr{E}$

(χ_{OO} = thermodynamic self-susceptibility of O).

The late time electrical current then is $j_\infty = \chi_{JO} \delta h = \left(\chi_{JO}^2 / \chi_{OO} \right) \mathscr{E}$

=> Generically seem to get dissipationless current, and hence infinite dc conductivity.

Transport in conventional metals

In clean Fermi liquids, the low-T conductivity arises due to relaxation of conserved momentum by

(a) irrelevant (umklapp) operators that transfer momentum to the lattice, or

(b) scattering off random impurities

More general examples :

$\vec{J}$ will overlap with all IR conserved operators with the same symmetry under G_{UV}.

These will generally lead to a $\delta(\omega)$ contribution to Re $\sigma(\omega)$.

Strange metal transport: hints from experiments

Experiments suggest that (in eg, cuprate strange metal) conductivity satisfies
$\sigma(\omega, T) \sim \frac{1}{T}\Sigma(\frac{\omega}{T})$ *at low* ω*,T with* $\Sigma(0)$ *finite.*

No intrinsic energy scale in the low energy theory (and hence set by T)

Low energy physics:

Scale invariant theory + (irrelevant) perturbations

The strange metal transport is a property of the scale invariant theory itself and is not determined by the irrelevant perturbations.

Strange metal transport

In particular the scale invariant theory has no $\delta(\omega)$ conductivity.

$$\sigma(\omega, T) = \cancel{D(T)\delta(\omega)} + \sigma_{inc}(\omega, T)$$

Linear resistivity, etc not determined by slow relaxation of nearly conserved quantities but is an intrinsic property of scale invariant low energy theory.

(Recall we are also assuming we can ignore random impurities)

A seeming paradox and its resolution

Two seemingly contradictory claims:

1. Strange metal transport is intrinsic (no delta function conductivity in fixed point theory)

2. Strange metal has emergent conserved quantities whose overlap with current is guaranteed by the anomaly.

Resolution: Susceptibilities of all conserved quantities (that overlap with current) diverge!

``Critical drag''

In $Re\ \sigma(\omega) = \frac{m^2}{4\pi\chi_{OO}}\delta(\omega) \quad (+ \quad Re\ \sigma_{inc}(\omega))$, if χ_{OO} diverges, the delta function is killed.

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Observables with diverging susceptibility

$\overrightarrow{J}$ will overlap with all IR conserved operators with the same symmetry under G_{UV}.

Their susceptibility must diverge at the strange metal fixed point.

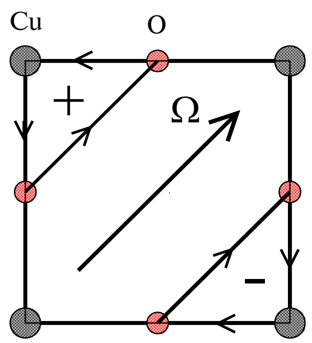
Such operators must be

- odd under time reversal, and inversion

- have zero crystal momentum

- transform as a vector under lattice rotations

These are the same symmetries as what is known as the Varma loop current order !!



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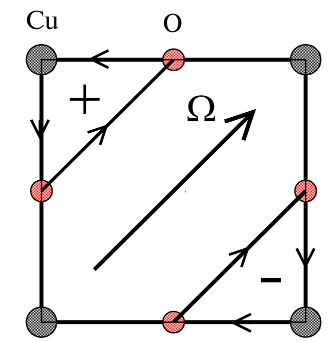
Loop currents and cuprates



Advocated by Chandra Varma (1990s- present) for other reasons.

Many *reports* of static loop current order in pseudo gap regime, and many *controversies.*

Other theories for loop currents:

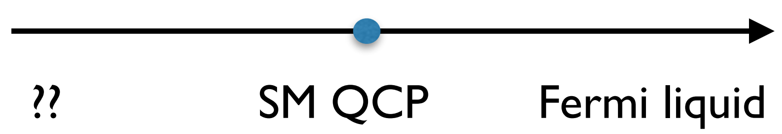
Agterberg, Melchert, Kashyap 2015 (``vestigial'' from PDW)

Chaterjee, Sachdev, Scheurer 2017 (emergent from fluctuating spin magnetism)

We give a completely different rationale for critically fluctuating order with the same symmetries in the strange metal!

Other experimental tests

1. Unlike in a fermi liquid, strange metal transport not associated with slow relaxation of conserved quantities



Strange metal quantum critical point:
the fermi liquid T^2 resistivity will not be part of a scaling function with the strange metal resistivity.

2. Quantum oscillations: signature of emergent continuous translation symmetry in strange metal

Summary

Charge conservation, lattice translation symmetries, and a tunable filling lead to strong constraints on the IR theory that must be satisfied by any putative non-Fermi liquid.

Anomalies and other topological structures play a crucial role, enabling model-independent statements.

Observed strange metal transport

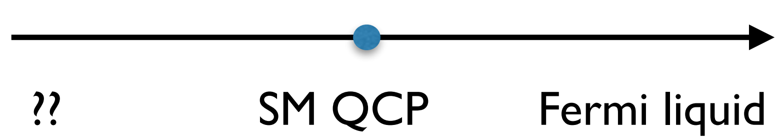
- not related to slow relaxation of nearly conserved quantities (unlike in a Fermi liquid)

- consistent with general constraints only if susceptibility of certain observables with same symmetry as loop current order parameter diverges.

How far can we go without committing to detailed dynamical models?

Other experimental tests

1. Unlike in a fermi liquid, strange metal transport not associated with slow relaxation of conserved quantities



Strange metal quantum critical point:
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2. Quantum oscillations: signature of emergent continuous translation symmetry in strange metal

Transport in clean metals

Separate the electrical conductivity into two contributions

$$\sigma(\omega, T) = \sigma_{coh}(\omega, T) + \sigma_{incoh}(\omega, T)$$

σ_{coh} = contribution from mixing of current with nearly conserved quantities.

A nearly conserved quantity: related to emergent symmetry of the low energy scale invariant theory.

(Irrelevant perturbations: Slow relaxation of the conserved quantity at a rate

$\Gamma \sim T^{\phi}$, with $\phi > 1$)