Controlling attainment of spontaneous ordering in many-body interacting systems

Shamik Gupta
Tata Institute of Fundamental Research, Mumbai, INDIA

Collaborators:

- **Theory:** A Acharya, R Majumder & R Chattopadhyay (TIFR); M Sarkar (Heidelberg)
- **Experiment:** P Parmananda & his group (IIT Bombay)

References:

- R Majumder, R Chattopadhyay and **SG**, Phys. Rev. E **109**, 064137 (2024)
- M Aravind, V Pachaulee, M Sarkar, I Tiwari, **SG**, and P Parmananda, Phys. Rev. E (Letter) **109**, L052302 (2024)

The main question

Many-body interacting systems

$\downarrow$

Phase transition between an ordered and a disordered phase

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Many-body interacting systems

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Phase transition between an ordered and a disordered phase

QUESTION: Can we induce order in the system by "minimally" tweaking the dynamics in a parameter regime in which the bare dynamics does not show order?

We address this question in the context of spontaneous synchronization

Spontaneous synchronization

Spontaneous coordination among interacting elements to act in unison

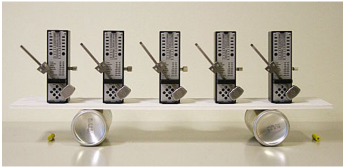
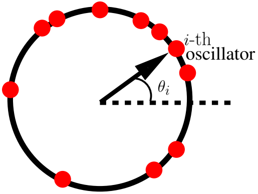


Photo Courtesy: Getty Images

- Synchronized firings of cardiac pacemaker cells
- Voltage oscillations in Josephson junctions arrays
- ....

Minimal framework: The Kuramoto model

1. N globally-coupled limit-cycle oscillators with distributed natural frequencies
2. θ_i: Phase
3. $\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i)$
4. K: Coupling constant, ω_i's: Natural frequencies, Unimodal distr. $g(\omega)$ with mean ω_0 *(Kuramoto (1975))*



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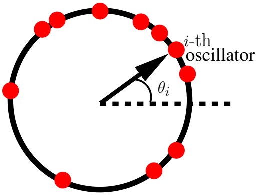
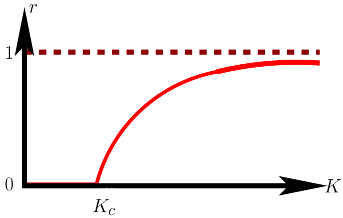
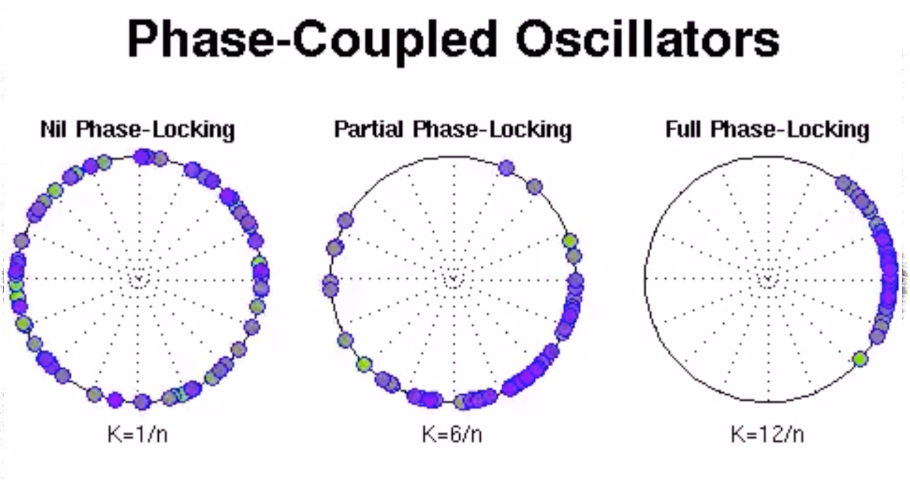


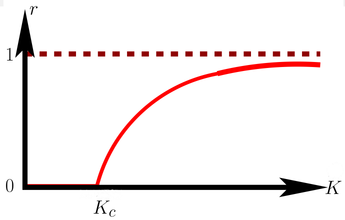
- $N \to \infty, t \to \infty$ limit:
  1. Define $R = re^{i\psi} \equiv \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$
  2. High K: Synchronized phase, $r \neq 0$
  3. Low K: Incoherent phase, $r = 0$
  4. "Phase transition" (Bifurcation) on tuning K
  5. $K_c = \frac{2}{\pi g(\omega_0)}$



The Kuramoto steady state



Main queries



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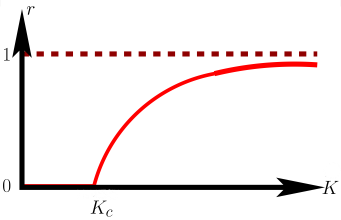

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YES!!, via

Stochastic Shuffling, or, **Subsystem Resetting**,

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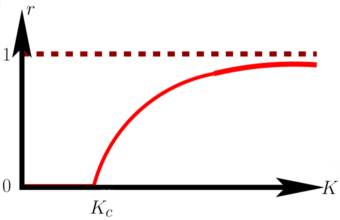

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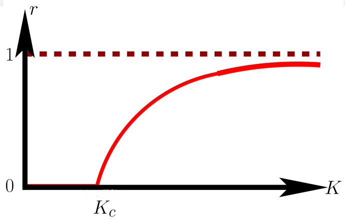

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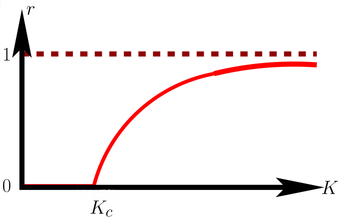

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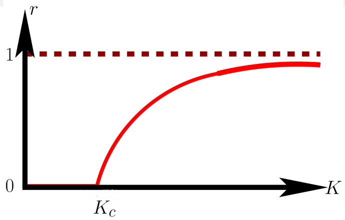

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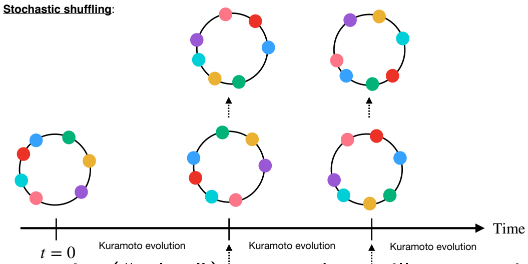
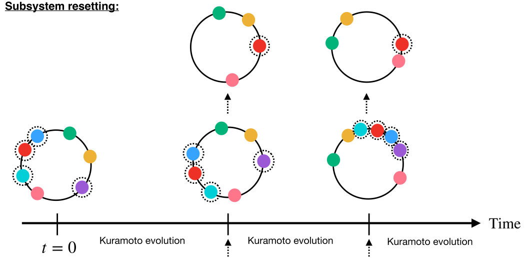
REST OF THE TALK: WHAT are these protocols? **HOW** do they work? **WHY** do they work?

Stochastic Shuffling and Subsystem Resetting



Shuffle the frequencies ("colors") among the oscillators at time intervals τ distributed as $p(\tau) = \lambda e^{-\lambda\tau}$



Reset a fixed subset of oscillator phases at time intervals τ with $p(\tau) = \lambda e^{-\lambda\tau}$

An ode to resetting

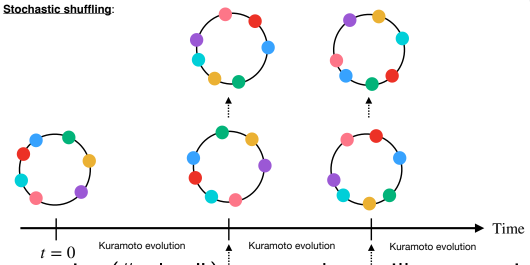
Restart is a simple and natural mechanism that has emerged as an over-reaching topic in physics, chemistry, biology, ecology, engineering and economics. Since the inaugural work of Evans and Majumdar (Evans M R and Majumdar S N 2011, Phys. Rev. Lett. **106***, 160601) a substantial amount of research has been carried out on stochastic resetting and its applications. This work spans different contexts starting from first-passage and search theory, stochastic thermodynamics, optimization theory, and all the way to quantum mechanics. Further connections have been made to animal foraging, protein-DNA interactions, coagulation-diffusion processes, chemical reaction processes, as well as to stock-market and population dynamics which display colossal crashes, i.e., resetting events.*

. . . J Phys. A Special Issue (2023)

Stochastic Resetting:
MAJUMDAR, Mallick, Rosso, Schehr, ADhar, Sengupta, Das, Basu, Krishnamurthy, Kundu, Pal, Sabhapandit, Kulkarni, many (all?) others (surely) in this room...

Review: Evans, Majumdar and Schehr, J. Phys. A **53**, 193001 (2020)

Stochastic Shuffling and Subsystem Resetting



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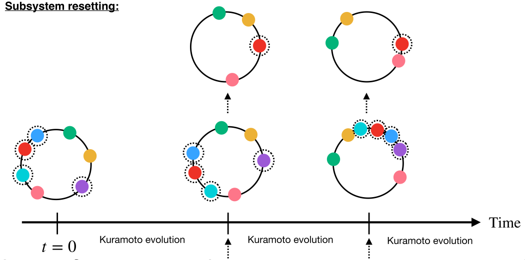

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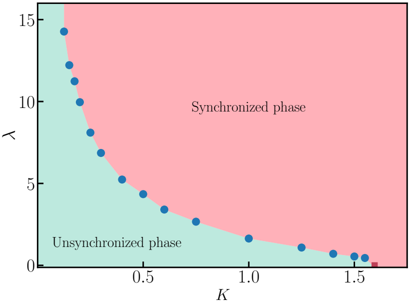
Stochastic Shuffling & Subsystem Resetting: Results

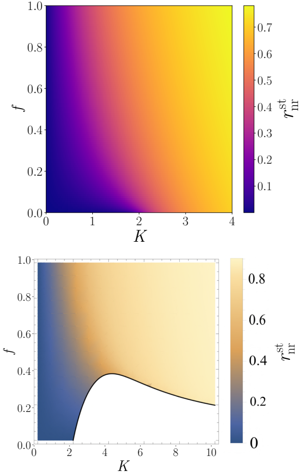
Stochastic Shuffling



λ: shuffling rate

$K_c \approx 1.6$ for bare model

Subsystem Resetting



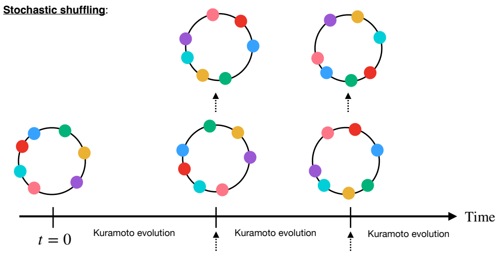
f: fraction of oscillators being reset

$\lambda \to \infty$ limit

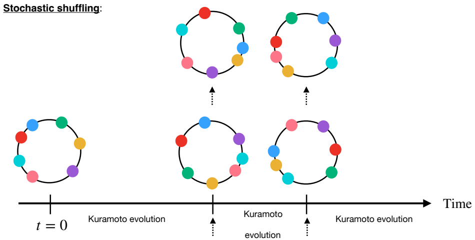
$K_c = 2$ for bare model

Stochastic Shuffling

Dynamical realization 1:

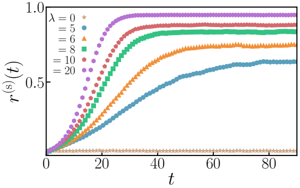


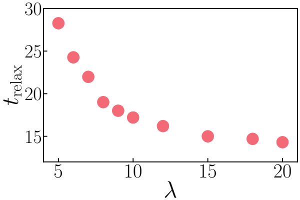
Dynamical realization 2:



Results for Gaussian $g(\omega)$

1. Relaxation to stationary state:





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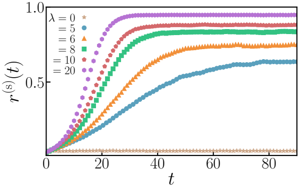

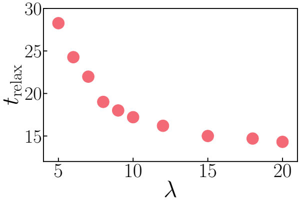

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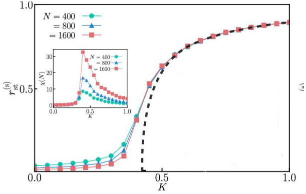
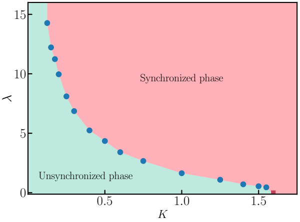
$$\chi(N) \equiv N[\langle (r_{\text{st}}^{(s)})^2\rangle - \langle r_{\text{st}}^{(s)}\rangle^2]$$





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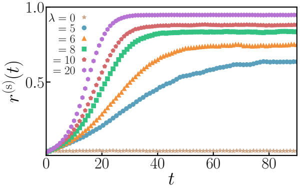

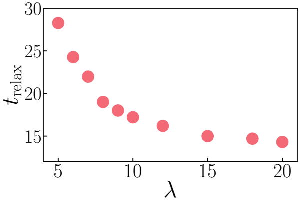

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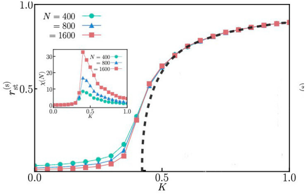

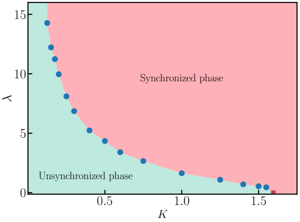

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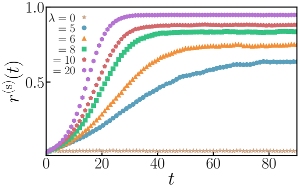

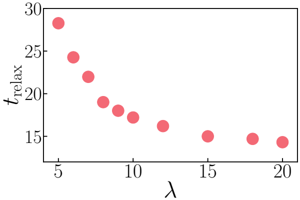

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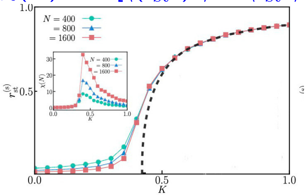

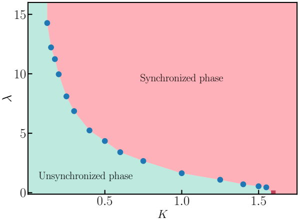

Synchronization under shuffling gets easier in every practical sense: lower coupling and less time to achieve synchrony;

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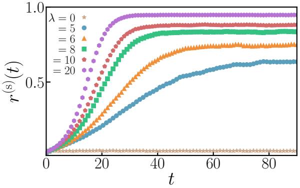

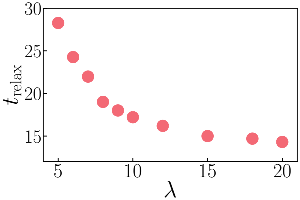

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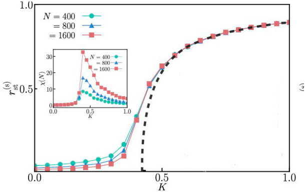

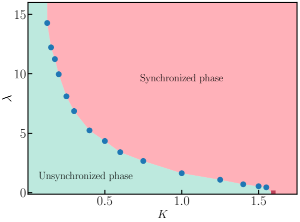

Synchronization under shuffling gets easier in every practical sense: lower coupling and less time to achieve synchrony; similar results for any $g(\omega)$ with finite variance and also with shuffling done at fixed time intervals

Stochastic Shuffling: Analysis

1. $N \to \infty$; Initial condition: $\theta_j(0) = \theta_0 \;\forall\; j$
2. $$R^{(\mathrm{s})}(t)|_{\{\theta_j(0)\}} = \underbrace{e^{-\lambda t} R(t)|_{\{\theta_j(0)\}}}_{\text{No shuffling since } t=0} + \underbrace{\lambda \int_0^t \mathrm{d}\tau \; e^{-\lambda \tau} R(t)|_{\{\theta_j(t-\tau)\}}}_{\text{Last shuffling at } t-\tau}$$

Inspired from *(Evans and Majumdar (2011))*

- First term: Result of dynamical evolution according to the bare Kuramoto model and with $\{\theta_j(0)\}$ as the initial condition
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3. $t \to \infty$: stationary state
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4. Requires θ_j as a function of t under dynamics of bare Kuramoto evolution interspersed with shuffling $\to$ Analytical solution not known
   $\Longrightarrow\ \{\theta_j(t-\tau)\}$ from simulations for large N

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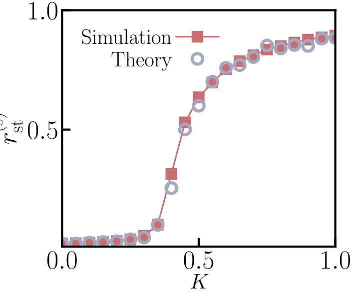
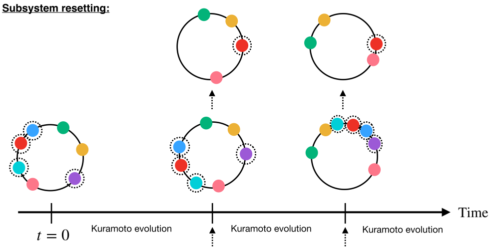
Gaussian $g(\omega)$:



Experiments with network of Wien Bridge oscillators: Qualitative agreement

Subsystem Resetting

Dynamical realization 1:



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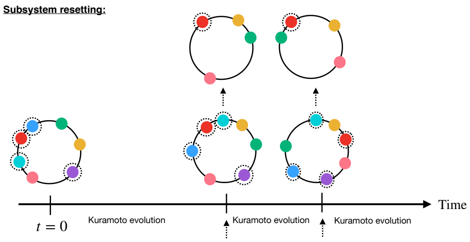

Order parameters of reset and non-reset oscillators

- N oscillators:
  Reset oscillators labelled $j = 1, 2, \ldots, n$;
  $j = (n+1), (n+2), \ldots, N$: Only Kuramoto evolution;
  $f \equiv n/N$: fraction of oscillators undergoing reset
- Initial configuration: $\theta_j = 0 \; \forall \; j$; reset oscillators undergoing reset to $\theta_j = 0$

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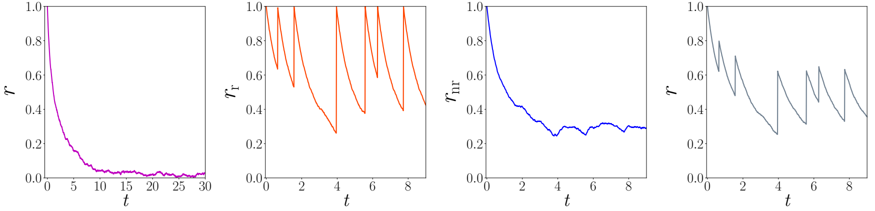
- $r_{\mathrm{r}}(t)e^{i\psi_{\mathrm{r}}(t)} \equiv \frac{1}{n}\sum_{j=1}^{n} e^{i\theta_j(t)}$
- $r_{\mathrm{nr}}(t)e^{i\psi_{\mathrm{nr}}(t)} \equiv \frac{1}{N-n}\sum_{j=n+1}^{N} e^{i\theta_j(t)}$
- $r = \sqrt{f^2 r_{\mathrm{r}}^2 + (1-f)^2 r_{\mathrm{nr}}^2 + 2f(1-f) r_{\mathrm{r}} r_{\mathrm{nr}} \cos(\psi_{\mathrm{r}} - \psi_{\mathrm{nr}})}$



Lorentzian $g(\omega)$ with zero mean and unit variance, $K = 1.5$ ($< K_c = 2.0$), $f = 0.5$, $\lambda = 0.5$

The case of infinite resetting rate

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- $\lambda \to \infty$: $\theta_j(t) = 0 \; \forall \; t$ and $j = 1, 2, \ldots, n$
- Non-reset oscillators:

$$\frac{d\theta_j}{dt} = \omega_j + \frac{K}{N} \sum_{l=1}^{N} \sin(\theta_l - \theta_j)$$

$$\downarrow$$

$$\frac{d\theta_j}{dt} = \omega_j - Kf \sin\theta_j + \frac{K}{N} \sum_{l=1}^{N} \sin(\theta_l - \theta_j)$$

- In terms of respective order parameters:
  $\frac{d\theta_j}{dt} = \omega_j - Kf \sin\theta_j + K(1-f) r_{\mathrm{nr}} \sin(\psi_{\mathrm{nr}} - \theta_j)$,
  $r_{\mathrm{r}} = 1, \psi_{\mathrm{r}} = 0$ at all times

The case of infinite resetting rate

- N oscillators:
  Reset oscillators labelled $j = 1, 2, \ldots, n$;
  $j = (n+1), (n+2), \ldots, N$: Only Kuramoto evolution;
  $f \equiv n/N$: fraction of oscillators undergoing reset
- Initial configuration: $\theta_j = 0 \; \forall \; j$; reset oscillators undergoing reset to $\theta_j = 0$
- $\lambda \to \infty$: $\theta_j(t) = 0 \; \forall \; t$ and $j = 1, 2, \ldots, n$
- Non-reset oscillators:

$$\frac{d\theta_j}{dt} = \omega_j + \frac{K}{N} \sum_{l=1}^{N} \sin(\theta_l - \theta_j)$$

$$\downarrow$$

$$\frac{d\theta_j}{dt} = \omega_j - Kf \sin\theta_j + \frac{K}{N} \sum_{l=1}^{N} \sin(\theta_l - \theta_j)$$

- In terms of respective order parameters:
  $\frac{d\theta_j}{dt} = \omega_j - Kf \sin\theta_j + K(1-f) r_{\rm nr} \sin(\psi_{\rm nr} - \theta_j)$,
  $r_{\rm r} = 1, \psi_{\rm r} = 0$ at all times
- Non-Hamiltonian dynamics of non-reset oscillators;
  Absence of noise $\to$ no Langevin-Fokker-Planck description

The case of infinite resetting rate

- $N \to \infty, n \to \infty$ for fixed and finite f
- $F(\theta, \omega, t)$: probability density to obtain an oscillator with phase θ and frequency ω;
  $\int_0^{2\pi} d\theta \; F(\theta, \omega, t) = g(\omega)$, $\int d\omega \int_0^{2\pi} d\theta \; F(\theta, \omega, t) = 1$

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- Number of oscillators with a given ω conserved by dynamics $\implies$
  1. Continuity equation: $\frac{\partial F}{\partial t} + \frac{\partial}{\partial \theta}\left(F \frac{d\theta}{dt}\right) = 0$
  2. $\frac{d\theta}{dt} = \omega + \frac{1}{2i}[(K(1-f)z_{\rm nr} + Kf)e^{-i\theta} - (K(1-f)z_{\rm nr}^* + Kf)e^{i\theta}]$
  3. $z_{\rm nr} = r_{\rm nr} e^{i\psi_{\rm nr}} = \int_{-\infty}^{\infty} \int_0^{2\pi} e^{i\theta} F(\theta, \omega, t) d\theta d\omega$

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- $F(\theta, \omega, t)$ is 2π periodic in θ $\implies$ Fourier expansion

$$F(\theta, \omega, t) = \frac{g(\omega)}{2\pi}\left[1 + \sum_{n=-\infty, n\neq 0}^{\infty} \tilde{F}_n(\omega, t)e^{in\theta}\right]$$

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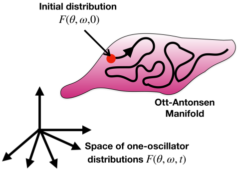

Special class of $F \to \widetilde{F}_n(\omega, t) = [\alpha(\omega, t)]^n$: defined on and remaining confined to Ott-Antonsen (OA) manifold under evolution *(Ott-Antonsen (2008))*
Dividend: two coupled first-order ordinary differential equations for $r_{\rm nr}(t)$ and $\psi_{\rm nr}(t)$

The case of infinite resetting rate

- OA ansatz $\Longrightarrow$ $\frac{\partial \alpha}{\partial t} = \frac{K}{2}[(1-f)z_{\rm nr}^* + f] - i\omega\alpha - \frac{K}{2}[(1-f)z_{\rm nr} + f]\alpha^2$
  $z_{\rm nr} = \int_{-\infty}^{\infty} \alpha^*(\omega, t) g(\omega) d\omega$

The case of infinite resetting rate

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  $z_{\mathrm{nr}} = \int_{-\infty}^{\infty} \alpha^*(\omega, t) g(\omega) d\omega$
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- $\frac{dz_{\mathrm{nr}}}{dt} = \frac{K}{2}[((1-f)z_{\mathrm{nr}} + f) - ((1-f)z_{\mathrm{nr}}^* + f)z_{\mathrm{nr}}^2] - (\sigma - i\omega_0)\, z_{\mathrm{nr}}$

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- Rescaling: $t \to \sigma t$, $K \to K/\sigma$, and $\omega_0 \to \omega_0/\sigma$
- $r_{\mathrm{nr}}^{'} = \frac{K(1-f)}{2} r_{\mathrm{nr}} \left(1 - r_{\mathrm{nr}}^2\right) - r_{\mathrm{nr}} + \frac{Kf}{2}\left(1 - r_{\mathrm{nr}}^2\right) \cos\psi_{\mathrm{nr}}$;
  $r_{\mathrm{nr}}\psi_{\mathrm{nr}}^{'} = -\left[-\omega_0 r_{\mathrm{nr}} + \frac{Kf}{2}\left(1 + r_{\mathrm{nr}}^2\right) \sin\psi_{\mathrm{nr}}\right]$

The case of infinite resetting rate

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  $r_{\rm nr}\psi_{\rm nr}^{'} = -\left[-\omega_0 r_{\rm nr} + \frac{Kf}{2}\left(1 + r_{\rm nr}^2\right)\sin\psi_{\rm nr}\right]$
- $r_{\rm nr} = 0$ never a stationary solution as soon as $f \neq 0$
  1. Resetting a vanishing fraction synchronizes the nonreset subsystem
  2. Synchronization transition of the bare model becomes a crossover

The case of infinite resetting rate

- OA ansatz $\implies$ $\frac{\partial \alpha}{\partial t} = \frac{K}{2}[(1-f)z_{\rm nr}^* + f] - i\omega\alpha - \frac{K}{2}[(1-f)z_{\rm nr} + f]\alpha^2$
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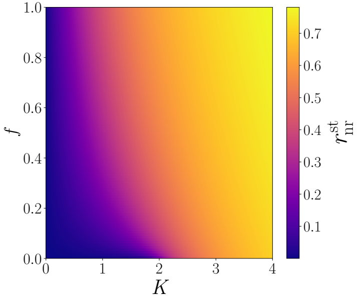
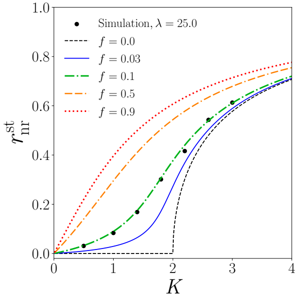
- $r_{\rm nr} = 0$ never a stationary solution as soon as $f \neq 0$
  1. Resetting a vanishing fraction synchronizes the nonreset subsystem
  2. Synchronization transition of the bare model becomes a crossover
- Long-time state depends on mean ω_0 of $g(\omega)$ (unlike the bare model)
  1. $\omega_0 = 0$: Stationary state $\implies$
     $\left(r_{\rm nr}^{\rm st}\right)^3 + \left(\frac{f}{1-f}\right)\left(r_{\rm nr}^{\rm st}\right)^2 + \left[\frac{2}{K(1-f)} - 1\right] r_{\rm nr}^{\rm st} - \left(\frac{f}{1-f}\right) = 0$
  2. $\omega_0 \neq 0$: $\frac{1}{1-(r_{\rm nr}^{\rm st})^2} = \frac{K(1-f)}{2} + \sqrt{\frac{K^2 f^2}{4}\frac{1}{(r_{\rm nr}^{\rm st})^2} - \frac{\omega_0^2}{\left(1+(r_{\rm nr}^{\rm st})^2\right)^2}}$;
     Stationary state provided $f > f_c$, with $K\left(\frac{f_c^2}{1-f_c}\right) = \omega_0^2 \frac{K(1-f_c)-2}{(K(1-f_c)-1)^2}$

The case of infinite resetting rate: $\omega_0 = 0$

- Stationary state:
  $$(r_{\rm nr}^{\rm st})^3 + \left(\frac{f}{1-f}\right)(r_{\rm nr}^{\rm st})^2 + \left[\frac{2}{K(1-f)} - 1\right] r_{\rm nr}^{\rm st} - \left(\frac{f}{1-f}\right) = 0$$





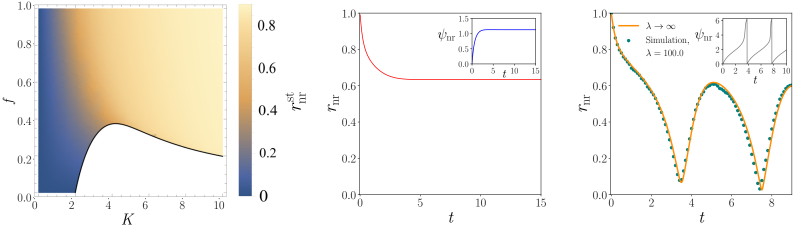
- Lorentzian $g(\omega)$ with unit variance

1. Non-reset subsystem has a synchronized stationary state at long times for any K and f
2. Synchronization transition as a function of K of the bare dynamics becomes a crossover in presence of subsystem resetting
3. Agreement between theory and simulations

The case of infinite resetting rate: $\omega_0 \neq 0$

- Stationary state: $\frac{1}{1-(r_{\rm nr}^{\rm st})^2} = \frac{K(1-f)}{2} + \sqrt{\frac{K^2 f^2}{4}\frac{1}{(r_{\rm nr}^{\rm st})^2} - \frac{\omega_0^2}{\left(1+(r_{\rm nr}^{\rm st})^2\right)^2}}$,

  provided $f > f_c$, with $K\left(\frac{f_c^2}{1-f_c}\right) = \omega_0^2 \frac{K(1-f_c)-2}{(K(1-f_c)-1)^2}$



- Lorentzian $g(\omega)$ with unit variance

1. Non-reset subsystem has a synchronized stationary state for any $K \leq K_c$ and for any f
2. For $K > K_c$, non-reset subsystem at long times has
   (i) for large f a synchronized stationary state, and
   (ii) for small f an oscillatory synchronized state with a non-zero time-independent time average
3. $\rightarrow$ Non-reset subsystem is synchronized at long times for any K and f
4. Agreement between theory and simulations

The case of infinite resetting rate

Main conclusion:

Non-reset subsystem may or may not have a stationary state depending on the values of the dynamical parameters, even when resetting happens all the time ($\lambda \to \infty$);

Non-reset subsystem always synchronized

Contrast with global resetting when the system always has a stationary state independent of the value of λ

(Sarkar and Gupta (2022))

The case of finite resetting rate

No resetting: Two coupled subsystems r and nr evolving according to bare Kuramoto dynamics

- Oscillators in the individual subsystems on respective OA manifolds
- $N \to \infty : g_{\mathrm{r}}(\omega) = g_{\mathrm{nr}}(\omega) = g(\omega)$
- Lorentzian $g(\omega)$
- 1. Reset subsystem:

     $\frac{dr_{\mathrm{r}}}{dt} = -\sigma r_{\mathrm{r}} + K \left(\frac{1-r_{\mathrm{r}}^2}{2}\right) [f r_{\mathrm{r}} + (1-f) r_{\mathrm{nr}} \cos\psi];$

     $\frac{d\psi_{\mathrm{r}}}{dt} = \omega_0 - K(1-f)\sin\psi \left(\frac{1+r_{\mathrm{r}}^2}{2r_{\mathrm{r}}}\right) r_{\mathrm{nr}}$

  2. Non-reset subsystem:

     $\frac{dr_{\mathrm{nr}}}{dt} = -\sigma r_{\mathrm{nr}} + K \left(\frac{1-r_{\mathrm{nr}}^2}{2}\right) [f r_{\mathrm{r}} \cos\psi + (1-f) r_{\mathrm{nr}}]; \quad \psi \equiv \psi_1 - \psi_2;$

     $\frac{d\psi_{\mathrm{nr}}}{dt} = \omega_0 + Kf \sin\psi \left(\frac{1+r_{\mathrm{nr}}^2}{2r_{\mathrm{nr}}}\right) r_{\mathrm{r}}$

Finite resetting rate: The case $\omega_0 = 0$

- $(r_{\rm r}(t), \psi_{\rm r}(t))$ and $(r_{\rm nr}(t), \psi_{\rm nr}(t))$: Random variables
- With initial condition $\psi_{\rm r}(0) = \psi_{\rm nr}(0) = 0$, one has $\psi_{\rm r}(t) = 0$ and $\psi_{\rm nr}(t) = 0$ for all times t
- During bare evolution between two resets:
  $\frac{dr_{\rm r}}{dt} = -\sigma r_{\rm r} + K\left(\frac{1-r_{\rm r}^2}{2}\right)[fr_{\rm r} + (1-f)r_{\rm nr}]$;
  $\frac{dr_{\rm nr}}{dt} = -\sigma r_{\rm nr} + K\left(\frac{1-r_{\rm nr}^2}{2}\right)[fr_{\rm r} + (1-f)r_{\rm nr}]$

Finite resetting rate: The case $\omega_0 = 0$

- $(r_{\rm r}(t), \psi_{\rm r}(t))$ and $(r_{\rm nr}(t), \psi_{\rm nr}(t))$: Random variables
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  $\frac{dr_{\rm nr}}{dt} = -\sigma r_{\rm nr} + K\left(\frac{1-r_{\rm nr}^2}{2}\right)[fr_{\rm r} + (1-f)r_{\rm nr}]$
- Realization average of change in order parameters in $[t, t+dt]$:
  $d\tilde{r}_{\rm r} = (1-\lambda dt)\, dr_{\rm r} + \lambda dt\,(1-r_{\rm r})$;
  $d\tilde{r}_{\rm nr} = dr_{\rm nr}$

Finite resetting rate: The case $\omega_0 = 0$

- $(r_\mathrm{r}(t), \psi_\mathrm{r}(t))$ and $(r_\mathrm{nr}(t), \psi_\mathrm{nr}(t))$: Random variables
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  $\frac{dr_\mathrm{nr}}{dt} = -\sigma r_\mathrm{nr} + K\left(\frac{1-r_\mathrm{nr}^2}{2}\right)[fr_\mathrm{r} + (1-f)r_\mathrm{nr}]$
- Realization average of change in order parameters in $[t, t+dt]$:
  $d\tilde{r}_\mathrm{r} = (1-\lambda dt)\, dr_\mathrm{r} + \lambda dt\,(1-r_\mathrm{r})$;
  $d\tilde{r}_\mathrm{nr} = dr_\mathrm{nr}$
- Exact evolution equation for realization-averaged order parameters of reset and non-reset subsystems:
  $\frac{d\bar{r}_\mathrm{r}}{dt} = -\sigma\bar{r}_\mathrm{r} + \frac{K}{2}\left[f\bar{r}_\mathrm{r} + (1-f)\bar{r}_\mathrm{nr} - f\overline{r_\mathrm{r}^3} - (1-f)\overline{r_\mathrm{r}^2 r_\mathrm{nr}}\right] + \lambda\,(1-\bar{r}_\mathrm{r})$;
  $\frac{d\bar{r}_\mathrm{nr}}{dt} = -\sigma\bar{r}_\mathrm{nr} + \frac{K}{2}\left[f\bar{r}_\mathrm{r} + (1-f)\bar{r}_\mathrm{nr} - f\overline{r_\mathrm{r} r_\mathrm{nr}^2} - (1-f)\overline{r_\mathrm{nr}^3}\right]$

Finite resetting rate: The case $\omega_0 = 0$

- Exact evolution equation for realization-averaged order parameters of reset and non-reset subsystems:
  $\frac{d\bar{r}_{\rm r}}{dt} = -\sigma \bar{r}_{\rm r} + \frac{K}{2}\left[f\bar{r}_{\rm r} + (1-f)\bar{r}_{\rm nr} - f\overline{r_{\rm r}^3} - (1-f)\overline{r_{\rm r}^2 r_{\rm nr}}\right] + \lambda\,(1-\bar{r}_{\rm r});$
  $\frac{d\bar{r}_{\rm nr}}{dt} = -\sigma \bar{r}_{\rm nr} + \frac{K}{2}\left[f\bar{r}_{\rm r} + (1-f)\bar{r}_{\rm nr} - f\overline{r_{\rm r} r_{\rm nr}^2} - (1-f)\overline{r_{\rm nr}^3}\right]$
- λ large but finite: $\overline{r_{\rm r} r_{\rm nr}^2} \approx \bar{r}_{\rm r}\bar{r}_{\rm nr}^2$; $\overline{r_{\rm r}^2 r_{\rm nr}} \approx \bar{r}_{\rm r}^2\bar{r}_{\rm nr}$; $\overline{r_{\rm nr}^3} \approx \bar{r}_{\rm nr}^3$; $\overline{r_{\rm r}^3} \approx \bar{r}_{\rm r}^3$

Finite resetting rate: The case $\omega_0 = 0$

- Exact evolution equation for realization-averaged order parameters of reset and non-reset subsystems:

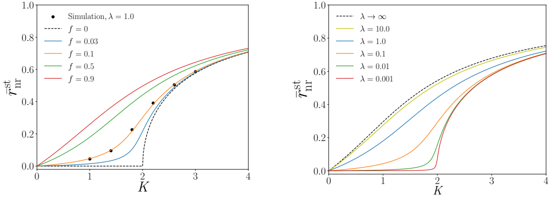
$$\frac{d\bar{r}_{\rm r}}{dt} = -\sigma\bar{r}_{\rm r} + \frac{K}{2}\left[f\bar{r}_{\rm r} + (1-f)\bar{r}_{\rm nr} - f\overline{r_{\rm r}^3} - (1-f)\overline{r_{\rm r}^2 r_{\rm nr}}\right] + \lambda\,(1-\bar{r}_{\rm r});$$

$$\frac{d\bar{r}_{\rm nr}}{dt} = -\sigma\bar{r}_{\rm nr} + \frac{K}{2}\left[f\bar{r}_{\rm r} + (1-f)\bar{r}_{\rm nr} - f\overline{r_{\rm r} r_{\rm nr}^2} - (1-f)\overline{r_{\rm nr}^3}\right]$$

- λ large but finite: $\overline{r_{\rm r} r_{\rm nr}^2} \approx \bar{r}_{\rm r}\bar{r}_{\rm nr}^2$; $\overline{r_{\rm r}^2 r_{\rm nr}} \approx \bar{r}_{\rm r}^2\bar{r}_{\rm nr}$; $\overline{r_{\rm nr}^3} \approx \bar{r}_{\rm nr}^3$; $\overline{r_{\rm r}^3} \approx \bar{r}_{\rm r}^3$

- $$(\bar{r}_{\rm r}^{\rm st})^3 + \left[\frac{1-f}{f}\right]\left((\bar{r}_{\rm r}^{\rm st})^2\,\bar{r}_{\rm nr}^{\rm st} - \bar{r}_{\rm nr}^{\rm st}\right) + \left[\frac{2(\lambda+\sigma)}{Kf} - 1\right]\bar{r}_{\rm r}^{\rm st} = \frac{2\lambda}{Kf};$$

$$(\bar{r}_{\rm nr}^{\rm st})^3 + \left[\frac{f}{1-f}\right]\left(\bar{r}_{\rm r}^{\rm st}\,(\bar{r}_{\rm nr}^{\rm st})^2 - \bar{r}_{\rm r}^{\rm st}\right) + \left[\frac{2\sigma}{K(1-f)} - 1\right]\bar{r}_{\rm nr}^{\rm st} = 0$$



- Lorentzian $g(\omega)$ with unit variance

Results similar to $\lambda \to \infty$ case:
Non-reset subsystem has a synchronized stationary state at long times,
Agreement between theory and simulations

Finite resetting rate: The case $\omega_0 \neq 0$

- Realization average of change in order parameters in $[t, t+dt]$:
  $d\tilde{r}_{\rm r} = (1-\lambda dt)\, dr_{\rm r} + \lambda dt\,(1-r_{\rm r})$;
  $d\tilde{r}_{\rm nr} = dr_{\rm nr}$;
  $d\tilde{\psi}_{\rm r} = (1-\lambda dt)\, d\psi_{\rm r} - \lambda dt \psi_{\rm r}$;
  $d\tilde{\psi}_{\rm nr} = d\psi_{\rm nr}$
- Exact evolution equation for realization-averaged order parameters of reset and non-reset subsystems:
  $\frac{d\bar{r}_{\rm r}}{dt} = -\sigma \bar{r}_{\rm r} + \lambda(1-\bar{r}_{\rm r}) + \frac{K}{2}\Big[f\,\bar{r}_{\rm r} + (1-f)\overline{r_{\rm nr}\cos(\psi_{\rm r}-\psi_{\rm nr})} -$
  $f\,\overline{r_{\rm r}^3} - (1-f)\overline{r_{\rm r}^2 r_{\rm nr}\cos(\psi_{\rm r}-\psi_{\rm nr})}\Big]$;

  $\frac{d\bar{r}_{\rm nr}}{dt} = -\sigma \bar{r}_{\rm nr} +$
  $\frac{K}{2}\left[f\,\overline{r_{\rm r}\cos(\psi_{\rm r}-\psi_{\rm nr})} + (1-f)\bar{r}_{\rm nr} - f\,\overline{r_{\rm r} r_{\rm nr}^2\cos(\psi_{\rm r}-\psi_{\rm nr})} - (1-f)\overline{r_{\rm nr}^3}\right]$;

  $\frac{d\bar{\psi}_{\rm r}}{dt} = \omega_0 - K(1-f)\overline{\sin(\psi_{\rm r}-\psi_{\rm nr})\left(\frac{1+r_{\rm r}^2}{2r_{\rm r}}\right) r_{\rm nr}} - \lambda\bar{\psi}_{\rm r}$;

  $\frac{d\bar{\psi}_{\rm nr}}{dt} = \omega_0 + Kf\overline{\sin(\psi_{\rm r}-\psi_{\rm nr})\left(\frac{1+r_{\rm nr}^2}{2r_{\rm nr}}\right) r_{\rm r}}$

Finite resetting rate: The case $\omega_0 \neq 0$

- Realization average of change in order parameters in $[t, t+dt]$:
  $d\tilde{r}_{\rm r} = (1-\lambda dt)\, dr_{\rm r} + \lambda dt\, (1 - r_{\rm r})$;
  $d\tilde{r}_{\rm nr} = dr_{\rm nr}$;
  $d\tilde{\psi}_{\rm r} = (1-\lambda dt)\, d\psi_{\rm r} - \lambda dt \psi_{\rm r}$;
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  $f\,\overline{r_{\rm r}^3} - (1-f)\overline{r_{\rm r}^2 r_{\rm nr}\cos(\psi_{\rm r}-\psi_{\rm nr})}\Big]$;
  $\frac{d\bar{r}_{\rm nr}}{dt} = -\sigma \bar{r}_{\rm nr} +$
  $\frac{K}{2}\left[f\,\overline{r_{\rm r}\cos(\psi_{\rm r}-\psi_{\rm nr})} + (1-f)\bar{r}_{\rm nr} - f\,\overline{r_{\rm r} r_{\rm nr}^2\cos(\psi_{\rm r}-\psi_{\rm nr})} - (1-f)\overline{r_{\rm nr}^3}\right]$;
  $\frac{d\bar{\psi}_{\rm r}}{dt} = \omega_0 - K(1-f)\overline{\sin(\psi_{\rm r}-\psi_{\rm nr})\left(\frac{1+r_{\rm r}^2}{2r_{\rm r}}\right) r_{\rm nr}} - \lambda \bar{\psi}_{\rm r}$;
  $\frac{d\bar{\psi}_{\rm nr}}{dt} = \omega_0 + Kf\overline{\sin(\psi_{\rm r}-\psi_{\rm nr})\left(\frac{1+r_{\rm nr}^2}{2r_{\rm nr}}\right) r_{\rm r}}$
- λ large but finite: $\overline{r_{\rm r} r_{\rm nr}^2} \approx \bar{r}_{\rm r}\bar{r}_{\rm nr}^2$; $\overline{r_{\rm r}^2 r_{\rm nr}} \approx \bar{r}_{\rm r}^2 \bar{r}_{\rm nr}$; $\overline{r_{\rm nr}^3} \approx \bar{r}_{\rm nr}^3$; $\overline{r_{\rm r}^3} \approx \bar{r}_{\rm r}^3$;
  $\overline{\cos(\psi_{\rm r}-\psi_{\rm nr})} \approx \cos(\bar{\psi}_{\rm r} - \bar{\psi}_{\rm nr})$

Finite resetting rate: The case $\omega_0 \neq 0$

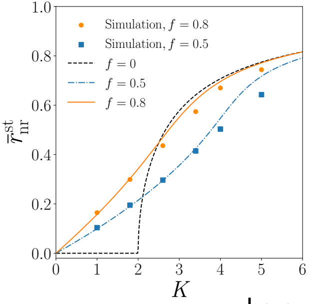
- $\frac{d\bar{r}_{\mathrm{r}}}{dt} = -\sigma \bar{r}_{\mathrm{r}} + \frac{K}{2}\left[f\bar{r}_{\mathrm{r}} + (1-f)\bar{r}_{\mathrm{nr}}\cos\left(\bar{\psi}_{\mathrm{r}} - \bar{\psi}_{\mathrm{nr}}\right) - f\bar{r}_{\mathrm{r}}^3 - (1-f)\bar{r}_{\mathrm{r}}^2\bar{r}_{\mathrm{nr}}\cos\left(\bar{\psi}_{\mathrm{r}} - \bar{\psi}_{\mathrm{nr}}\right)\right] + \lambda(1-\bar{r}_{\mathrm{r}})$;

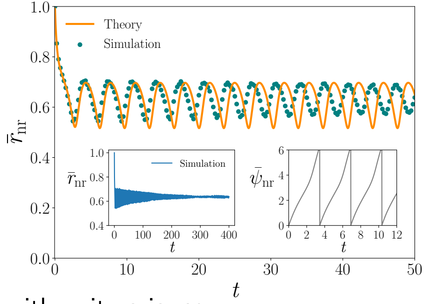
  $\frac{d\bar{r}_{\mathrm{nr}}}{dt} = -\sigma \bar{r}_{\mathrm{nr}} + \frac{K}{2}\left[f\bar{r}_{\mathrm{r}}\cos\left(\bar{\psi}_{\mathrm{r}} - \bar{\psi}_{\mathrm{nr}}\right) + (1-f)\bar{r}_{\mathrm{nr}} - f\bar{r}_{\mathrm{r}}\bar{r}_{\mathrm{nr}}^2\cos\left(\bar{\psi}_{\mathrm{r}} - \bar{\psi}_{\mathrm{nr}}\right) - (1-f)\bar{r}_{\mathrm{nr}}^3\right]$;

  $\frac{d\bar{\psi}_{\mathrm{r}}}{dt} = \omega_0 - K(1-f)\sin\left(\bar{\psi}_{\mathrm{r}} - \bar{\psi}_{\mathrm{nr}}\right)\left(\frac{1+\bar{r}_{\mathrm{r}}^2}{2\bar{r}_{\mathrm{r}}}\right)\bar{r}_{\mathrm{nr}} - \lambda\bar{\psi}_{\mathrm{r}}$;

  $\frac{d\bar{\psi}_{\mathrm{nr}}}{dt} = \omega_0 + Kf\sin\left(\bar{\psi}_{\mathrm{r}} - \bar{\psi}_{\mathrm{nr}}\right)\left(\frac{1+\bar{r}_{\mathrm{nr}}^2}{2\bar{r}_{\mathrm{nr}}}\right)\bar{r}_{\mathrm{r}}$

Finite resetting rate: The case $\omega_0 \neq 0$

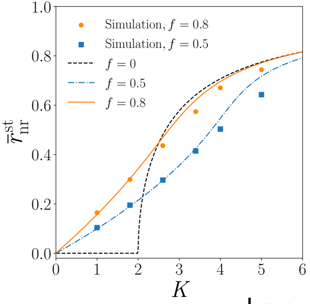


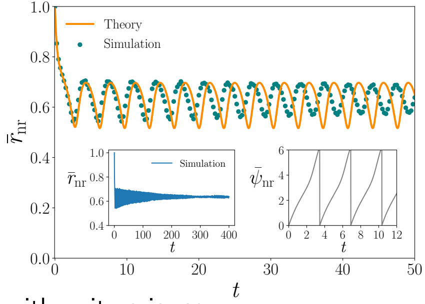


Lorentzian $g(\omega)$ with unit variance

- (i) for $K \leq K_c$:
  theory and simulations agree $\rightarrow$ synchronized stationary state for all f
  (ii) for $K > K_c$:
  1. large f: theory $\implies$ synchronized stationary state; qualitative agreement with simulations for large f
  2. small f: theory $\implies$ oscillatory synchronized state at long times
     simulations $\implies$ $\bar{r}_{\mathrm{nr}}$ oscillating with decaying amplitude and eventually settling to a synchronized stationary state

Finite resetting rate: The case $\omega_0 \neq 0$





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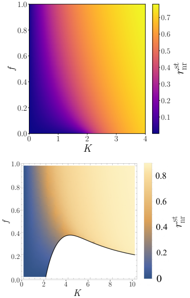
Also studied subsystem resetting for Gaussian $g(\omega)$, using an extension of OA-ansatz *(Campa (2022))*, and obtaining qualitatively similar results

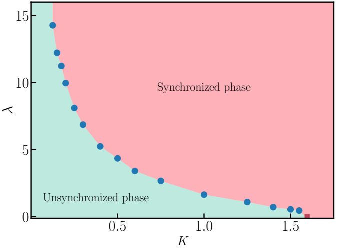
Conclusions

- **Stochastic Shuffling** and **Subsystem Resetting**: Two efficient mechanisms to induce order in many-body interacting systems

Subsystem Resetting



Stochastic Shuffling



λ: shuffling rate;
$K_c \approx 1.6$ for bare model

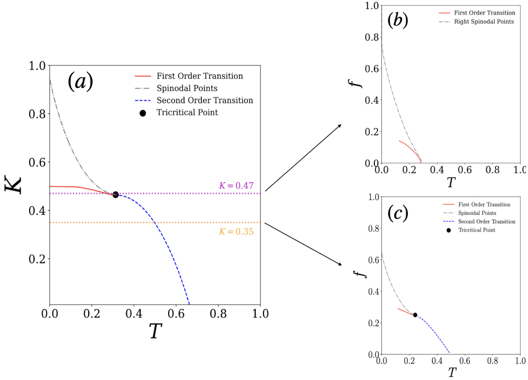
f: fraction of oscillators being reset;
$\lambda \to \infty$ limit;
$K_c = 2$ for bare model

Conclusions

- **Subsystem Resetting**: Allows to access phase diagrams without having to tune coupling constants *(A Acharya, R Majumder, SG (in preparation))*

$$H = K\sum_{i=1}^{N} S_i^2 - \frac{1}{2N}\sum_{i,j=1}^{N} S_i S_j; \quad S_i = 0, \pm 1; \;\; K > 0$$

(Blume, Emery, Griffiths (1971))



- **Future directions**: Stochastic shuffling in spin-glass systems, Subsystem resetting in quantum systems