Global heating from local cooling (& vice versa)

Shovan Dutta

Raman Research Institute

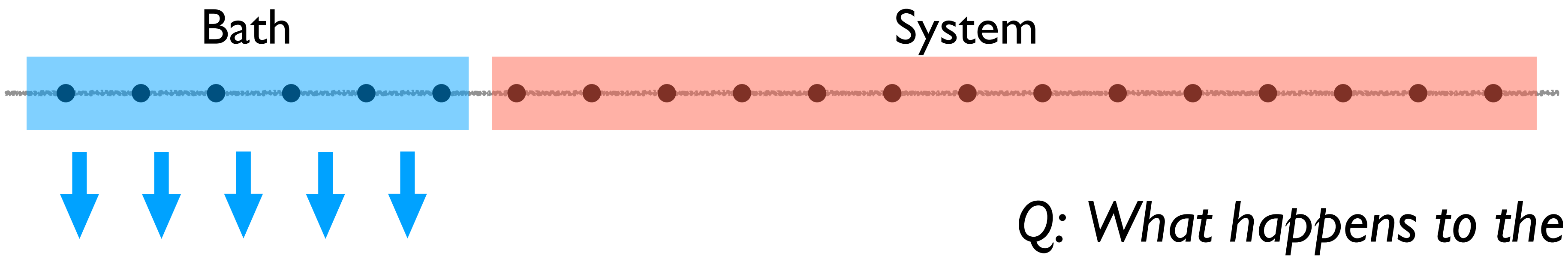
In collaboration with: Jaswanth Uppalapati (IISc)
Paul McClarty (CNRS)
Masud Haque (TU Dresden)

सत्यमेव जयते
विज्ञान एवं प्रौद्योगिकी विभाग
DEPARTMENT OF
SCIENCE & TECHNOLOGY

ISPCM’24

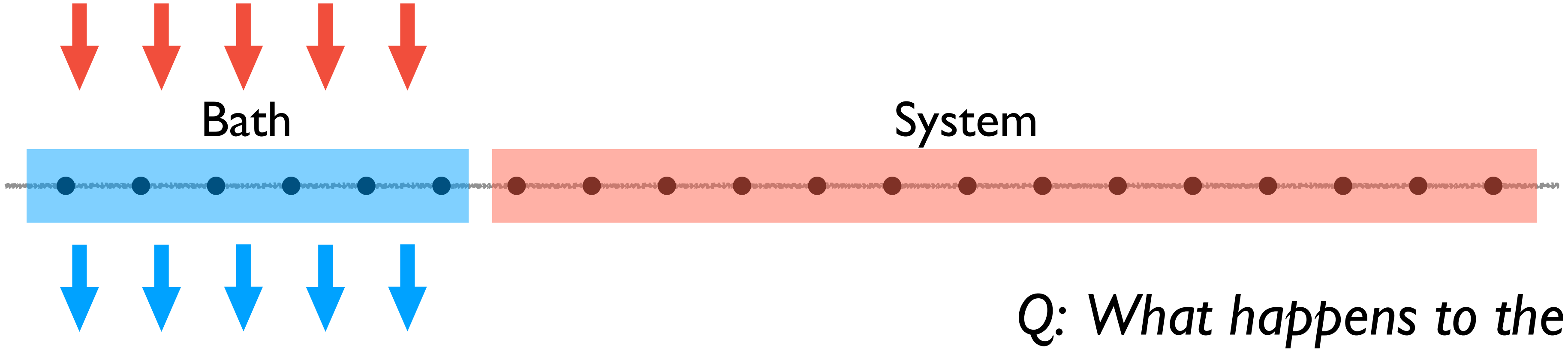
RRI

The setup



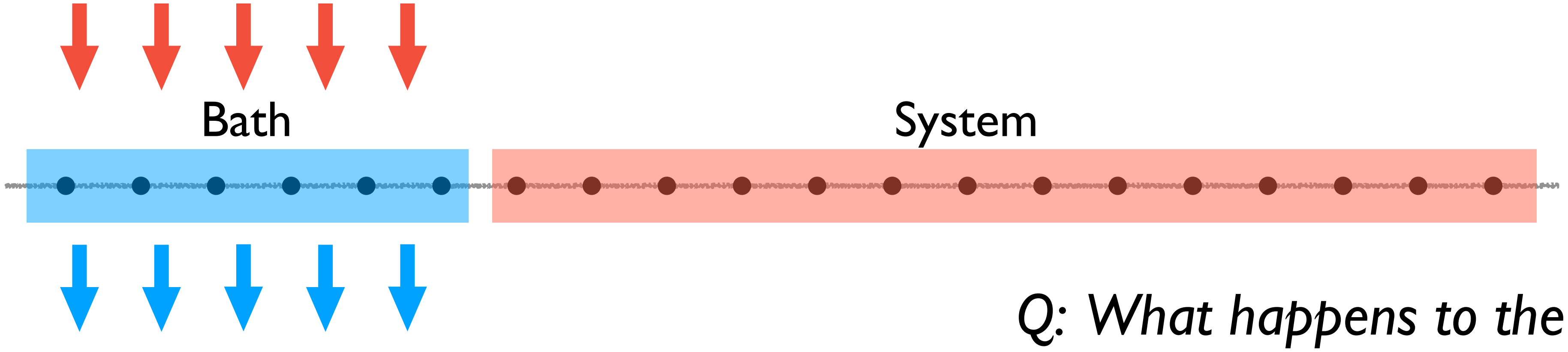
Q: What happens to the system?

The setup



Q: What happens to the system?

The setup



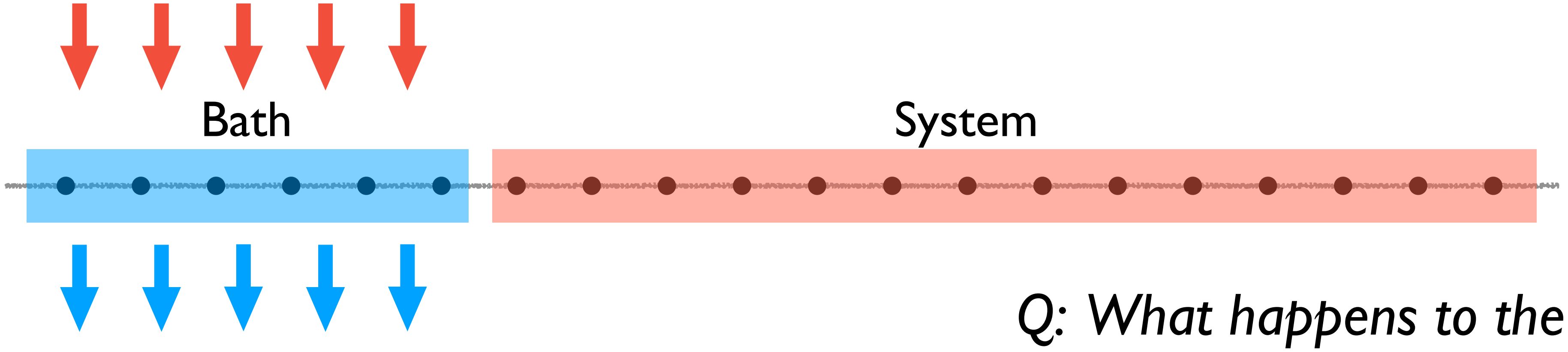
Q: What happens to the system?

Free-fermion bath + Lindblad dissipators — system reaches bath temperature for

- Infinite bath w/ bandwidth larger than system
- Weak and generic system-bath coupling
- Weak dissipation rates

Reichental, Klempner, Kafri, Podolsky,
PRB 97, 134301 (2018)

The setup



Q: What happens to the system?

Free-fermion bath + Lindblad dissipators — system reaches bath temperature for

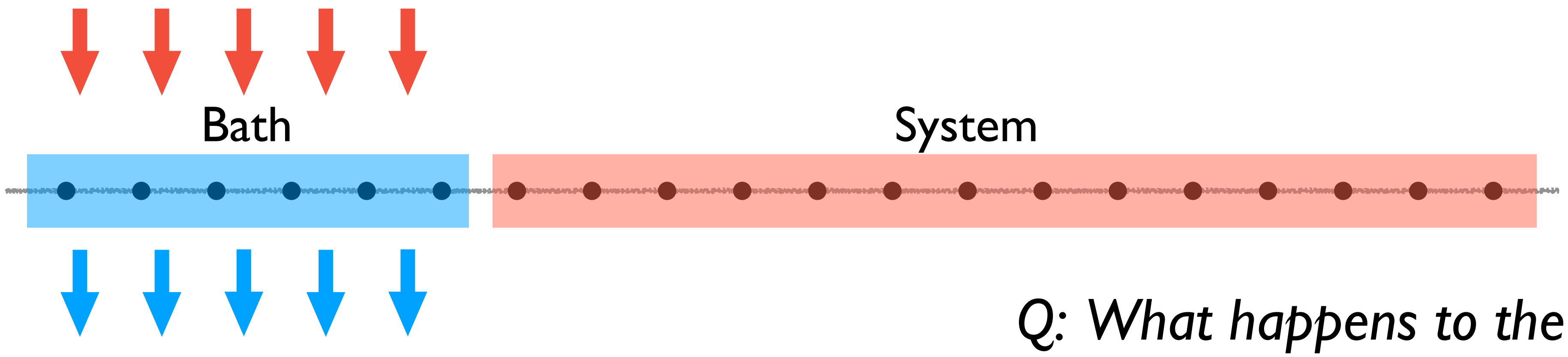
- Infinite bath w/ bandwidth larger than system
- Weak and generic system-bath coupling
- Weak dissipation rates

Reichental, Klempner, Kafri, Podolsky, PRB 97, 134301 (2018)

Numerical simulations: sympathetic cooling works best in above limits

Raghunandan, Wolf, Ospelkaus, Schmidt, Weimer, Sci. Adv. 6, eaaw9268 (2020)
Zanoci, Yoo, Swingle, PRB 108, 035156 (2023)
Tupkary, Dhar, Kulkarni, Purkayastha, PRA 107, 062216 (2023)
Palmero, Xu, Guo, Poletti, PRE 100, 022111 (2019)

The setup



Q: What happens to the system?

Free-fermion bath + Lindblad dissipators — system reaches bath temperature for

- Infinite bath w/ bandwidth larger than system
- Weak and generic system-bath coupling
- Weak dissipation rates

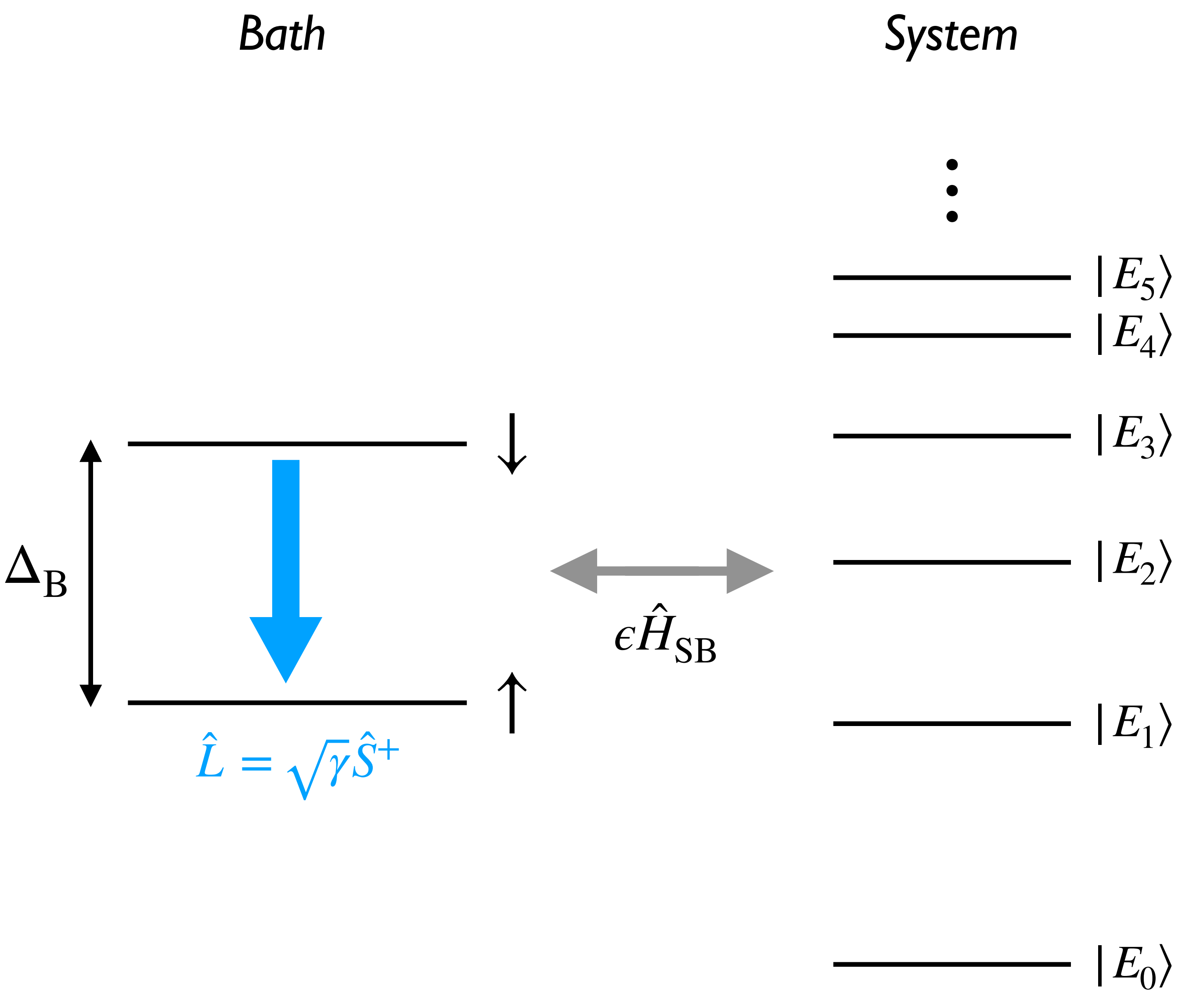
Reichental, Klempner, Kafri, Podolsky,
PRB 97, 134301 (2018)

Numerical simulations: sympathetic cooling works best in above limits

Raghunandan, Wolf, Ospelkaus, Schmidt, Weimer, Sci. Adv. 6, eaaw9268 (2020)
Zanoci, Yoo, Swingle, PRB 108, 035156 (2023)
Tupkary, Dhar, Kulkarni, Purkayastha, PRA 107, 062216 (2023)
Palmero, Xu, Guo, Poletti, PRE 100, 022111 (2019)

Here: Spectacular failure in presence of symmetry — heating by cooling

Generic case for single-qubit $T = 0$ bath



Bath

System

Δ_{B}

$\downarrow$

$\uparrow$

$\hat{L} = \sqrt{\gamma}\hat{S}^{+}$

$\epsilon\hat{H}_{\mathrm{SB}}$

$\vdots$

$|E_5\rangle$

$|E_4\rangle$

$|E_3\rangle$

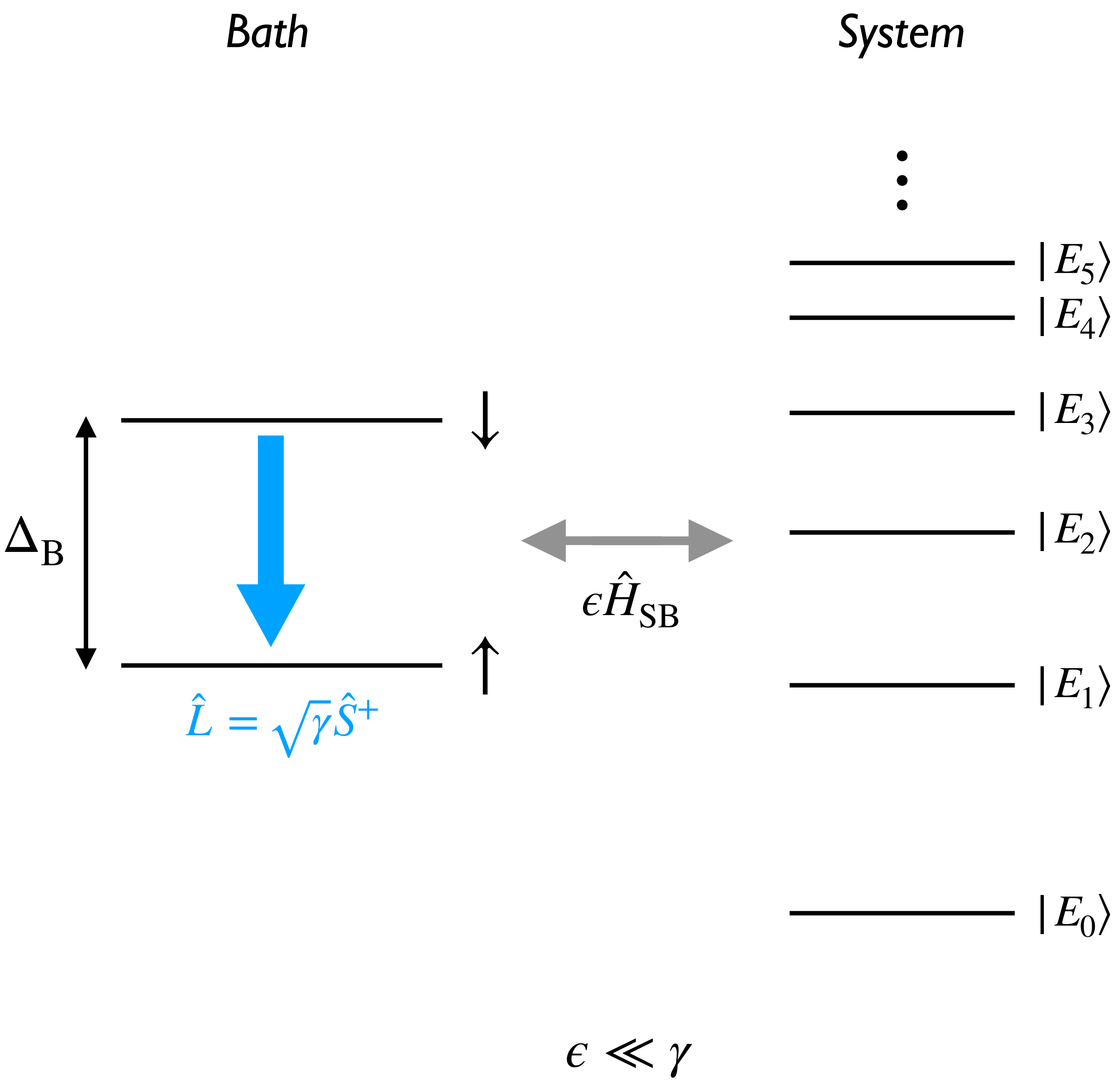
$|E_2\rangle$

$|E_1\rangle$

$|E_0\rangle$

$\epsilon \ll \gamma$

Generic case for single-qubit $T = 0$ bath

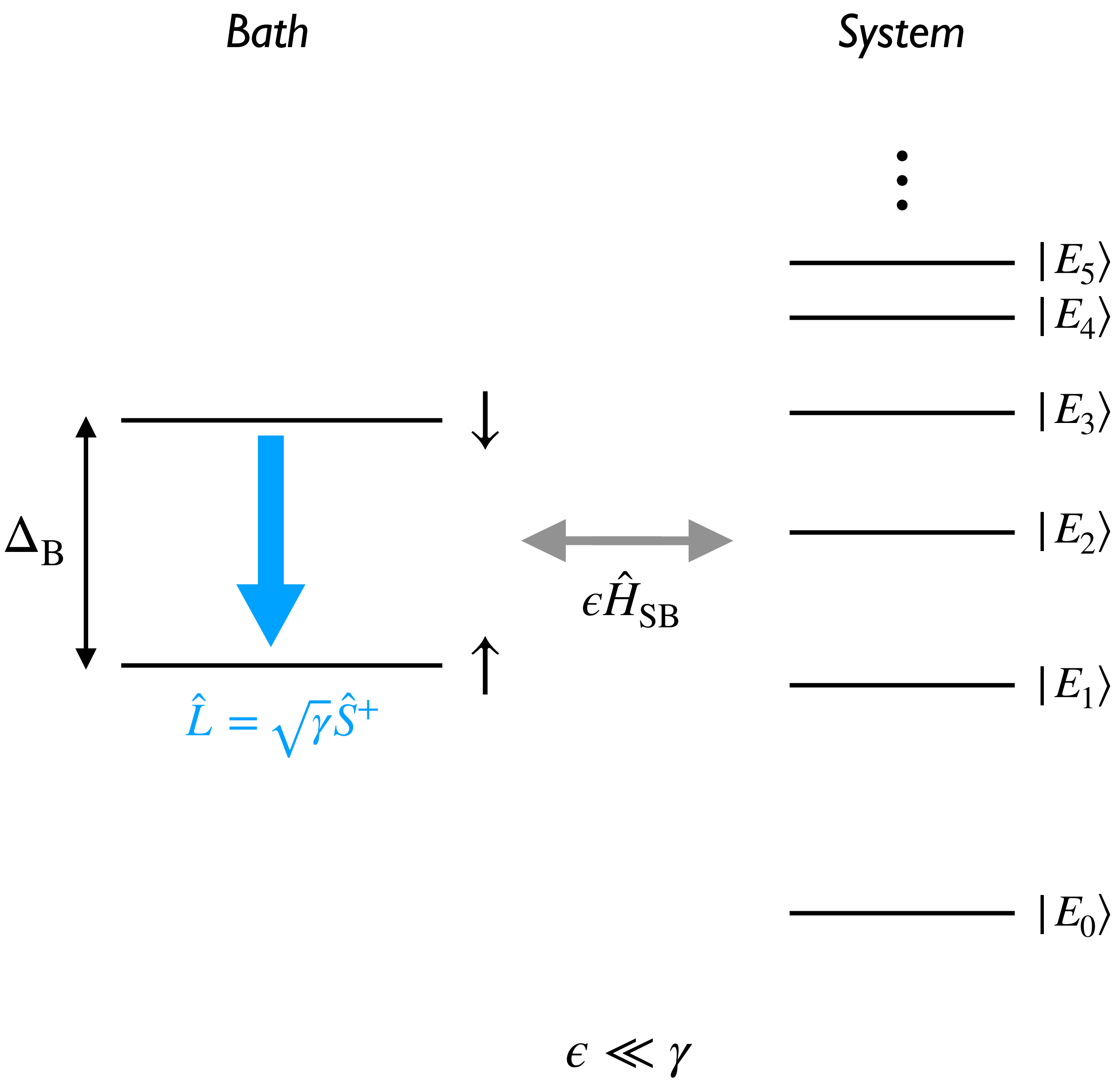


Steady state given by transitions between system eigenstates due to $\hat{H}_{\mathrm{SB}}$

Perturbed eigenstates of $\hat{H}_{\mathrm{total}}$:

$$|\uparrow \otimes E_i\rangle_p = |\uparrow \otimes E_i\rangle + \epsilon \sum_j c_{i,j} |\downarrow \otimes E_j\rangle$$
$$+ \epsilon \sum_{j \neq i} d_{i,j} |\uparrow \otimes E_j\rangle + O(\epsilon^2)$$

Generic case for single-qubit $T = 0$ bath



Steady state given by transitions between system eigenstates due to $\hat{H}_{SB}$

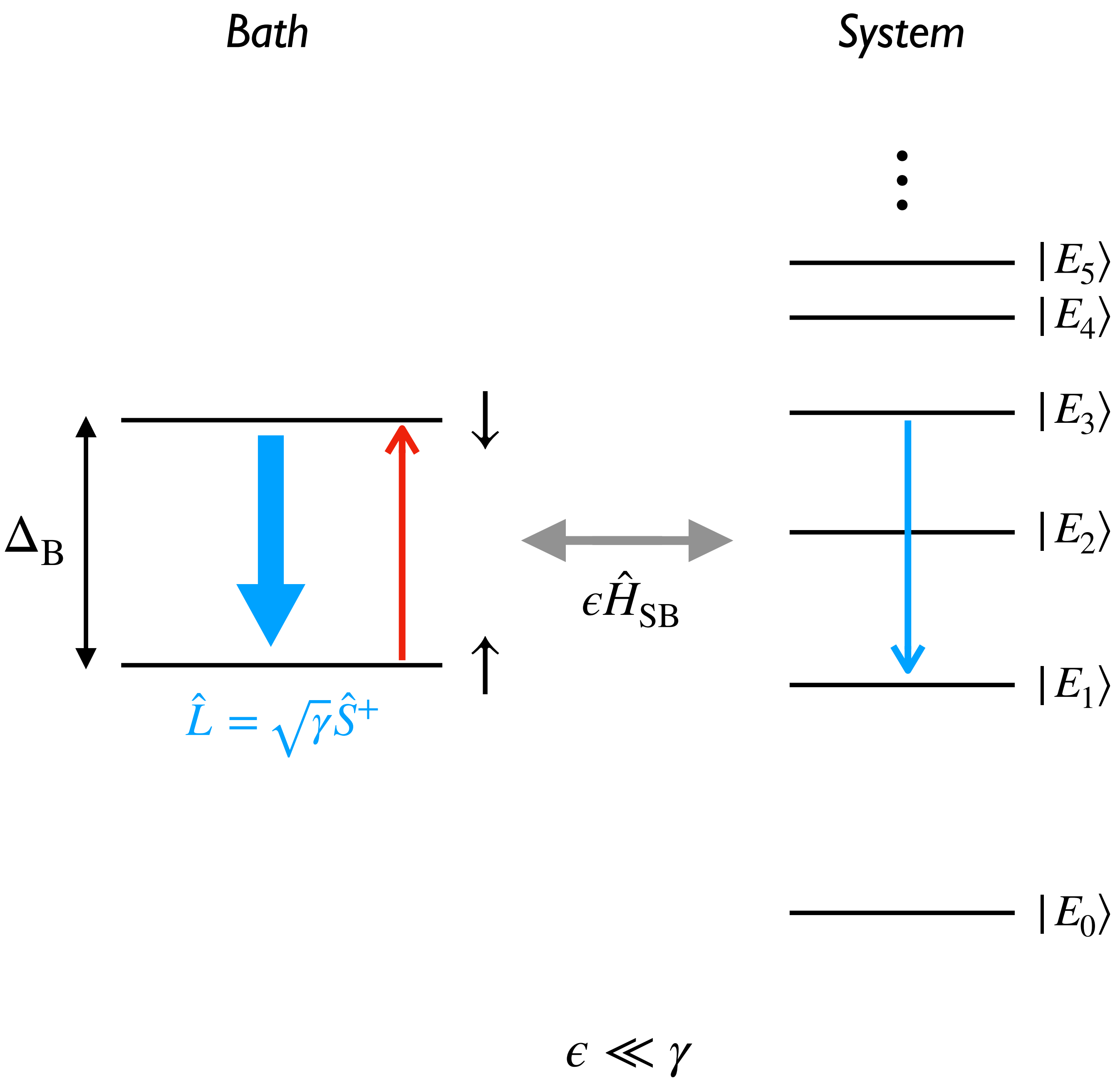
Perturbed eigenstates of $\hat{H}_{\text{total}}$:

$$|\uparrow \otimes E_i\rangle_p = |\uparrow \otimes E_i\rangle + \epsilon \sum_j c_{i,j} |\downarrow \otimes E_j\rangle + \epsilon \sum_{j \neq i} d_{i,j} |\uparrow \otimes E_j\rangle + O(\epsilon^2)$$

$\implies$ Transition rates

$$R_{i\to j} \approx \left| {}_p\langle \uparrow \otimes E_j \middle| \hat{L} \middle| \uparrow \otimes E_i \rangle_p \right|^2 \approx \gamma\epsilon^2 \frac{\left| \langle \downarrow \otimes E_j \middle| \hat{H}_{SB} \middle| \uparrow \otimes E_i \rangle \right|^2}{\left(E_i - E_j - \Delta_B\right)^2}$$

Generic case for single-qubit $T = 0$ bath



Steady state given by transitions between system eigenstates due to $\hat{H}_\mathrm{SB}$

Perturbed eigenstates of $\hat{H}_\mathrm{total}$:

$$|\uparrow \otimes E_i\rangle_p = |\uparrow \otimes E_i\rangle + \epsilon \sum_j c_{i,j} |\downarrow \otimes E_j\rangle + \epsilon \sum_{j \neq i} d_{i,j} |\uparrow \otimes E_j\rangle + O(\epsilon^2)$$

$\Longrightarrow$ Transition rates

$$R_{i \to j} \approx \left| {}_p\langle \uparrow \otimes E_j \middle| \hat{L} \middle| \uparrow \otimes E_i \rangle_p \right|^2 \approx \gamma \epsilon^2 \frac{\left| \langle \downarrow \otimes E_j \middle| \hat{H}_{SB} \middle| \uparrow \otimes E_i \rangle \right|^2}{\left(E_i - E_j - \Delta_\mathrm{B}\right)^2}$$

Resonant cooling for $E_i - E_j = \Delta_\mathrm{B}$

Heating from cooling — simplest case

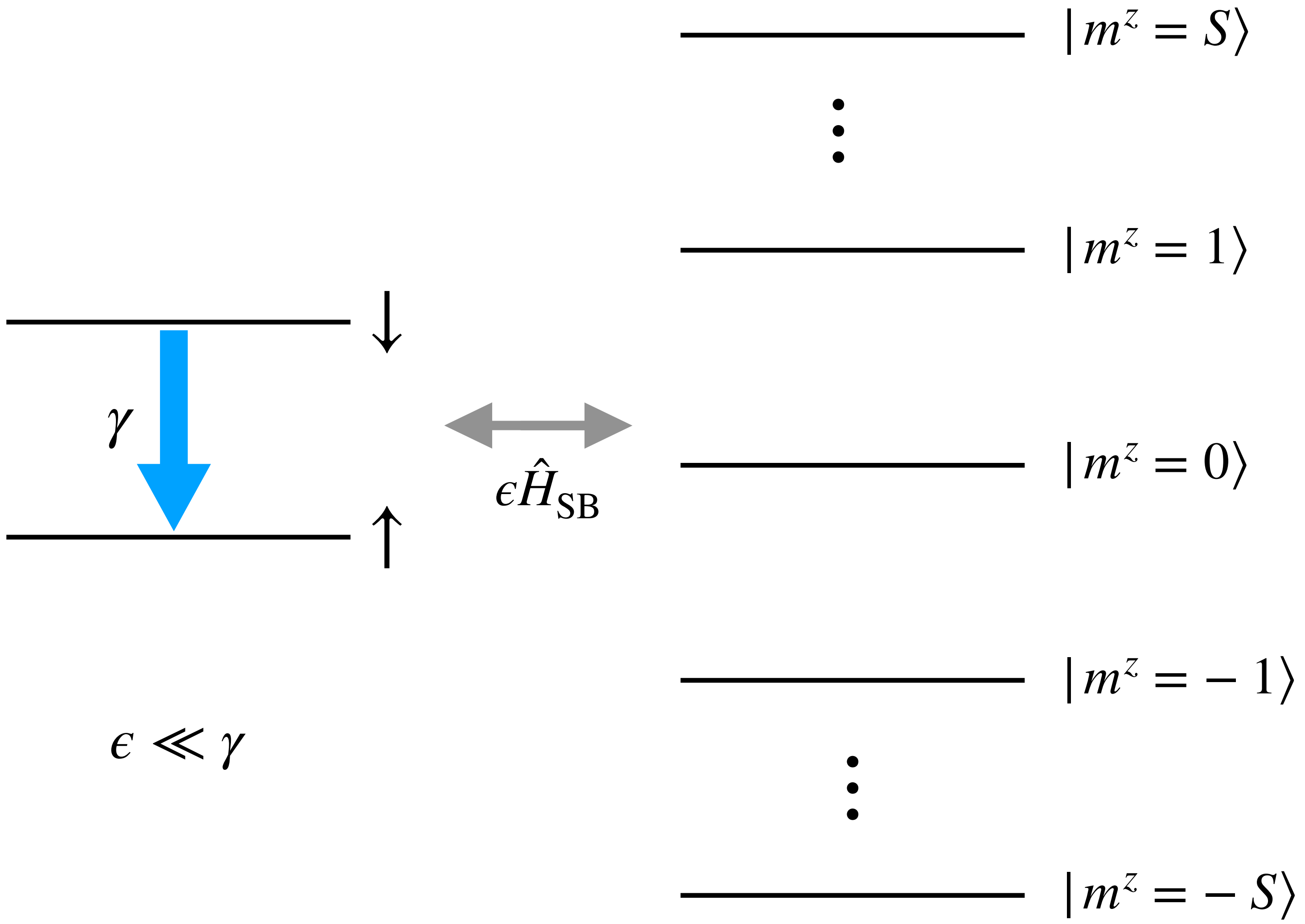
Bath

$\hat{H}_{\rm B} = -\,\Delta_{\rm B}\hat{\sigma}^z$

System

$\hat{H}_{\rm S} = +\,\Delta_{\rm S}\hat{S}^z$

$\hat{H}_{\rm SB} = \hat{\sigma}^+\hat{S}^- + \hat{\sigma}^-\hat{S}^+$



$|m^z = S\rangle$

$|m^z = 1\rangle$

$\downarrow$

γ

$\epsilon\hat{H}_{\rm SB}$

$|m^z = 0\rangle$

$\uparrow$

$|m^z = -\,1\rangle$

$\epsilon \ll \gamma$

$|m^z = -\,S\rangle$

Heating from cooling — simplest case

Bath

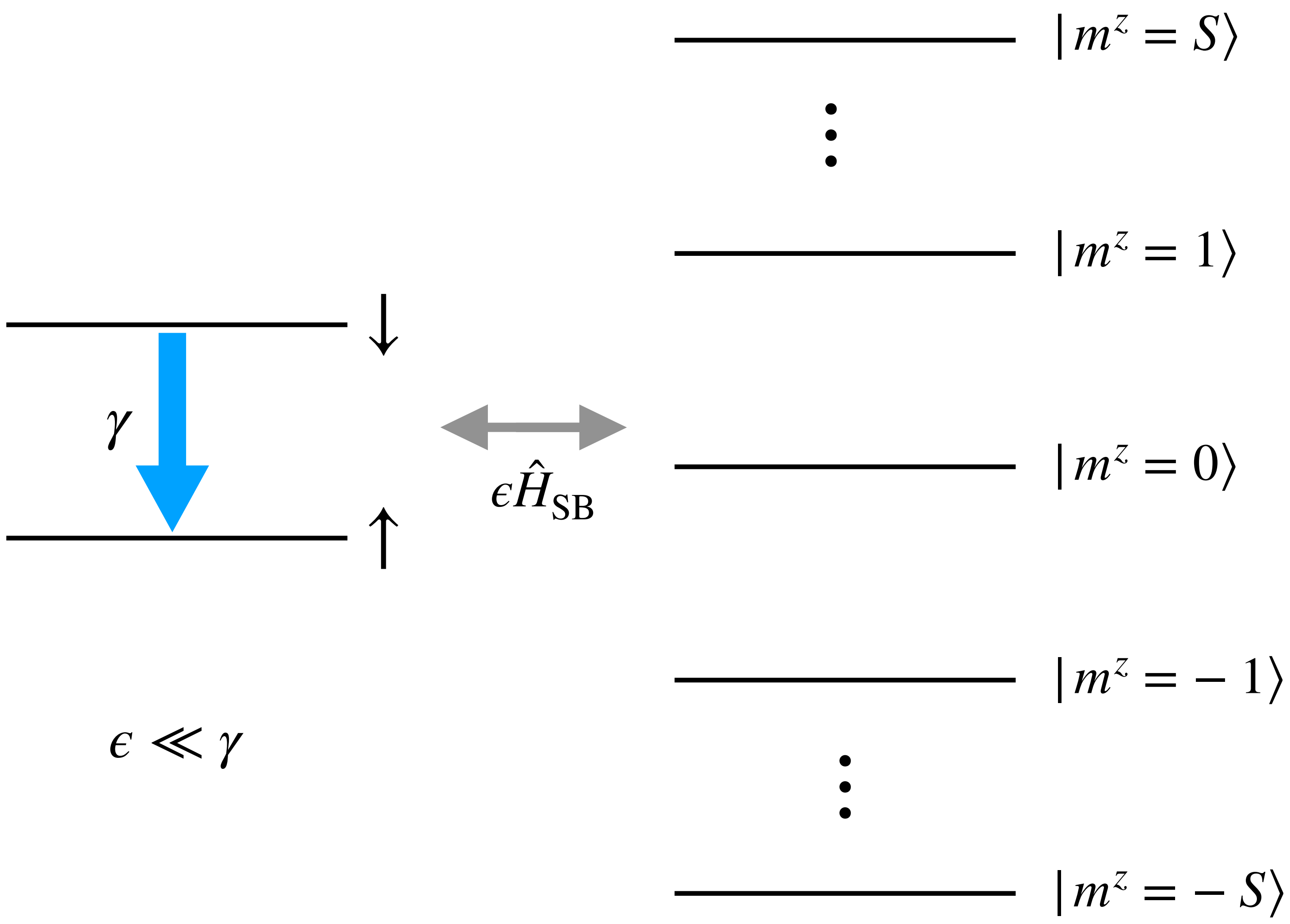
$\hat{H}_{\mathrm{B}} = -\Delta_{\mathrm{B}} \hat{\sigma}^z$

System

$\hat{H}_{\mathrm{S}} = +\Delta_{\mathrm{S}} \hat{S}^z$

$\hat{H}_{\mathrm{SB}} = \cancel{\hat{\sigma}^+ \hat{S}^-} + \hat{\sigma}^- \hat{S}^+$

bath spin is ↑



$|m^z = S\rangle$

$\vdots$

$|m^z = 1\rangle$

$|m^z = 0\rangle$

$|m^z = -1\rangle$

$\vdots$

$|m^z = -S\rangle$

↓

γ

↑

$\epsilon \hat{H}_{\mathrm{SB}}$

$\epsilon \ll \gamma$

Heating from cooling — simplest case

Bath

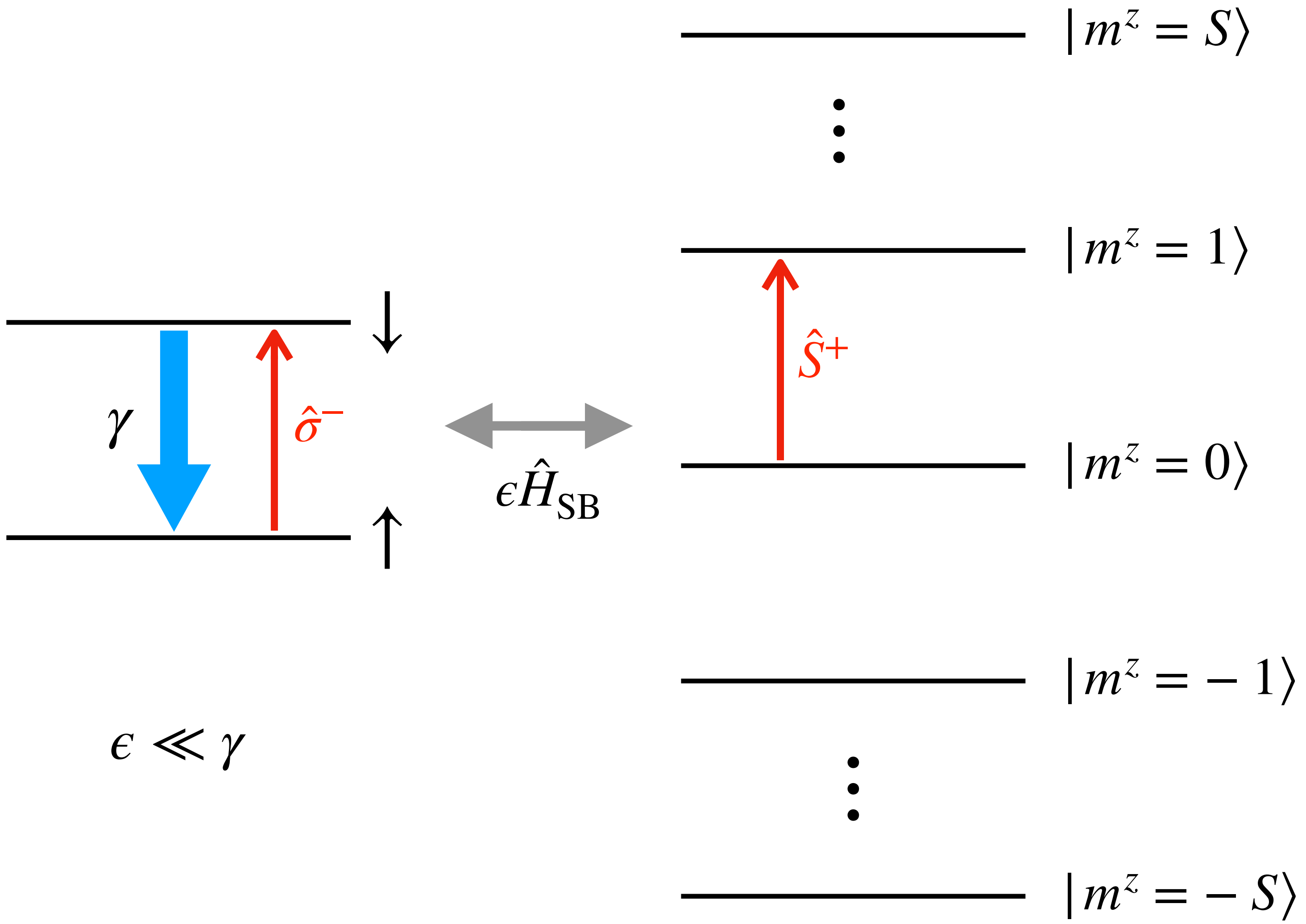
$\hat{H}_{\mathrm{B}} = -\Delta_{\mathrm{B}}\hat{\sigma}^z$

System

$\hat{H}_{\mathrm{S}} = +\Delta_{\mathrm{S}}\hat{S}^z$

$\hat{H}_{\mathrm{SB}} = \cancel{\hat{\sigma}^+\hat{S}^-} + \hat{\sigma}^-\hat{S}^+$

bath spin is ↑



$\epsilon \ll \gamma$

Heating from cooling — simplest case

Bath

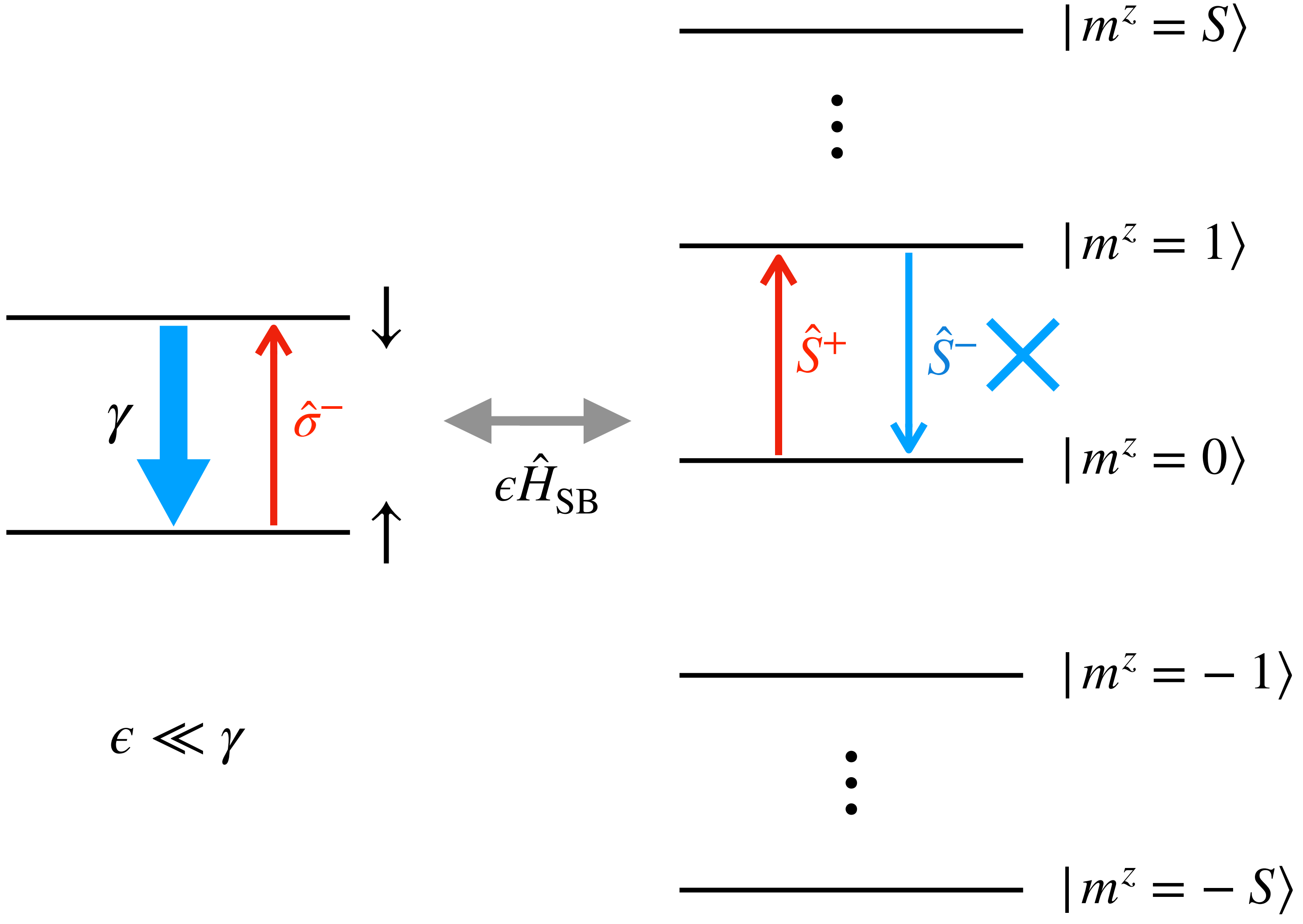
$\hat{H}_{\mathrm{B}} = -\Delta_{\mathrm{B}}\hat{\sigma}^z$

System

$\hat{H}_{\mathrm{S}} = +\Delta_{\mathrm{S}}\hat{S}^z$

$\hat{H}_{\mathrm{SB}} = \cancel{\hat{\sigma}^+\hat{S}^-} + \hat{\sigma}^-\hat{S}^+$

bath spin is ↑



No resonance — S^z_{total} conserved

[$U(1)$ symmetry]

$\epsilon \ll \gamma$

Heating from cooling — simplest case

Bath

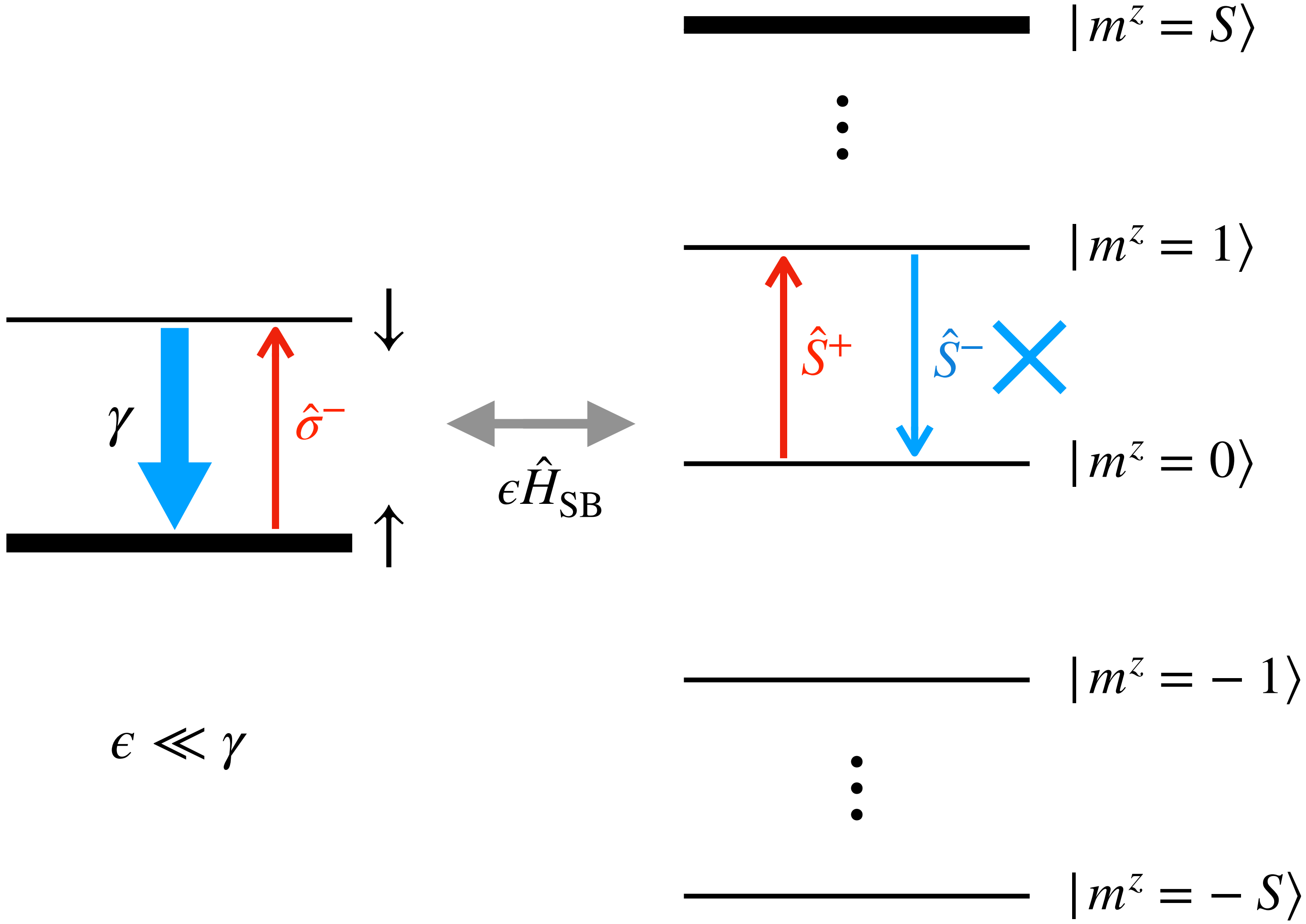
$\hat{H}_{\mathrm{B}} = -\Delta_{\mathrm{B}}\hat{\sigma}^z$

System

$\hat{H}_{\mathrm{S}} = +\Delta_{\mathrm{S}}\hat{S}^z$

$\hat{H}_{\mathrm{SB}} = \cancel{\hat{\sigma}^+\hat{S}^-} + \hat{\sigma}^-\hat{S}^+$

bath spin is ↑



$\downarrow$

γ

$\hat{\sigma}^-$

$\uparrow$

$\epsilon\hat{H}_{\mathrm{SB}}$

$|m^z = S\rangle$

$|m^z = 1\rangle$

$\hat{S}^+$

$\hat{S}^-$ ✕

$|m^z = 0\rangle$

$|m^z = -1\rangle$

$|m^z = -S\rangle$

$\epsilon \ll \gamma$

No resonance — S^z_{total} conserved

[$U(1)$ symmetry]

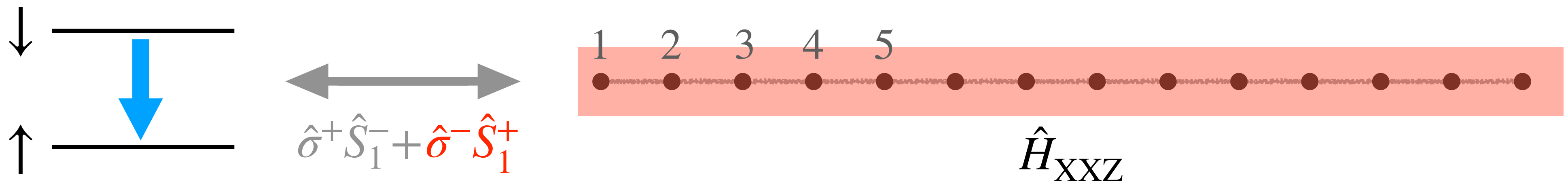
$\Longrightarrow$ System heats to max energy!

(& vice versa)

Also valid for nonuniform spacings

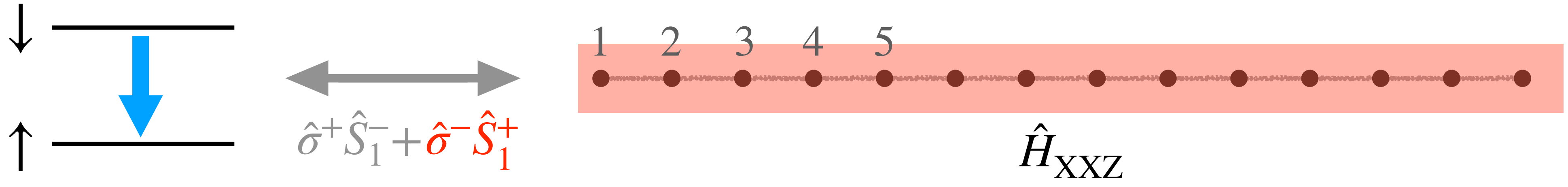
Heating from cooling — extended system

Same argument for generic system that preserves total S^z (e.g. XXZ chain)



Heating from cooling — extended system

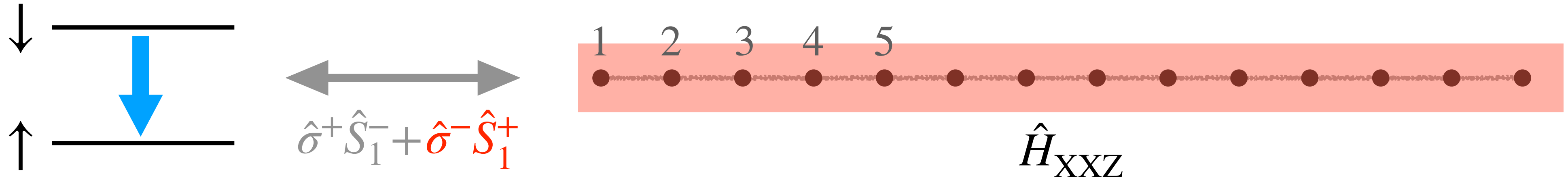
Same argument for generic system that preserves total S^z (e.g. XXZ chain)



$\Longrightarrow$ system heats to $|\uparrow\uparrow\ldots\uparrow\rangle$ — which can be anywhere in the spectrum

Heating from cooling — extended system

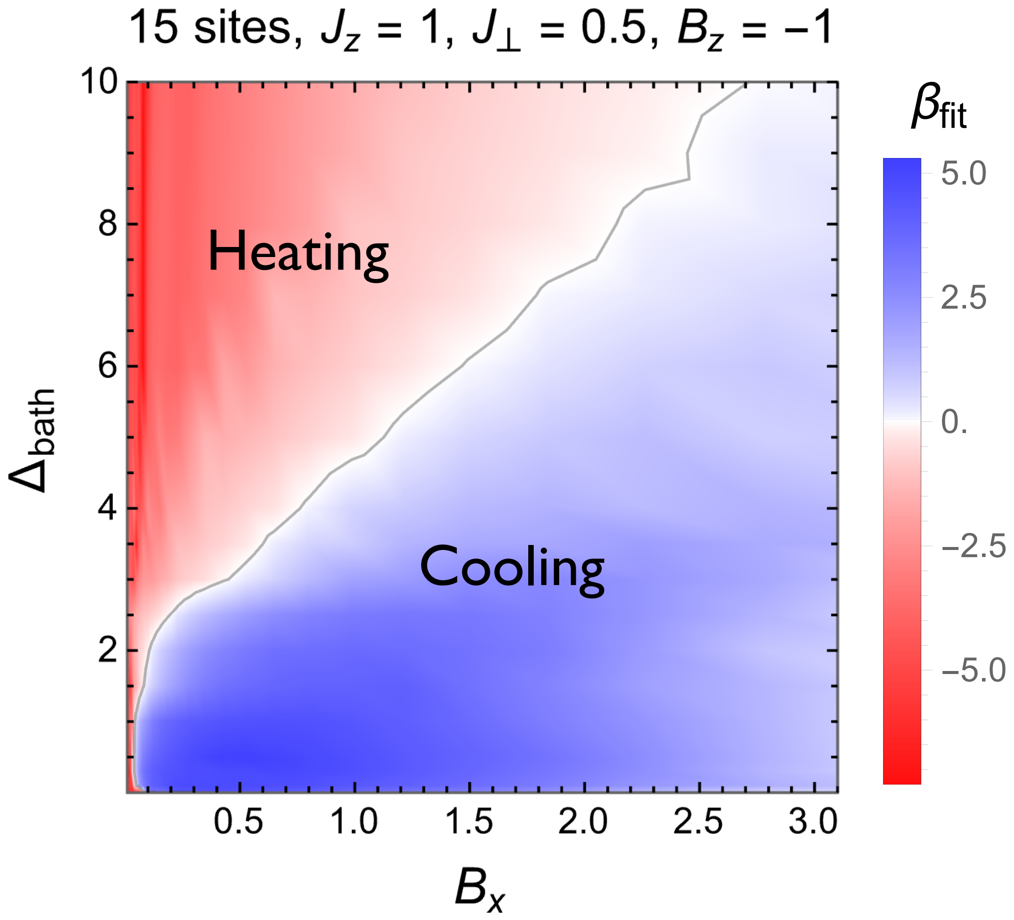
Same argument for generic system that preserves total S^z (e.g. XXZ chain)



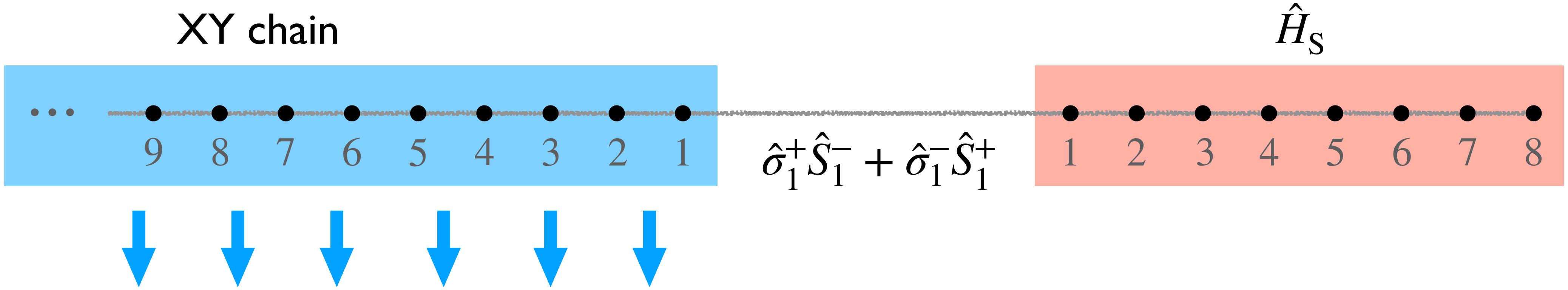
$\Longrightarrow$ system heats to $|\uparrow\uparrow\ldots\uparrow\rangle$ — which can be anywhere in the spectrum

Effect of symmetry breaking:

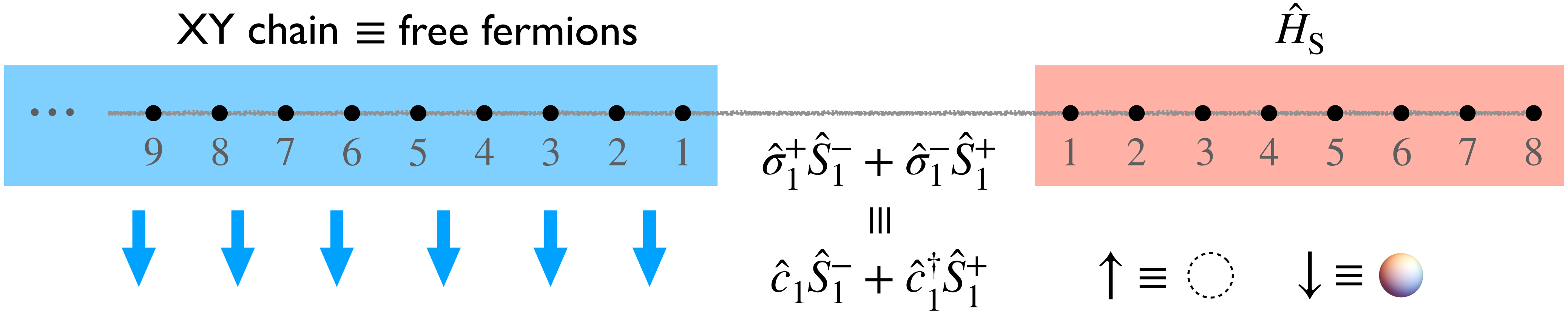
$$\hat{H}_{\mathrm{XXZ}} = \sum_i \Big[J_z\hat{S}_i^z\hat{S}_{i+1}^z + J_\perp\big(\hat{S}_i^x\hat{S}_{i+1}^x + \hat{S}_i^y\hat{S}_{i+1}^y\big) - B^z\hat{S}_i^z - \hat{B}^x\hat{S}_i^x\Big]$$


15 sites, $J_z = 1$, $J_\perp = 0.5$, $B_z = -1$

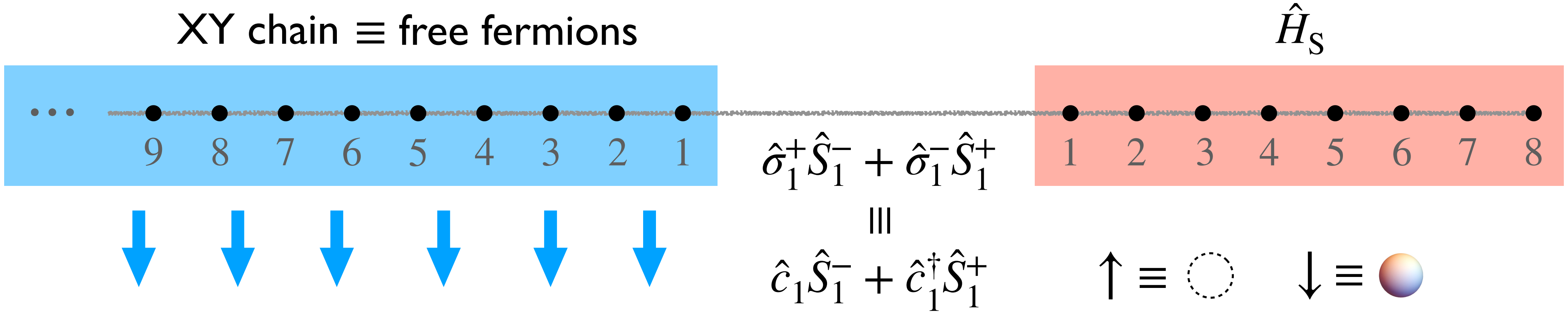
Heating from cooling — infinite bath



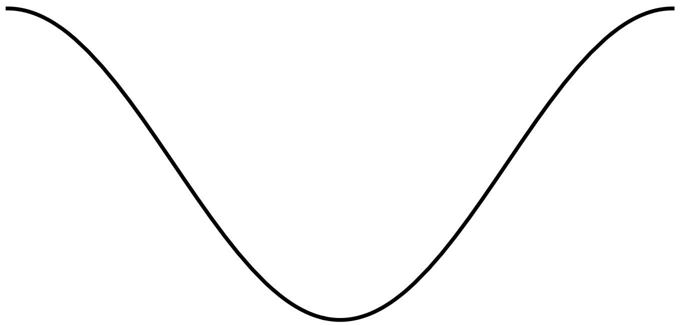
Heating from cooling — infinite bath



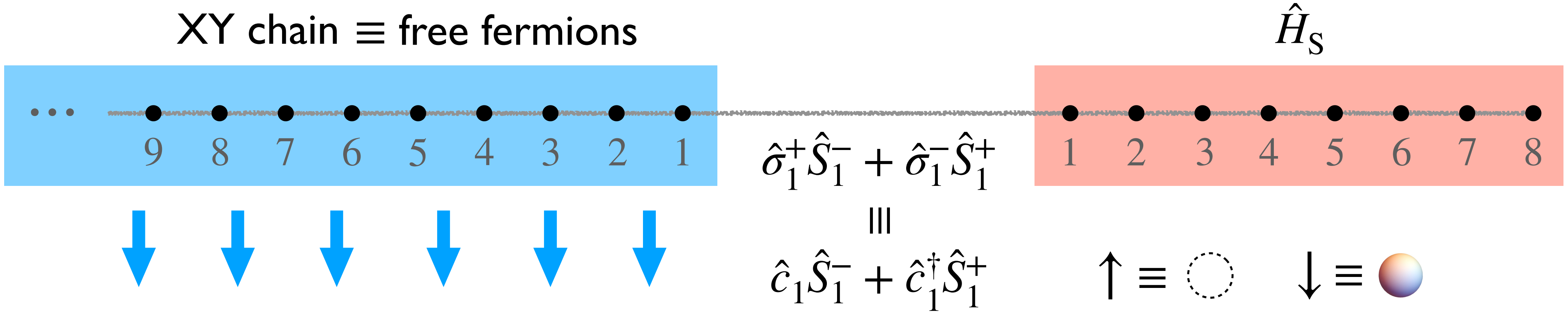
Heating from cooling — infinite bath



Bath spectrum for single-particle excitations: $\varepsilon_b(k) = -2J\cos k$



Heating from cooling — infinite bath



XY chain ≡ free fermions

$\hat{H}_\mathrm{S}$

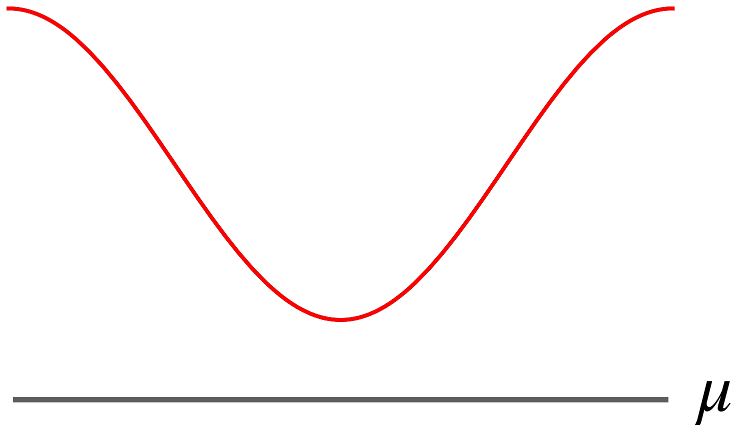
$$\hat{\sigma}_1^+ \hat{S}_1^- + \hat{\sigma}_1^- \hat{S}_1^+$$

$$|||$$

$$\hat{c}_1 \hat{S}_1^- + \hat{c}_1^\dagger \hat{S}_1^+$$

↑ ≡ ◌   ↓ ≡ ●

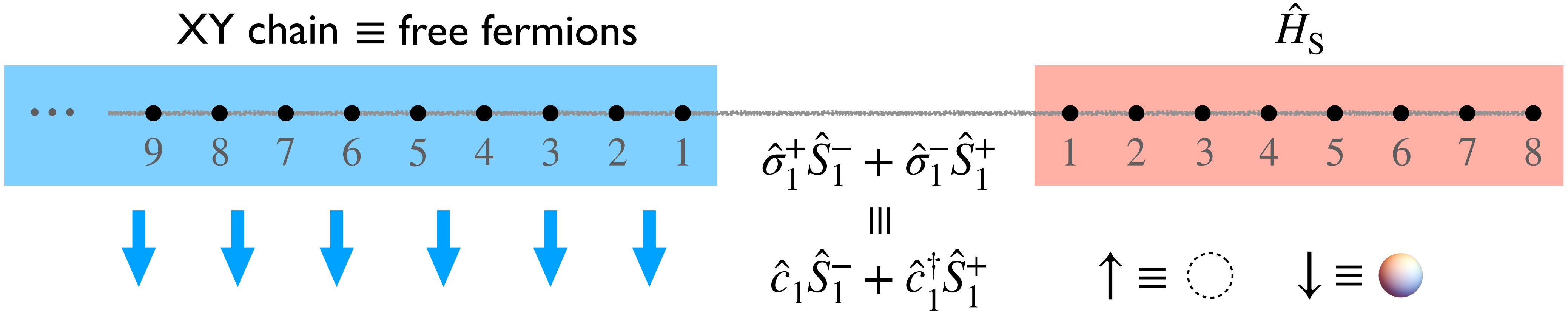
Bath spectrum for single-particle excitations: $\varepsilon_\mathrm{b}(k) = -2J\cos k$



Only particle excitations: $\hat{c}_1 \hat{S}_1^- + \hat{c}_1^\dagger \hat{S}_1^+$

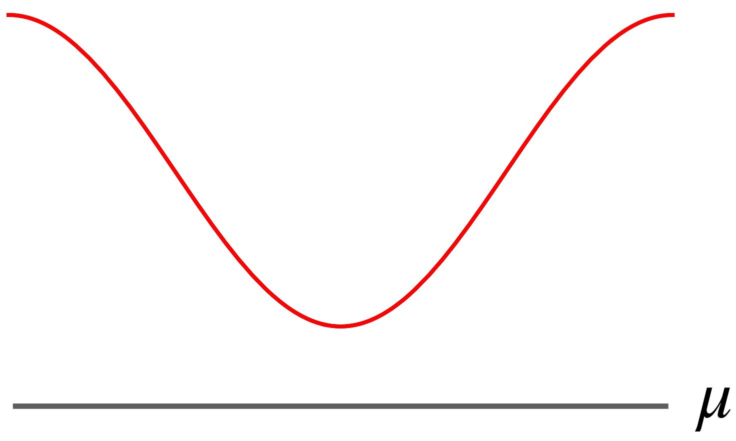
$\Longrightarrow$ Heats to maximum S^z

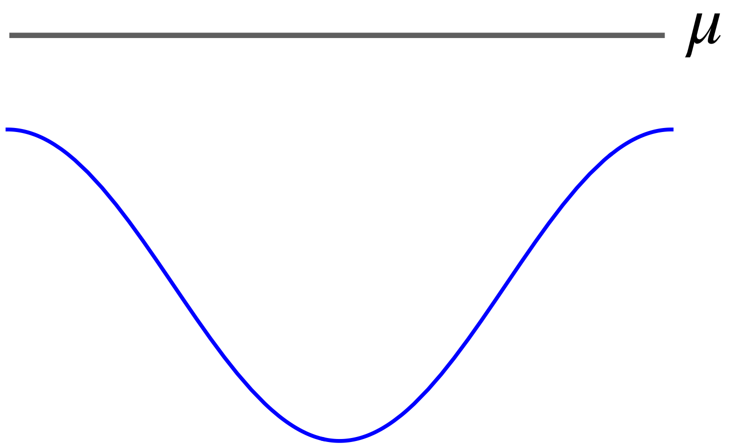
Heating from cooling — infinite bath



$$\hat{\sigma}_1^+ \hat{S}_1^- + \hat{\sigma}_1^- \hat{S}_1^+ \equiv \hat{c}_1 \hat{S}_1^- + \hat{c}_1^\dagger \hat{S}_1^+$$

Bath spectrum for single-particle excitations: $\varepsilon_b(k) = -2J\cos k$





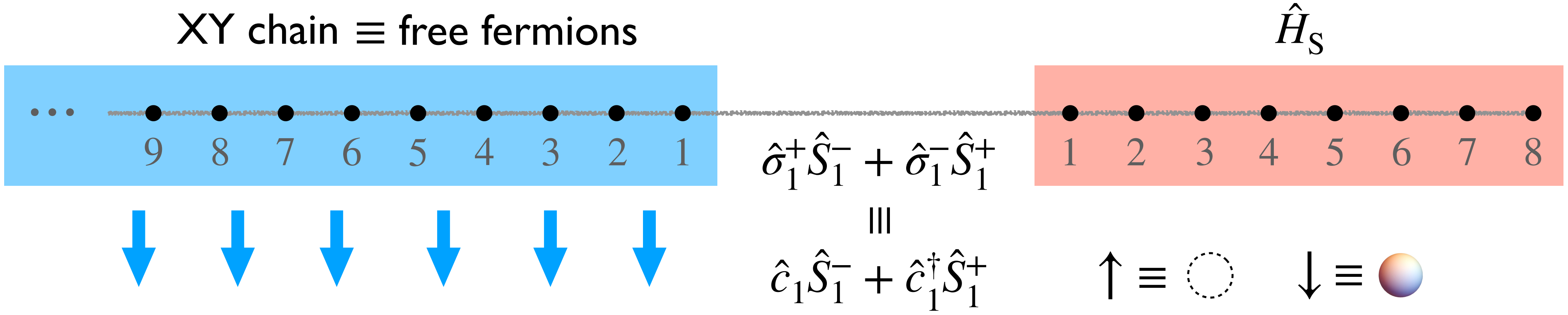
Only particle excitations: $\hat{c}_1 \hat{S}_1^- + \hat{c}_1^\dagger \hat{S}_1^+$

$\Longrightarrow$ Heats to maximum S^z

Only hole excitations: $\hat{c}_1 \hat{S}_1^- + \hat{c}_1^\dagger \hat{S}_1^+$

$\Longrightarrow$ Cools to minimum S^z

Heating from cooling — infinite bath

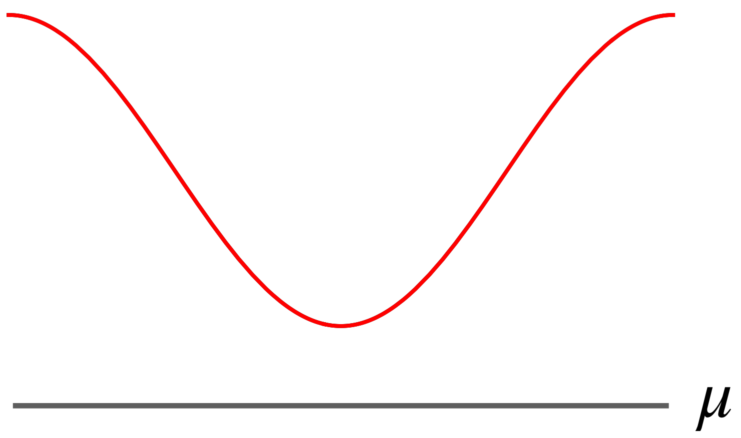


$\hat{\sigma}_1^{+}\hat{S}_1^{-} + \hat{\sigma}_1^{-}\hat{S}_1^{+}$

$\equiv$

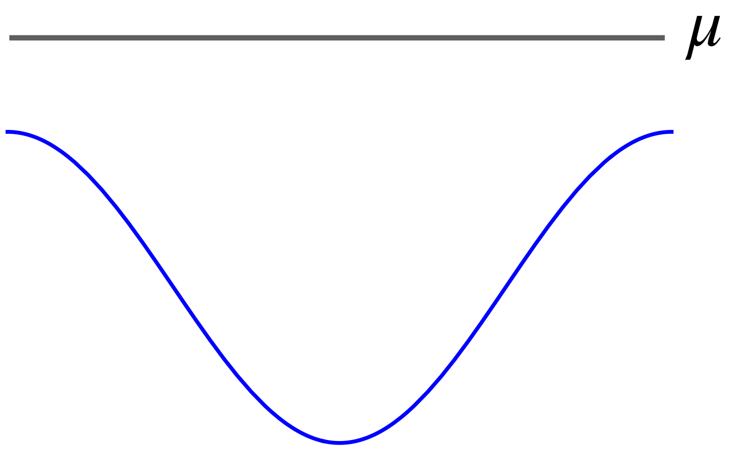
$\hat{c}_1\hat{S}_1^{-} + \hat{c}_1^{\dagger}\hat{S}_1^{+}$

Bath spectrum for single-particle excitations: $\varepsilon_{\mathrm{b}}(k) = -2J\cos k$



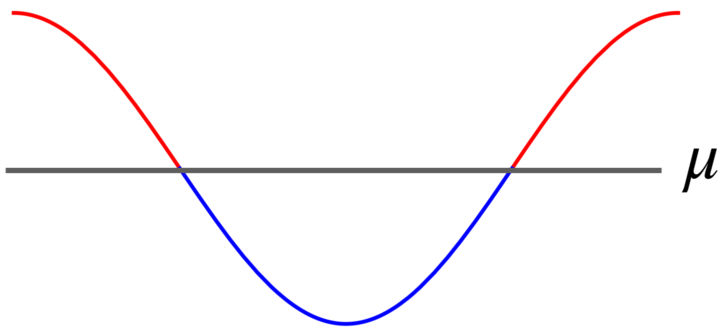
Only particle excitations: $\hat{c}_1\hat{S}_1^{-} + \hat{c}_1^{\dagger}\hat{S}_1^{+}$

$\Longrightarrow$ Heats to maximum S^z



Only hole excitations: $\hat{c}_1\hat{S}_1^{-} + \hat{c}_1^{\dagger}\hat{S}_1^{+}$

$\Longrightarrow$ Cools to minimum S^z



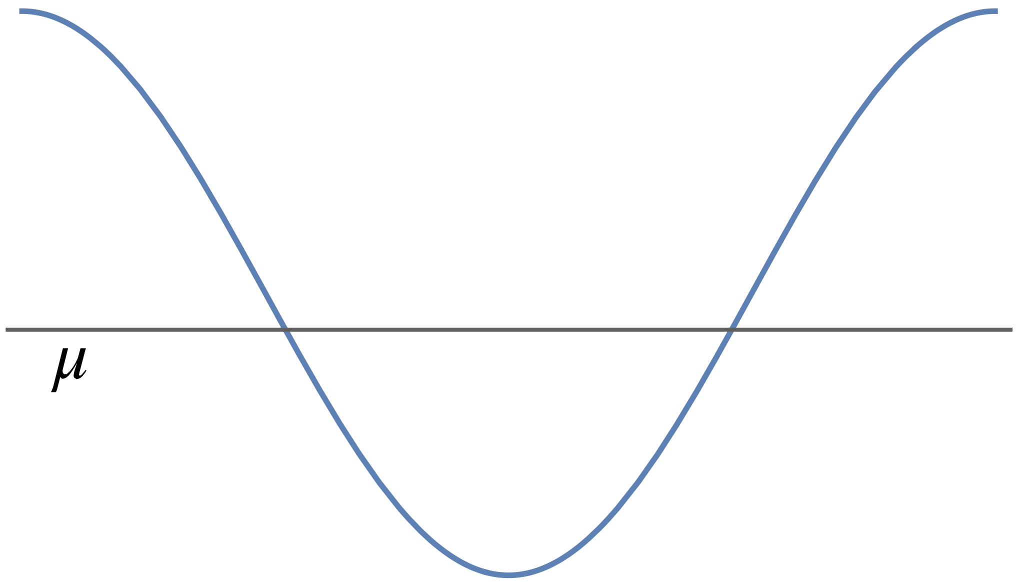
Both: $\hat{c}_1\hat{S}_1^{-} + \hat{c}_1^{\dagger}\hat{S}_1^{+}$

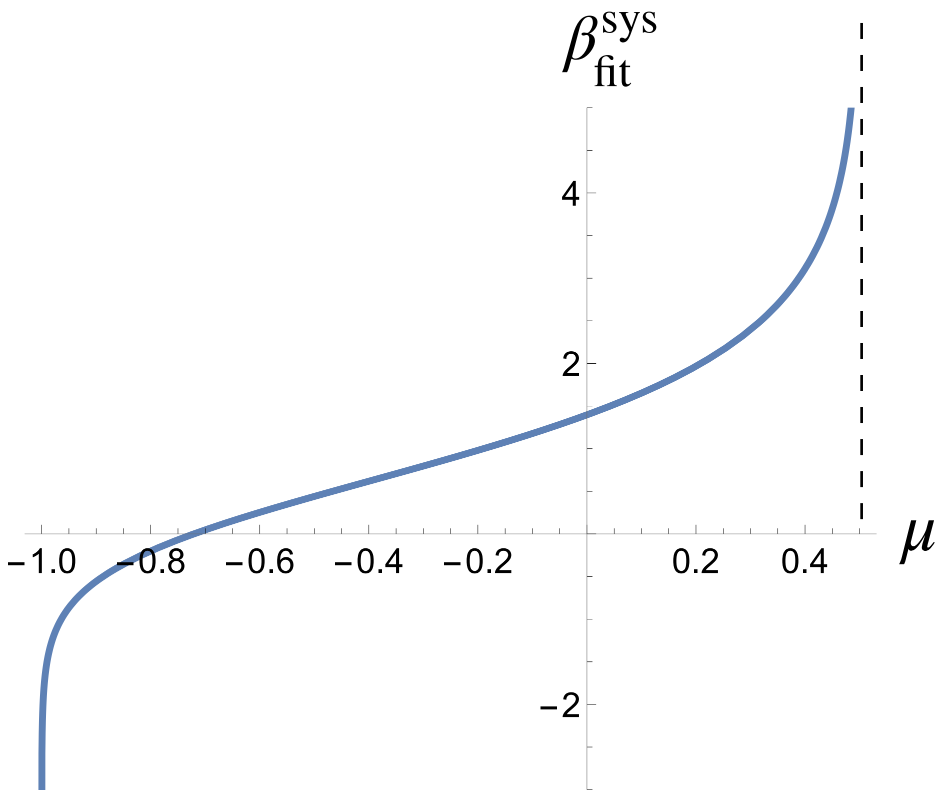
$\Longrightarrow$ Intermediate state

Free-fermion bath + N-level system

$\hat{H}_\mathrm{S} = \Delta_\mathrm{S} \hat{S}^z$

$\varepsilon_\mathrm{b}(k) = -\cos k$



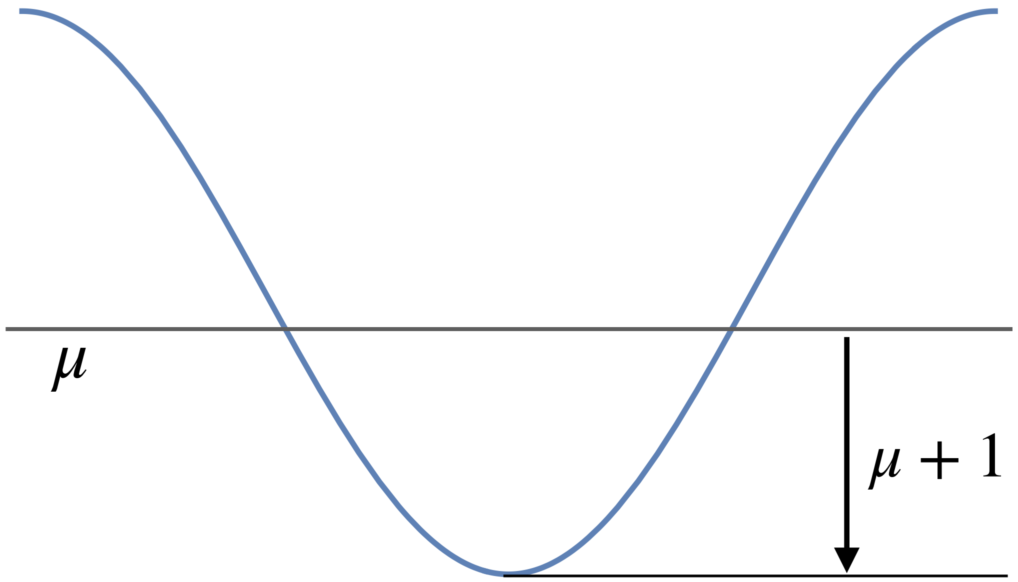


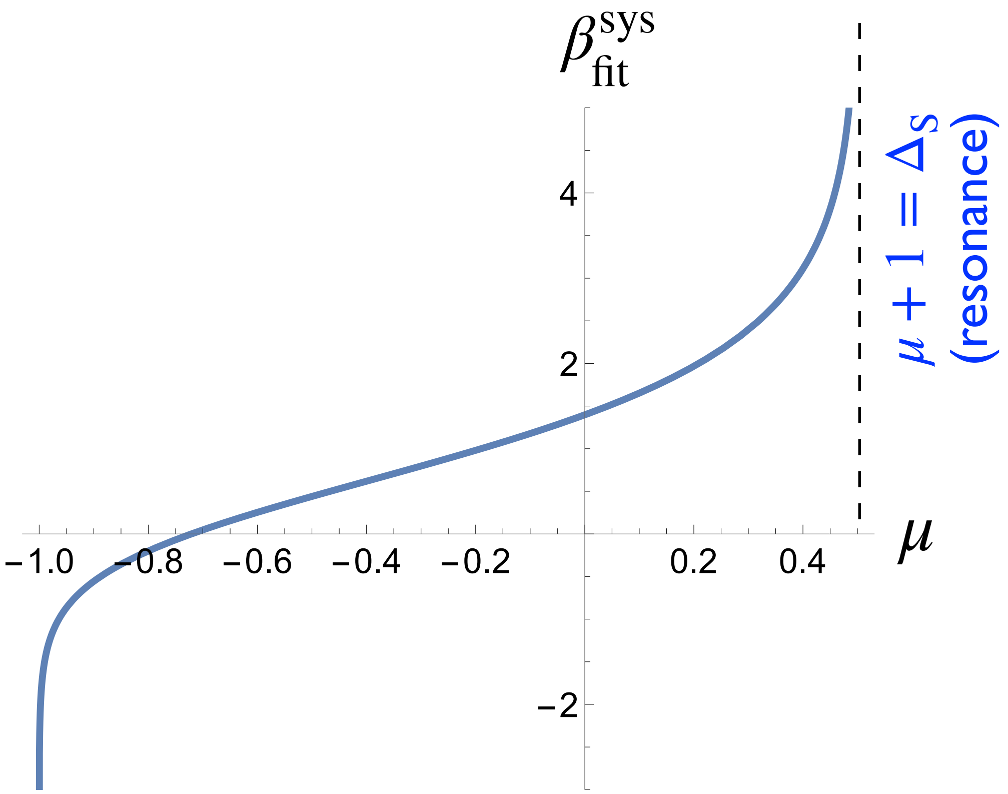
$\Delta_\mathrm{S} = 1.5$

Free-fermion bath + N-level system

$\hat{H}_{\mathrm{S}} = \Delta_{\mathrm{S}} \hat{S}^z$

$\varepsilon_{\mathrm{b}}(k) = -\cos k$





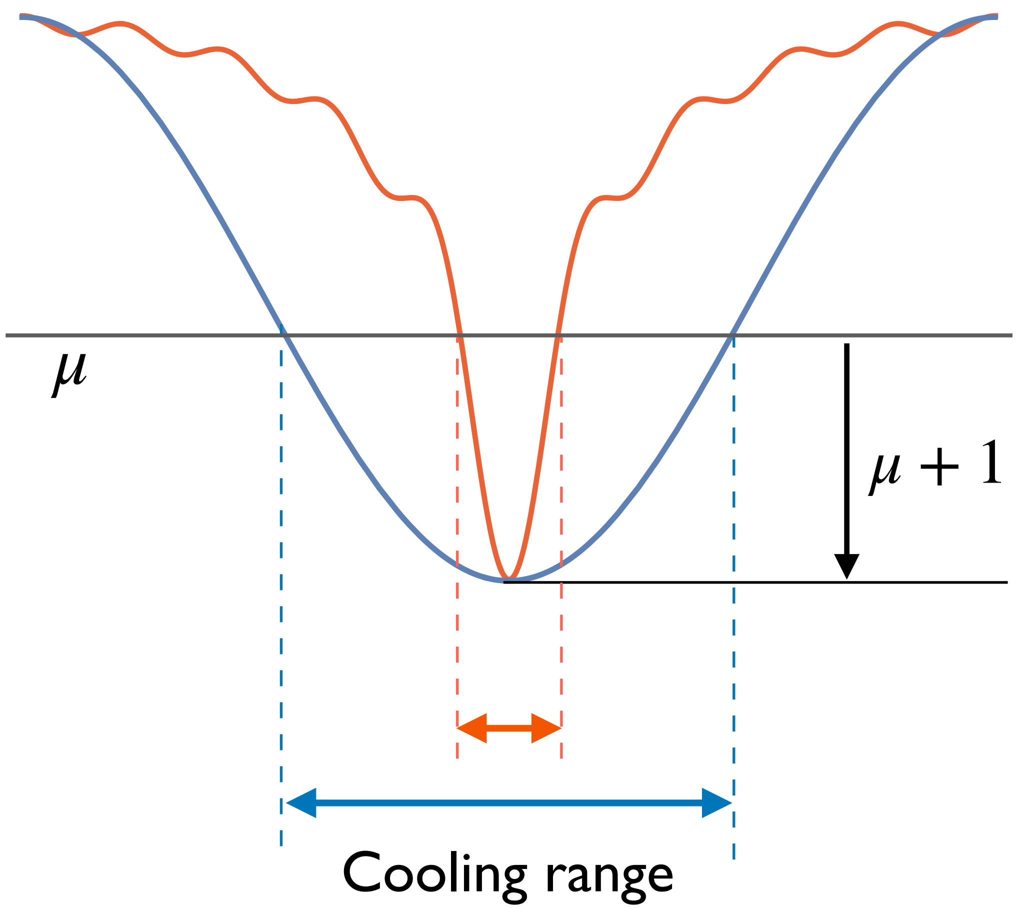
$\Delta_{\mathrm{S}} = 1.5$

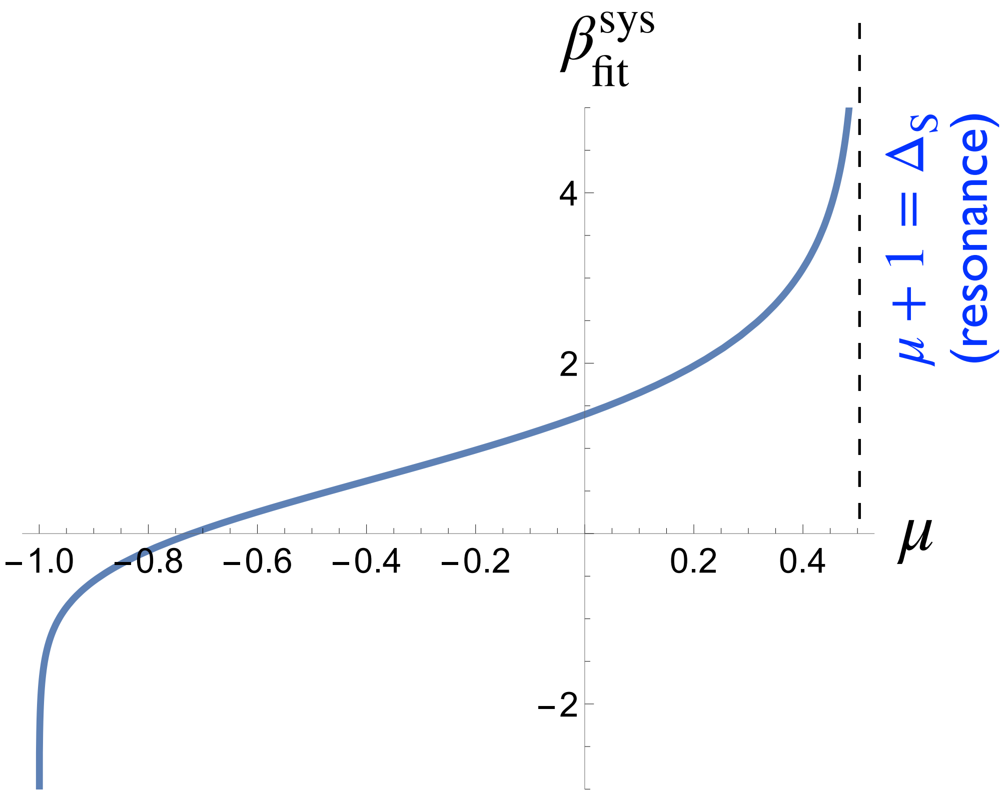
Free-fermion bath + N-level system

$\hat{H}_{\mathrm{S}} = \Delta_{\mathrm{S}} \hat{S}^z$

$\varepsilon_{\mathrm{b}}(k) = -\cos k$

$\varepsilon_{\mathrm{b}}(k) \sim -\sum_{n=1}^{9} \frac{1}{n} \cos(nk)$ Fewer modes cool





$\Delta_{\mathrm{S}} = 1.5$

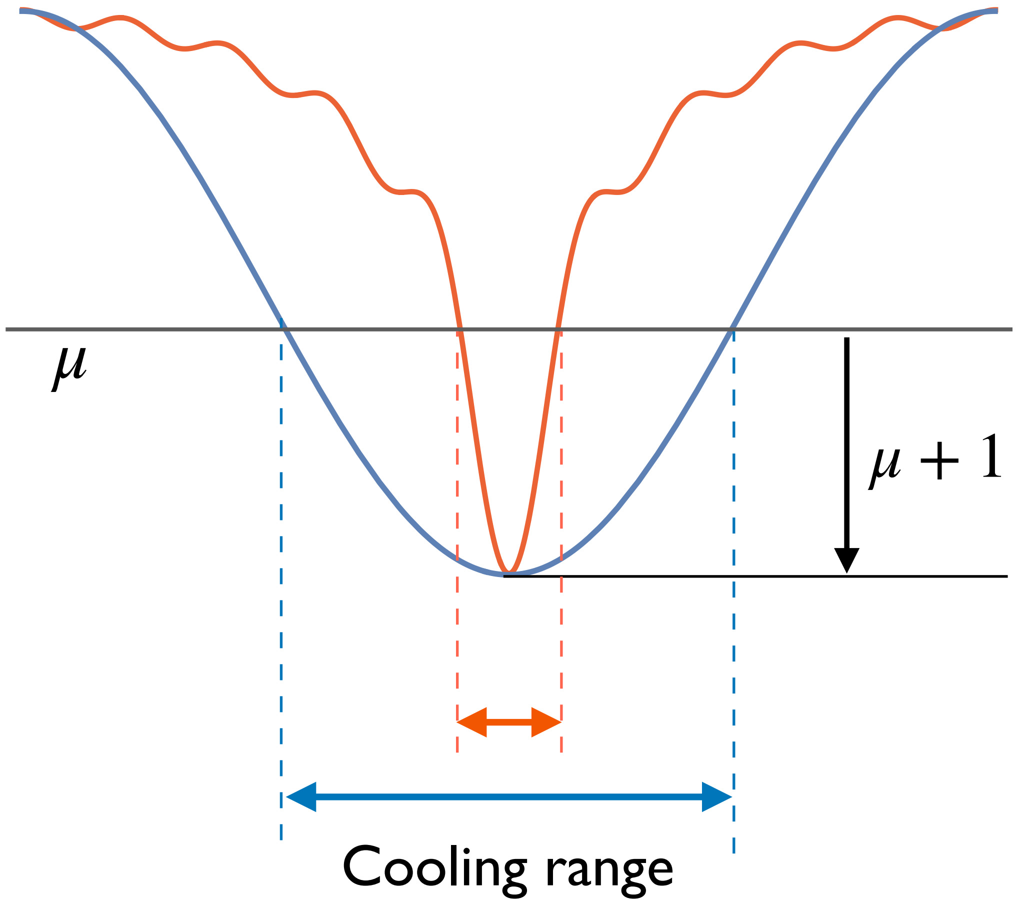
Free-fermion bath + N-level system

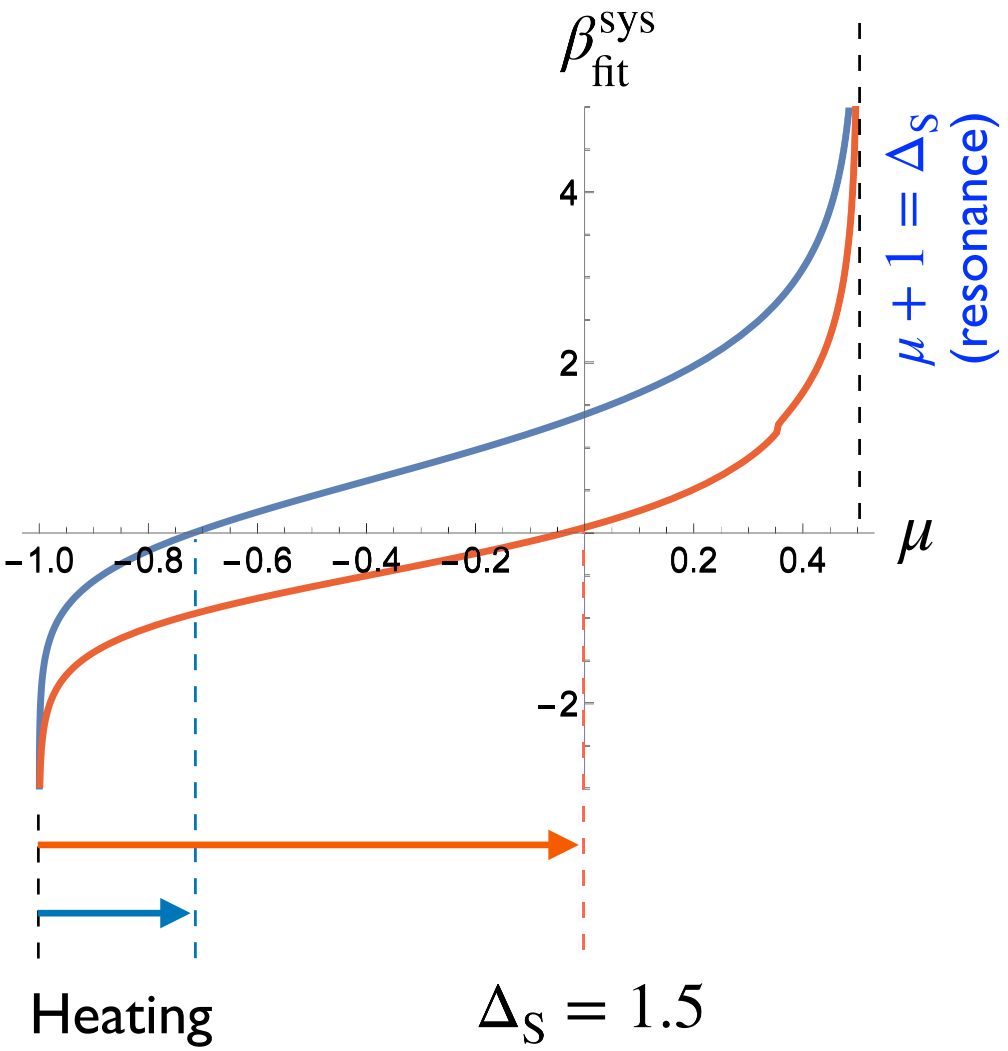
$$\hat{H}_{\mathrm{S}} = \Delta_{\mathrm{S}} \hat{S}^{z}$$

$$\varepsilon_{\mathrm{b}}(k) = -\cos k$$

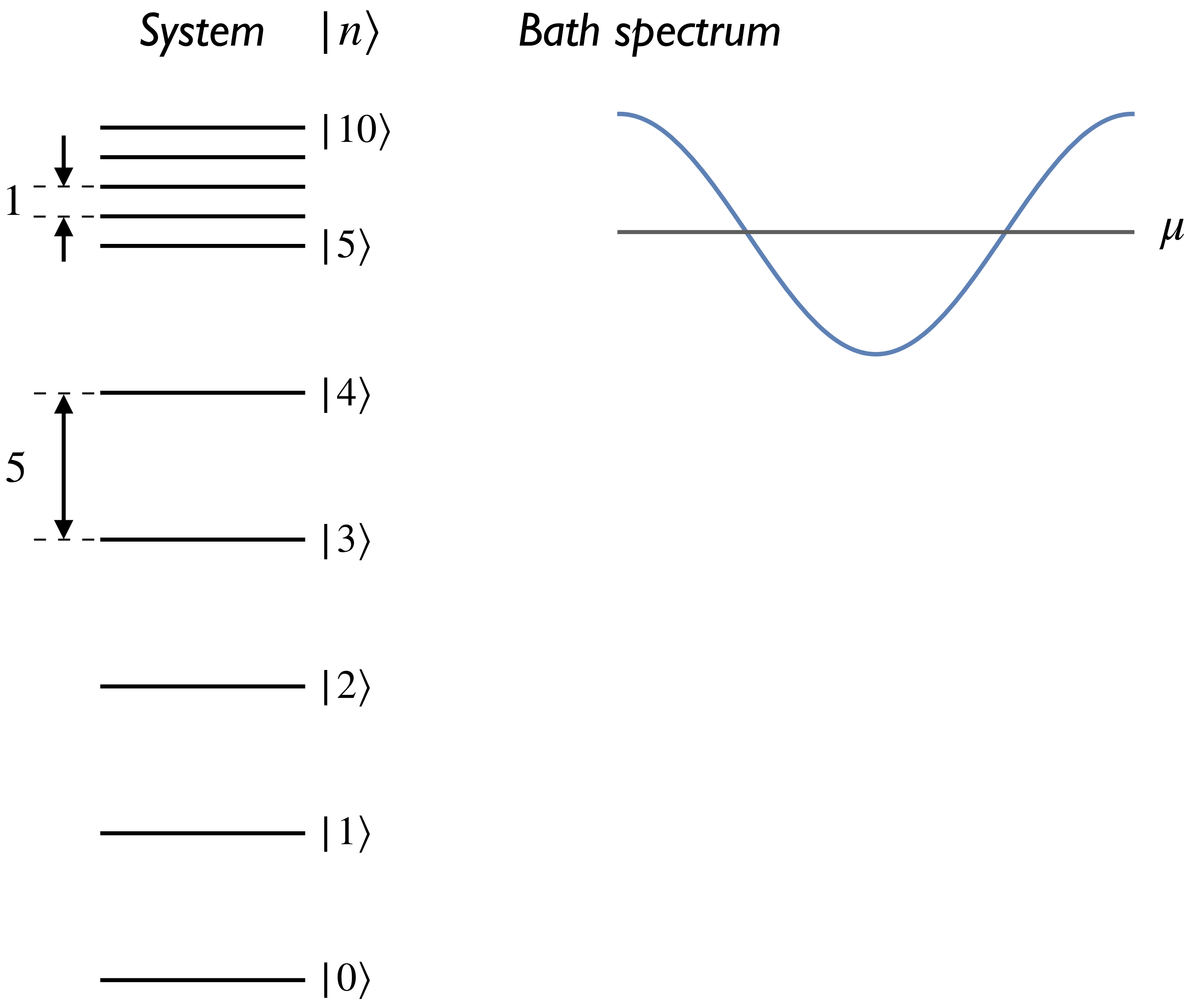
$$\varepsilon_{\mathrm{b}}(k) \sim -\sum_{n=1}^{9} \frac{1}{n} \cos(nk)$$

Fewer modes cool $\Longrightarrow$ more heating





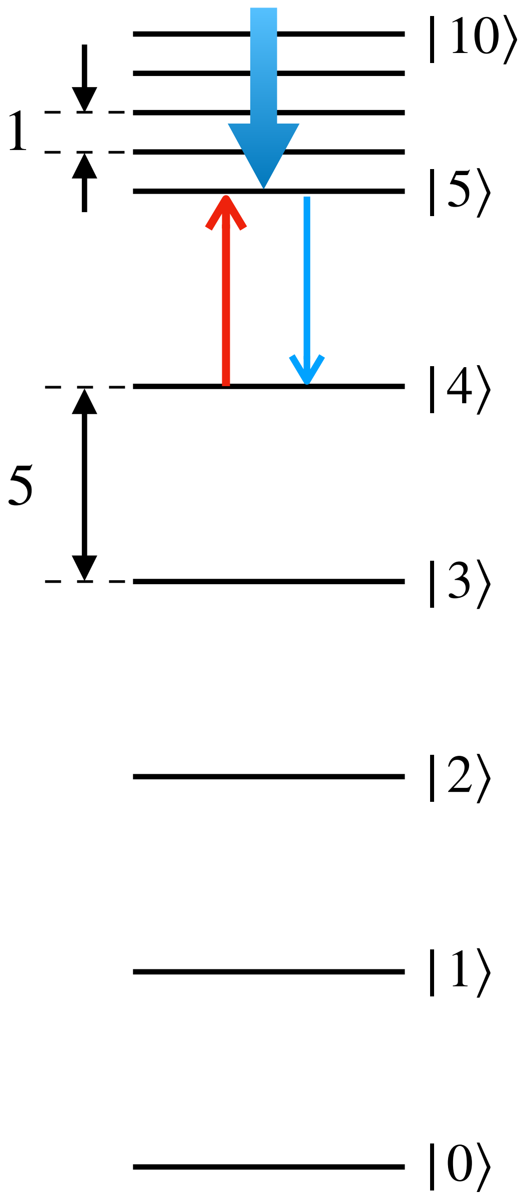
Preparing mid-spectrum nonclassical state



“Nonlinear cavity”

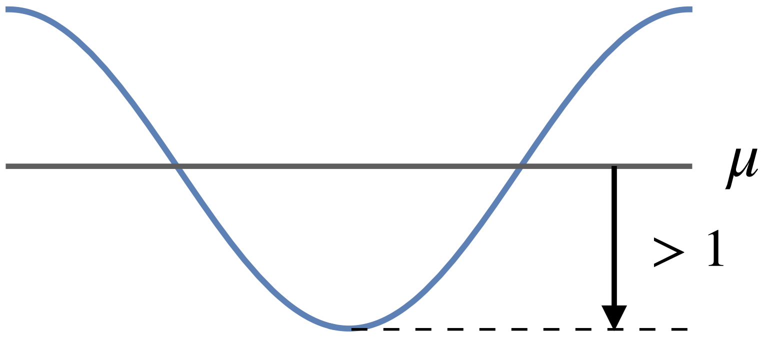
Preparing mid-spectrum nonclassical state

System $|n\rangle$

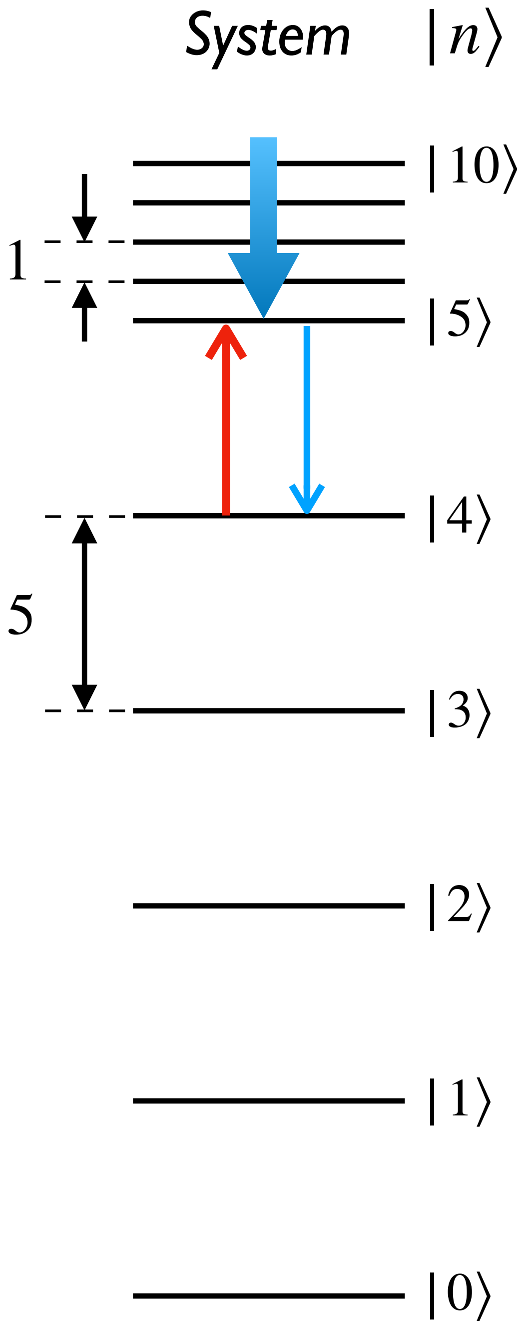


“Nonlinear cavity”

Bath spectrum

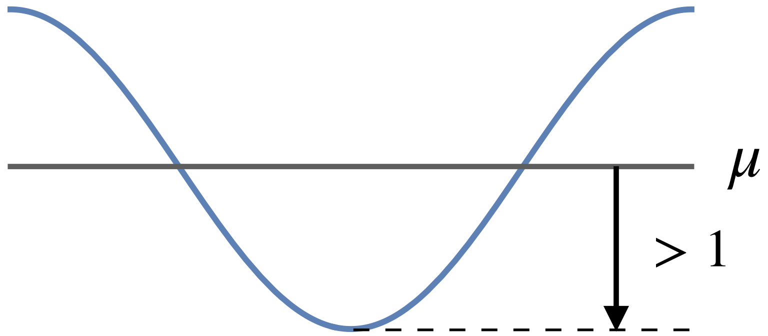


Preparing mid-spectrum nonclassical state

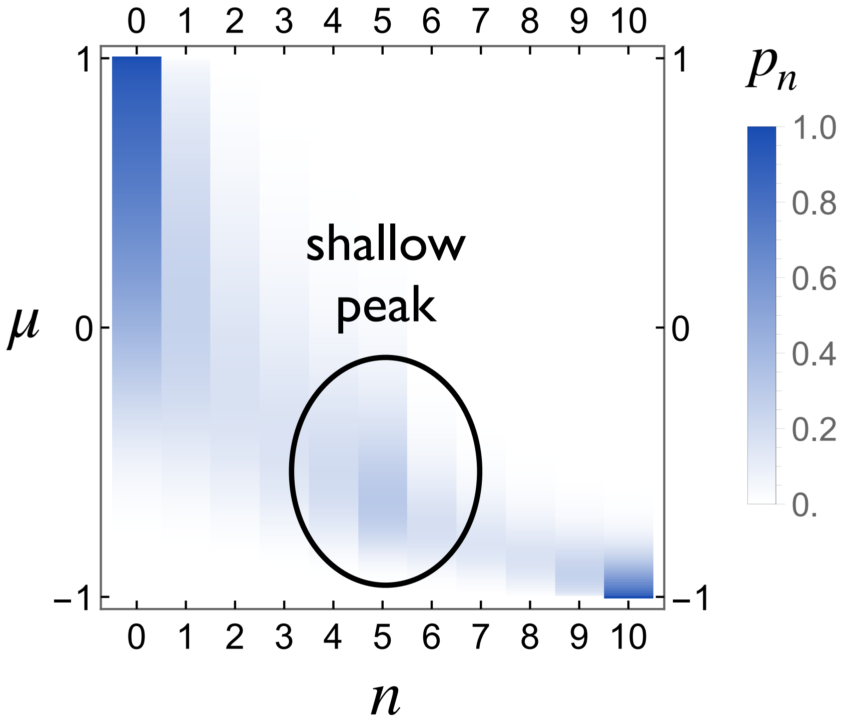


“Nonlinear cavity”

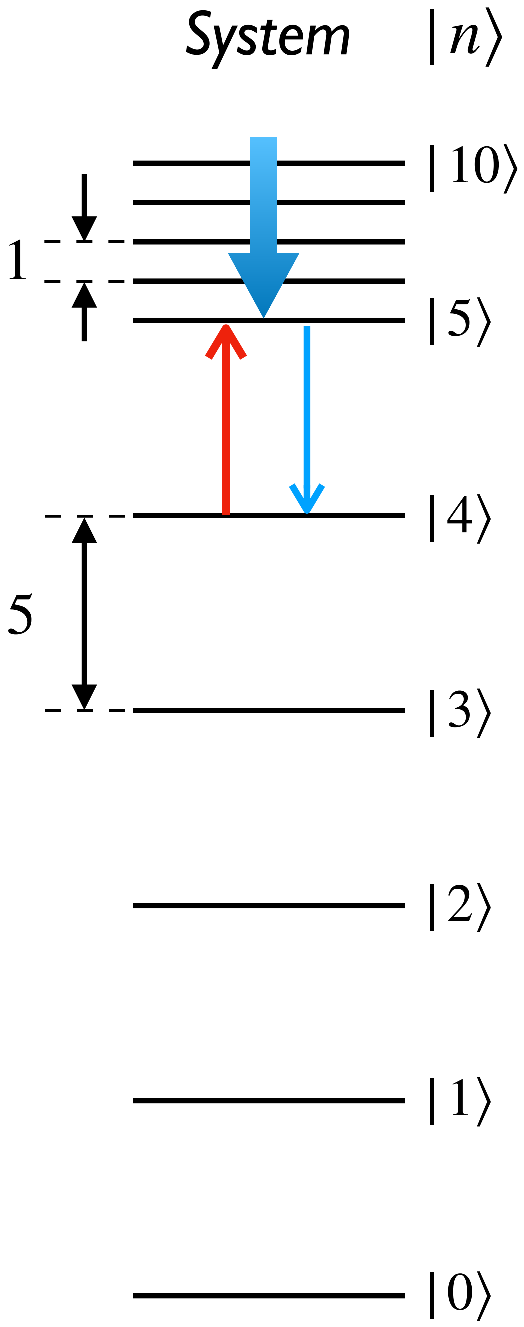
Bath spectrum



Steady-state occupations

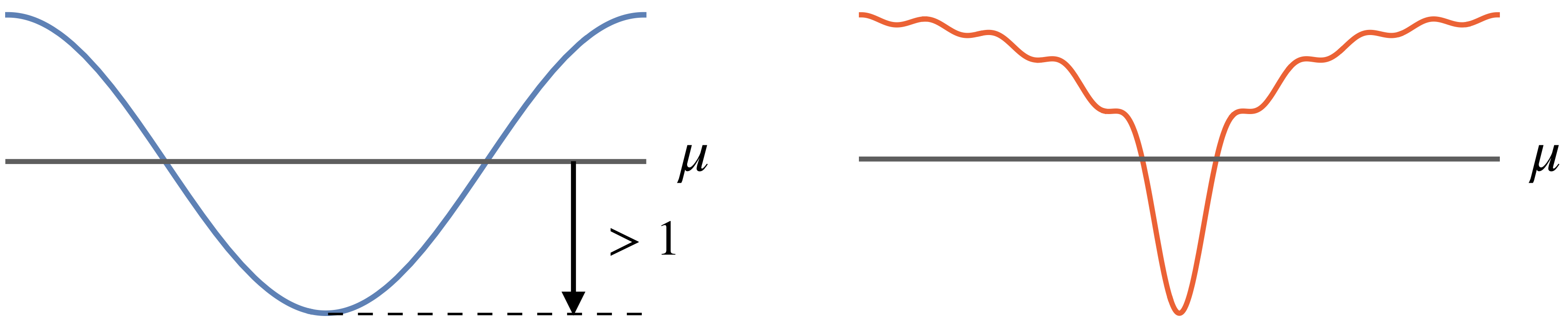


Preparing mid-spectrum nonclassical state

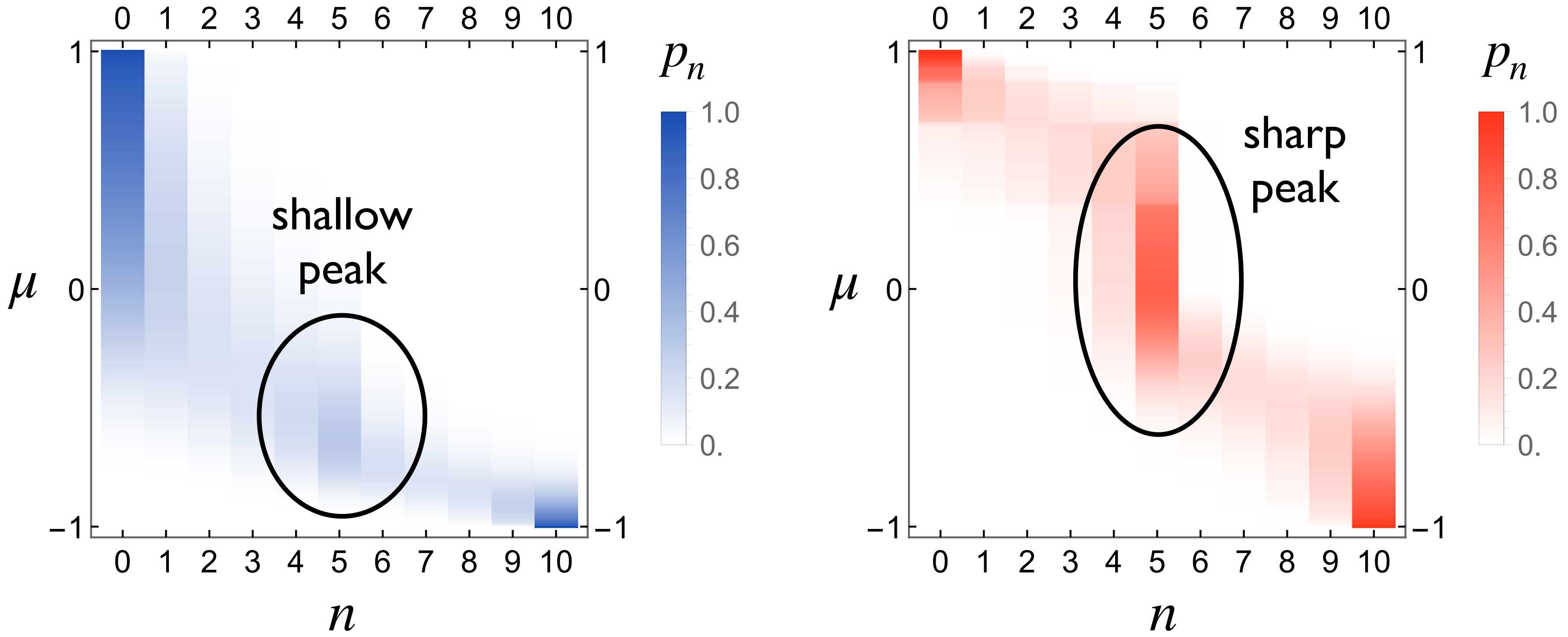


“Nonlinear cavity”

Bath spectrum



Steady-state occupations

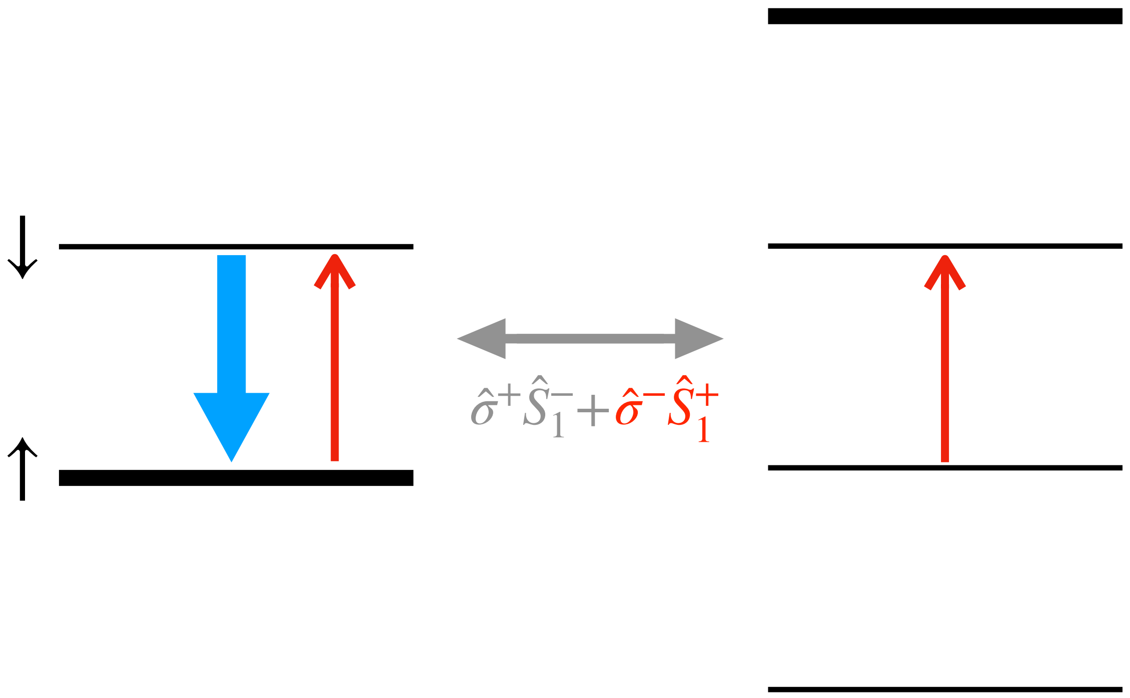


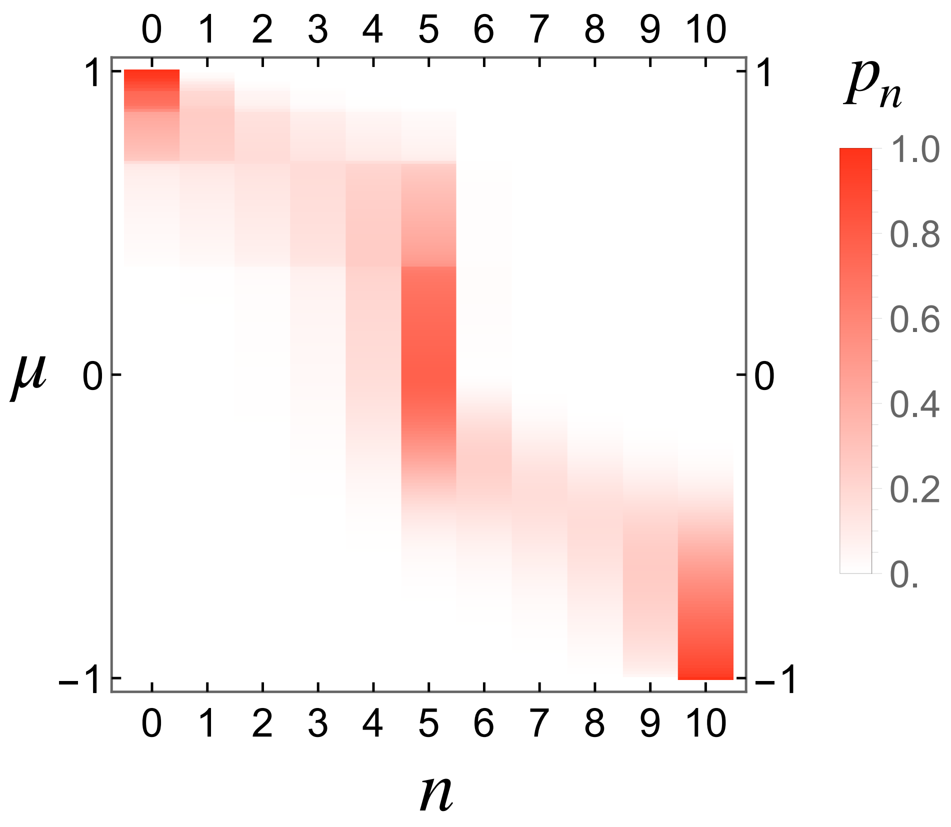
5-photon Fock state

Summary

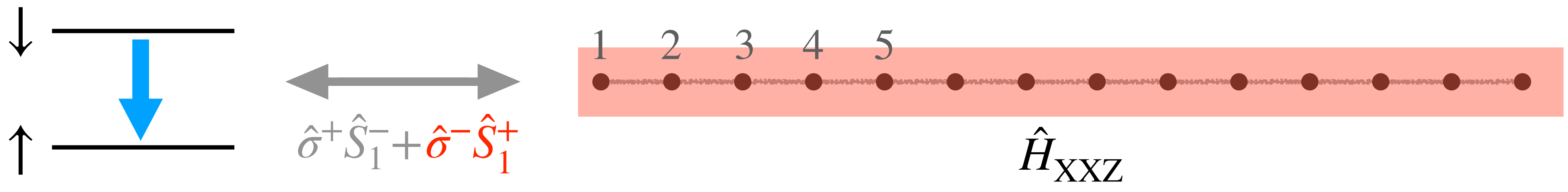
Thank you :)

- Breakdown of thermalization due to conserved “charge” related to energy
- Global heating by local cooling (& vice versa)
- N-photon Fock state & other non-thermal states



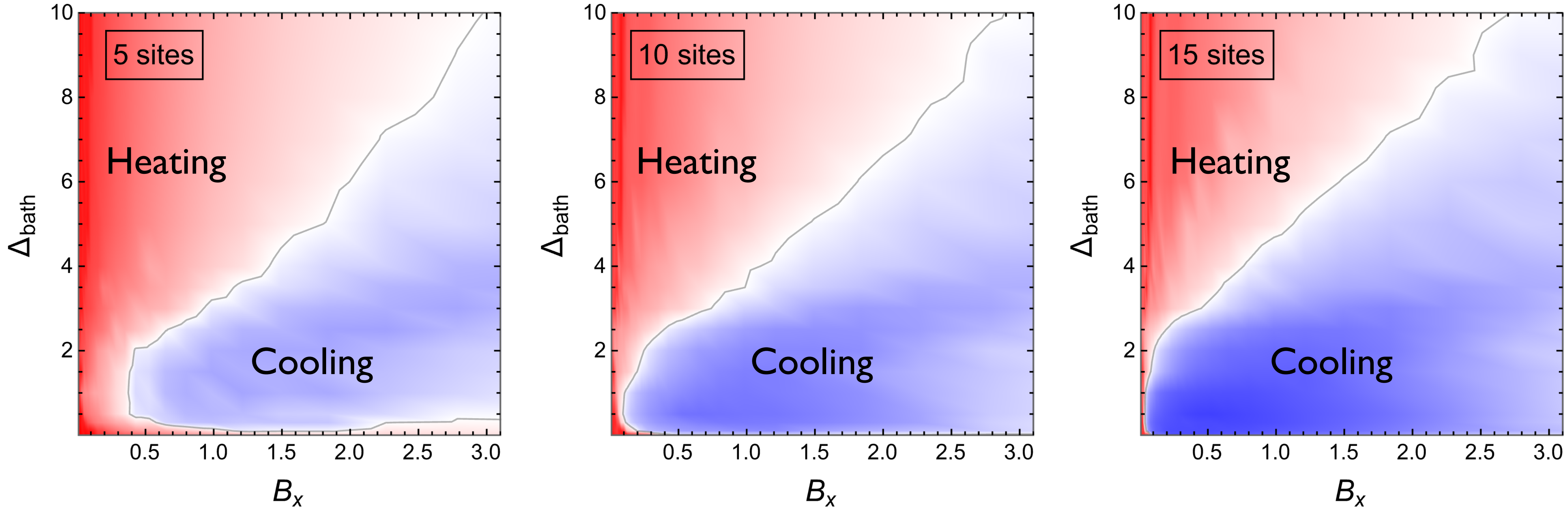


Supplement: Symmetry breaking in $\hat{H}_{\mathrm{S}}$



$$\hat{H}_{\mathrm{XXZ}}=\sum_{i}\left[J_{z}\hat{S}_{i}^{z}\hat{S}_{i+1}^{z}+J_{\perp}\left(\hat{S}_{i}^{x}\hat{S}_{i+1}^{x}+\hat{S}_{i}^{y}\hat{S}_{i+1}^{y}\right)-B^{z}\hat{S}_{i}^{z}-\hat{B}^{x}\hat{S}_{i}^{x}\right]$$

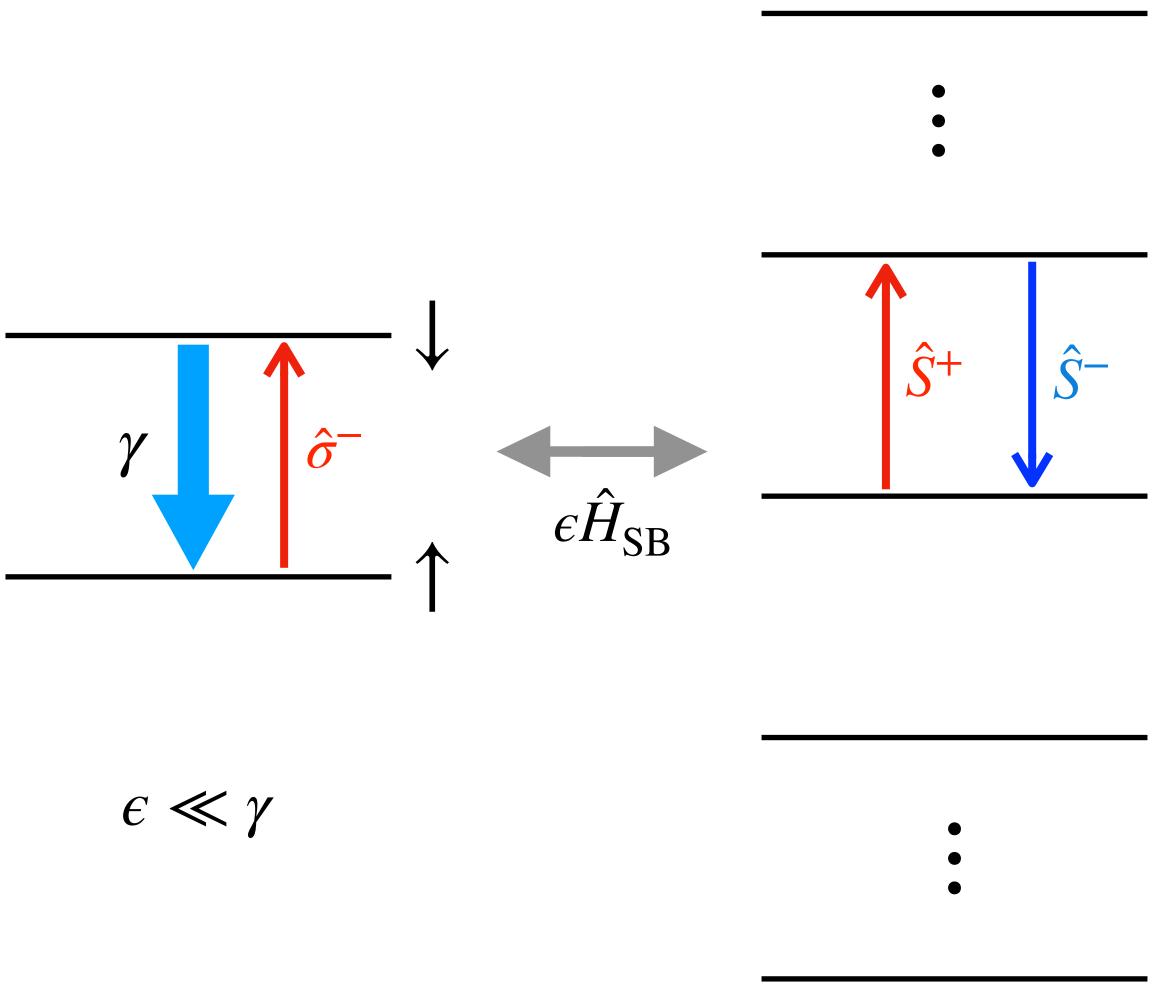
$J_z = 1,\ J_\perp = 0.5,\ B_z = -1$



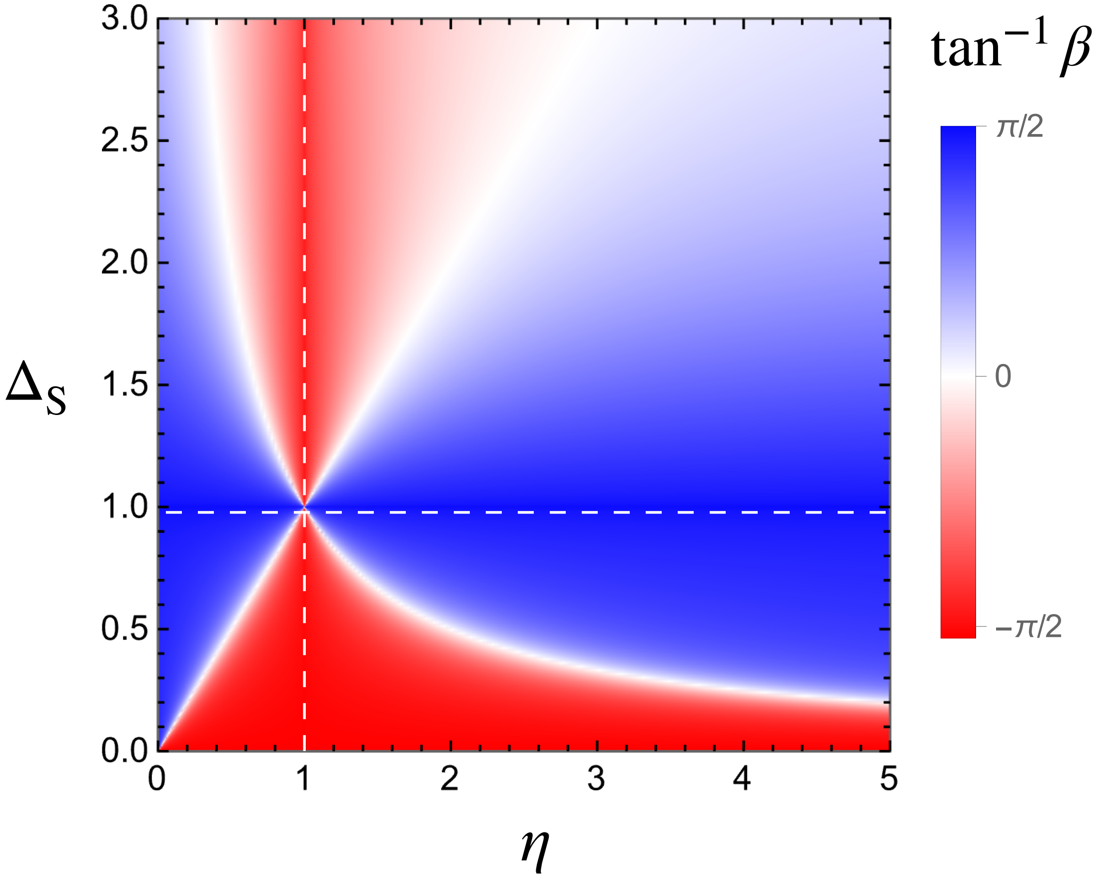
Supplement: Symmetry breaking in $\hat{H}_{\mathrm{SB}}$

$$\hat{H}_{\mathrm{B}} = -\,\hat{\sigma}^z \qquad \hat{H}_{\mathrm{S}} = \Delta_{\mathrm{S}}\hat{S}^z \qquad \hat{H}_{\mathrm{SB}} = \hat{\sigma}^x\hat{S}^x + \eta\,\hat{\sigma}^y\hat{S}^y$$

$$= \frac{1+\eta}{2}\left(\hat{\sigma}^+\hat{S}^- + \hat{\sigma}^-\hat{S}^+\right) + \frac{1-\eta}{2}\left(\hat{\sigma}^+\hat{S}^+ + \hat{\sigma}^-\hat{S}^-\right)$$



$\epsilon \ll \gamma$



$$\beta = \frac{2}{\Delta_{\mathrm{S}}}\ln\left|\frac{1-\eta}{1+\eta}\cdot\frac{1+\Delta_{\mathrm{S}}}{1-\Delta_{\mathrm{S}}}\right|$$