Scaling of Fock Space Propagator across Many-body Localization Transition

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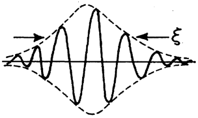
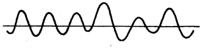
Localization





$$H = t \sum_{<i,j>} \left(\hat{c}_i^\dagger \hat{c}_j + \hat{c}_j^\dagger \hat{c}_i \right) + \sum_i \varepsilon_i \hat{n}_i \quad ; \qquad \varepsilon_i \in [-W, W]$$

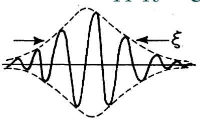
P. W. Anderson, Phys. Rev. (1958)

Inverse participation ratio

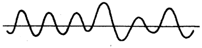
$$IPR_n = \sum_i |\psi_n(i)|^4$$

Localization

$IPR \sim \mathcal{O}(1)$



$IPR \sim \frac{1}{L}$



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Many-Body Localization

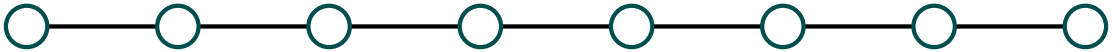
- Localization $\Rightarrow$ Some memory of local initial conditions preserved
- What happens in interacting system?

  No thermalization for strong enough disorder in one dimension.
- Anderson Localization — Exponentially localized eigenstates
- Many body localization — Non-ergodic extended eigenstates

Non-ergodic extended states
Wave function occupies L^D number of sites $(0 < D < 1)$
Fraction of sites occupied by wavefunction L^{D-1}

A. De Luca, A. Scardicchio, EPL (2013); N. Macé et. al., PRL (2019);
S. Roy and D. Logan, PRB (2021)

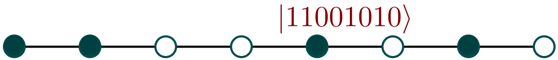
Fock space representation



$$\mathcal{H} = t\sum_{i=1}^{L}\left(\hat{c}_i^\dagger \hat{c}_{i+1} + \hat{c}_{i+1}^\dagger \hat{c}_i\right) + \sum_{i=1}^{L}\epsilon_i \hat{n}_i + V\sum_{i=1}^{L}\hat{n}_i\hat{n}_{i+1}$$

$$\epsilon_i \in [-W, W]; \qquad t = 0.5, \quad V = 1$$

Fock space representation



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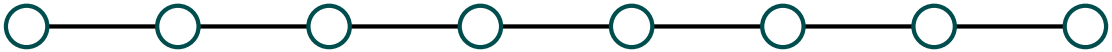
Basis states: $|I\rangle = |n_1, n_2, n_3, \ldots, n_L\rangle$; $n_i = 0$ or 1

Fock space representation



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$$\boxed{\mathcal{H} = \sum_{I,J} T_{IJ}|I\rangle\langle J| + \sum_{I}\mathcal{E}_I|I\rangle\langle I|} \qquad \mathcal{N}_F = \binom{L}{\frac{L}{2}}$$

Tight-binding model with correlated disorder on Fock space

A. Altland and T. Micklitz, PRL(2017); S. Ghosh et. al. PRB (2019);

S. Roy and D. Logan, PRB (2020)

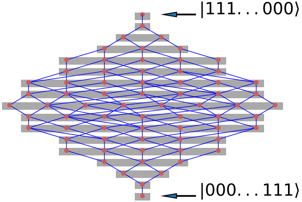
Feenberg Self-energy

Many body resolvent: $G = \dfrac{\mathbb{I}}{E + i\eta - \mathcal{H}}$

η : Mean level spacing

Feenberg self-energy: $\Delta_I = -\mathrm{Im}[S_I(E)]$

$$\left[G^+_{II}(E)\right]^{-1} = E^+ - V_I - S_I(E)$$



$$\mathcal{H} = \sum_{I,J} T_{IJ}|I\rangle\langle J| + \sum_I \mathcal{E}_I |I\rangle\langle I|$$

We use a recursive technique to calculate G

Feenberg Self-energy

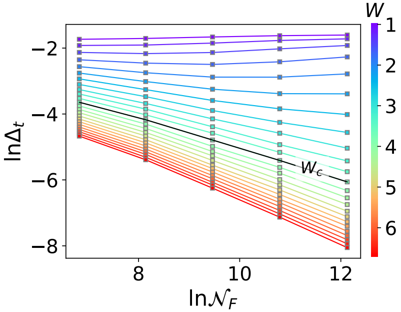
$$\Delta_t = \exp\left[\langle \log \Delta_I \rangle_{I,\{\varepsilon_i\}}\right] \qquad \Delta_t \begin{cases} \sim \mathcal{O}(1) & \text{Thermal phase} \\ \to 0 \quad \text{as} \quad \mathcal{N}_F \to \infty & \text{MBL phase} \end{cases}$$

D. Logan and S. Welsh, PRB (2019)

$$\Delta_t \propto \mathcal{N}_F^{-1+D_s} \quad \text{for} \quad \eta \sim \mathcal{N}_F^{-1}$$

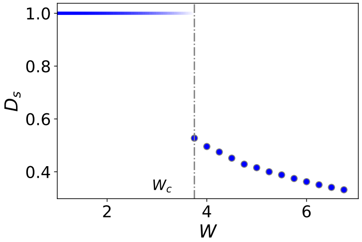
$$D_s \to \quad \text{Fractal dimension,} \quad 0 < D_s < 1$$

Feenberg Self-energy

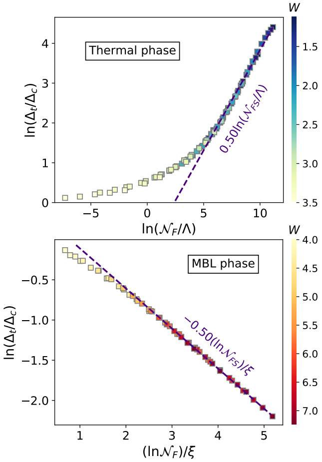


$$\Delta_{typ} \sim \mathcal{O}(1)$$

$$\Delta_{typ} \propto \mathcal{N}_F^{-(1-D_s)}$$



Scaling of self energy

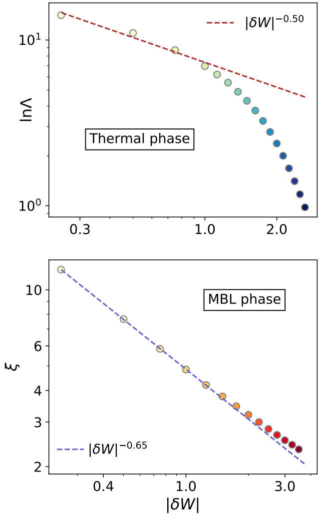


Scaling ansatz for $\ln\left(\frac{\Delta_t}{\Delta_c}\right)$:

$$\begin{cases} \mathcal{F}\left(\frac{N_F}{\Lambda}\right); & W < W_c \\ \mathcal{G}\left(\frac{\ln N_F}{\xi}\right); & W > W_c \end{cases}$$

$\Delta_t \sim \mathcal{O}(1);\qquad \Delta_c \sim N_F^{-(1-D_c)} \Rightarrow \mathcal{F}(x) \sim (1-D_c)\ln x$ for $x >> 1$

Scaling of Self-energy



$$\Lambda \sim \exp\left[\frac{b}{(\delta W)^{\alpha}}\right]$$

$$\xi \sim |\delta W|^{-\beta}$$

N. Macé et.al., PRL (2019); S. Roy and D. Logan, PRB (2021);

Summary

- The finite size scaling of Δ_t is associated with a Fock space volume scale Λ which has an essential singularity at the critical point.
- In the MBL phase, the finite size scaling is associated with a diverging correlation lenght ξ which diverges with a power law at the transition.
- The multifractality of the non-ergodic extended states in the MBL phase gets captured in the system size scaling of Δ_t.
- The fractal dimension D_s changes discontinously across the MBL transition.
- The fractal dimension $D_s < 1$ at the critical point W_c.

arXiv:2203.07415