Periodically driven Langevin systems with vanishingly small viscous drives

Sreedhar Dutta

IISER Thiruvananthapuram

ISPCM 3-5 April 2024, Bangalore

Shakul Awasthi and SD (arXiv:2402.16512)

Summary

- Periodically driven thermodynamic systems can exist in time-dependent nonequilibrium states (Oscillating states)
- Prototypical example:
  Driven under-damped Brownian particle
  - Viscous drive $\gamma(t)$; Noise drive $D(t)$
  - Potential drive $U(x;\lambda)$ with $\lambda \to \lambda(t)$
- These OS are characterized by $T_b = D/\gamma$ and γ.
  - What are the properties of OS for given T_b as $\gamma \to 0$?
- Vanishing viscous limit is singular.
  - We have a singular perturbation theory
- We find two classes of OS behaviour in low viscous limit

Driven under-damped Brownian motion

- Fokker-Planck equation:

$$\frac{\partial p}{\partial t} = \left[-\frac{\partial}{\partial x} v - \frac{\partial}{\partial v} \left(-\gamma v + f \right) + D \frac{\partial^2}{\partial v^2} \right] p$$

- Harmonic $f_h = -kx$ and Anharmonic: $f_{ah} = \sum \lambda_n x^n$
- When undriven, the large-time distribution is equilibrium
- When driven
  - viscous drive $\gamma(t)$; noise drive $D(t)$,
  - mechanical drive: $k(t)$ and $\{\lambda_n(t)\}$

then the large-time state is Oscillating state (OS):

$$\lim_{N \to \infty} p(x, v, NT + t) = p_{os}(x, v, t)$$

Low-viscous drives

- Perturbative parameter α
  $\gamma_\alpha = \alpha\gamma(t)$ and $T_b = D_\alpha/\gamma_\alpha$
- Inertial term is important; effects of drive are drastic.
- The limit is singular:

$$\lim_{\alpha\to 0} \text{Fokker-Planck} = \text{Liouville}$$

- large t and small α do not commute:

$$\lim_{\alpha\to 0}\lim_{N\to\infty} \neq \lim_{N\to\infty}\lim_{\alpha\to 0}$$

Singular perturbation theory

- Alternative way of identifying OS is demand periodicity:

$$P(t+T;\alpha;P_{in}) = P(t;\alpha;P_{in})$$

- Follow the standard perturbative steps
  - $L_{FP} = L_0 + \alpha L_1$ and $P(t) = \sum \alpha^n P^{(n)}(t)$
  - Solve the hierarchy of equations for arbitrary $P_{in}^{(n)}$ where $P_{in} = \sum \alpha^n P_{in}^{(n)}$.
- Impose the non-standard periodicity demand (Floquet theory)
  - Periodicity at $O(\alpha^n)$ uniquely determines OS to $(n-1)$-th order.
- The algorithm can be employed to moments dynamics, to determine OS moments and in turn P_{os}.

Two distinct cases

- Consider $U(x) = k_0 x^2$ with drives $\gamma(t)$ and $T_b(t)$ of period $T = 2\pi/\omega$.
- Case 1: When $k_0 \neq n_0^2\omega^2/4$
  - $P_{os} \to P_{eq}$ as $\alpha \to 0$ with T_{eff}.
  - Steady temperature T_s within in spite of varying $T_b(t)$ without.
  - For small α, $T_s = T_s(t)$
- Case 2: When $k_0 = n_0^2\omega^2/4$
  - As $\alpha \to 0$, P_{os} resonates with time-period T/n_0
  - For small α, OS become T-periodic
- The results hold beyond harmonic potentials.