Quantum entanglement
in
quantum matter

QM100
International Centre for Theoretical Sciences, Bengaluru
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PHYSICS
HARVARD

From quantum mechanics to quantum matter

- Bose (1924): Photons are *bosons*: an arbitrary number of *indistinguishable* photons can be present at each wavelength, and this alone is sufficient to explain Planck's black body spectrum.

- Einstein (1924): If there were non-relativistic bosonic particles, then they would undergo Bose-Einstein condensation at low temperature *i.e.* a finite fraction of them would have zero momentum.

- Sommerfeld (1927): Electrons are *fermions*: metals can be described by a degenerate quantum gas of nearly-free electrons.

- Bardeen, Cooper, Schrieffer (1957): Low temperature superconductors can be described as Bose-Einstein condensates of electron pairs.

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- Today: Quantum matter exhibits many *emergent phenomena*, related to **quantum entanglement**. These are crucial to understand modern quantum materials, such as the high temperature superconductors, and the quantum properties of black holes.

Spin liquids with an energy gap

Spin liquids without an energy gap

Experiments on spin liquids

Metals without quasiparticles: the SYK model

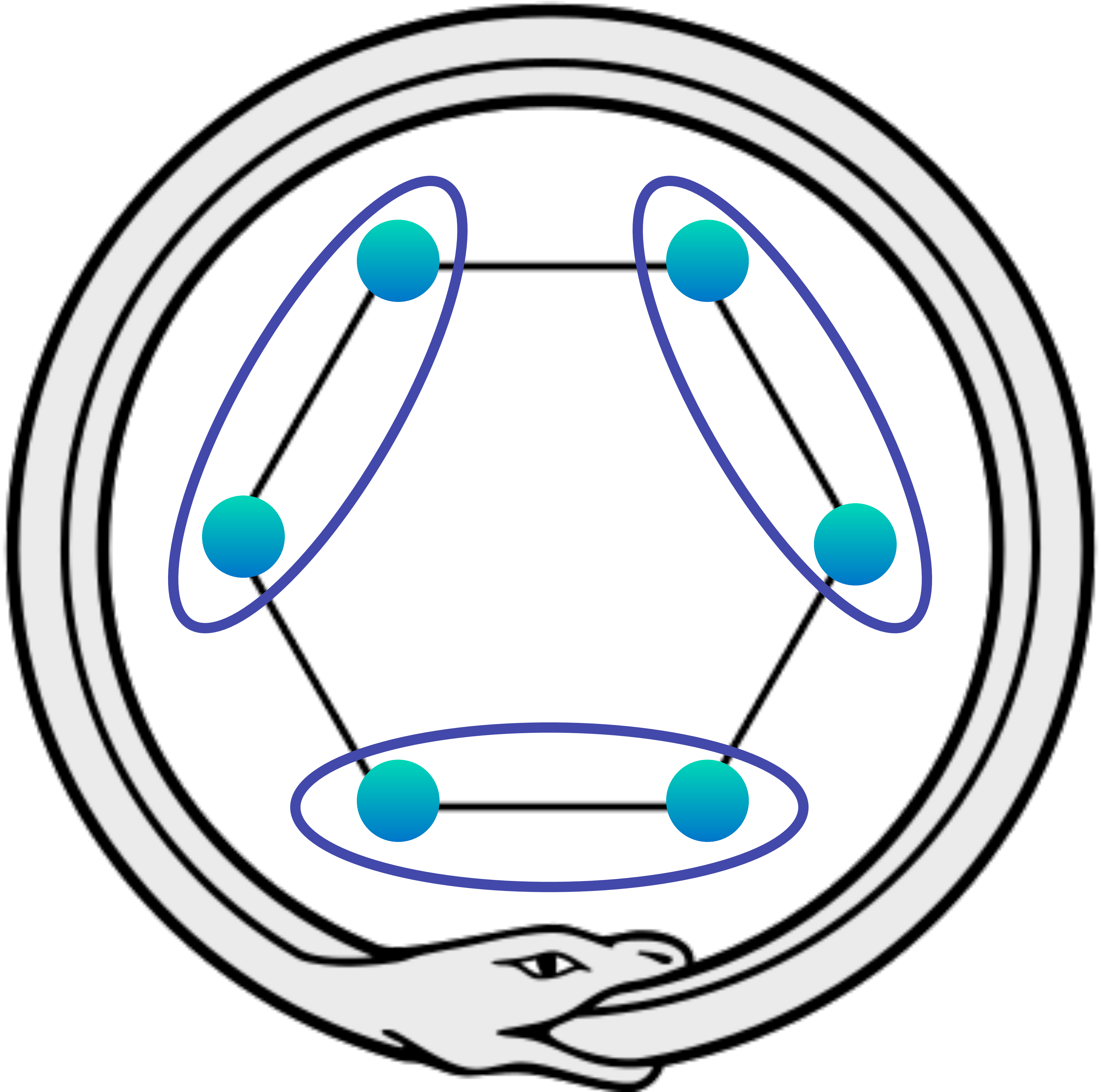
From the SYK model to black holes

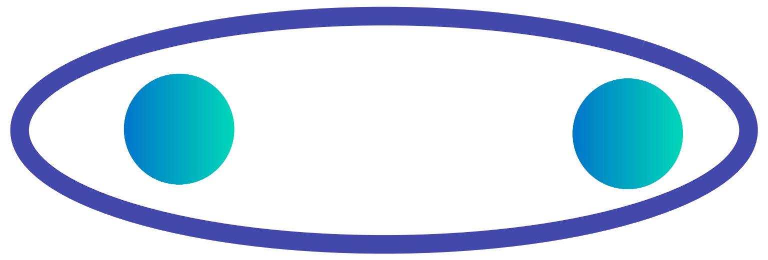
From the SYK model to the universal 2d-YSYK theory of strange metals

Theories of spin liquids with an energy gap

Kekule’s spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*



 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Benzene

*Wikipedia

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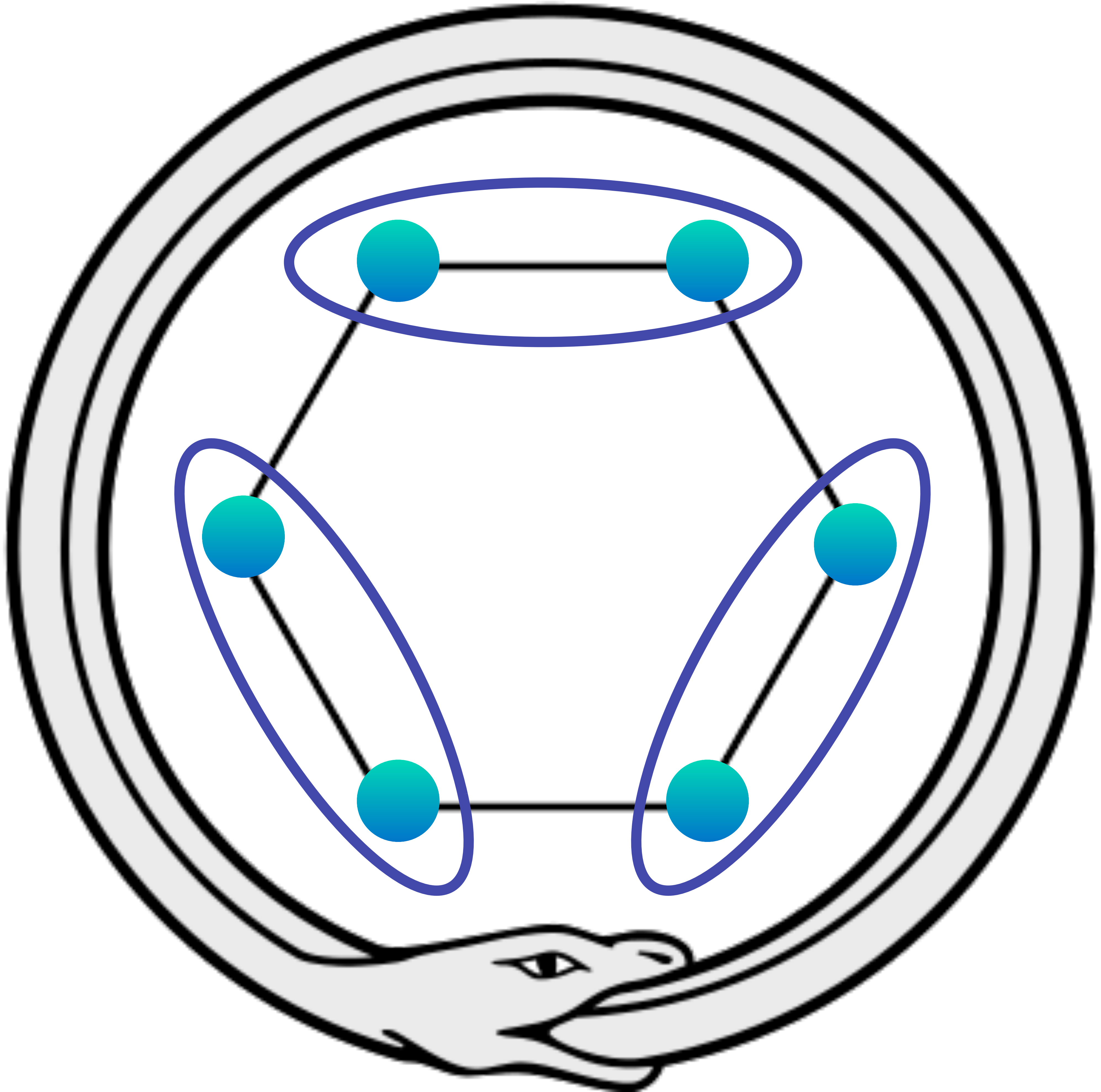

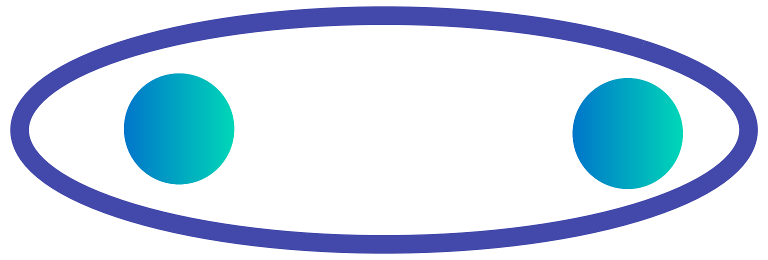 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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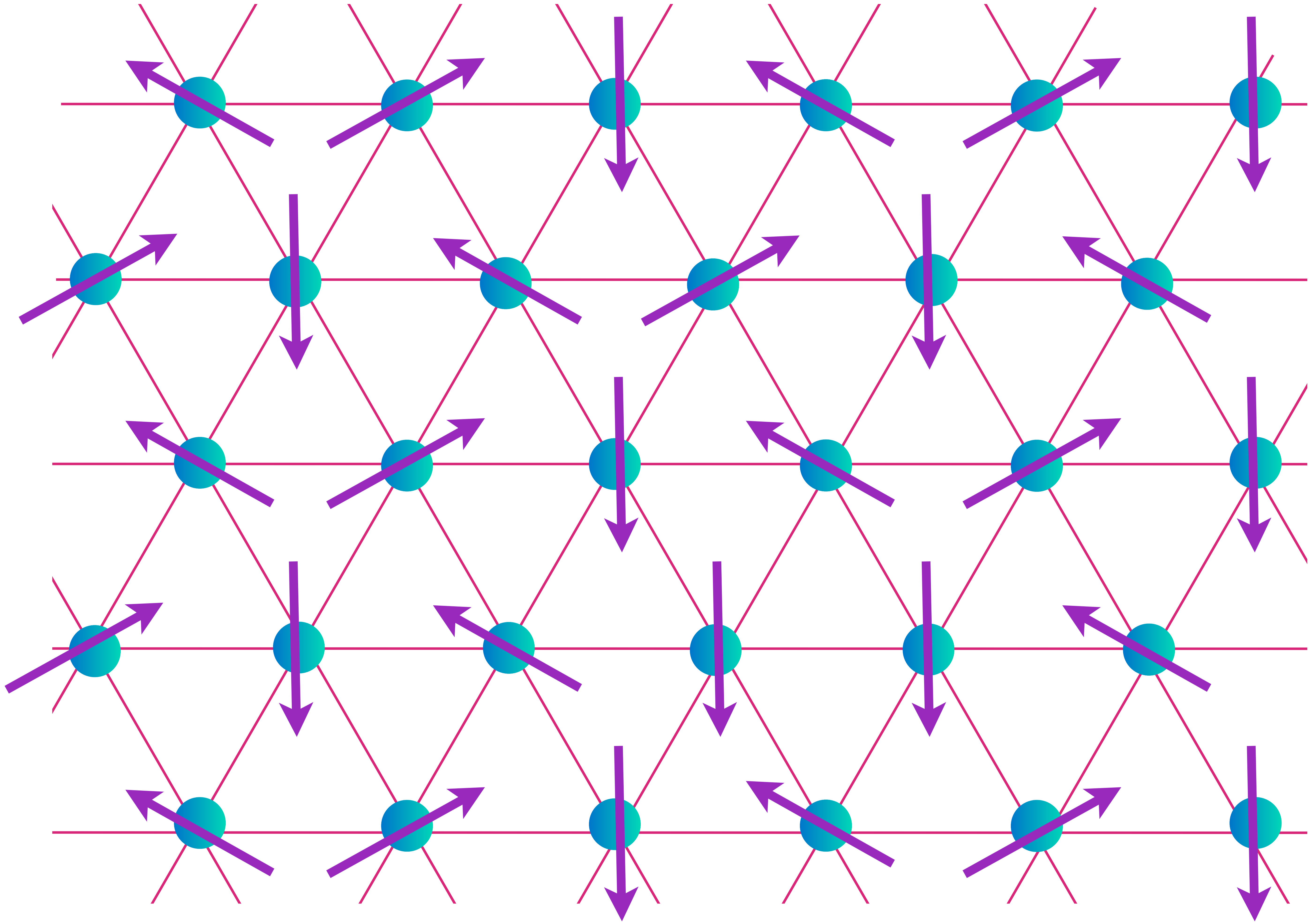
Triangular lattice antiferromagnet

Spin model with S=1/2 per unit cell

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



$$[S_\alpha, S_\beta] = i\epsilon_{\alpha\beta\gamma} S_y$$

$$S_\alpha^2 = S(S+1);$$

$$S = 1/2$$

$$S_z \left|\uparrow\right\rangle = (1/2) \left|\uparrow\right\rangle$$

$$S_z \left|\downarrow\right\rangle = -(1/2) \left|\downarrow\right\rangle$$

on each site i

Nearest-neighbor model has non-collinear Neel order

Spin liquid: resonating valence bonds

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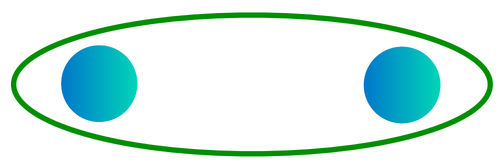

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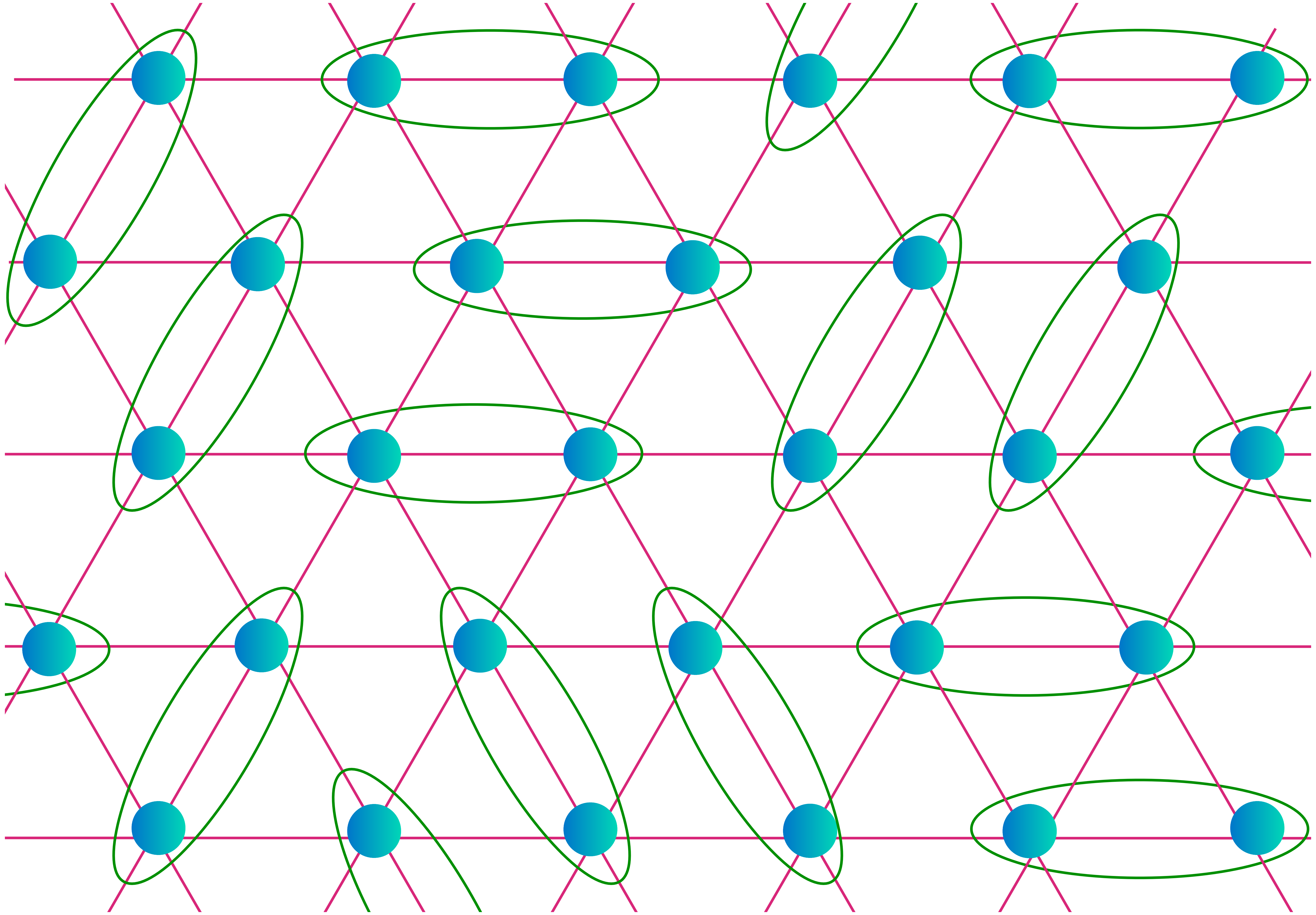

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$\mathcal{D} \rightarrow$ dimer covering of lattice

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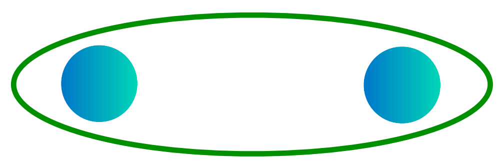
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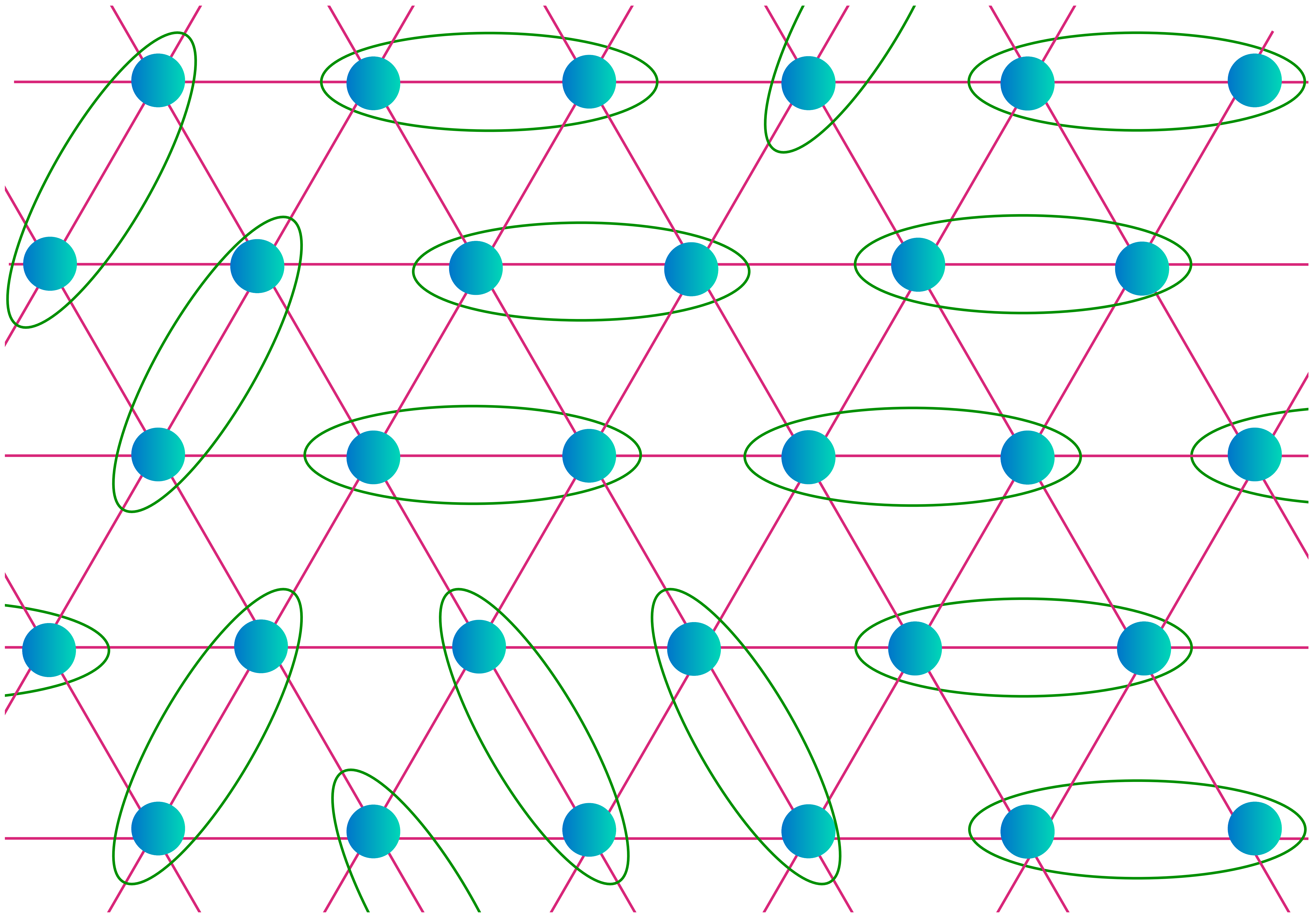

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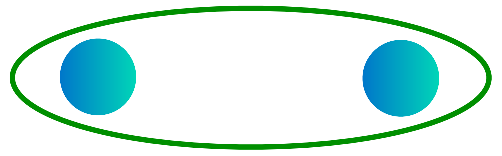

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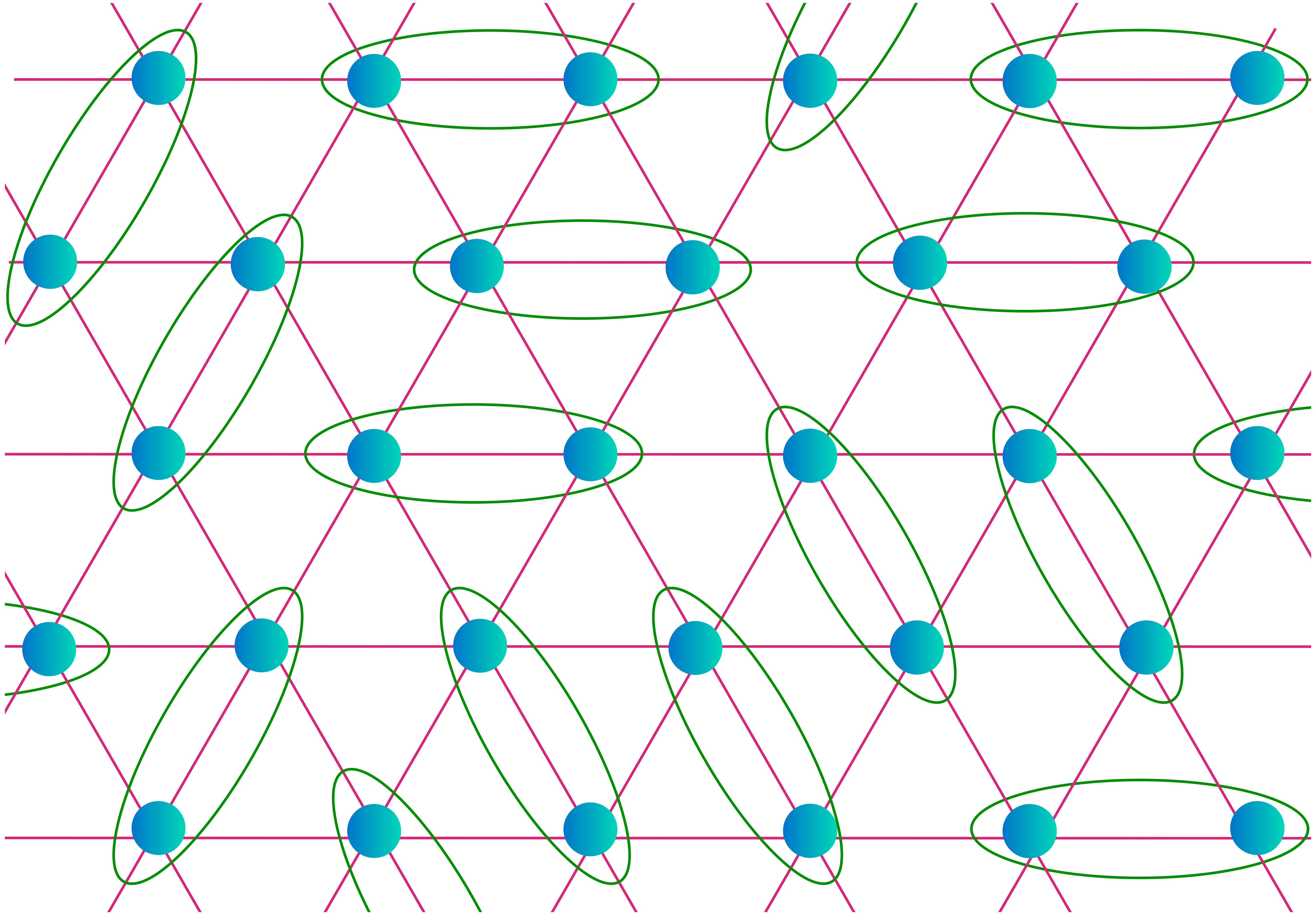

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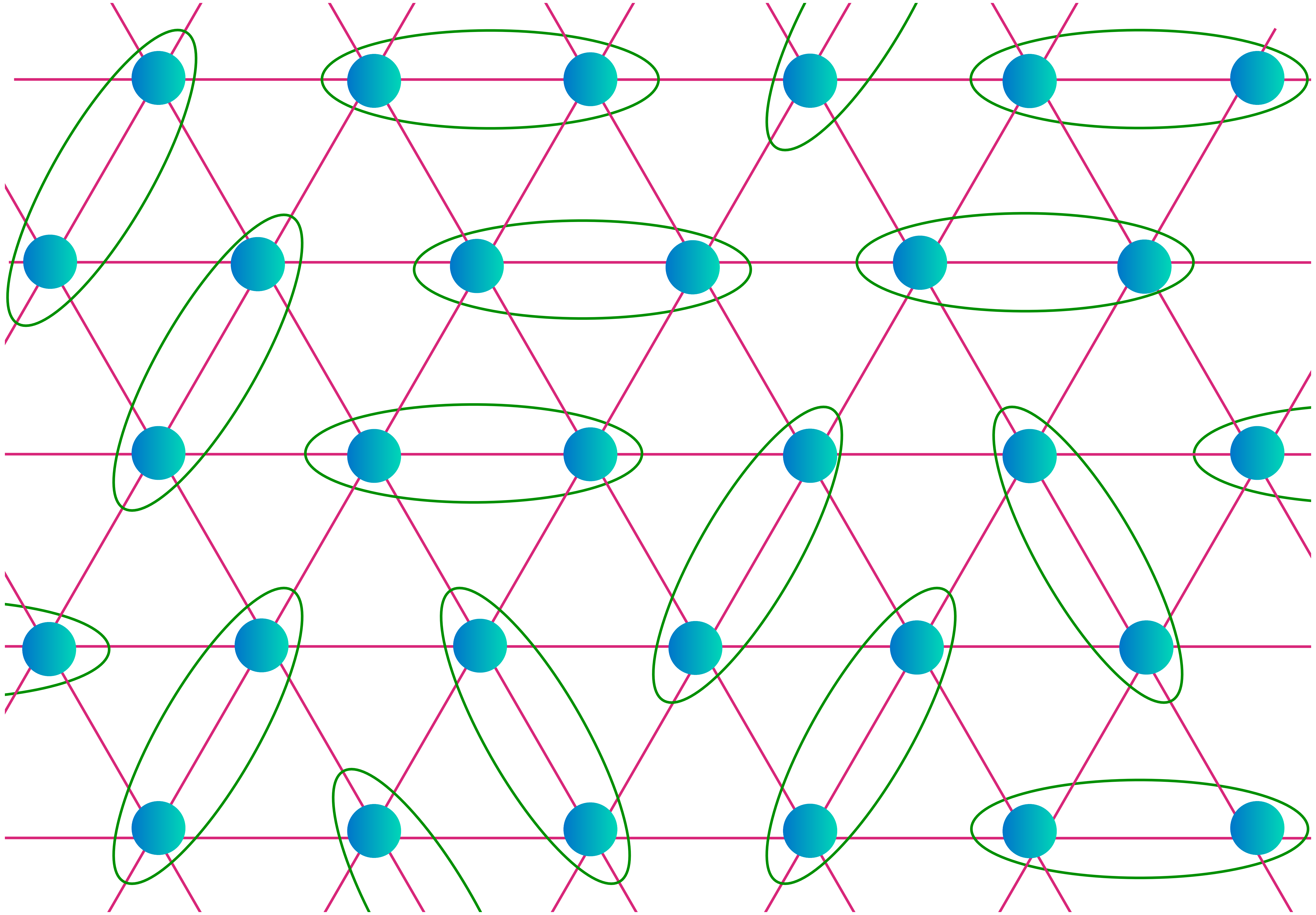

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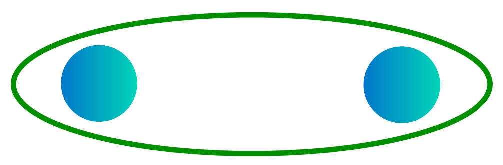

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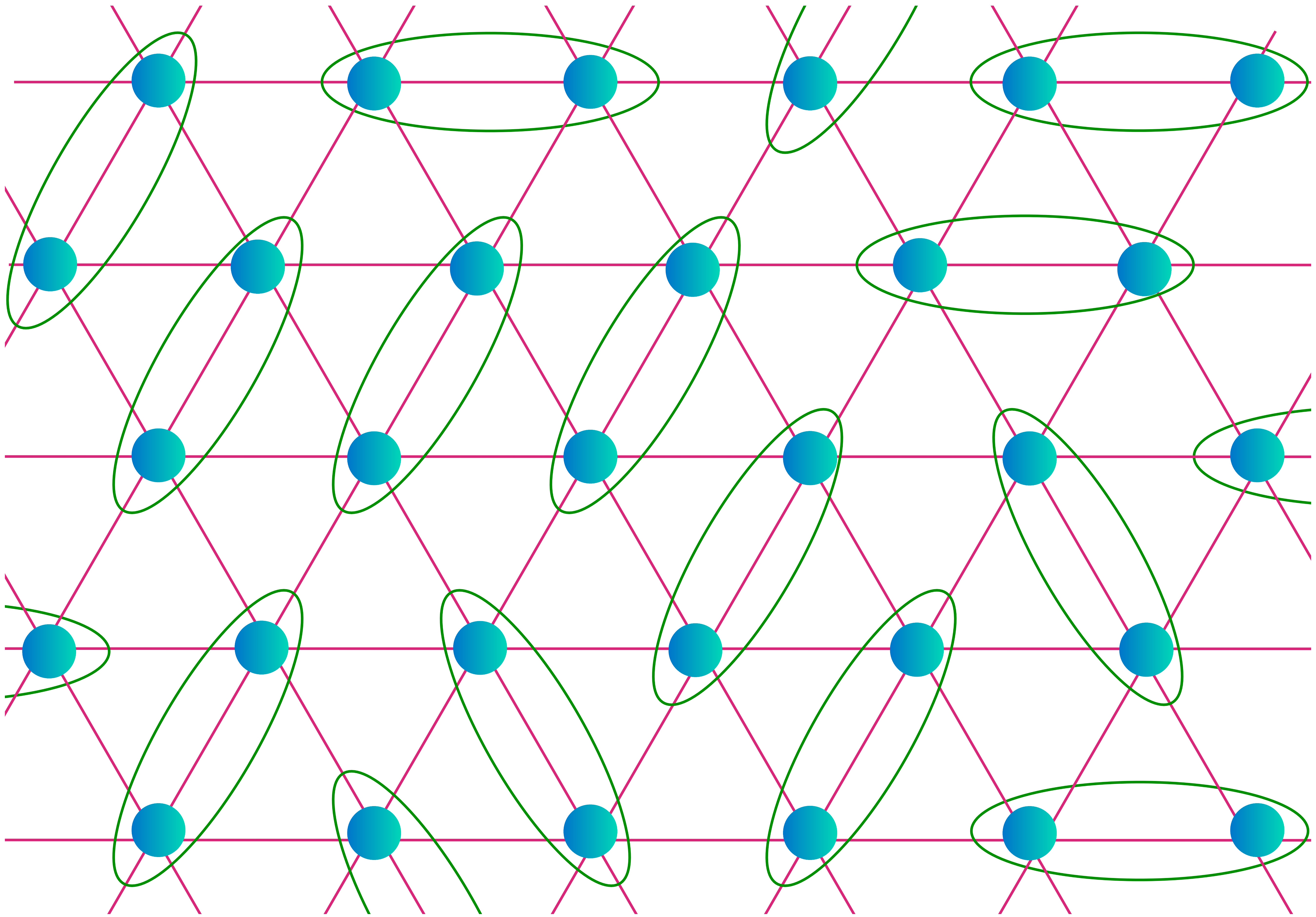

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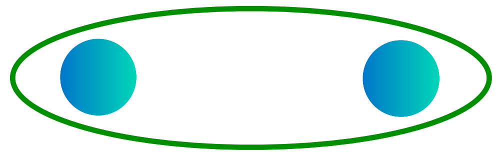

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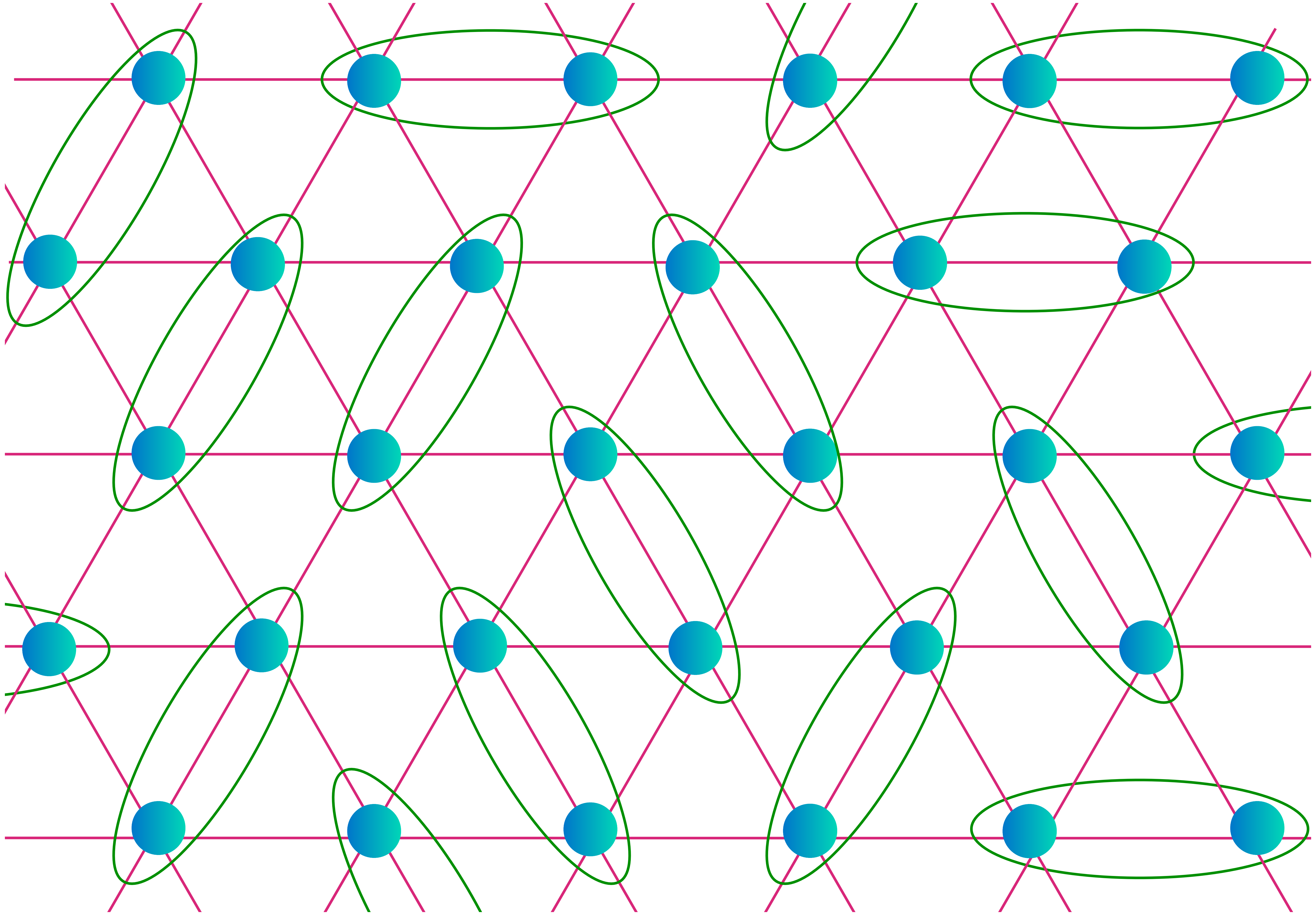

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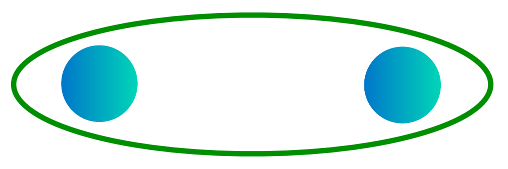

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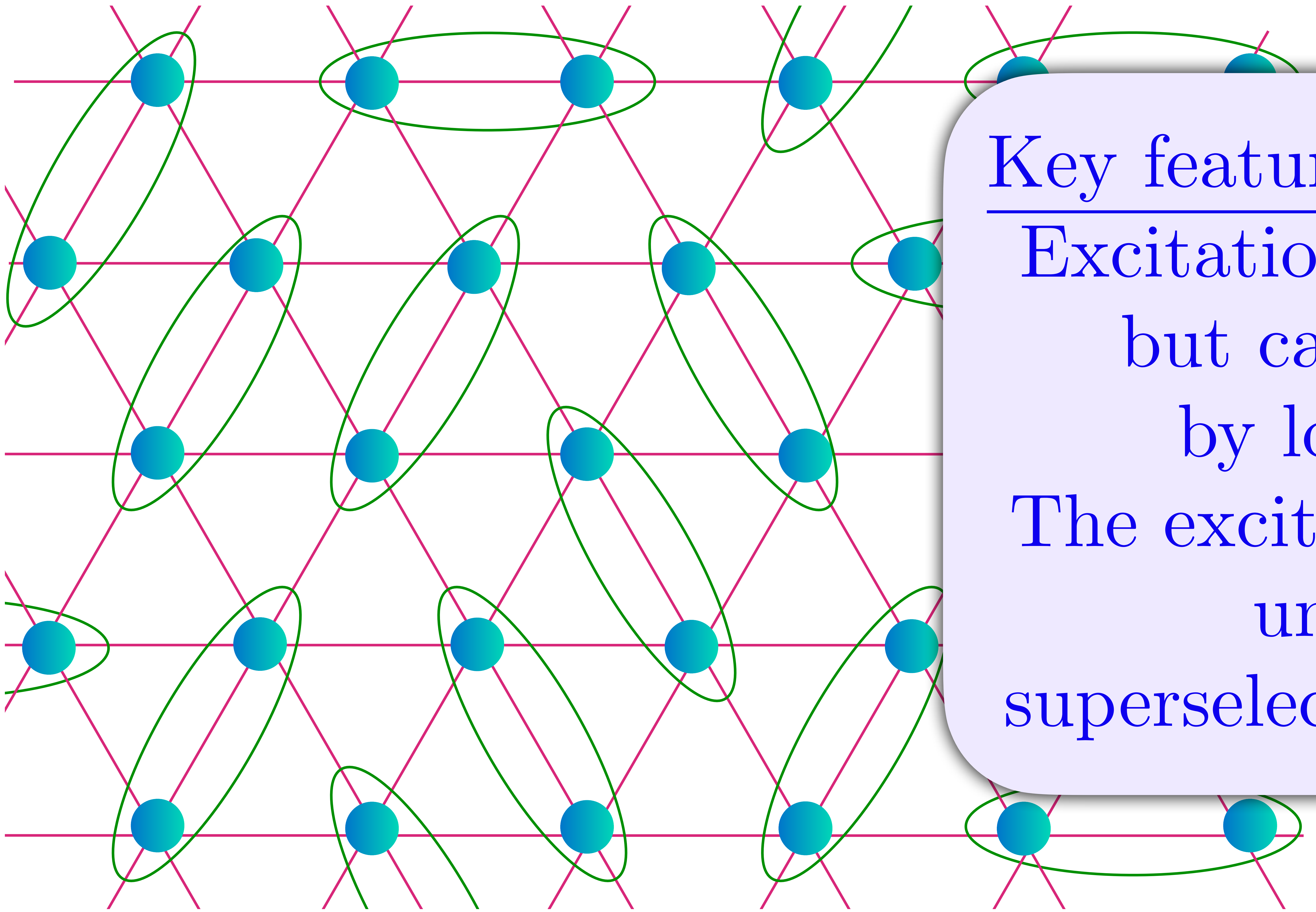


Key feature: fractionalization.

Excitations are particle-like, but cannot be created by local operators. The excitations are classified under distinct superselection/anyon sectors.

RVB: Z_2 spin liquid

Read and Sachdev (1990); Wen (1991)

The simplest stable spin liquid (which need not break time-reversal) is the deconfined phase of a $\mathbb{Z}_2$ gauge theory. There are excitations which cannot be created by any local spin operators: "spinons" which carry unit $\mathbb{Z}_2$ electric charges, and 'vison' excitations which carry π $\mathbb{Z}_2$ magnetic flux.

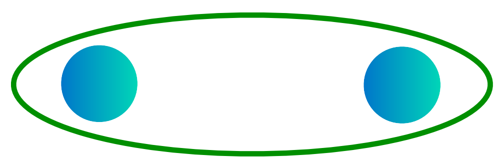
Anyon	e (spinon)	ϵ (spinon)	m (vison)
Self-statistics	boson	fermion	boson
Spin	1/2	1/2	0

Any pair of e, ϵ, m are mutual semions.

This structure ("unitary modular tensor category") is the same as that found in Kitaev's toric code (1997).

RVB: Z_2 spin liquid

Fractionalized excitations: a "spinon" with spin S=1/2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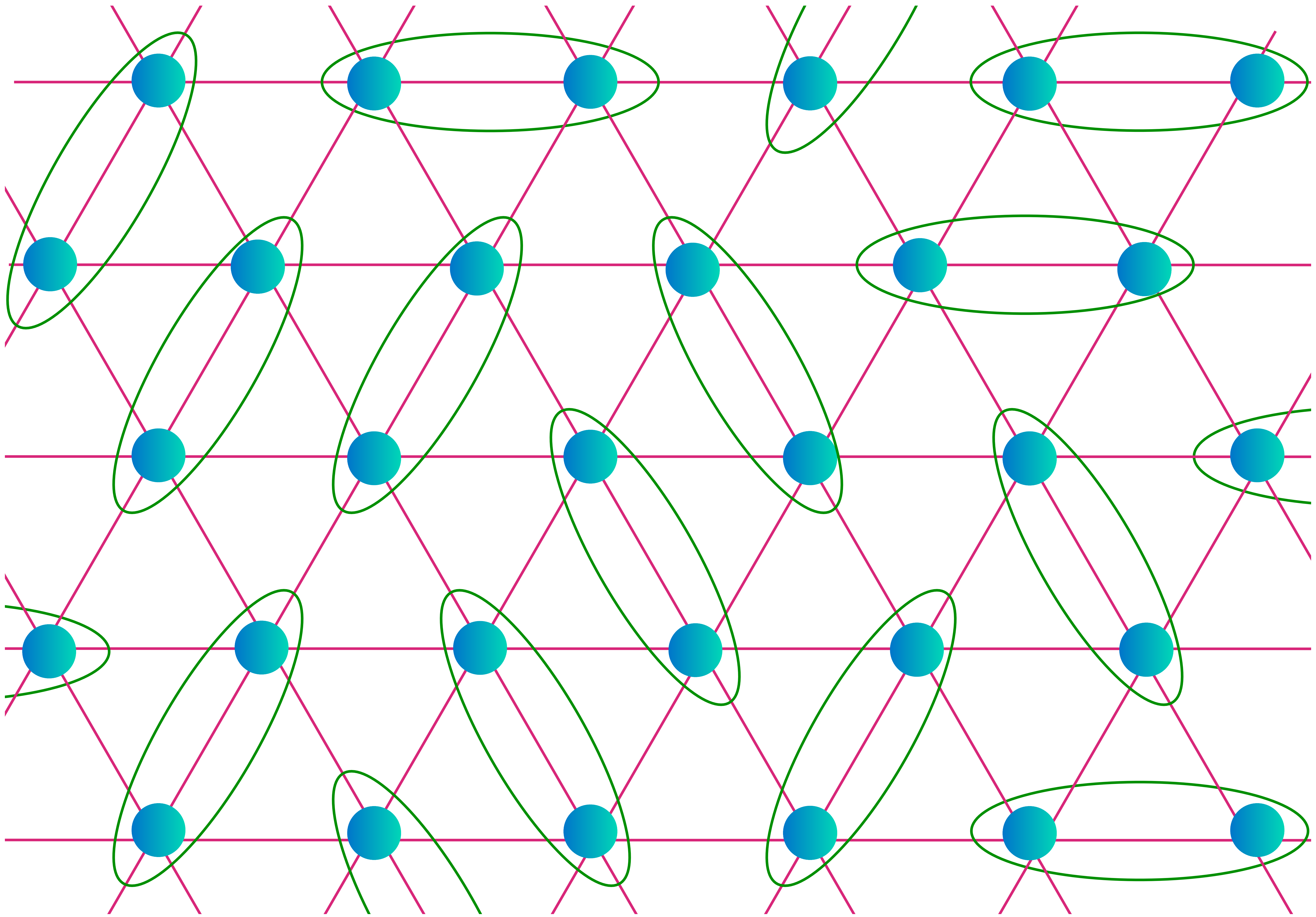

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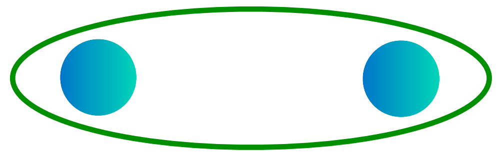
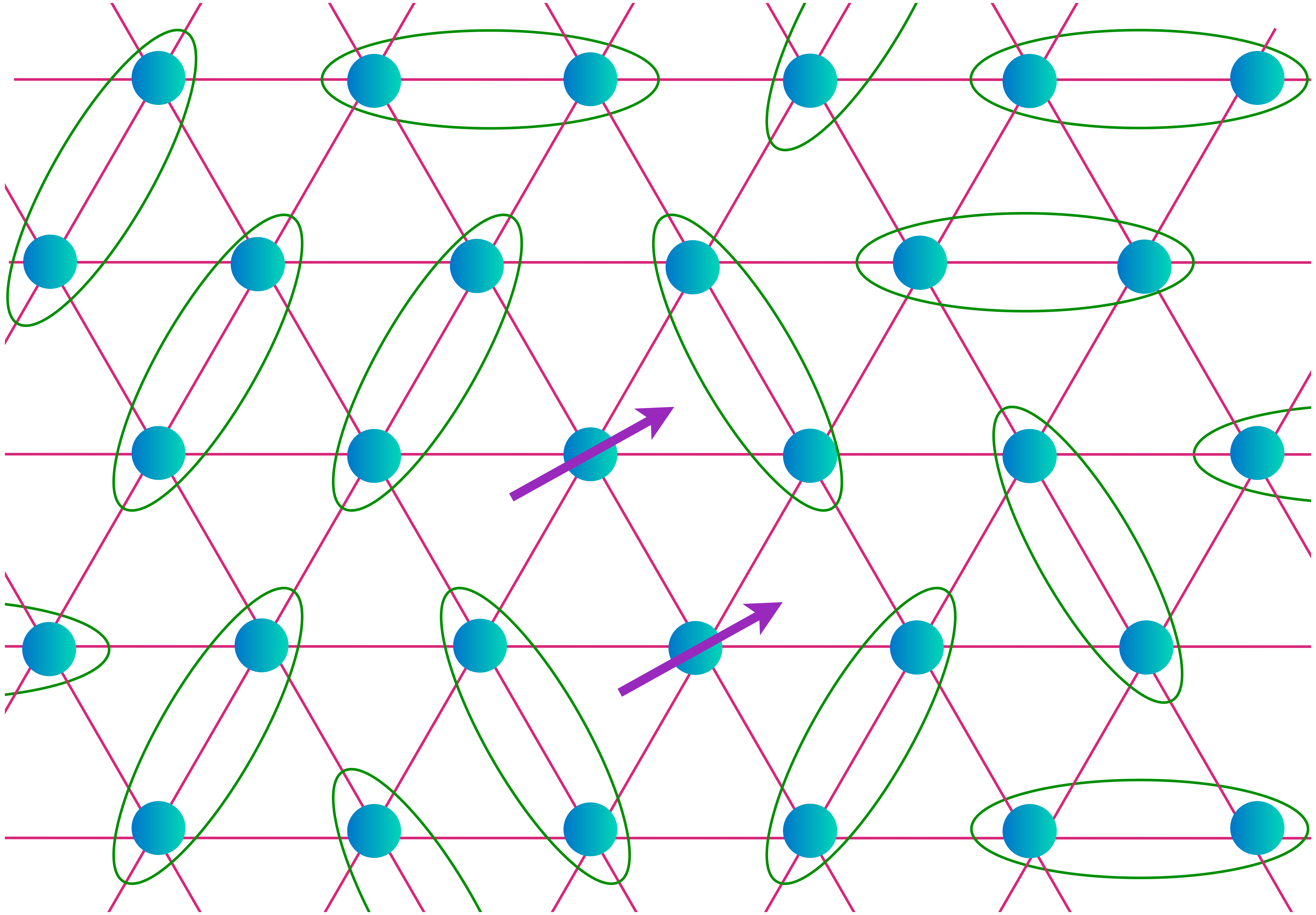
 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



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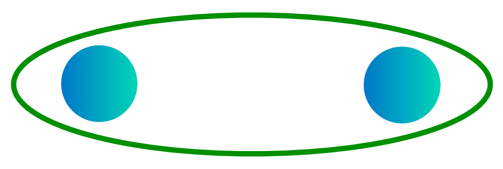

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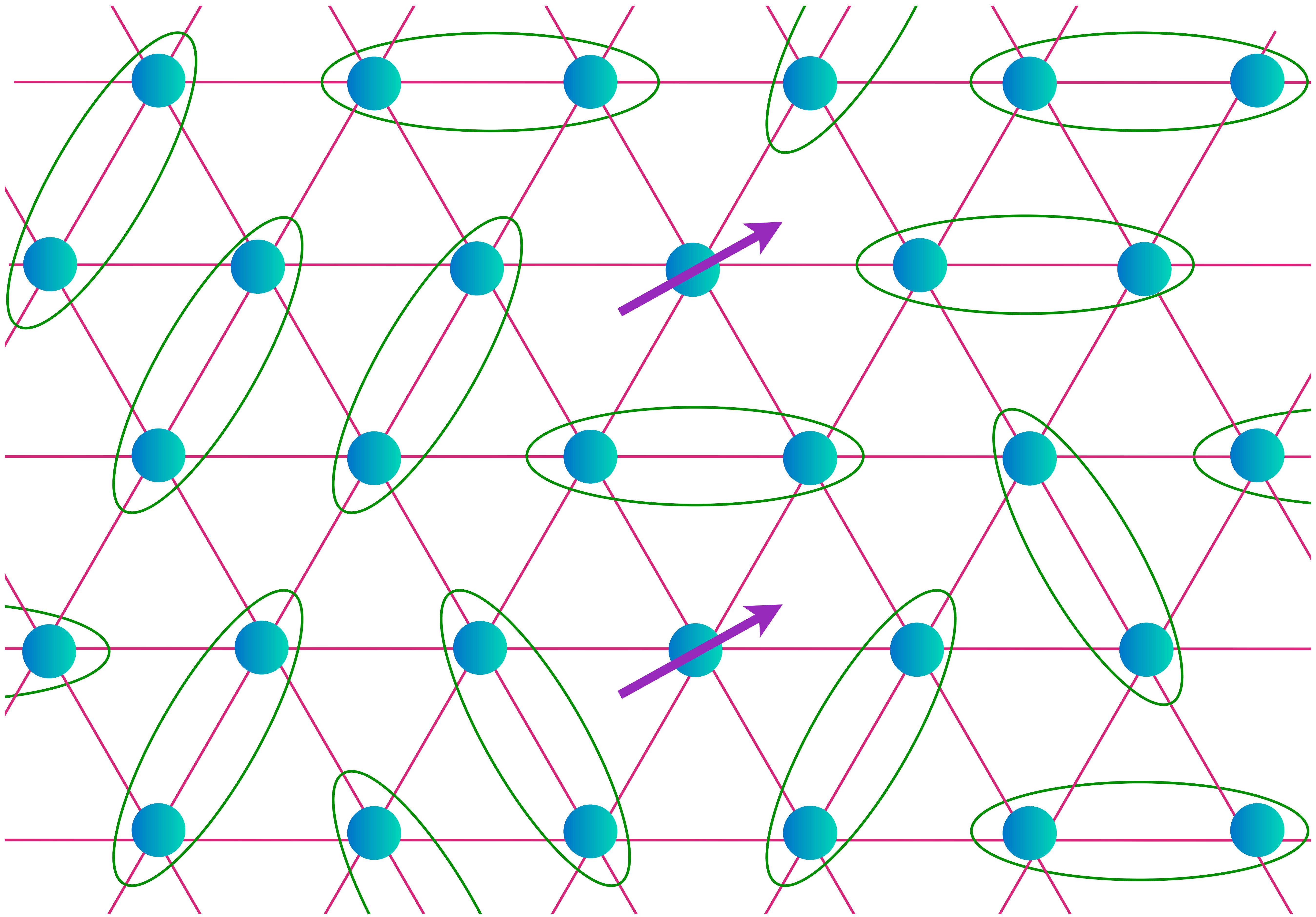

Kivelson, Baskaran…..

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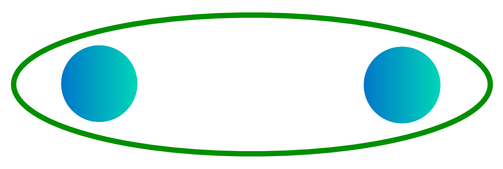

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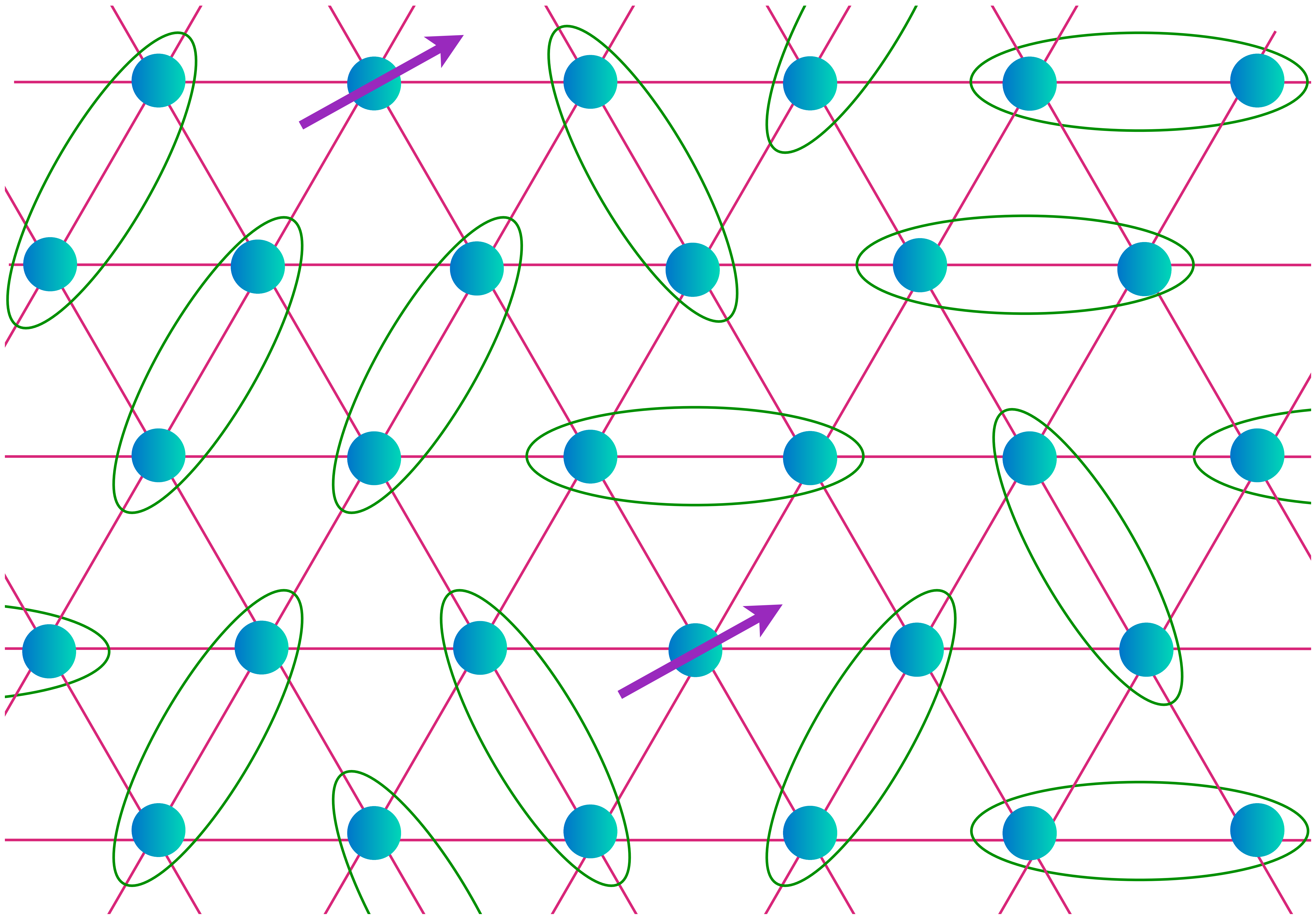

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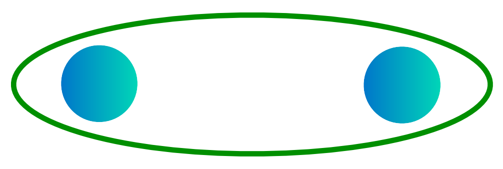

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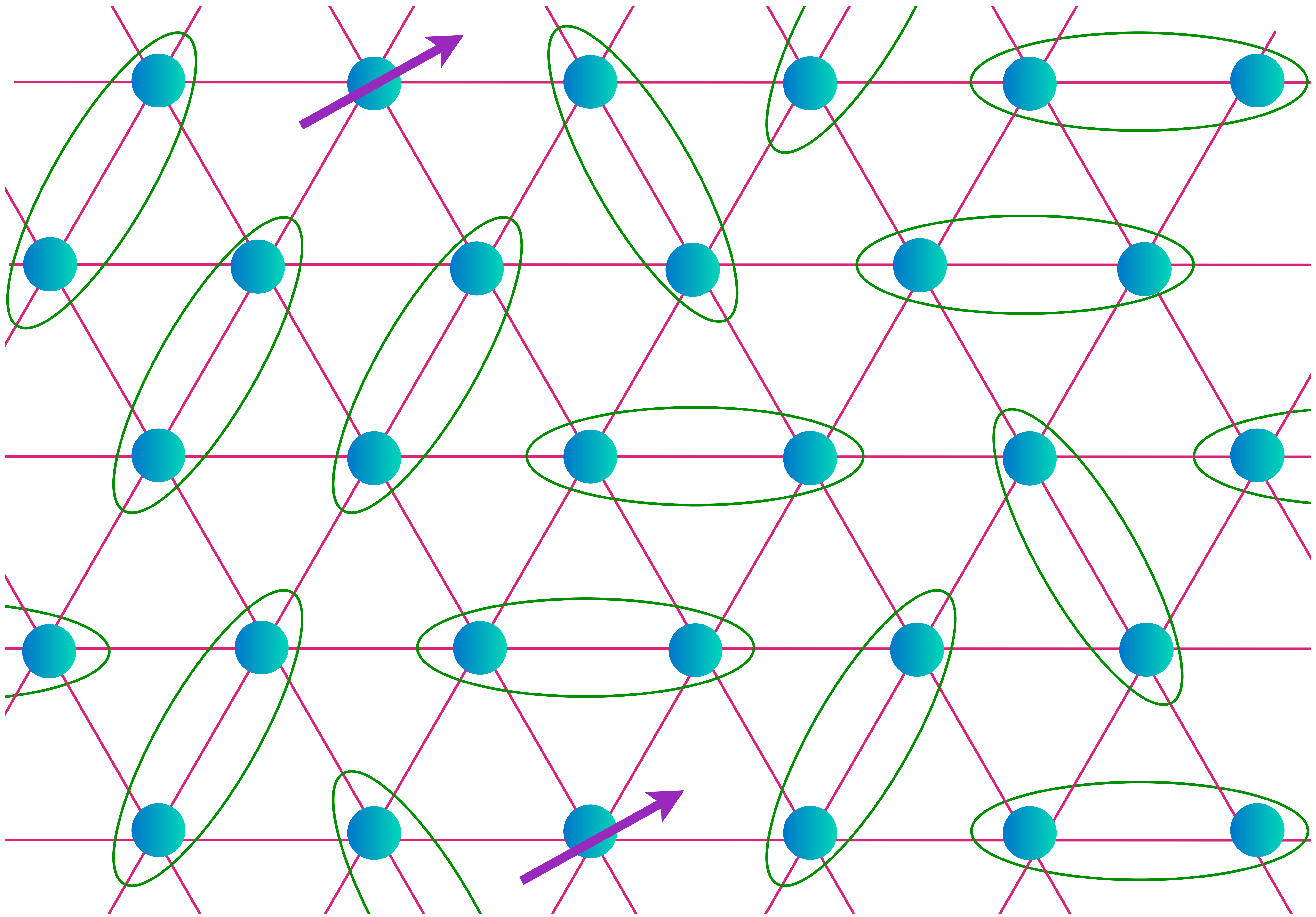

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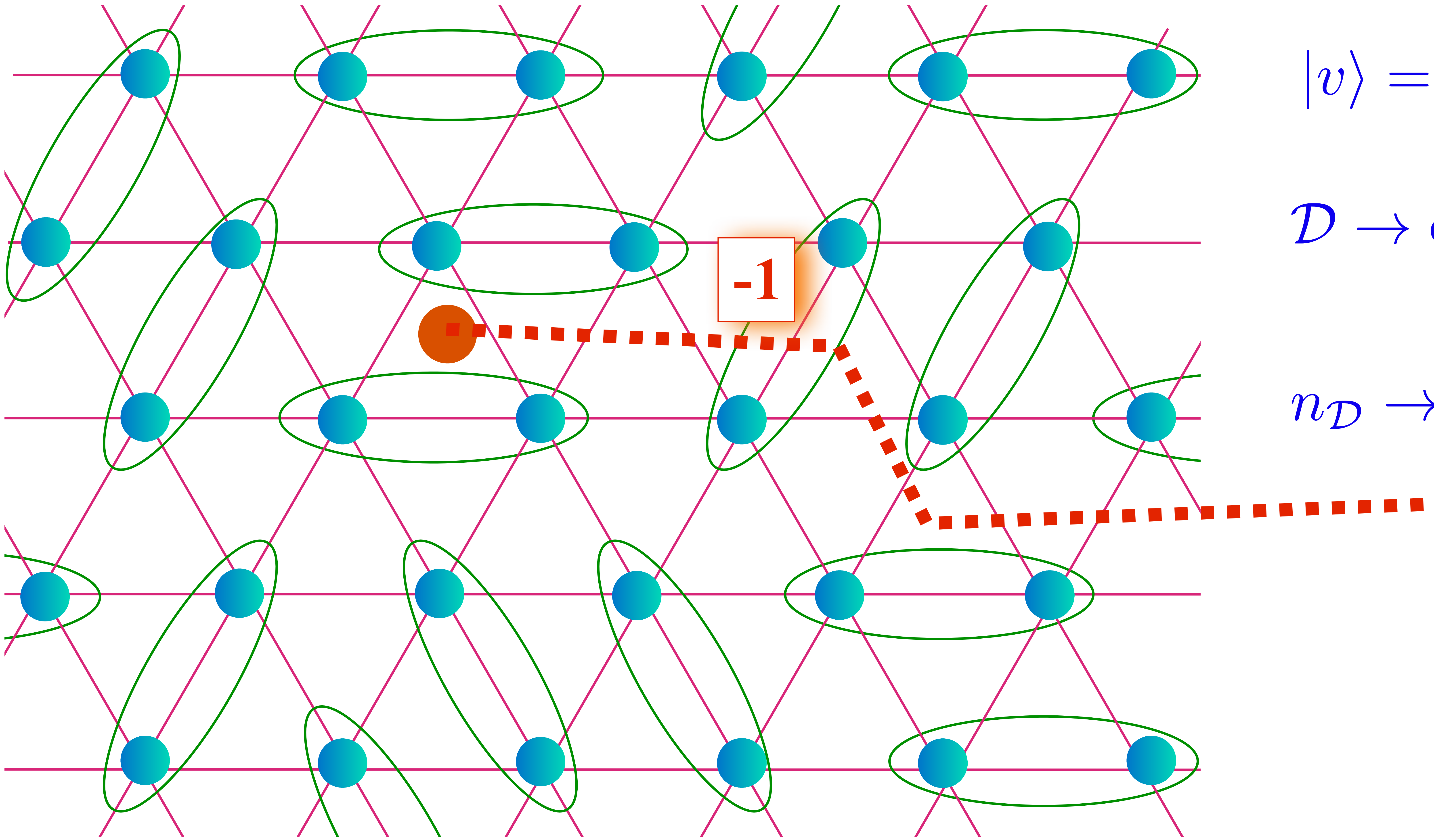
Z_2 "vortex" with spin S=0:
a vison (*m* particle)



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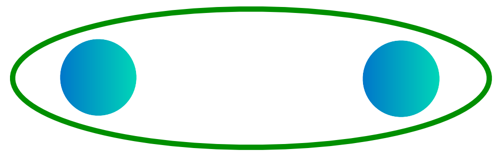

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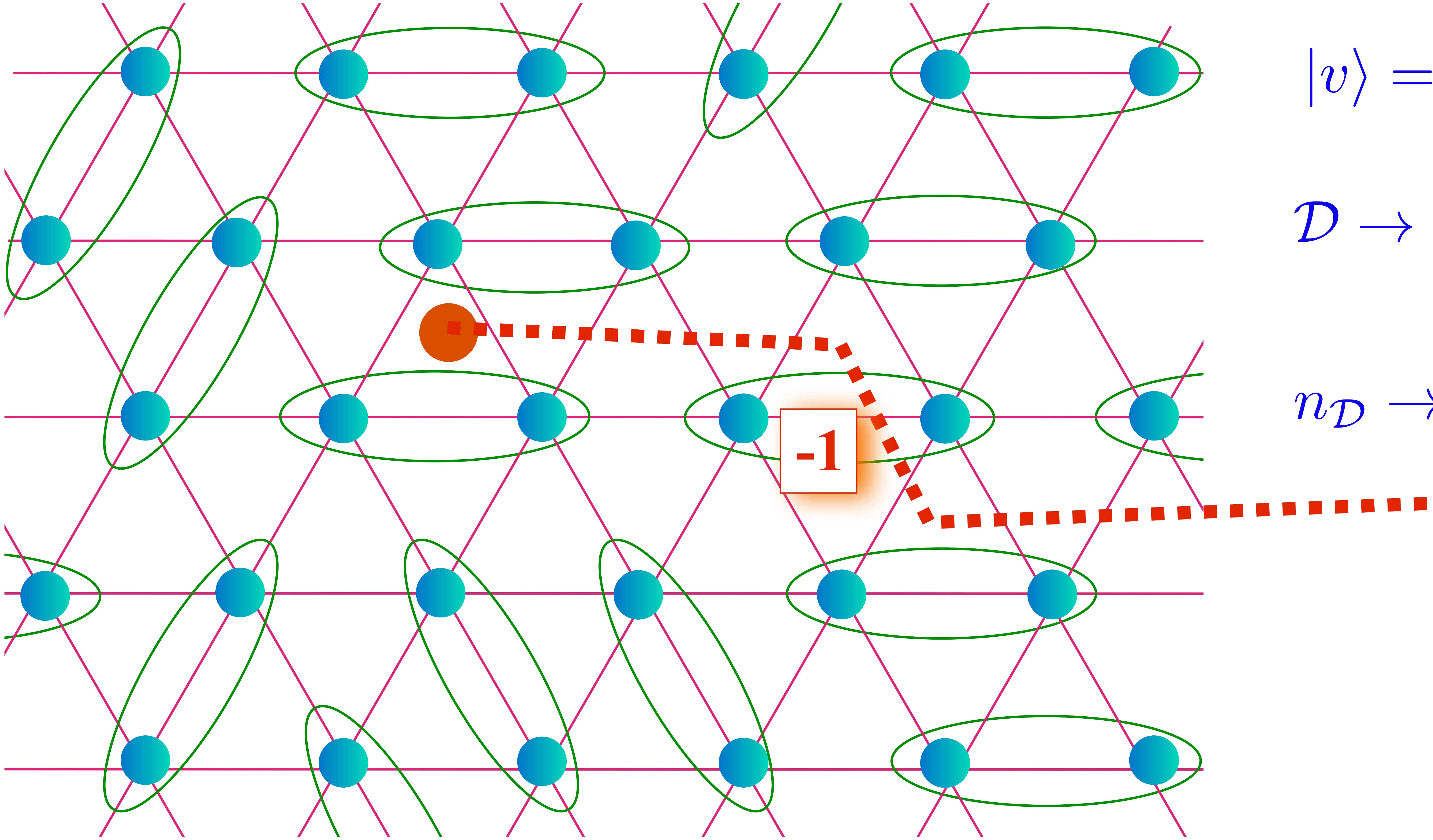

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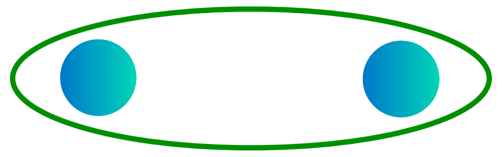

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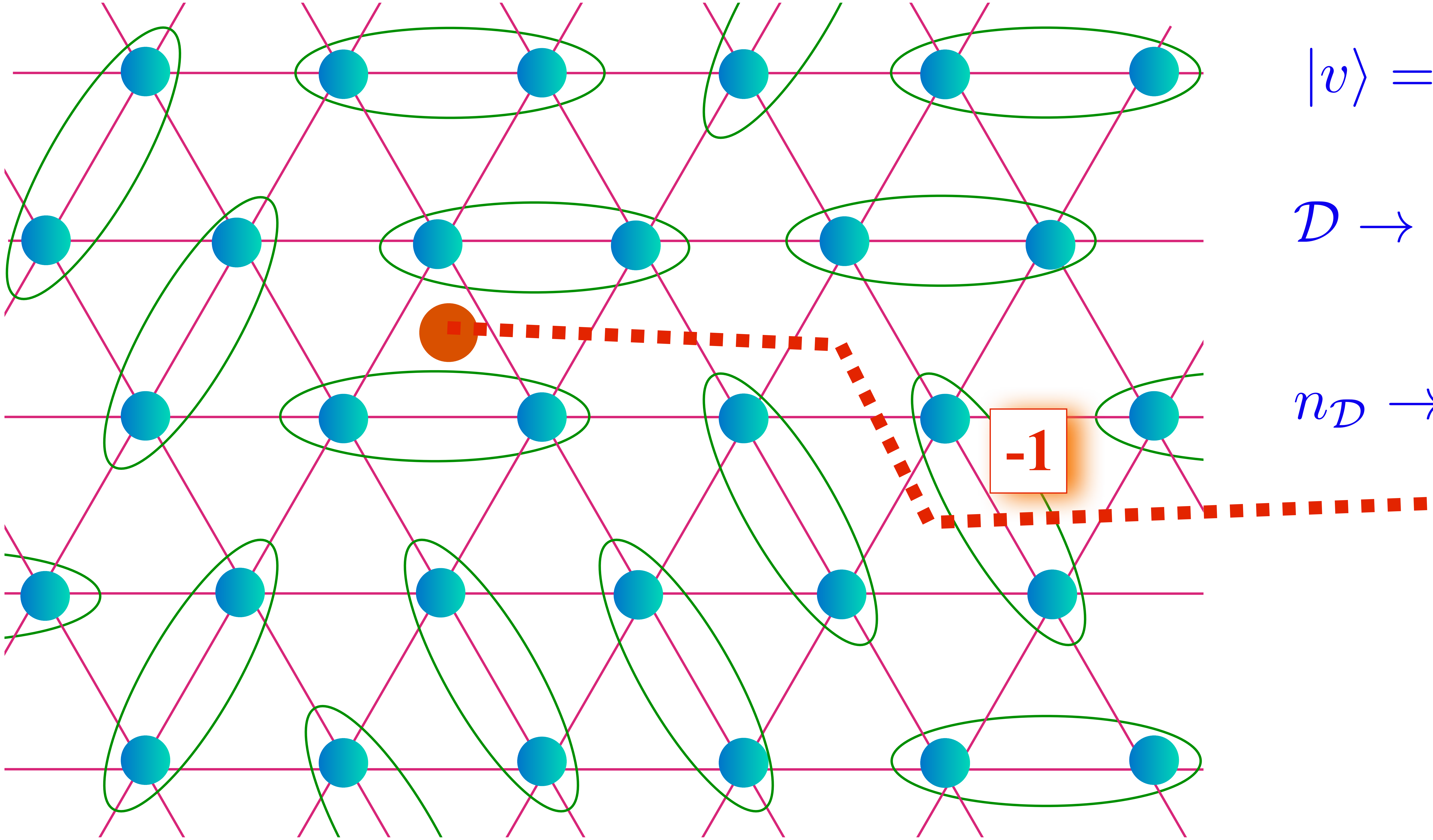

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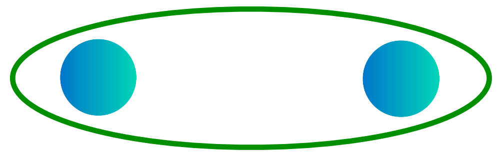

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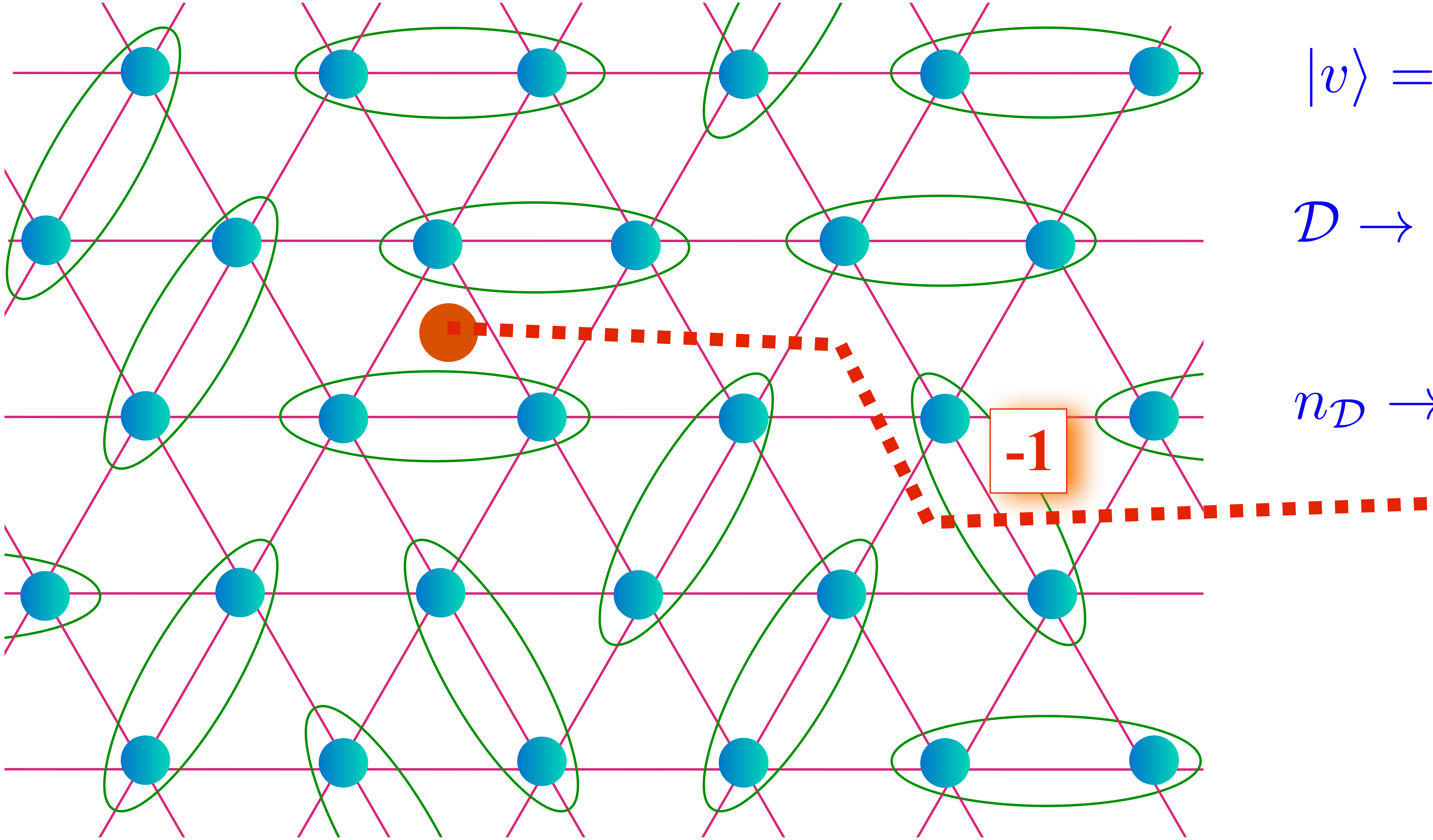

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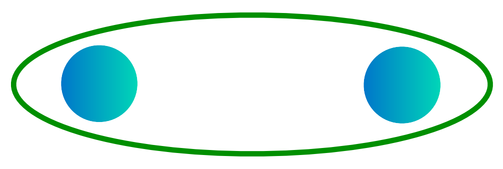

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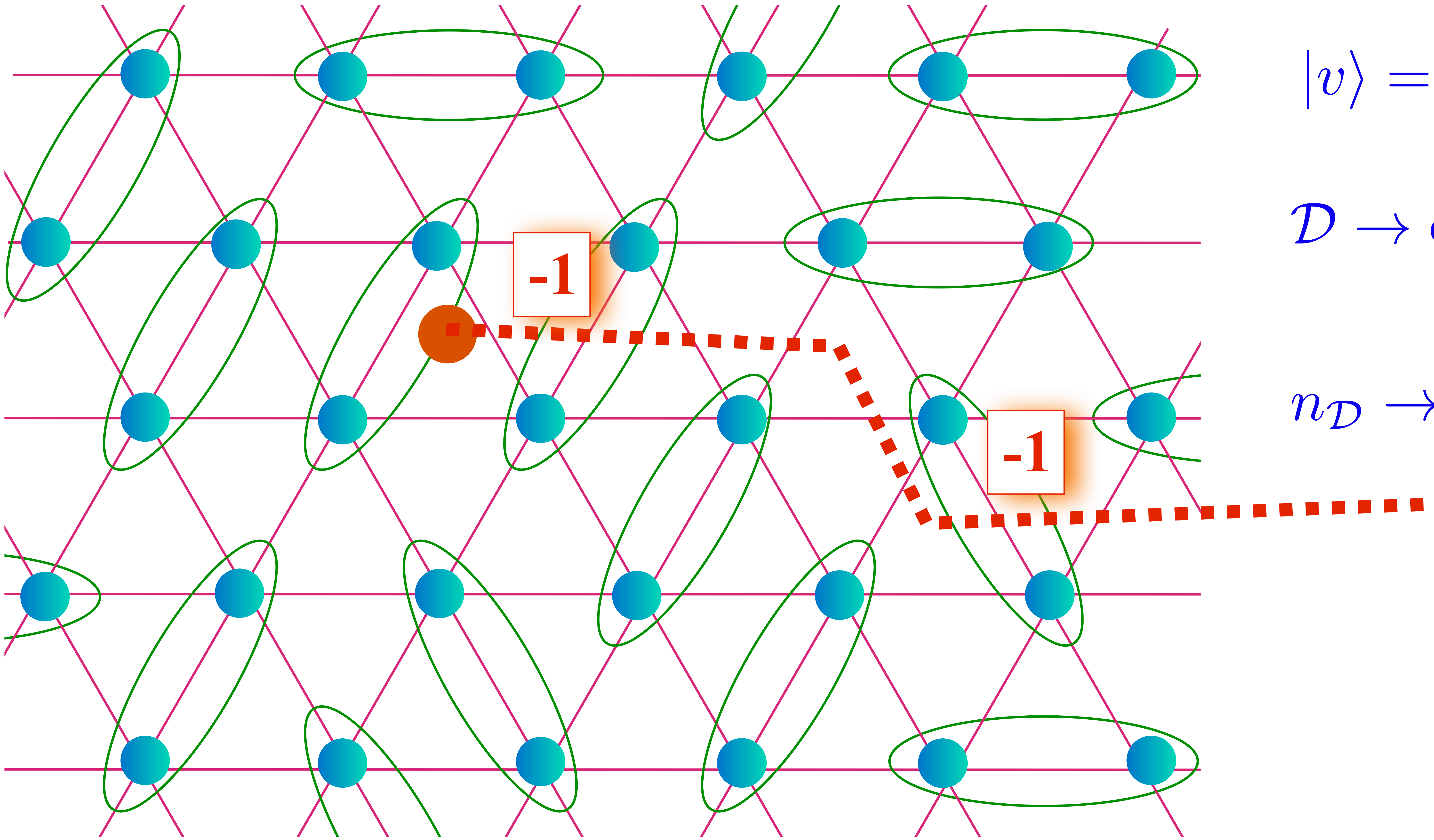

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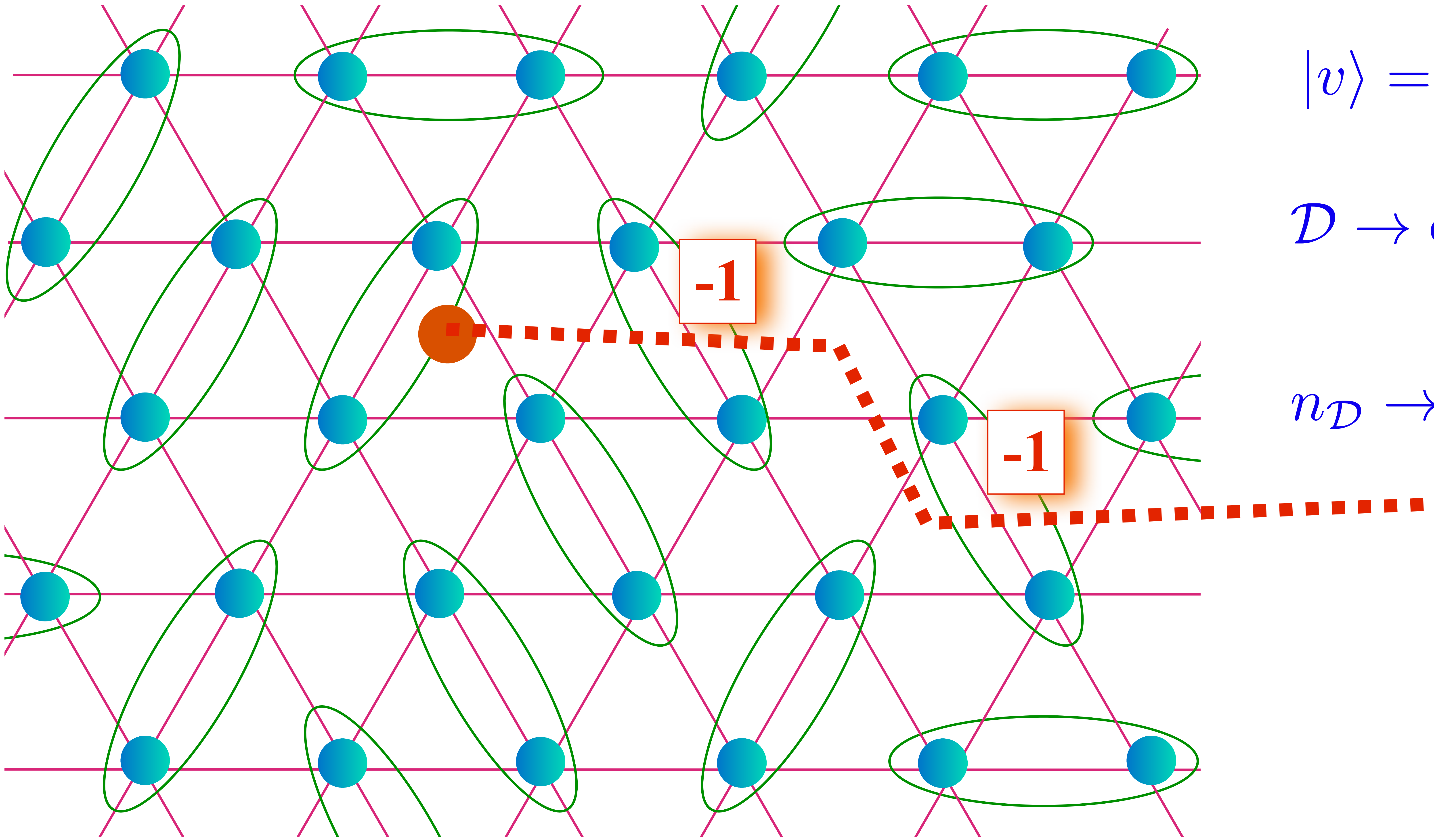

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Other gapped spin liquids

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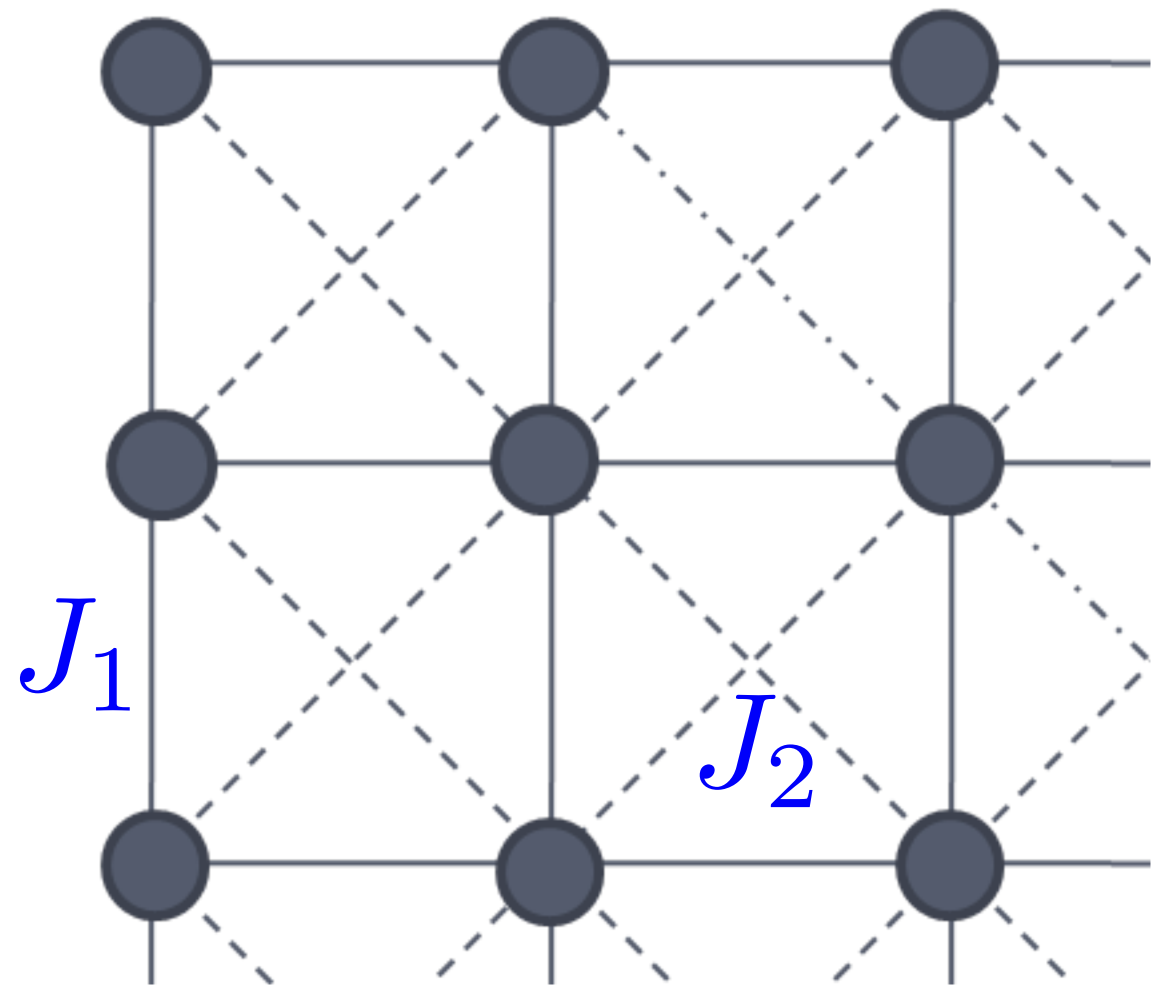
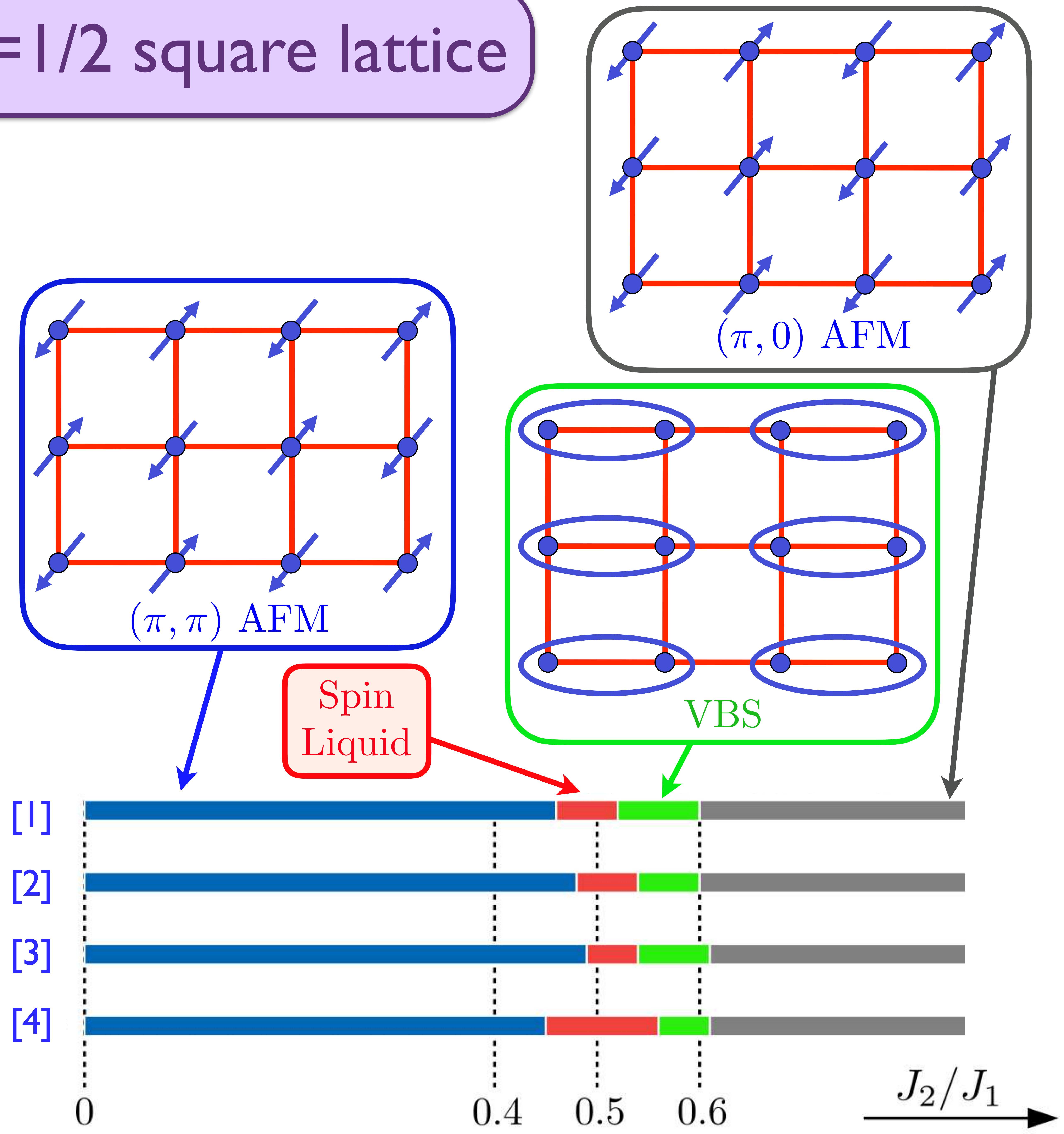
- Kitaev's non-Abelian Ising anyons (2006). A solvable honeycomb lattice model with XX, YY, ZZ interactions along three directions realizes a $\mathbb{Z}_2$ spin liquid in which the ϵ fermions have the spectrum of massless, relativistic Majoranas. Turning on a time-reversal breaking perturbation gaps the Majorana fermions, and the visons acquire a zero mode which turns them into non-Abelian anyons.

Theories of spin liquids without an energy gap

S=1/2 square lattice

$$H = J_1 \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$





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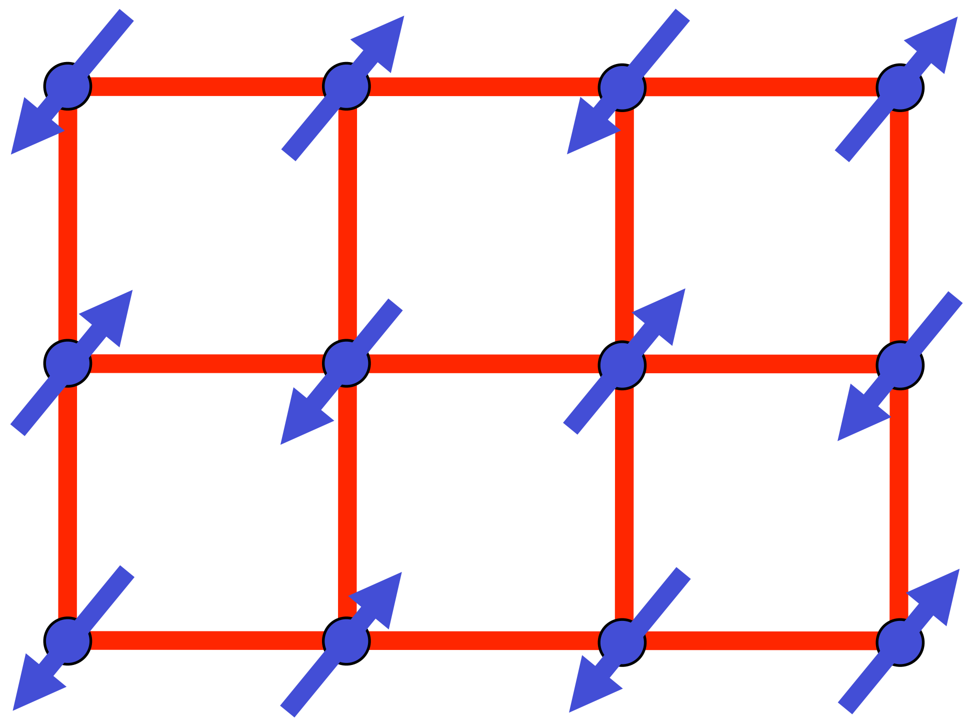
2. F. Ferrari and F. Becca, *Phys. Rev. B* **102**, 014417 (2020)
3. Y. Nomura and M. Imada, *Phys. Rev. X* **11**, 031034 (2021)
4. W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, *Science Bulletin* **67**, 1034 (2022)

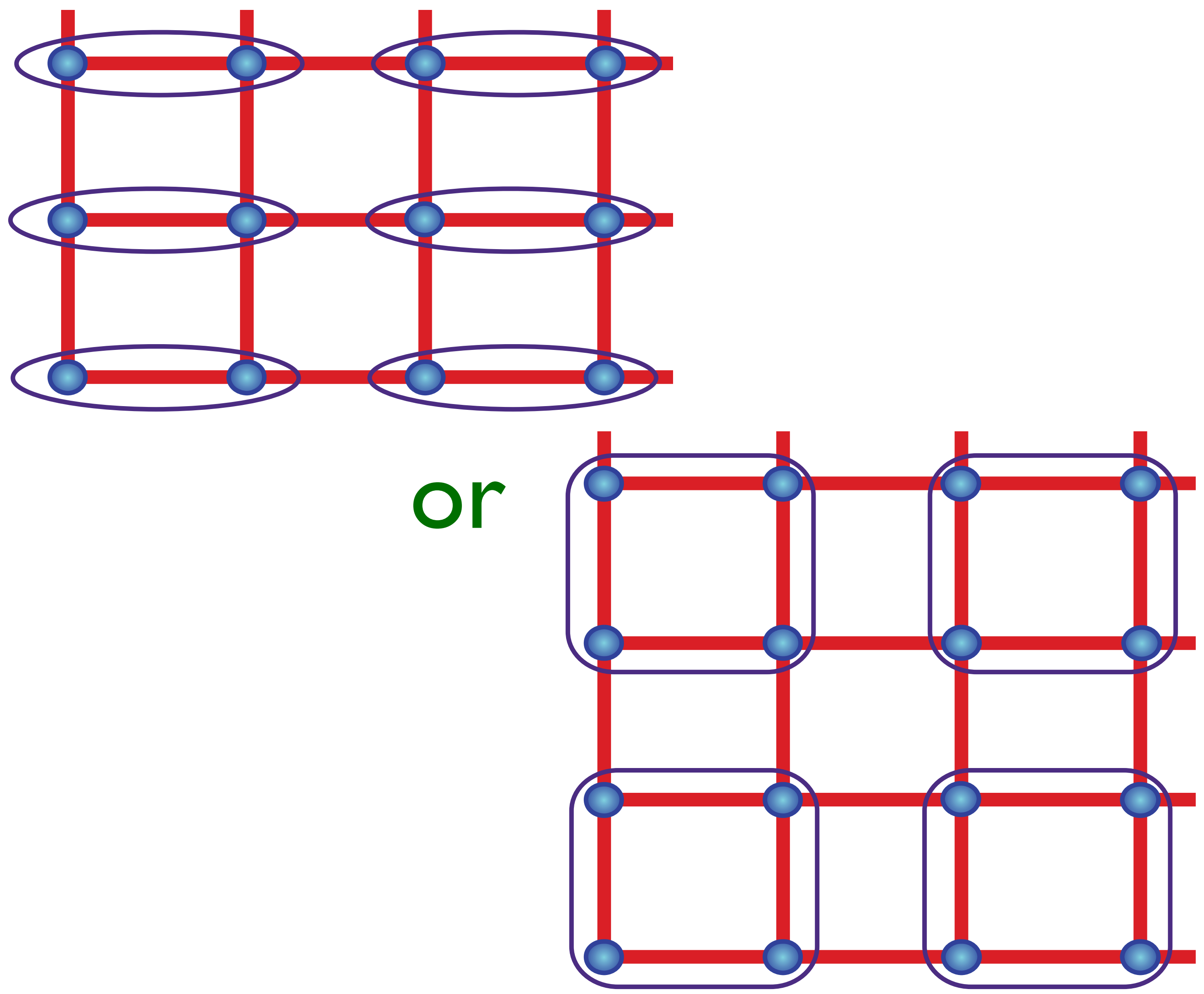
S=1/2 square lattice

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}\,, \quad \sum_{\alpha=1}^{N=2} b_{i\alpha}^\dagger b_{i\alpha} = n_b = 2S$$





$\langle b_\alpha \rangle \neq 0$:
Néel order

$\langle b_\alpha \rangle = 0$:
Valence bond solid (VBS)

N/n_b

Mean-field spin liquid
with gapped bosonic spinons.

Low energy $\mathbb{CP}^1$ U(1) gauge theory

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}^\dagger$$

$$\mathcal{L} = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989)
N. Read and S. Sachdev, Phys. Rev. B **42**, 4568 (1990)

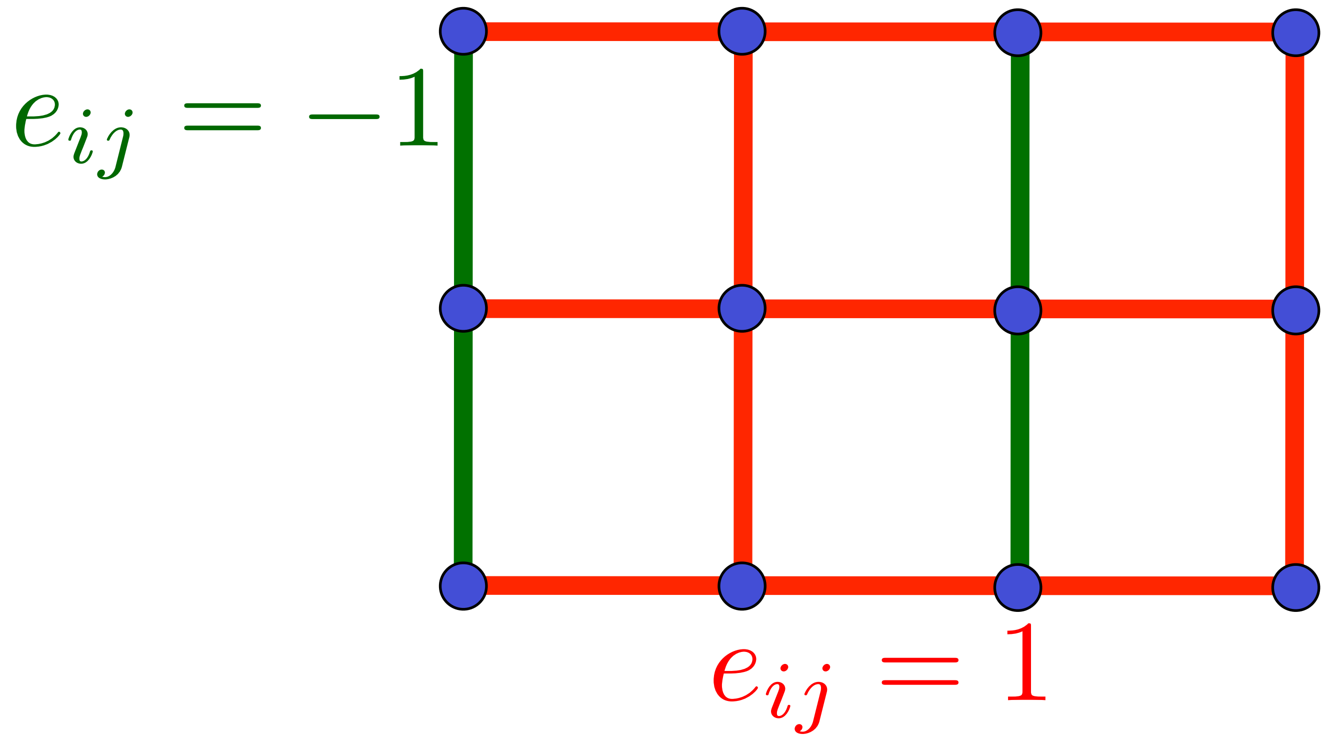
S=1/2 square lattice

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger fermions

$$\boldsymbol{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta}\,, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

π-flux Spin liquid



π-flux mean-field theory
with gapless spinons at 2 Dirac points.

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

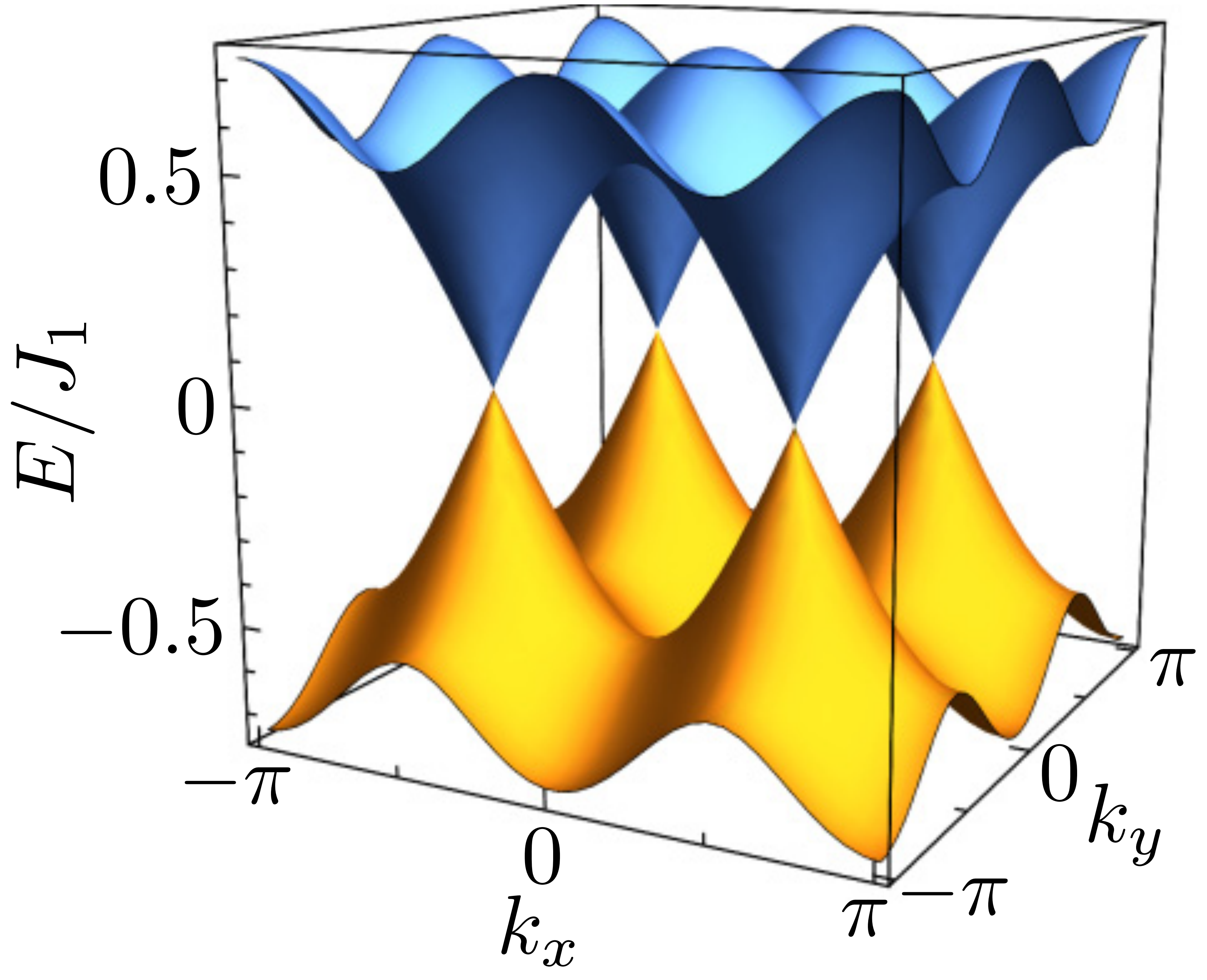
$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right) \quad , \quad \varepsilon_{\mathbf{k}} = 2J\sqrt{\sin^2(k_x) + \sin^2(k_y)}$$

SU(2) QCD with $N_f = 2$ massless fermions; $\mathcal{L} = i\overline{\Psi}_s \gamma_\mu D_\mu \Psi_s$.

S=1/2 square lattice

Characterization of quantum spin liquids and their spinon band structures via functional renormalization

We apply this approach to the antiferromagnetic J_1-J_2 Heisenberg model on the square lattice and to the antiferromagnetic nearest-neighbor Heisenberg model on the kagome lattice. For the J_1-J_2 model, we find that in the regime of maximal frustration a SU(2) π-flux state with Dirac spinons yields the largest mean-field amplitudes.



square $J_2/J_1 = 0.55$

M. Hering, J. Sonnenschein, Y. Iqbal and J. Reuther, PRB **99**, 100405 (2019)

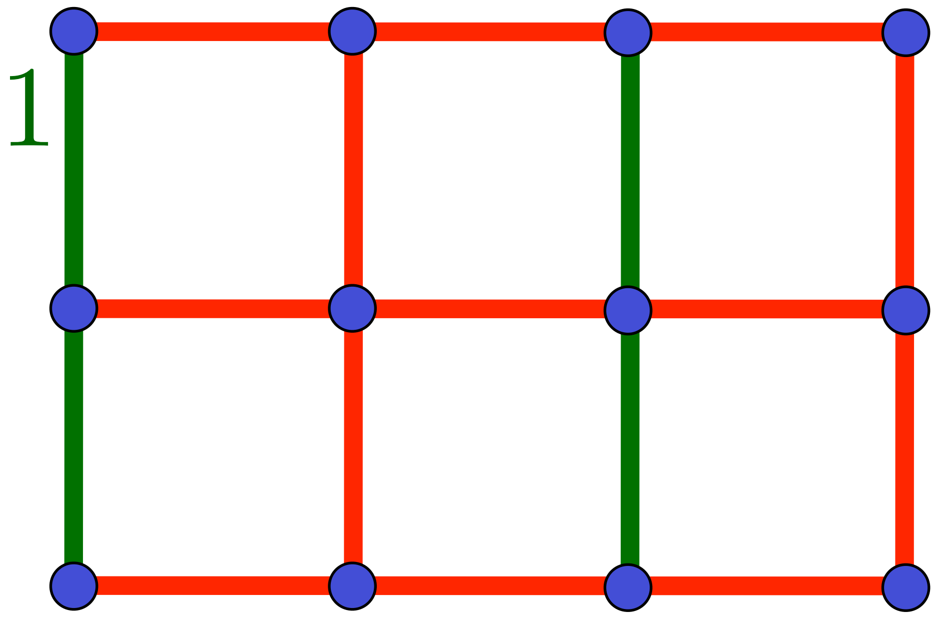
S=1/2 square lattice

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

π-flux Spin liquid

Schwinger fermions

$$\boldsymbol{S}_i = \frac{1}{2} f^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta}\,, \quad \sum_{\alpha=\uparrow,\downarrow} f^\dagger_{i\alpha} f_{i\alpha} = 1$$



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I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

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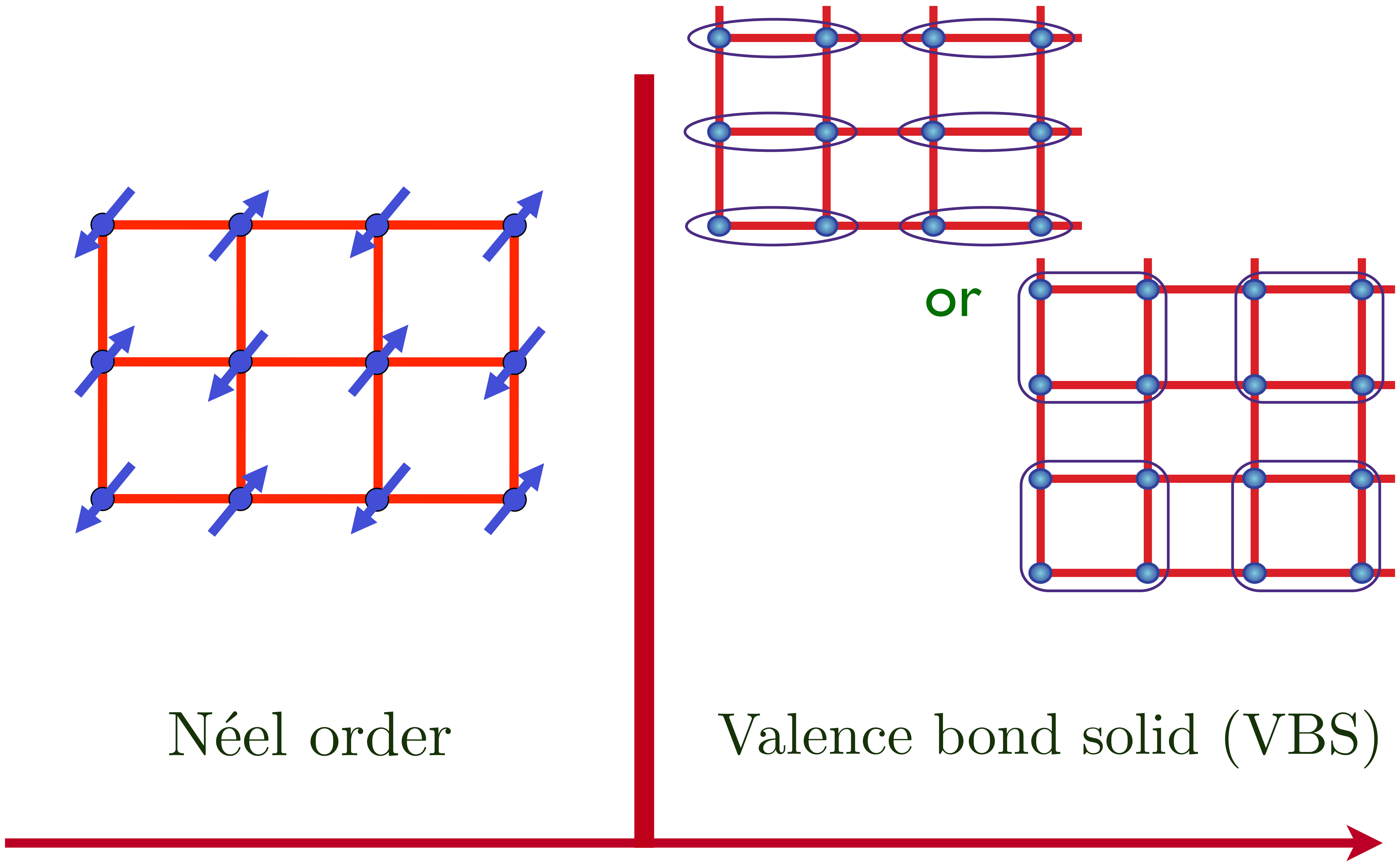

Schwinger fermions

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SU(2) QCD with $N_f = 2$ massless fermions; $\mathcal{L} = i\overline{\Psi}_s \gamma_\mu D_\mu \Psi_s$.
Confining instability to precisely the Néel and VBS orders of $\mathbb{CP}^1$ theory.

S=1/2 square lattice

Bosonic spinons:
$\mathbb{CP}^1$ U(1) gauge theory

Nearly-critical S=1/2 square lattice anti-ferromagnet

$\mathrm{SU}(2)_N$ gauge theory of $N_f = 2$ fundamental, massless, Dirac fermions.

Obtained from a saddle-point of fermionic spinons moving in π-flux.

SO(5) non-linear σ-model of Néel/VBS orders with $k = 1$ WZW term

A. Tanaka and X. Hu, *Phys. Rev. Lett.* **95**, 036402 (2005); T. Senthil and M.P.A. Fisher *Phys. Rev. B* **74**, 064405 (2006); Ying Ran and X.-G. Wen, cond-mat/0609620; C. Wang, A. Nahum, M. A. Metlitski, C. Xu, and T. Senthil, *Phys. Rev. X* **7**, 031051 (2017)

S=1/2 square lattice

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Many numerical works show that deconfined critical theory applies over a substantial length scale, but ultimately confines at the longest distances.

Zheng Zhou, Liangdong Hu, Wei Zhu, and Yin-Chen He, PRX **14**, 021044 (2024); S. M. Chester and N. Su, PRL **132**, 111601 (2024).
B.-B. Chen, X. Zhang, Y. Wang, K. Sun, and Z. Y. Meng, arXiv:2307.05307;
J. Takahashi, H. Shao, B. Zhao, W. Guo, and A. W. Sandvik, arXiv:2405.06607.

S=1/2 triangular and kagome lattices

- Hasting (2000), Wen (2002): Gapless spin liquid described by $N_f = 4$ QED: possible conformal U(1) gauge theory.

S=1/2 triangular and kagome lattices

- Hasting (2000), Wen (2002): Gapless spin liquid described by $N_f = 4$ QED: possible conformal U(1) gauge theory.

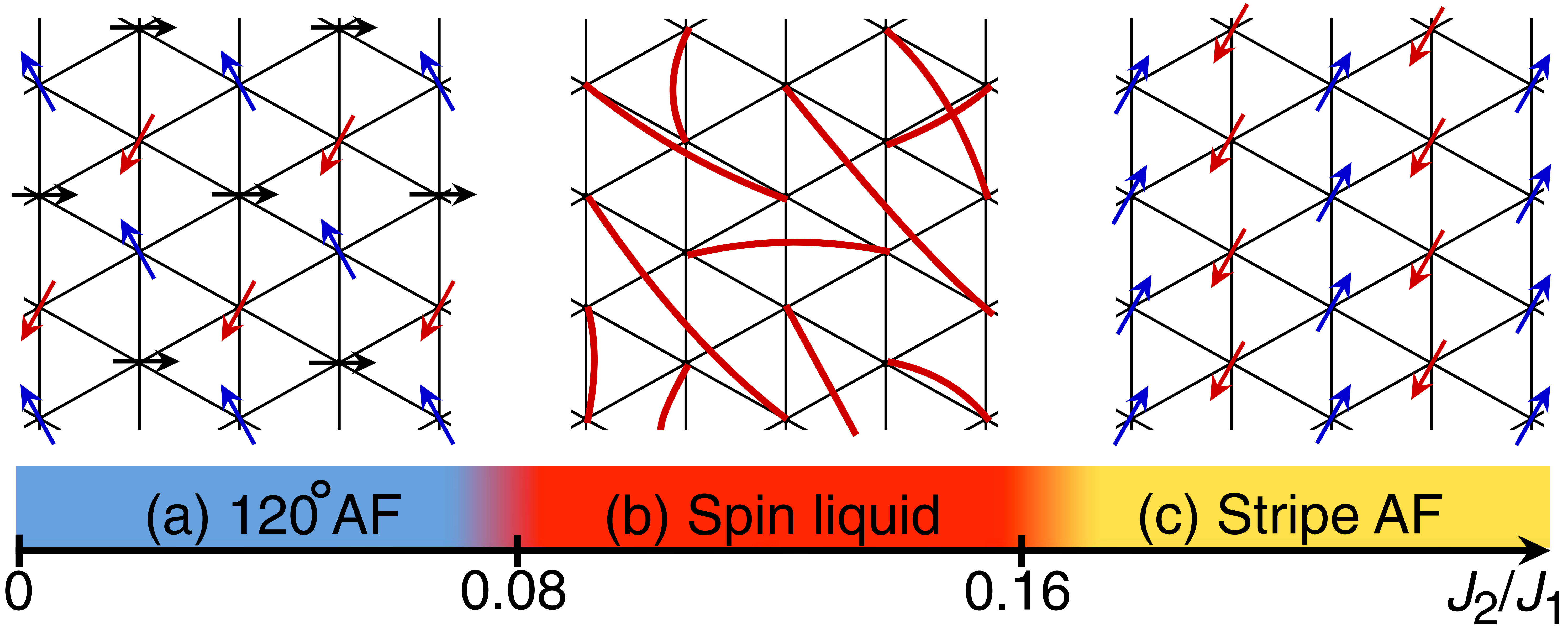
- Xue-Yang Song, Yin-Chen He, Vishwanath, Chong Wang (2020): No trivial $q = 1$ monopole, unlike the U(1) 'staggered-flux' state for the square lattice. So monopole proliferation instability is unlikely.

S=1/2 triangular and kagome lattices

Spin liquid nature in the Heisenberg J_1-J_2 triangular antiferromagnet

Yasir Iqbal,[1,*] Wen-Jun Hu,[2,†] Ronny Thomale,[1,‡] Didier Poilblanc,[3,§] and Federico Becca[4,‖]

PHYSICAL REVIEW B **93**, 144411 (2016)

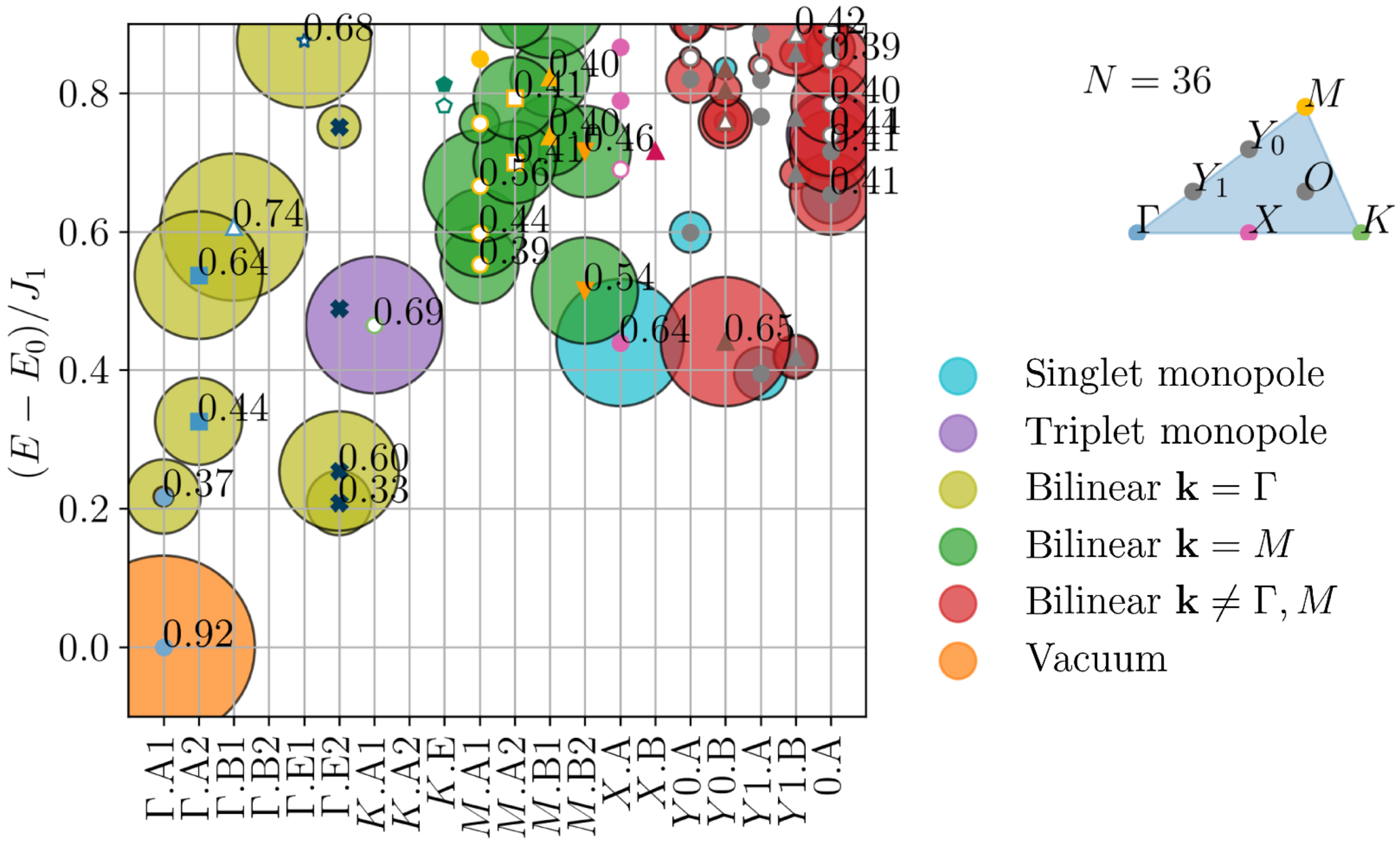


S=1/2 triangular and kagome lattices

PHYSICAL REVIEW X **14,** 021010 (2024)

Quantum Electrodynamics in 2+1 Dimensions as the Organizing Principle of a Triangular Lattice Antiferromagnet

Alexander Wietek,[1,2,*] Sylvain Capponi,[3]
and Andreas M. Läuchli[4,5]



S=1/2 triangular and kagome lattices

PHYSICAL REVIEW X **14,** 021010 (2024)

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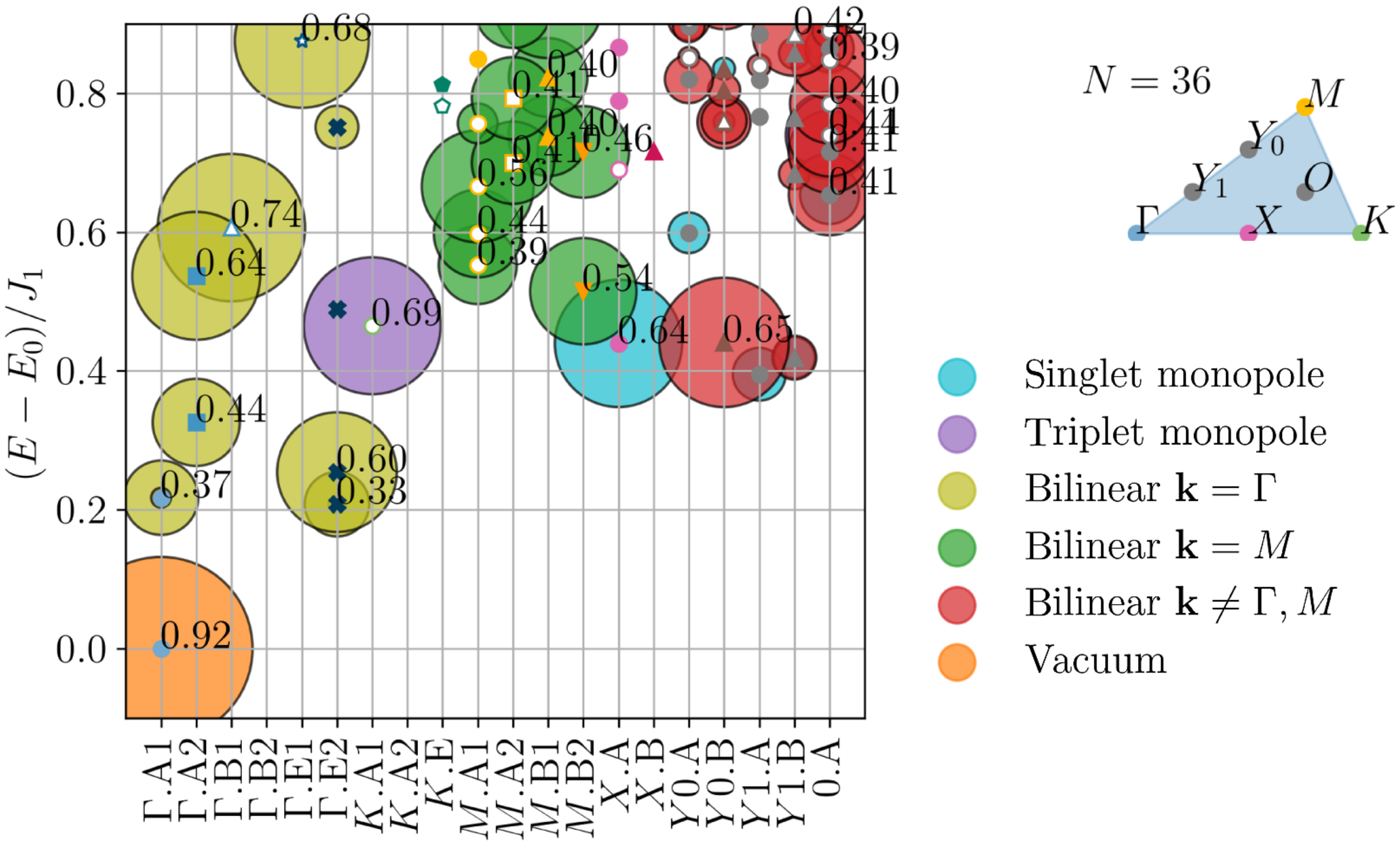
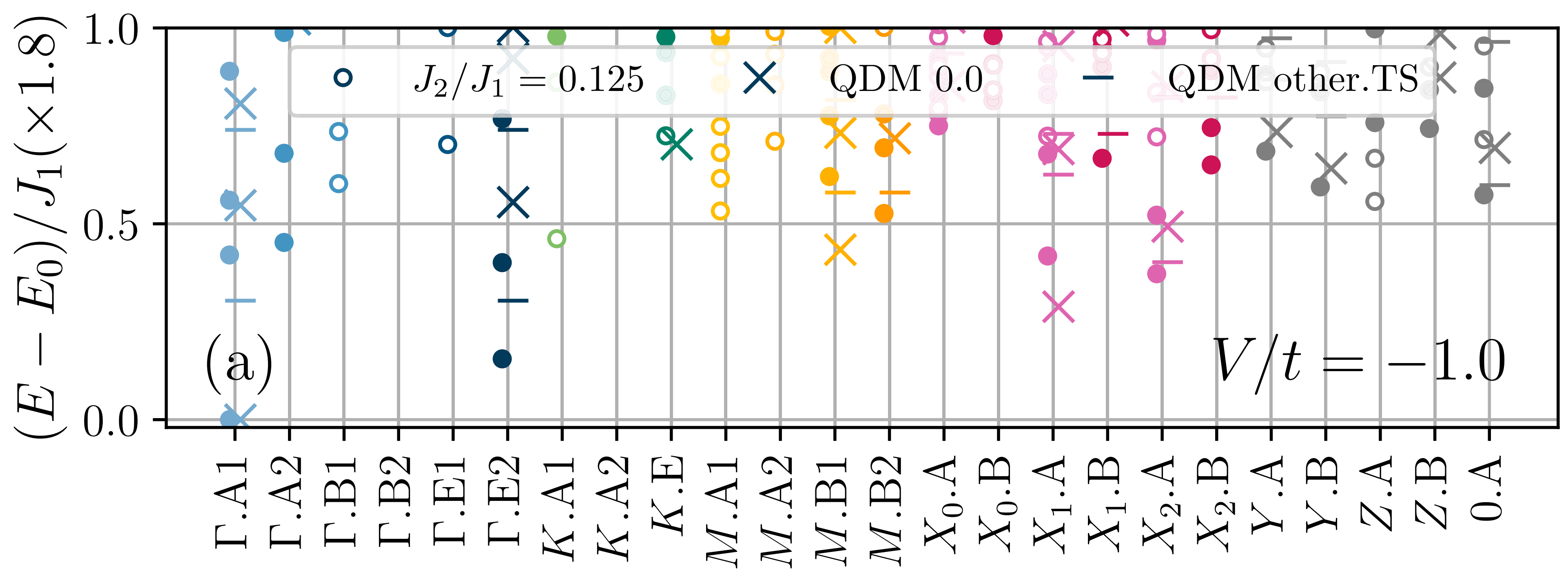
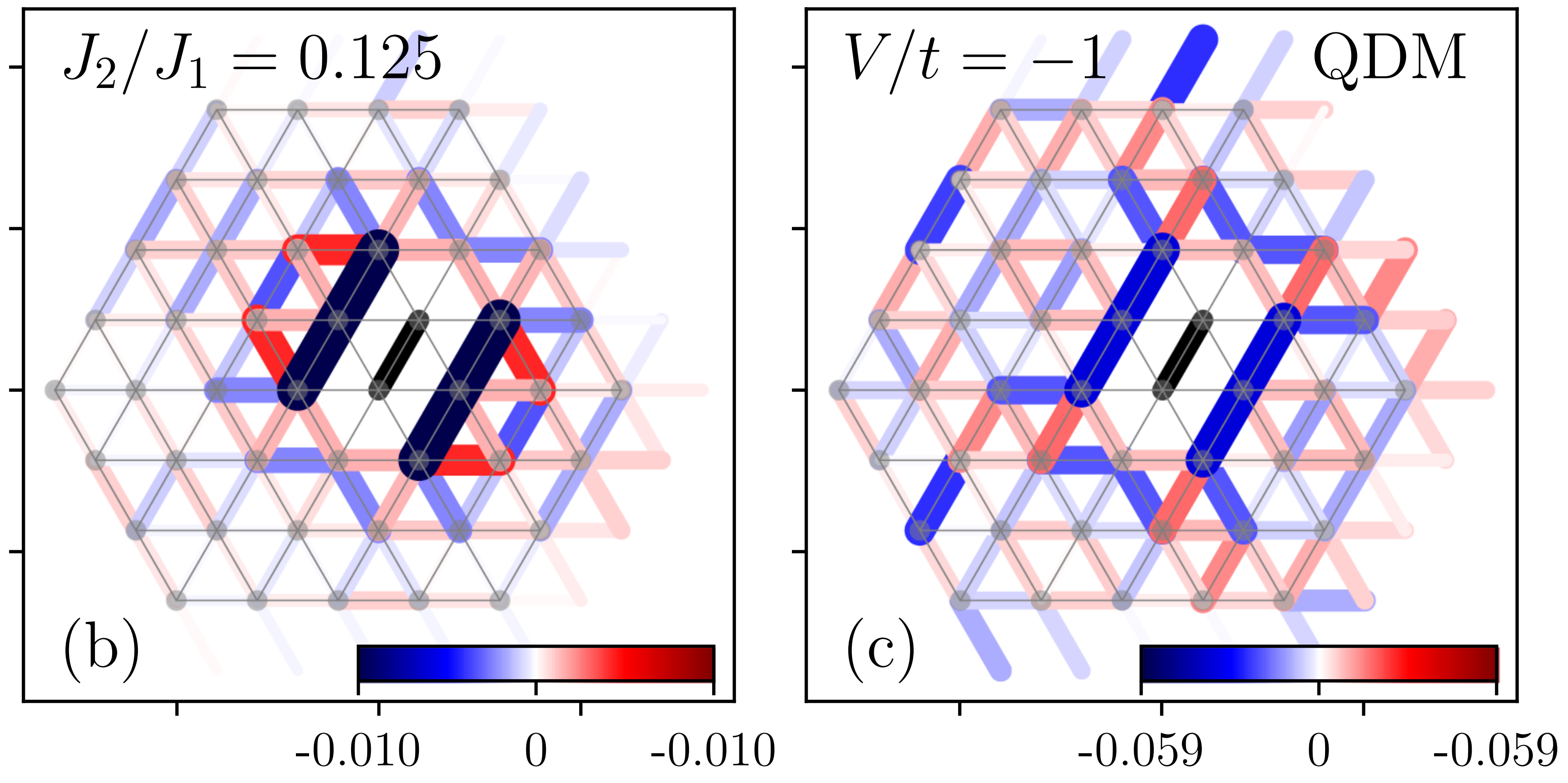






The low-lying states match those of the quantum dimer model, indicating a nearly-critical charge-2 Higgs field in $N_f = 4$ QED.

S=1/2 triangular and kagome lattices

Sign-problem-free QMC of odd $\mathbb{Z}_2$ gauge theory $s_{j,j+\hat{\mu}}$ coupled to bosonic spinons $z_{j\alpha}$.

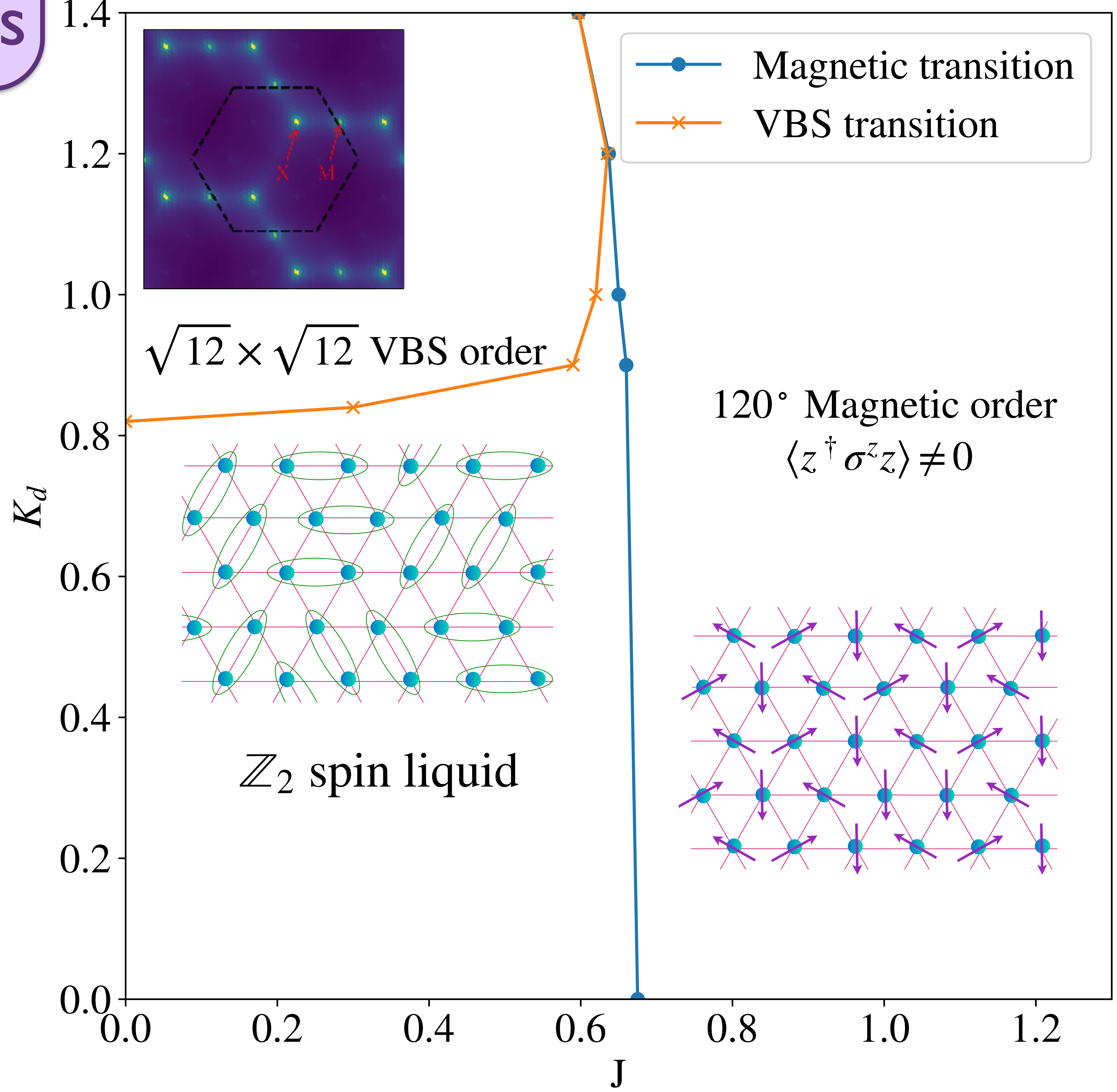
$$\mathcal{Z} = \sum_{s_{j,j+\hat{\mu}}=\pm 1} \prod_j \int dz_{j\alpha} \delta \left(\sum_\alpha \delta z_{j\alpha}^2 - 1 \right)$$

Berry phase

$$\times \left[\prod_j s_{j,j+\tau} \right] \exp\left(-H[z_\alpha, s]\right)$$

$$H[z_\alpha, s] = -\frac{J}{2} \sum_{\langle j,\mu \rangle} s_{j,j+\hat{\mu}} \left(z^*_{j,\alpha} z_{j+\hat{\mu},\alpha} + \text{ c.c} \right)$$

$$- K \sum_{\triangle\square} \prod_{\triangle\square} s_{j,j+\hat{\mu}}$$



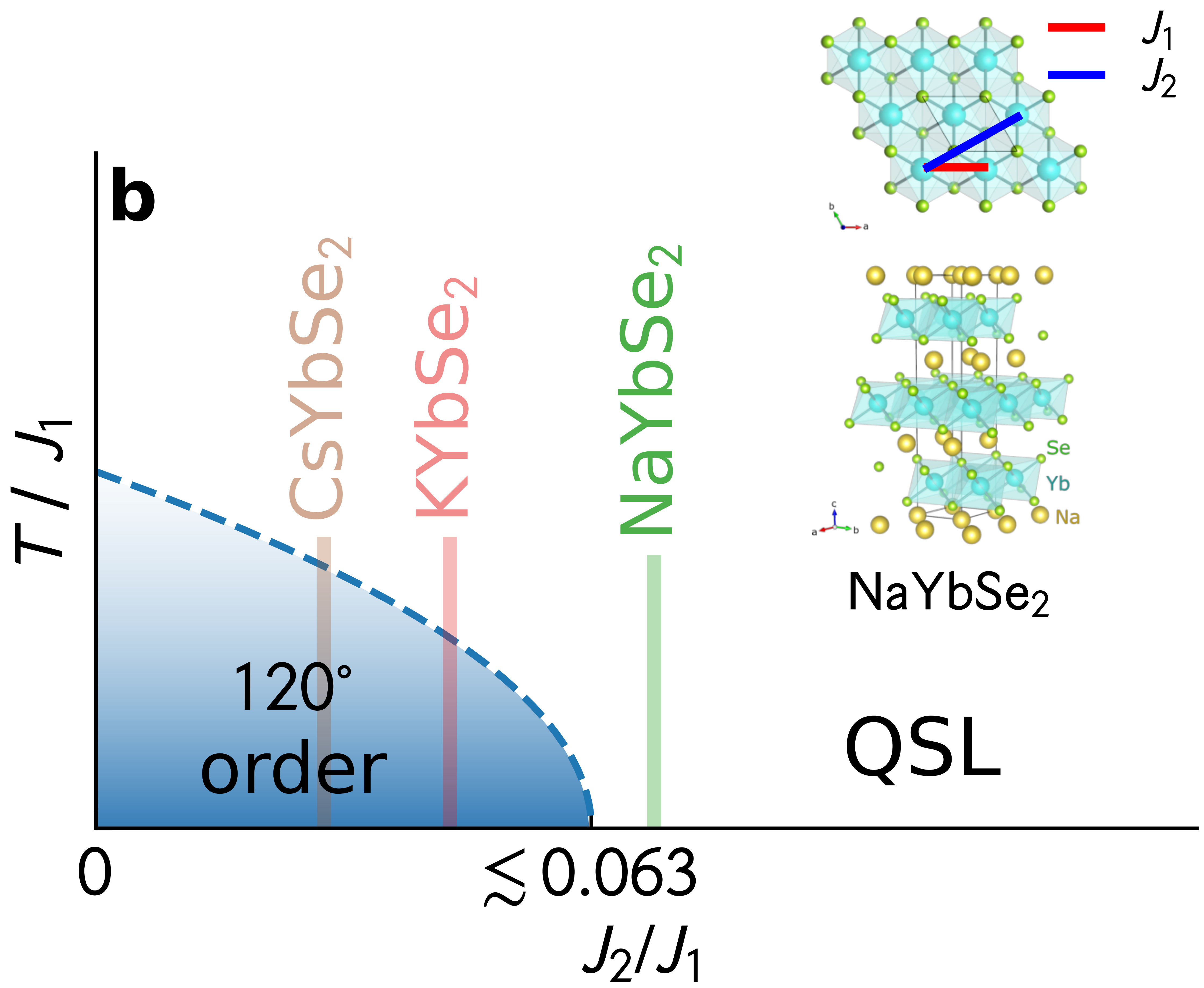
H. Shackleton and S. Sachdev, arXiv:2311.01572.

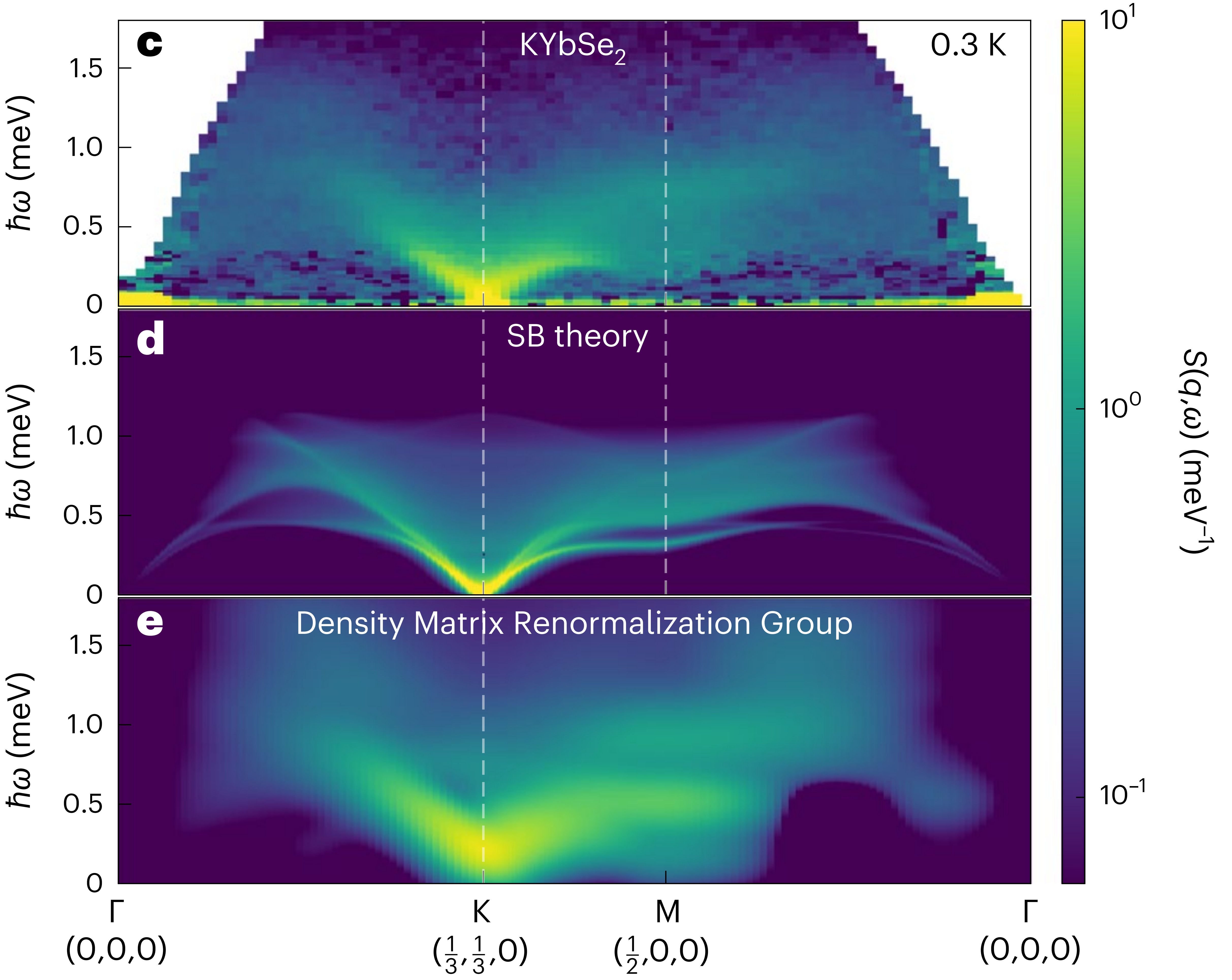
Experiments on spin liquids

Proximate spin liquid and fractionalization in the triangular antiferromagnet $KYbSe_2$

A. O. Scheie[1], E. A. Ghioldi[2,3], J. Xing[4], J. A. M. Paddison[4], N. E. Sherman[5,6], M. Dupont[5,6], L. D. Sanjeewa[7,8], Sangyun Lee[9], A. J. Woods[9], D. Abernathy[1], D. M. Pajerowski[1], T. J. Williams[1], Shang-Shun Zhang[10], L. O. Manuel[3], A. E. Trumper[3], C. D. Pemmaraju[11], A. S. Sefat[4], D. S. Parker[4], T. P. Devereaux[11,12], R. Movshovich[9], J. E. Moore[5,6], C. D. Batista[2,13] & D. A. Tennant[2,13]

Nature Physics **20**, 74 (2024)

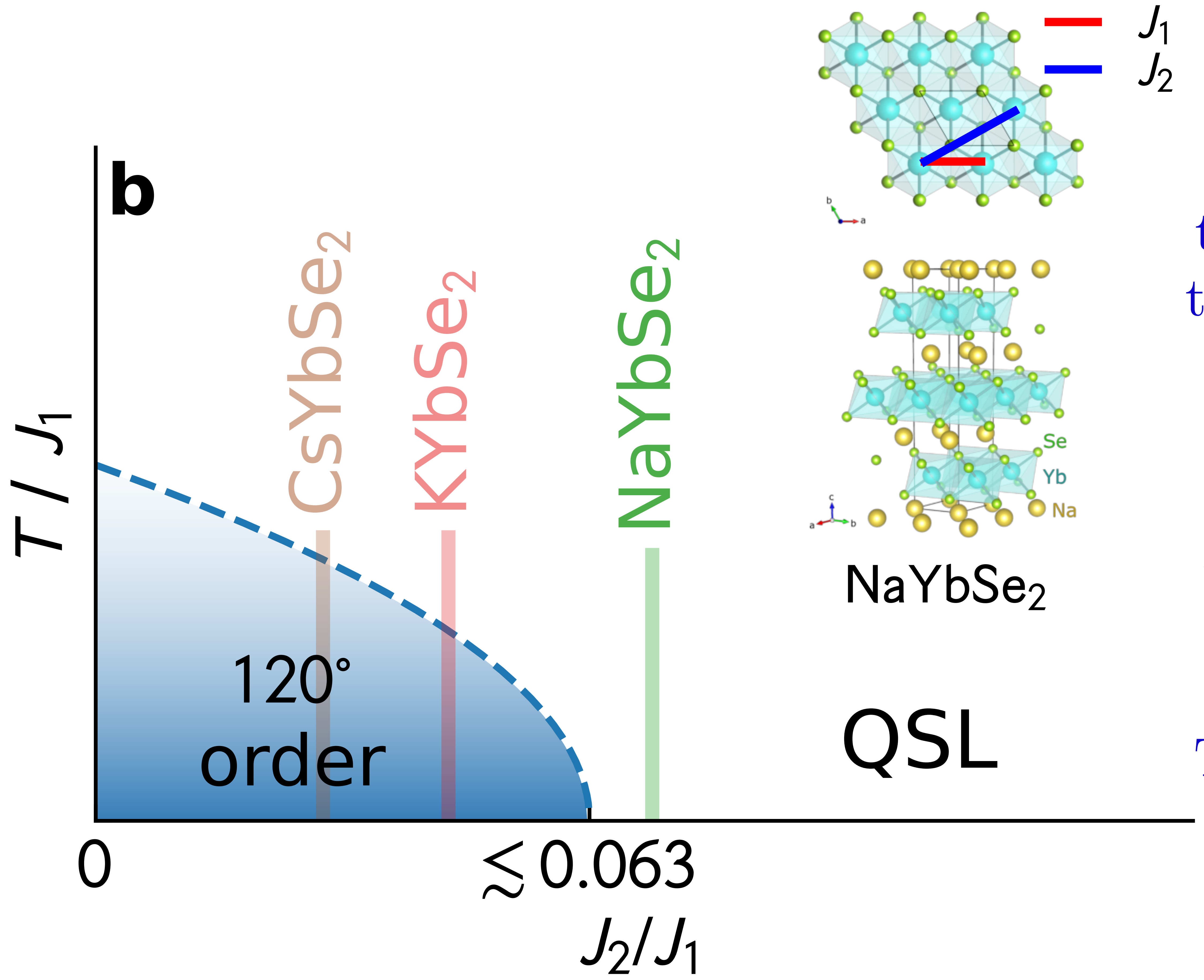




Spectrum and low-energy gap in triangular quantum spin liquid $NaYbSe_2$

A. O. Scheie,[1,*] Minseong Lee,[2,†] Kevin Wang,[3] P. Laurell,[4] E. S. Choi,[5] D. Pajerowski,[6] Qingming Zhang,[7] Jie Ma,[8] H. D. Zhou,[4] Sangyun Lee,[2] S. M. Thomas,[1] M. O. Ajeesh,[1] P. F. S. Rosa,[1] Ao Chen,[9] Vivien S. Zapf,[2] M. Heyl,[9] C. D. Batista,[4,6] E. Dagotto,[4,10] J. E. Moore,[3,‡] and D. Alan Tennant[4,11,§]

arXiv:2406.17773

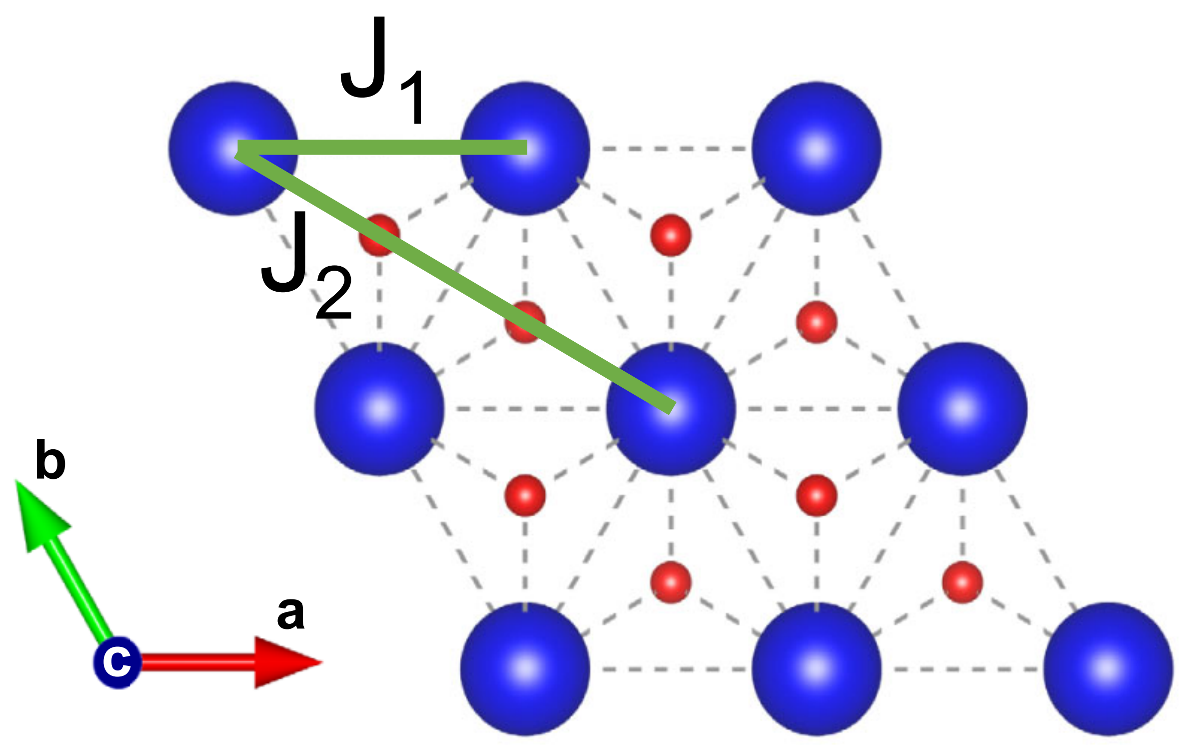


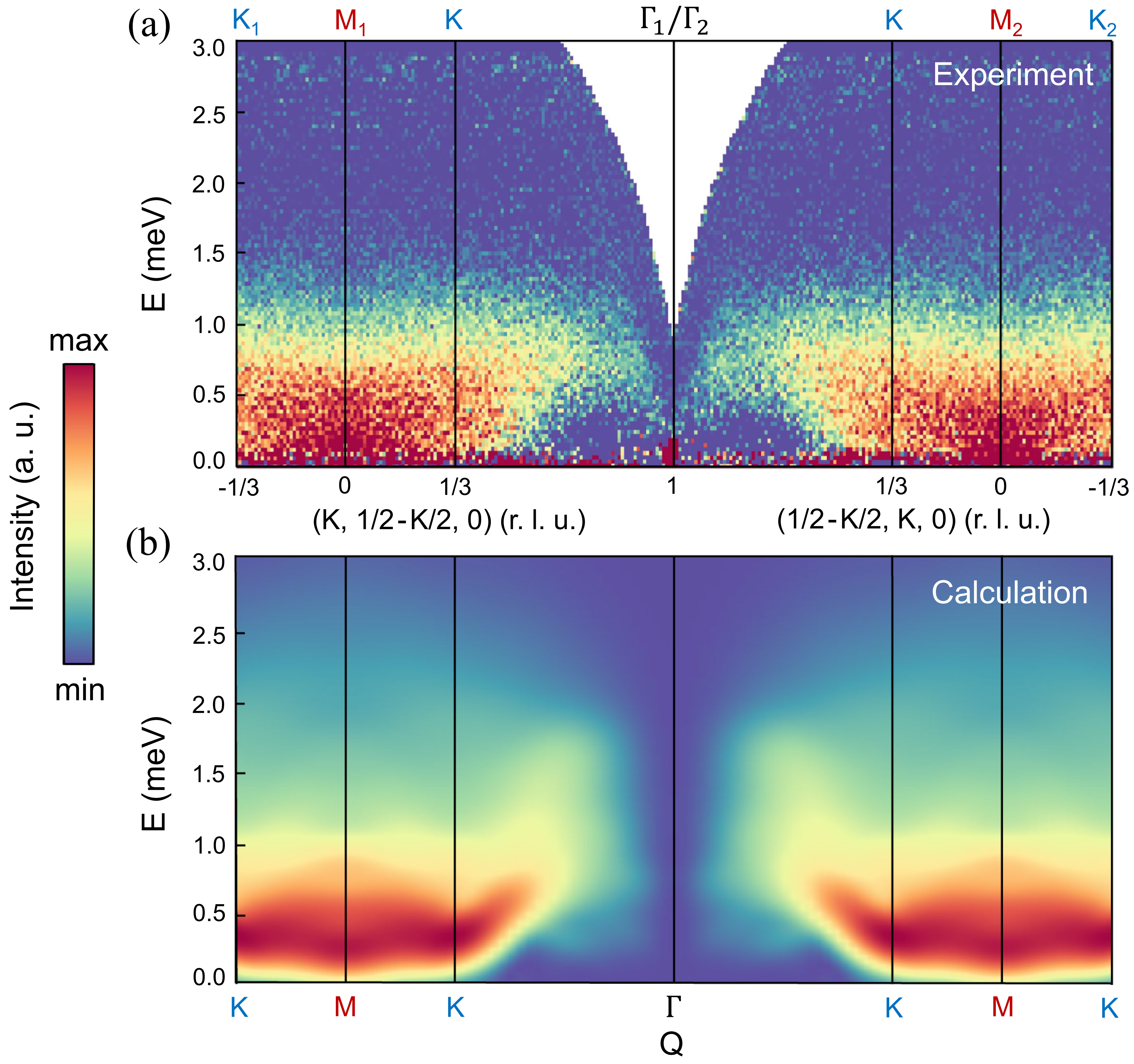
We observe a continuum of (neutron) scattering, which is reproduced by matrix product simulations, and no phase transition is detected in any bulk measurements. Comparison to heat capacity simulations suggest the material is within the Heisenberg spin liquid phase. AC Susceptibility shows a signifi- cant 23 mK downturn, indicating a gap in the magnetic spectrum. The combination of a gap with no detectable magnetic order, comparison to theoretical models, and comparison to other $AYbSe_2$ compounds all strongly indicate $NaYbSe_2$ is within the quantum spin liquid phase. The gap also allows us to rule out a gapless Dirac spin liquid, with a gapped $\mathbb{Z}_2$ liquid the most natural explanation.

Evidence of Dirac Quantum Spin Liquid in $YbZn_2GaO_5$

Rabindranath Bag[1], Sijie Xu[1], Nicholas E. Sherman[2,3], Lalit Yadav[1], Alexander I. Kolesnikov[4], Andrey A. Podlesnyak[4], Eun Sang Choi[5], Ivan da Silva[6], Joel E. Moore[2,3] and Sara Haravifard[1,7,*]

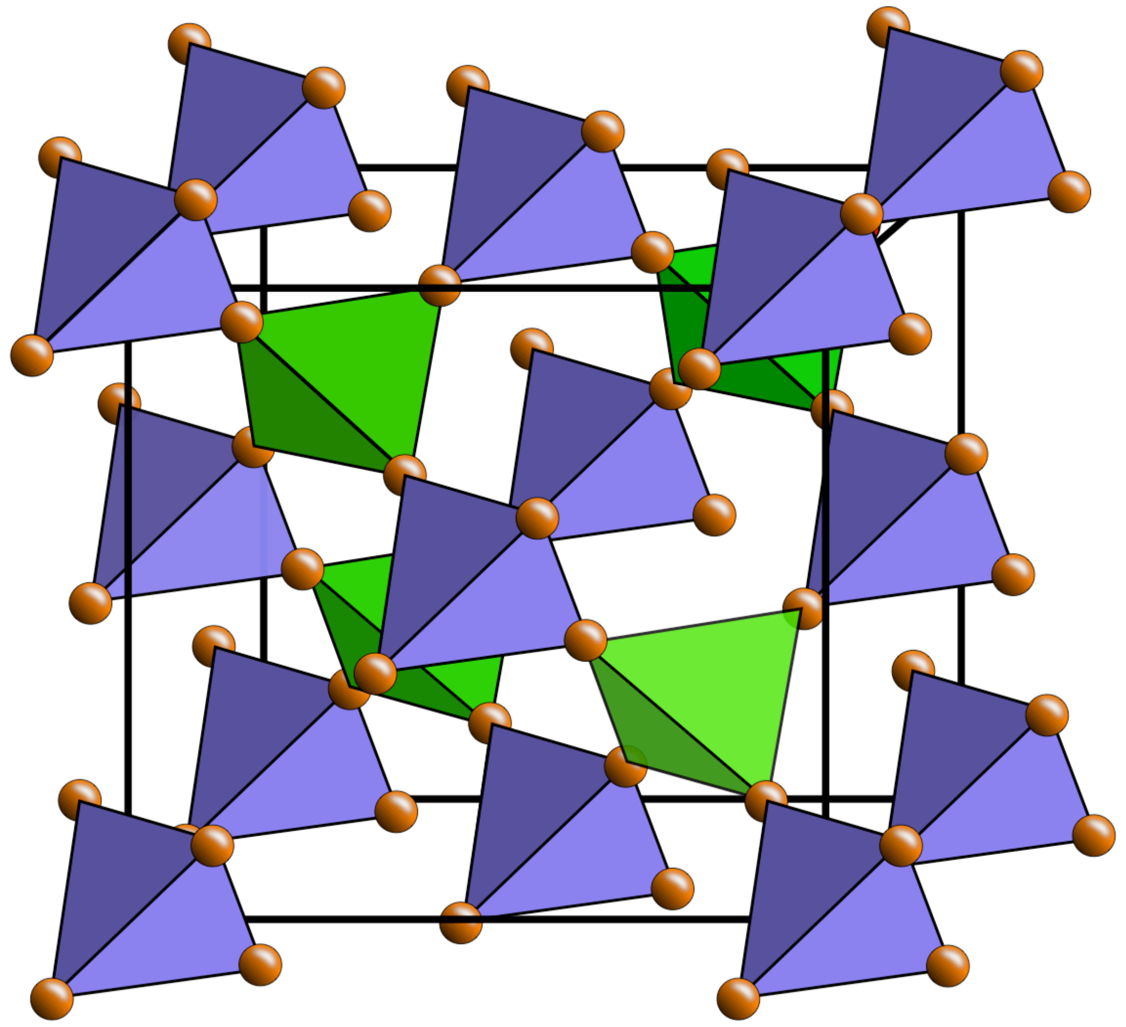
PHYSICAL REVIEW LETTERS **133,** 266703 (2024)





Quantum Spin Ice

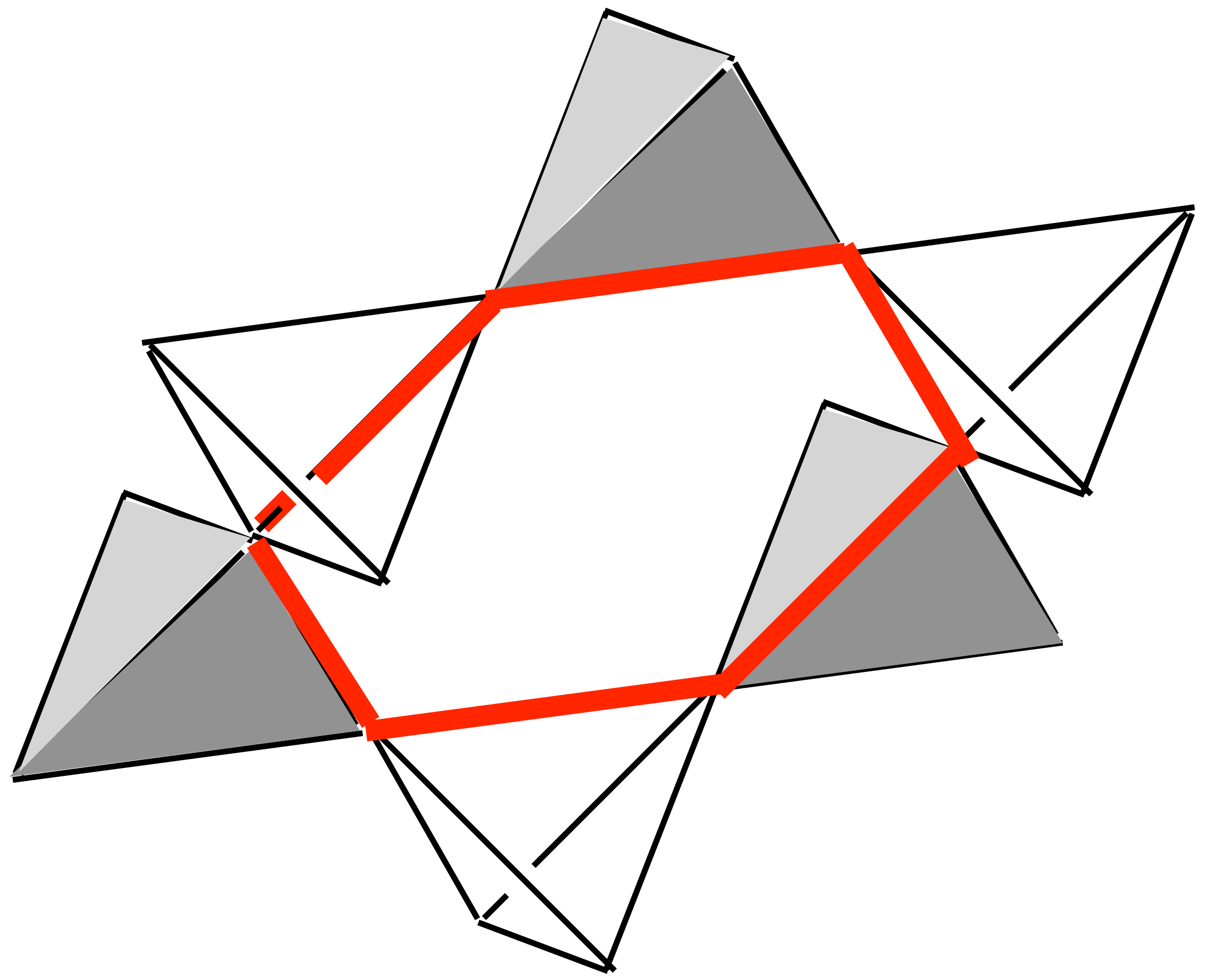
$$\mathcal{H}_I = J_z \sum_{\langle ij \rangle} S_i^z S_j^z = \frac{J_z}{2} \sum_{\triangleleft} (S^z_{\triangleleft})^2 \quad + \text{constant}$$



$$S^z_{\triangleleft} = \sum_{i \in \triangleleft} S_i^z = 0 \quad \text{2-in/2-out}$$

$$\mathcal{H}' = -\frac{J_\perp}{2} \sum_{\langle ij \rangle} (S_i^+ S_j^- + h.c.)$$

Hermele, Balents, Fisher ’03



$J_z \gg J_\perp$ degenerate perturbation theory

$$\mathcal{H}_{eff} = -J_{ring} \sum_{\hexagon} (S_1^+ S_2^- S_3^+ S_4^- S_5^+ S_6^- + h.c.)$$

$$J_{ring} = 12 J_\perp^3 / J_z^2$$

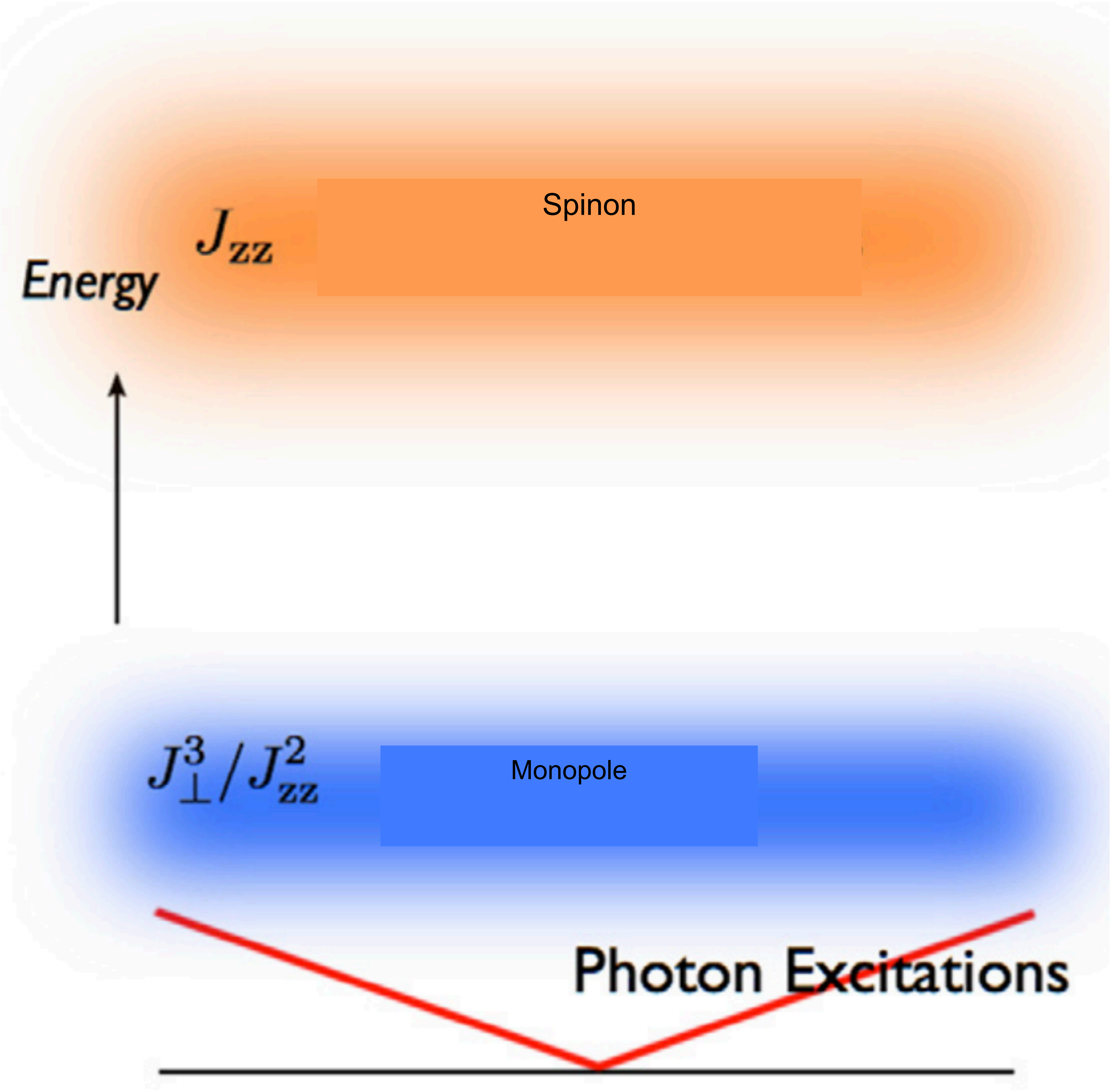
Banerjee, Isakov, Damle, YBK ’08

Benton, Sikora, Shannon ’12

QMC

Excitations in the deconfined phase: U(1) quantum spin liquid (Quantum Spin Ice)

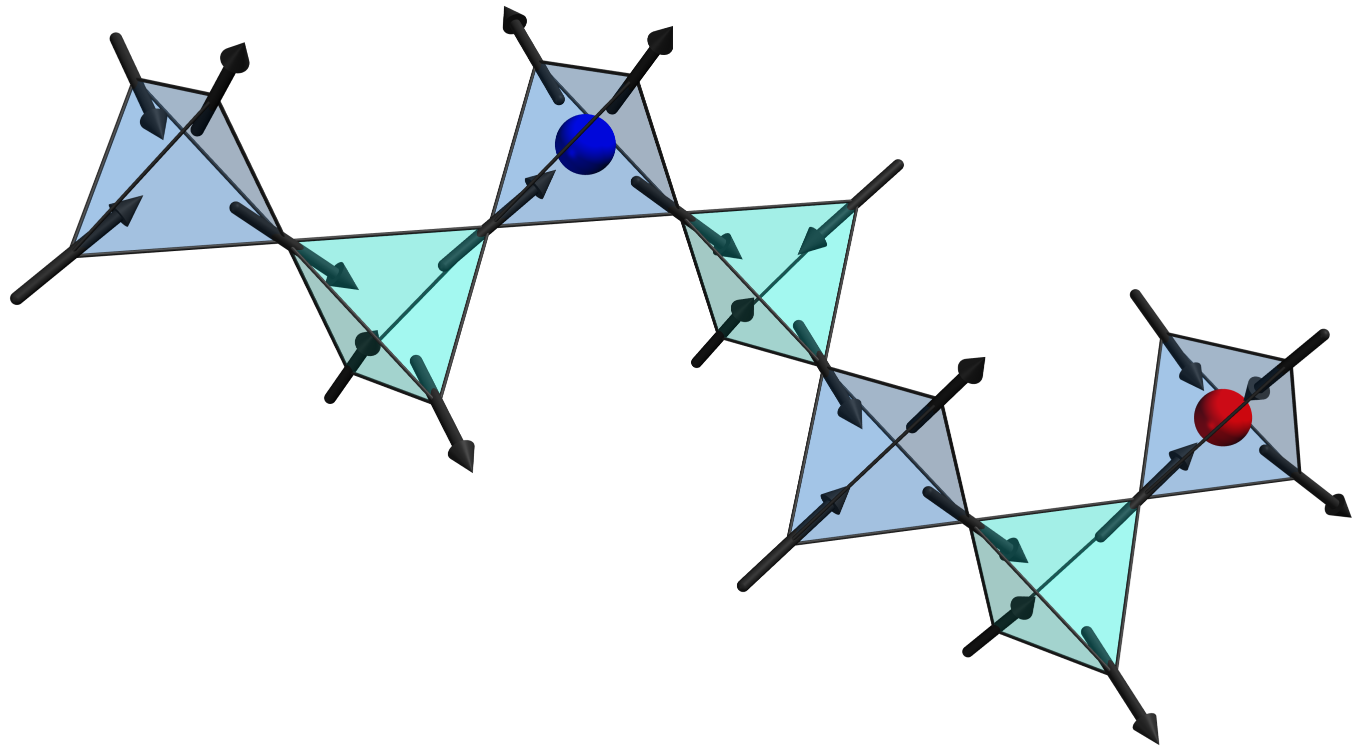
$$\mathcal{H} = \frac{U}{2}\sum_{\langle \mathbf{rr}' \rangle}\left(E^2_{\mathbf{rr}'} - \frac{1}{4}\right) + \frac{K}{2}\sum_{\hexagon}[(\nabla \times A)_{\hexagon}]^2$$



Gingras & McClarty (2014)

electric monopoles (spinons)

$$2\Delta_{\text{spinon}} \sim J_z$$



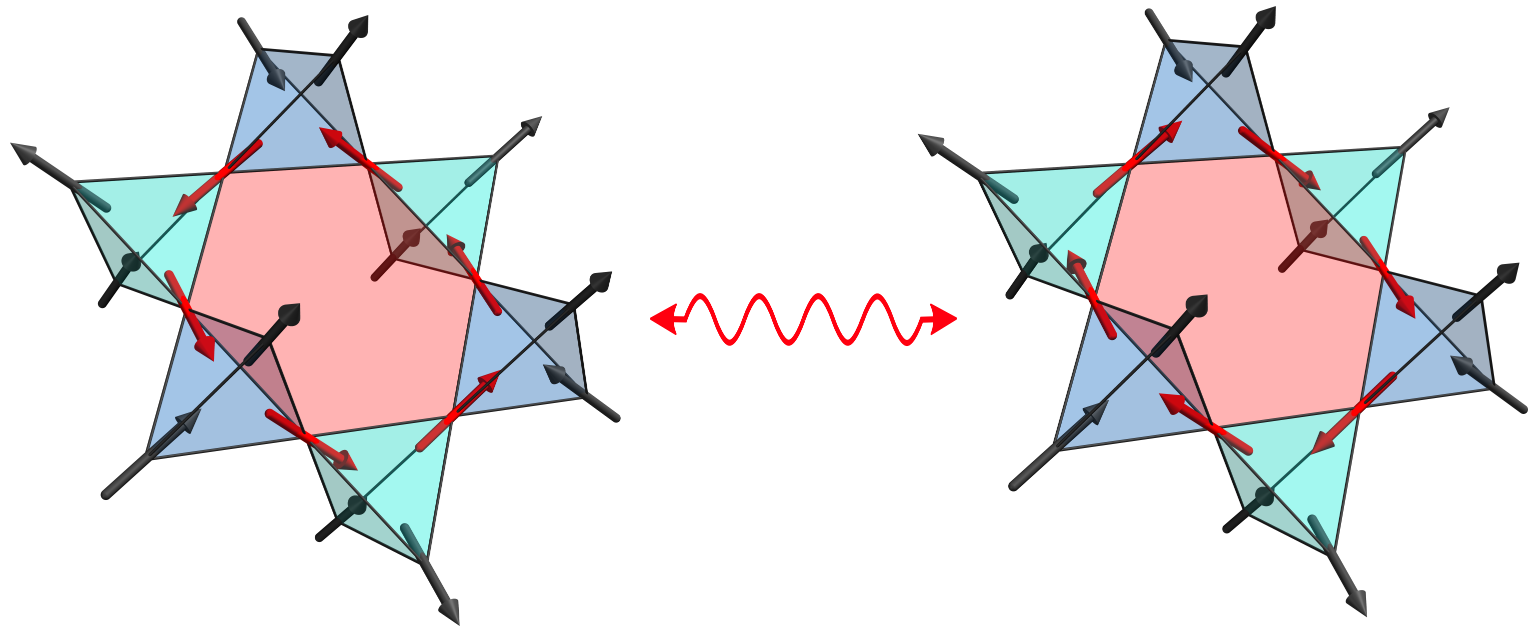
magnetic monopoles (visons)

$$\Delta_{\text{mon}} \sim J^3_\pm / J^2_z$$

emergent photons

$$\omega(\mathbf{k}) \approx c|\mathbf{k}| \qquad c \propto \sqrt{UK}a_0/\hbar$$

$$C(T) \propto \frac{1}{c^3}T^3$$

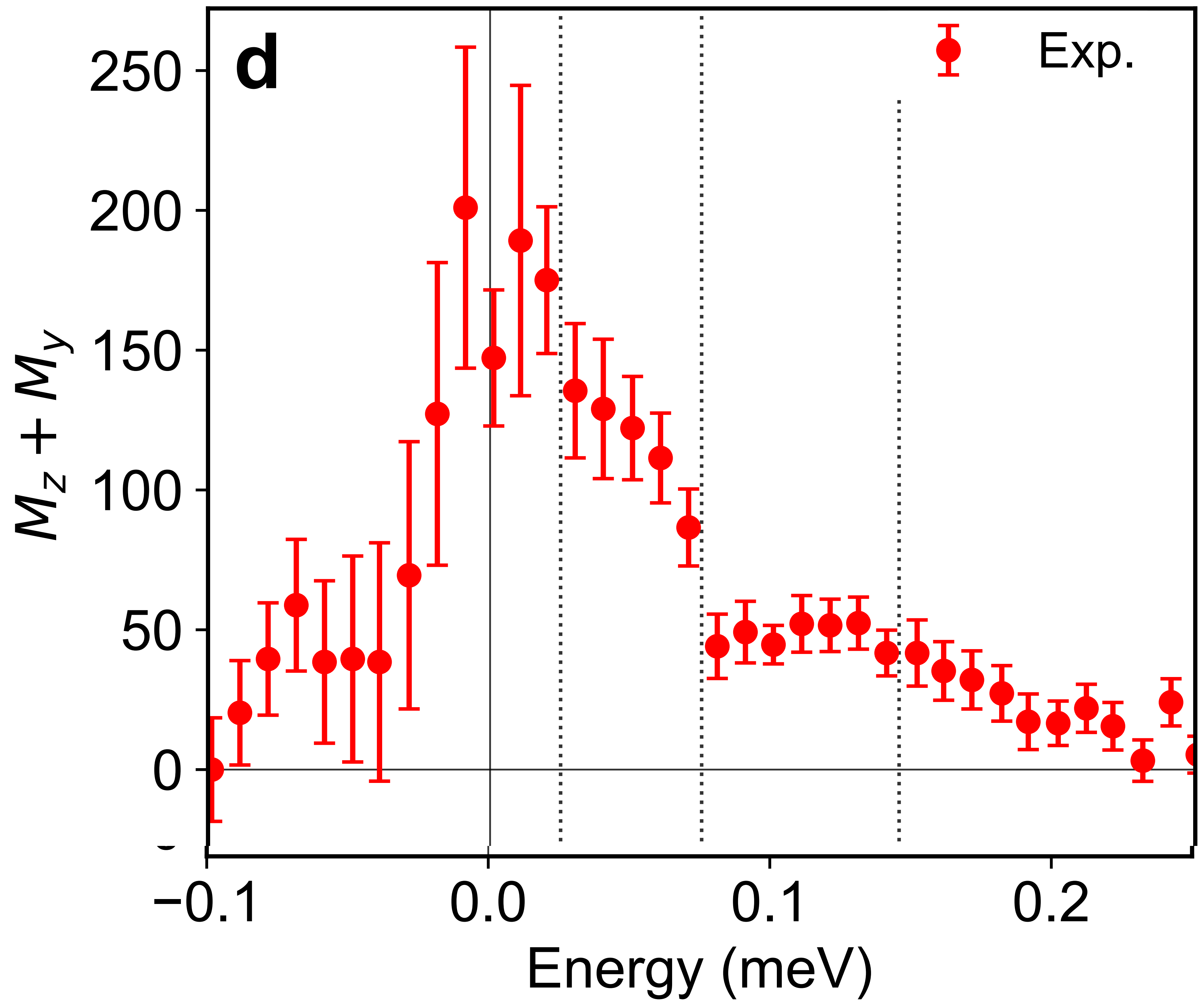


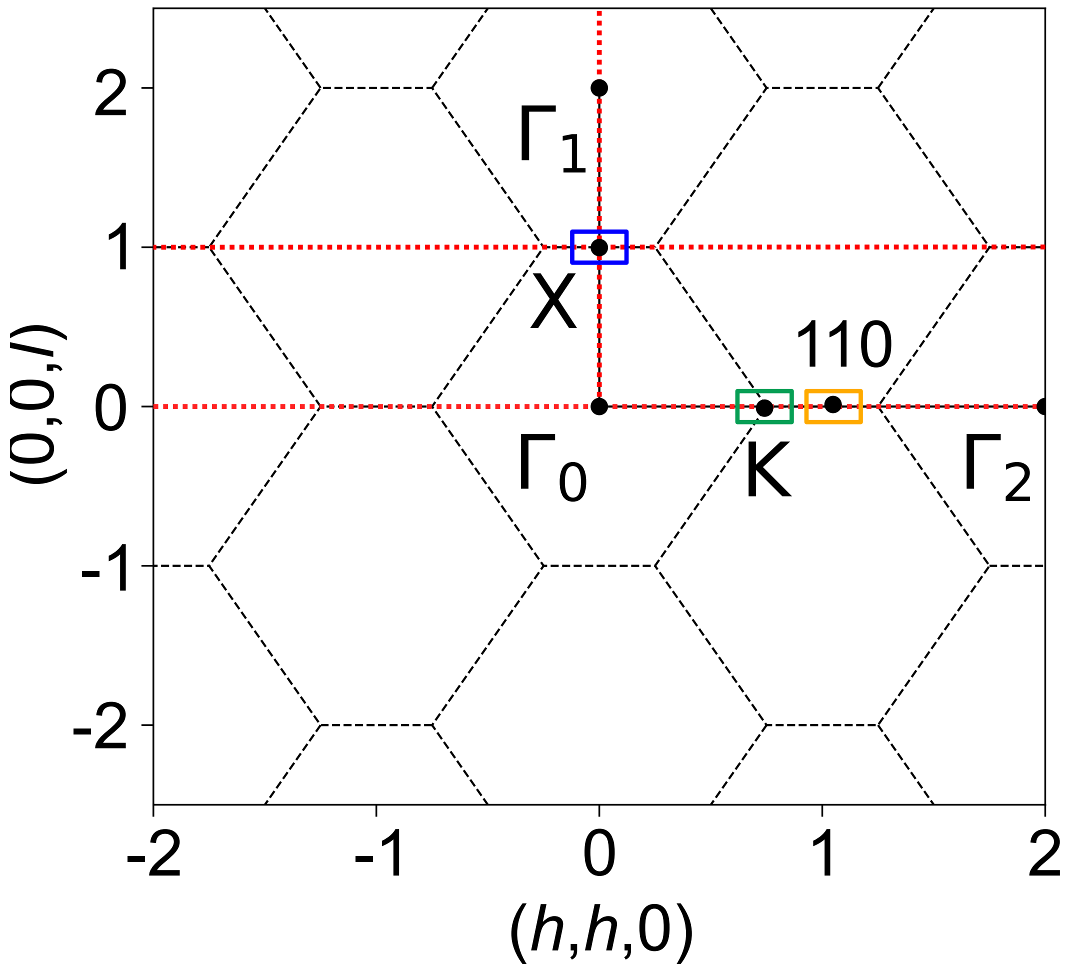
Polarized neutron scattering   Inelastic

50mK

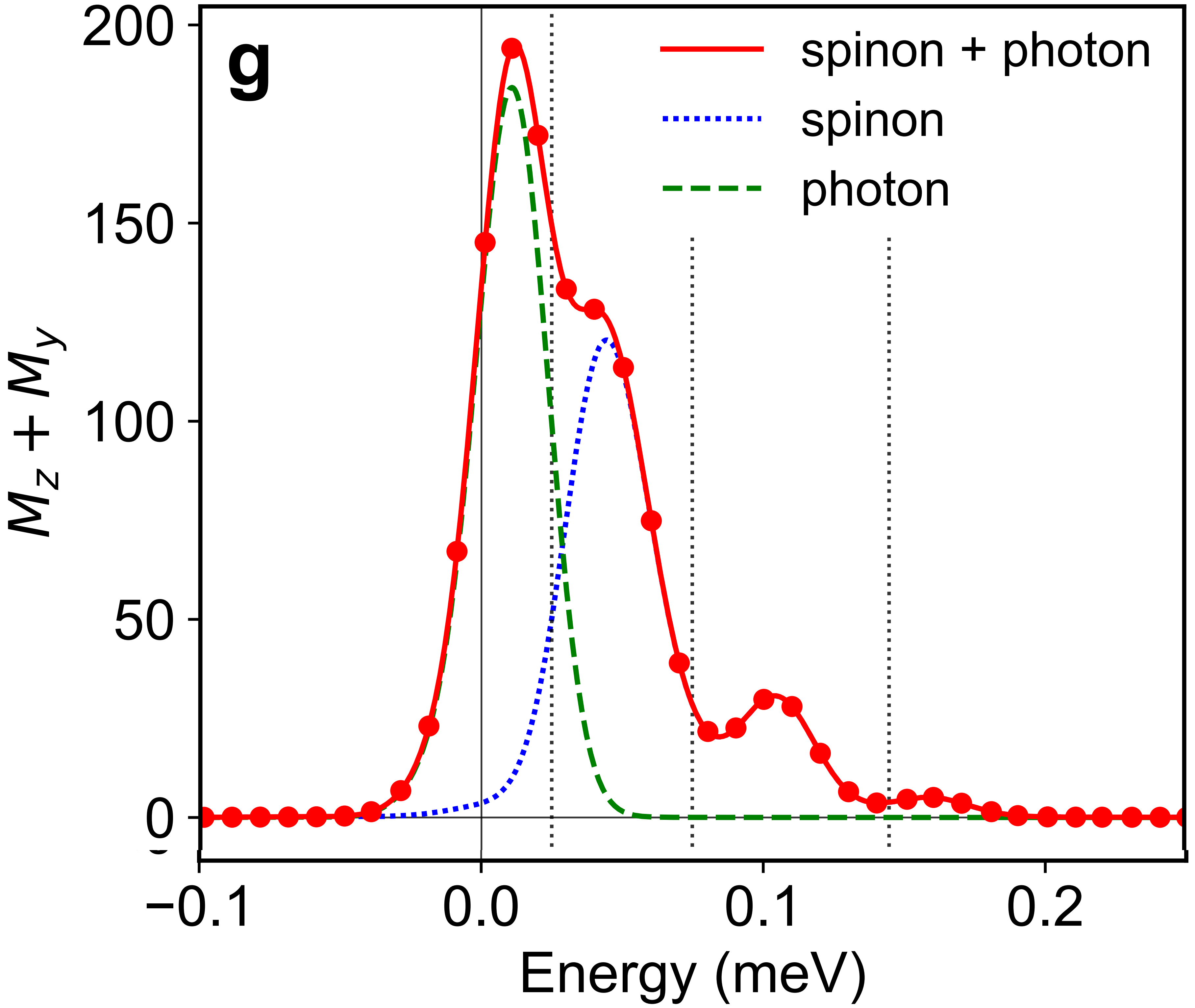
$$\sigma \sim \cos^2\theta \ \langle S^z S^z \rangle + \sin^2\theta \ \langle S^x S^x \rangle$$

X point, Q = (0, 0, 1)





theory fit



$Ce_2Zr_2O_7$

Bin Gao, Félix Desrochers, David W. Tam, Paul Steffens, Arno Hiess, Yixi Su, Sang-Wook Cheong, Yong Baek Kim, and Pengcheng Dai, arXiv:2404.04207

Dipolar Ising $J_x > J_y$

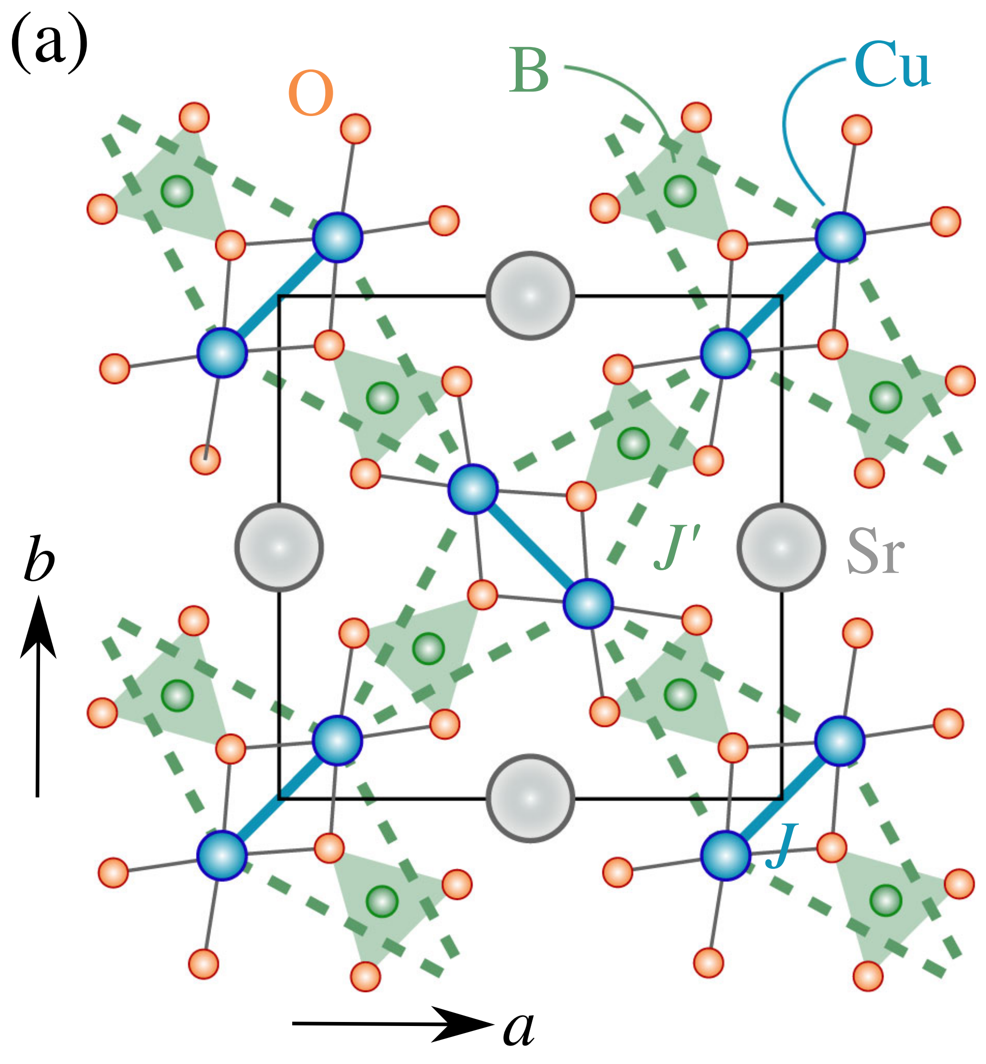
$S^x \sim E$ $S^z \sim \Phi^\dagger \Phi$

$\theta \neq 0$

Proximate deconfined quantum critical point in $SrCu_2(BO_3)_2$

Yi Cui[1]†, Lu Liu[2,3]†, Huihang Lin[1]†, Kai-Hsin Wu[4], Wenshan Hong[2], Xuefei Liu[1], Cong Li[1], Ze Hu[1], Ning Xi[1], Shiliang Li[2,5,6], Rong Yu[1,7]*, Anders W. Sandvik[4,2]*, Weiqiang Yu[1,7]*

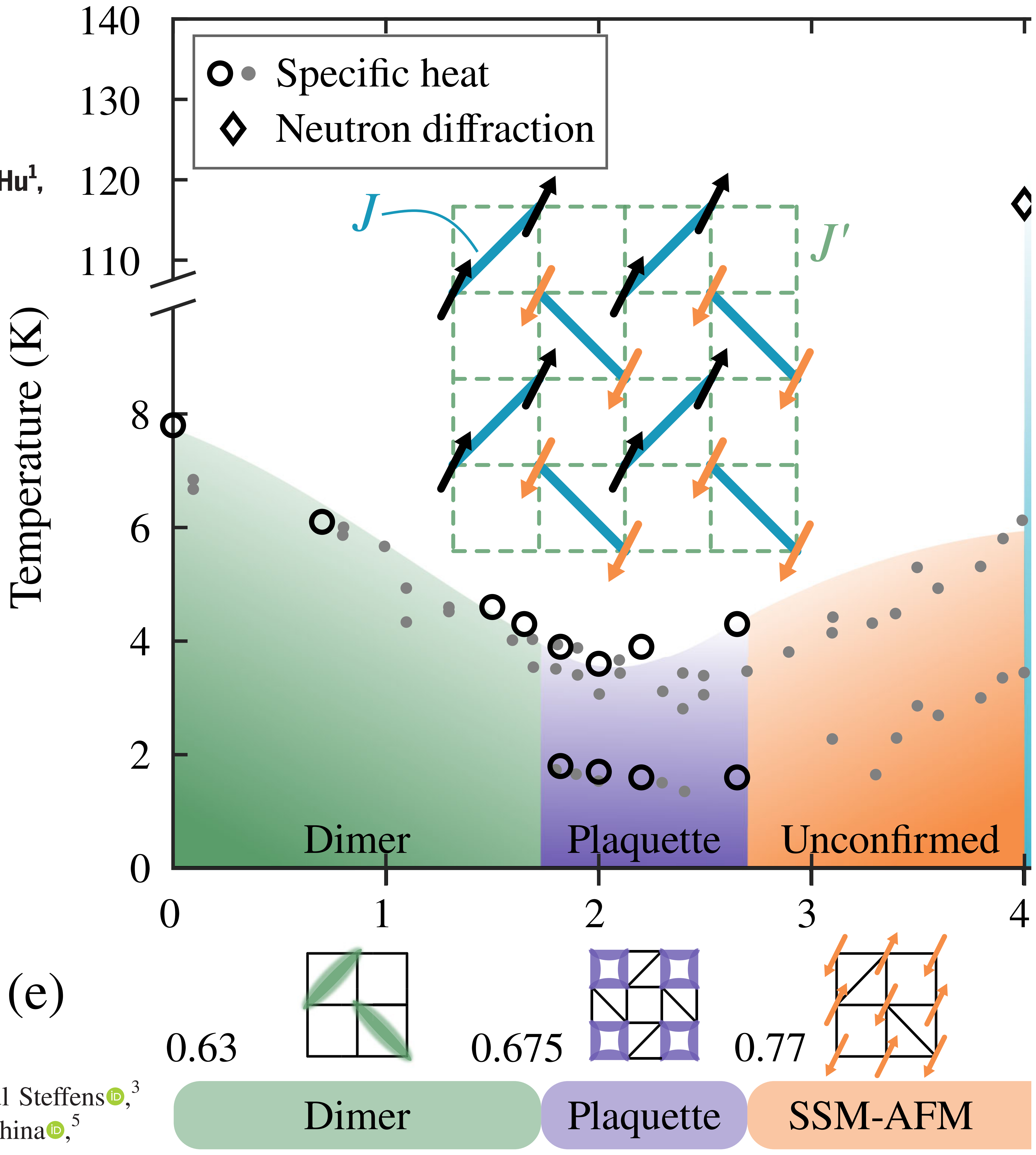
Science **380**, 1179–1184 (2023)

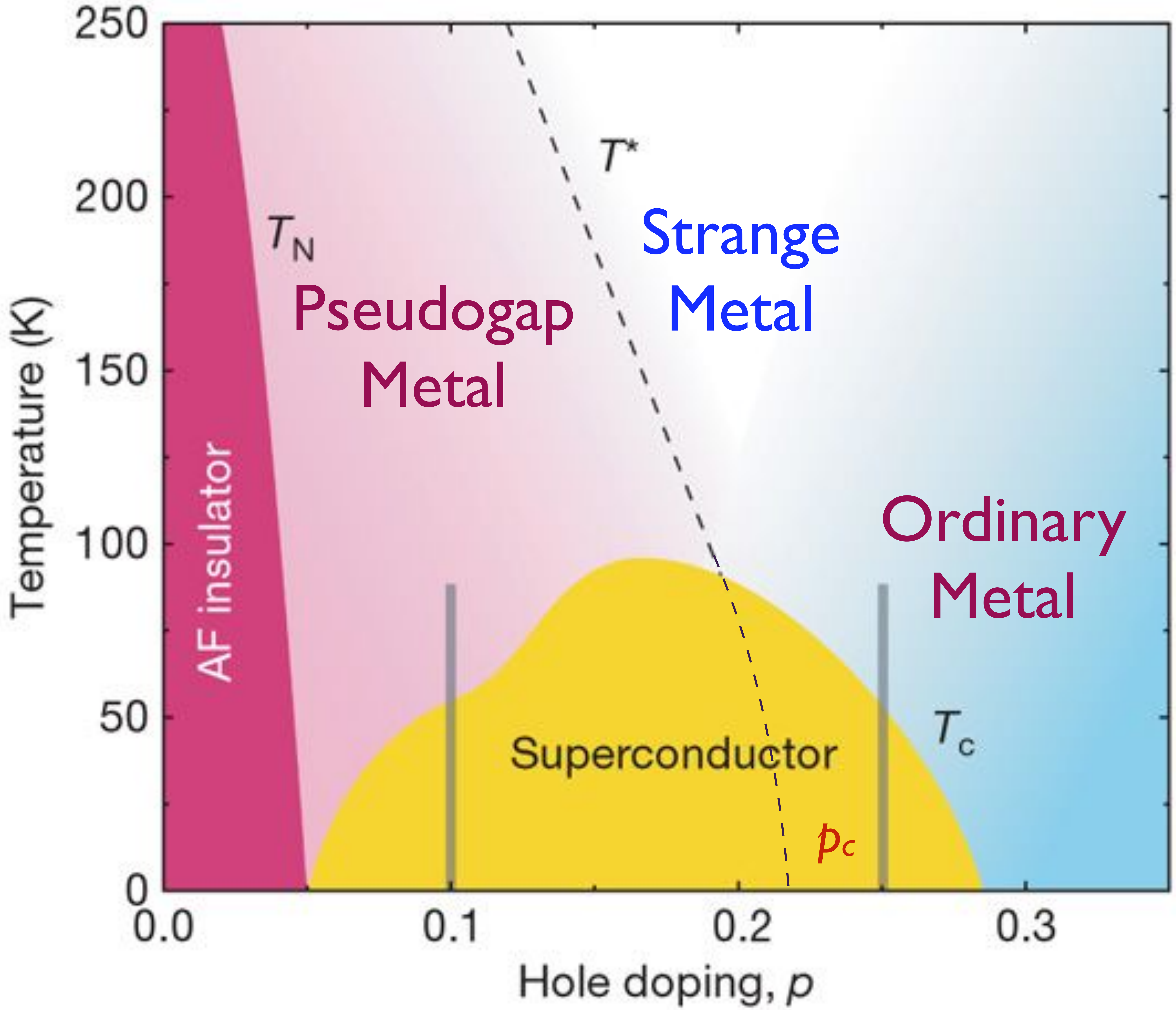


PHYSICAL REVIEW LETTERS **133**, 246702 (2024)

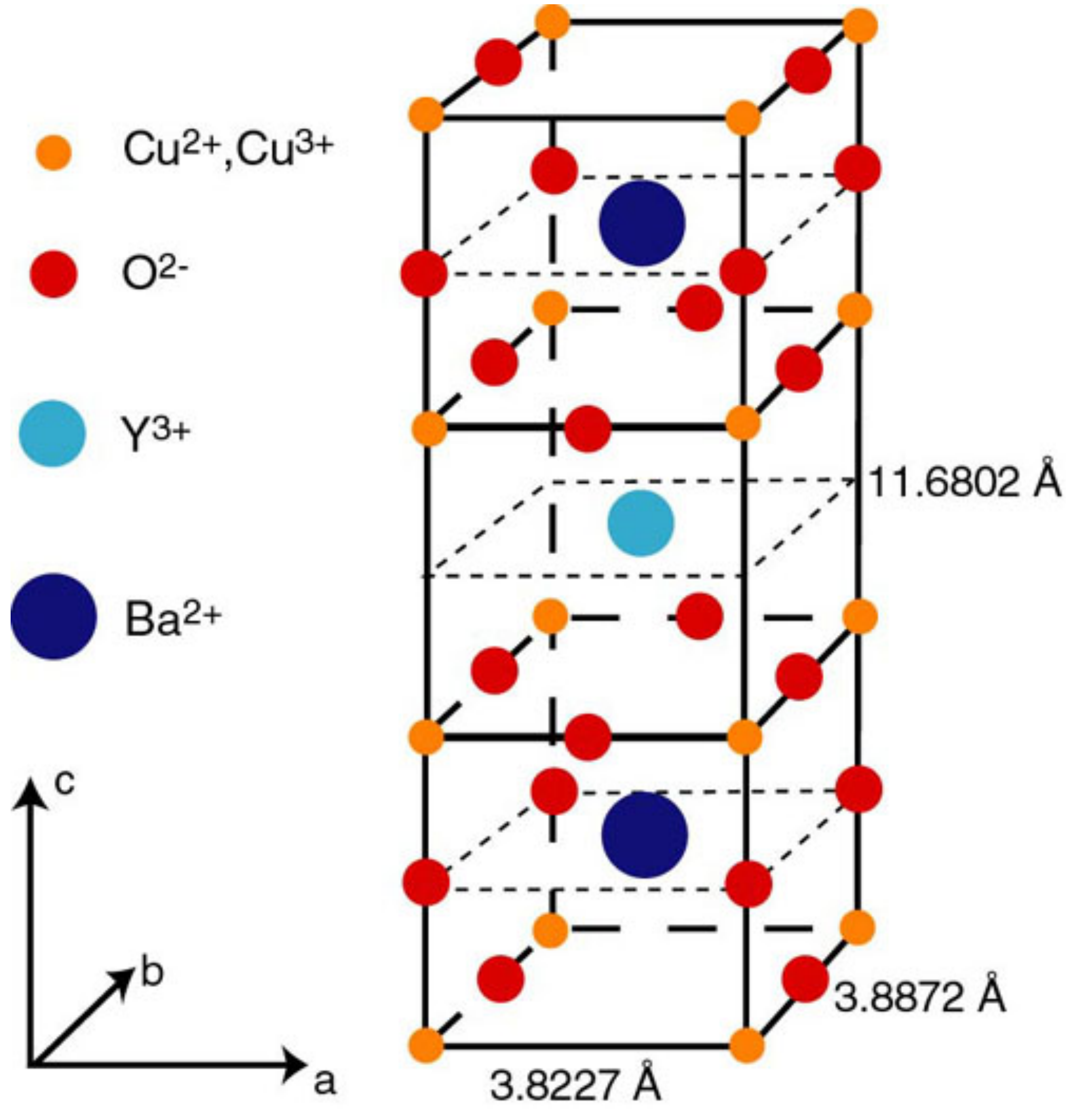
Spin Waves and Three Dimensionality in the High-Pressure Antiferromagnetic Phase of $SrCu_2(BO_3)_2$

Ellen Fogh,[1,*] Gaétan Giriat,[1] Mohamed E. Zayed,[2] Andrea Piovano,[3] Martin Boehm,[3] Paul Steffens,[3] Irina Safiulina,[3] Ursula B. Hansen,[3] Stefan Klotz,[4] Jian-Rui Soh,[1] Ekaterina Pomjakushina,[5] Frédéric Mila,[6] Bruce Normand,[1,7] and Henrik M. Rønnow[1]



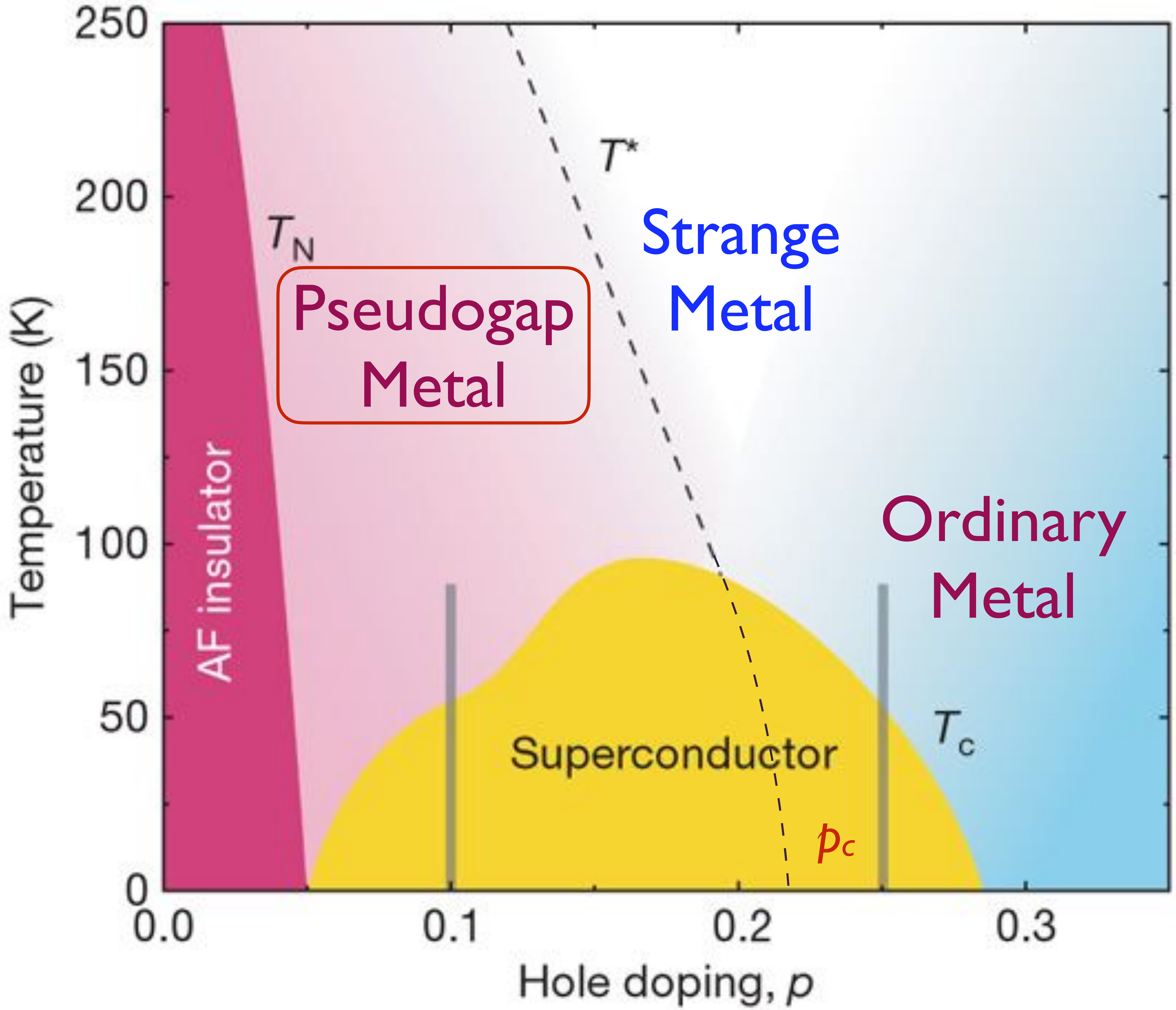


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
Pseudogap
Metal
T^*
Strange
Metal
Ordinary
Metal
Superconductor
T_c
p_c

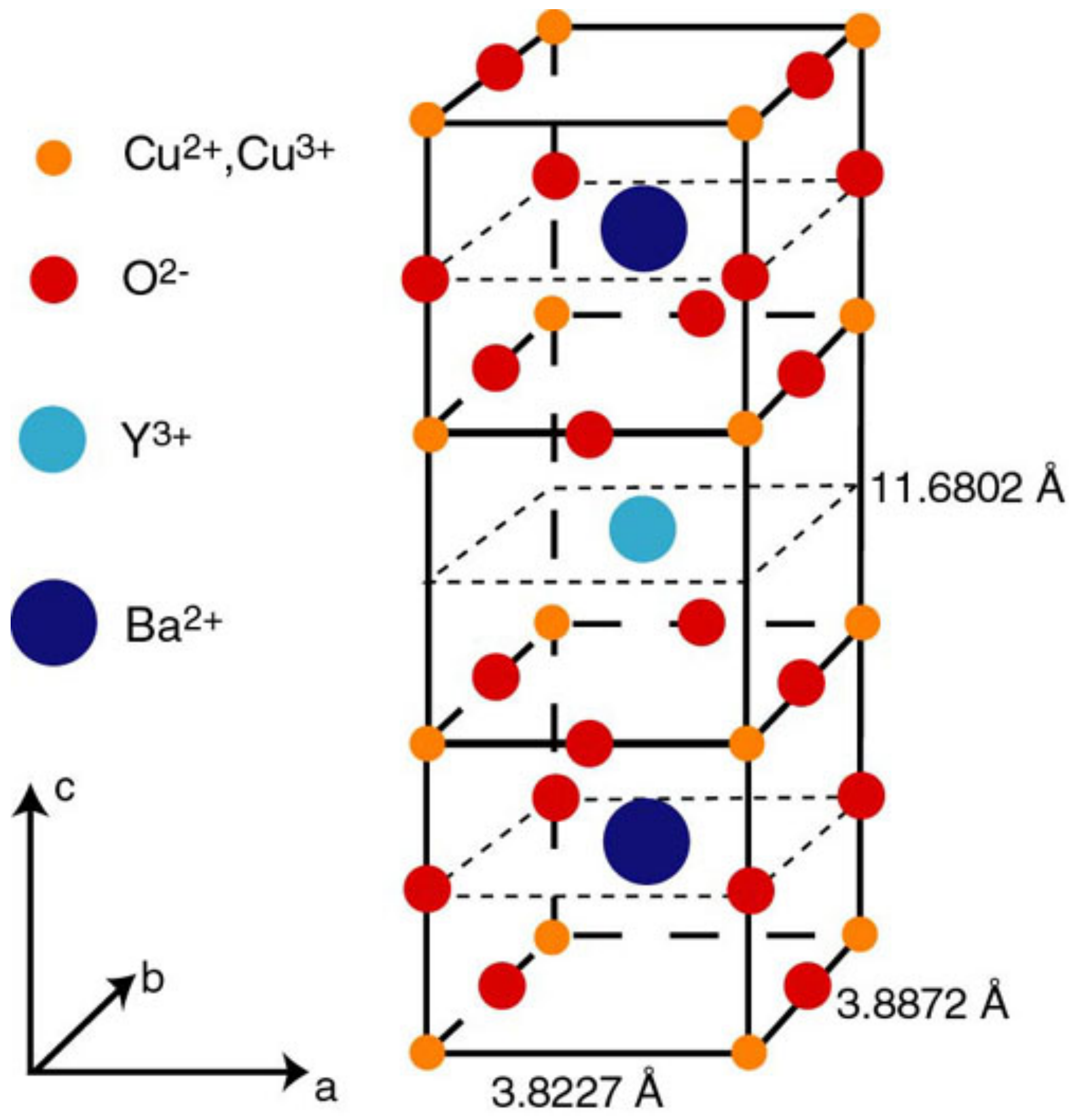


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$



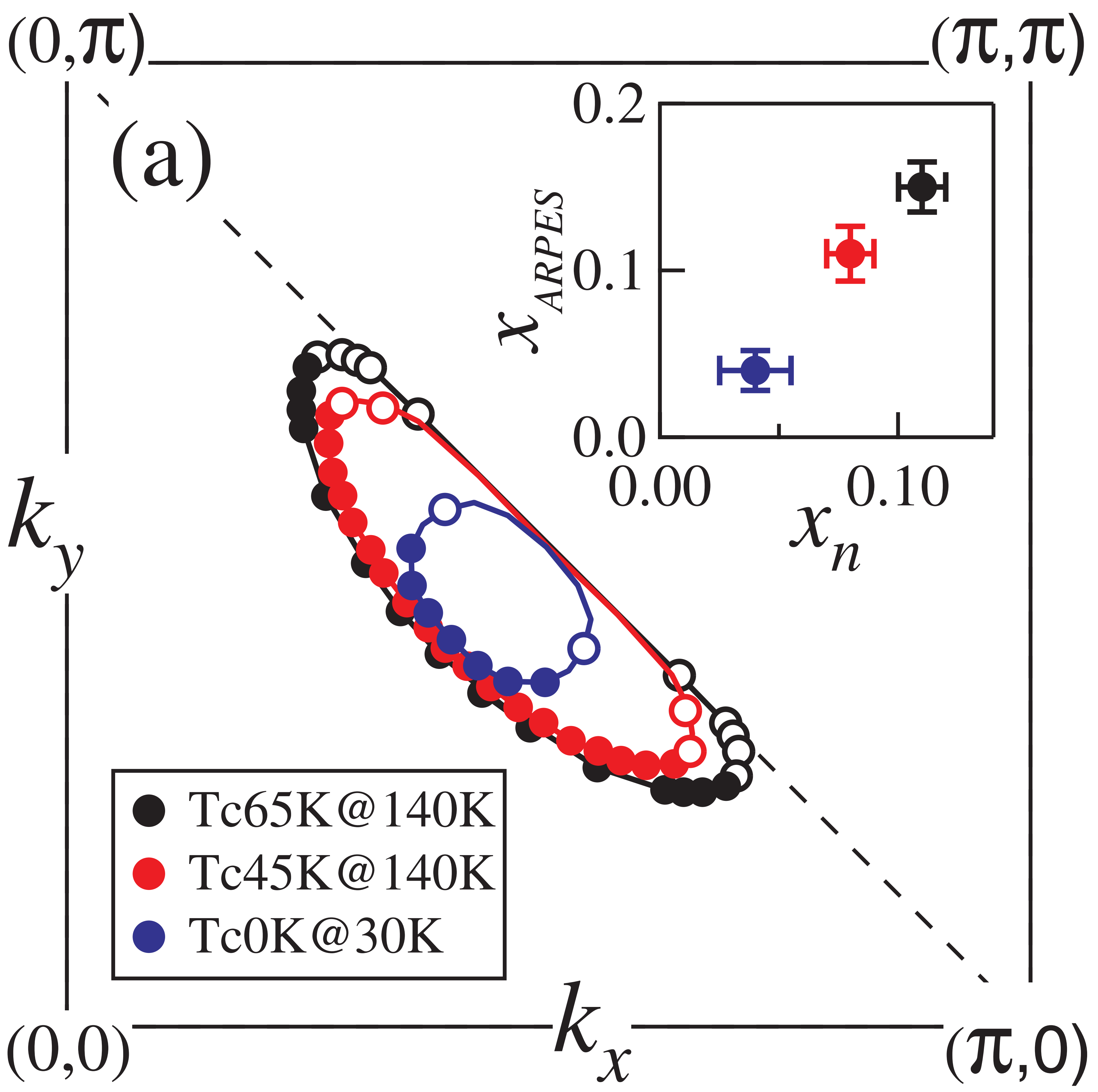
250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Pseudogap Metal
Strange Metal
Ordinary Metal
Superconductor
T_c
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$

Photoemission expts in cuprates in pseudogap metal



Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).

Non-Luttinger volume Fermi surfaces from various self-consistent Green's function approaches.

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, *Phys. Rev. B* **73**, 174501 (2006).
T. D. Stanescu and G. Kotliar, *Phys. Rev. B* **74**, 125110 (2006).
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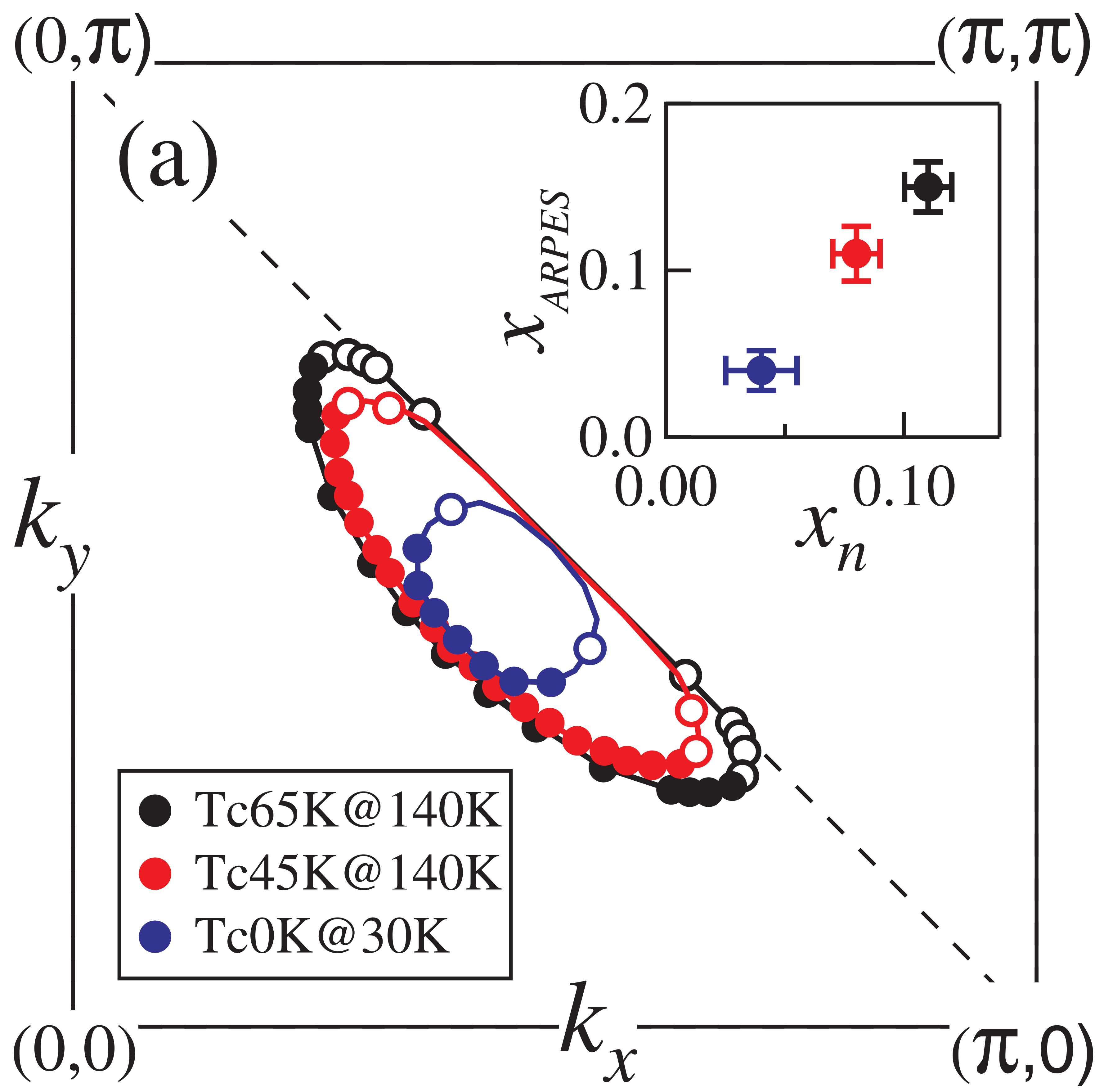
S. Sakai, Y. Motome, M. Imada, *Phys. Rev. Lett.* **102**, 056404 (2009).
J. Skolimowski and M. Fabrizio, *Phys. Rev. B* **106**, 045109 (2022).
N. Wagner….A. Georges, G. Sangiovanni, *Nature Communication* **14**, 7531 (2023)
Jinchao Zhao, Gabriele La Nave, Philip Phillips, *Phys. Rev. B* **108**, 165135
Jing-Yu Zhao, Zheng-Yu Weng, arXiv:2309.11556

Photoemission expts in cuprates in pseudogap metal



Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).

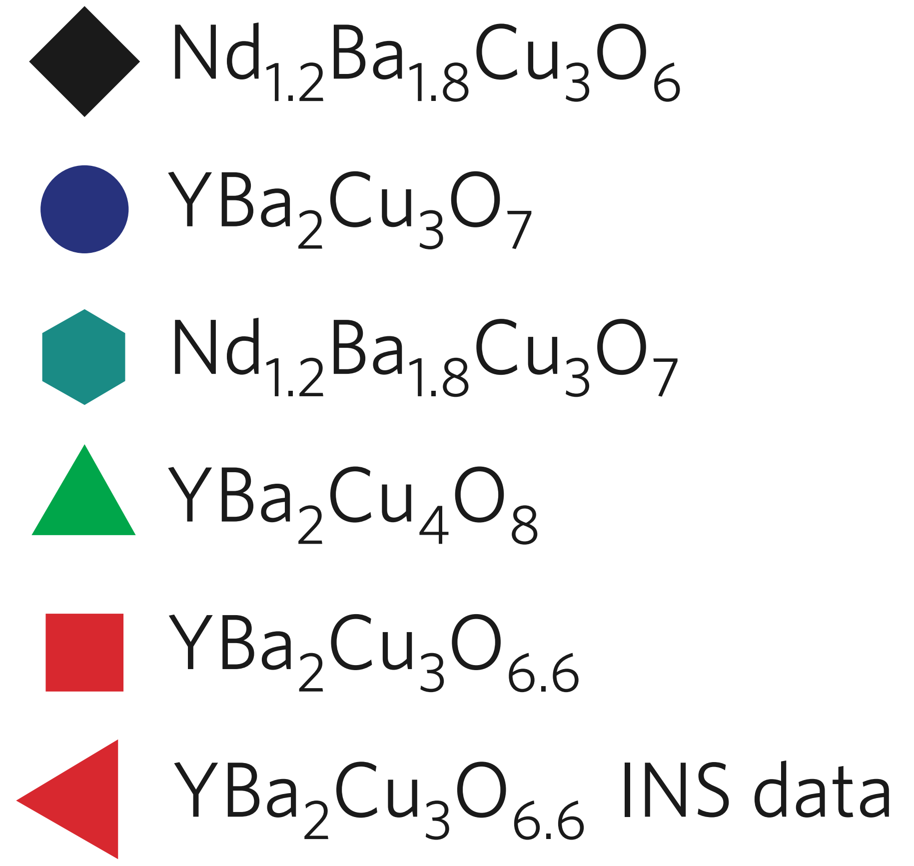
Non-Luttinger volume Fermi surfaces from various self-consistent Green's function approaches.

Oshikawa's topological Luttinger argument implies that non-Luttinger Fermi surfaces must be accompanied by fractionalized spinon excitations

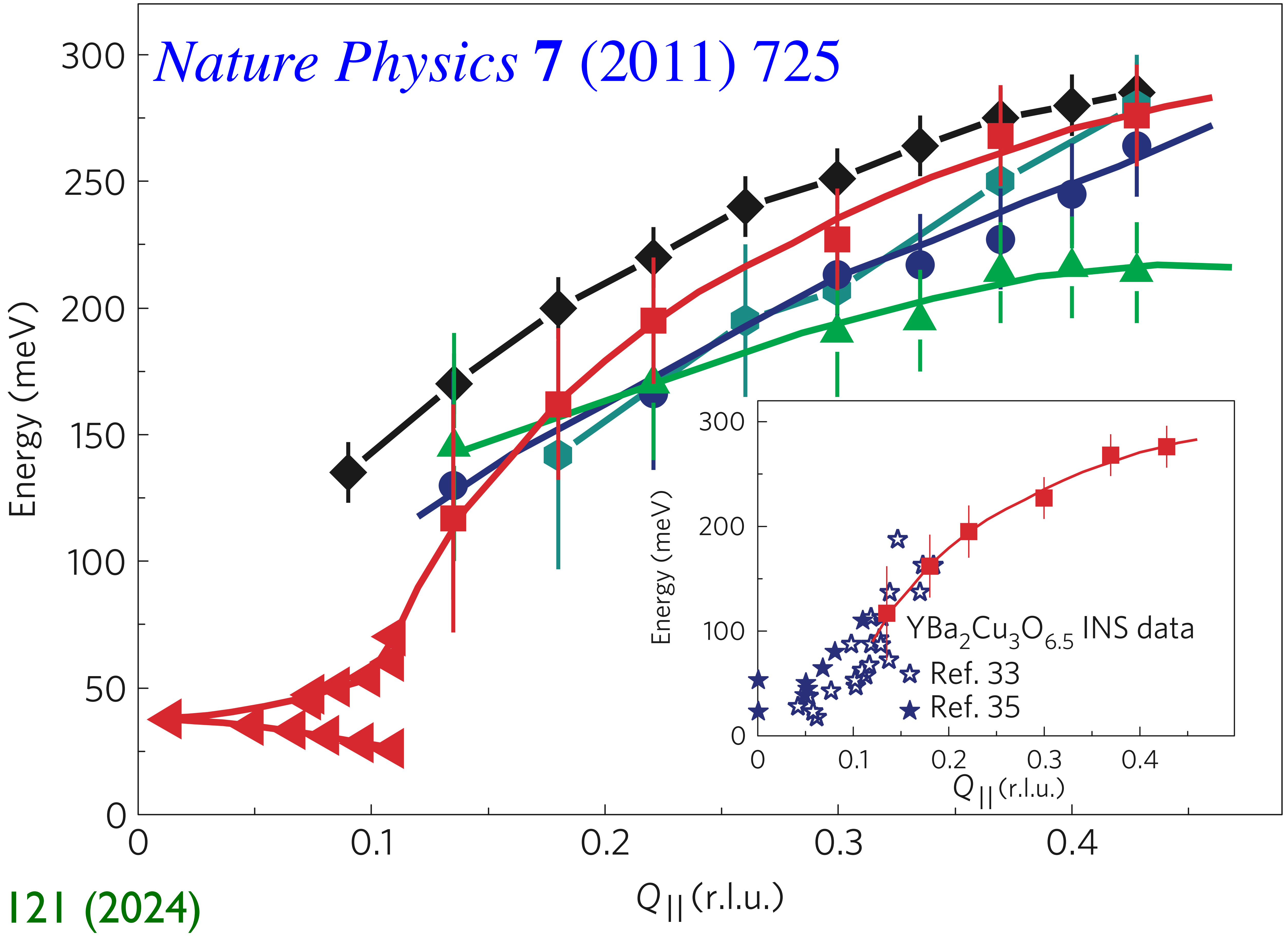
T. Senthil, M. Vojta, S.S., PRB **69**, 035111 (2004)
R. K. Kaul, A. Kolezhuk, M. Levin, S. S., T. Senthil, PRB **75** , 235122 (2007)
Y. Qi, S. S., PRB **81**, 115129 (2010)
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Intense paramagnon excitations in a large family of high-temperature superconductors

M. Le Tacon[1]*, G. Ghiringhelli[2], J. Chaloupka[1], M. Moretti Sala[2], V. Hinkov[1,3], M. W. Haverkort[1], M. Minola[2], M. Bakr[1], K. J. Zhou[4], S. Blanco-Canosa[1], C. Monney[4], Y. T. Song[1], G. L. Sun[1], C. T. Lin[1], G. M. De Luca[5], M. Salluzzo[5], G. Khaliullin[1], T. Schmitt[4], L. Braicovich[2] and B. Keimer[1]*



- Difficult to have intense paramagnons from a small Fermi surface.
- Spin waves only present at low energies in the presence of antiferromagnetic order
- Most natural interpretation is a spinon continuum, similar to that observed on the triangular lattice in $KYbSe_2$



Pietro Bonetti, Maine Christos and S.S. (BCS), PNAS **121**, e2418633121 (2024)

Anisotropic damping and wave vector dependent susceptibility of the spin fluctuations in $La_{2-x}Sr_xCuO_4$ studied by resonant inelastic x-ray scattering

H. C. Robarts, M. Barthélemy, K. Kummer, M. García-Fernández, J. Li, A. Nag, A. C. Walters, K. J. Zhou, and S. M. Hayden

PHYSICAL REVIEW B **100**, 214510 (2019)

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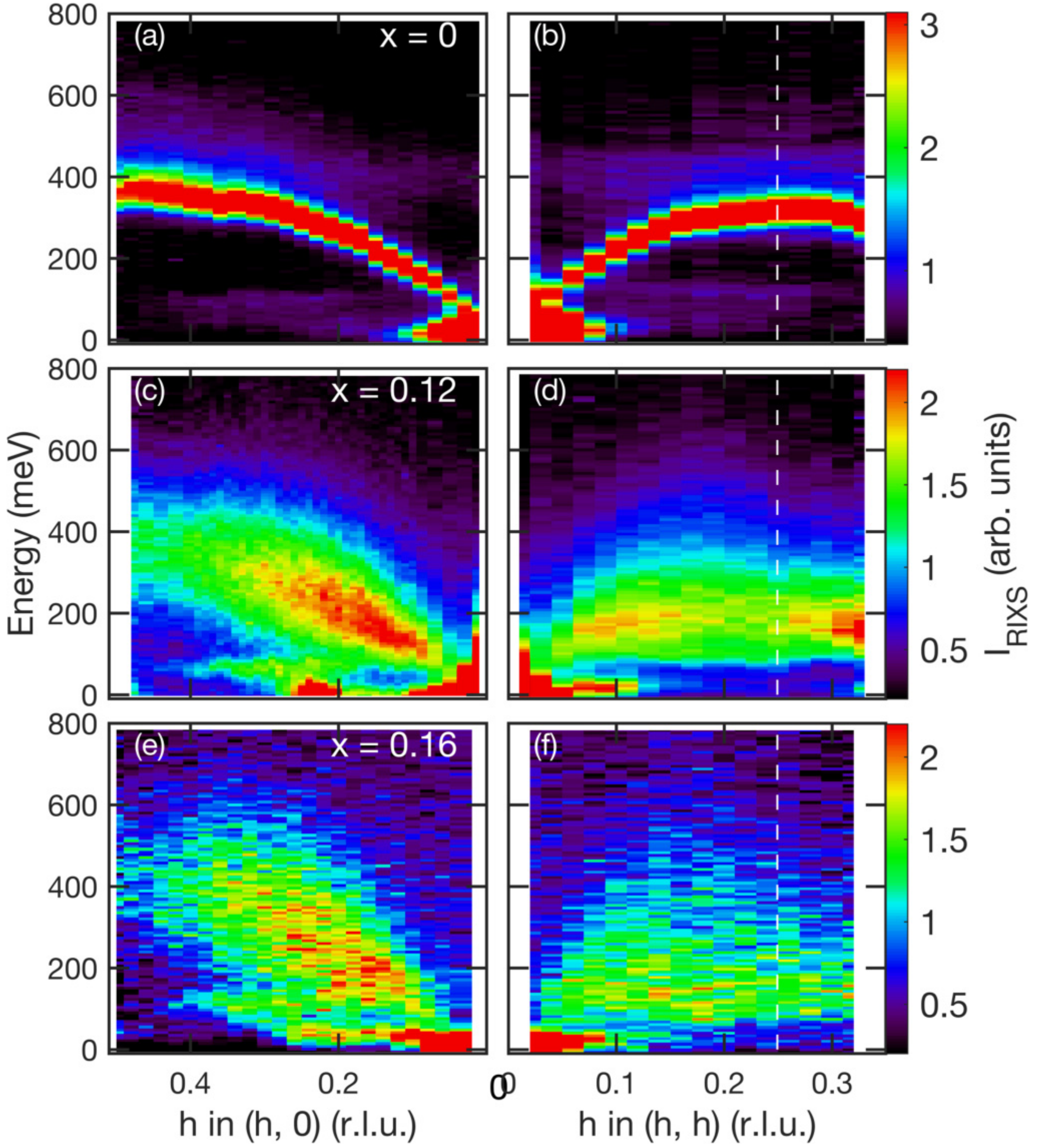
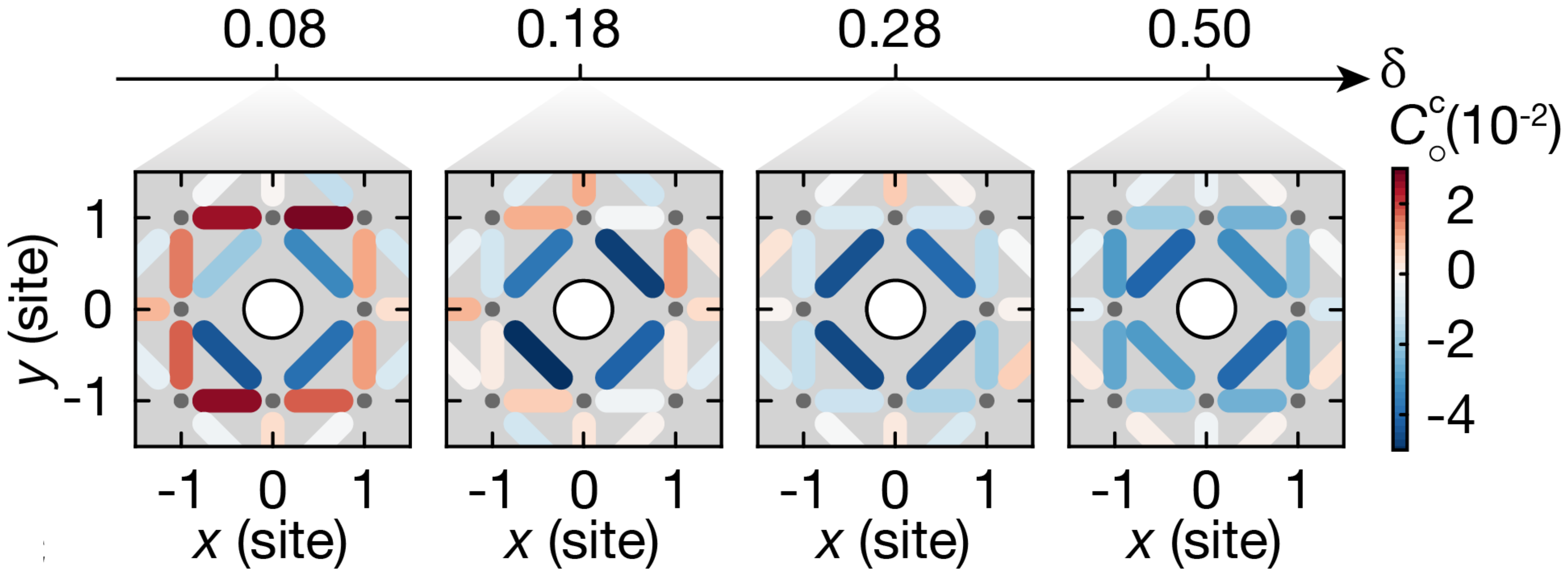


FIG. 2. I_{RIXS} intensity maps as a function of **Q** in LSCO $x = 0$ ($T \approx 20$ K), 0.12, and 0.16 ($T \approx 30$ K).

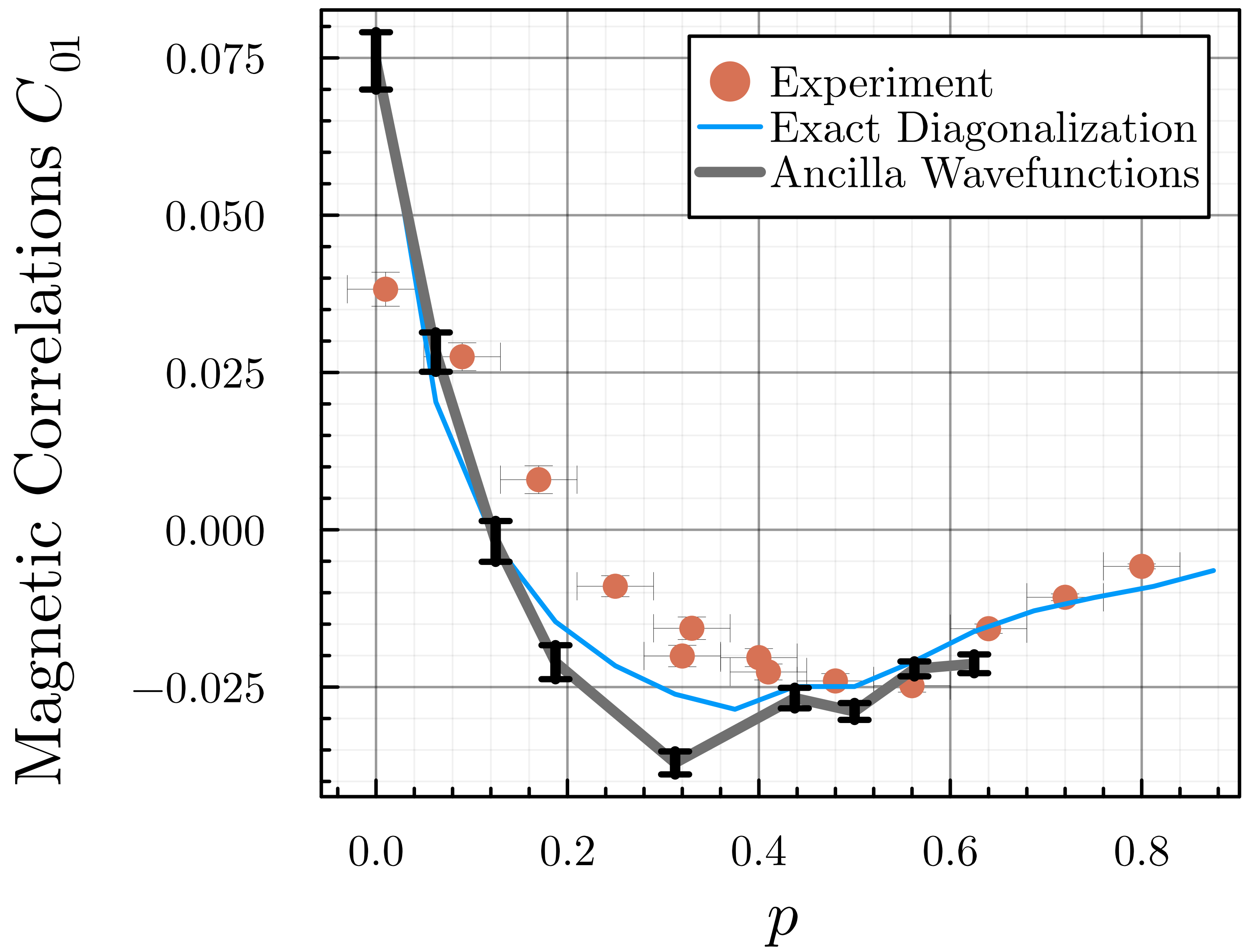
Pietro Bonetti, Maine Christos and S.S. (BCS), PNAS **121**, e2418633121 (2024)

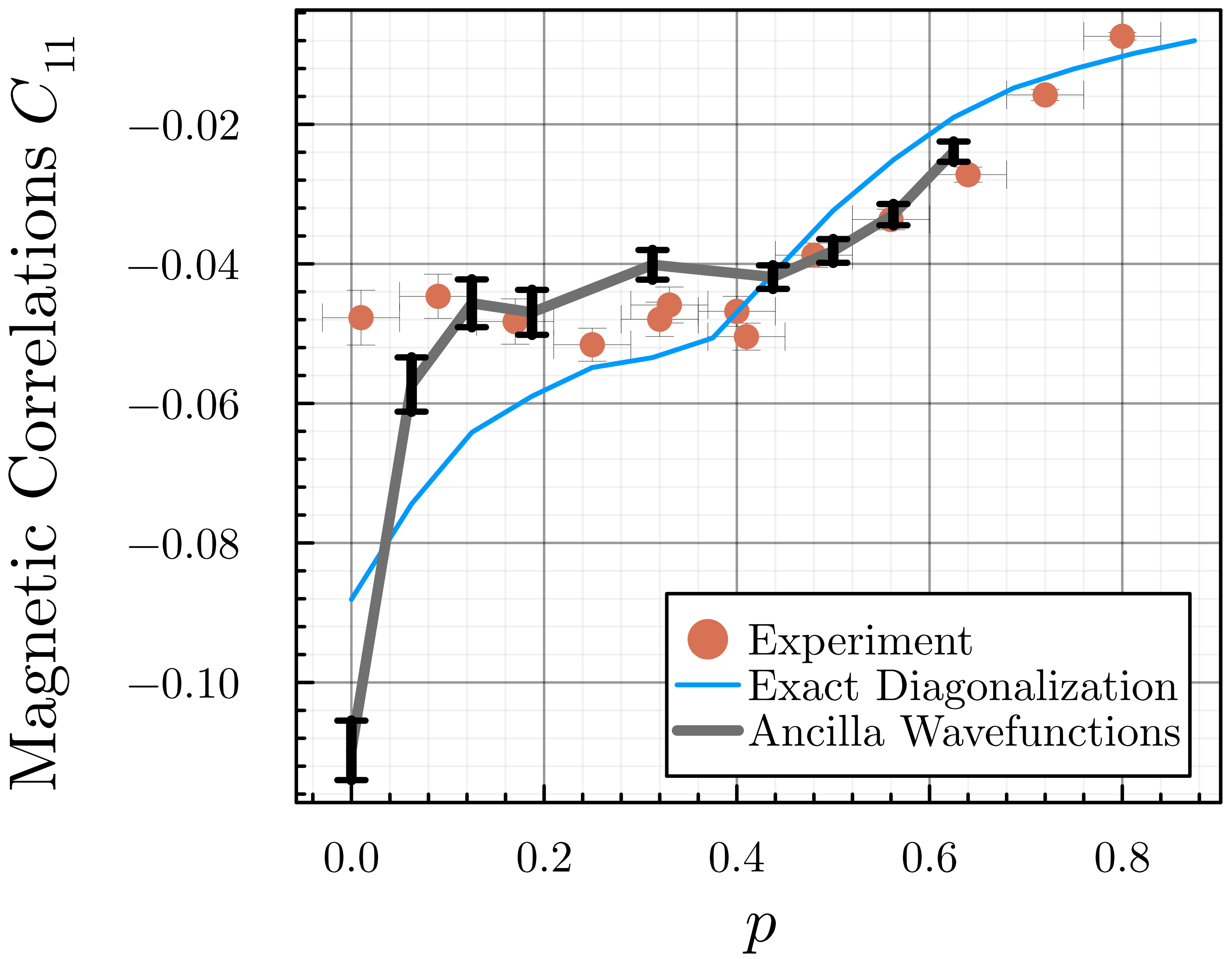


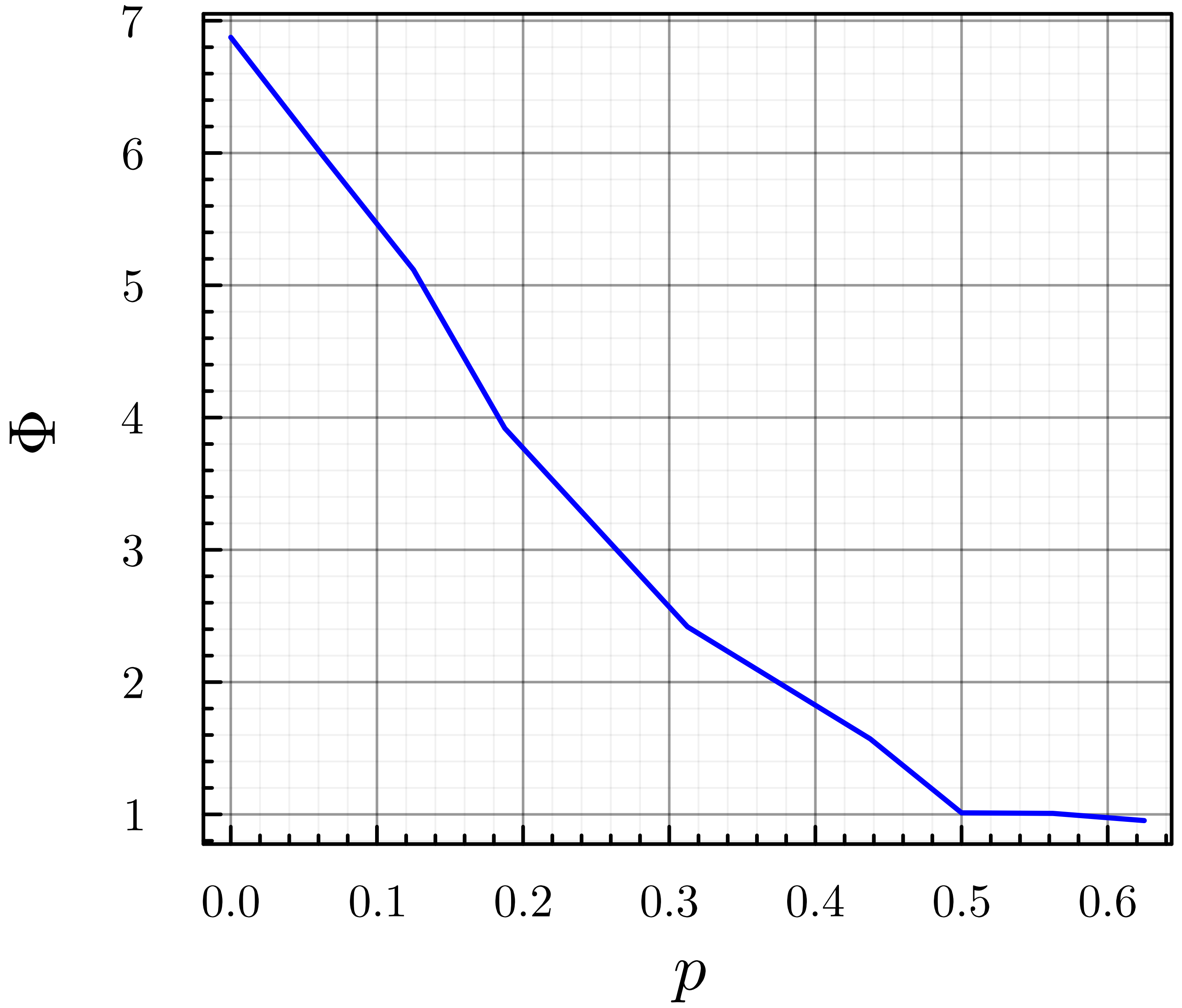
Microscopic evolution of doped Mott insulators from polaronic metal to Fermi liquid

Joannis Koepsell, Dominik Bourgund, Pimonpan Sompet, Sarah Hirthe, Annabelle Bohrdt, Yao Wang, Fabian Grusdt, Eugene Demler, Guillaume Salomon, Christian Gross, Immanuel Bloch

Science **374** (2021) 82







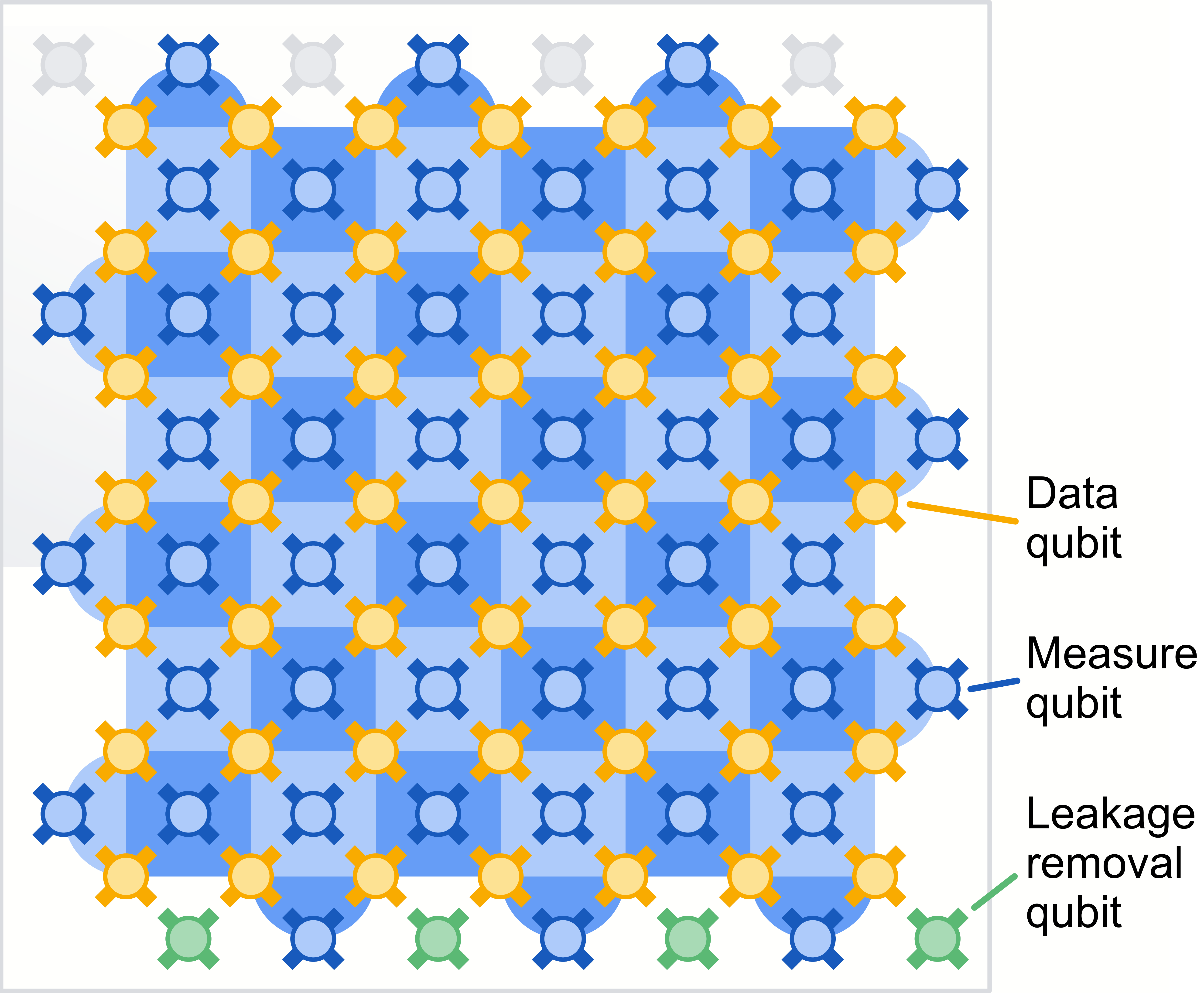
H. Shackleton and Shiwei Zhang, arXiv:2408.02190
Tobias Müller, Yasir Iqbal, S.S., Ronny Thomale, arXiv:2408.01492

Quantum error correction below the surface code threshold

Google Quantum AI and Collaborators

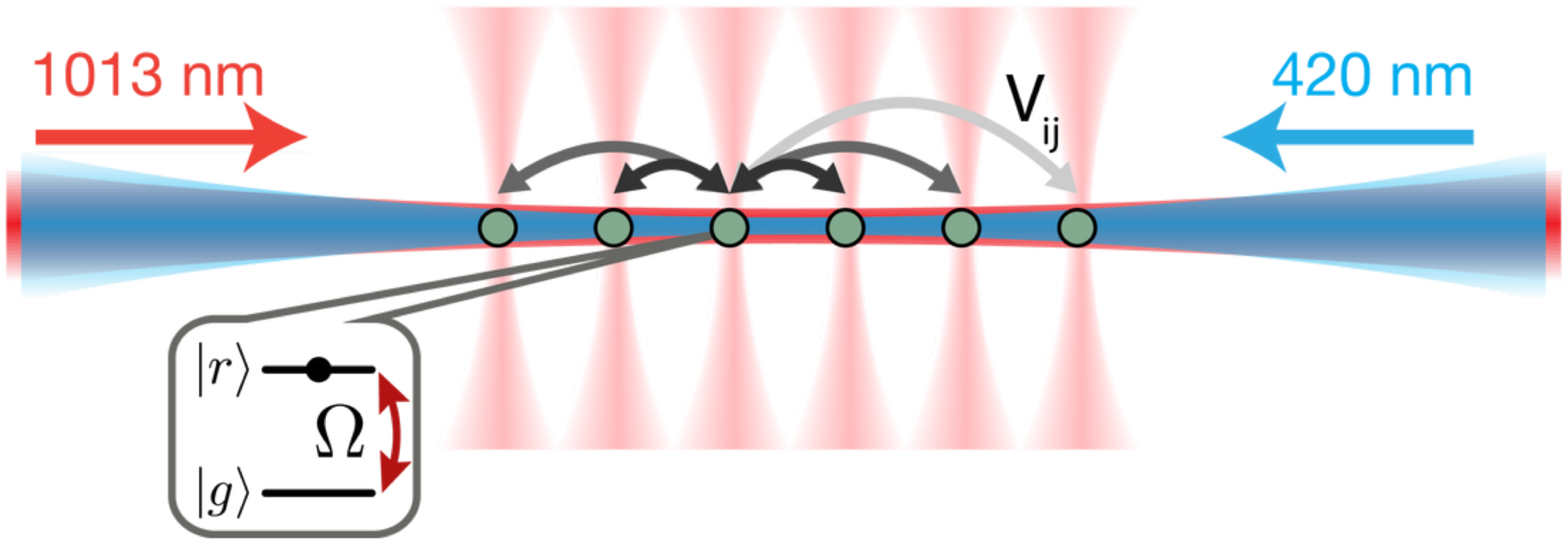
Nature 2024

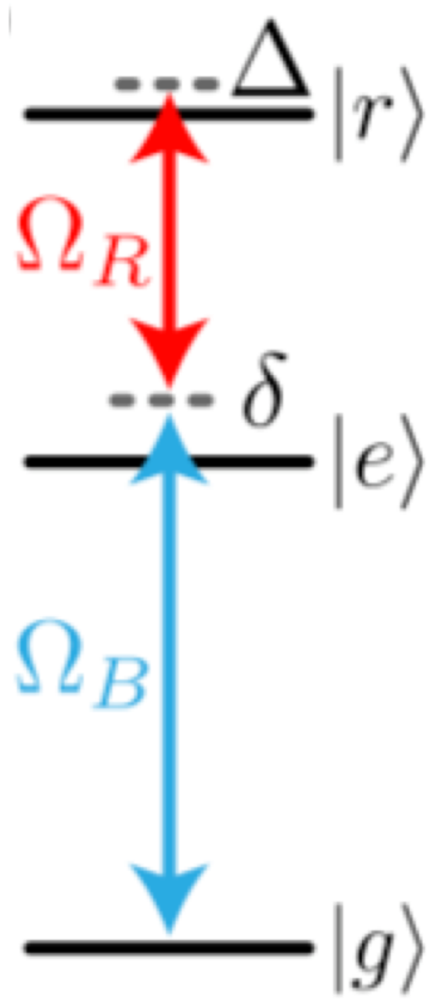
Realization of the quantum entanglement of the Z_2 spin liquid



Rydberg quantum simulator

Zoller, Lukin, Browaeys.....





$$|g\rangle \equiv |0\rangle$$

$$|r\rangle \equiv B^\dagger |0\rangle$$

$$\mathcal{H} = \sum_\ell \left[\frac{\Omega}{2} \left(B_\ell + B_\ell^\dagger \right) - \Delta n_\ell \right] + \sum_{\ell < \ell'} V_{|\ell - \ell'|} n_\ell n_{\ell'}$$

$$n_\ell \equiv B_\ell^\dagger B_\ell$$

$$n_\ell = 0, 1 \quad \text{'hard core' bosons}$$

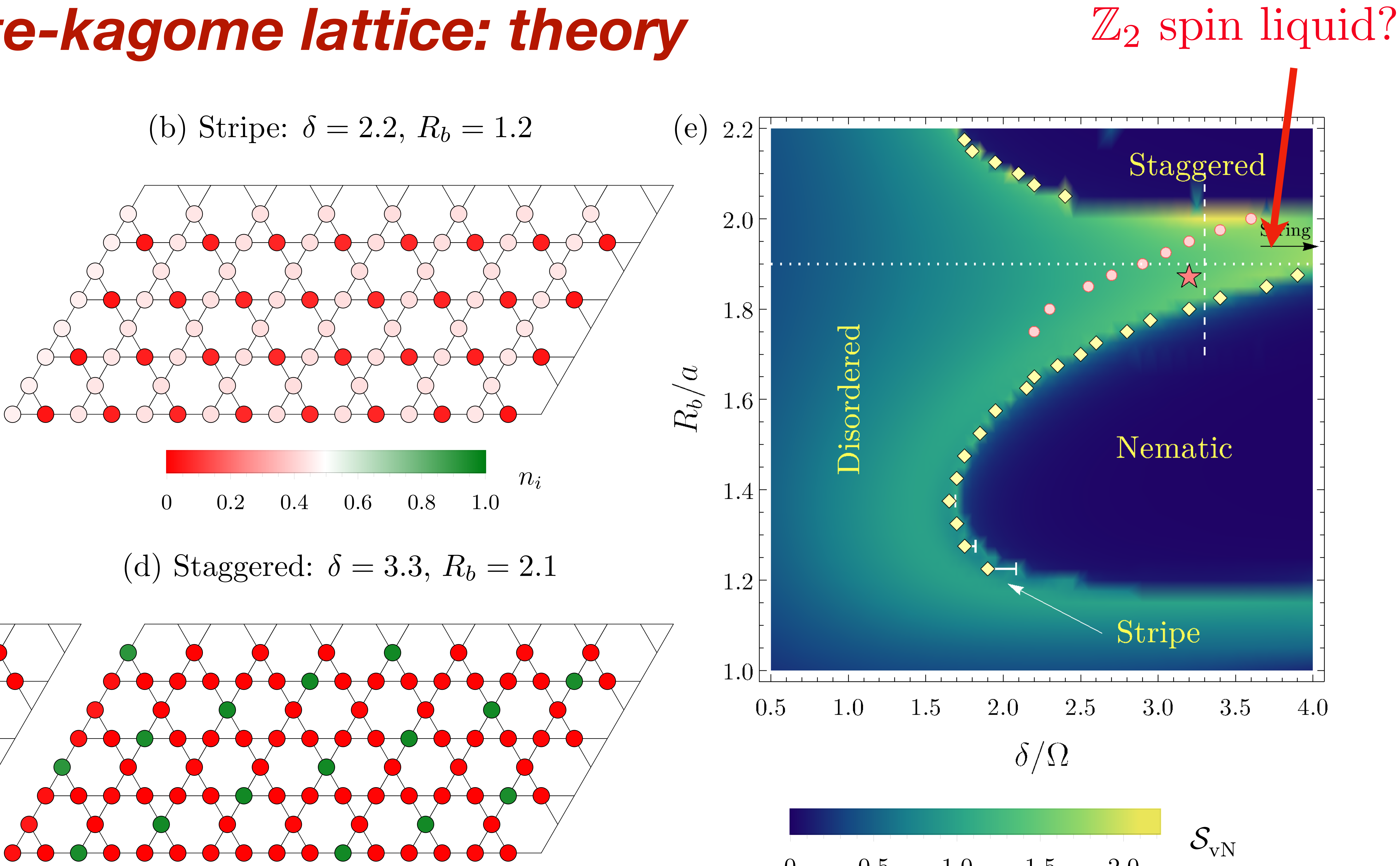
$$V_{|\ell-\ell'|} \sim \frac{1}{|\ell - \ell'|^6}$$

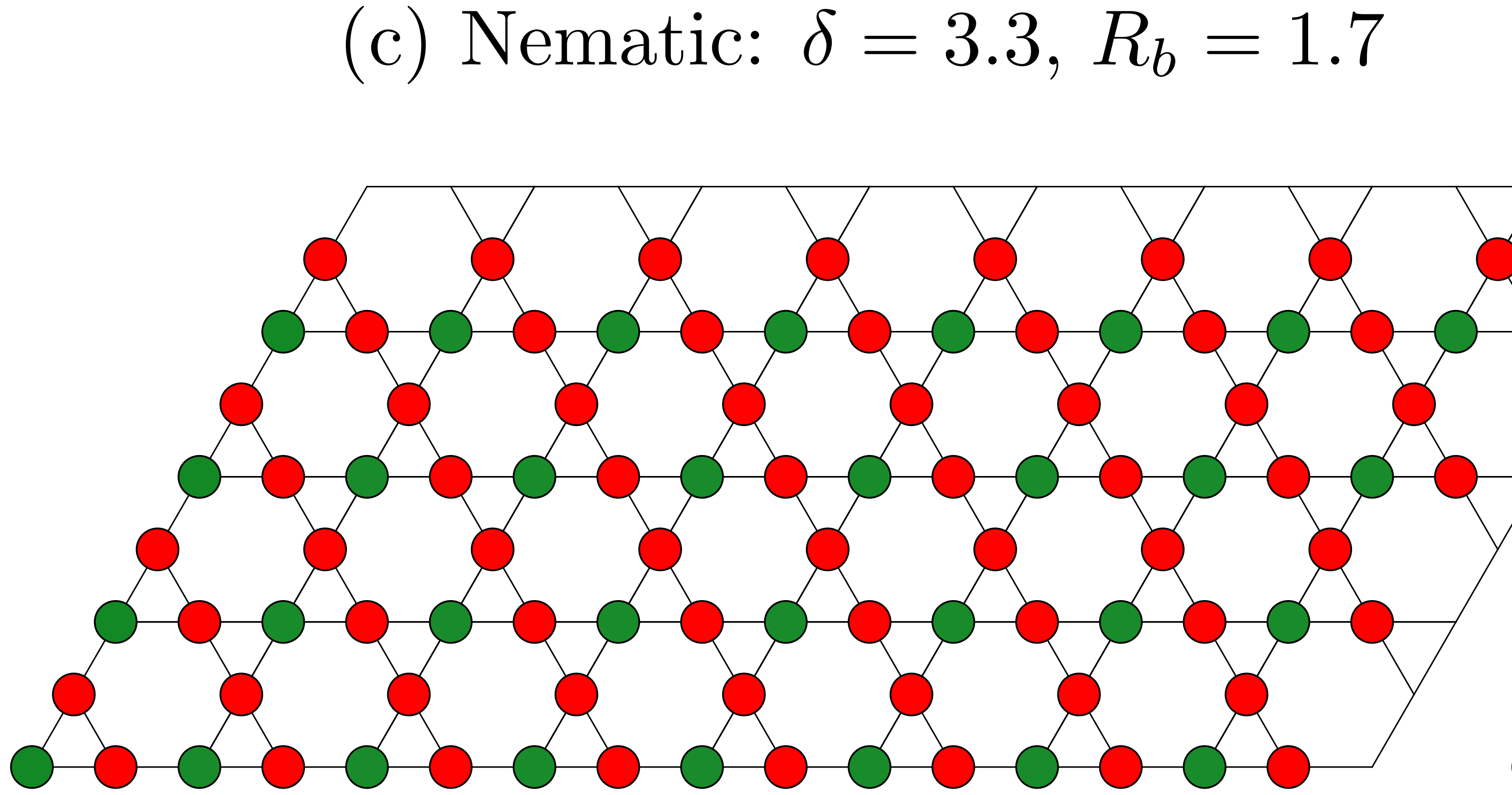
FSS model (PXP model is a special case)

S. Sachdev, K. Sengupta, and S.M. Girvin, PRB **66**, 075128 (2002)
P. **F**endley, K. **S**engupta, S. **S**achdev, PRB **69**, 075106 (2004)

Rydberg atoms on site-kagome lattice: theory

R. Samajdar





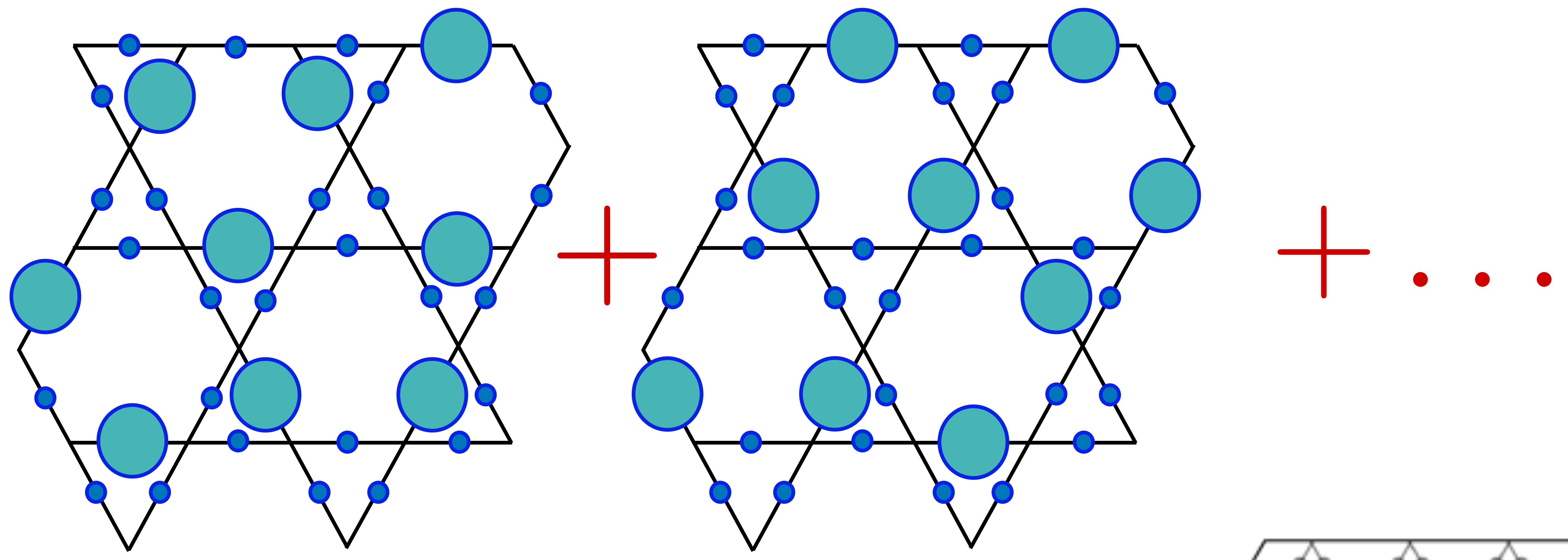
R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and S. Sachdev, PRL **124**, 103601 (2020)
S. Ebadi, Tout T. Wang, H. Levine, A. Keesling, G. Semeghini, A. Omran, D. Bluvstein, R. Samajdar, H. Pichler, Wen Wei Ho, Soonwon Choi, S. Sachdev, M. Greiner, V. Vuletić, and M. D. Lukin, Nature **595**, 227 (2021)
R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and S. Sachdev, PNAS **118**, e2015785118 (2021)

Probing Topological Spin Liquids on a Programmable Quantum Simulator

G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T. T. Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletic, M. D. Lukin, Science **374**, 1242 (2021).

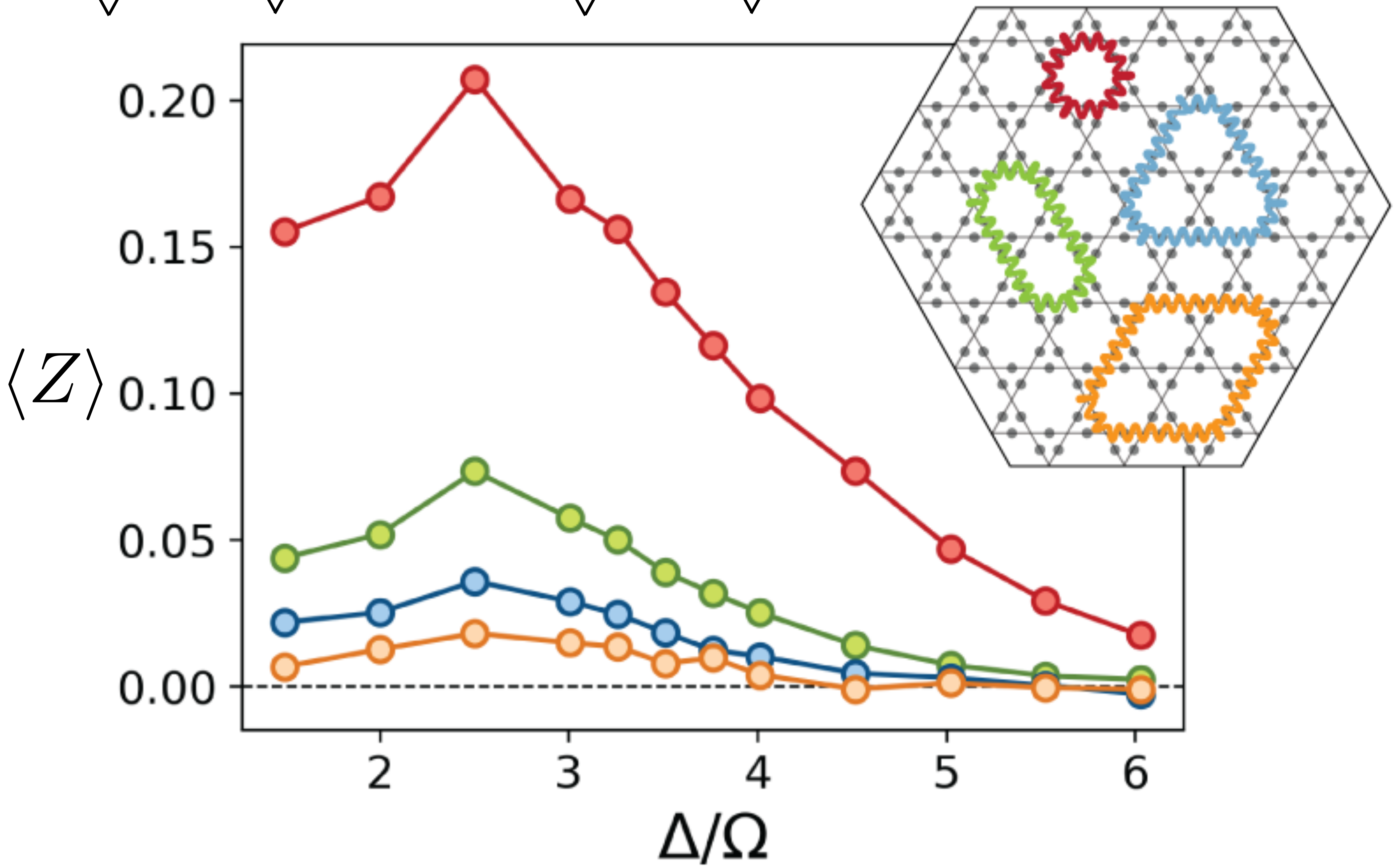
Rydberg atoms on the link-kagome lattice: experiment

$$|\Psi\rangle =$$



$$+ \; \ldots$$

Evidence for $\mathbb{Z}_2$ spin liquid correlations

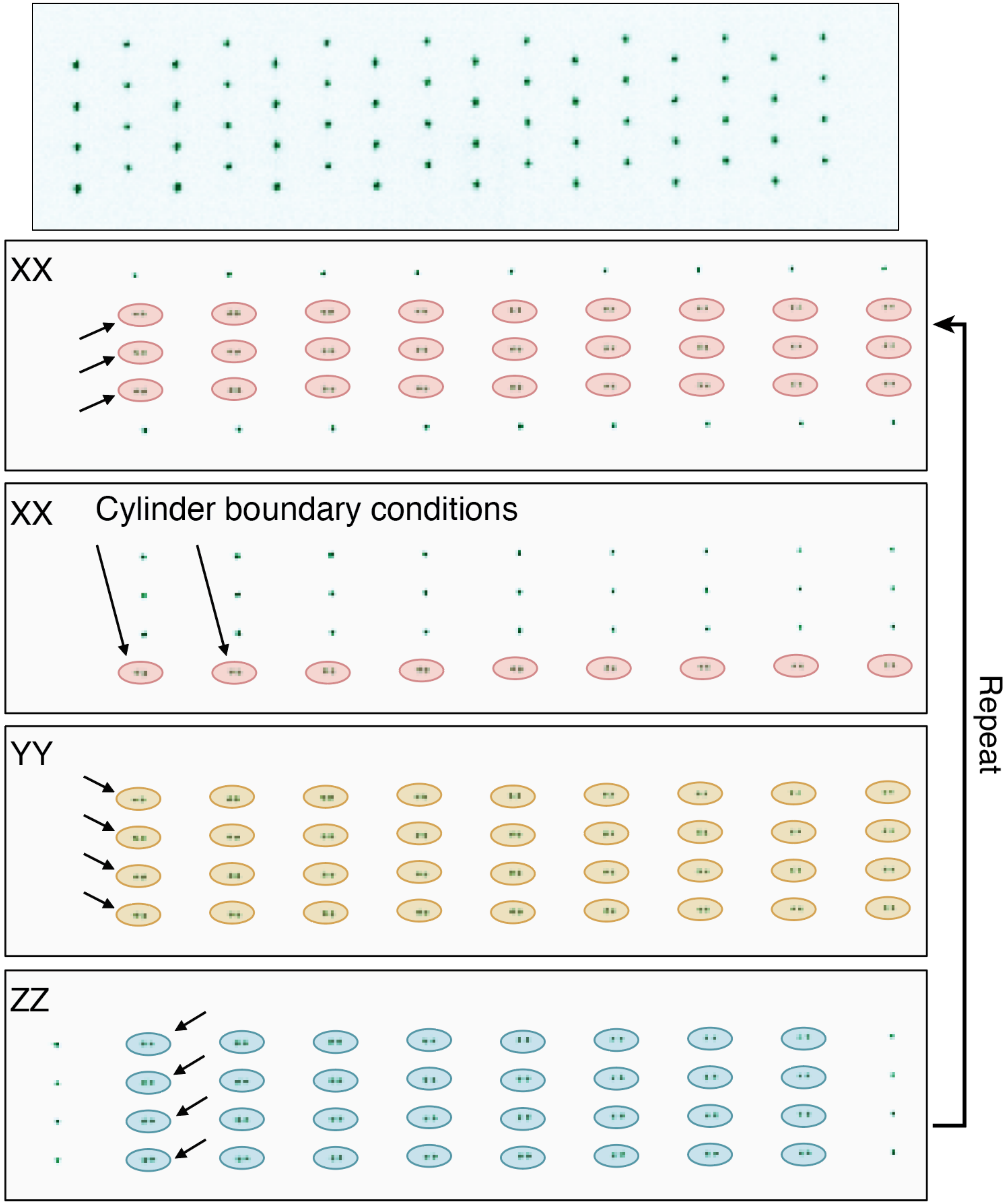


Probing topological matter and fermion dynamics on a neutral-atom quantum computer

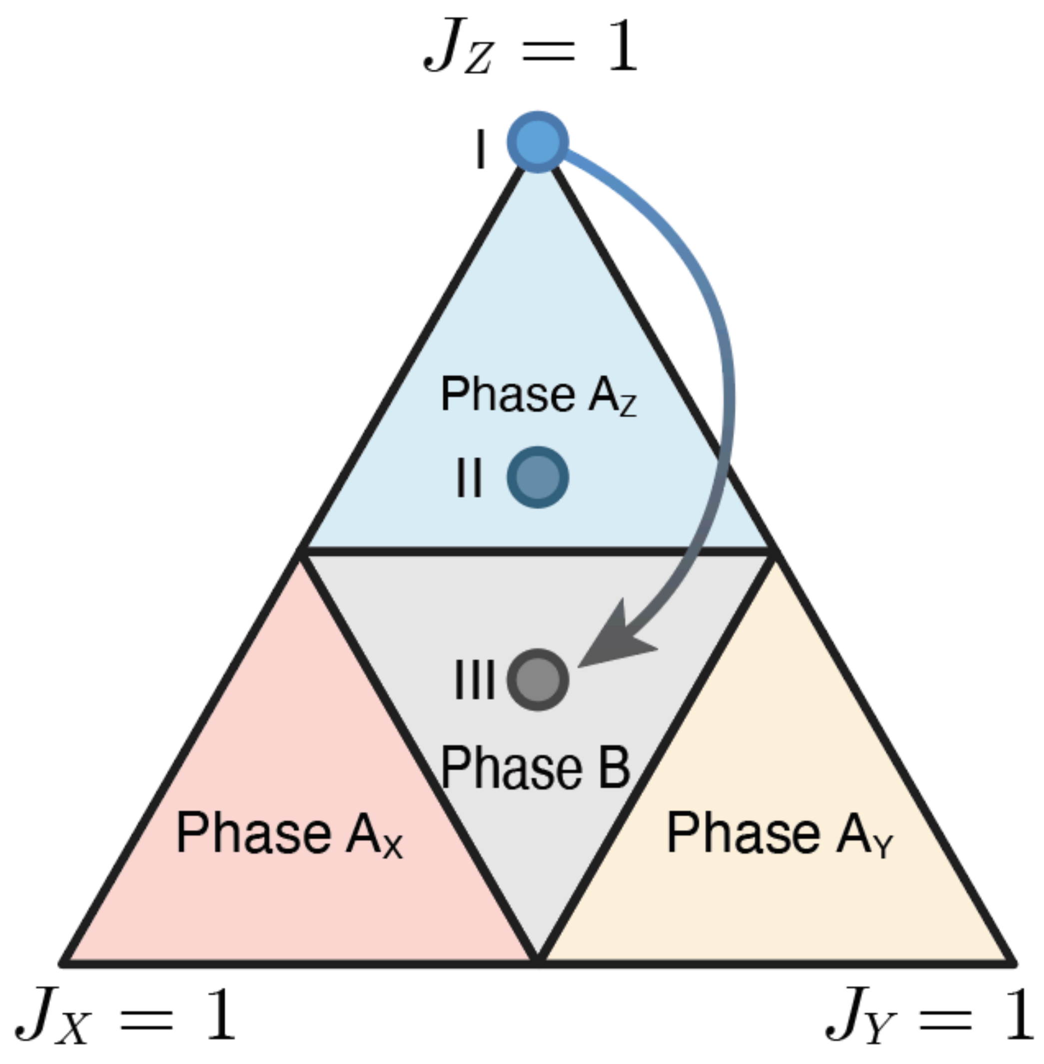
Simon J. Evered, Marcin Kalinowski, Alexandra A. Geim, Tom Manovitz, Dolev Bluvstein, Sophie H. Li, Nishad Maskara, Hengyun Zhou, Sepehr Ebadi, Muqing Xu, Joseph Campo, Madelyn Cain, Stefan Ostermann, Susanne F. Yelin, Subir Sachdev, Markus Greiner, Vladan Vuletic, and Mikhail D. Lukin,
submitted (2025)

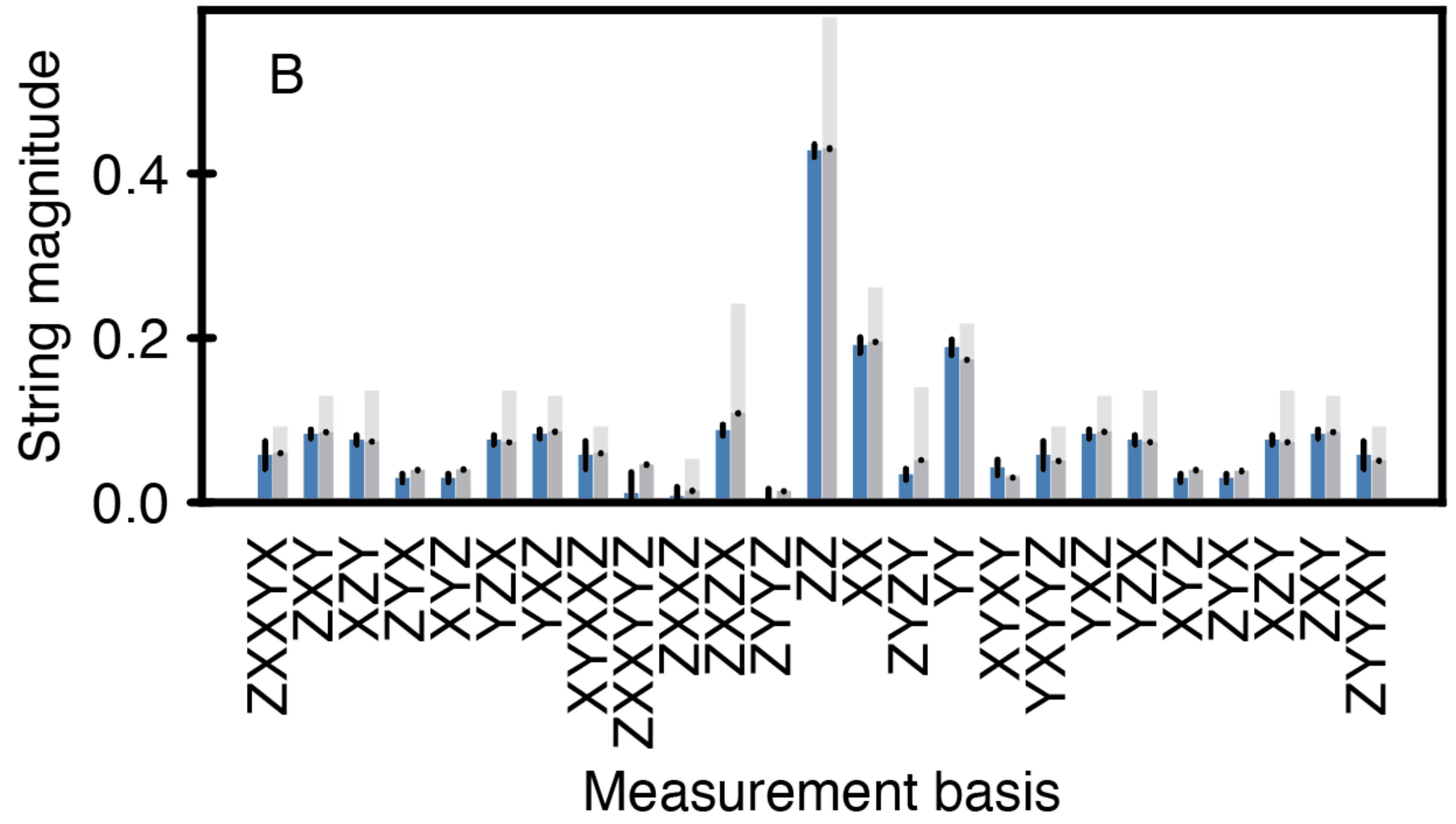
- Create $\mathbb{Z}_2$ spin liquid by active error correction of fluxes
- Apply cyclic time evolution under XX, YY, ZZ operators.
- Measure Chern number of ϵ fermions

$\Rightarrow$ Implies Kitaev's non-Abelian Ising anyon state.

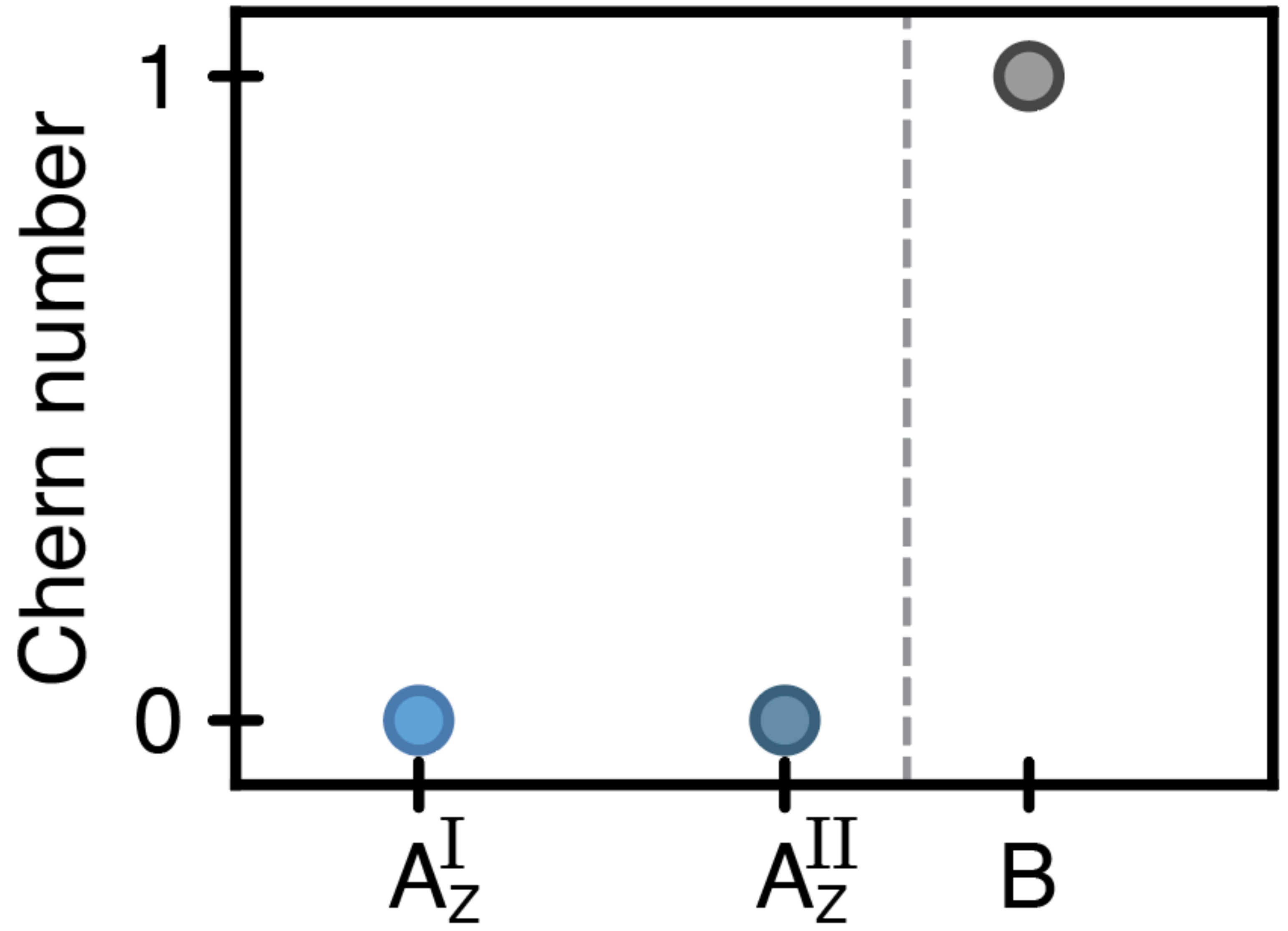


Measuring Chern number with string data

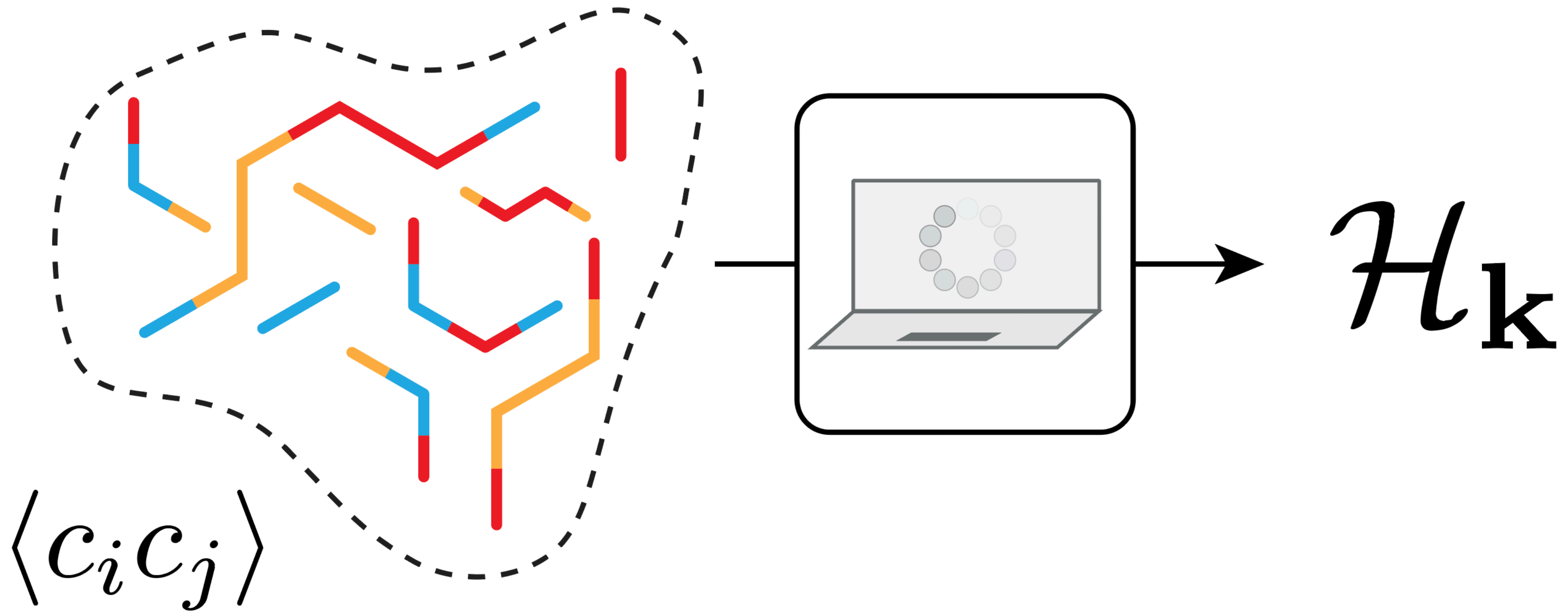




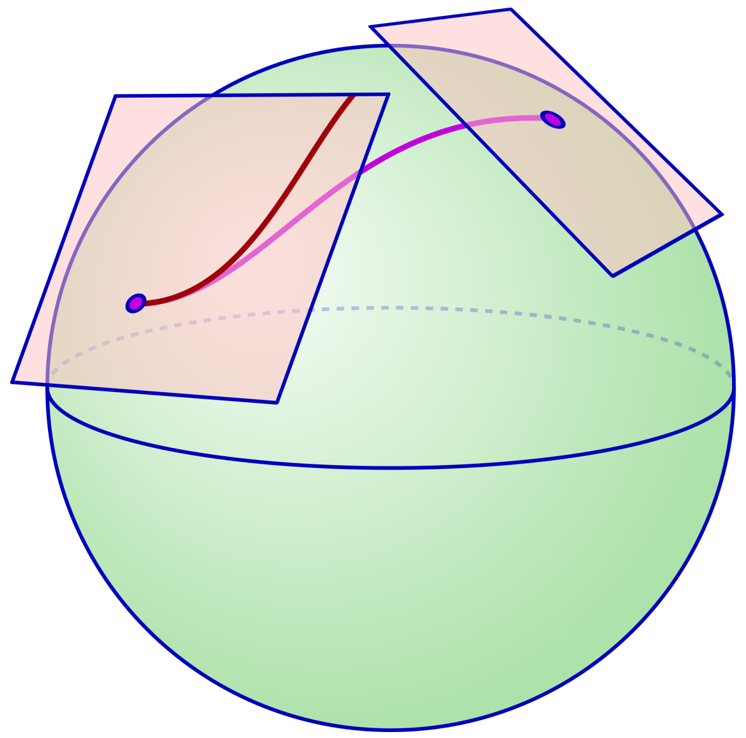
Odd Chern number in *B* phase



Use string data to learn free-fermion Hamiltonian



Evaluate Chern number

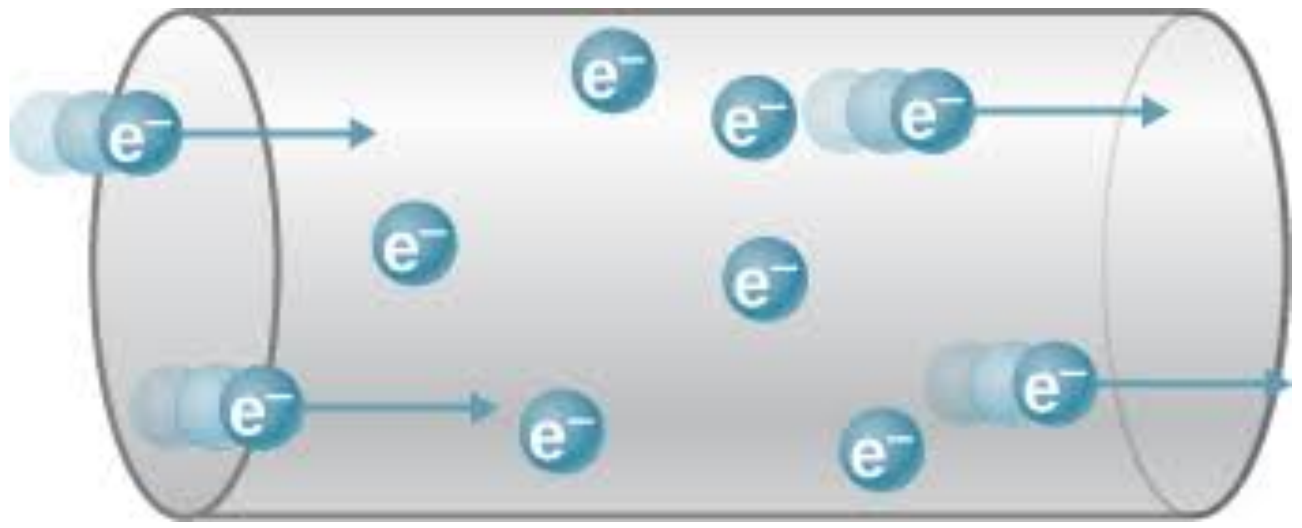


Probing topological matter and fermion dynamics on a neutral-atom quantum computer

Simon J. Evered, Marcin Kalinowski, Alexandra A. Geim, Tom Manovitz, Dolev Bluvstein, Sophie H. Li, Nishad Maskara, Hengyun Zhou, Sepehr Ebadi, Muqing Xu, Joseph Campo, Madelyn Cain, Stefan Ostermann, Susanne F. Yelin, Subir Sachdev, Markus Greiner, Vladan Vuletic, and Mikhail D. Lukin, submitted (2025)

Metals without quasiparticles: the SYK model

Current flow with electrons in ordinary metals

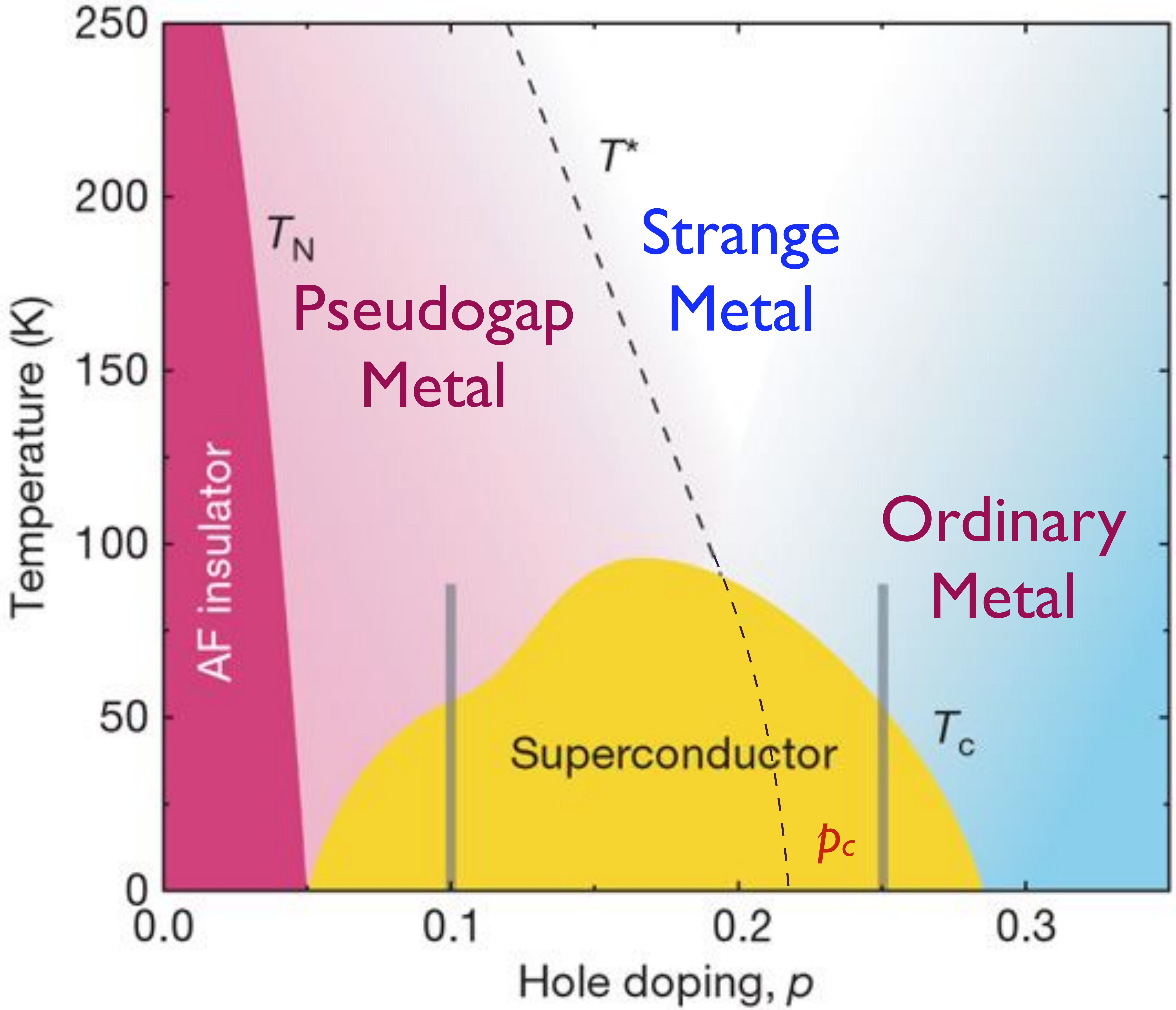


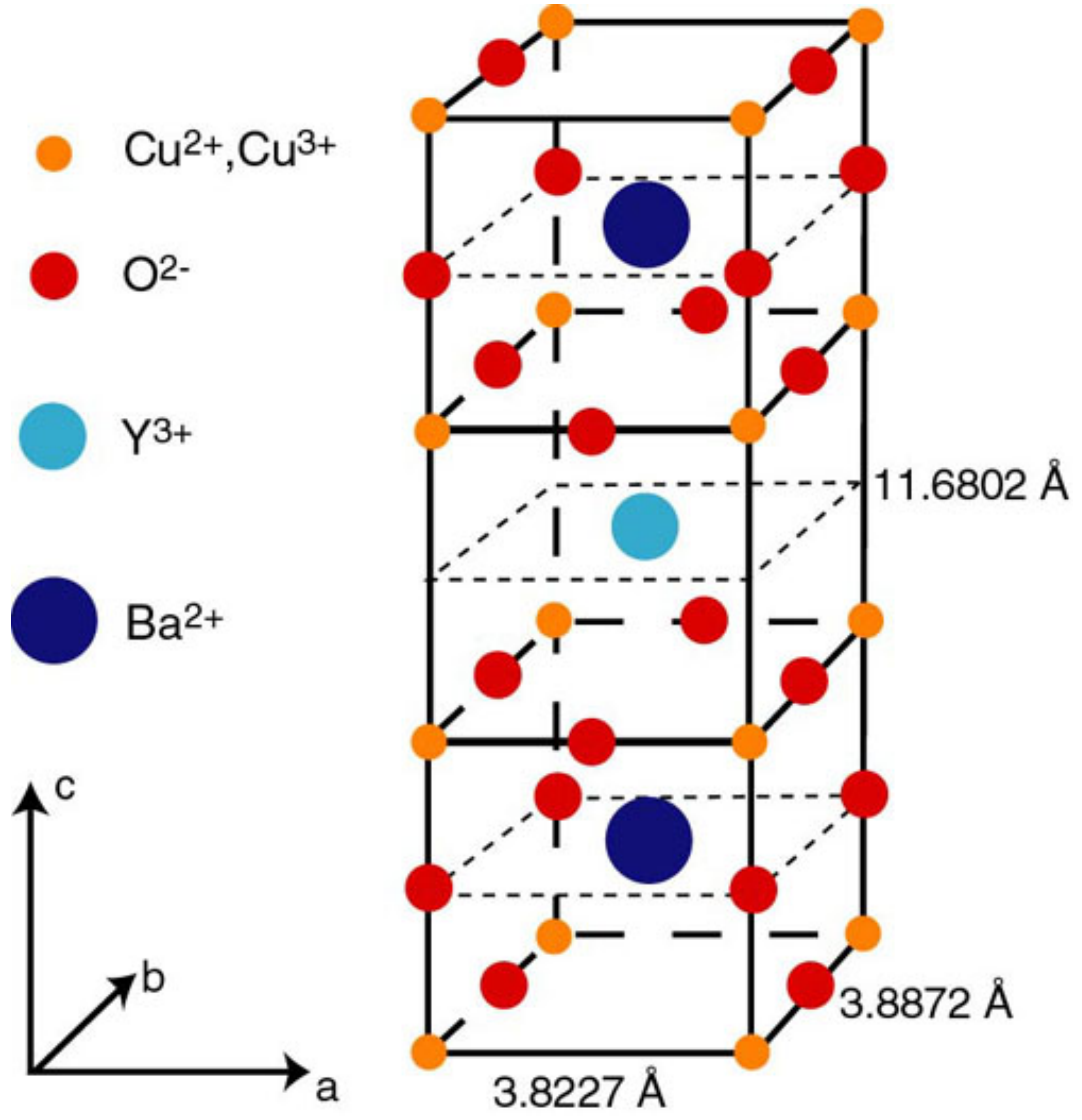
Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/(UT)^2$ (U is the strength of interactions),
resistivity $\rho(T) = \rho(0) + AT^2$

The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

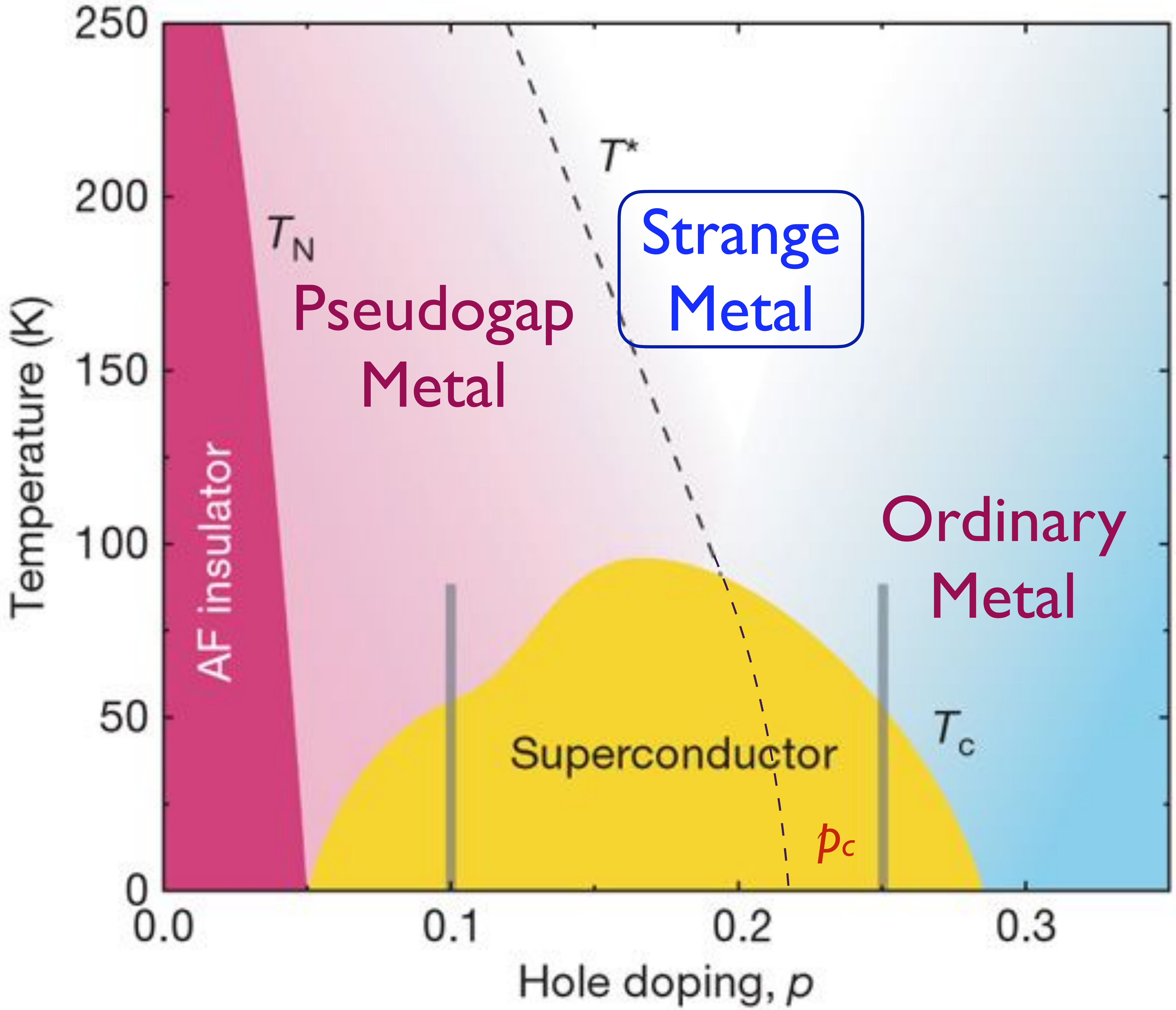
The long scattering time implies that individual electrons are well-defined.

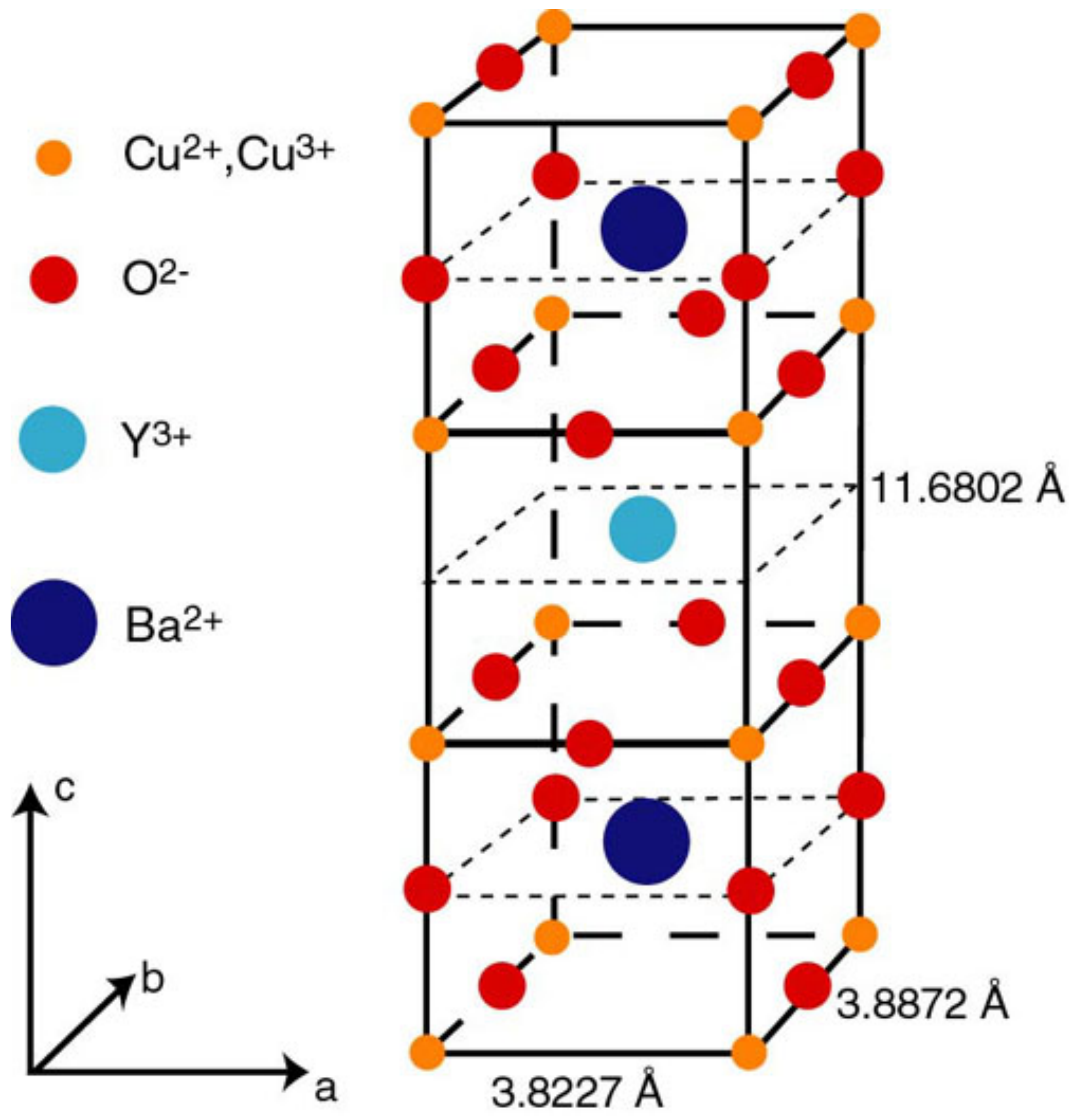
The motion of electrons is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
Pseudogap Metal
T*
Strange Metal
Ordinary Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$$YBa_2Cu_3O_{6+x}$$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Pseudogap
Metal
Strange
Metal
Ordinary
Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$

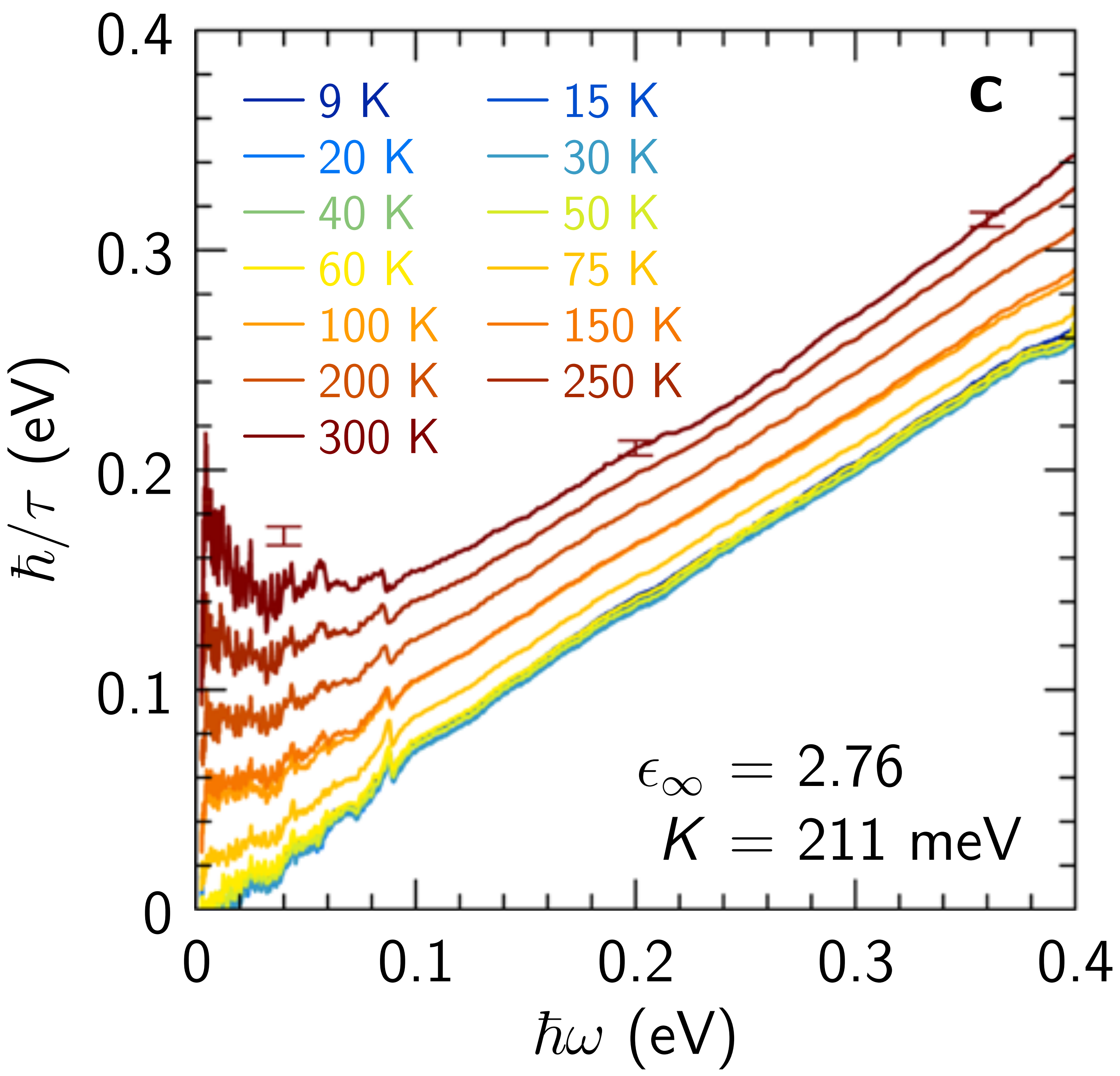
Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

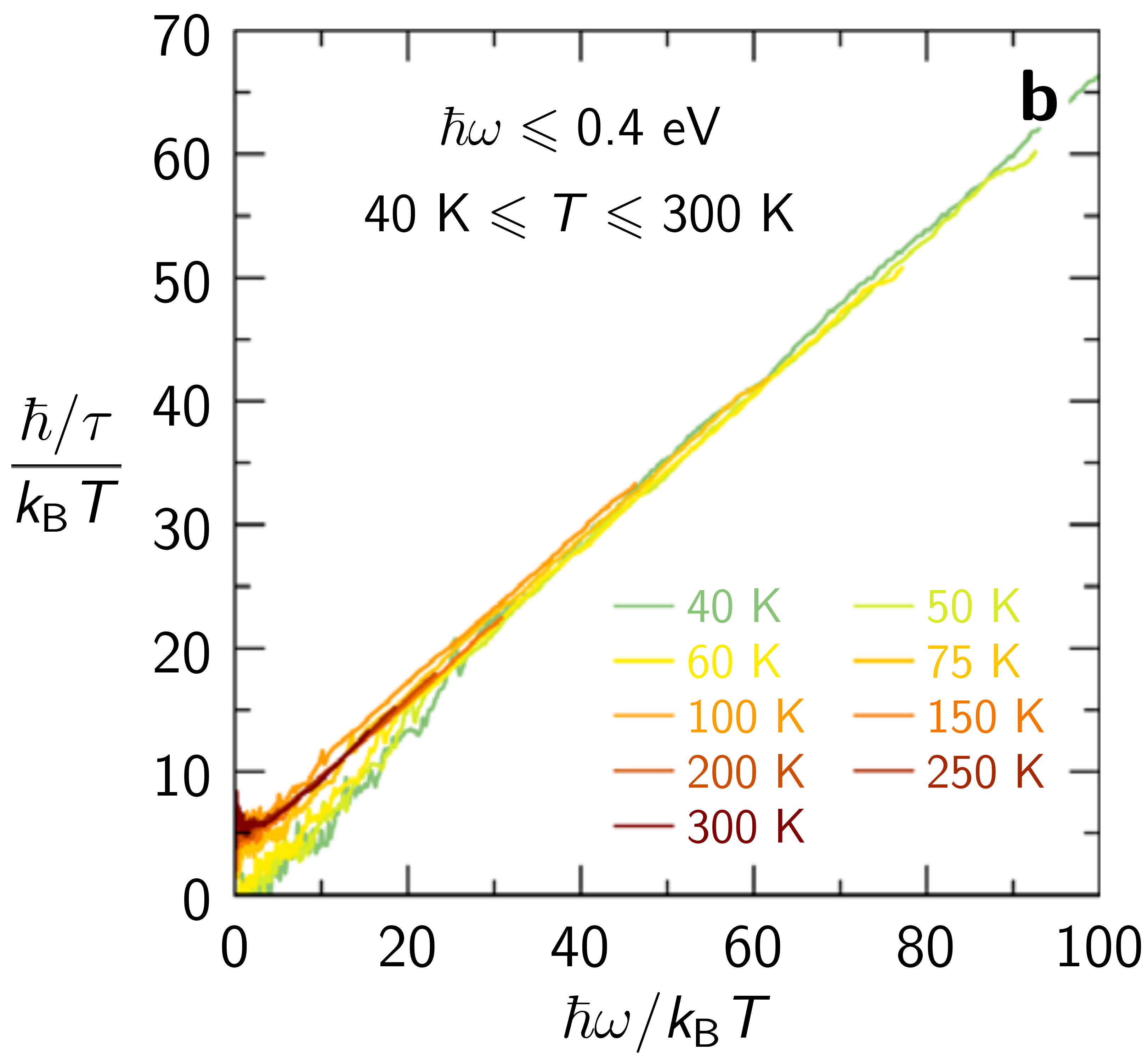
B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$

Planckian dynamics!



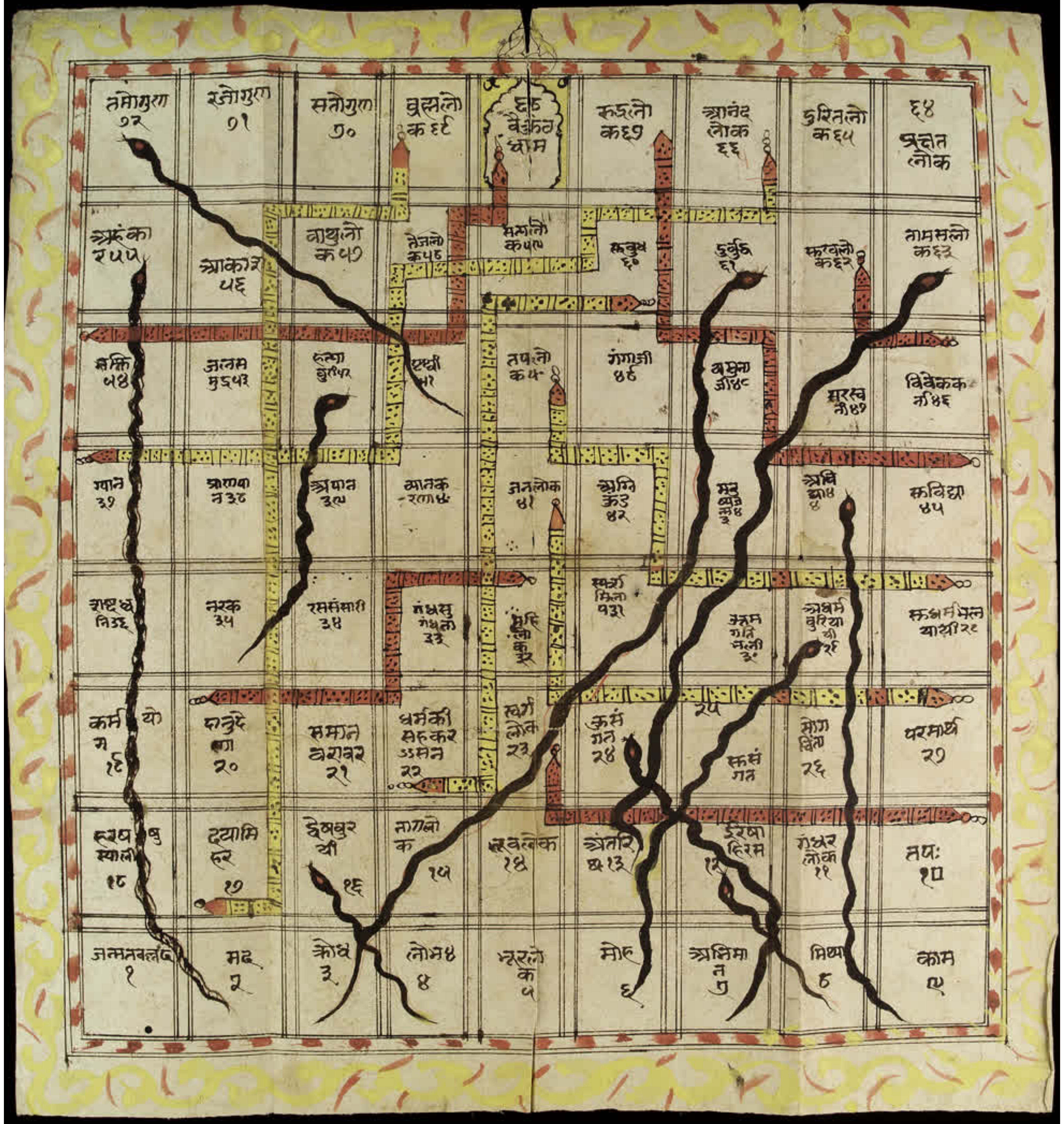


$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

Needed,
to solve open problems in the theory of superconductivity and black holes:

A solvable model of quantum entanglement of $3, 4, 5, \ldots \infty$ particles

The Sachdev-Ye-Kitaev model of many-particle entanglement



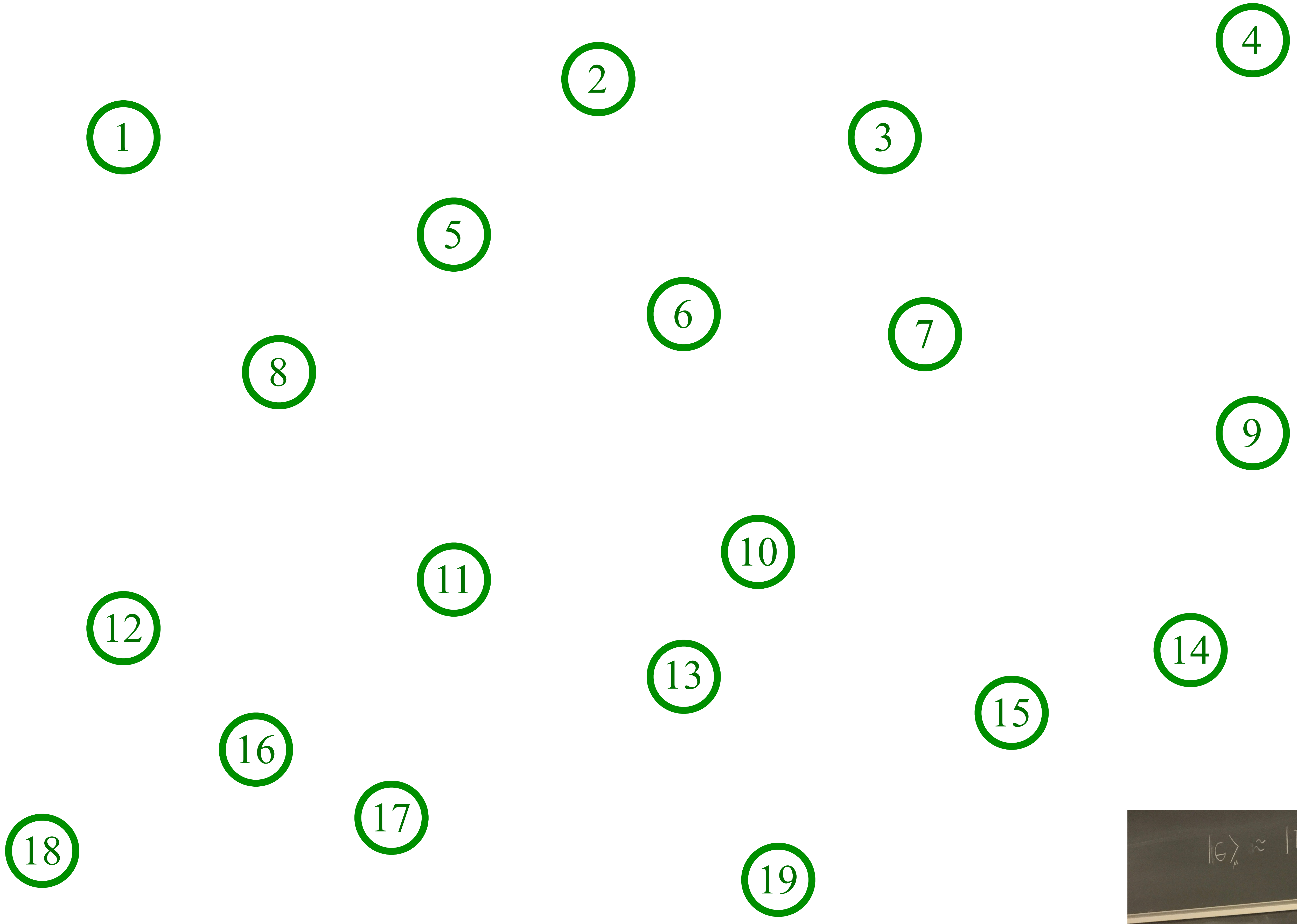
My spooky dream (1992)*

Ancient Indian game of Snakes and Ladders

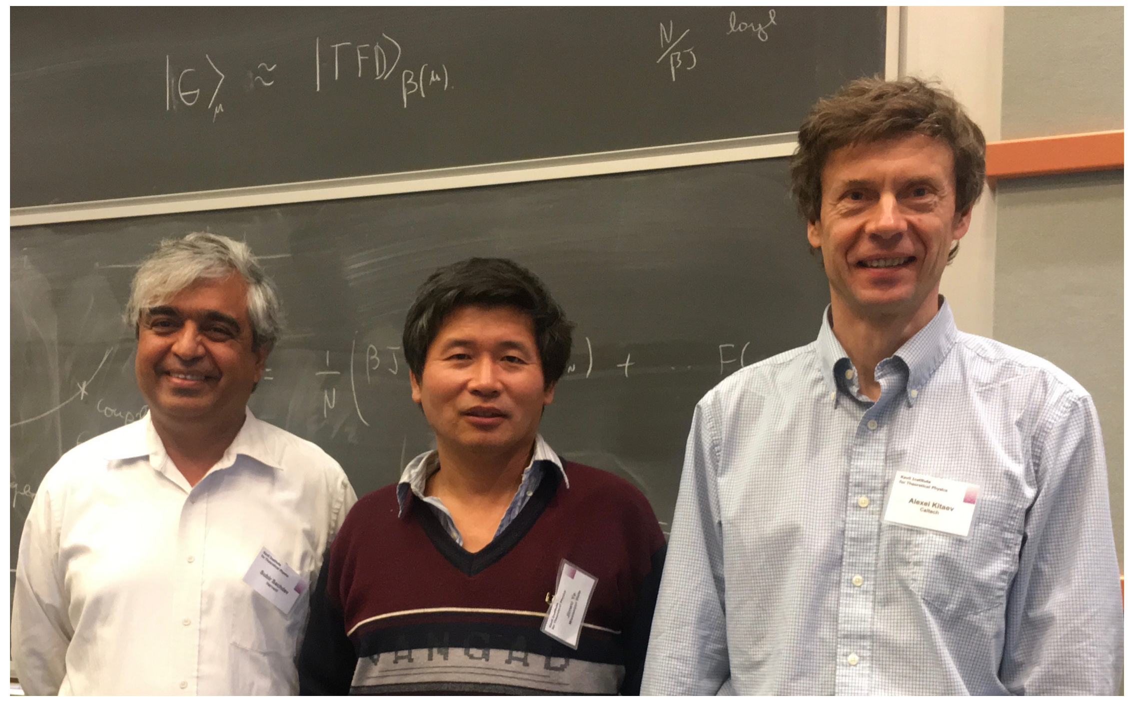
*Not true

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

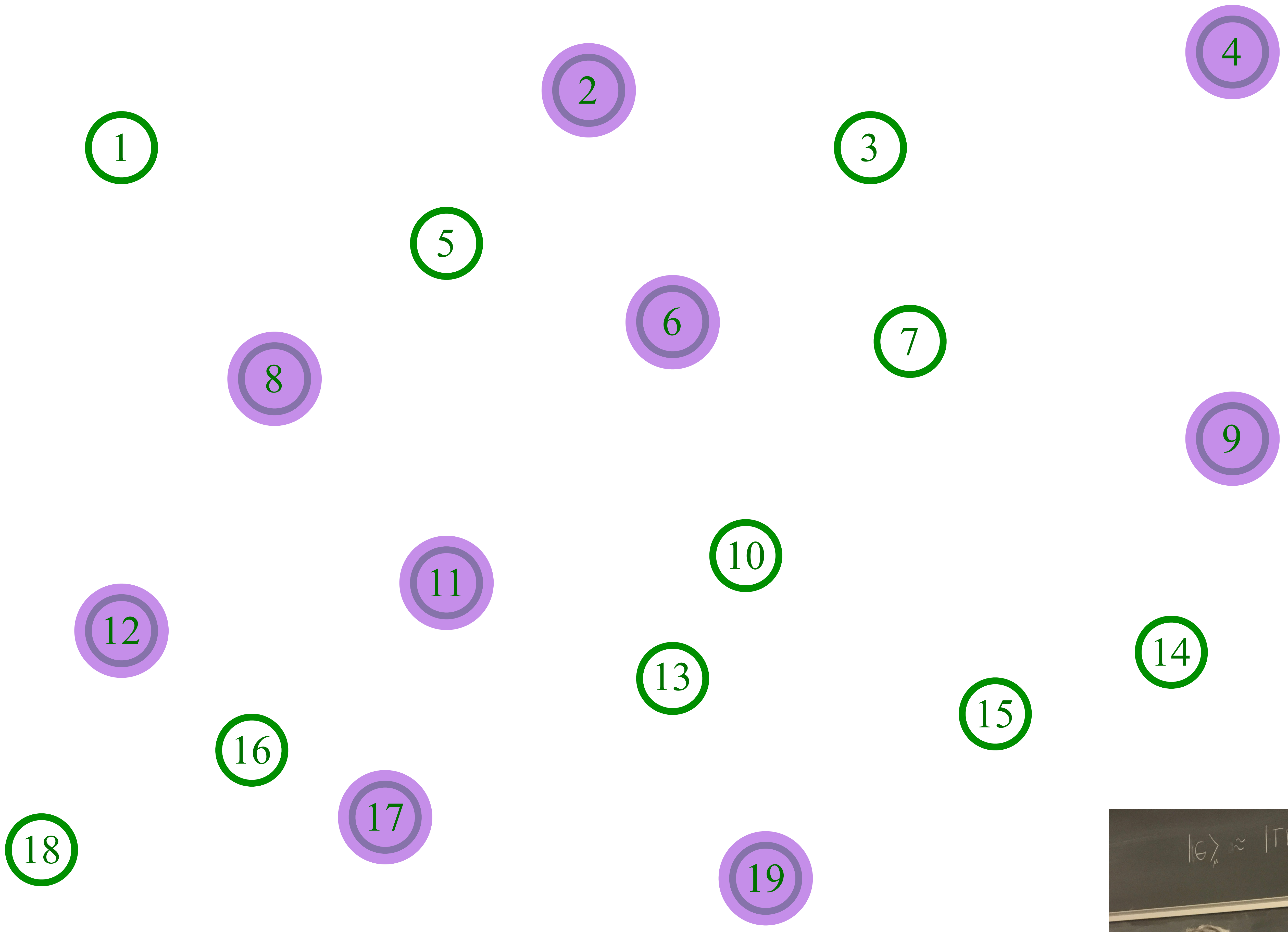


Pick a set of random positions

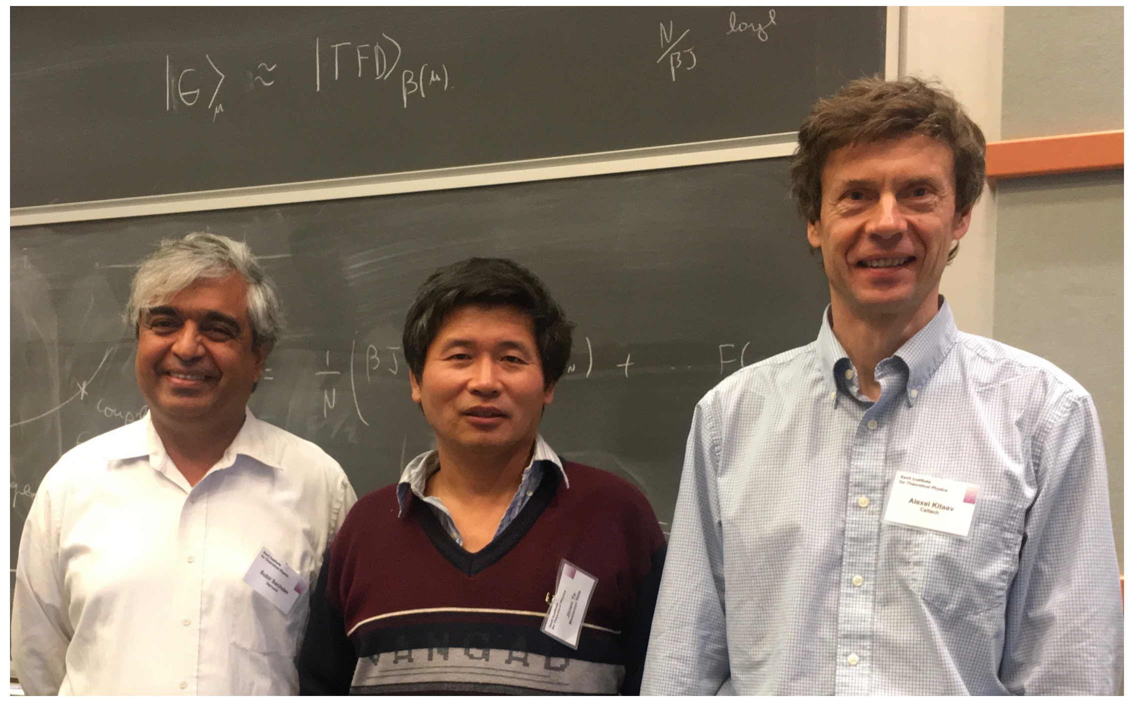


The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)



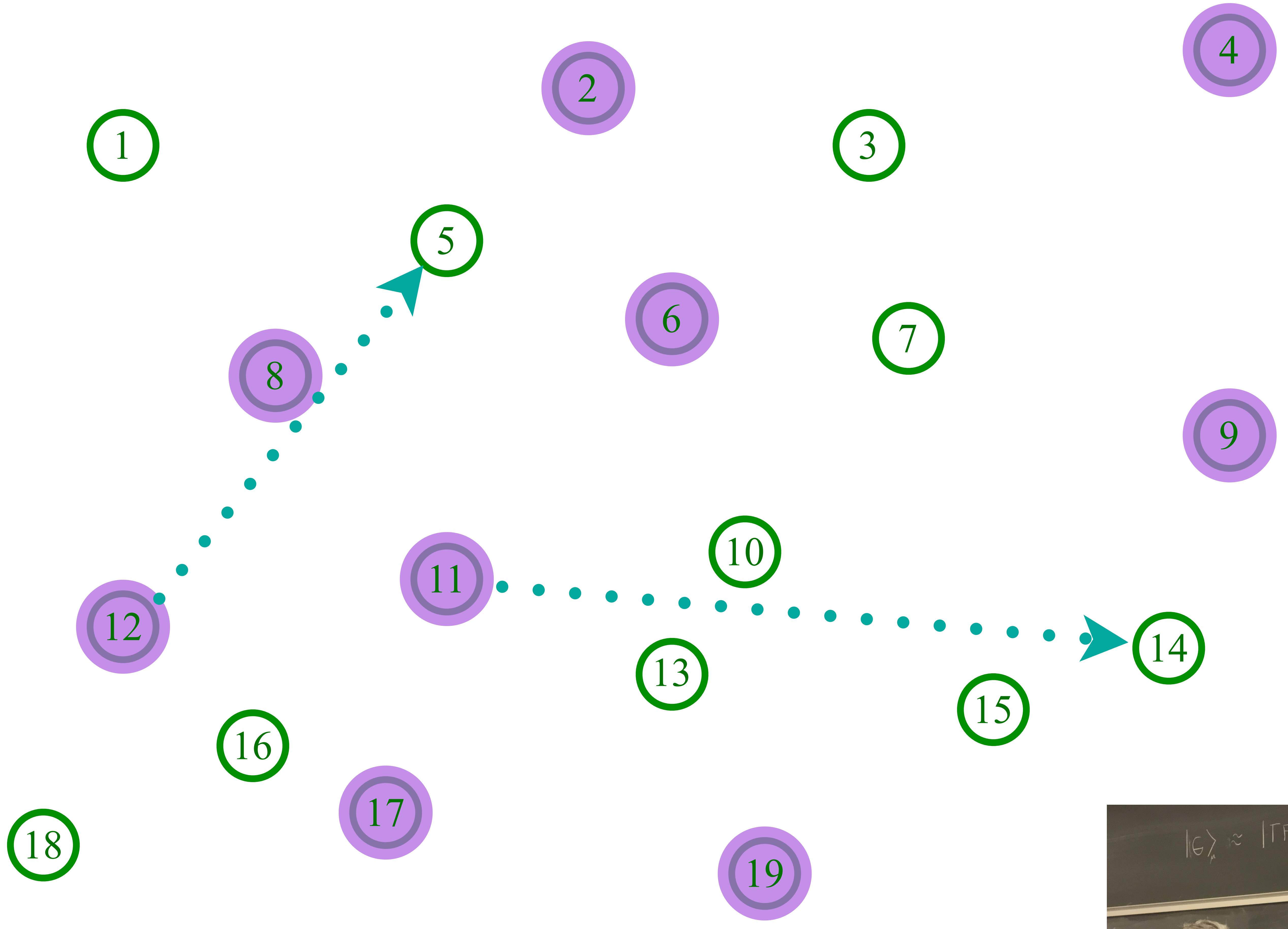
Place electrons randomly on some sites



The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



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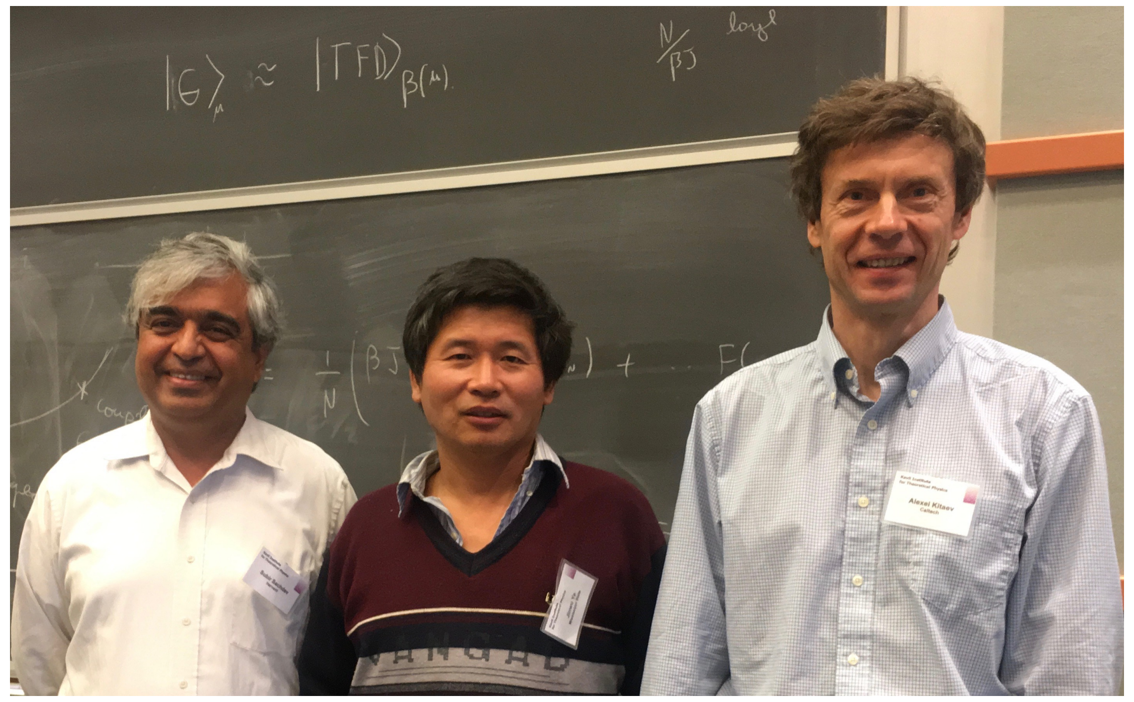

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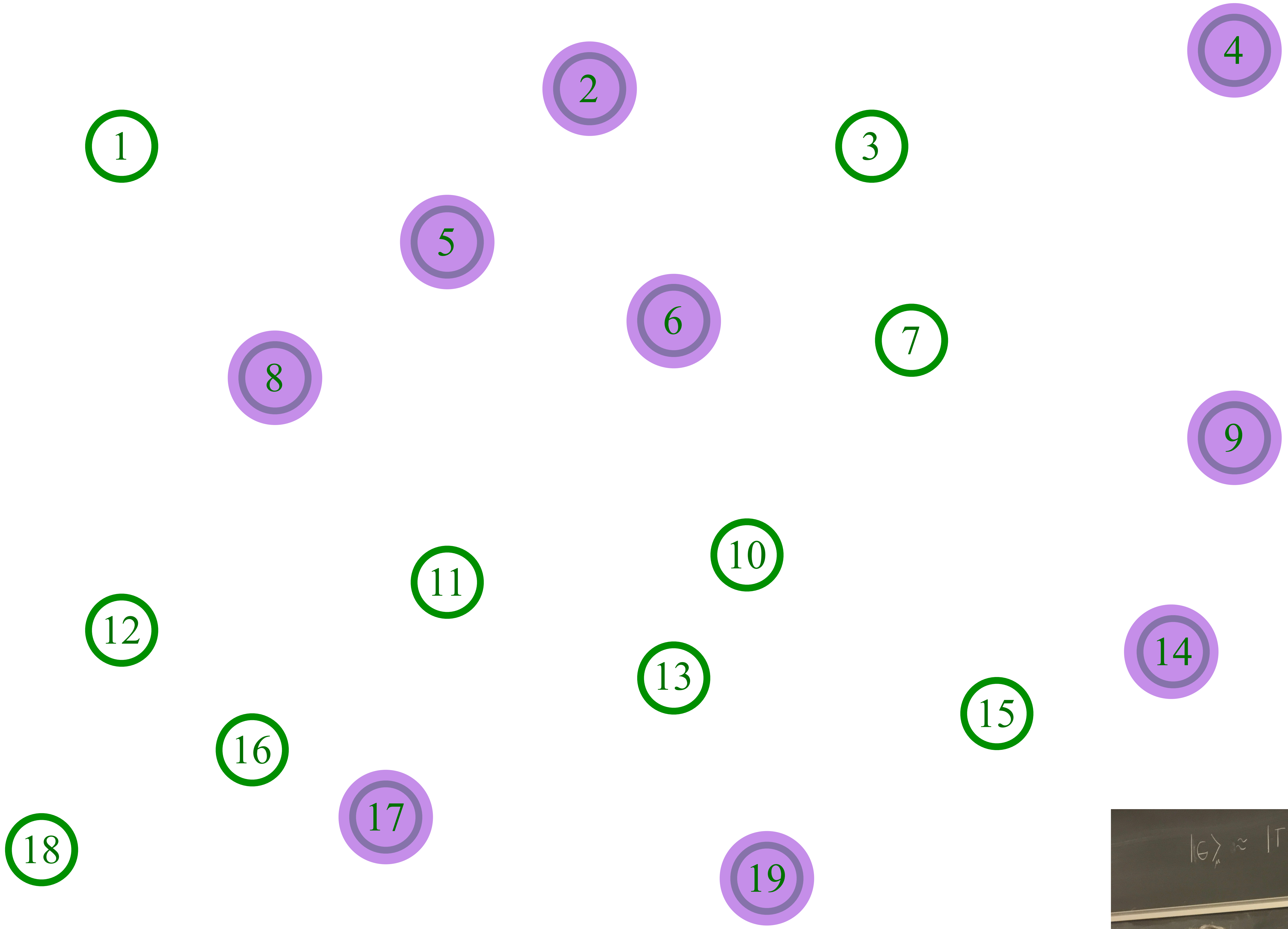
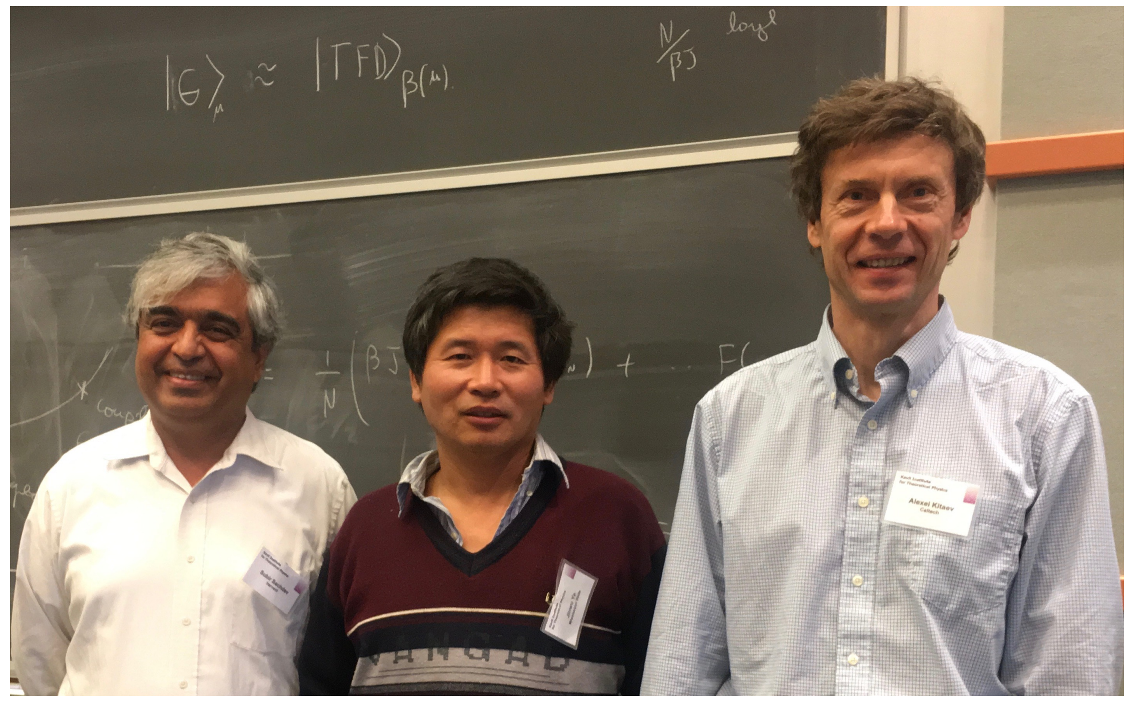
Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



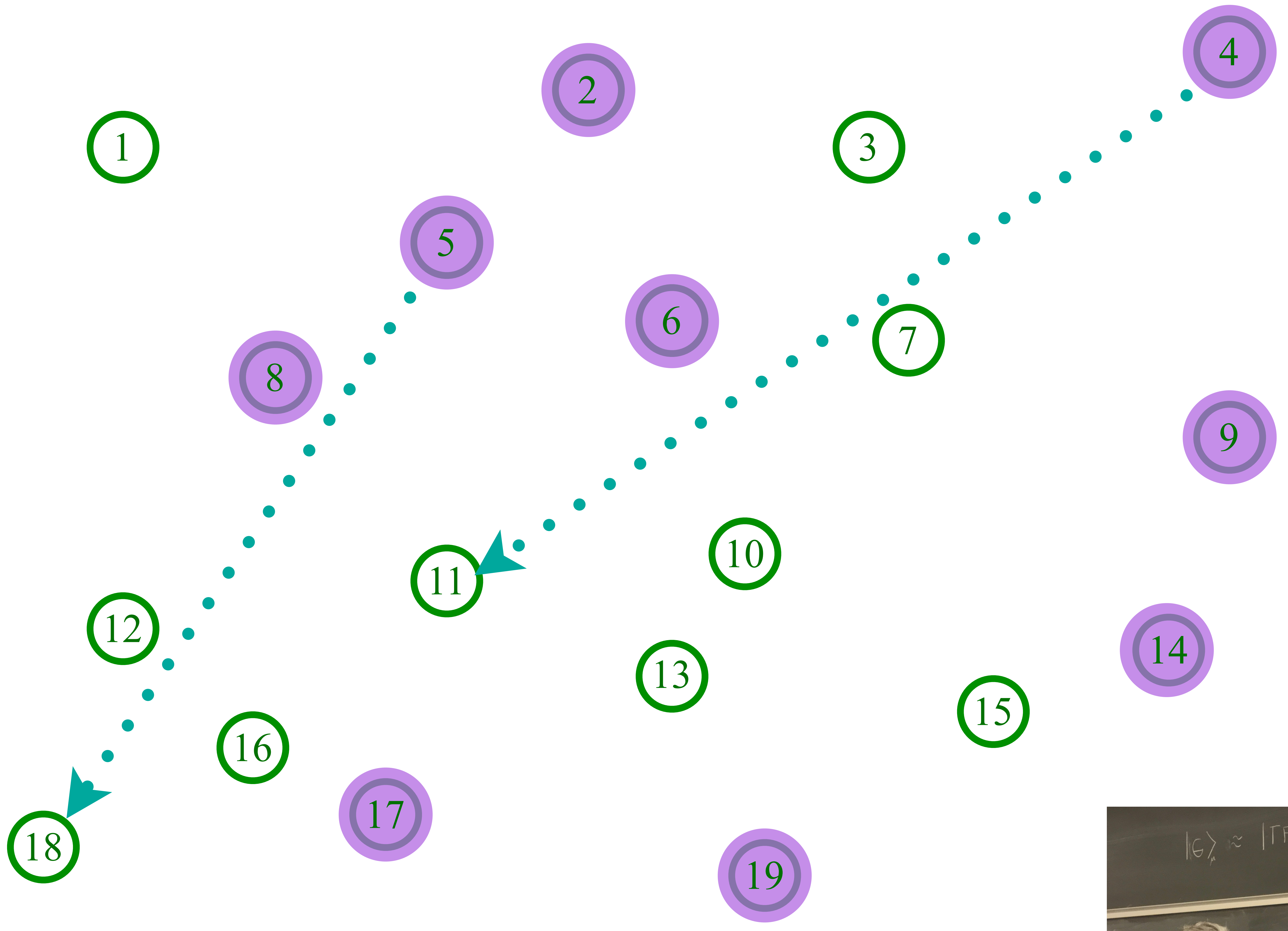
Entangle electrons pairwise randomly



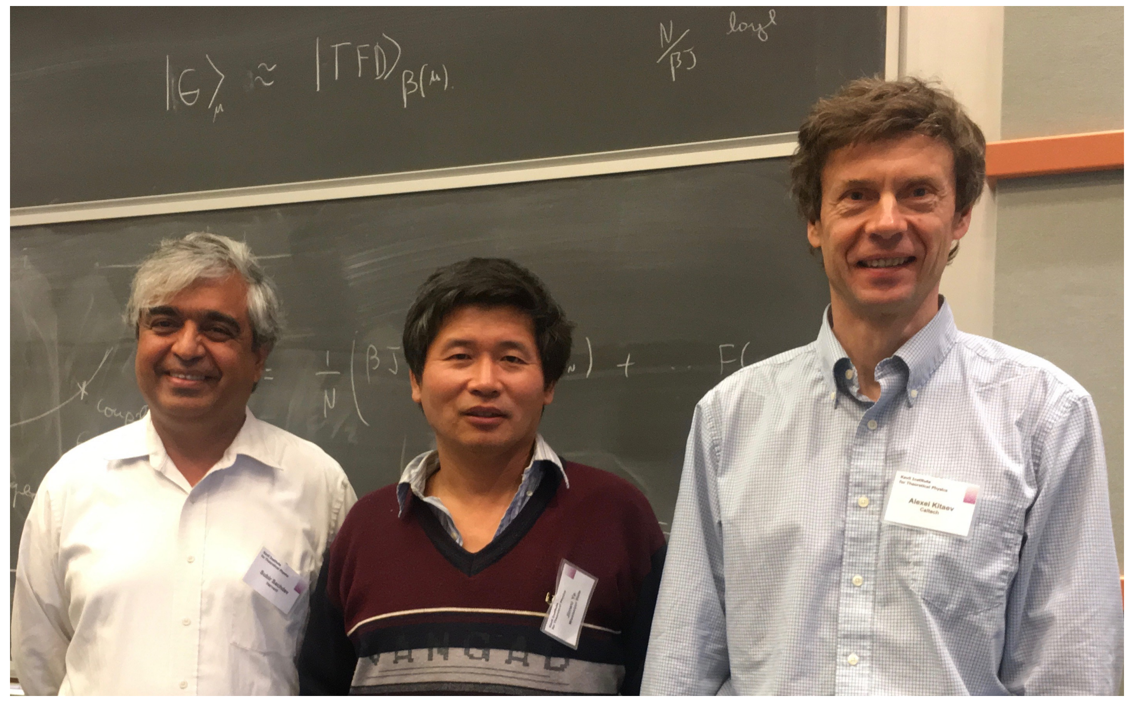
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



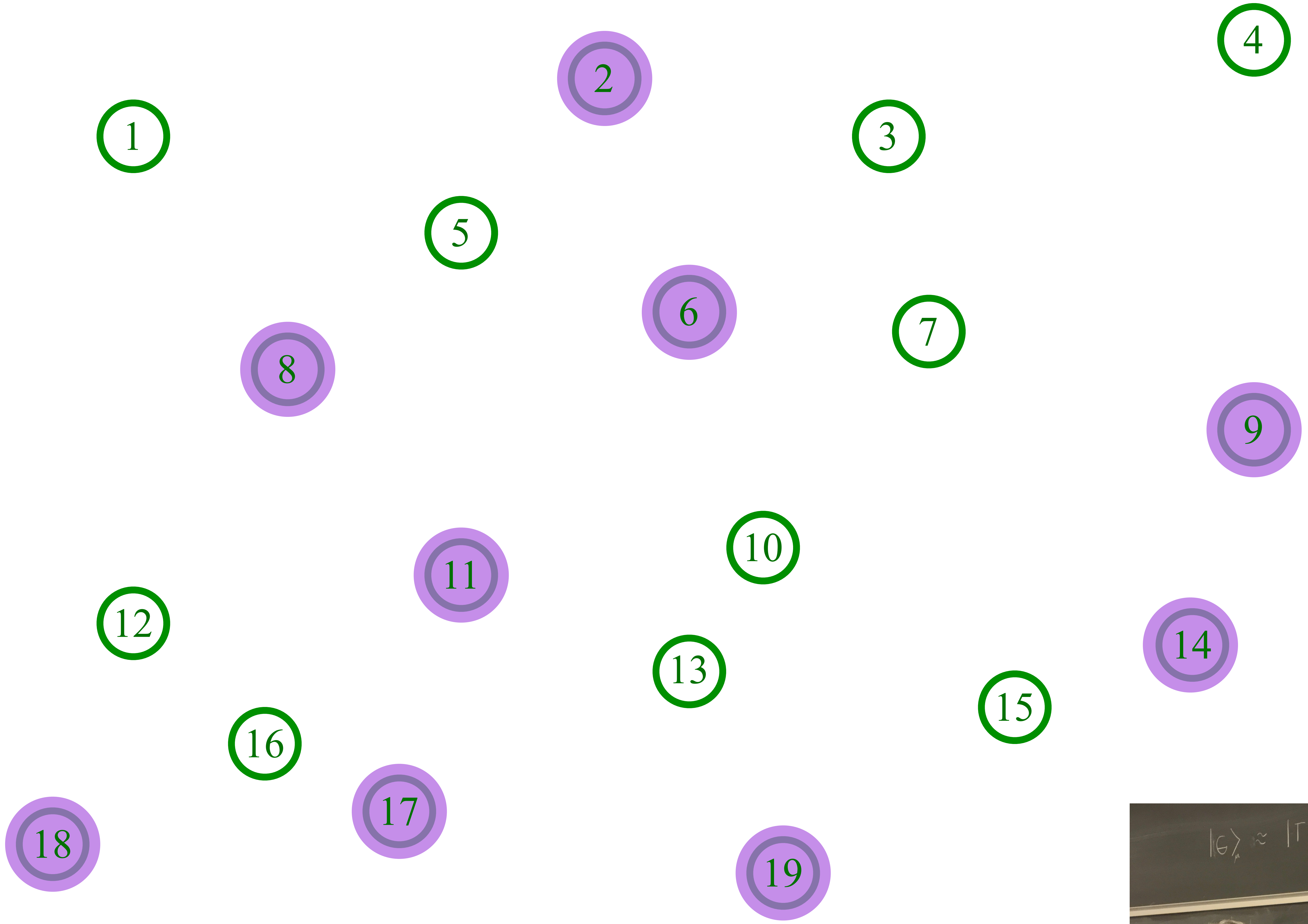
Entangle electrons pairwise randomly



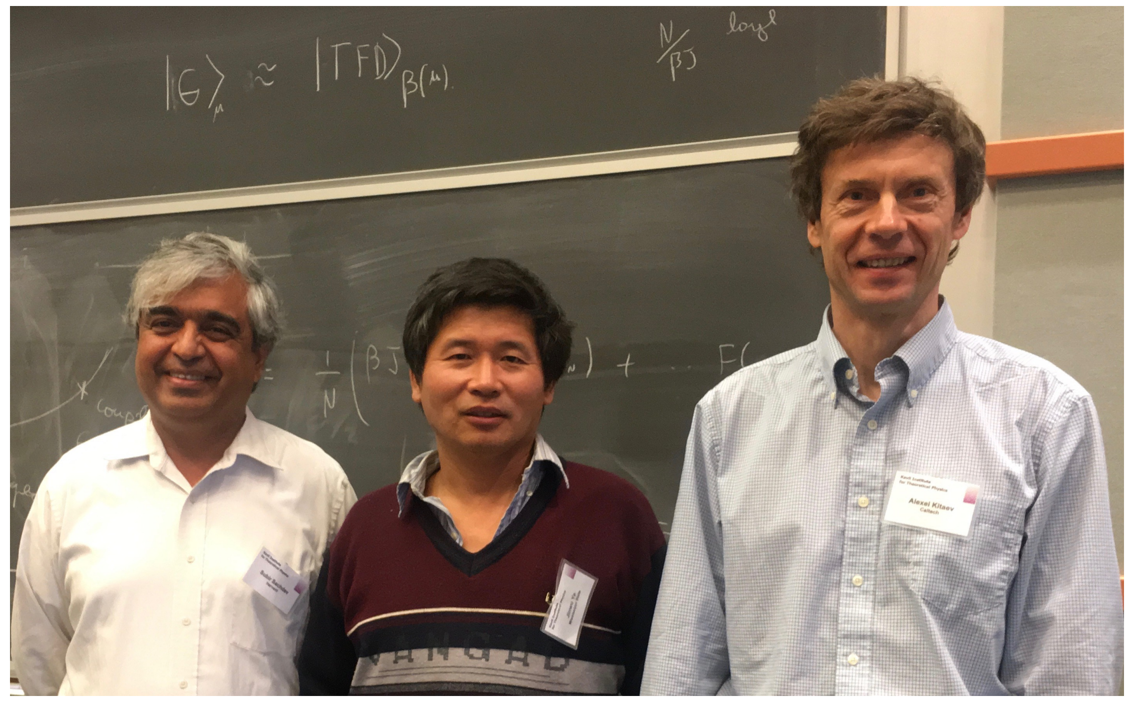
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



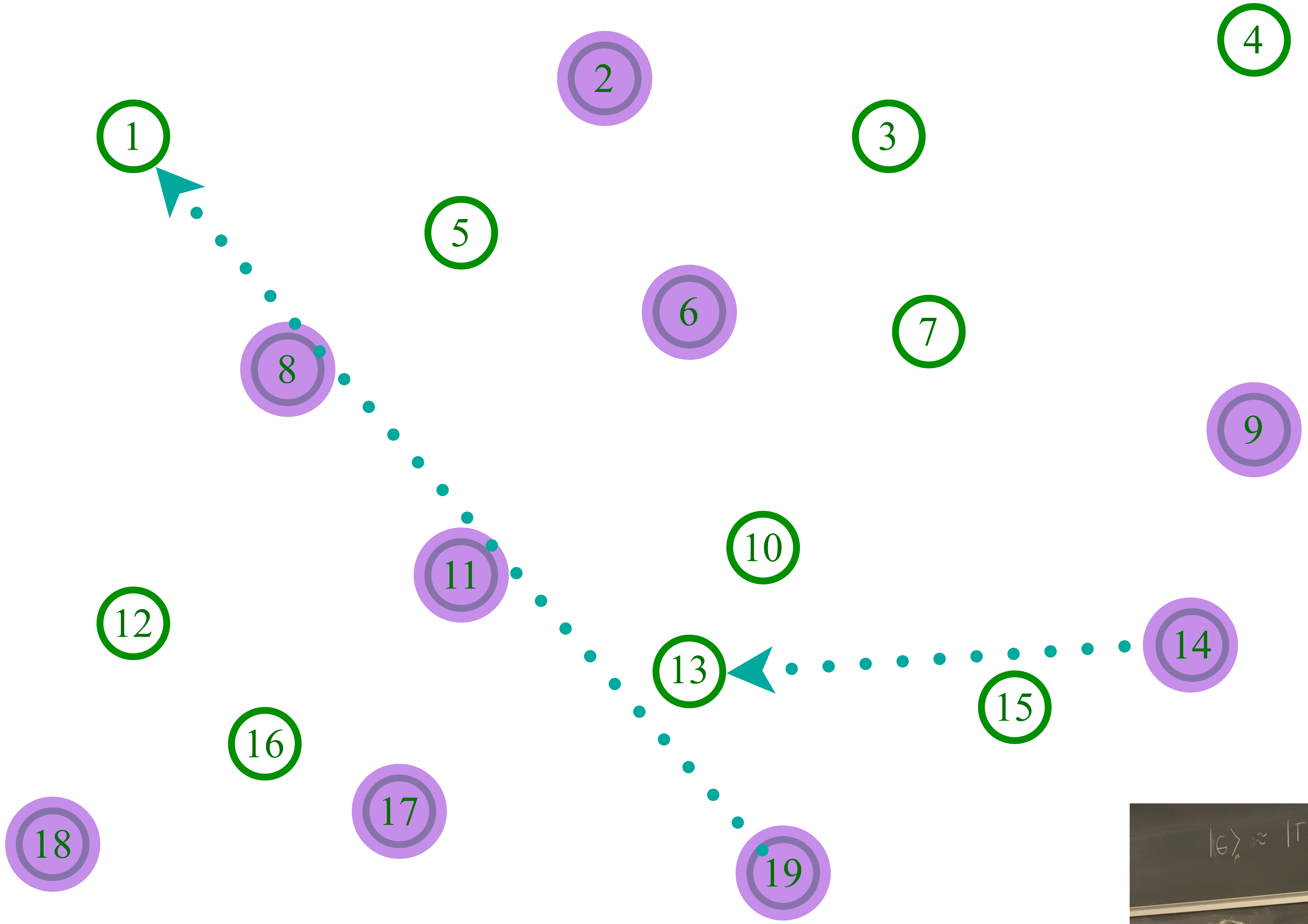
Entangle electrons pairwise randomly



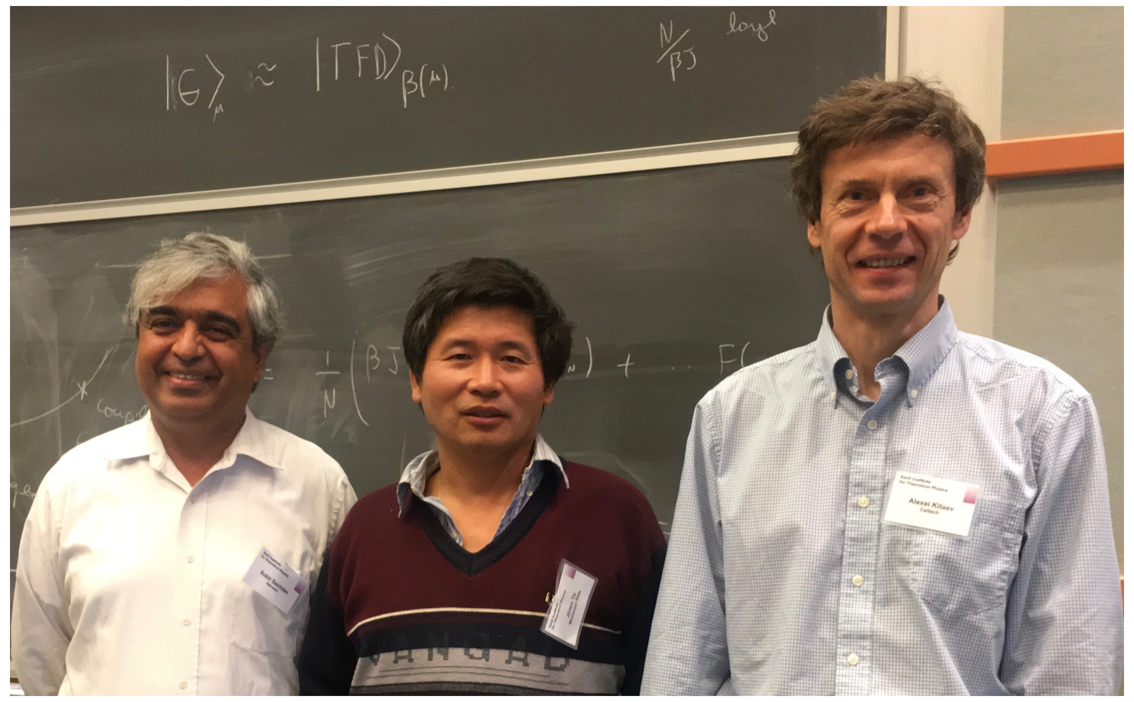
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



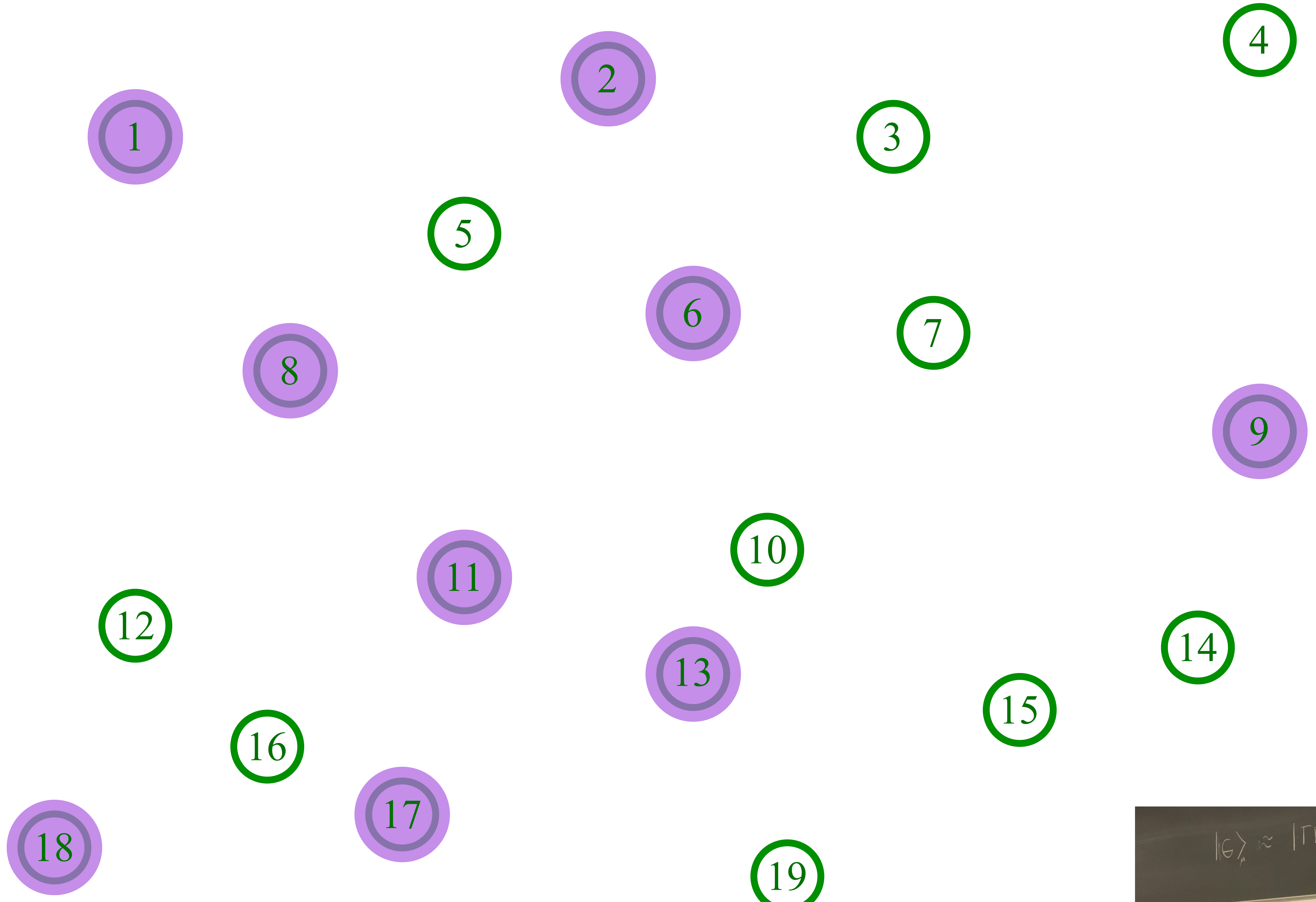
Entangle electrons pairwise randomly



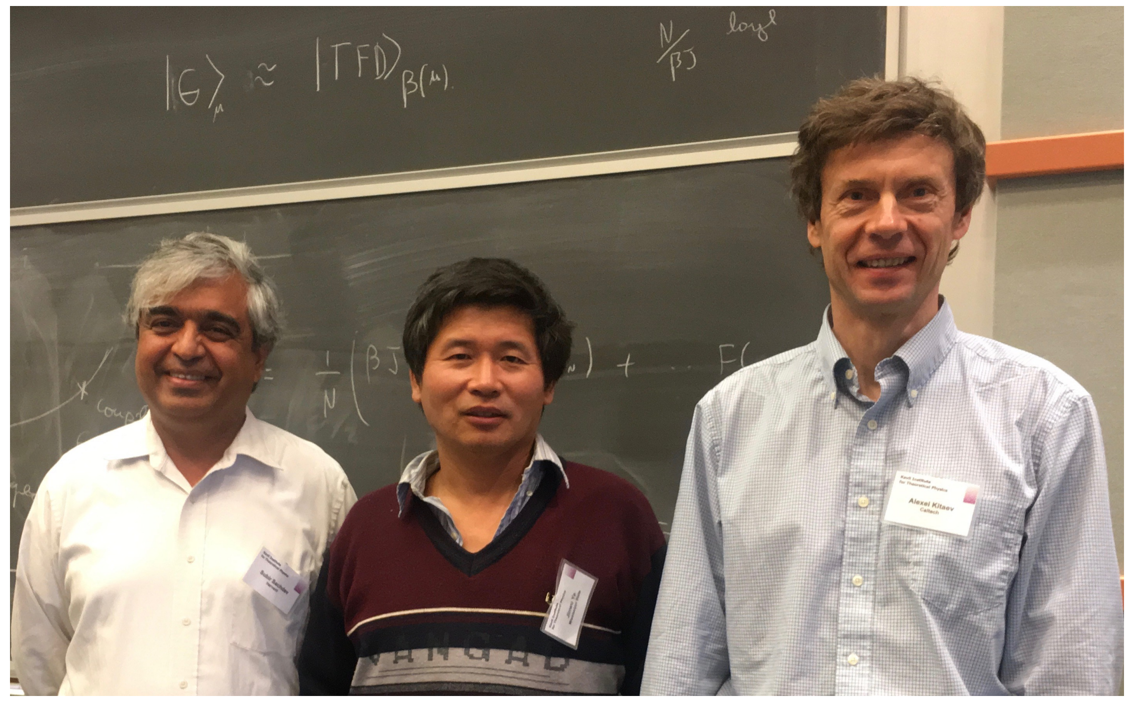
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



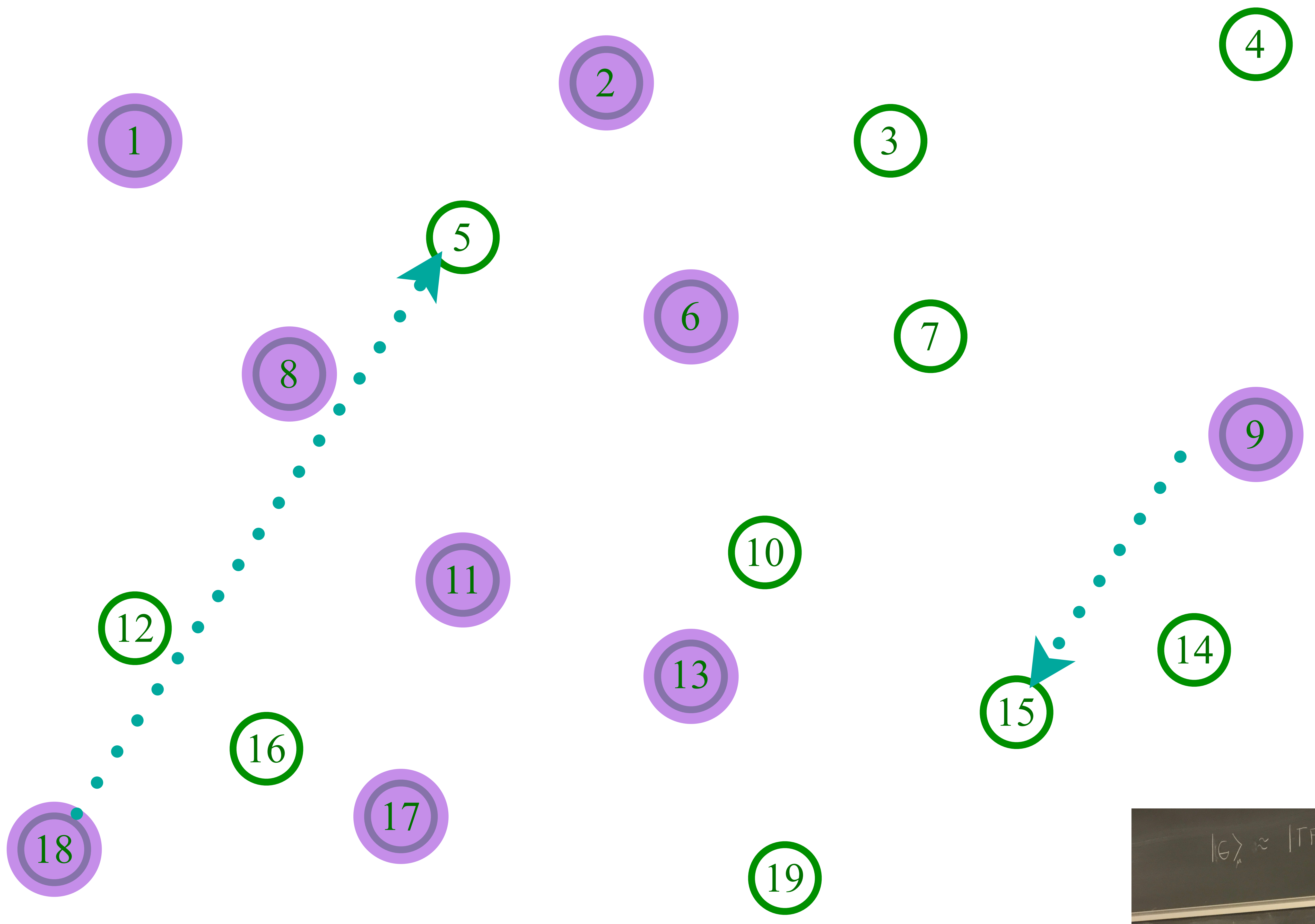
Entangle electrons pairwise randomly



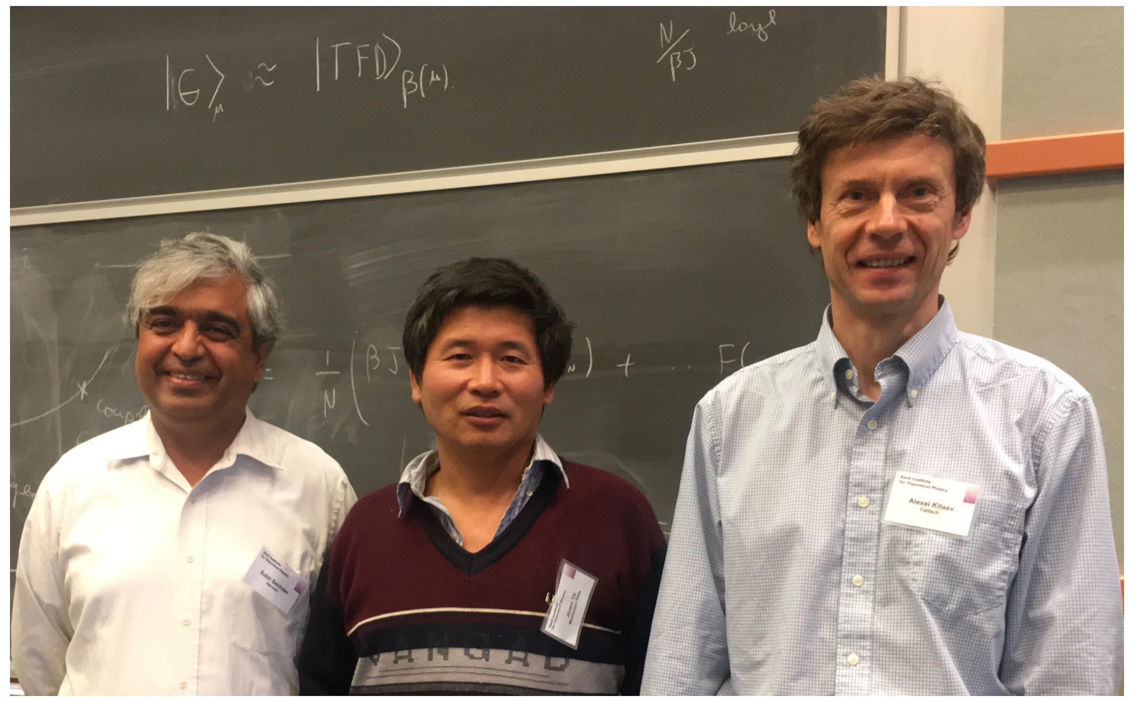
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$U_{9,18;5,15}$



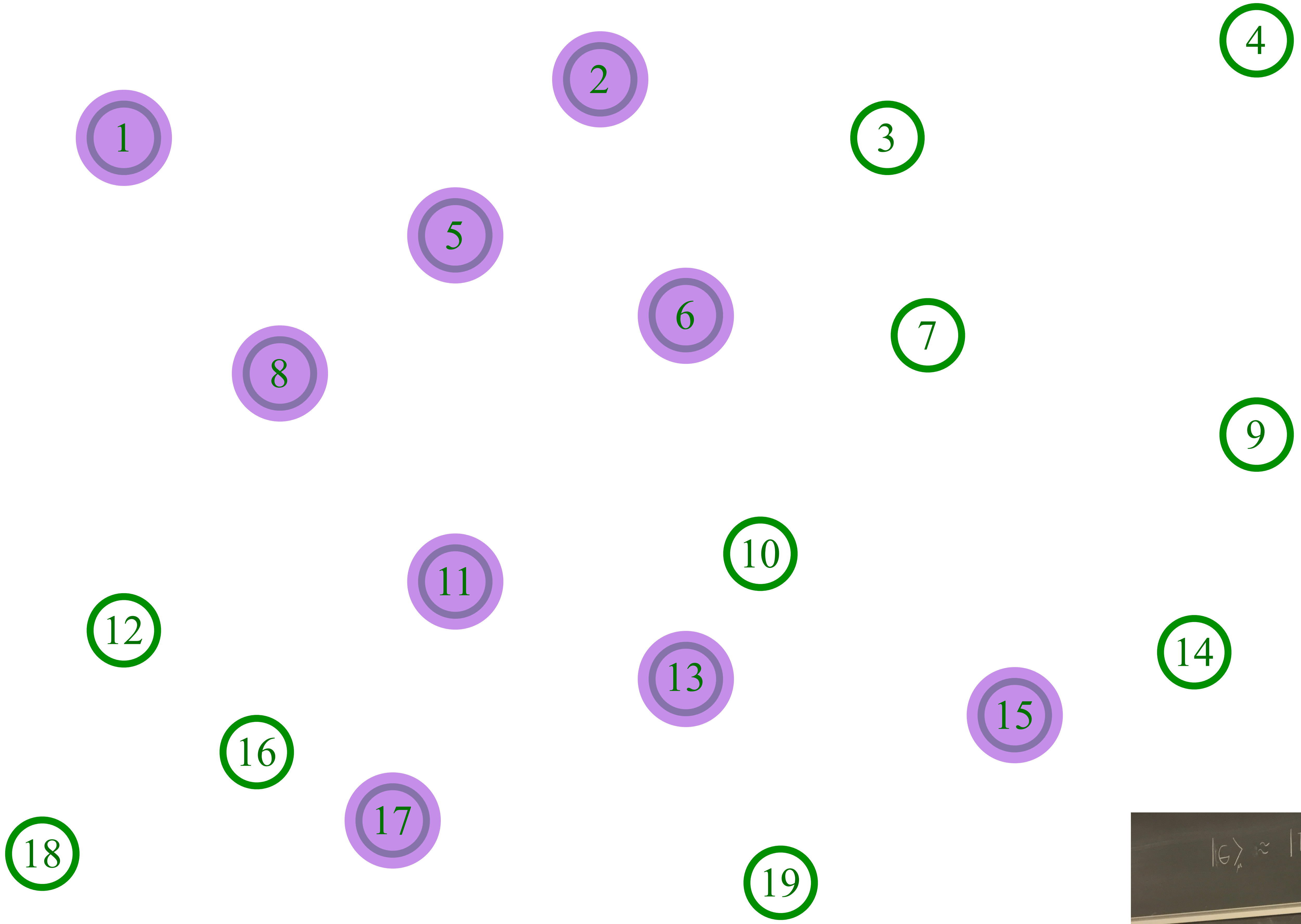
Entangle electrons pairwise randomly



The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



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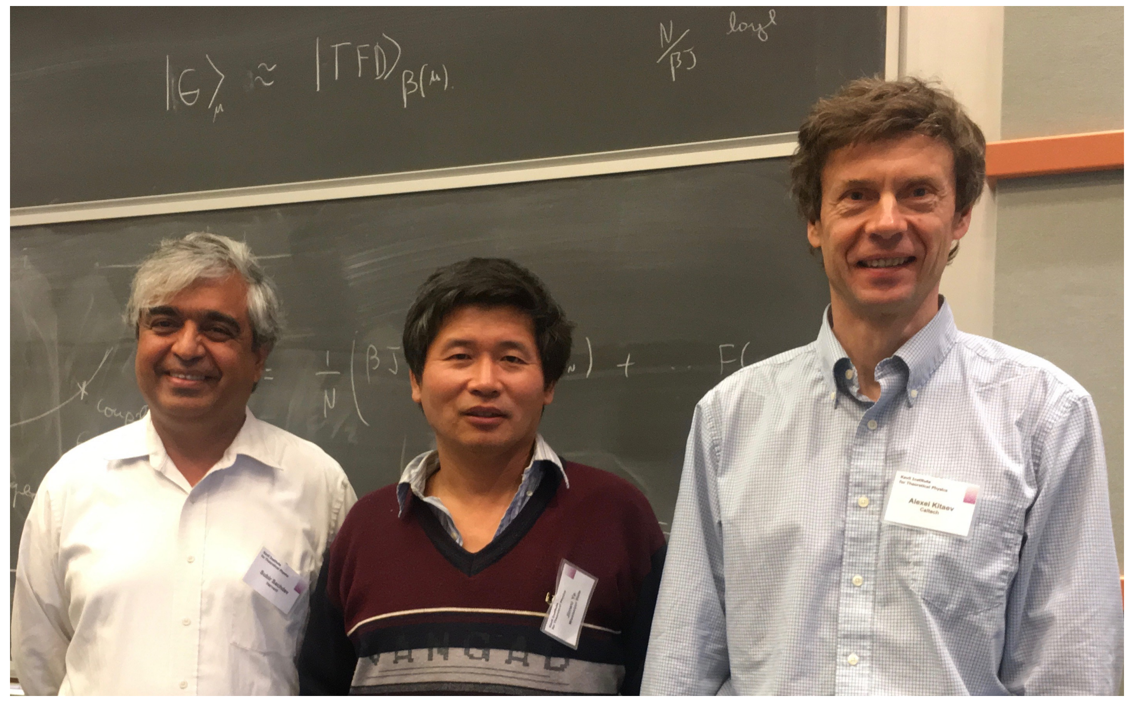
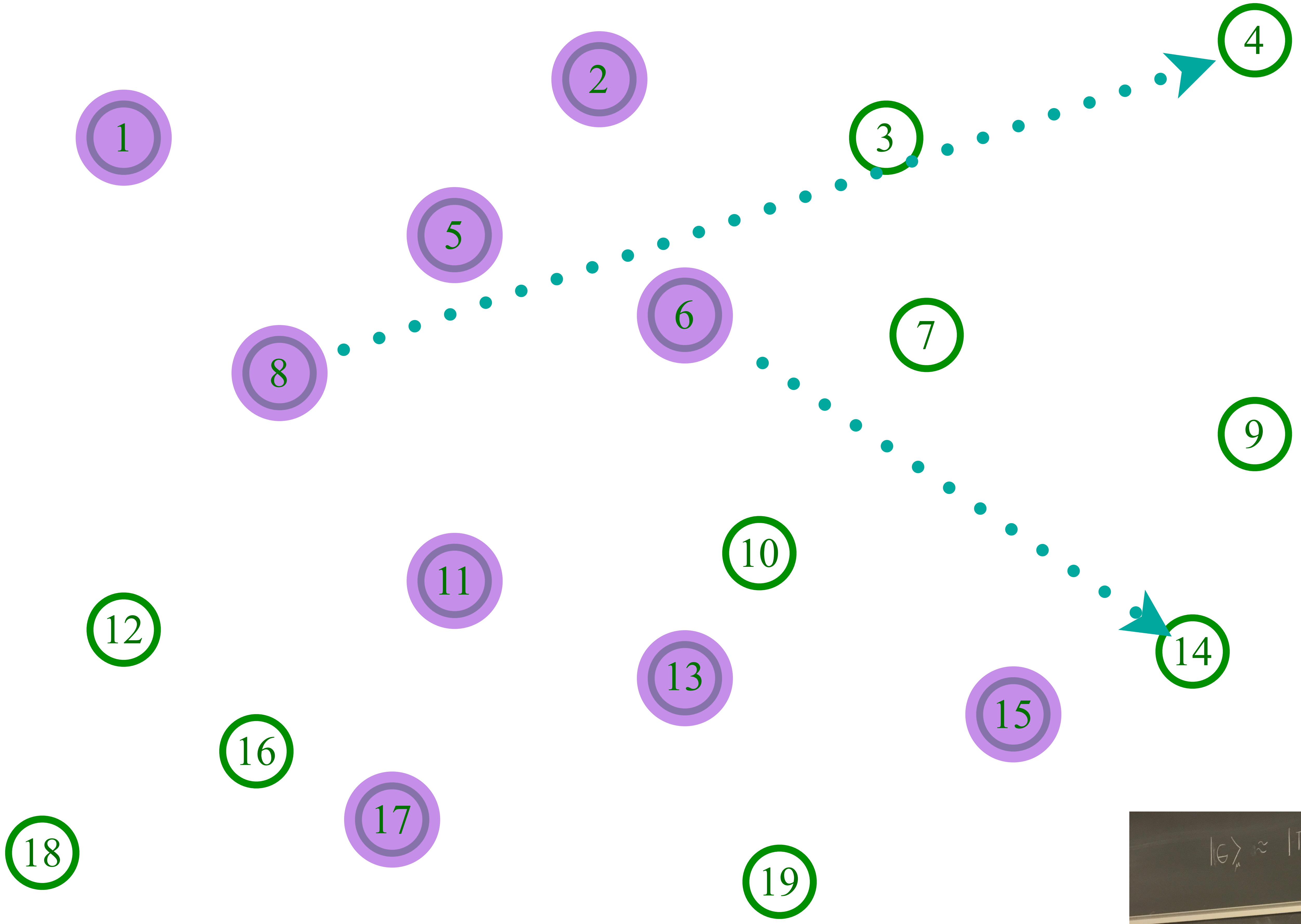


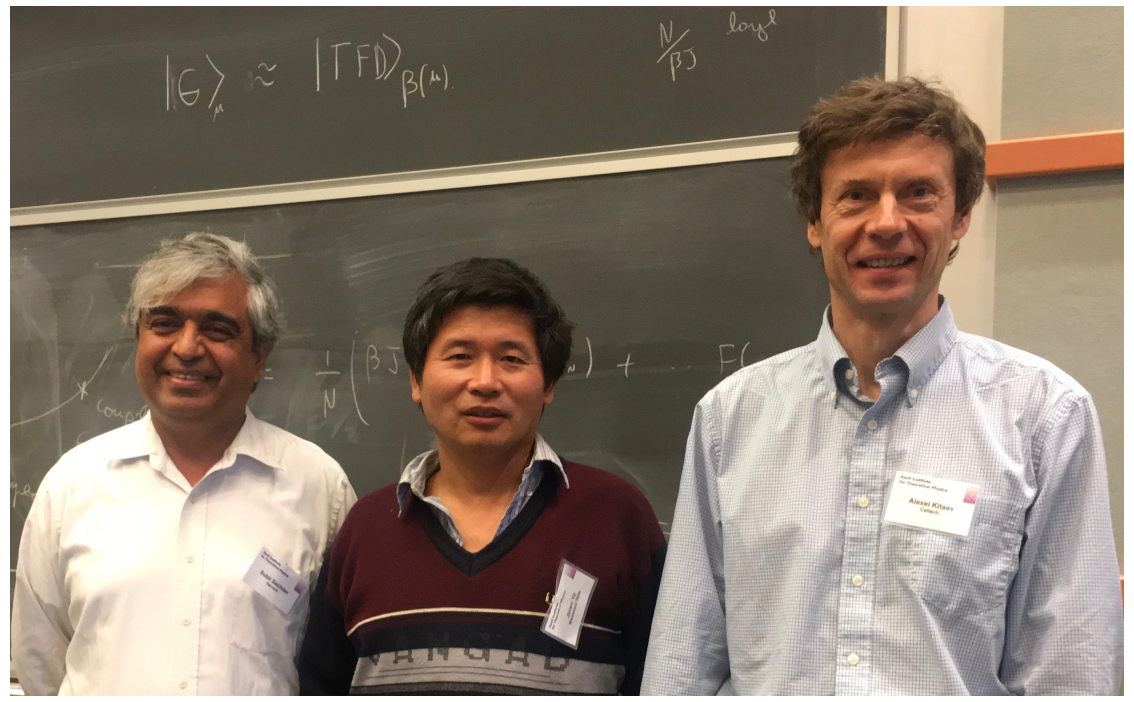
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



Entangle electrons pairwise randomly



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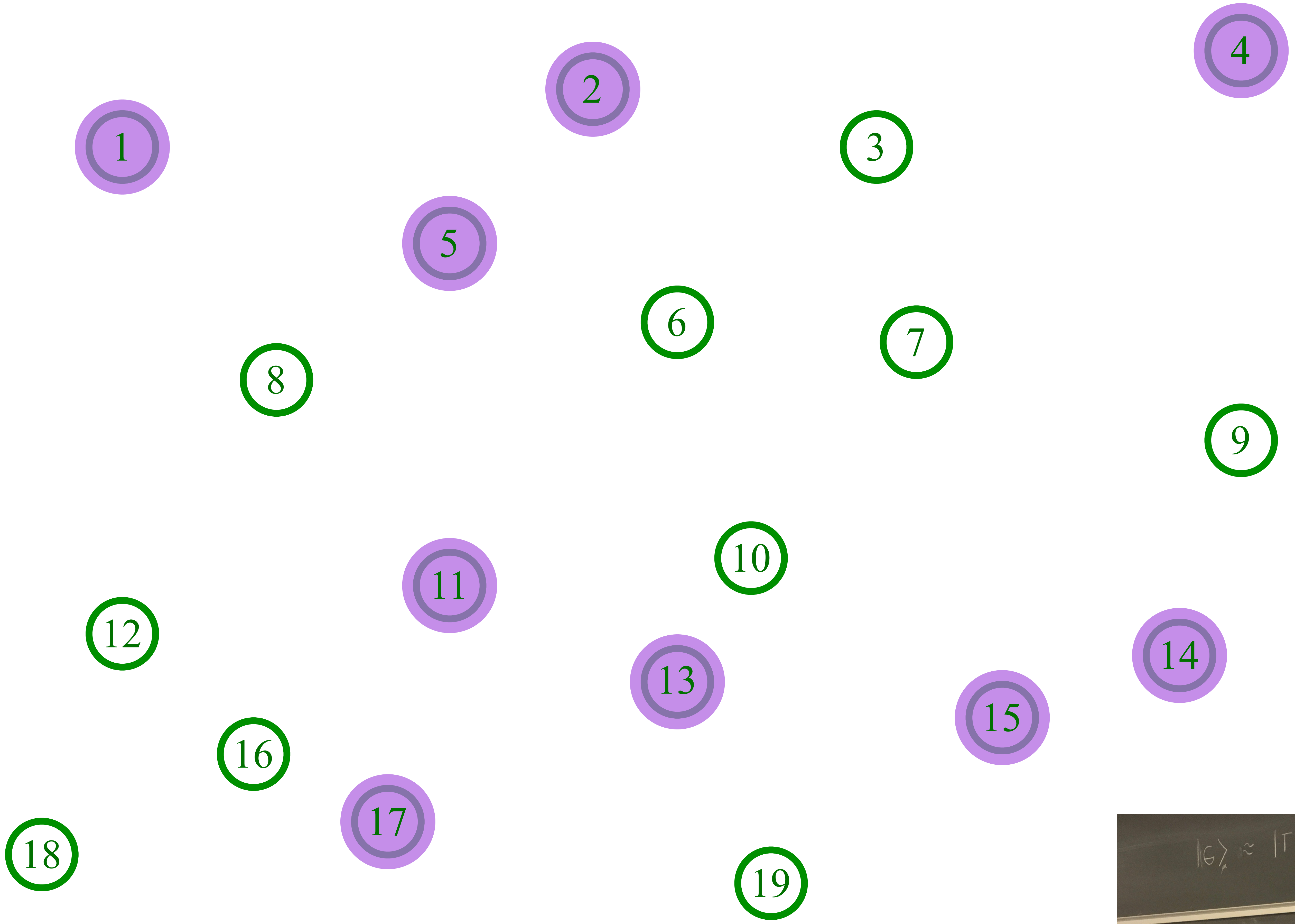
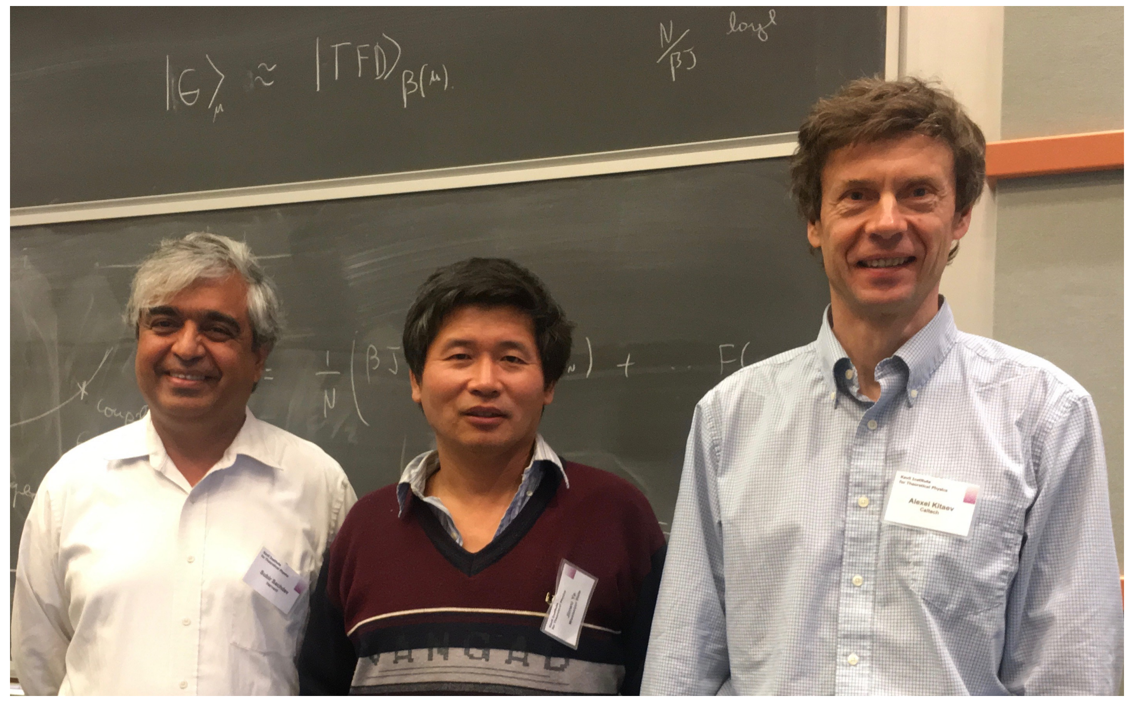


Entangle electrons pairwise randomly



The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

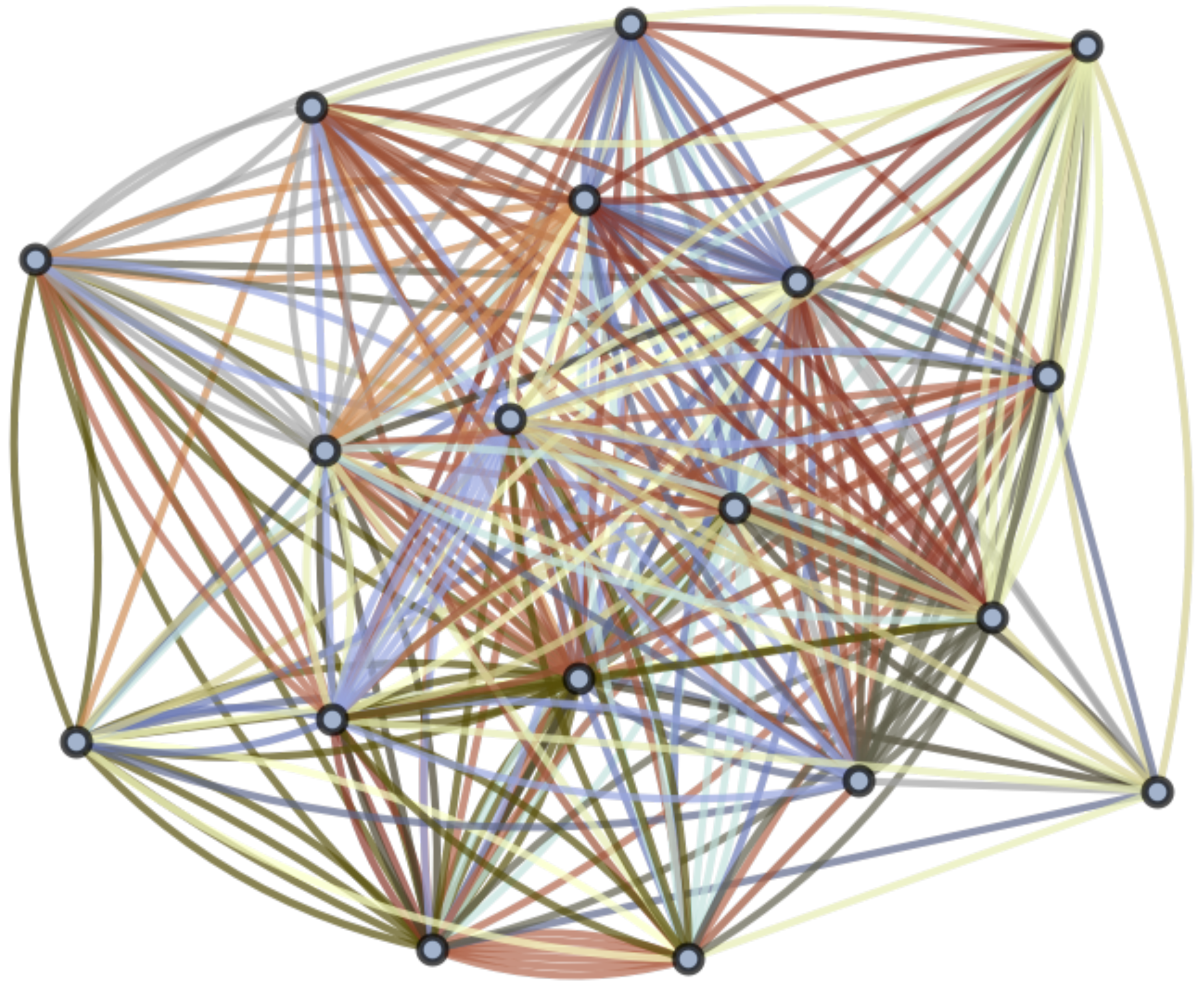
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

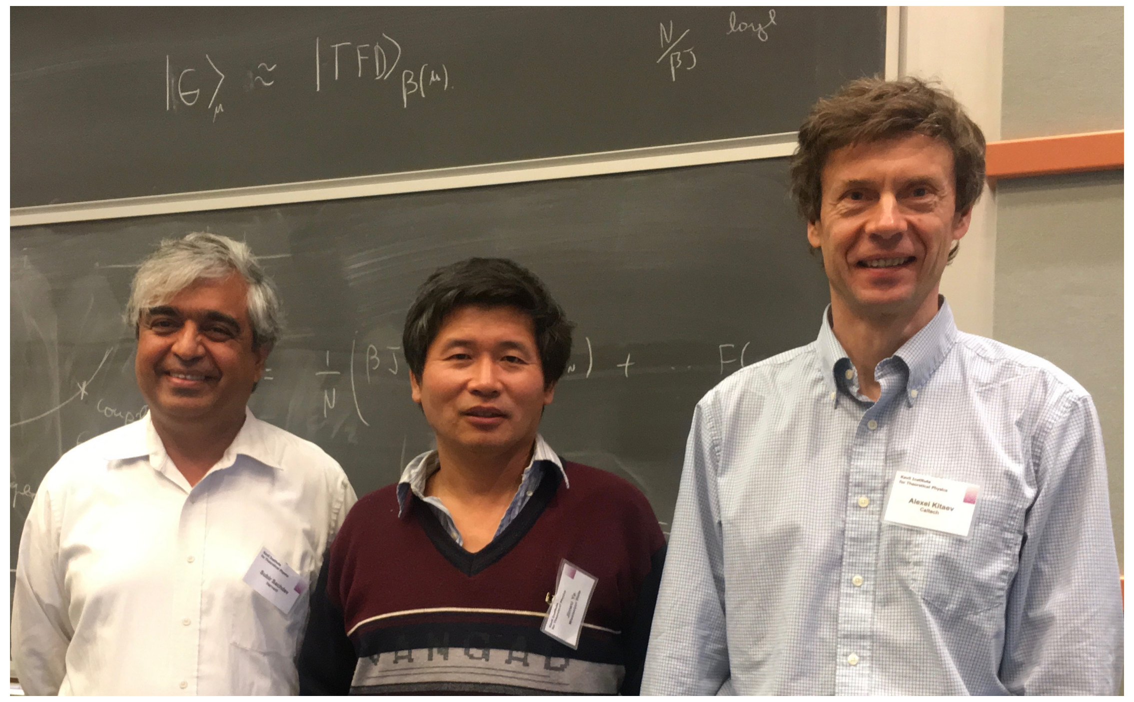
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha \, ; \quad [\mathcal{H}, \mathcal{Q}] = 0 \, ; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





A simple model of a metal with quasiparticles

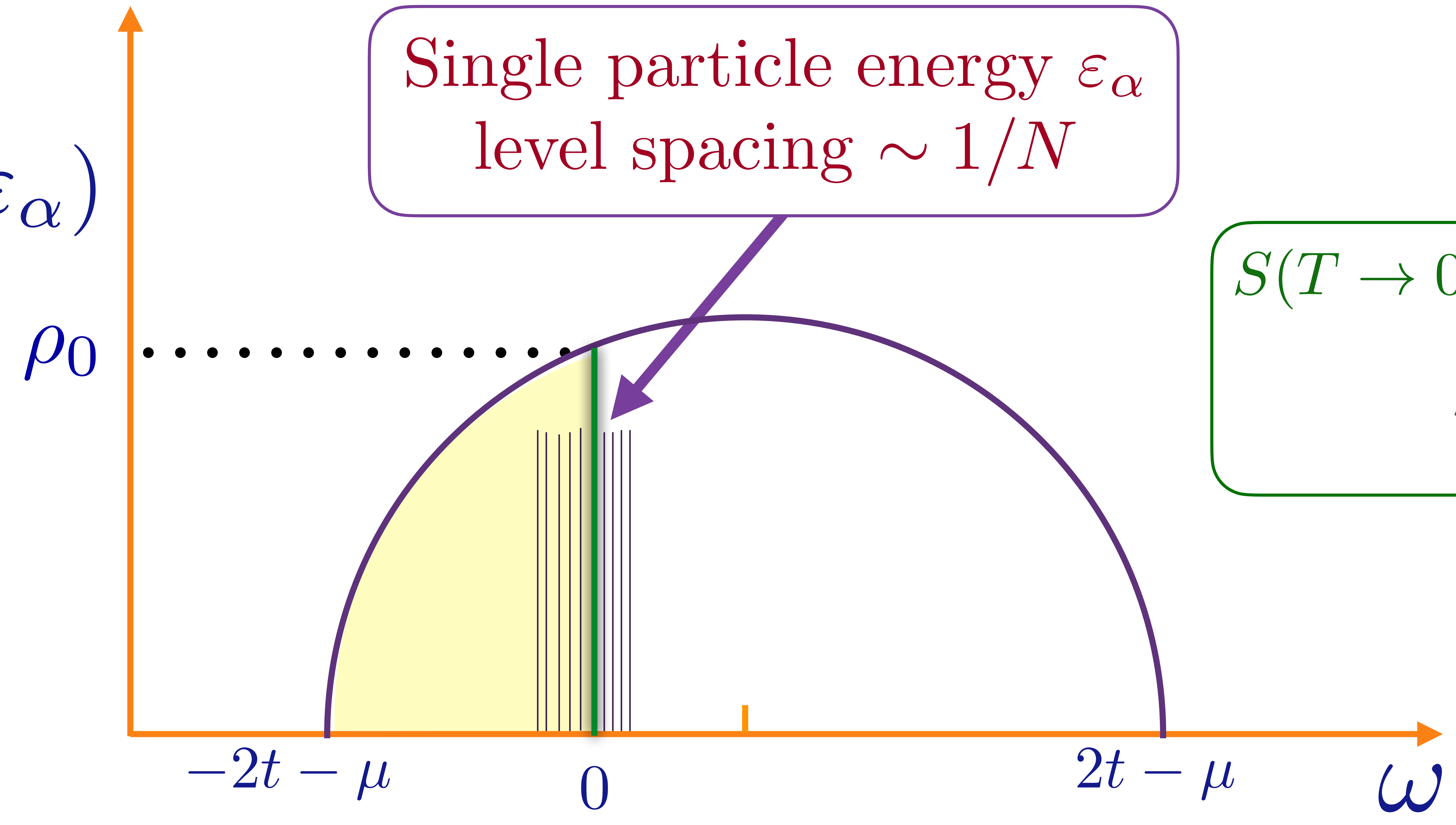
$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

$\rho(\omega) = \frac{1}{N} \sum_\alpha \delta(\omega - \varepsilon_\alpha)$

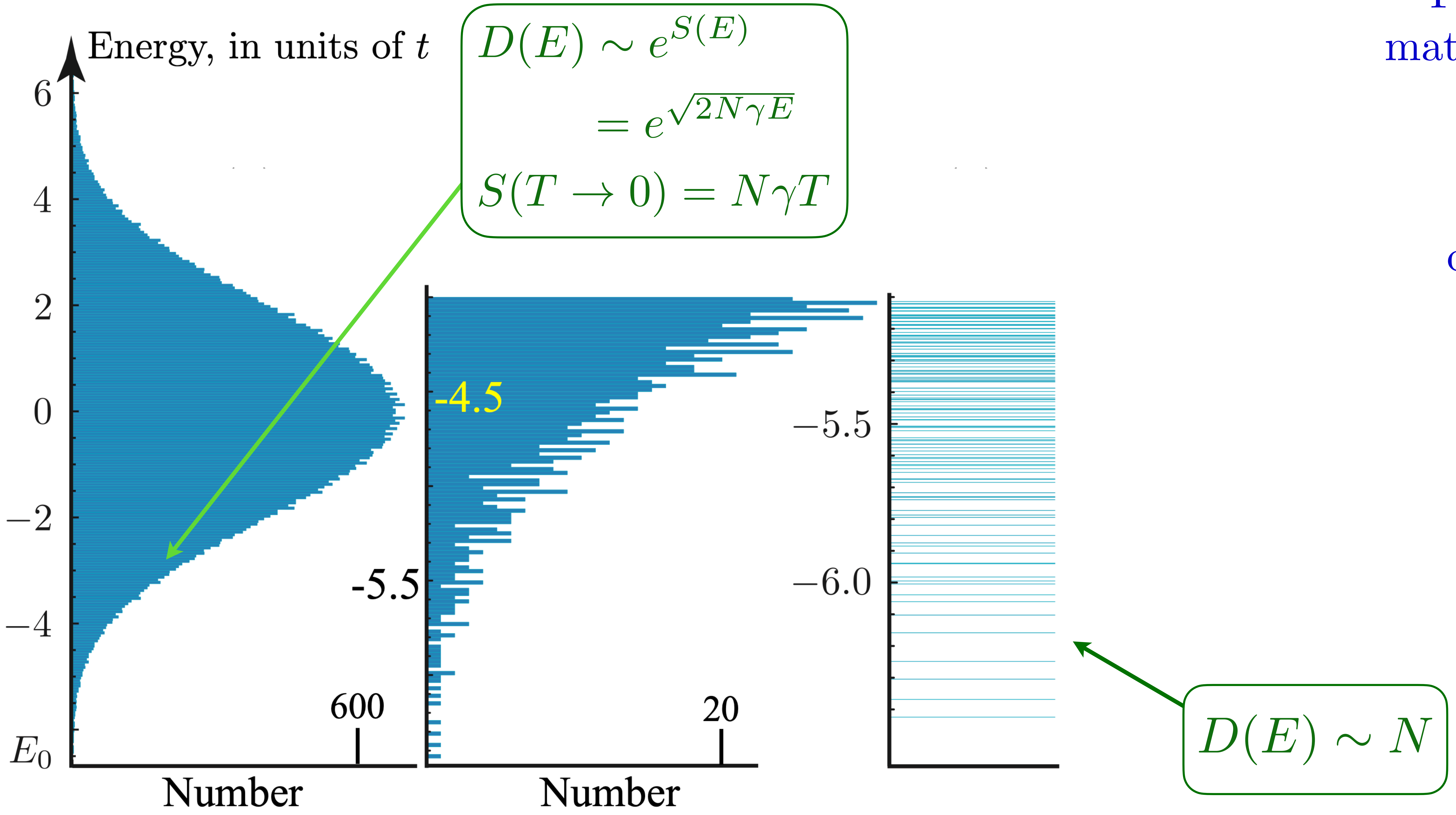


$S(T \to 0) = N\gamma T$

$\gamma = \frac{\pi^2}{3} \rho_0$

Many-body density of states

$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow$ Many body eigenvalue



For random matrix model: $E_0 + E_i = \sum_\alpha n_\alpha \varepsilon_\alpha$ $n_\alpha = 0, 1,$ occupation number

Random matrix model

The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

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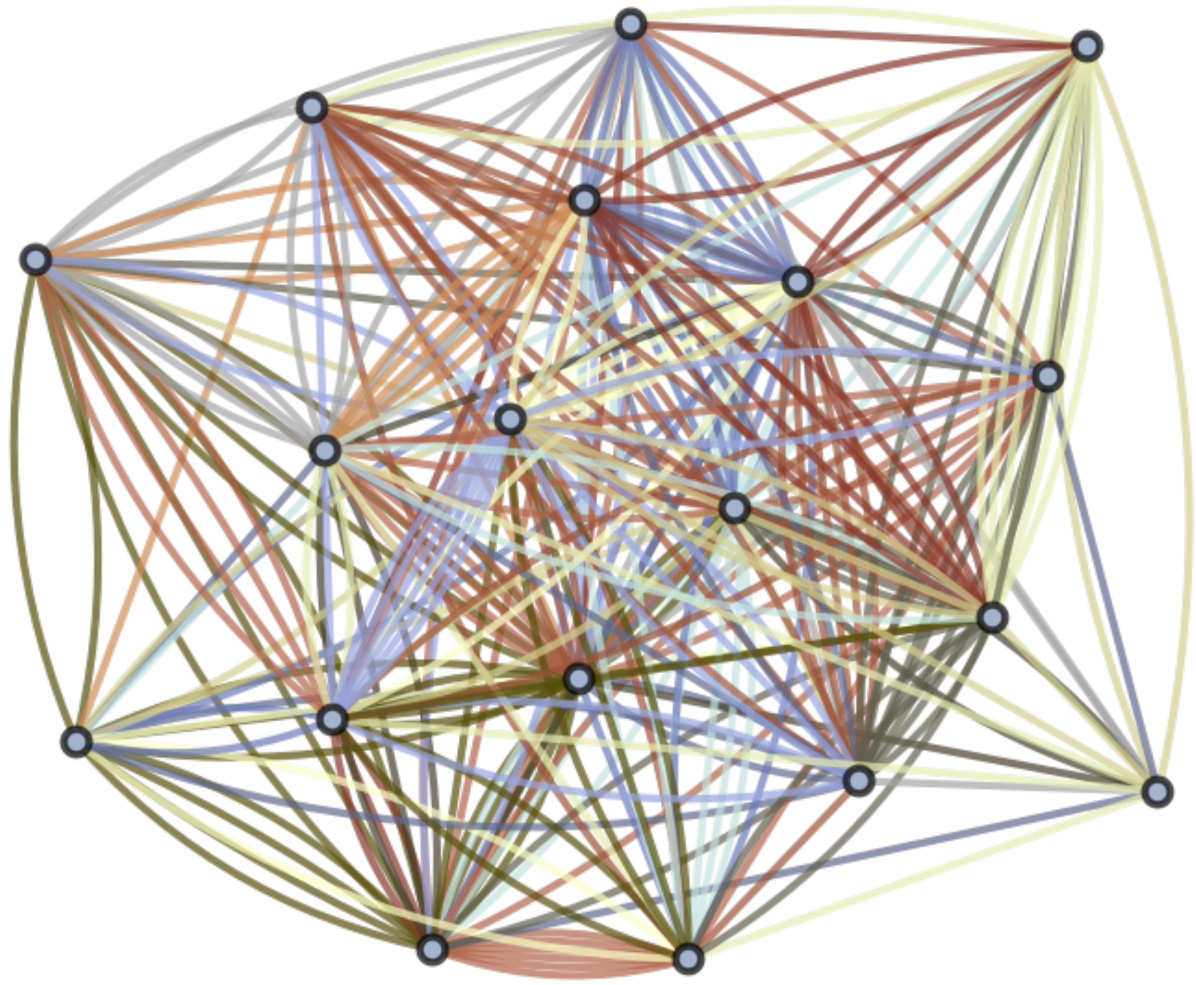
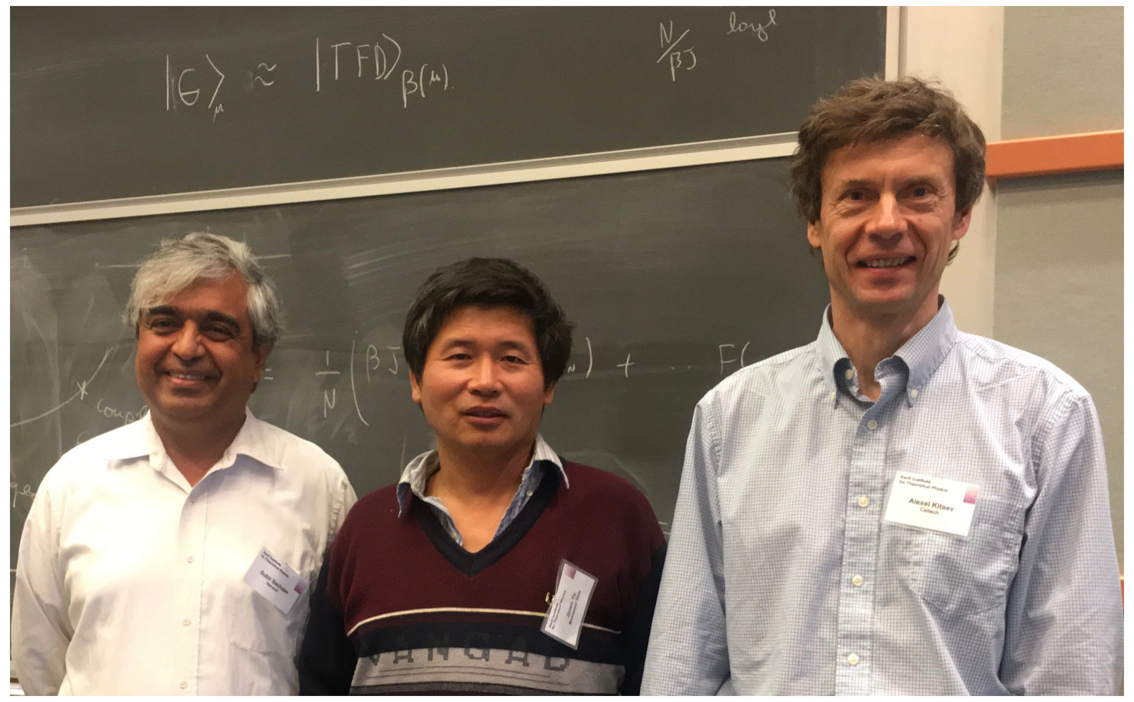
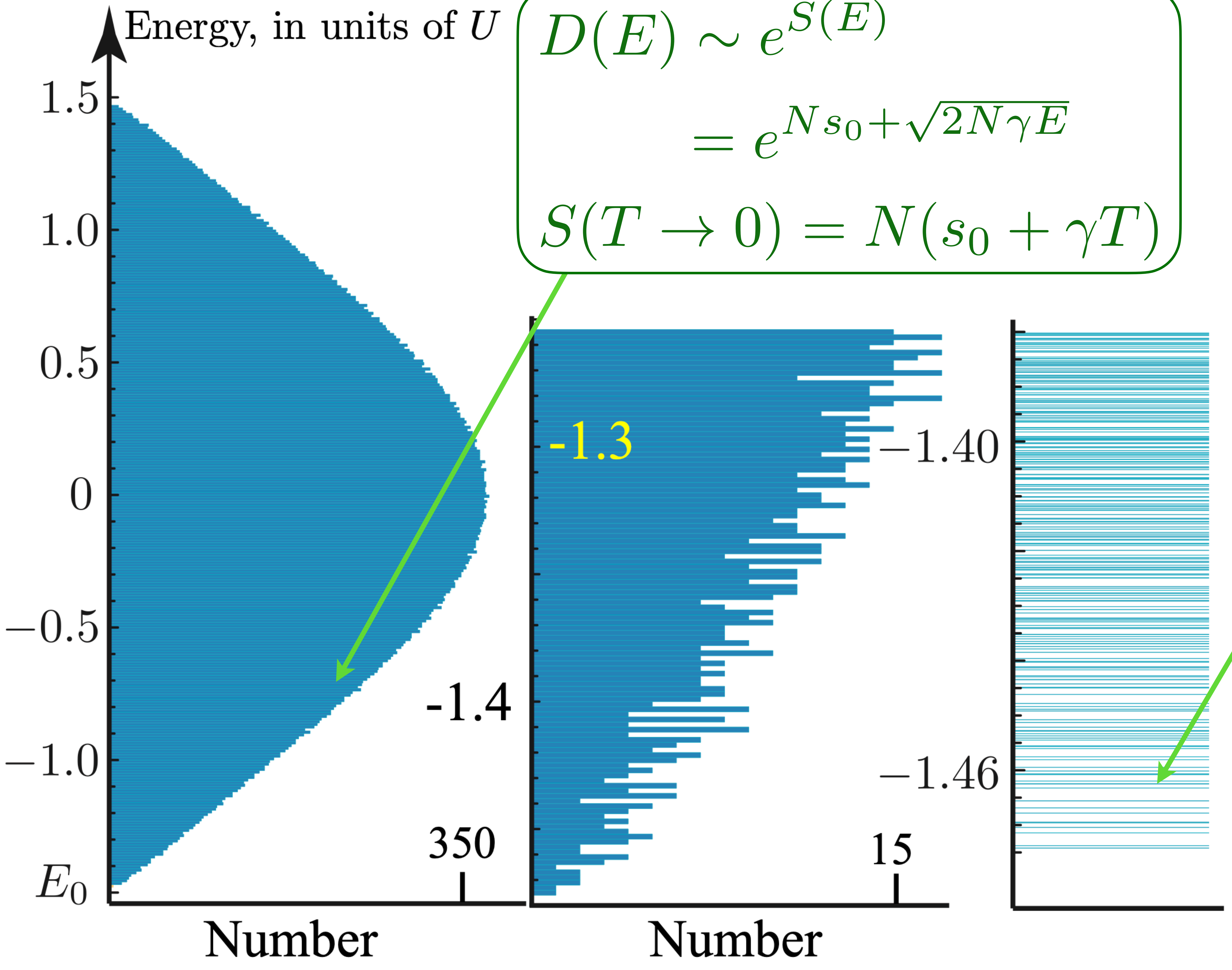




Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



$$D(E) \sim e^{S(E)}$$
$$= e^{N s_0 + \sqrt{2 N \gamma E}}$$
$$S(T \to 0) = N(s_0 + \gamma T)$$

$$D(E) \sim 2\, e^{N s_0} \sqrt{2 N \gamma E}$$

No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

$$s_0 = 0.464848\ldots$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Complex SYK model

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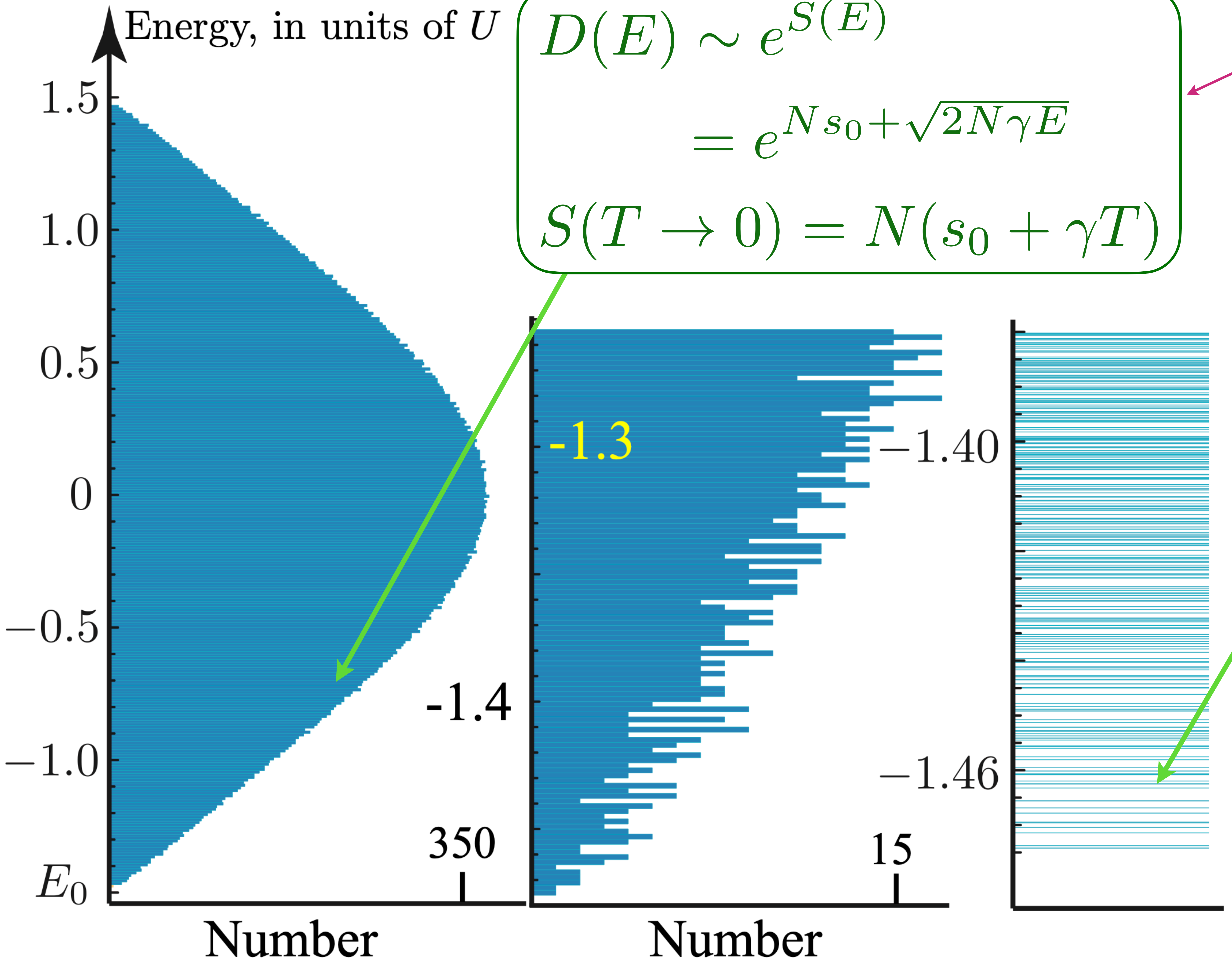
$$S(T \to 0) = N(s_0 + \gamma T)$$

$$D(E) \sim 2\, e^{N s_0} \sinh(\sqrt{2N\gamma E})$$

$$e^{-F(T)/T} = \int_0^\infty dE D(E) e^{-E/T}$$
$$S(T) = -\partial F / \partial T$$

$$D(E) \sim 2\, e^{N s_0} \sqrt{2N\gamma E}$$



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Complex SYK model

The SYK model

Consequences of emergent time-reparameterization and conformal symmetries in low-energy theory in 0+1 spacetime dimensions:

1. Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right) \text{ independent of } U.$$

No bosons, fermions, anyons ...

S. Sachdev and J. Ye, PRL **70**, 3339 (1993); A. Georges and O. Parcollet PRB **59**, 5341 (1999)

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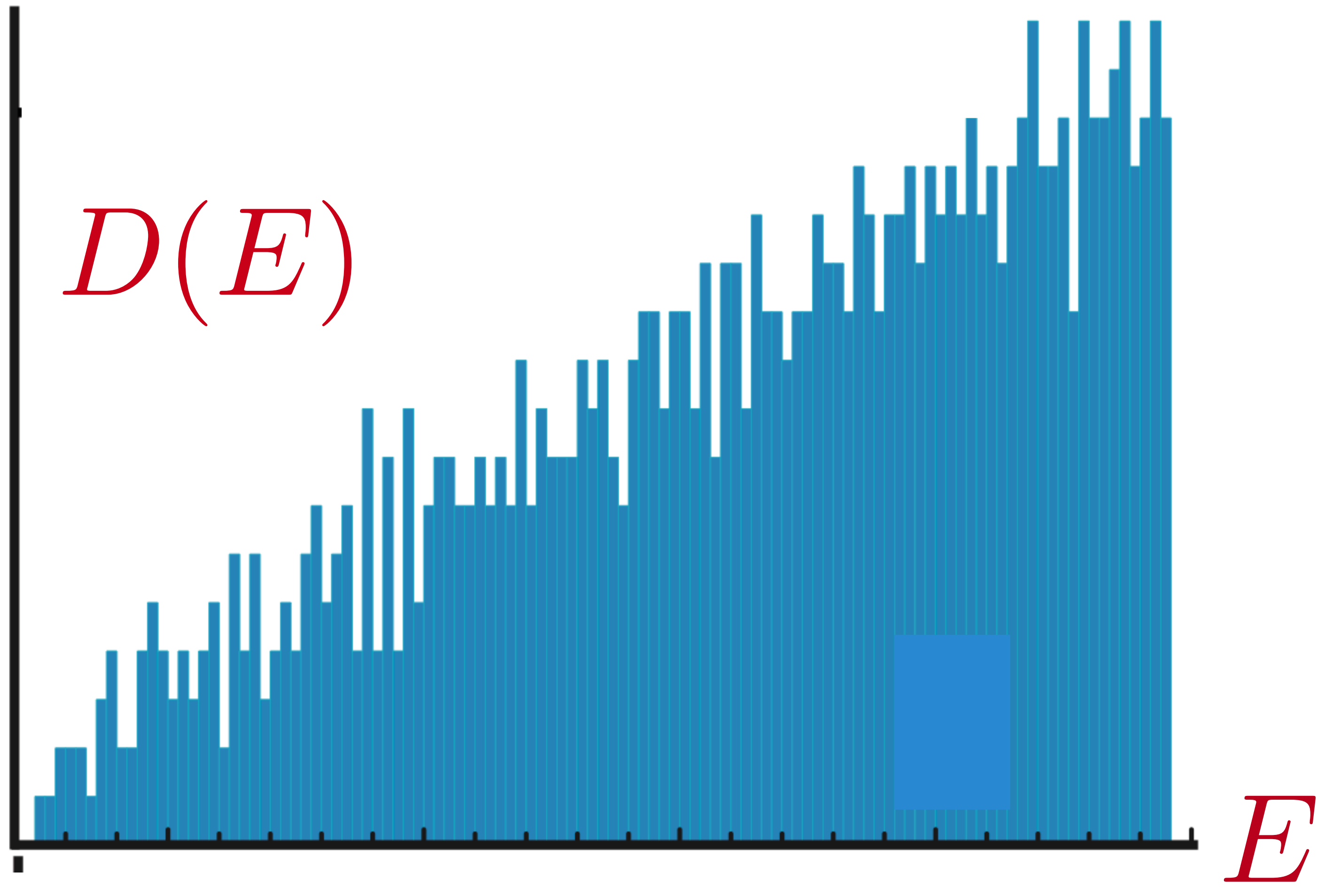
No bosons, fermions, anyons . . .

2. Zero temperature entropy without exponential ground state degeneracy!

$$\lim_{T\to 0}\lim_{N\to\infty}\frac{1}{N}S(T) = s_0 \quad , \quad D(E\to 0) = e^{Ns_0}\sinh(\sqrt{2N\gamma E})$$

$$s_0 = 0.4648476991708051074 9\ldots \text{ for } \mathcal{Q} = 1/2.$$



A. Georges, O. Parcollet, and S. Sachdev (**GPS**), Physical Review B **63**, 134406 (2001)

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

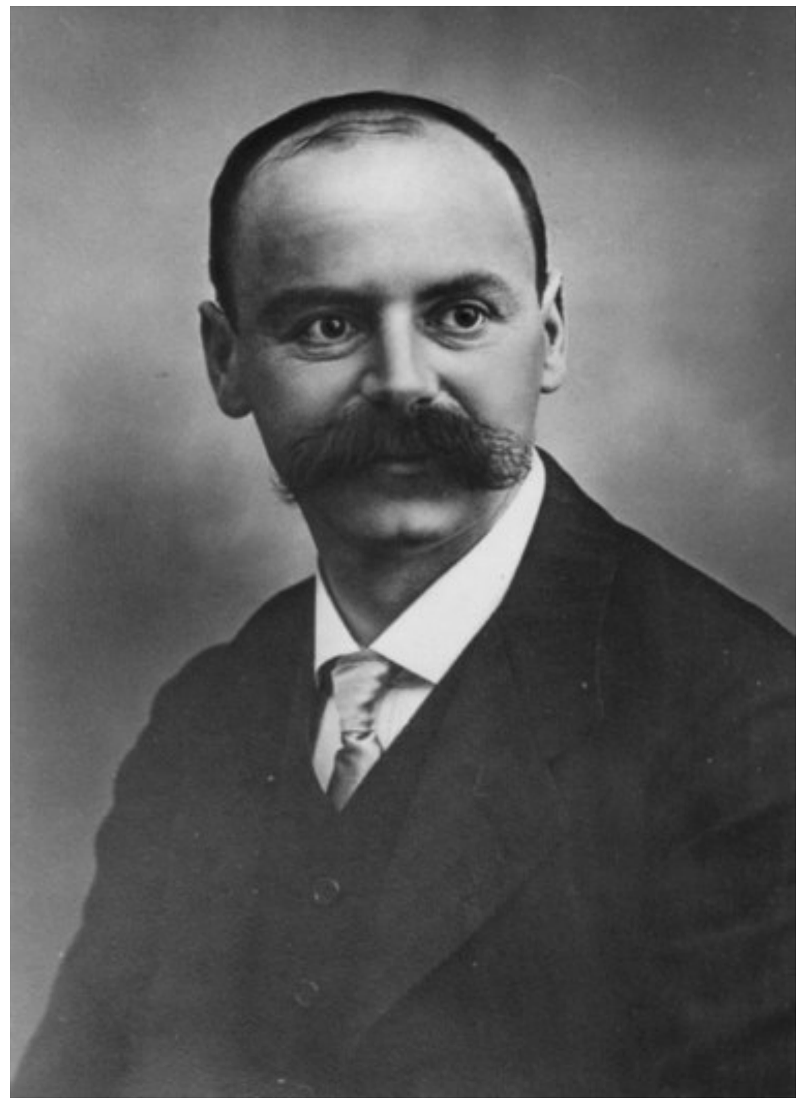
A solvable model of multi-particle quantum entanglement.

No quasiparticles: yields a metal in which current is carried not by individual electrons, but by an entangled "quantum soup"

From
the SYK model
to
black holes

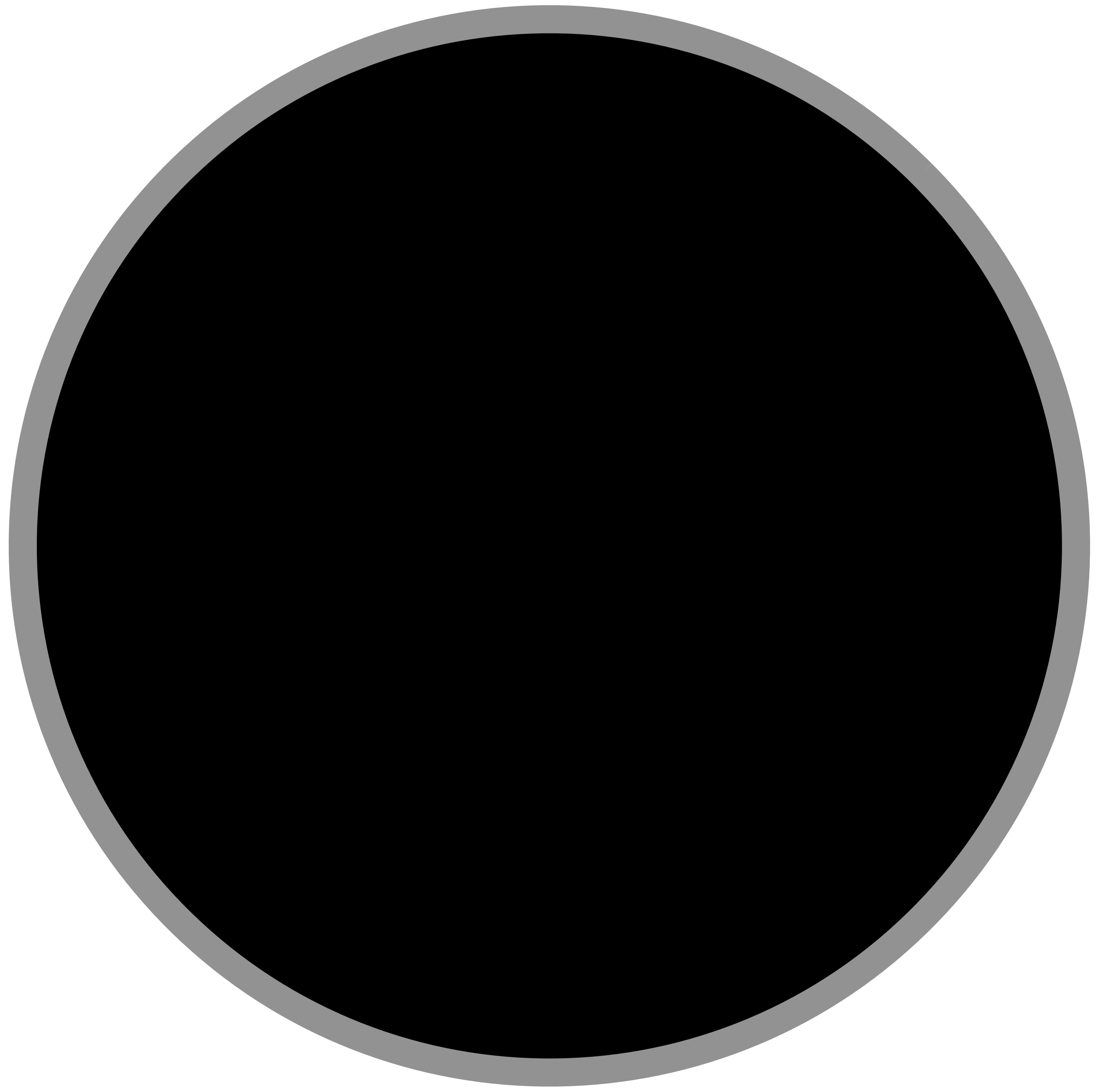
Black Holes

Objects so dense that light is gravitationally bound to them.



Karl Schwarzschild (1916)

Horizon radius $R = \dfrac{2GM}{c^2}$



G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

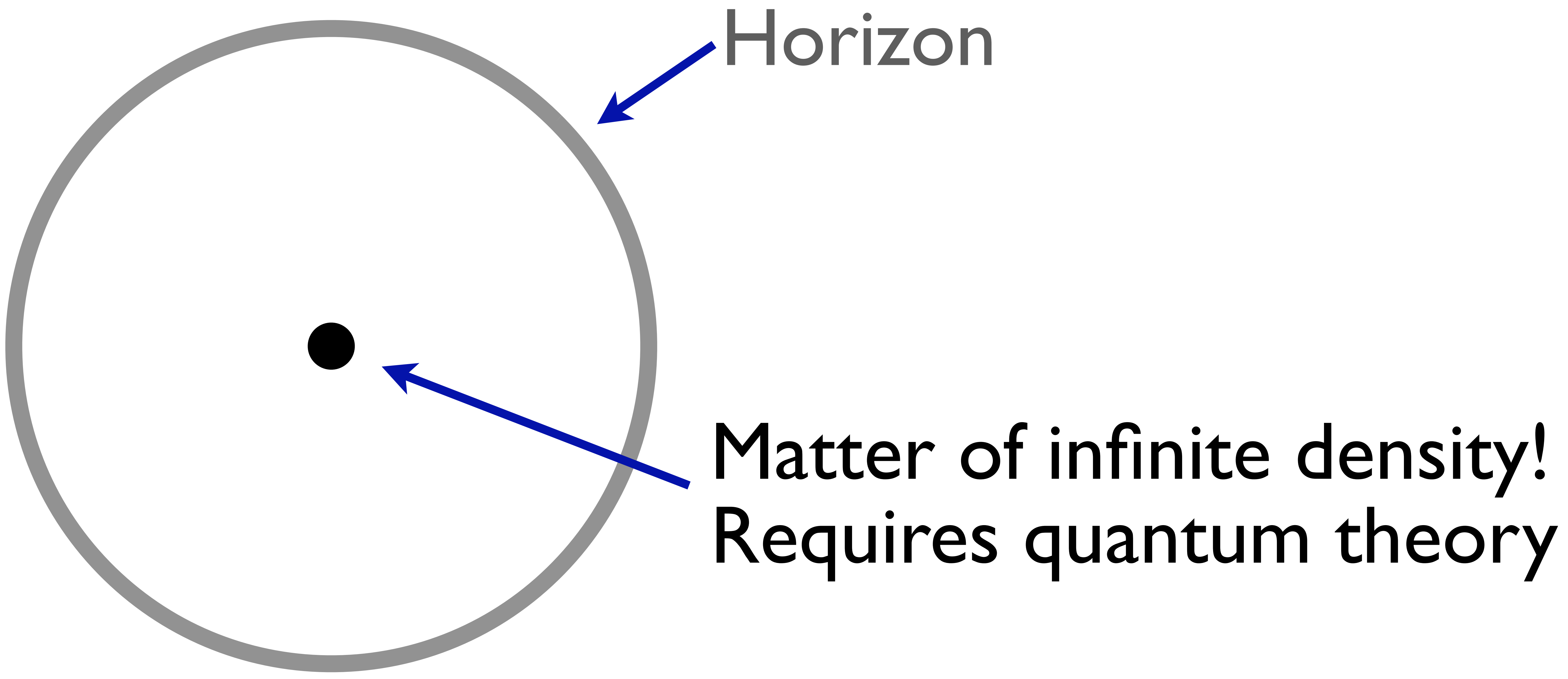
The supermassive black hole lurking at the heart of the Milky Way – Sagittarius A* contains about 4.3 million solar masses

$$R = 1.3 \times 10^{11} \mathrm{m}$$
$$\approx \text{earth's orbit}$$

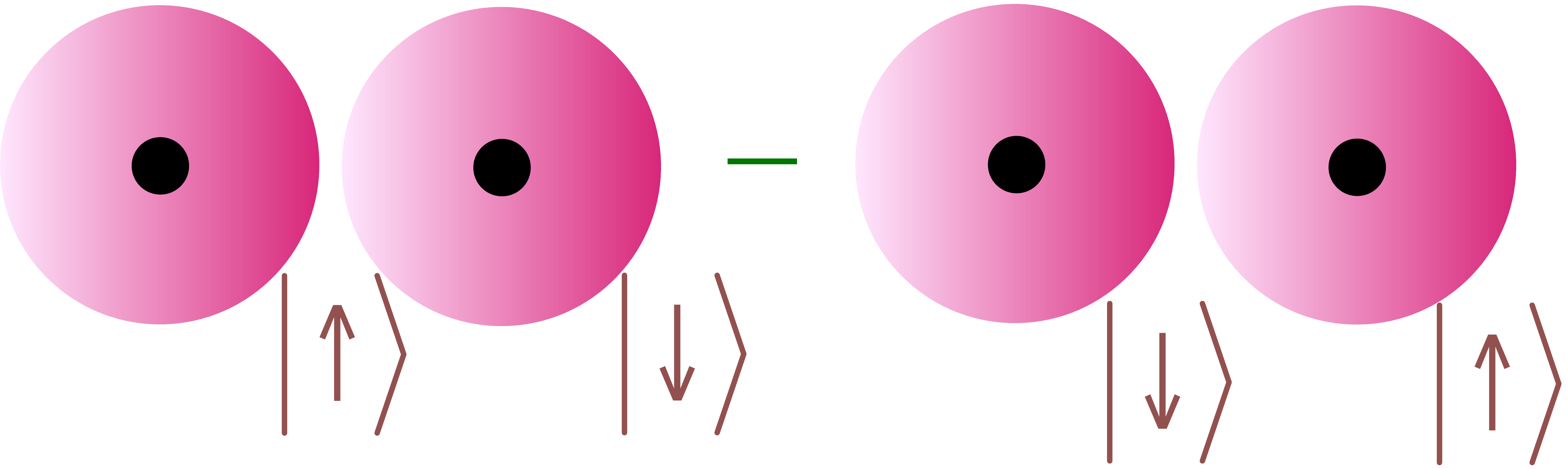
Event Horizon Telescope
May 12, 2022

What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



Quantum Entanglement across a black hole horizon



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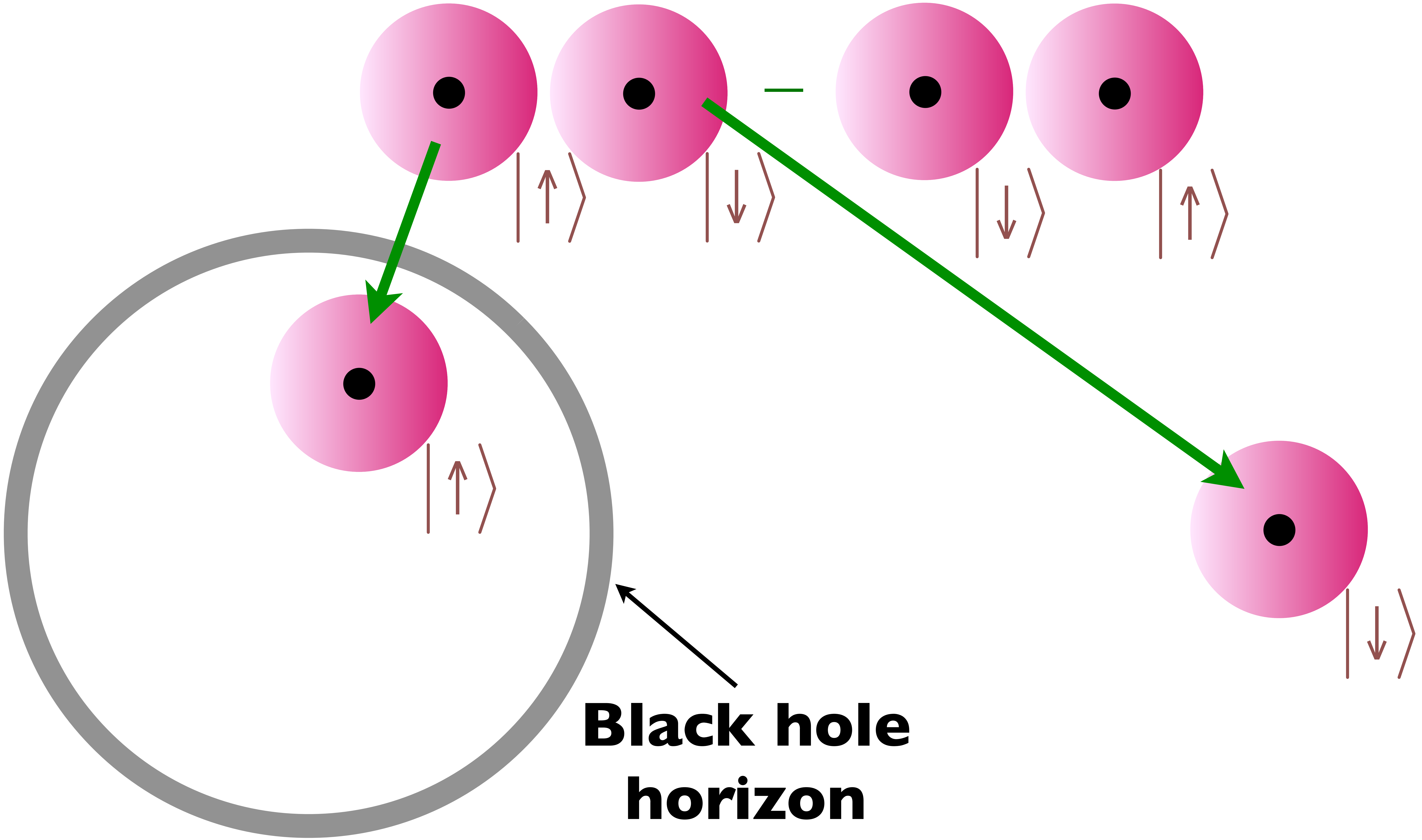

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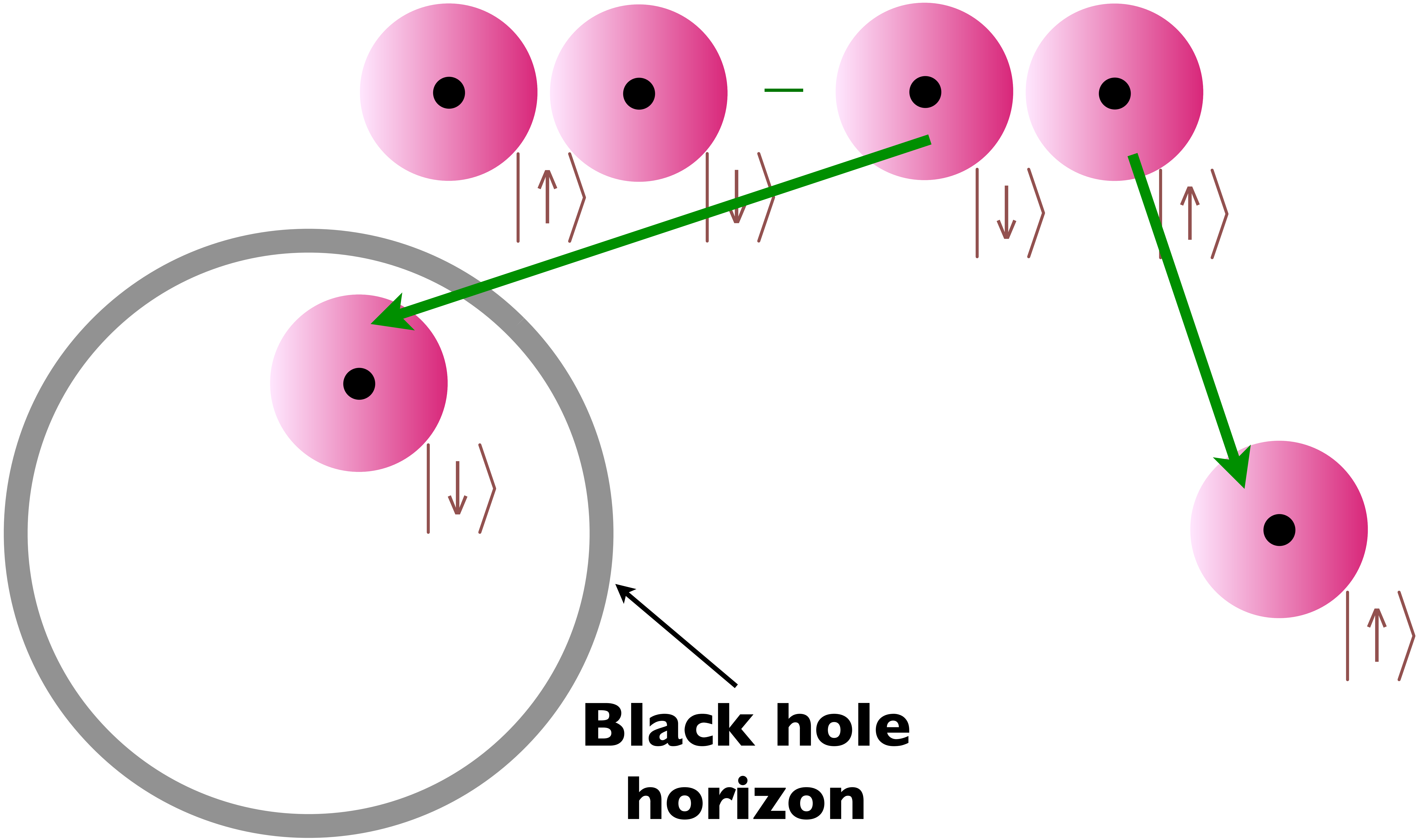
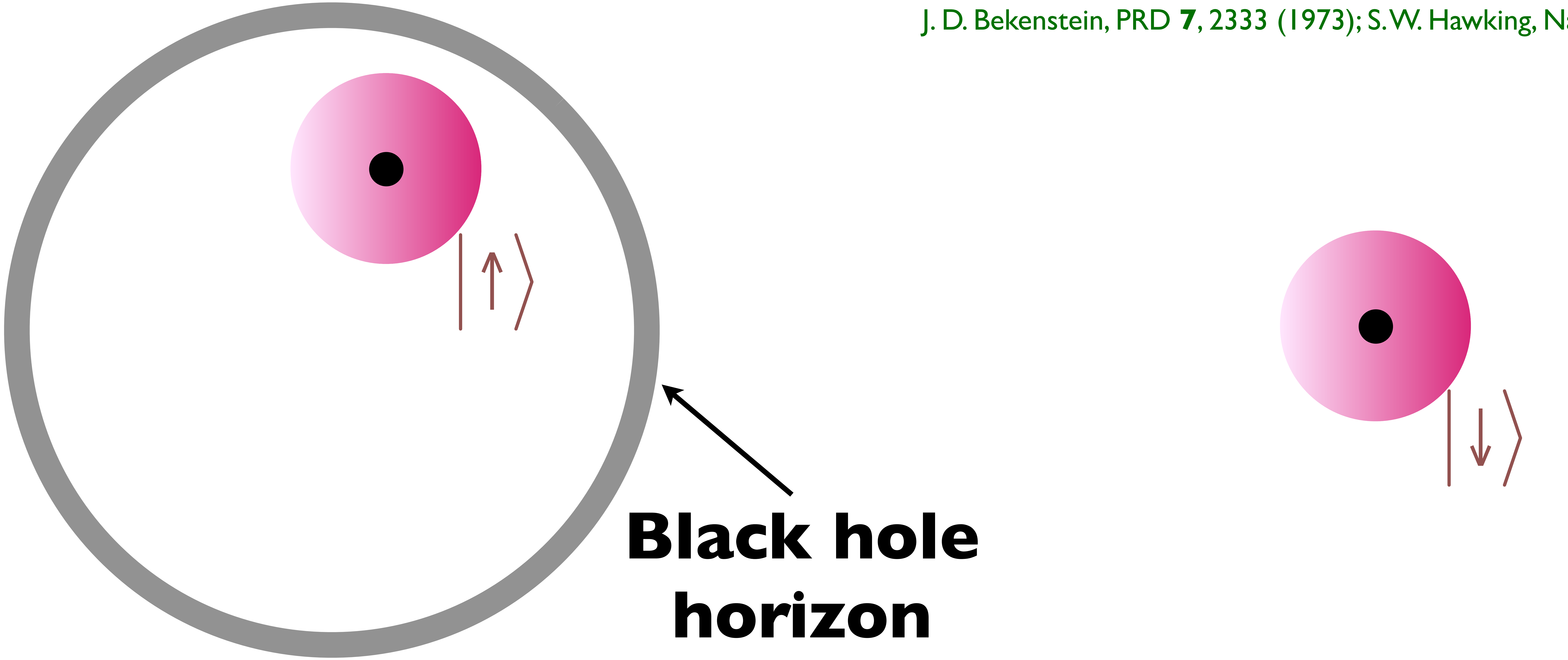


Quantum Entanglement across a black hole horizon

Bekenstein, Hawking: Black holes have a temperature and an entropy!

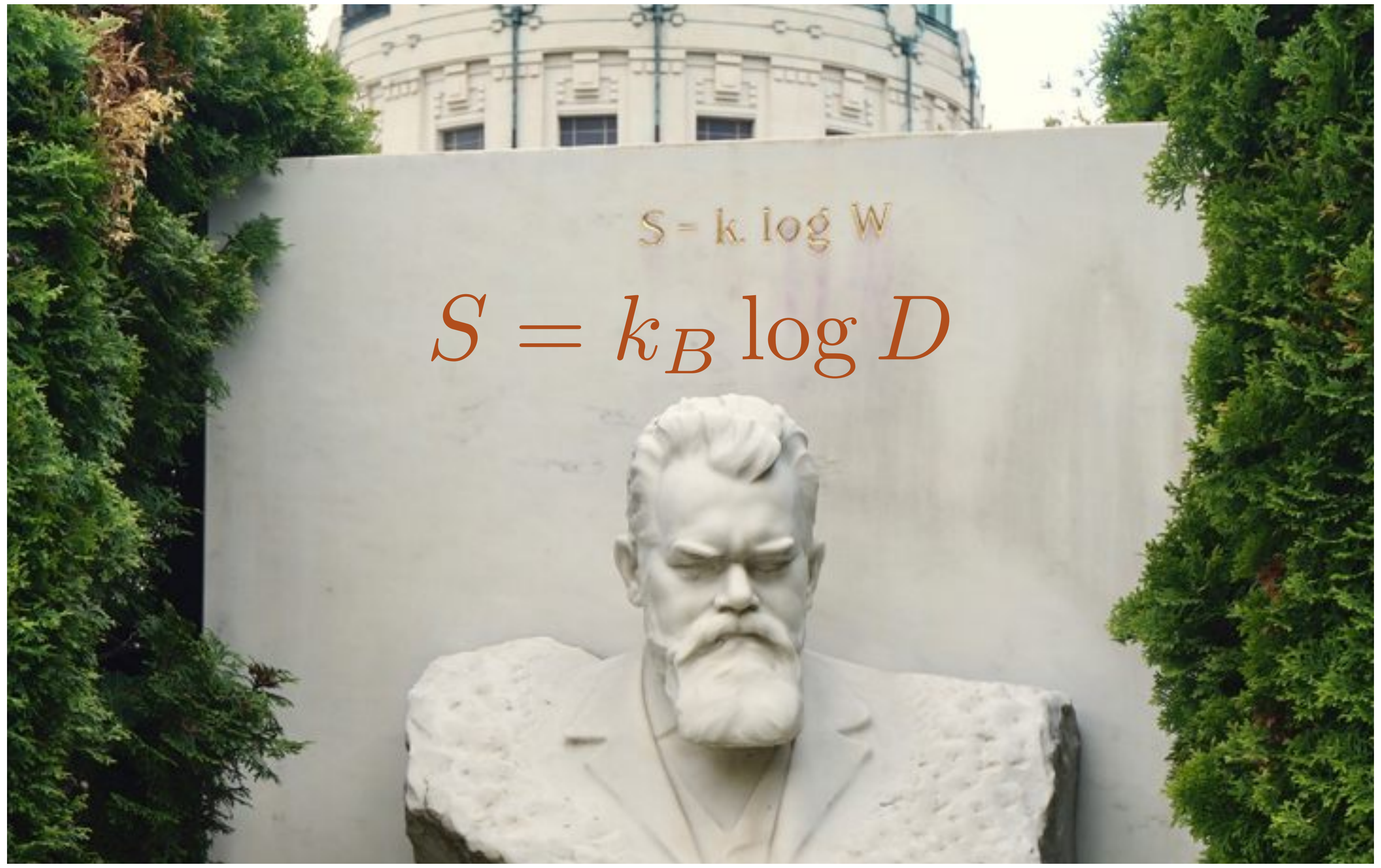
To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.

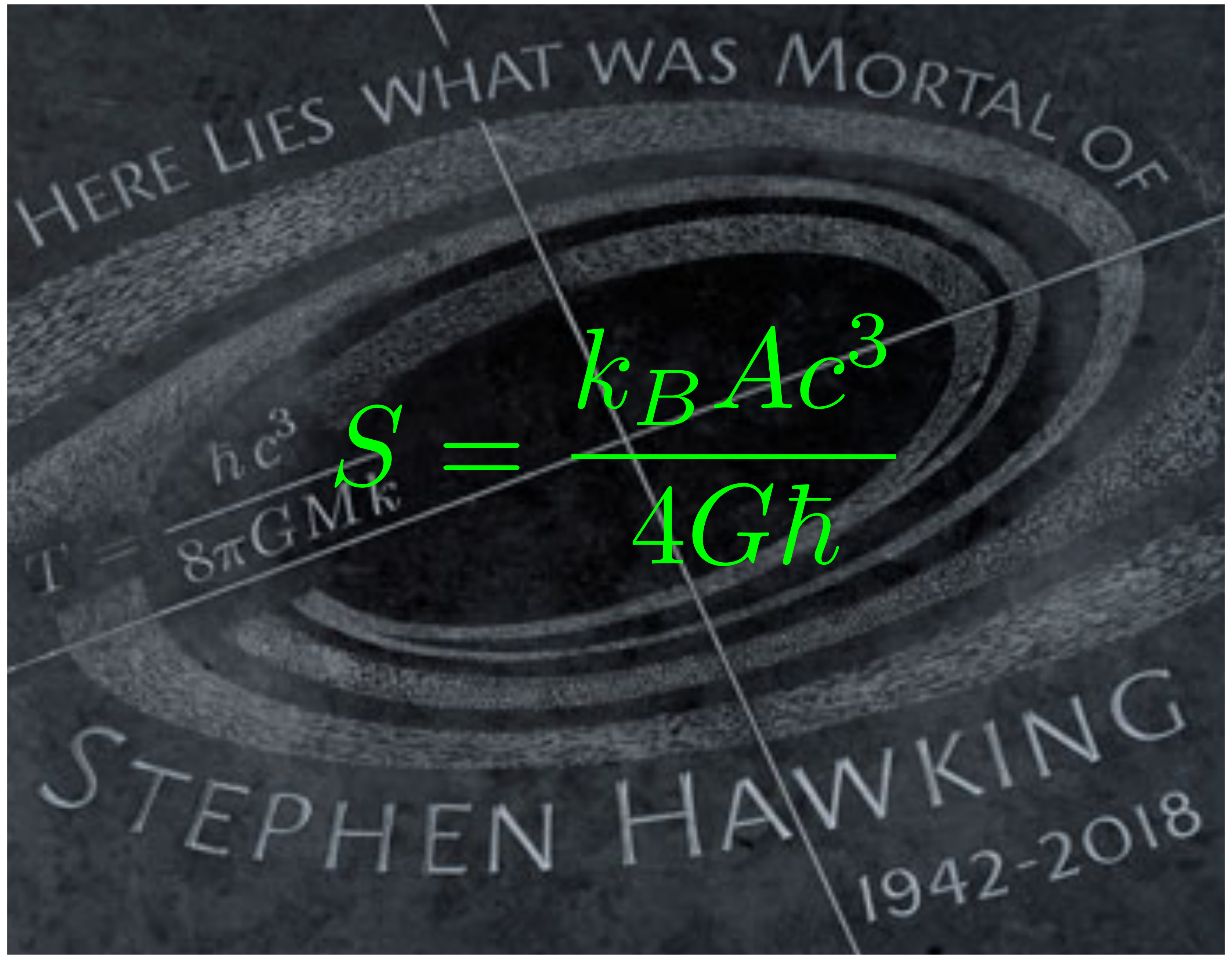
J. D. Bekenstein, PRD **7**, 2333 (1973); S.W. Hawking, Nature **248**, 30 (1974)



Quantum Black Holes

- Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* $D(E)$ [the quantum analog of Boltzmann's W] matches Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics, $S(E) = k_B \log D(E)$?





Connections between the SYK model and black holes

- Black hole 'ring-down' or 'quasinormal mode damping' or 'chaos' times are Planckian $\sim \hbar/(k_B T)$

C.V. Vishveshwara, Nature **227**, 936 (1970)

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

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- Charged black holes have a non-zero Bekenstein-Hawking entropy in the limit $T \to 0$:

  $S_{BH} = A_0 c^3/(4\hbar G)$ where $A_0 = 2GQ^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

  Also applies to rotating neutral black holes.

U. Moitra, S.K. Sake, S.P. Trivedi and V. Vishal, JHEP 11 (2019) 047.
D. Kapec, A. Sheta, A. Strominger and C. Toldo, PRL 133 (2024) 021601
M. Kolanowski, D. Marolf, I. Rakic, M. Rangamani and G.J. Turiaci, arXiv:2409.16248

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

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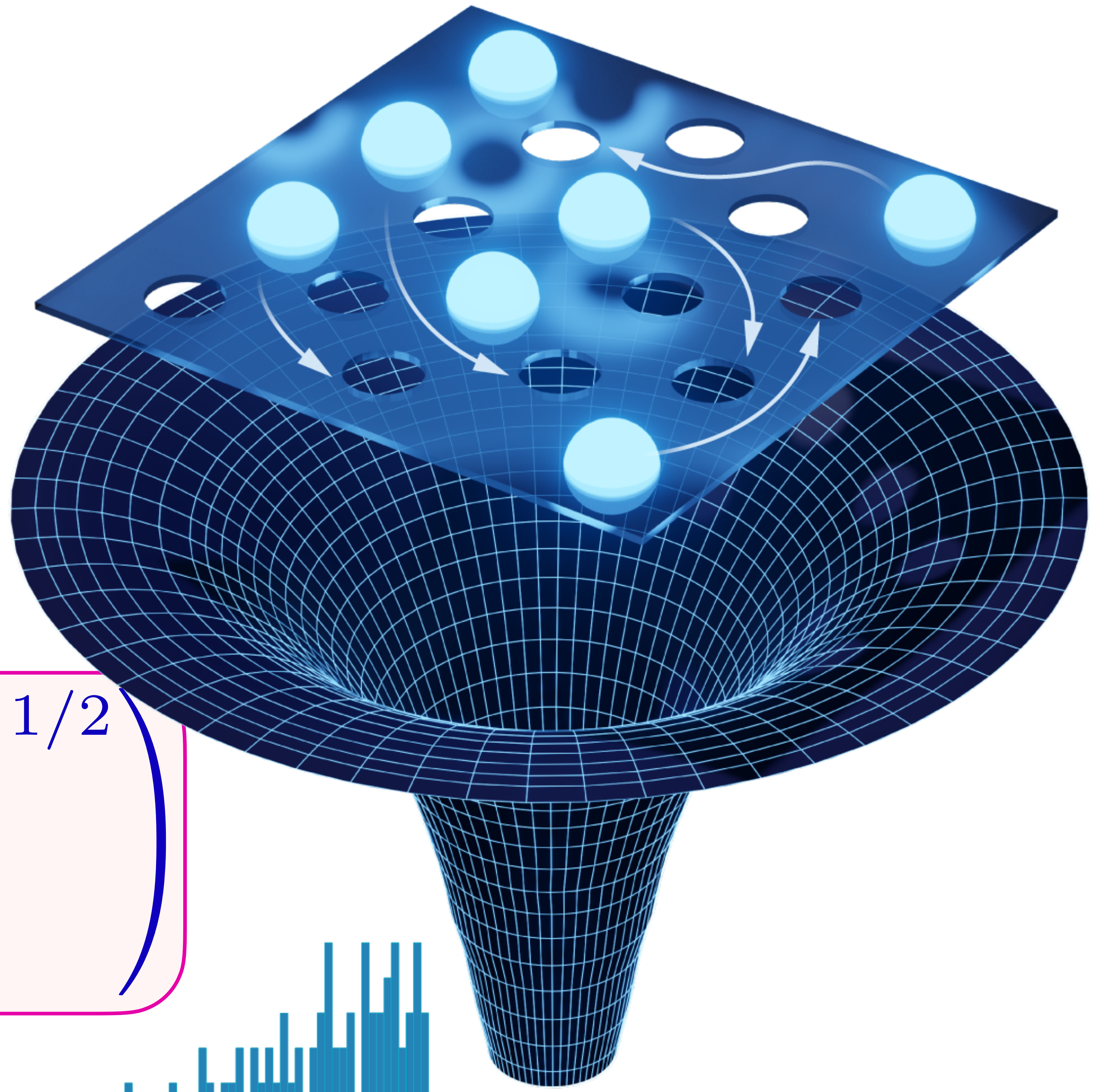
  Also applies to rotating neutral black holes.

- The example of the SYK model implies that S_{BH} is *not* realized by an exponentially large ground state degeneracy (as is the case in all earlier string-theoretic computations).

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

D(E) of charged black holes from the SYK model



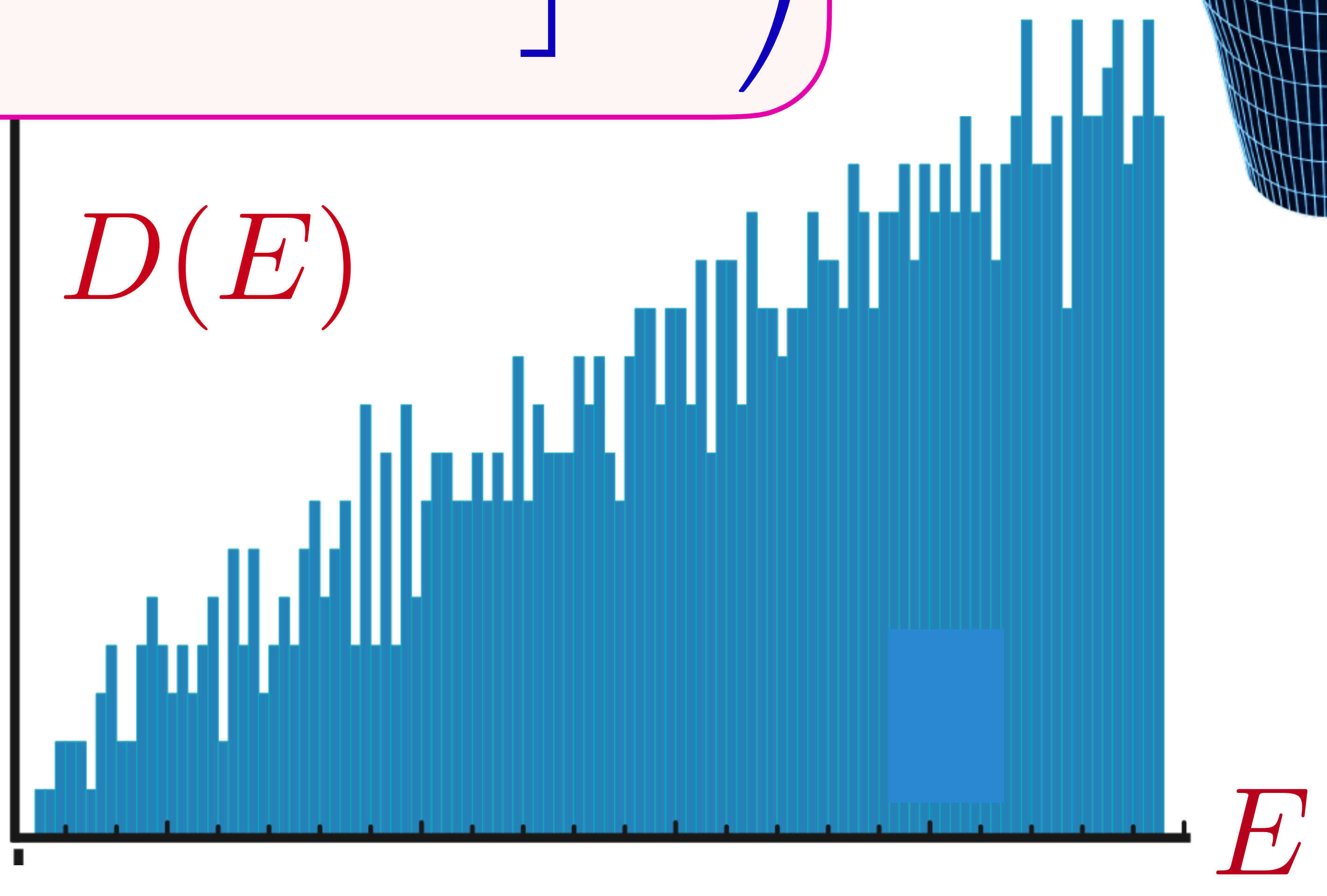
- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T = 0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy E is

Bekenstein-Hawking

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} {A_0}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

Iliesiu, Murthy, Turiaci (2022)

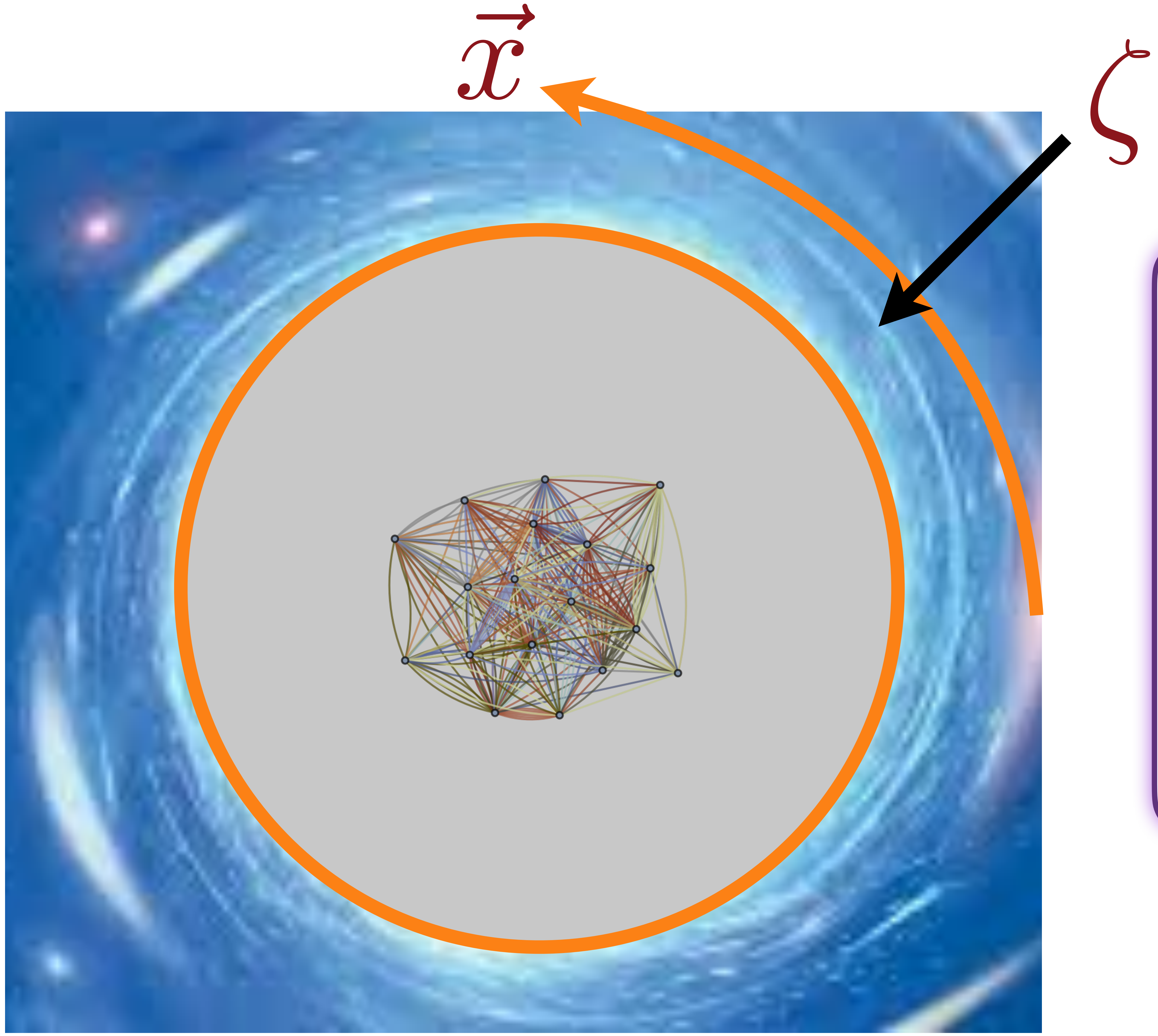
$f_{\rm smooth}(E)$: developments from the SYK model



Similar remarks apply to rotating neutral black holes.

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

Quantum simulation of charged black holes by the SYK model



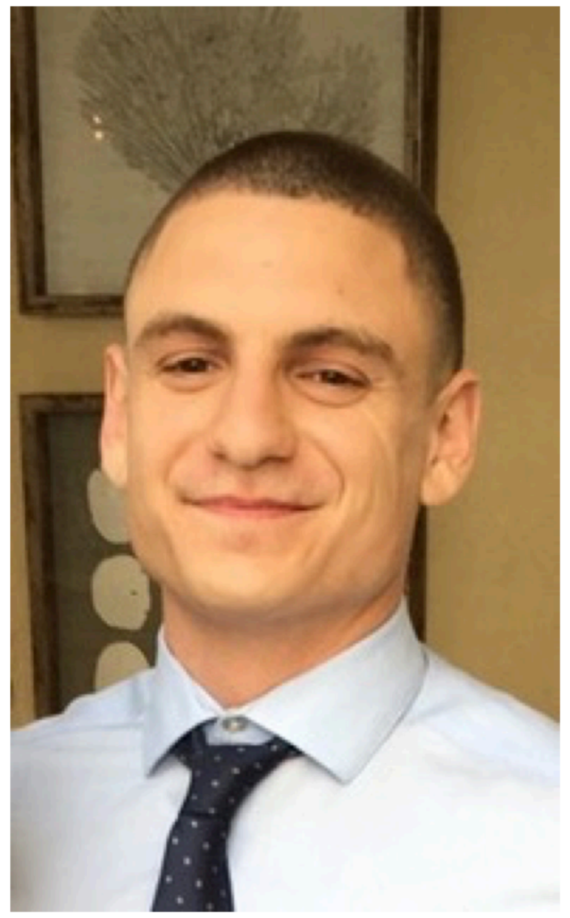
The SYK model simulates the low energy properties of the interior of the black hole for an outside observer in ζ-τ co-ordinates.

From the SYK model to the universal 2d-YSYK theory of strange metals

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

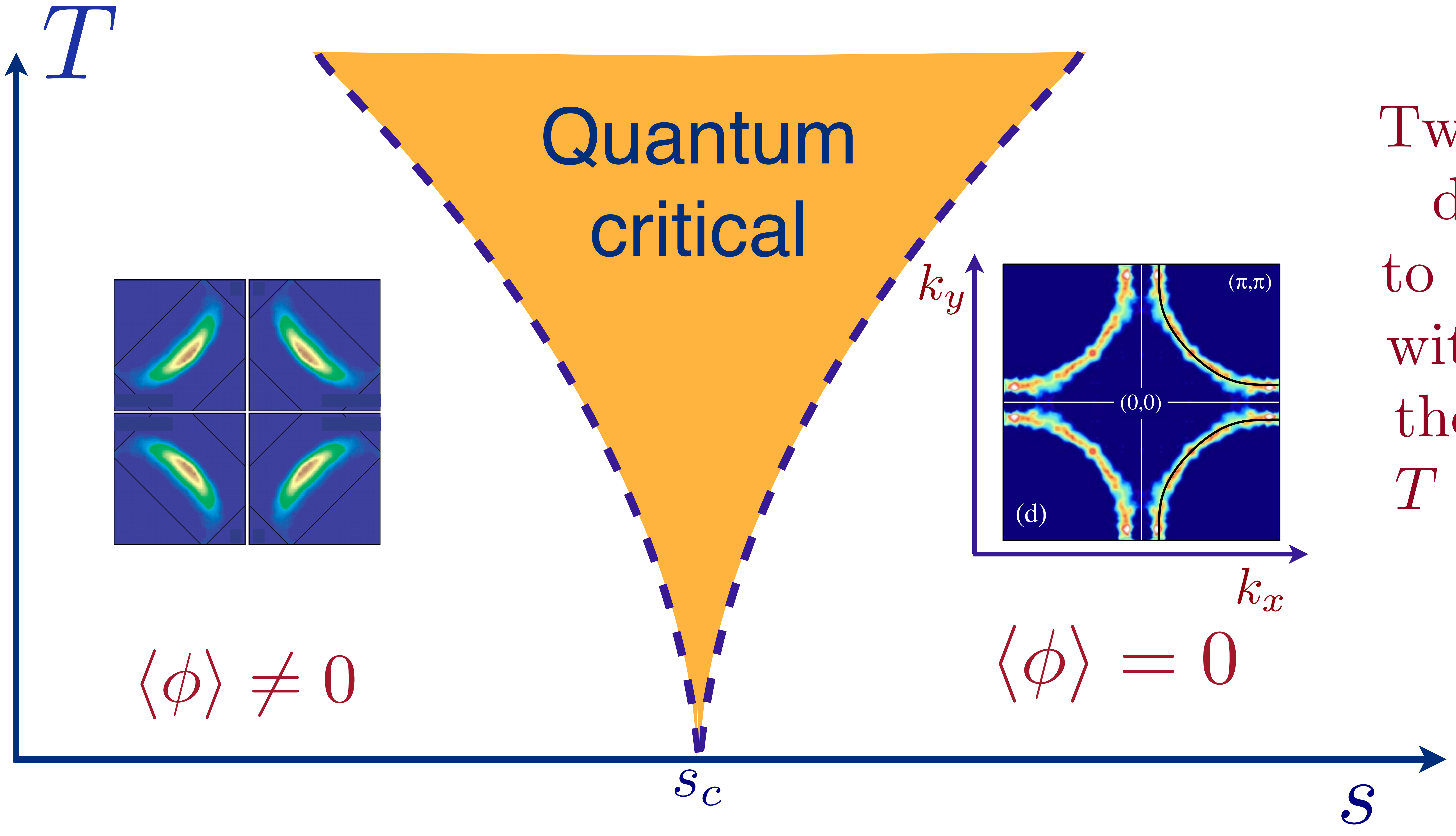
Aavishkar Patel
Flatiron

Haoyu Guo
Cornell



Ilya Esterlis
Wisconsin

Quantum phase transition of Fermi surface change



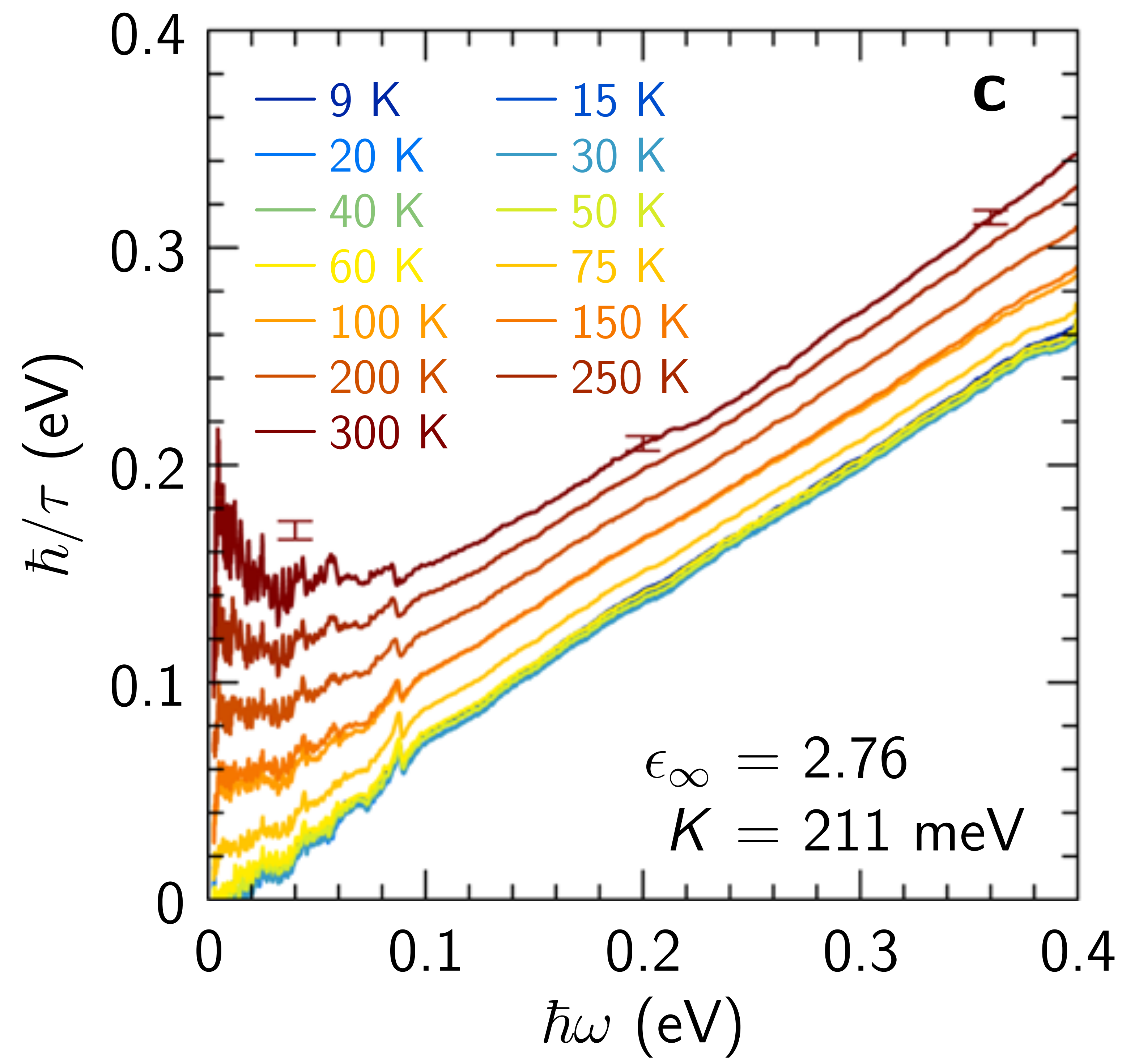
Two-dimensional YSYK model describes electrons coupled to a boson ϕ driving the QPT, with spatial randomness in s_c, the position of the underlying $T = 0$ quantum critical point.

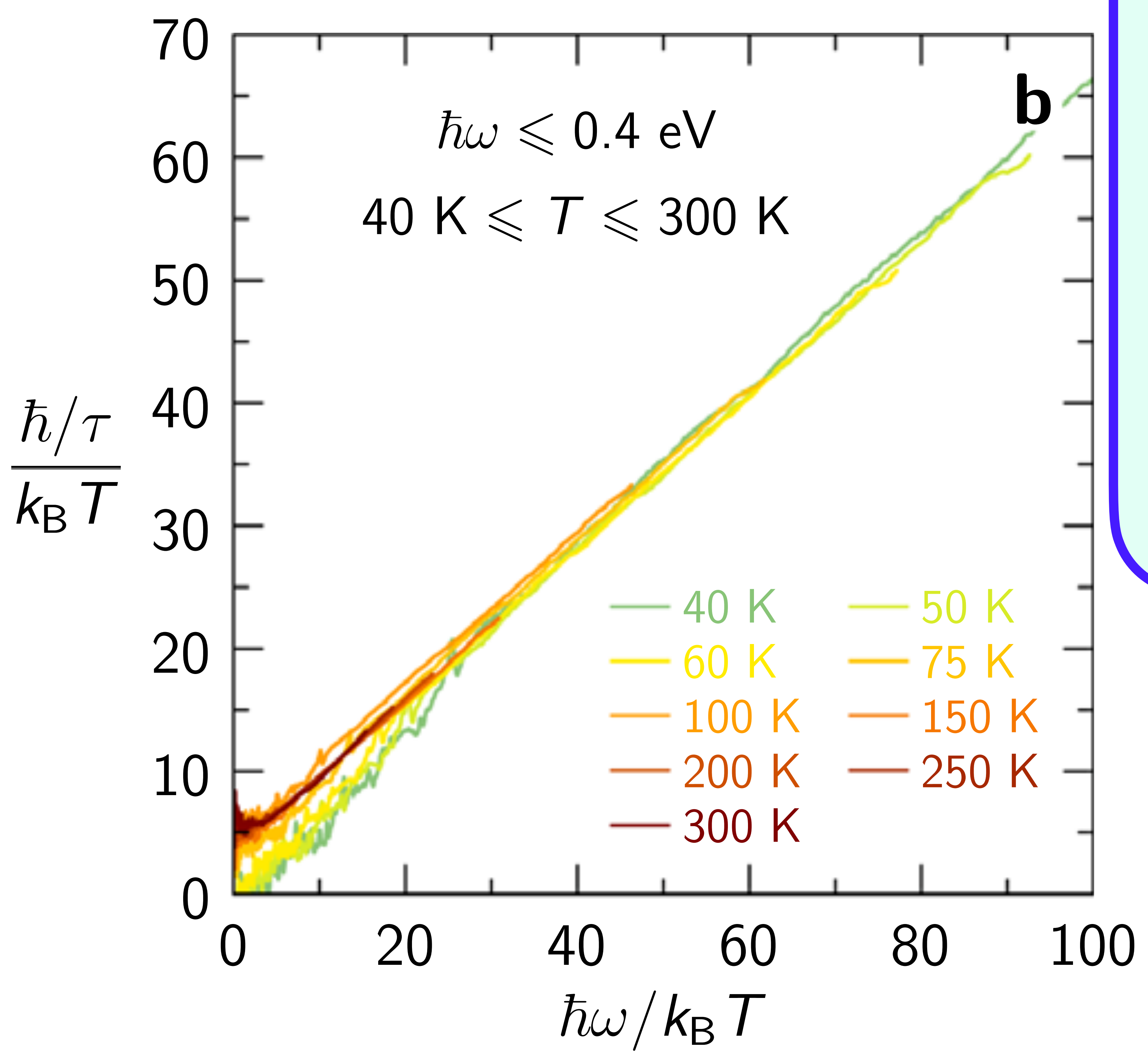
Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\dfrac{m^*(\omega)}{m} + i\dfrac{\hbar}{\tau(\omega)}}$$





Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T}F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy

$S(T \to 0) \sim T\ln(1/T)$.

$La_{2-x}Sr_xCuO_4$

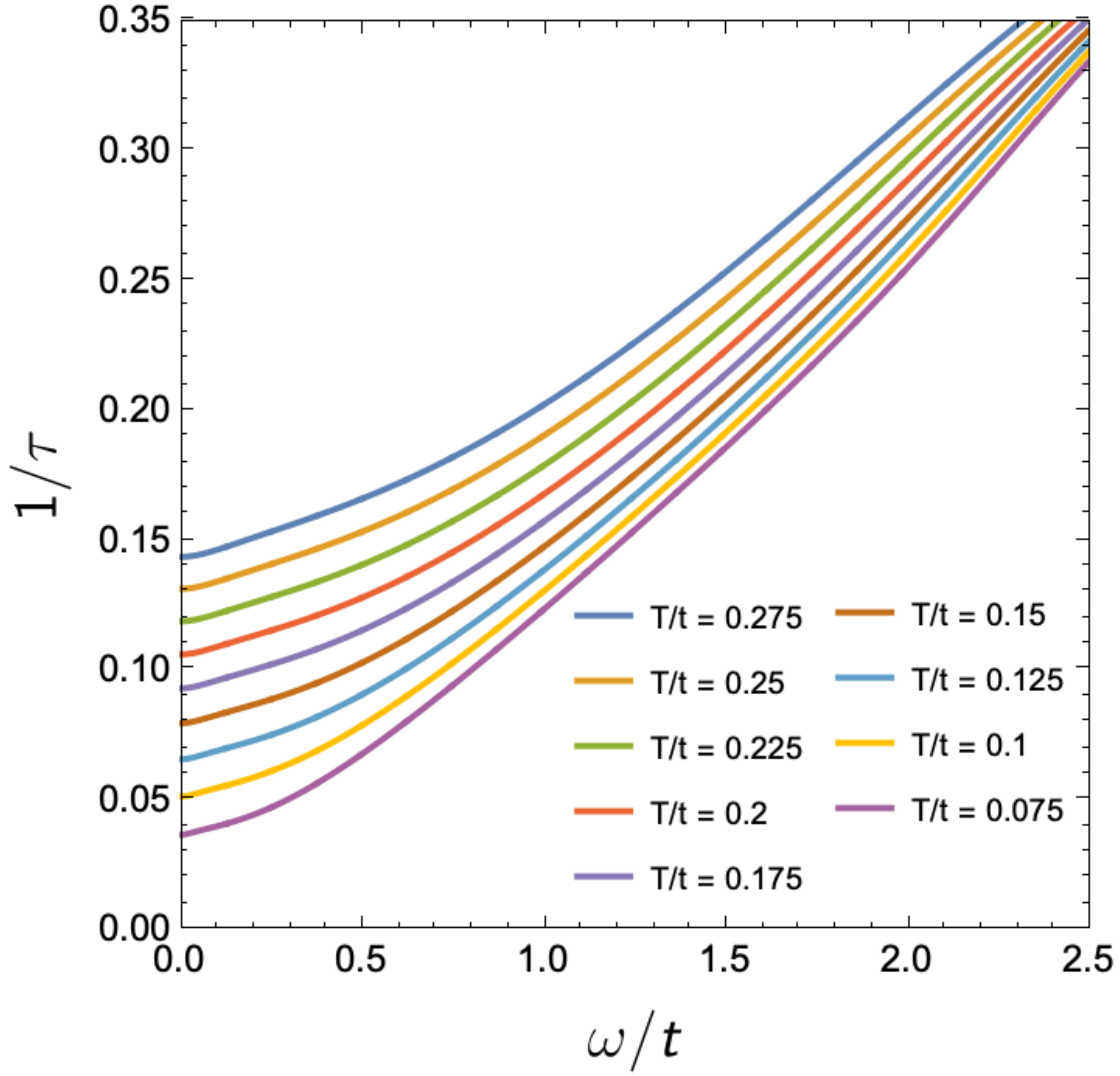
$p = 0.24$

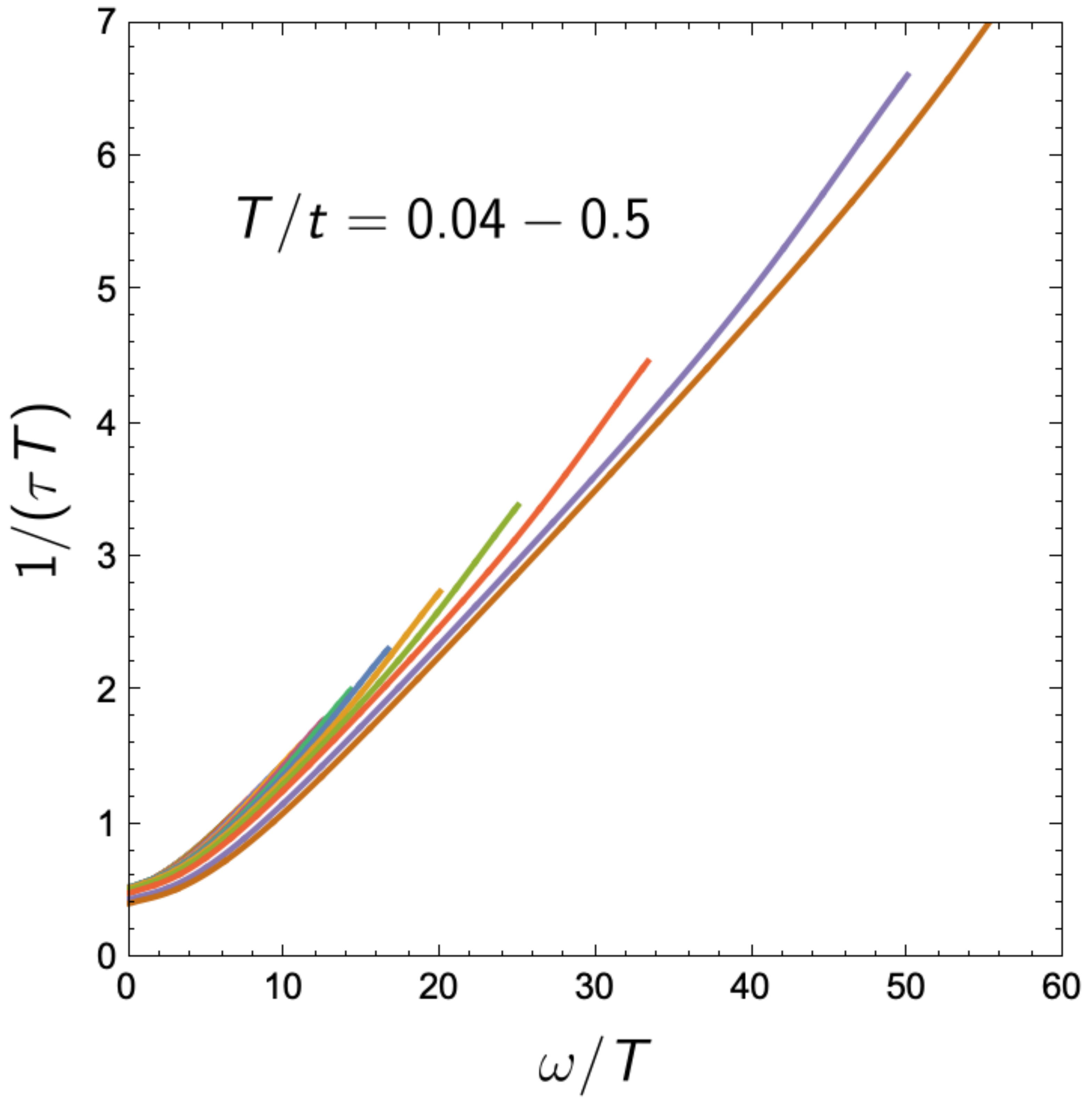
$T_c = 19$ K

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

$$\sigma(\omega) = i\frac{e^2 K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$





Planckian dynamics!

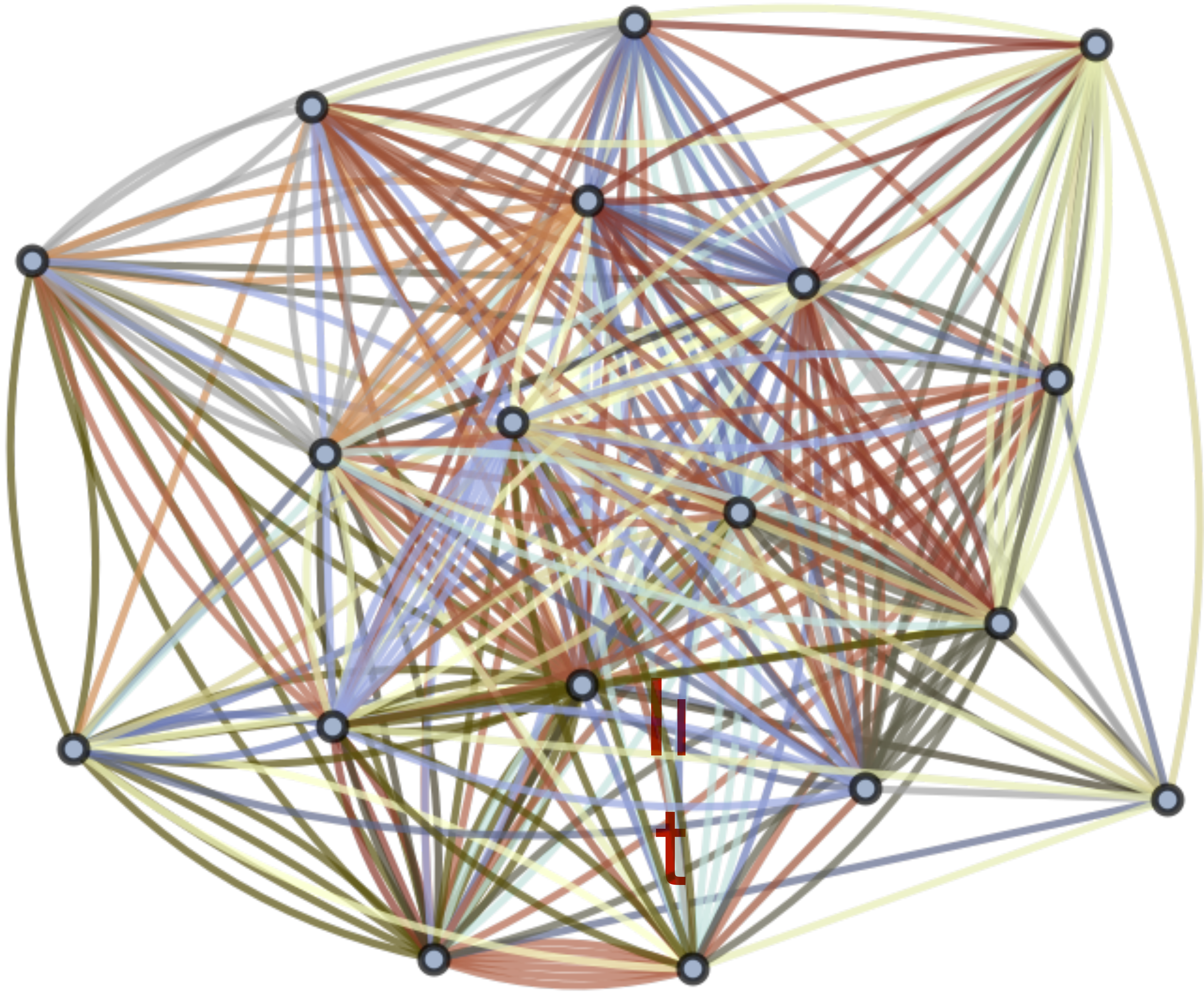
$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy $S(T \to 0) \sim T\ln(1/T)$ in 2d-YSYK model (unlike zero temperature entropy in SYK model).

Recap

The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



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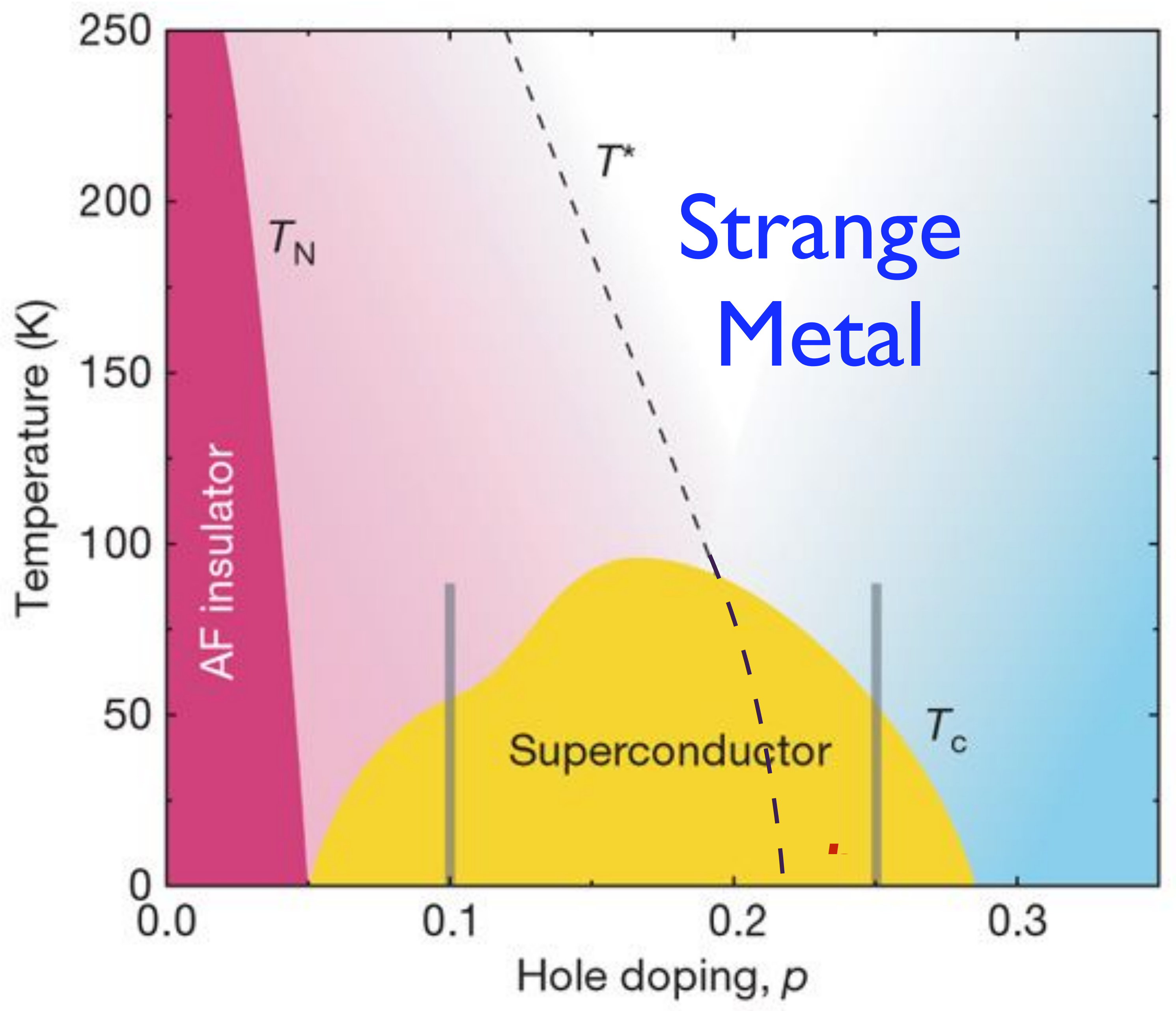
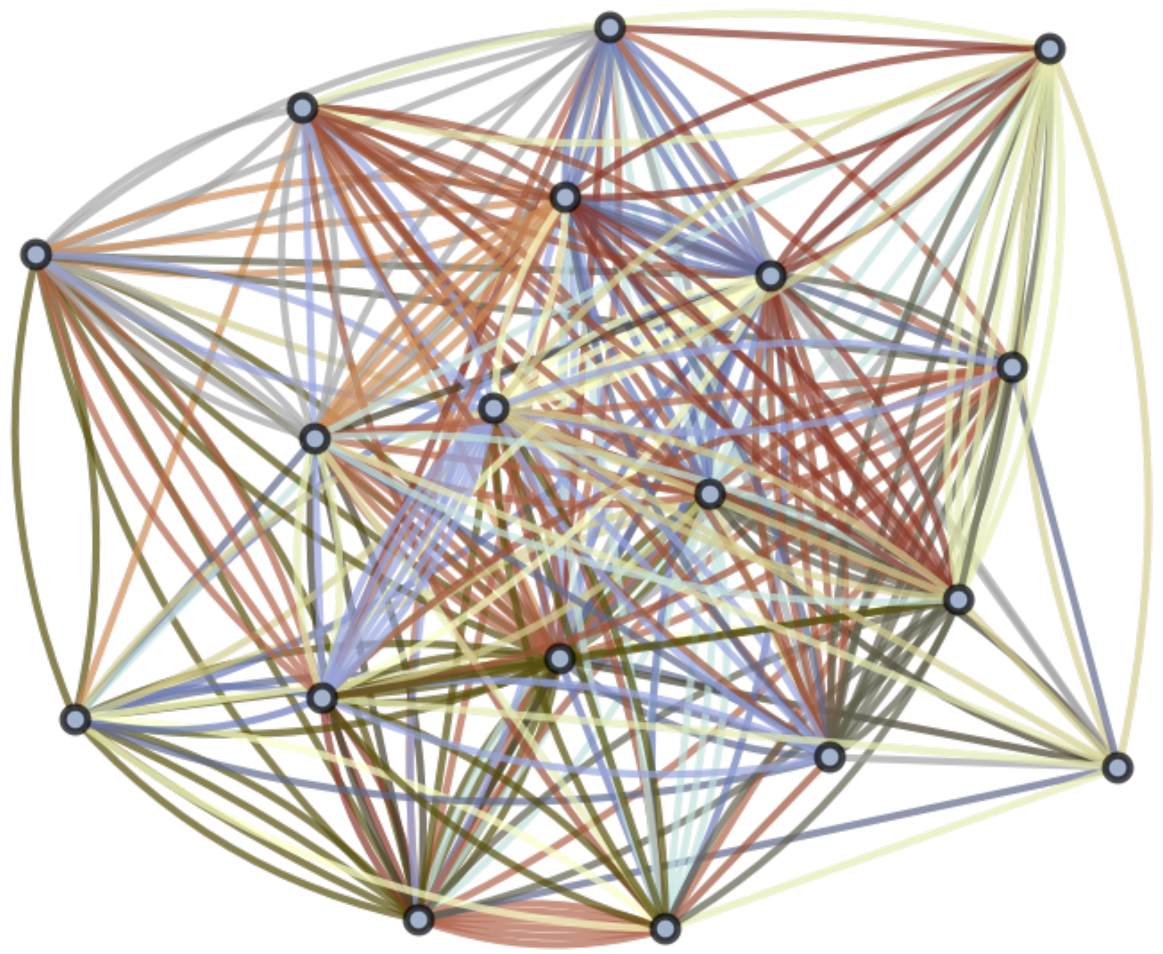
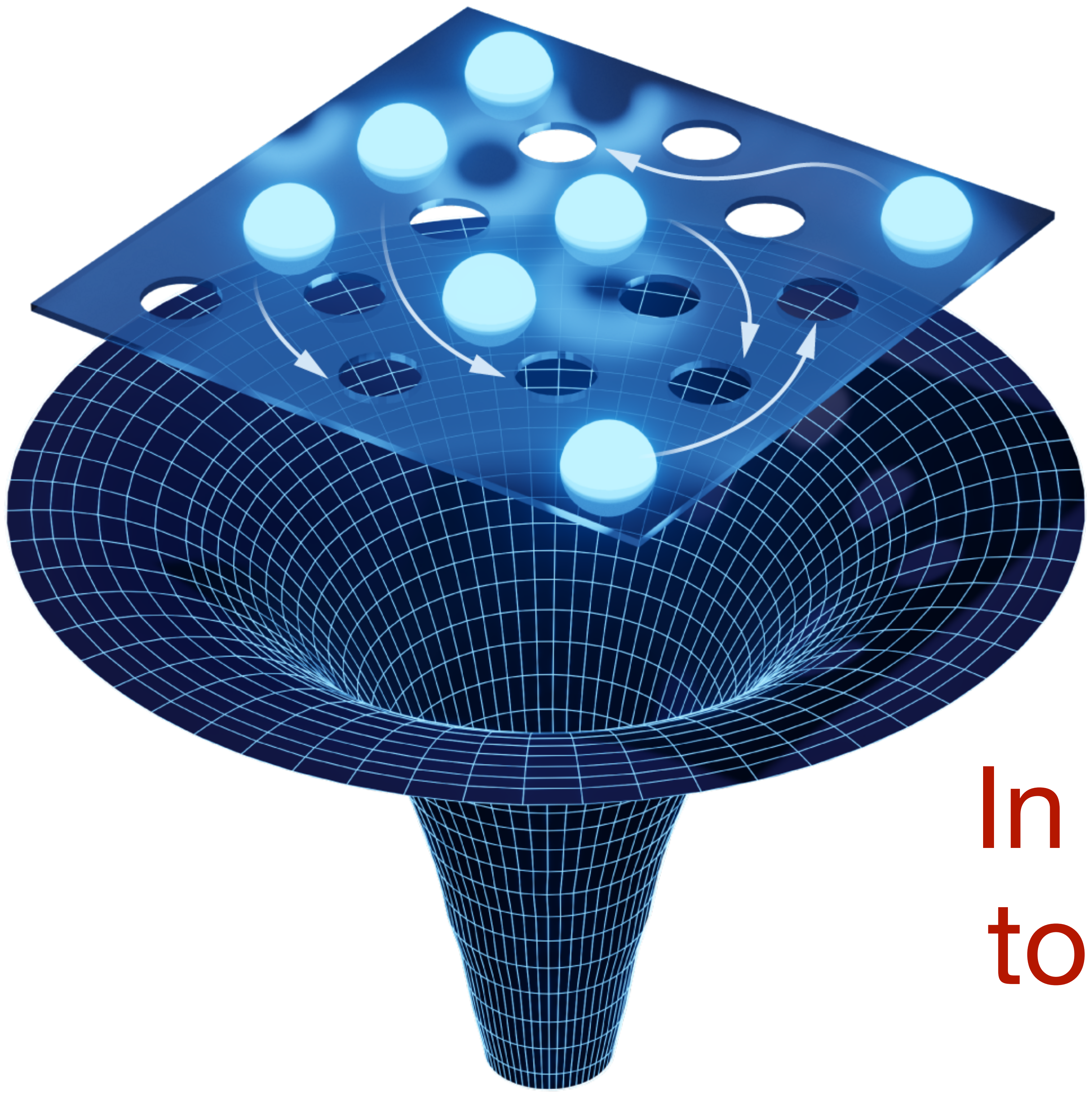


A 2d-YSYK theory describes the **strange metal** behavior of numerous quantum materials

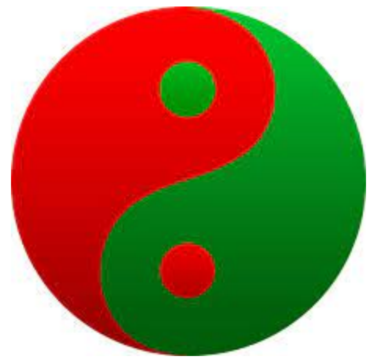


The Sachdev-Ye-Kitaev (SYK) model



The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

A 2d-YSYK theory describes the **strange metal** behavior of numerous quantum materials



In a *dual* set of variables the SYK model has led to the computation of the low energy density of states of ***charged black holes***