A Quantum Field Theoretical Study of Correlated Quantum Ising model with Longer Range Interaction

Sujit Sarkar
Poornaprajna Institute of Scientific Research
4 Sadashivanagar, Bangalore-5600 80

Statiscal Physics Community Meeting 2023
3rd February

Ackonwledgement: Library (Raman Research Institute)

Outline

1. Introduction

2. Basic Aspects of Model Hamiltonian

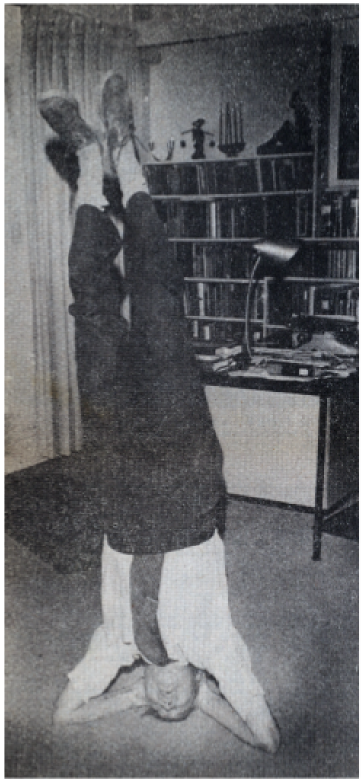
3. Results

4. Conclusions

A Few Words on Quantum Ising Model

This is remarkable for a simple model of

the electronic structure of a magnet. Devised in 1920 by Wilhelm Lenz, Ernst Ising's doctoral supervisor, it was given to Ising and solved by him in 1925 [3]. Ising solved a one-dimensional model, by way of transfer matrix. The solution is simple, but unfortunately, there is no phase transition in one-dimension, making the classical one-dimensional Ising model not interesting. In two- dimensions, we observe a phase transition from a paramagnet —disordered spins, no magnetisation —to a ferromagnet at temperatures below a critical point. On the other hand, in two-dimensions, the interactions become too complex to solve for analytically with any ease.



Ising 1976

Motivation

(1). The physics of quantum Ising model (QIM) has studied extensively in literature but the physics of strong correlation for has not explored.

(2). This model Hamiltonian have already been studied from the perspective of topological phases but the main motivation of our studyis to explore the physics from the perspective of correlated quantum many body system.

(3). This model Hamiltonian has three competiting interaction terms, the most interesting feature of this study is to show how the behaviour of RG flow lines for these three competiting interaction and finally leads to the different emergence phases.

Model Hamiltonian

$$H = -\sum_i (\mu\sigma_i{}^x + \lambda_1\sigma_i{}^z\sigma_{i+1}{}^z + \lambda_2\sigma_i{}^x\sigma_{i-1}{}^z\sigma_{i+1}{}^z). \qquad (1)$$

Here μ is the transverse field, λ_1 is the two spin interaction of nearest-neighbour (NN) sites and λ_2 is the three spin interactions. Thus we term this model Hamiltonian as lqIm with three spin interaction.

It is well known to us for qIm, the system is in disorder quantum phase when the transverse field exceed the FM coupling, we term this disorder quantum phase as dqpI. The coupling λ_2 is related with the three sites (i-1, i and i+1), the left (i-1) and right (i+1) sites are related with the σ_z operator and the middle site (i) is with the σ_x operator. It is very clear from this interaction term that next-nearest-neighbour (NNN) sites are related with the FM interaction with spin alignment in the Z direction. But NN sites are related with the *XZ* interaction, i.e, the spin flipping occurs at the site i. This interaction introduce the frustration in the system and finally leads to the disorder quantum phase.

Another Presentation of Model Hamiltonian

$$H = -\sum_{i=1}^{N} \sigma_{i-1}^{x} \sigma_{i}^{z} \sigma_{i+1}^{x} - h \sum_{i=1}^{N} \sigma_{i}^{z} + \lambda_y \sum_{i=1}^{N} \sigma_{i}^{y} \sigma_{i+1}^{y} + \lambda_x \sum_{i=1}^{N} \sigma_{i}^{x} \sigma_{i+1}^{x},$$

where σ_n^α ($\alpha = x, y, z$) are the Pauli matrices acting on the site n of the lattice and we impose periodic boundary conditions

$$H = -\sum_i (\mu \sigma_i^x + \lambda_1 \sigma_i^z \sigma_{i+1}^z + \lambda_2 \sigma_{i-1}^z \sigma_i^x \sigma_{i+1}^z)$$

$\downarrow$ $\sigma_i^x \to \sigma_i^z, \quad \sigma_i^z \to -\sigma_i^x$

(This is achieved by $U = \prod_j e^{i\frac{\pi}{4}\sigma_j^y}$,

because $U\sigma_i^x U^\dagger = \sigma_i^z$, $U\sigma_i^z U^\dagger = -\sigma_i^x$)

$$H = -\sum_i (\mu \sigma_i^z + \lambda_1 \sigma_i^x \sigma_{i+1}^x + \lambda_2 \sigma_{i-1}^x \sigma_i^z \sigma_{i+1}^x)$$

This is equivalent to Eq.(2) of arXiv:1112.4414

$$H = -h \sum_i \sigma_i^z - \sum_i \sigma_{i-1}^x \sigma_i^z \sigma_{i+1}^x + \lambda_x \sum_i \sigma_i^x \sigma_{i+1}^x + \lambda_y \sum_i \sigma_i^y \sigma_{i+1}^y,$$

if we identify $(\mu, \lambda_1, \lambda_2)$ with $(h, -\lambda_x, 1)$.

(NOTE : λ_y in Eq.(2) should be set 0.)

Bosonized Hamiltonian

$$H = H_0 + \frac{\lambda_1}{2}\int \cos[4\sqrt{\pi}\phi(r)]dr -$$
$$\mu \int \cos[2\sqrt{\pi}\phi(r)]\cos[\sqrt{\pi}\theta(r)]dr$$
$$-\lambda_2 \int \cos[2\sqrt{\pi}\phi(r)]\cos[\sqrt{\pi}\theta(r)]\,(\partial_r\phi(r))^2\,dr \quad (2)$$

where $H_0 = \frac{v}{2}\int[(\partial_r\sqrt{K}\phi(r))^2]dr$ with $v = \lambda_1$. $\theta(x)$ is the dual field of $\phi(x)$ and satisfy the following commutation relation , $[\phi(r), \partial_r\theta(r')] = -i\pi\delta(r - r')$.

The bosonized form of the model Hamiltonian consists four terms. The first term is the kinetic energy term and the rest three terms present the sine-Gordon coupling terms. It is to be noted that the starting Hamiltonian (eq. 1) has no K, term but it appears after the continuum field theoretical calculation

$$H = -\sum_i (\mu S_i^x + \lambda_1 S_i^z S_{i+1}^z + \lambda_2 S_i^x S_{i-1}^z S_{i+1}^z), \quad (11)$$

The field, ϕ, corresponds to the spin fluctuations and θ is

). ϕ and θ represent the bosonic fluctuation of the system.

$$S_n^x = [\cos(2\sqrt{\pi K}\phi(r)) + (-1)^n] \cos[\sqrt{\frac{\pi}{K}}\theta(r)] \quad (12)$$

$$S_n^y = [\cos(2\sqrt{\pi K}\phi(r)) + (-1)^n] \sin[\sqrt{\frac{\pi}{K}}\theta(r)] \quad (13)$$

$$S_n^z = (-1)^n \cos(2\sqrt{\pi K}\phi(r)) + \sqrt{\frac{\pi}{K}}\partial_r\phi(r). \quad (14)$$

Here K is the Luttinger liquid parameter. The physics of low-dimensional quantum many body condensed matter system is enriched with its new and interesting emergent behavior. $K < 1$ and $K > 1$ and $K = 1$ characterizes the repulsive, attractive interactions and non-interacting, respectively

effect of strong correlation on qIm and lqIm through the Luttinger liquid parameter.

Sine-Gordon model Hamiltonian

We notice that the nearest-neighbour (NN) coupling term (λ_1) is related with the a single sine-Gordon coupling term of the field (ϕ (x)). The transverse field is the product of two sine-Gordon coupling terms of $\phi(x)$ and $\theta(x)$ which are dual to the each other. But we notice that for the longer range coupling with three spin interaction is also product of two sine-Gordon coupling terms augmented with a part of the kinetic energy term from the field $\phi(x)$. These extra sine-Gordon coupling terms for the longer range interaction over the quantum Ising model give the enrich quantum physics over the qIm.

Renormalization Group Study

our model Hamiltonian contains three strongly relevant and mutually nonlocal perturbations over the Gaussian (critical) theory. In such a situation, the strong coupling fixed point is usually determined by the most relevant perturbation whose amplitude grows up according to its Gaussian scaling dimensions and it is not much affected by the less relevant coupling terms. However, this is not the general rule if the operators exclude each other. In this case, the interplay between the three competing relevant operators (here μ, λ_1 and λ_2) are the three competing relevant operators, which are related with dual fields $\theta(x)$ and $\phi(x)$ can produce a novel quantum phase transition through a critical point or a critical line [23,24]. Therefore, the present study based on RG equations will give us the appropriate results for these model Hamiltonian. Now we present the RG equations for the present study:

Renormalization Group Equations
for Correlated Quantum Ising Models

Renormalization Group Equation for Correlated Quantum Ising Model and Extended Correlated Quantum Ising Model.

$$\frac{d\lambda_1}{dl} = (2-4K)\lambda_1 + \frac{\mu^2}{8}(2K - \frac{1}{2K})$$
$$\frac{d\mu}{dl} = (2-K-\frac{1}{4K})\mu + \lambda_1 \mu K$$
$$\frac{dK}{dl} = -\lambda_1^2 K^2$$

$$\frac{d\lambda_1}{dl} = (2-4K)\lambda_1 + \frac{\mu^2}{8}(2K - \frac{1}{2K})$$
$$\frac{d\lambda_2}{dl} = (2-K-\frac{1}{4K})\lambda_2 + \frac{\lambda_1 \lambda_2 K}{\pi}$$
$$\frac{d\mu}{dl} = (2-K-\frac{1}{4K})\mu + \lambda_1 \mu K$$
$$\frac{dK}{dl} = -\lambda_1^2 K^2 \qquad (4)$$

Renormalization Group Equations With Out Transverse Field

$$\frac{d\lambda_1}{dl} = (2 - 4K)\lambda_1$$
$$\frac{d\lambda_2}{dl} = (2 - K - \frac{1}{4K})\lambda_2 + \frac{\lambda_1 \lambda_2 K}{\pi}$$
$$\frac{dK}{dl} = -\lambda_1^2 K^2 \qquad (5)$$

Benchmarking the results

$$H = -\lambda_1 \sum_i (g{\sigma_i}^x + {\sigma_i}^z {\sigma_{i+1}}^z),$$

where $g = \frac{\mu}{\lambda_1}$.

A nonzero g allows for tunnelling between the up and the down states of the spin with an amplitude proportional to g. The eigenvalues of ${\sigma_i}^z$ are ± 1 and the corresponding eigenstates can be labelled as $|\uparrow>$ and $|\downarrow>$. This model Hamiltonian is quite well studied in the quantum condensed matter physics. Now we state the main results very briefly in the different limits.

(A). In the case of $g << 1$, the tunnelling between the $|\uparrow>$ and the $|\downarrow>$ states can be neglected. This leads to a ferromagnetically ordered state in the z-direction with the Z_2 symmetry broken. The fundamental excitations away from this ground state correspond to domain walls between lines of flipped spins. For $g = 0$, each spin configuration is an eigenstate of the Hamiltonian as we have already seen in the classical Ising model. As we increase g to a small but nonzero value, the domain walls become mobile even at zero temperature. This leads to the development of zero-point motion and quantum kinetics.

(B). In the case of $g >> 1$, the ground state is an eigenstate of ${\sigma^i}_x$. It is well known the two eigenstates of ${\sigma^i}_x$.

The state where all spins are aligned with the transverse field (i.e. they are all in the $|\rightarrow>_i$ configuration) corresponds to the ground state of a quantum paramagnet with no spontaneously broken symmetry. The system can be excited out of the ground state by flipping spins in the direction opposite to the external field (i.e. into the $|\leftarrow>_i$. configuration). These flipped spins then correspond to quasiparticles which are stationary for $g = 1$ and

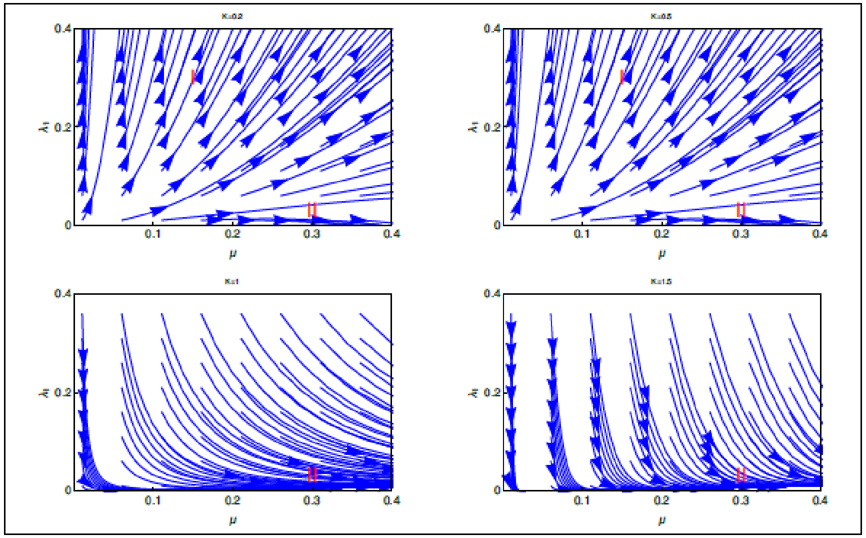


Fig. 1. (Color online.) This figure shows the behaviour of renormalization group flow lines for the couplings λ_1 and μ (eq. 3) for the different initial values of K as depicted in figures.

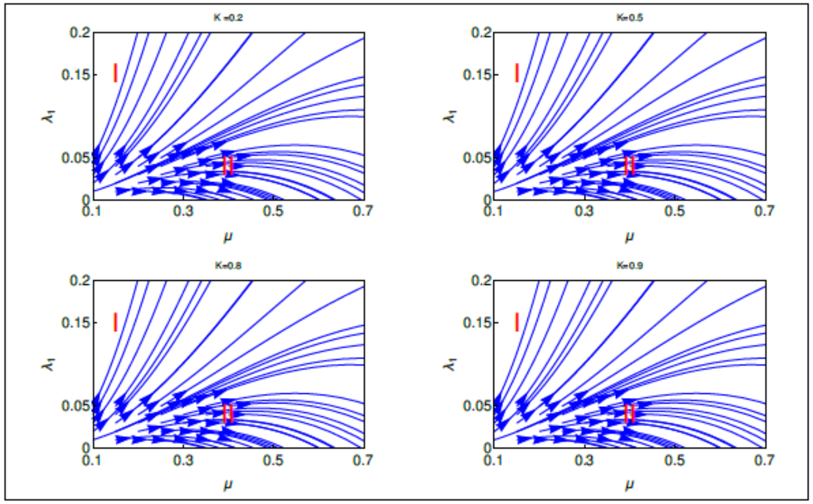


Fig. 2. (Color online.) This figure shows the behaviour of renormalization group flow lines for the couplings λ_1 and μ (eq. 3) for the different initial values of K as depicted in figures.

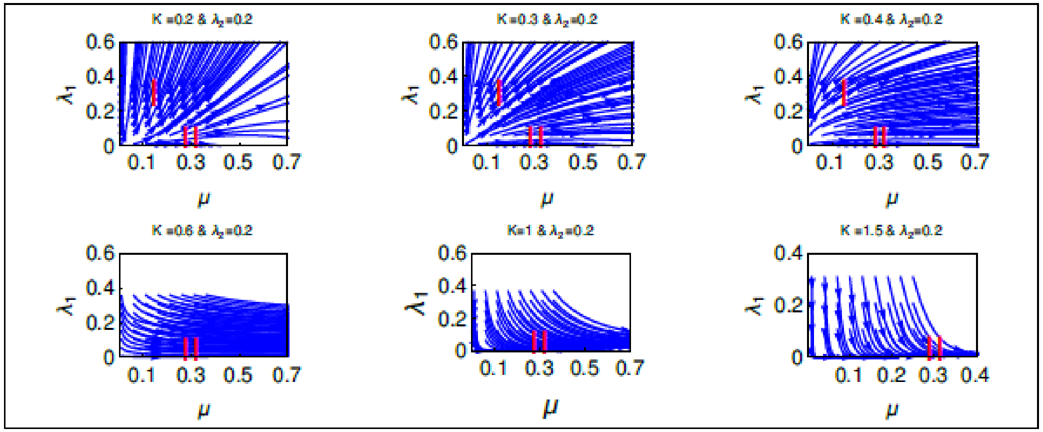


Fig. 3. (Color online.) This figure shows the behaviour of renormalization group flow lines for the couplings λ_1 and μ (eq. 4) for the different initial values of K and λ_2 as depicted in the figure.

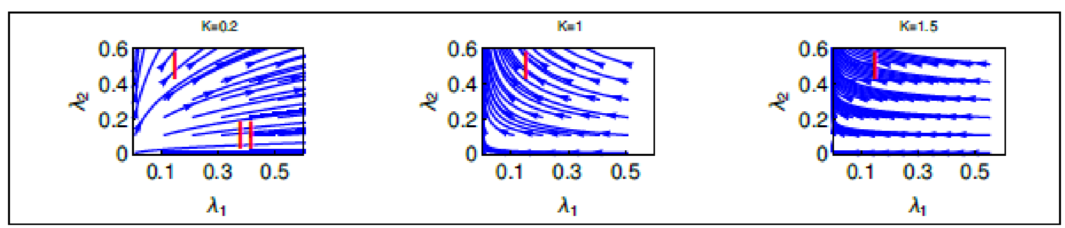


Fig. 4. (Color online.) This figure show the behaviour of renormalization group flow lines for the couplings λ_2 and λ_1 (eq. 4) for the different initial values of K as depicted in figures.

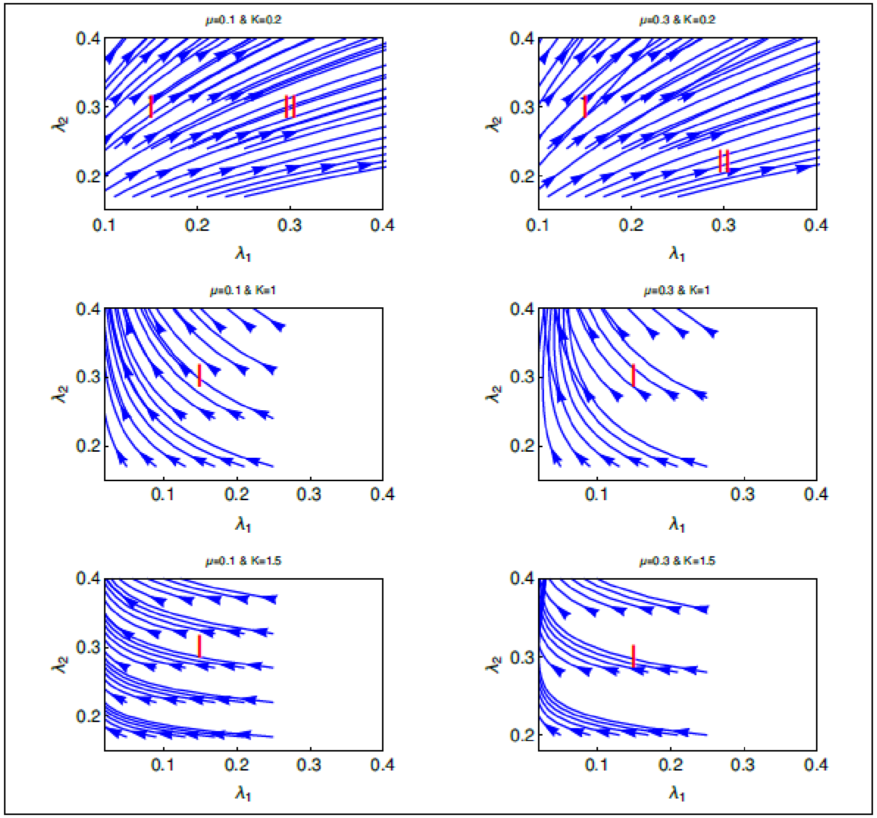


Fig. 5. (Color online.) Shows the behaviour of renormalization group flow lines for the couplings λ_2 and λ_1 for the different initial values of K and μ. This figure consists of three rows for different initial values of K. We present the RG flow lines based on eq.4 . The left and right figures are respectively for $\mu = 0.1$ and $\mu = 0.3$.

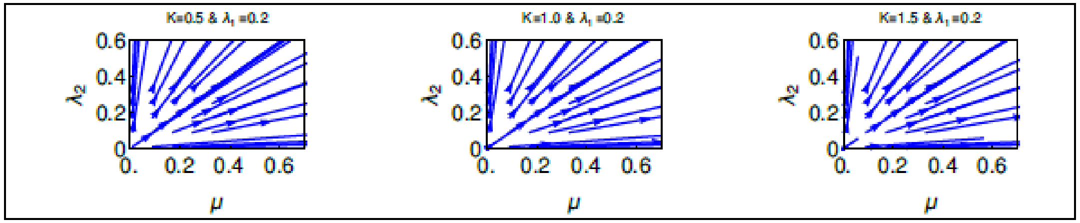


Fig. 6. (Color online.) This figure shows the behaviour of renormalization group flow lines for the couplings λ_2 and μ (eq. 4) for the different initial values of K and λ_1 as depicted in figures.

Scaling Analysis

It is well known that the critical theory is invariant under the rescaling. Then the singular part of the free energy density satisfies the following scaling relations [1].

$$f_s[\mu, \lambda_1, \lambda_2] = {\lambda_1}^{2/(2-4K)} f_s[1, {\lambda_1}^{-(2-4K)/(2-K-1/4K)}\lambda_2, {\lambda_1}^{-(2-4K)/(2-K-1/4K)}\mu] \quad (6)$$

The scaling equation for λ_1 and λ_2 is the following:

$$f_s[\lambda_1, \lambda_2] = {\lambda_1}^{2/(2-4K)} f_s[1, {\lambda_1}^{-(2-4K)/(2-K-1/4K)}\lambda_2] \quad (7)$$

The scaling equation for λ_1 and μ is the following:

$$f_s[\lambda_1, \mu] = {\lambda_1}^{2/(2-4K)} f_s[1, {\lambda_1}^{-(2-4K)/(2-K-1/4K)}\mu] \quad (8)$$

The scaling equation for λ_2 and μ is the following:

$$f_s[\lambda_2, \mu] = {\lambda_2}^{2/(2-K-1/4K)} f_s[1, {\lambda_2}^{-(2-K-1/4K)/(2-K-1/4K)}\mu] \quad (9)$$

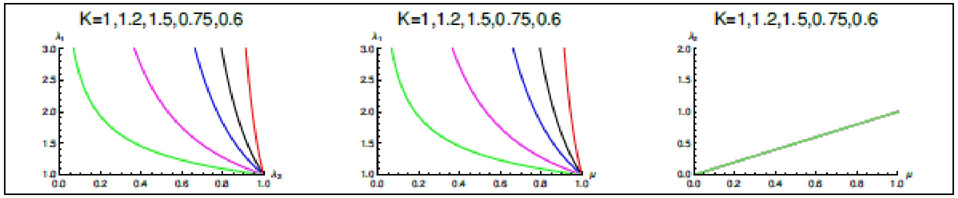


Fig. 7. (Color online.) Shows the results for scaling analysis based on the equations (6) and (9). The Green, Magenta, Blue, Black and Red lines are respectively for $K = 0.6, 0.75, 1, 1.2$ and 1.5.

Results

(1). We have found two emergent quantum disorder phase. One is due to the transverse field and the other is due to the three spin interaction term.

(2). Three different kinds of mixed phases due to the following sources.

(a). Coexistence of transverse field flow and FM coupling.

(b). Coexietence of ferromagnetc coupling and three spin interactions.

(C). Coexistence of three spin interactions and transverse field.

(3). We derive three scaling relations for our model Hamiltonian. Scaling theory results is consistent with the results of RG study.

Conclusions

(1). Quantum Ising Chain: Order ferromagnetic phase to disorder quantum paramagnetic phase transition occurs only for strongly correlated regime.

(2). Longer Range Quantum Ising Chain: Order ferromagnetic phase to disorder quantum paramagnetic phase transition occurs for more correlated regime.

(3). We have predicted three different regimes of coexistence phase in three different regime of strongly correlated phase.

(4). Our analysis of scaling consistent with the results of quantum field theoretical RG results.

Thank You