Classical limit of measurement-induced transition in many-body chaos in integrable and non-integrable oscillator chains

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8th Indian Statistical Physics Community Meeting
ICTS, February 2, 2023

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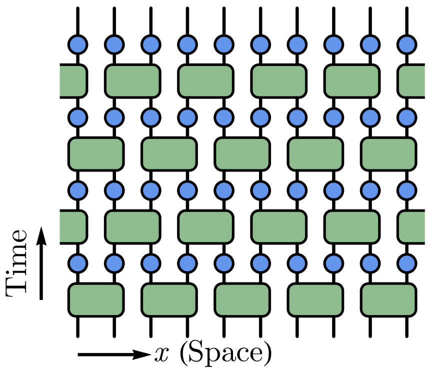
S. Ruidas & SB, arXiv:2210.03760

Measurement-induced phase transition (MIPT)

Random quantum circuits with non-unitary evolution



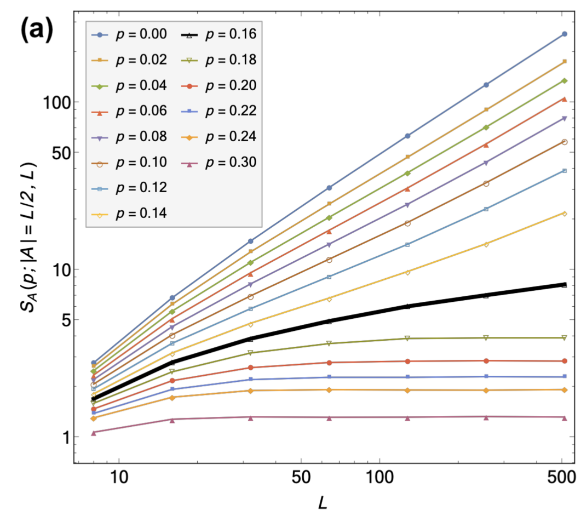
Unitary evolution + measurements

local projective or weak measurements

Tune measurement rate, measurement strength, etc., ..

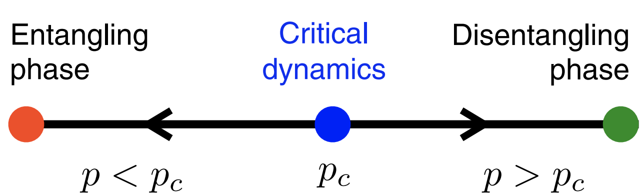
Other systems – non-interacting fermions, interacting bosons, Luttinger liquids, ..

Skinner et al. (2019), Bao et al. (2019);
Li et al. (2019), Jian et al. (2019),
Gullans et al. (2020), Nahum et al. (2021)
Sang et al. (2021), ...



"Chaotic"
Volume law
$S_A \sim L^d$

"Non chaotic"
Area law
$S_A \sim L^{d-1}$



Chaotic to non-chaotic phase transitions

Transition in steady state from volume-law to area-law entanglement

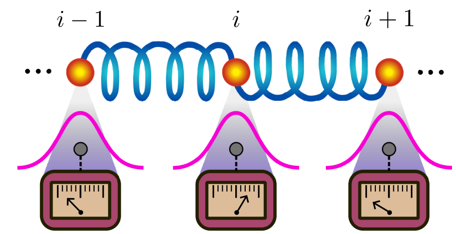
Can there be “measurement-induced phase transition” in the “classical limit”?

Outline/Summary

○ Semiclassical limit of a model of continuous weak measurements

⇒ Stochastic Langevin equation

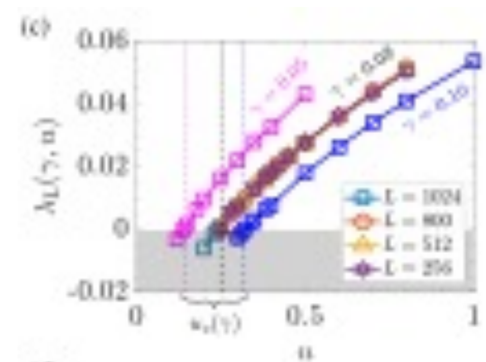
noise/dissipation ∝ “measurement strength”

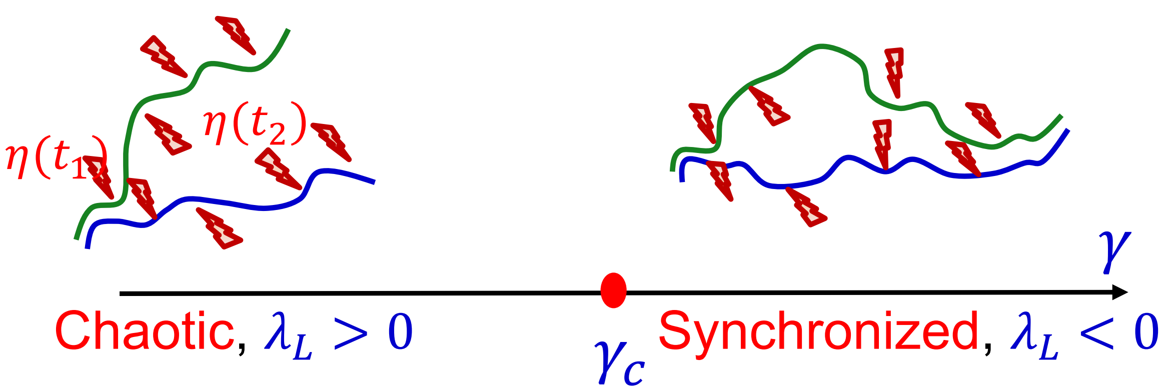


○ Noise/measurement induced chaotic to non-chaotic transition in coupled oscillators

Stochastic synchronization transition

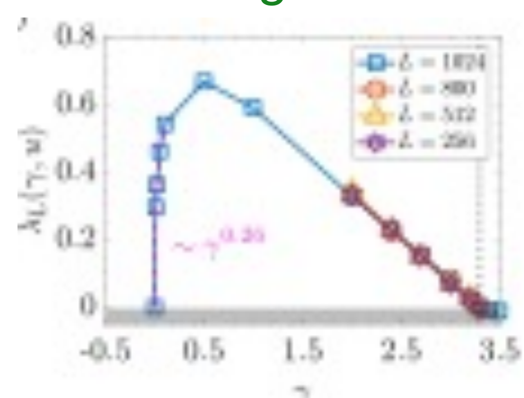
Non-integrable





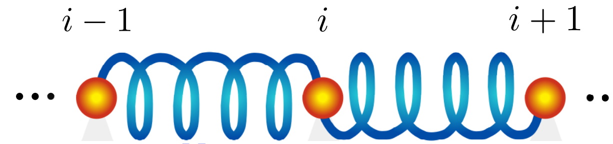
Noise/dissipation (measurement) strength

Integrable



Quantum model of weak measurements

Caves and Milburn, Phys. Rev. A 36 (1987)

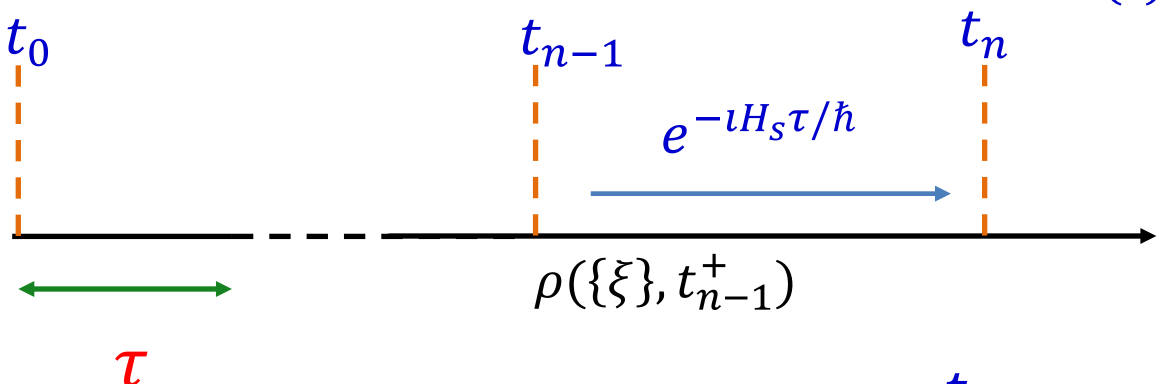


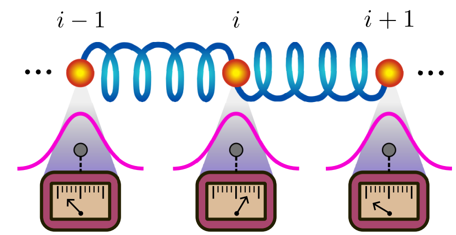
$$H_s = \sum_{i=1}^{N} \frac{\hat{p}_i^2}{2m} + V(\{\hat{x}_i\})$$

System under repeated weak measurements in intervals of τ

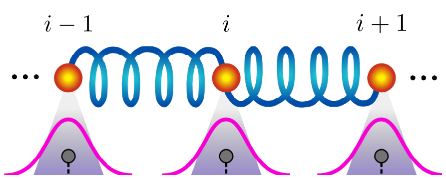
$$t_n = n\tau$$

$$H(t) = H_s + \sum_{i,n} \delta(t - t_n)\, \hat{x}_i\, \hat{p}_{in}$$





t_n^-

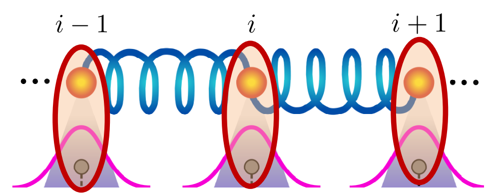


Meters

$\hat{\xi}_{in}, \hat{p}_{in}$

$$\psi(\xi_{in}) \sim \exp\left(-\frac{\xi_{in}^2}{2\sigma}\right)$$

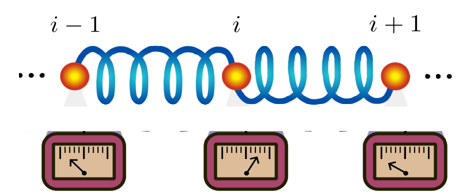
t_n



Apply

$$\sum_i \delta(t - t_n)\, \hat{x}_i\, \hat{p}_{in}$$

t_n^+

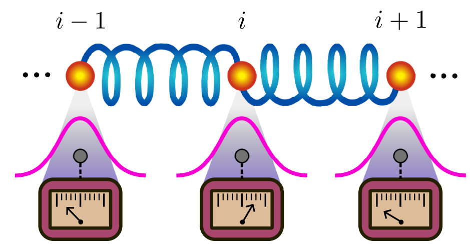


Readings

$\{\xi_{in}\}$

Projective measurements

Time Evolution



Measurement + feedback

Limit of continuous weak measurement

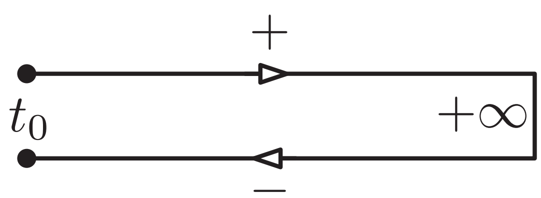
$\sigma \to \infty, \tau \to 0$ with Δ finite

$\Delta = \sigma\tau$

Measurement strength Δ^{-1}

Schwinger-Keldysh path integral

$$Tr[\rho(\{\xi(t)\})] = \int \mathcal{D}x \, e^{\frac{\imath S[\{\xi(t)\},x(t)]}{\hbar}}$$



Action

$$S[\{\xi\},x] = \int_{-\infty}^{\infty} dt \sum_{s=\pm} s \left[\sum_i \left\{ \frac{m}{2} \dot{x}_{is}^2 + m\gamma \, \dot{x}_{is}\xi_i + \frac{\imath s\hbar}{2\Delta}(x_{is} - \xi_i)^2 \right\} - V(\{x_{is}\}) \right]$$

Semiclassical limit, small $\hbar$

Expand in $\hbar$ while scaling $\Delta \sim \hbar^2$

$\Rightarrow$ Stochastic Langevin equation

$$\frac{d^2 x_i}{dt^2} + \gamma \frac{dx_i}{dt} = \frac{1}{m}\left[-\frac{\partial V(\{x_i\})}{\partial x_i} + \eta_i(t)\right]$$

$$\langle \eta_i(t)\eta_j(t')\rangle = 2m\gamma T\delta_{ij}\delta(t-t')$$

Noise strength $\sim \gamma T \sim \frac{\hbar^2}{\Delta}$

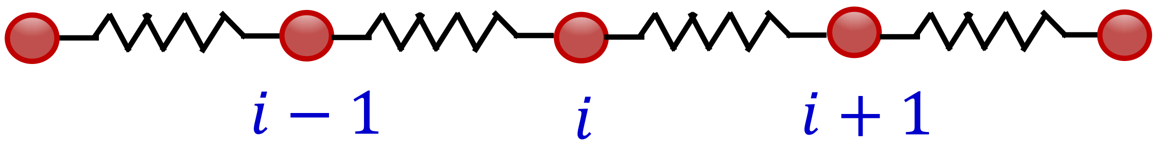
$\propto$ measurement strength

Effective temperature $T \sim \frac{\hbar^{\frac{3}{2}}}{\sqrt{\Delta}} \sim \sqrt{\hbar}$

Long-time steady state (non-equilibrium steady state) $\Rightarrow$

Classical Boltzmann distribution $\sim \exp\left[-\frac{H_s(\{x_i, p_i\})}{T}\right]$

Integrable and non-integrable anharmonic chains of oscillators



Two models:

1. Non-integrable model
   Anharmonic coupled oscillators

$$V(\{x_i\}) = \sum_i \left[\frac{k}{2}(x_{i+1}-x_i)^2 + \frac{u}{4}(x_{i+1}-x_i)^4\right]$$

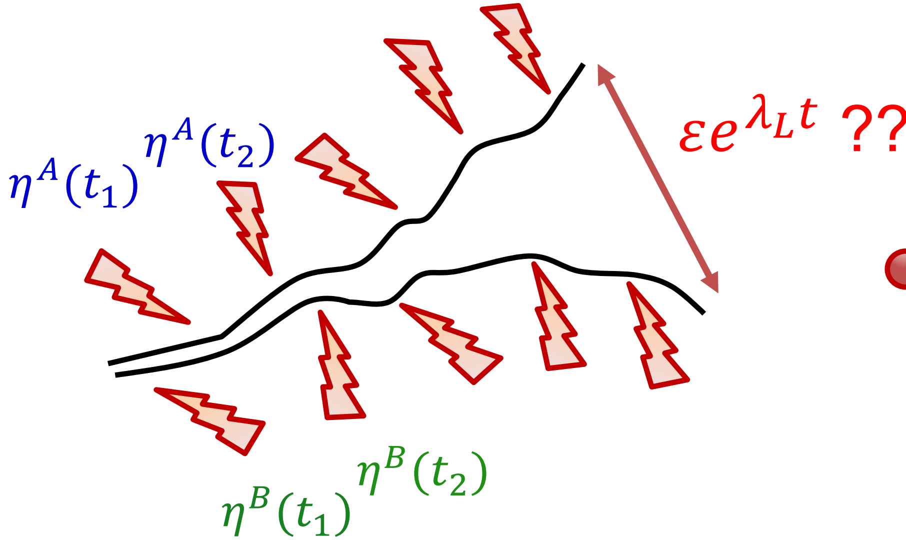
2. Integrable model
   Toda chain

$$V(\{x_i\}) = \sum_i \left[\frac{a}{b}e^{-b(x_{i+1}-x_i)} + a(x_{i+1}-x_i) - \frac{a}{b}\right]$$

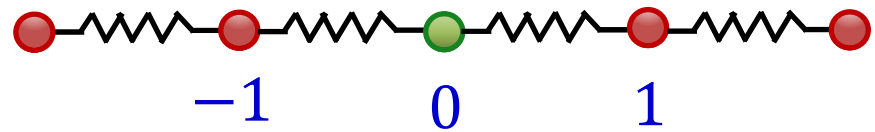
N constants of motion

Can one meaningfully define chaos in the presence of noise?

System is randomly kicked at each instant of time.



Noise strength, $\gamma \neq 0$



$$x_i^A(0) - x_i^B(0) = \varepsilon \delta_{i,0}$$

Take exactly the same noise realizations for the two copies

$$\{\eta_i^A(t)\} = \{\eta_i^B(t)\} \quad \forall t$$

Momentum OTOC

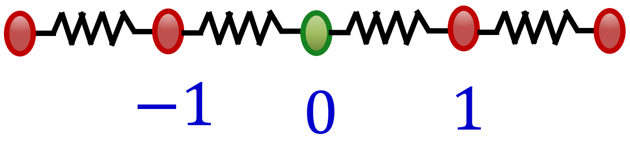
$$D(i,t) = \left\langle \left(p_i^A(t) - p_i^B(t) \right)^2 \right\rangle_{T,\{\eta\}}$$

with perturbation at $i = 0, t = 0$

- Thermal initial condition at temperature T is generated using Langevin dynamics

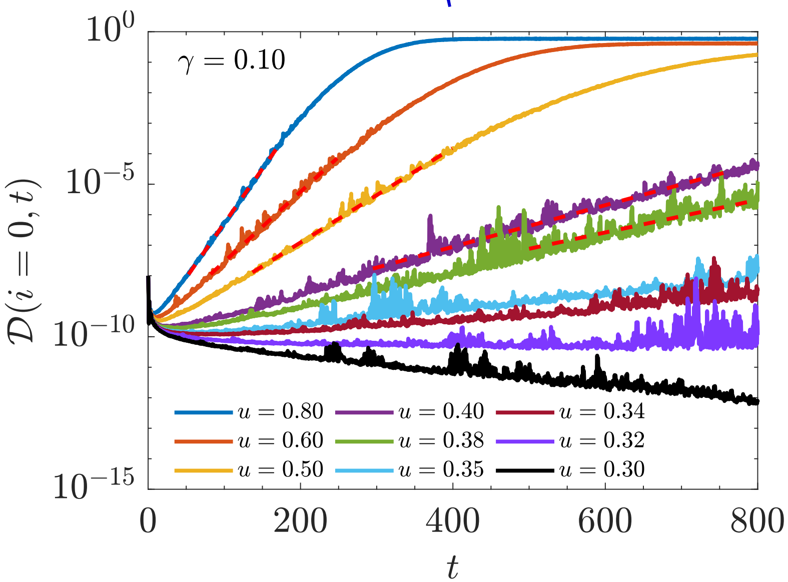
Noise-induced chaotic to non-chaotic transition

Non-integrable model, Anharmonic oscillators

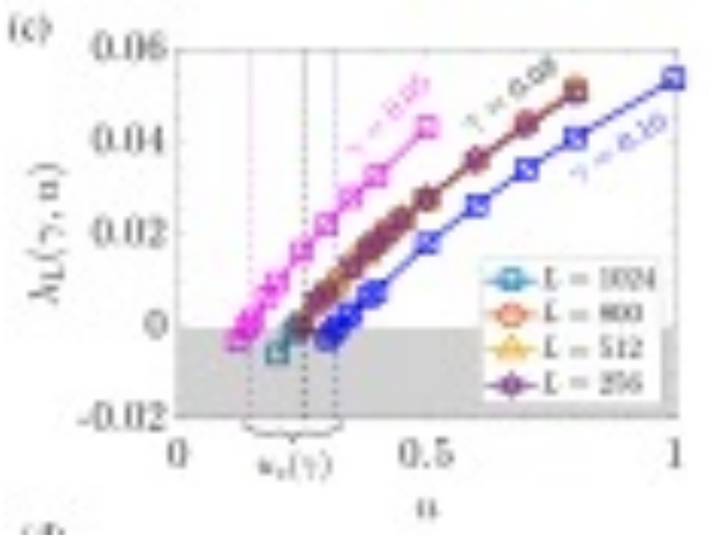


$$V(\{x_i\}) = \sum_i \left[\frac{k}{2}(x_{i+1} - x_i)^2 + \frac{u}{4}(x_{i+1} - x_i)^4\right]$$

OTOC $D(i,t) = \left\langle \left(p_i^A(t) - p_i^B(t)\right)^2 \right\rangle_{T,\{\eta\}}$



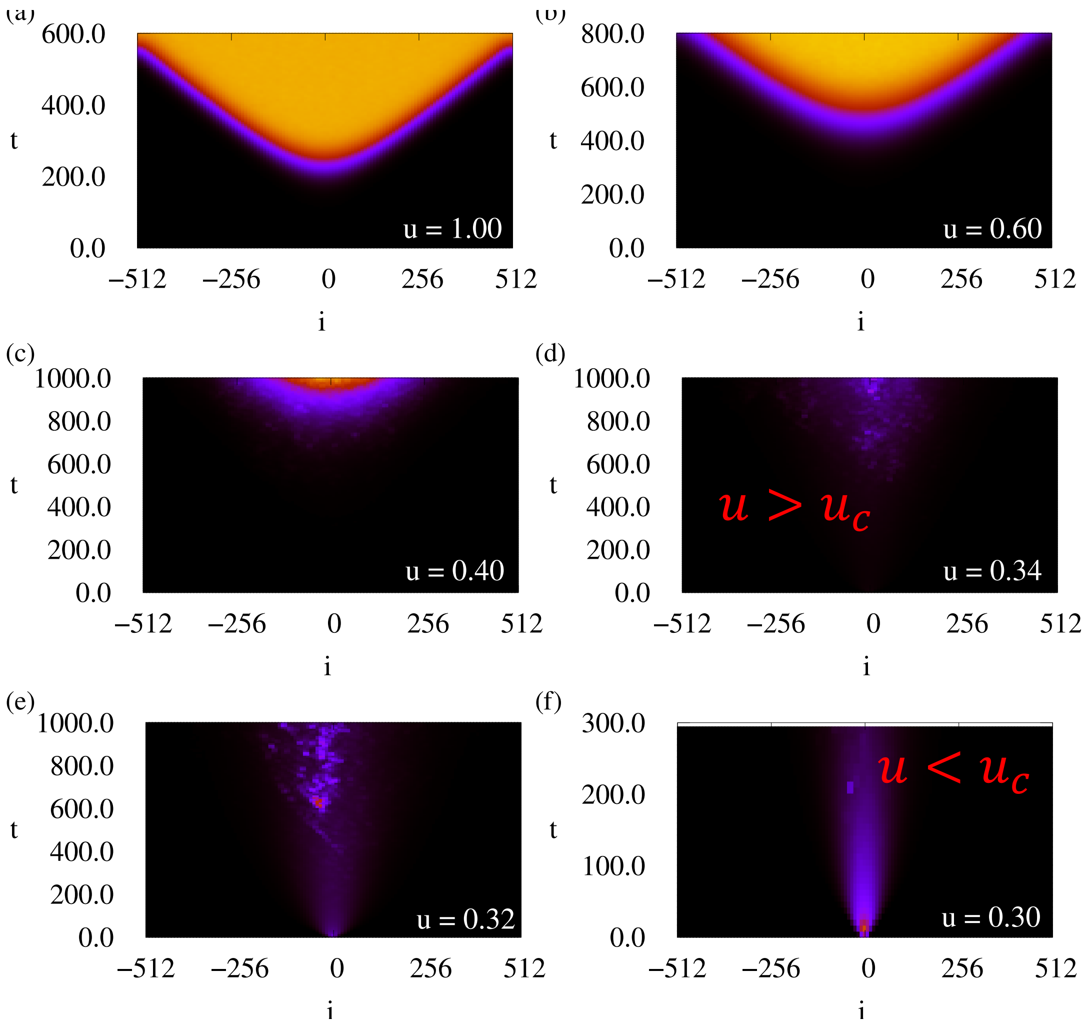
- Harmonic limit $(u = 0)$ is non-chaotic.
- Transition from exponential growth to exponential decay as a function of decreasing or u/γ



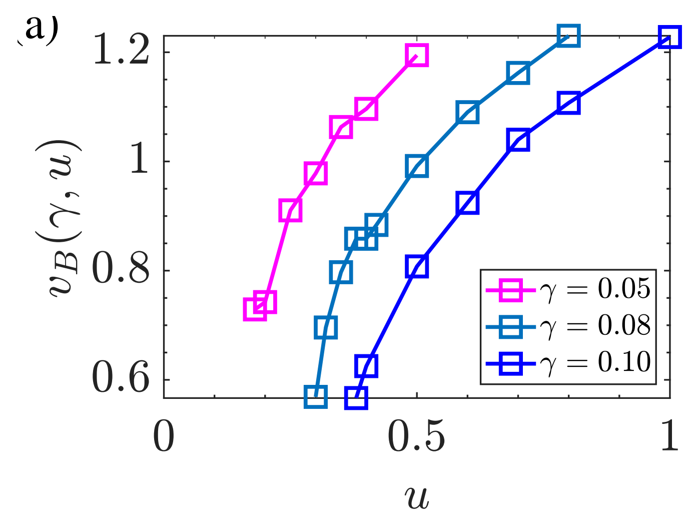
Lyapunov exponent

- $\lambda_L > 0 \rightarrow \lambda_L < 0$ for $u < u_c(\gamma)$ or $\gamma < \gamma_c(u)$

Light cone and butterfly velocity



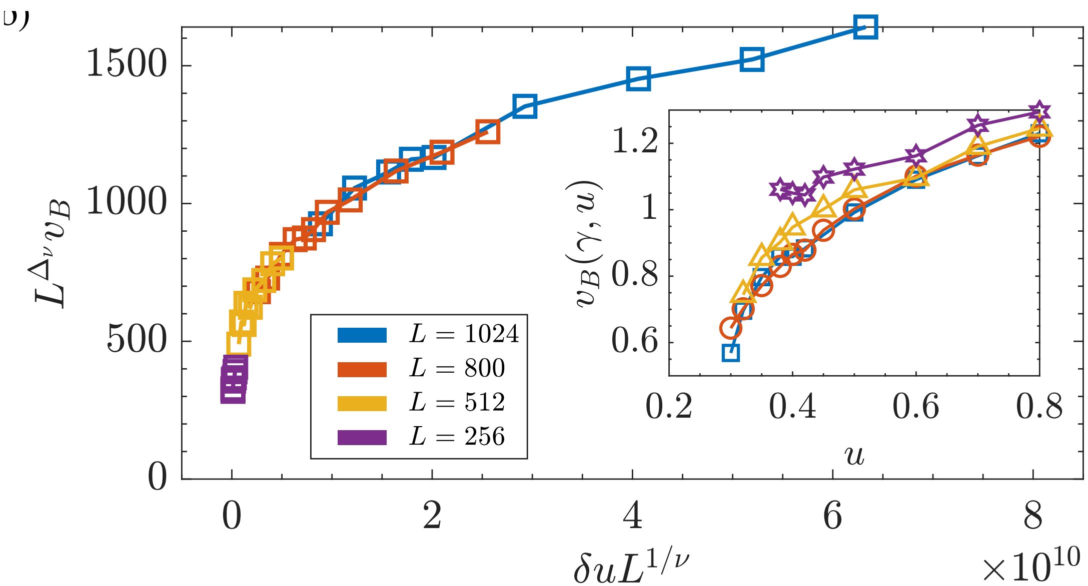
Butterfly velocity



- $v_B \to 0$ for $u \lesssim u_c(\gamma)$.

- Light cone is destroyed for $u < u_c(\gamma)$.

Dynamical transition and finite-size scaling



$$\delta u = u - u_c > 0$$

$$v_B(u, L) = L^{-\frac{\beta}{\nu}} \mathcal{F}(\delta u\, L^{1/\nu})$$

$$v_B \sim (\delta u)^{\beta}$$
$$\xi \sim (\delta u)^{-\nu}$$

$\beta \simeq 0.28, \qquad \nu \simeq 0.3$

- The transition shows critical scaling.

- The critical exponents do not match with known universality classes like directed percolation (DP) or multiplicative noise (MN)

Recent works on chaotic transition in classical systems

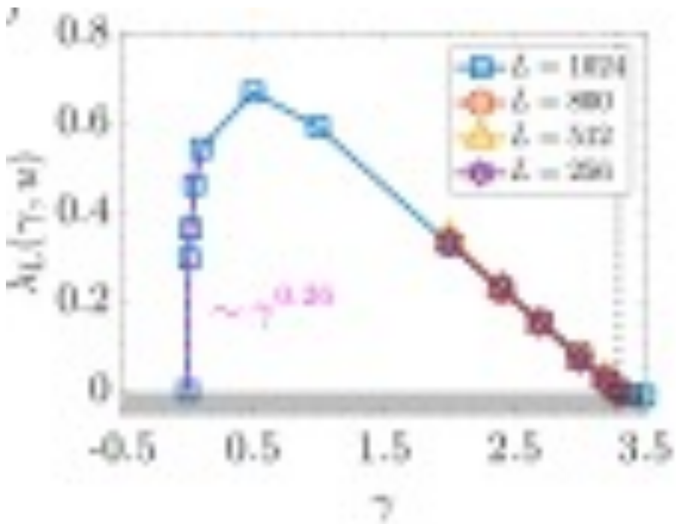
Willsher et al. PRB (2022); Deger et al. PRLs (2022)

- DP universality class

Noise-induced chaotic to non-chaotic transitions in Toda chain

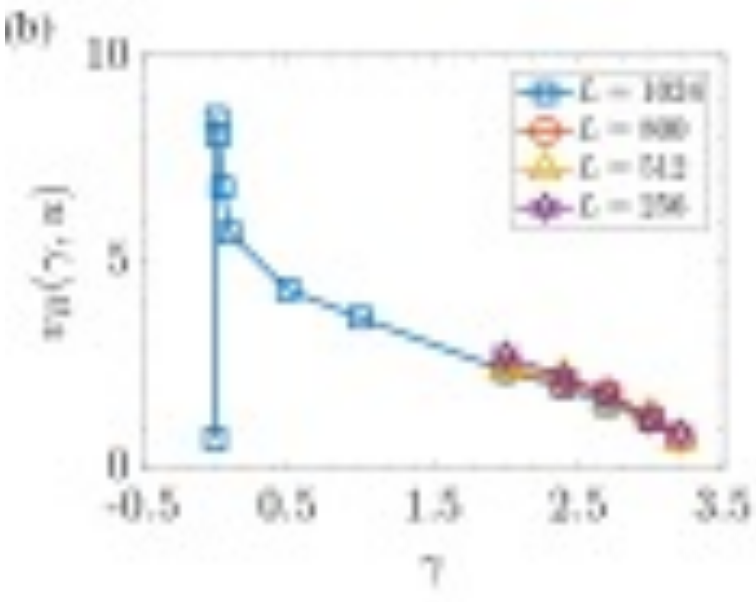
Integrable model $V(\{x_i\}) = \sum_i \left[\frac{a}{b} e^{-b(x_{j+1}-x_j)} + a(x_{j+1}-x_j) - \frac{a}{b}\right]$

Lyapunov exponent



- $\lambda_L \to 0$, $v_B \to$ large in the integrable limit $\gamma \to 0$.
- $\lambda_L, v_B \to 0$ for $\gamma > \gamma_c$.

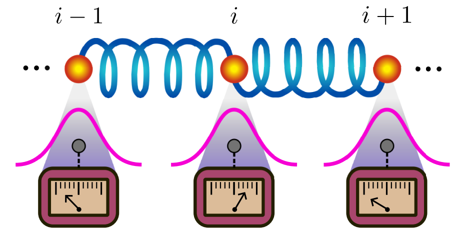
Butterfly velocity



- Weak noise induces weak chaos in integrable model

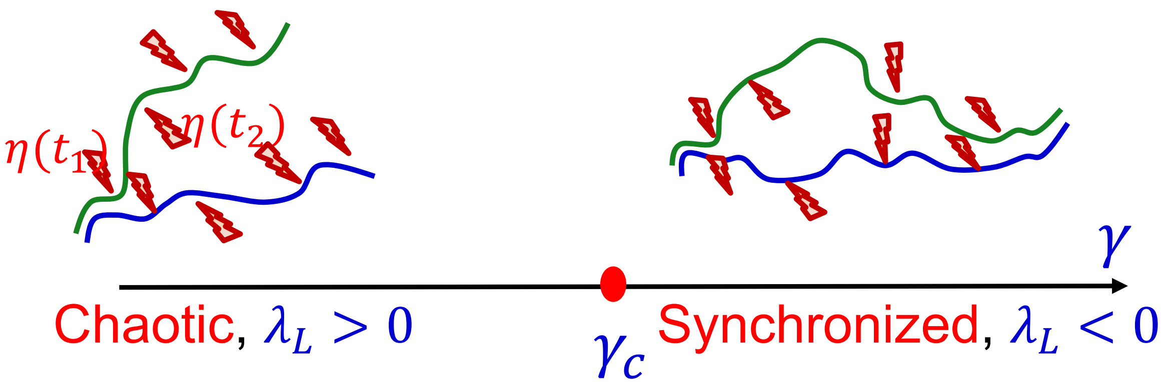
Lam and Kurchan, J. Stat. Phys. 156 (2014)

Summary and conclusion



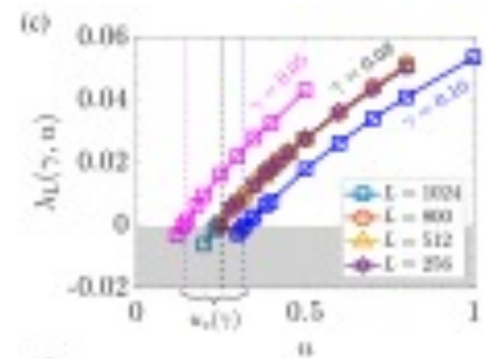
- Semiclassical limit of a model of continuous weak measurements
  - ⇒ Stochastic Langevin equation
  - noise/dissipation ∝ "measurement strength"

- Noise/measurement induced chaotic to non-chaotic transition
  - Stochastic synchronization transition

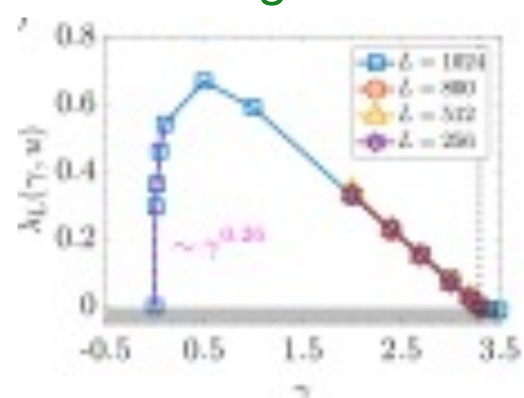


Noise/dissipation (measurement) strength

Non-integrable



Integrable



Thank You!