

Detecting gravitational waves using Cosmic Microwave Background

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Tata Institute of Fundamental Research

HEARING BEYOND THE STANDARD MODEL WITH COSMIC SOURCES OF
GRAVITATIONAL WAVES

ICTS-TIFR,
January 6th, 2025



THE SPECTRUM OF GRAVITATIONAL WAVES

Observatories & experiments

Ground-based experiment

Space-based observatory

Pulsar timing array

Cosmic microwave background polarisation



Timescales



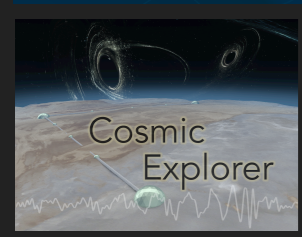
Frequency (Hz)



Cosmic sources

Cosmic fluctuations in the early Universe

#lisa



THE SPECTRUM OF GRAVITATIONAL WAVES

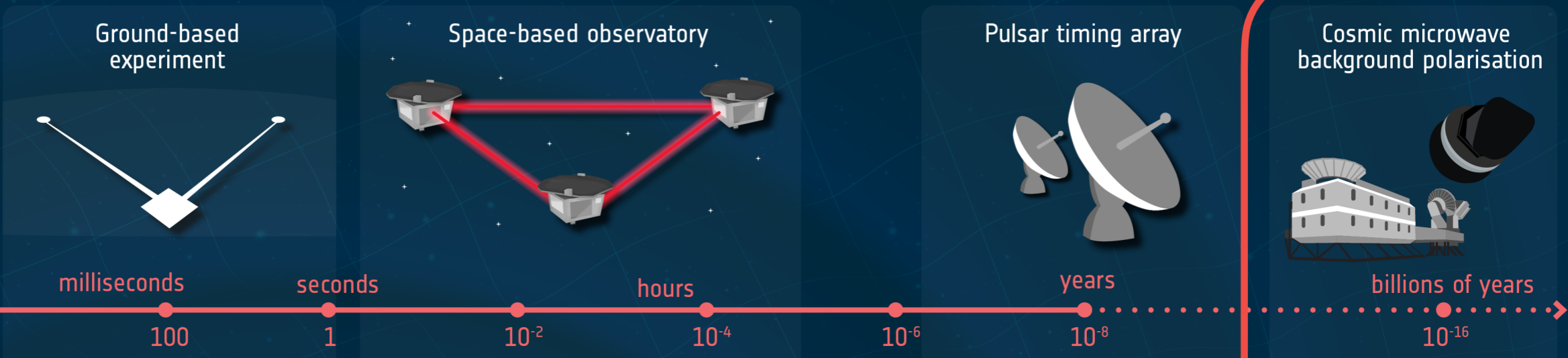
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Cosmic sources

Cosmic fluctuations in the early Universe

Supernova

Pulsar

Compact object falling onto a supermassive black hole

Merging supermassive black holes

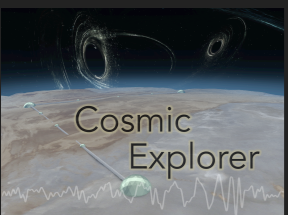
Merging neutron stars in other galaxies

Merging stellar-mass black holes in other galaxies

Merging white dwarfs in our Galaxy

Today

#lisa



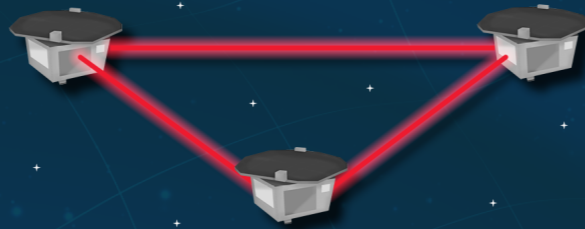
THE SPECTRUM OF GRAVITATIONAL WAVES

Observatories & experiments

Ground-based experiment



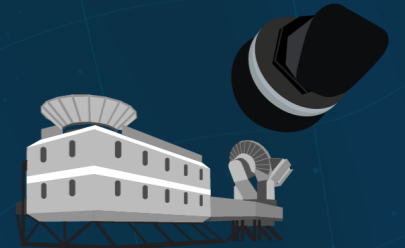
Space-based observatory



Pulsar timing array



Cosmic microwave background polarisation



Timescales

milliseconds

seconds

hours

years

billions of years

Frequency (Hz)

100

1

10⁻²

10⁻⁴

10⁻⁶

10⁻⁸

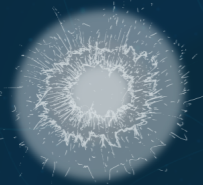
10⁻¹⁶

Tomorrow's talk

Cosmic fluctuations in the early Universe

Tomorrow's talk

Cosmic sources



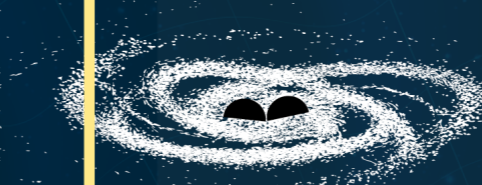
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Compact object falling onto a supermassive black hole



Merging supermassive black holes



Merging neutron stars in other galaxies

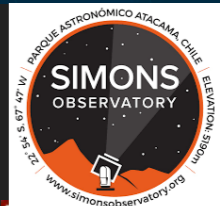
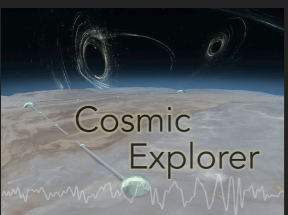


Merging stellar-mass black holes in other galaxies

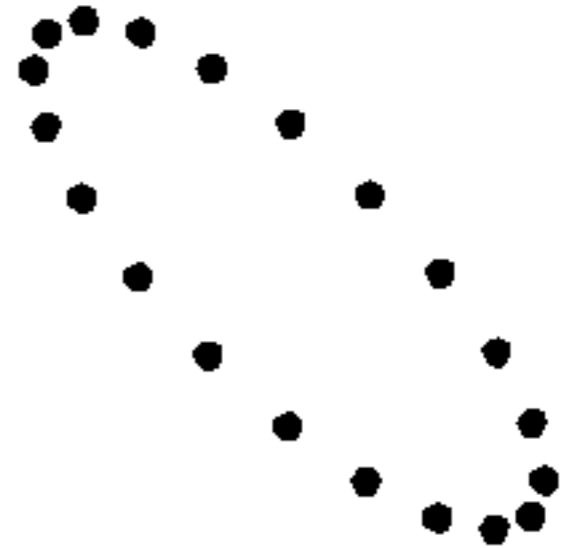
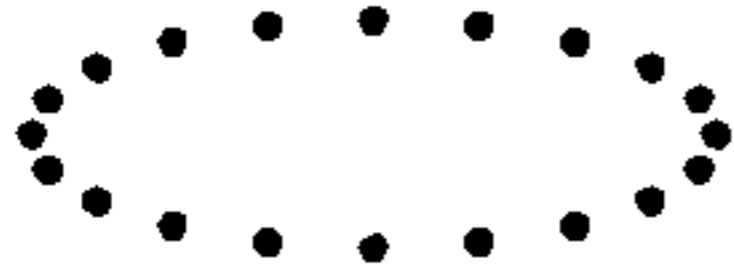


Merging white dwarfs in our Galaxy

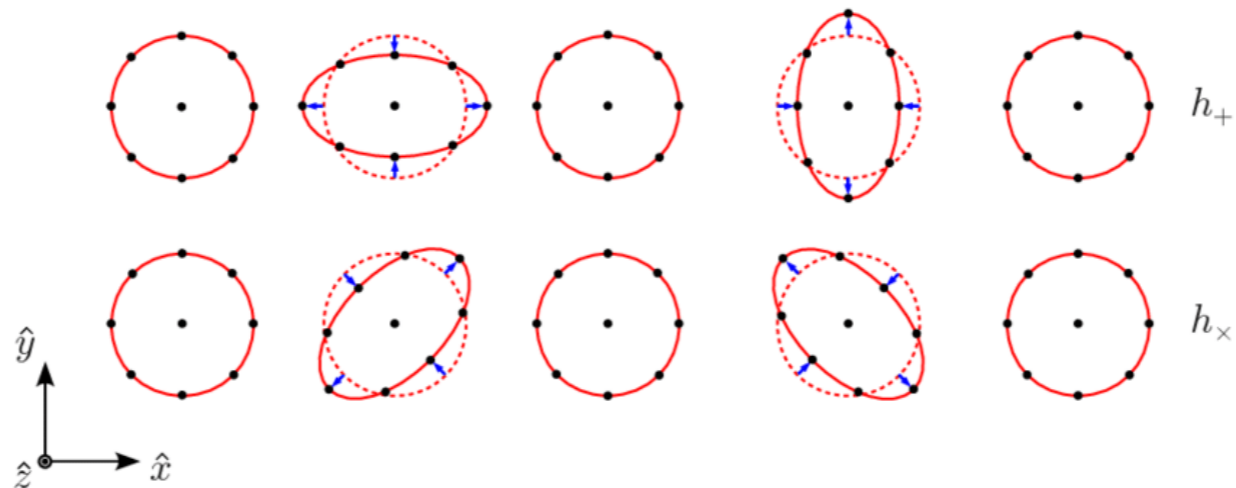
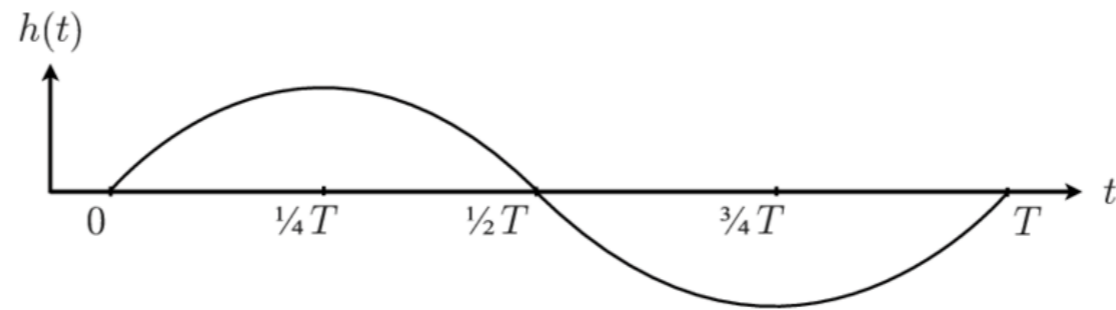
#lisa



IMPRINTS OF GRAVITATIONAL WAVES



IMPRINTS OF GRAVITATIONAL WAVES



IMPRINTS OF GRAVITATIONAL WAVES CAN BE MEASURED BY A DIFFERENTIAL MEASUREMENT

THE SPECTRUM OF GRAVITATIONAL WAVES

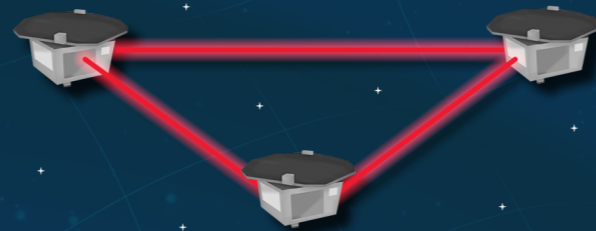


Observatories & experiments

Ground-based experiment



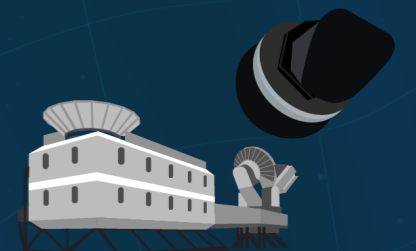
Space-based observatory



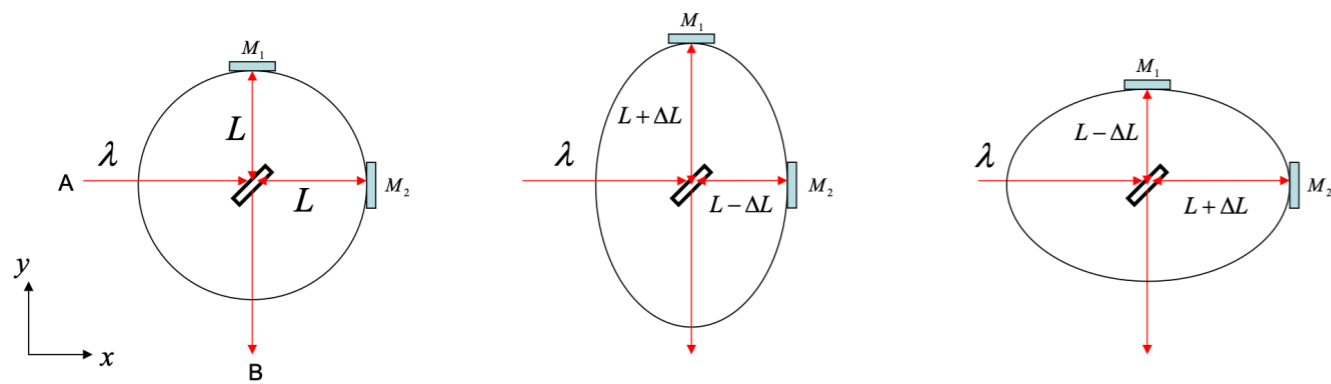
Pulsar timing array



Cosmic microwave background polarisation



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THE SPECTRUM OF GRAVITATIONAL WAVES



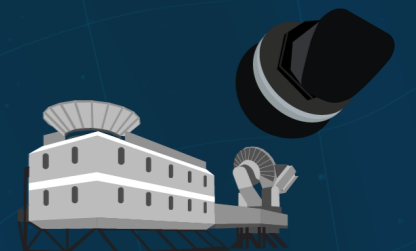
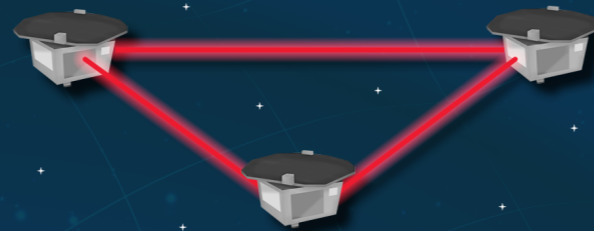
Observatories & experiments

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Cosmic microwave background polarisation



Timescales

milliseconds

seconds

hours

years

billions of years

Frequency (Hz)

100

1

10^{-2}

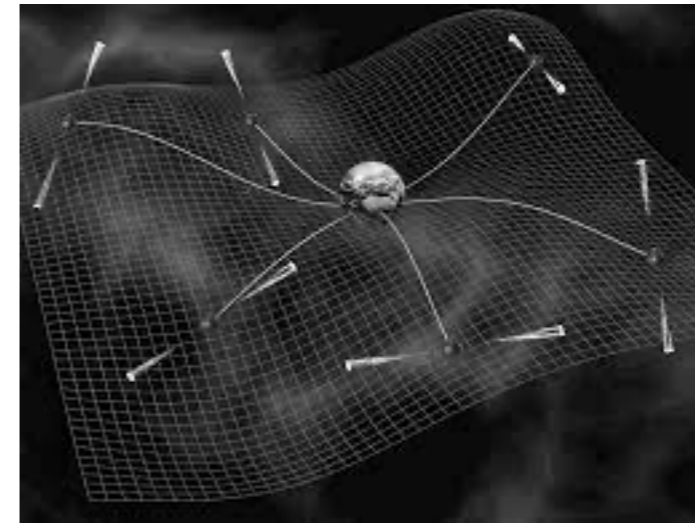
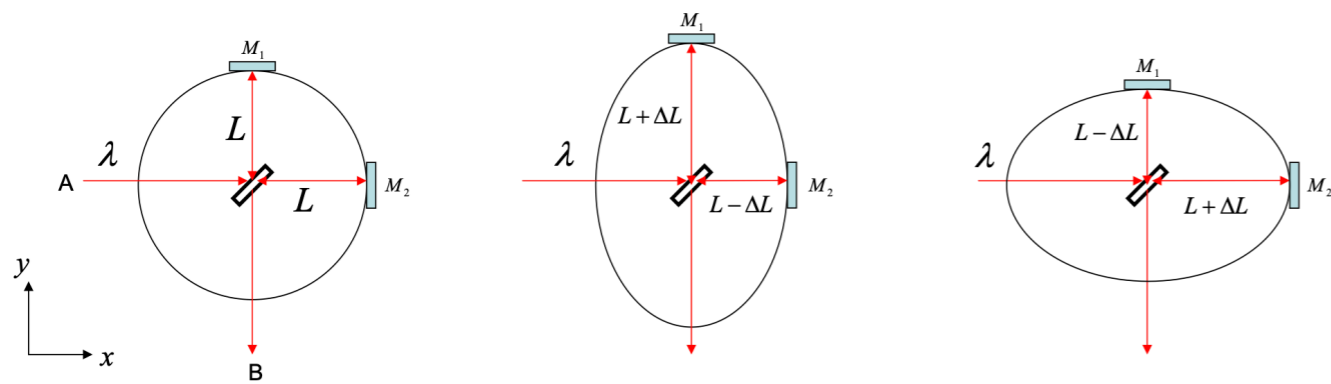
10^{-4}

10^{-6}

10^{-8}

10^{-16}

IMPRINTS OF GRAVITATIONAL WAVES CAN BE MEASURED BY A DIFFERENTIAL MEASUREMENT



THE SPECTRUM OF GRAVITATIONAL WAVES



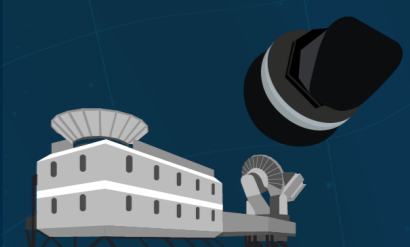
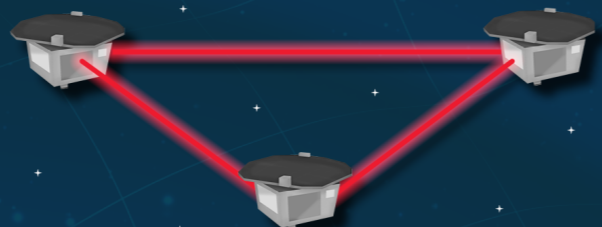
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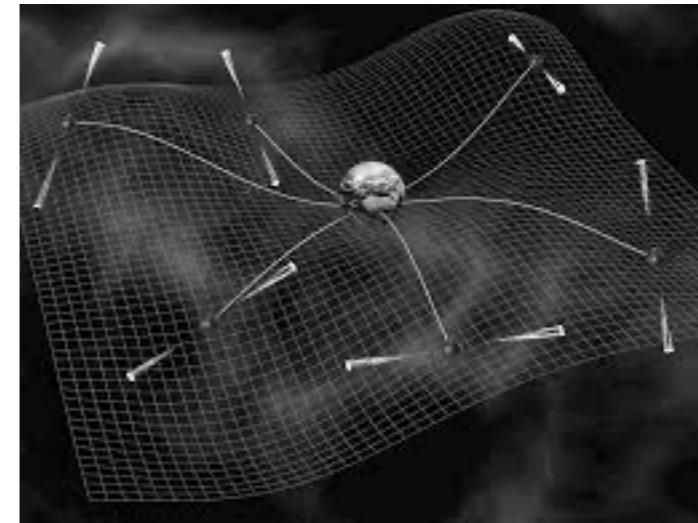
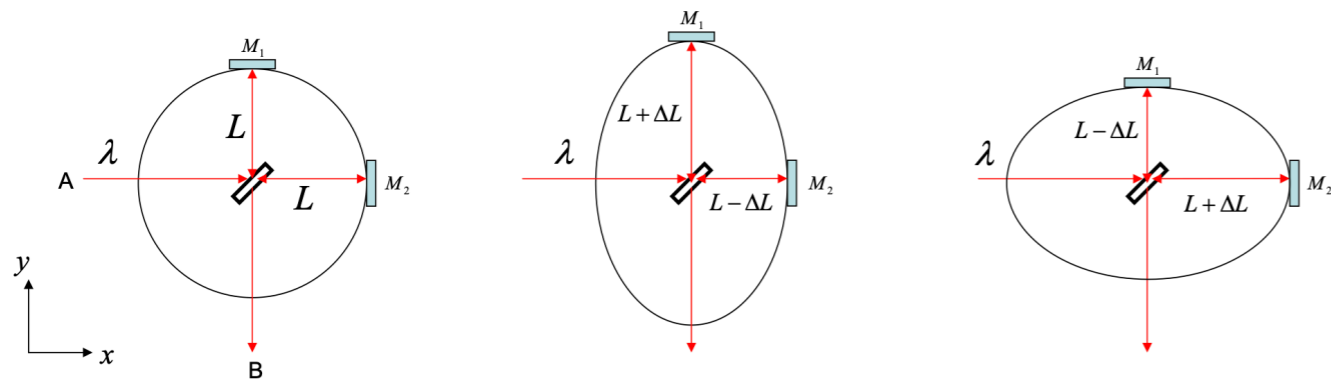
10^{-4}

10^{-6}

10^{-8}

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IMPRINTS OF GRAVITATIONAL WAVES CAN BE MEASURED BY A DIFFERENTIAL MEASUREMENT



Using Cosmic Probes at the Largest Scales.

THE SPECTRUM OF GRAVITATIONAL WAVES



Observatories & experiments

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Space-based observatory

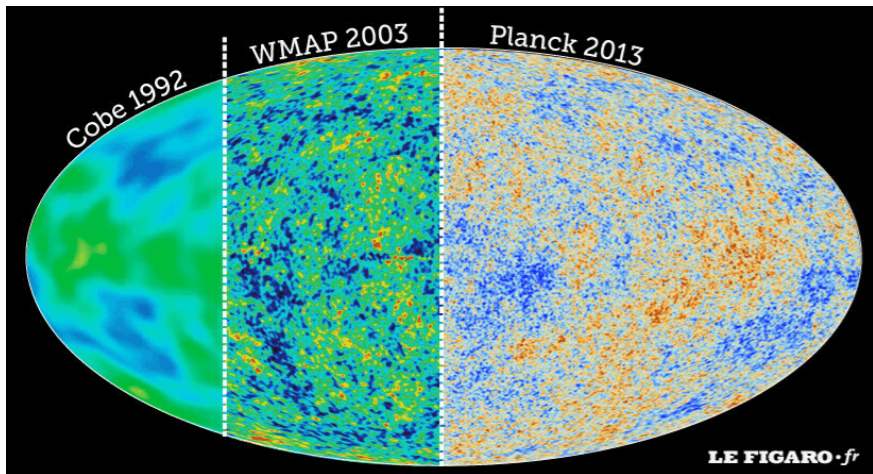
Pulsar timing array

Cosmic microwave background polarisation



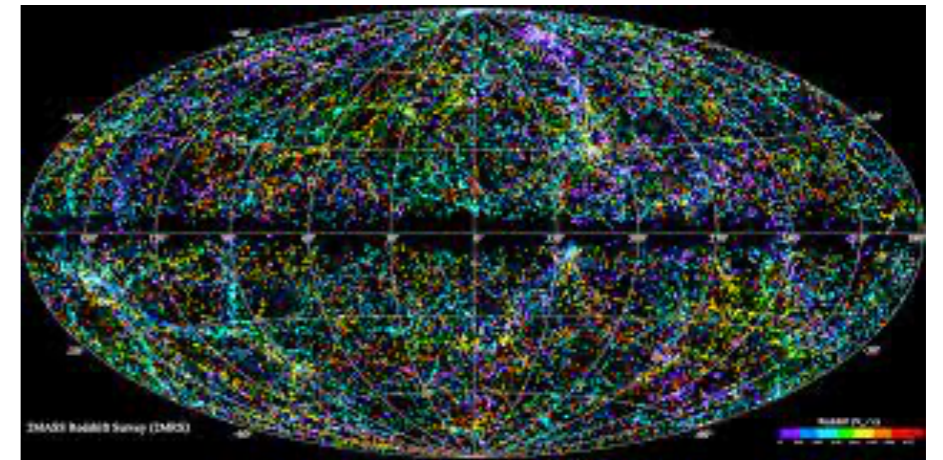
IMPRINTS OF GRAVITATIONAL WAVES USING COSMIC PROBES AT THE LARGEST SCALES.

Cosmic Microwave Background



Feasible in interesting physics regime **with** existing technology

Galaxy Distribution



Feasible in-principle

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milliseconds

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10^{-2}

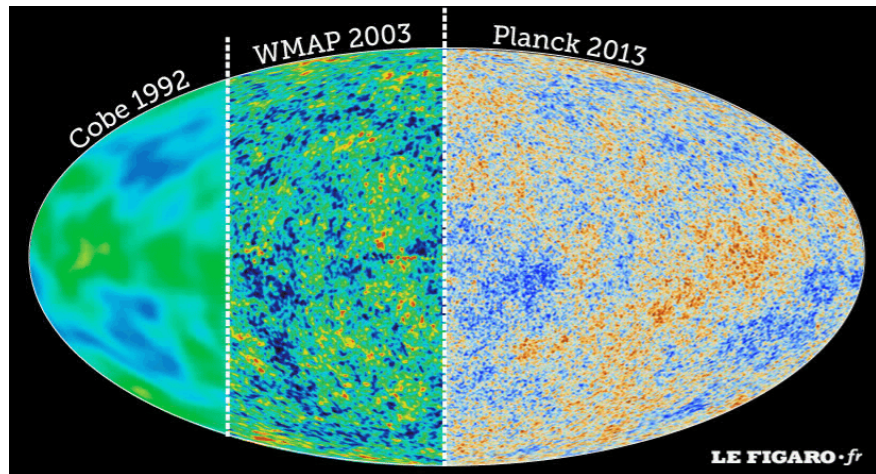
10^{-4}

10^{-6}

10^{-8}

10^{-16}

CMB AS A PROBE TO GRAVITATIONAL WAVES



What is the interaction between CMB and GW?

What are the observables of this interaction?

What is the expected strength of the observables?

What is the prospect of detection?

THE SPECTRUM OF GRAVITATIONAL WAVES

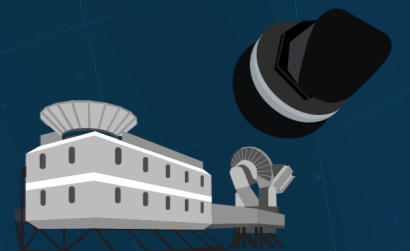
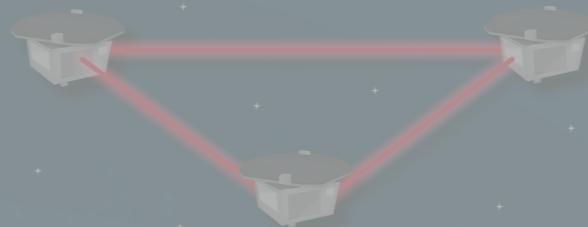
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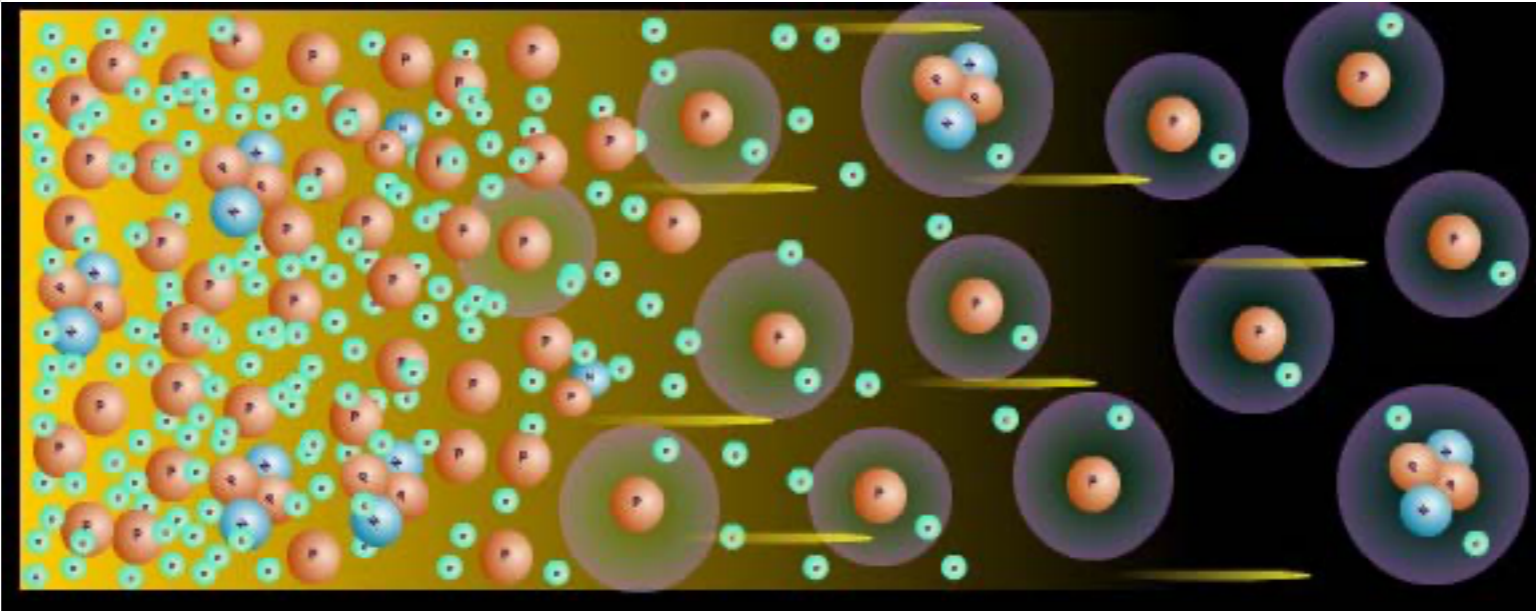
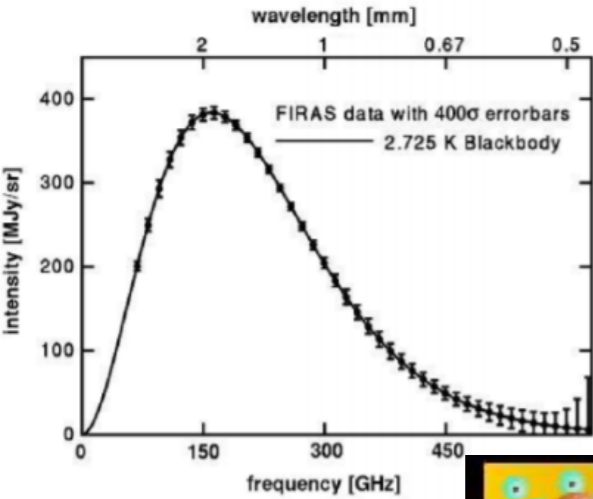
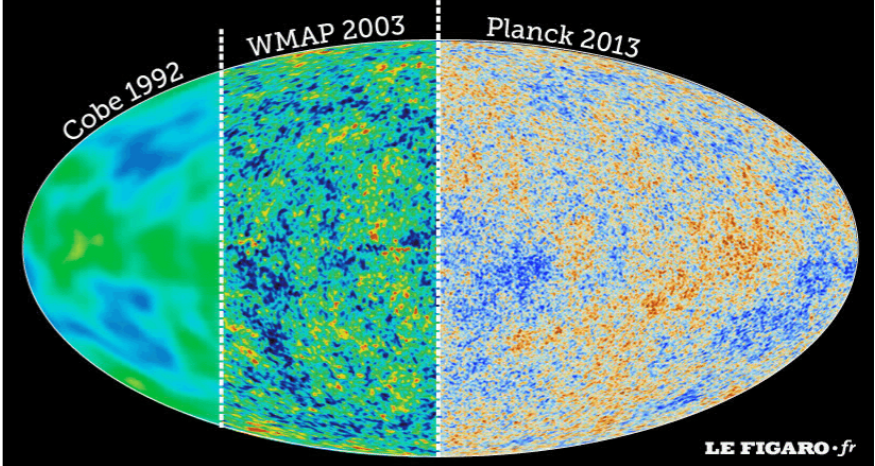
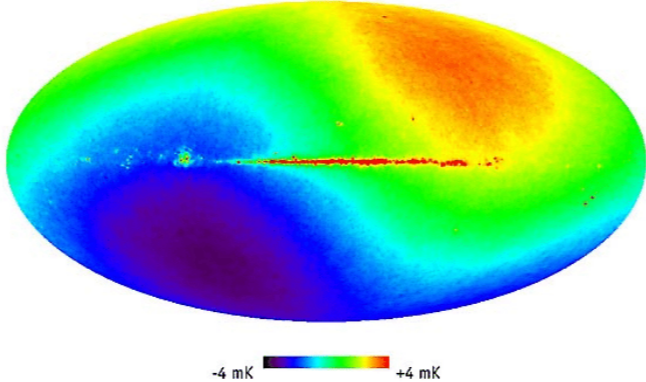
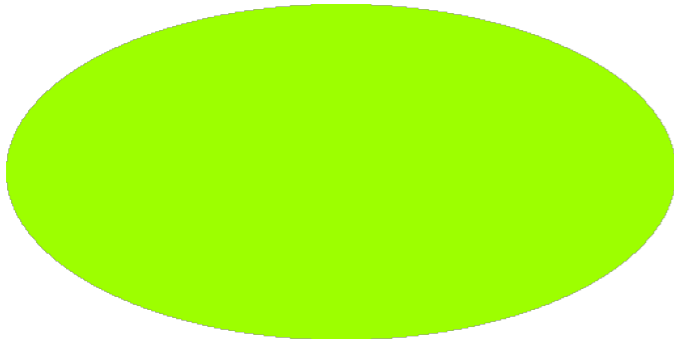
10^{-8}

10^{-16}

CMB as a Probe to Gravitational Waves

What is the interaction between CMB and GW?

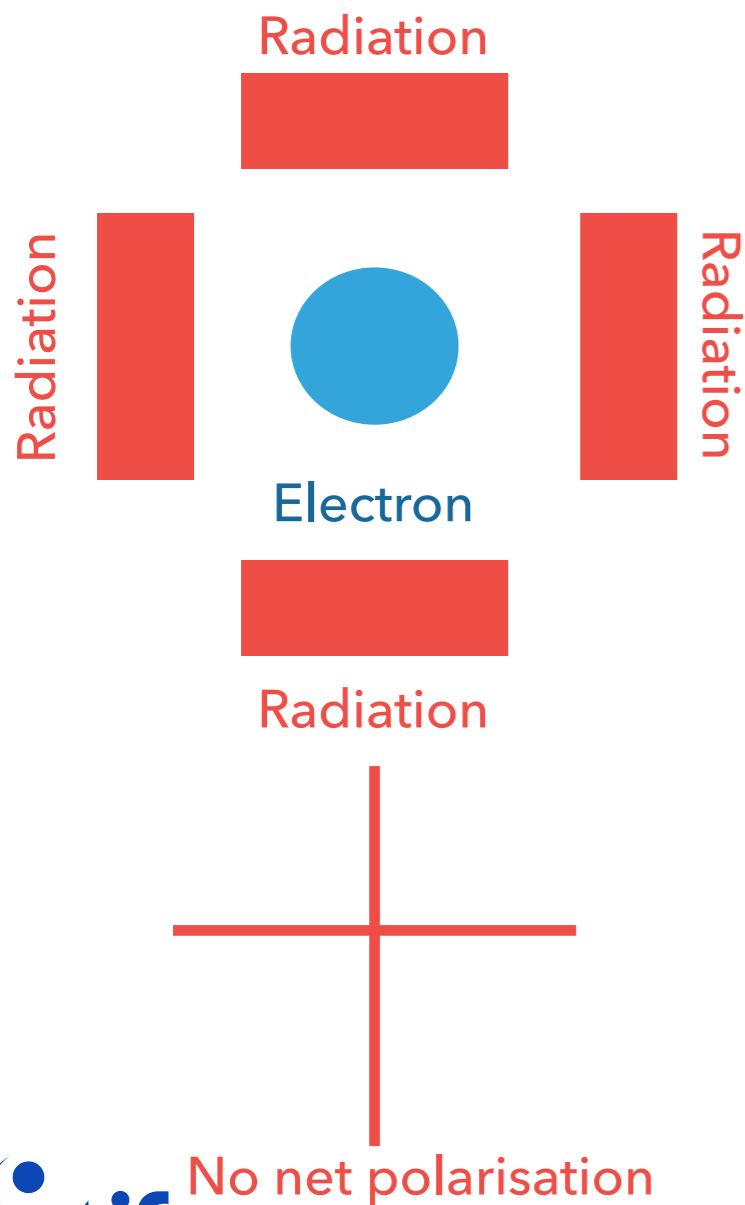
COSMIC MICROWAVE BACKGROUND TEMPERATURE AND ITS FLUCTUATIONS



COSMIC MICROWAVE BACKGROUND POLARISATION

Can CMB be Polarised by Thomson scattering?

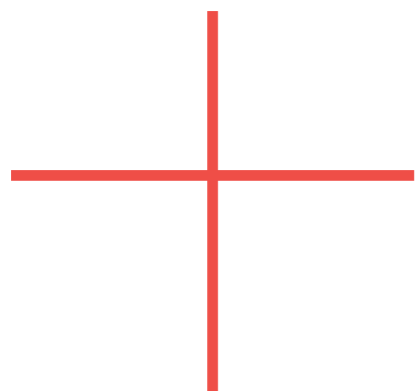
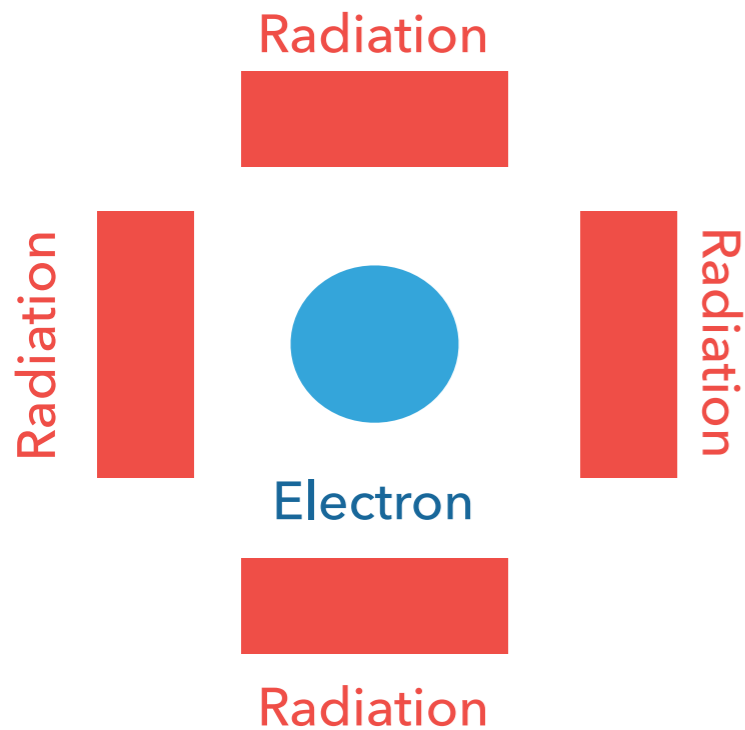
Isotropic radiation bath



COSMIC MICROWAVE BACKGROUND POLARISATION

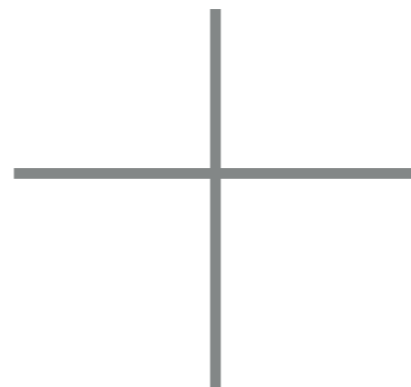
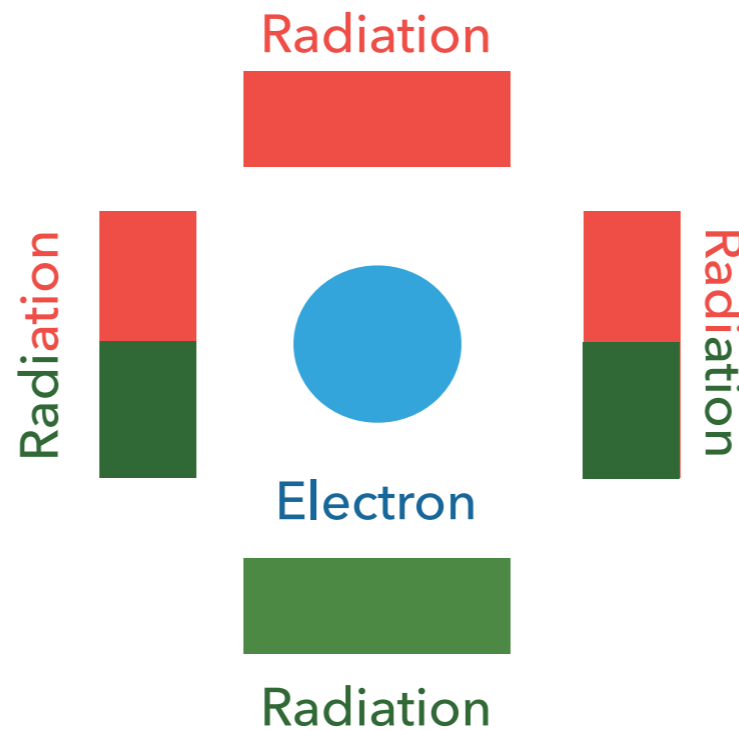
Can CMB be Polarised by Thomson scattering?

Isotropic radiation bath



No net polarisation

dipole radiation bath

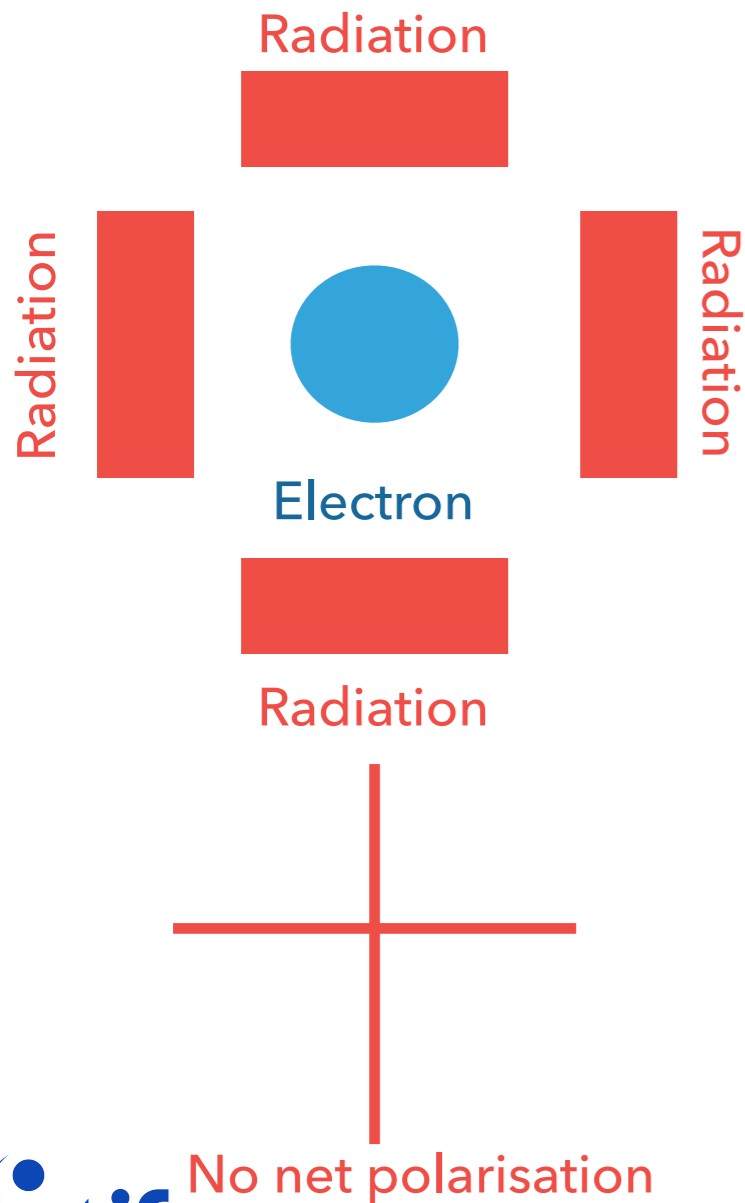


No net polarisation
Suvodip Mukherjee, 2025

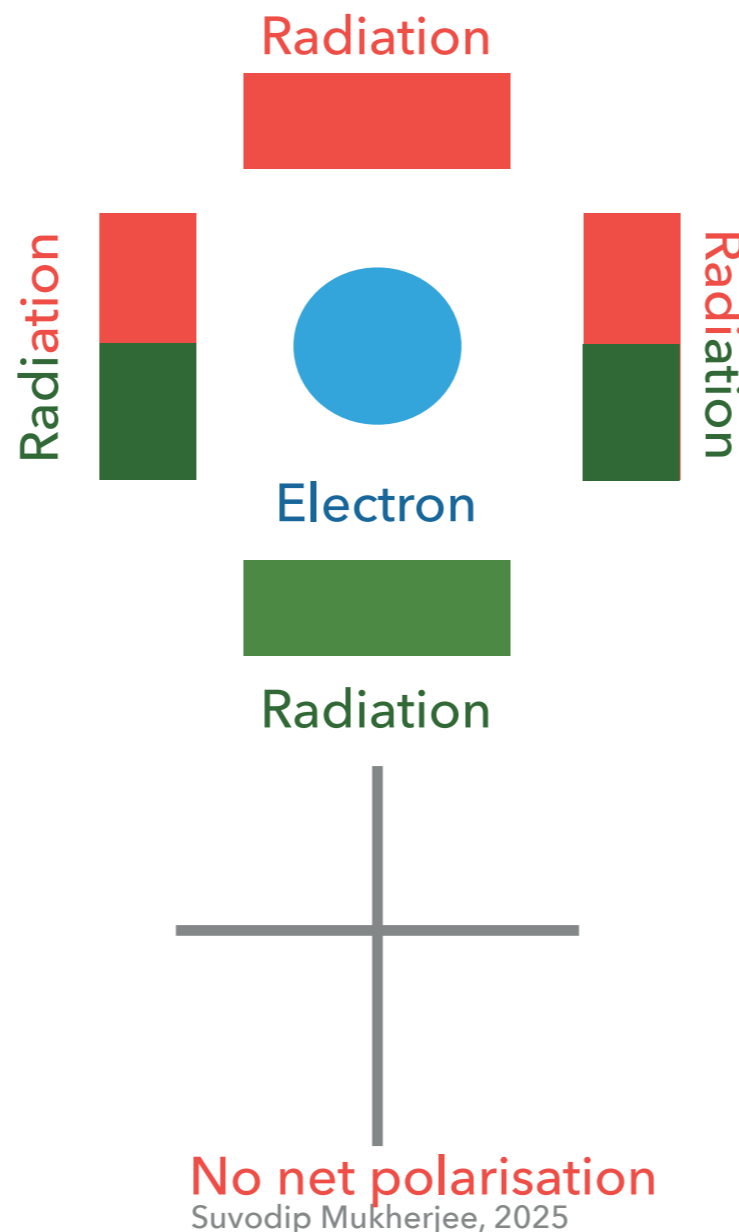
COSMIC MICROWAVE BACKGROUND POLARISATION

Can CMB be Polarised by Thomson scattering?

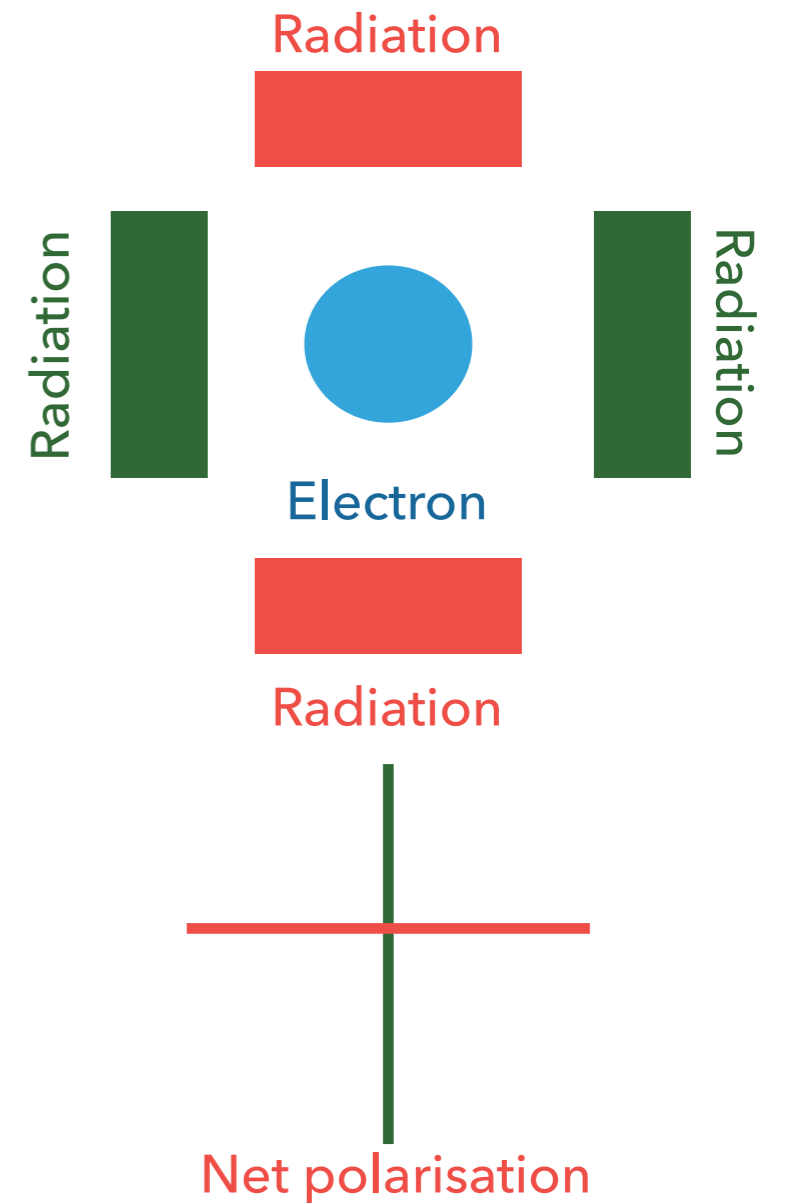
Isotropic radiation bath



dipole radiation bath

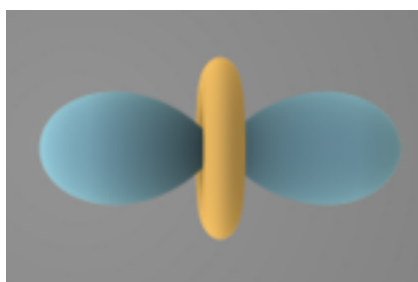
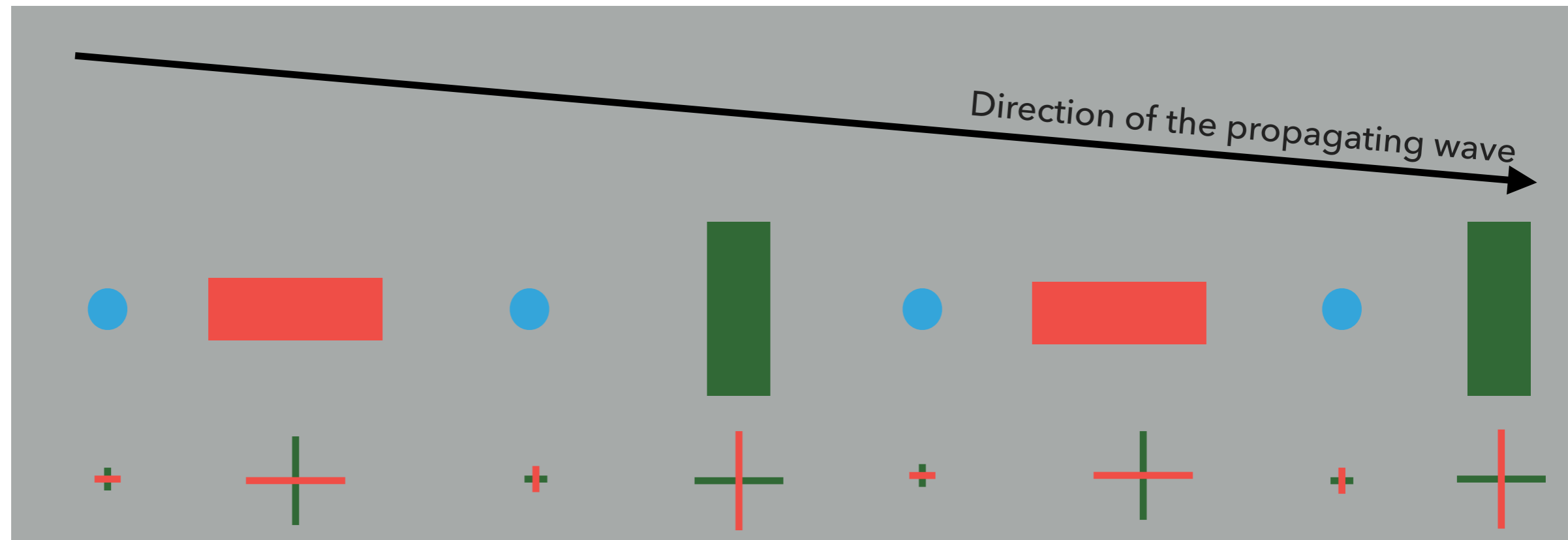


Quadrupolar radiation bath



COSMIC MICROWAVE BACKGROUND POLARISATION

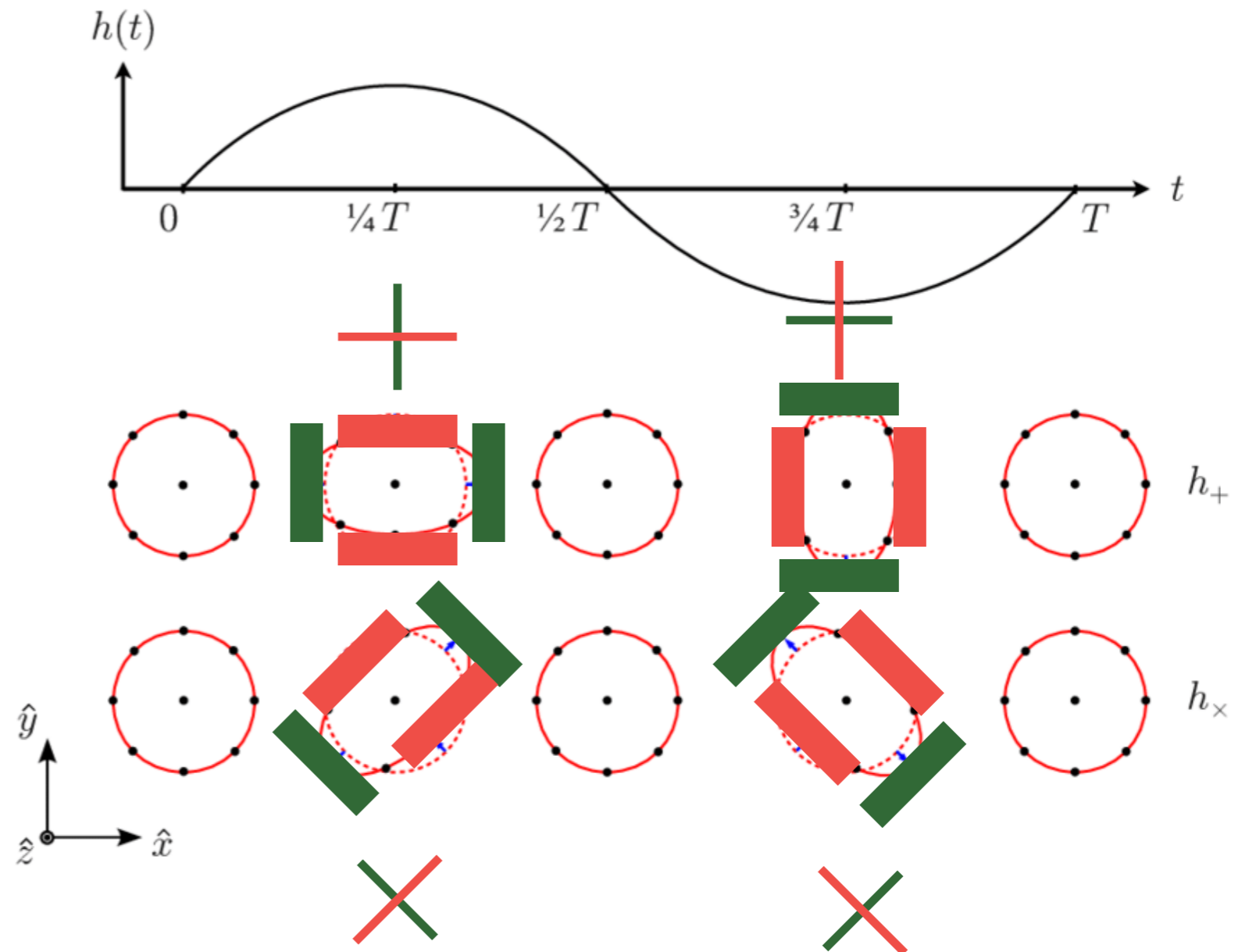
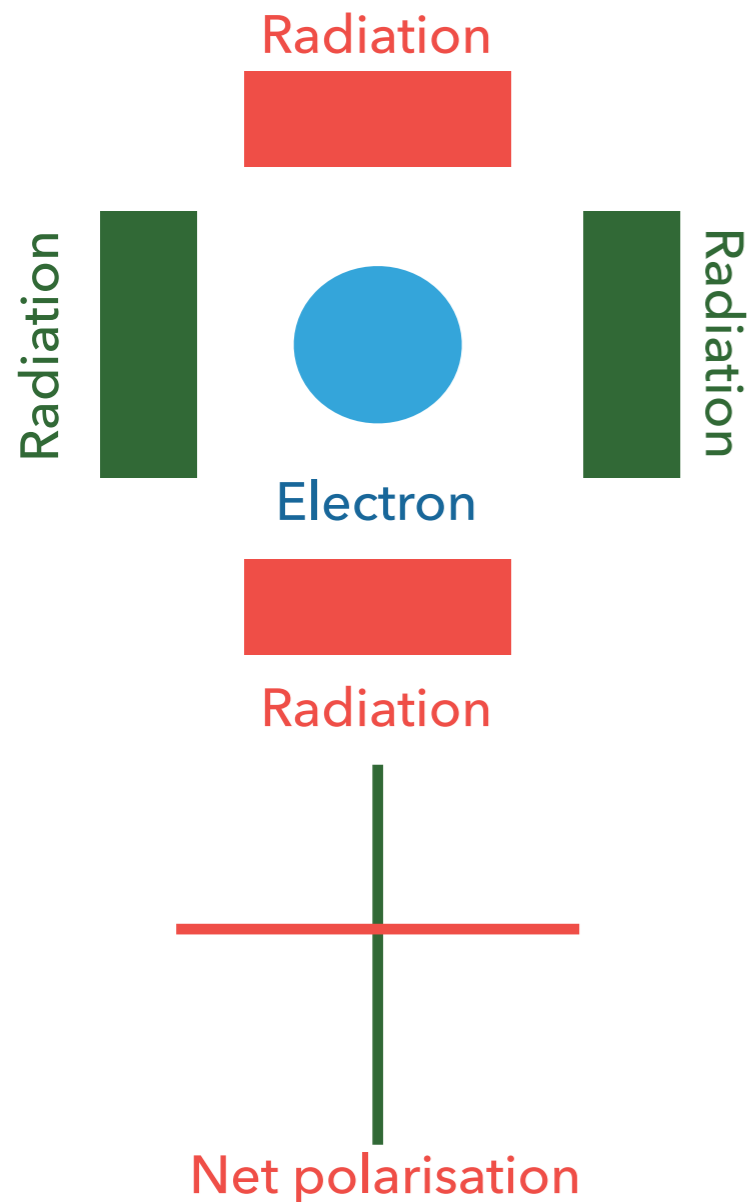
Projection of GW on the sky



COSMIC MICROWAVE BACKGROUND POLARISATION

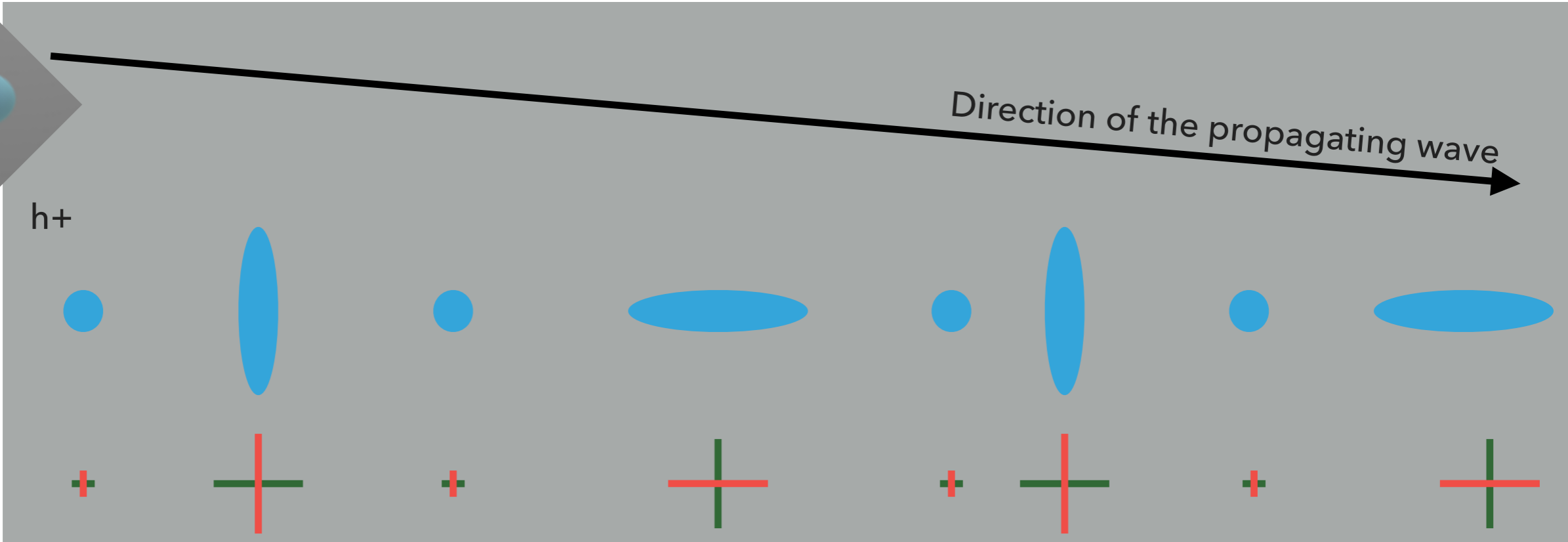
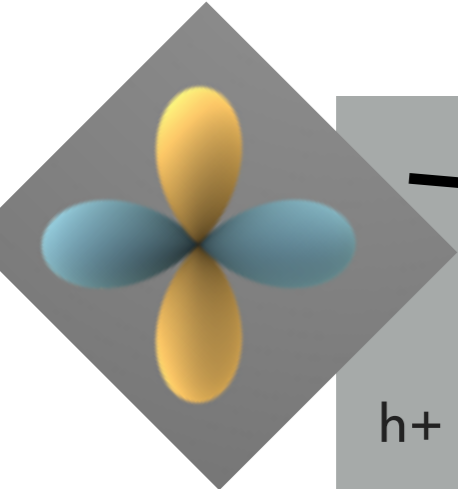
Can GW also generate quadrupolar fluctuation?

Quadrupolar radiation bath



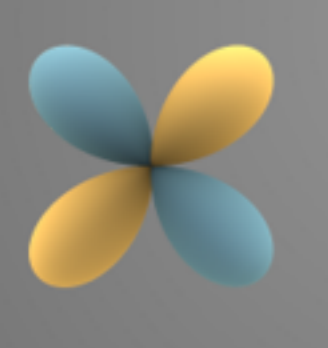
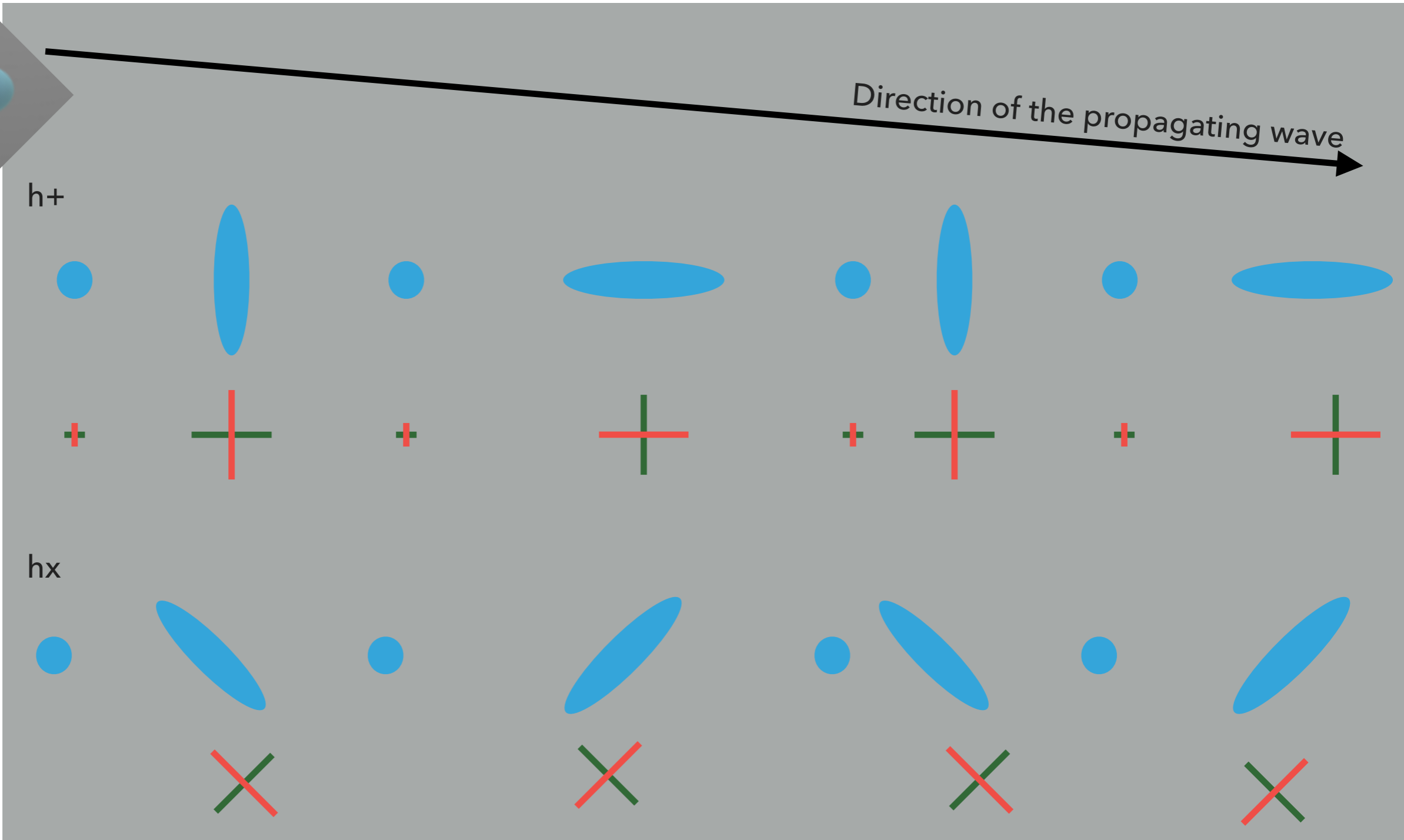
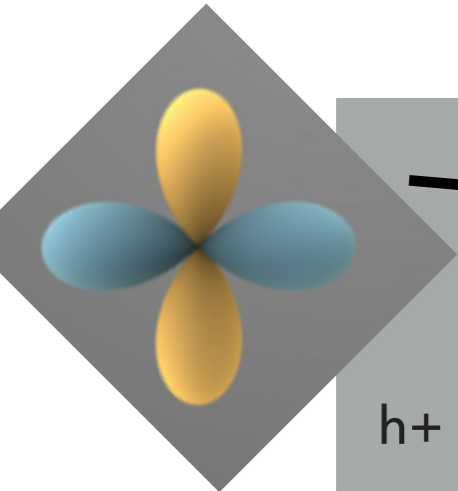
COSMIC MICROWAVE BACKGROUND POLARISATION

Projection of GW on the sky



COSMIC MICROWAVE BACKGROUND POLARISATION

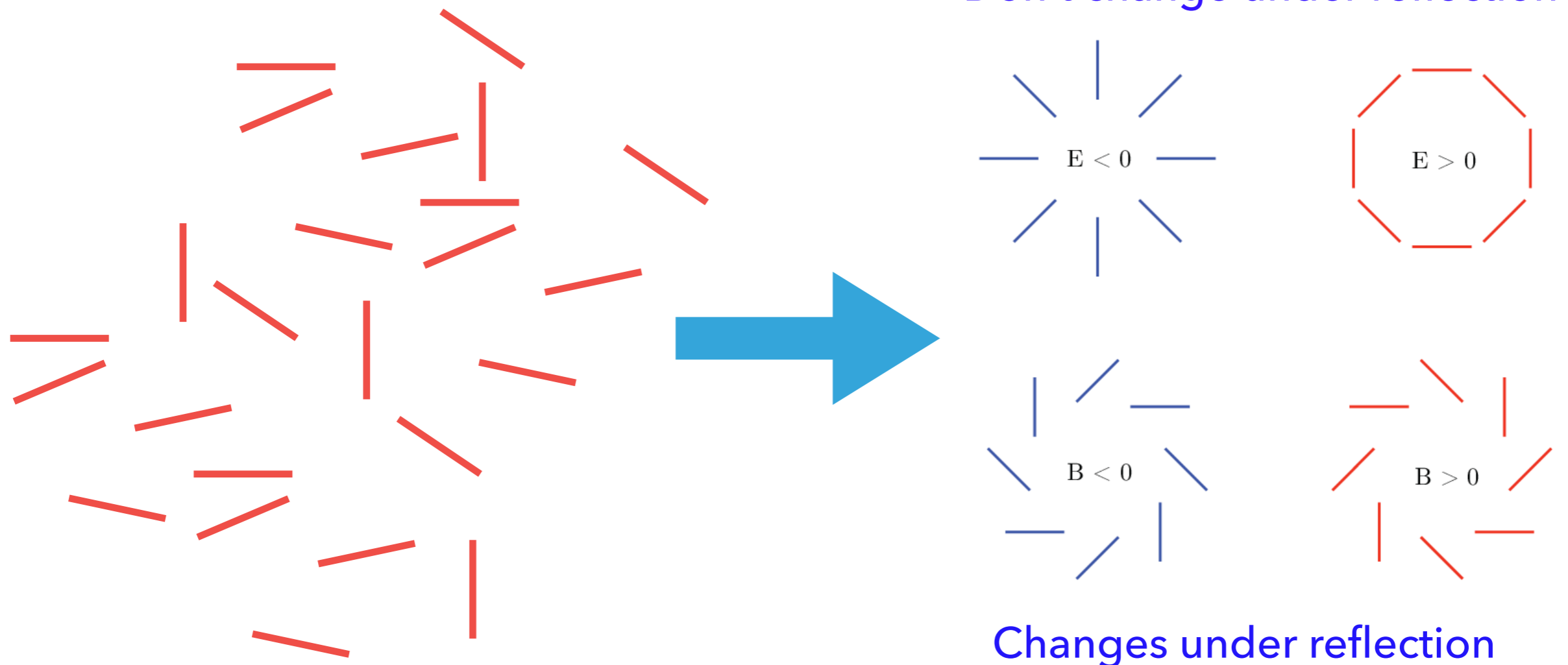
Projection of GW on the sky



COSMIC MICROWAVE BACKGROUND POLARISATION

Identifying polarisation pattern on the sky

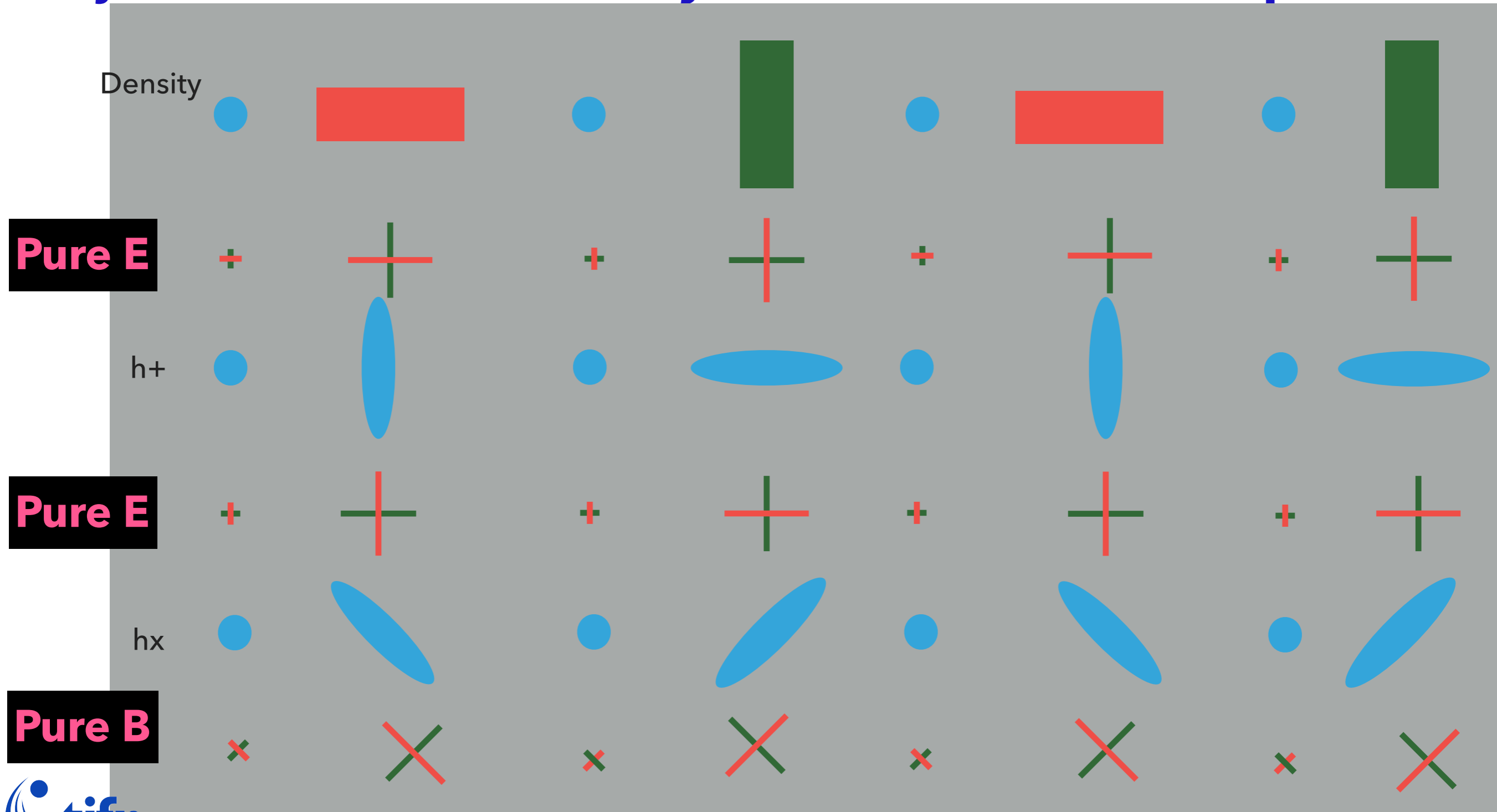
(In a coordinate independent way)



$Q(n)$ and $U(n)$

COSMIC MICROWAVE BACKGROUND POLARISATION

Projection of GW on the sky: E-mode and B-mode patterns



CMB as a Probe to Gravitational Waves

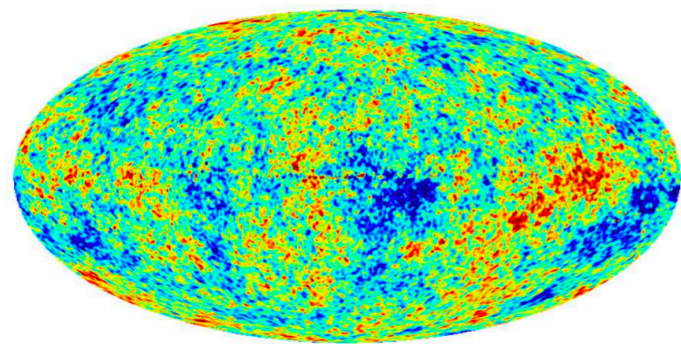
What are the observables of this interaction?

OBSERVABLES OF GW PERTURBATIONS ON B-MODE

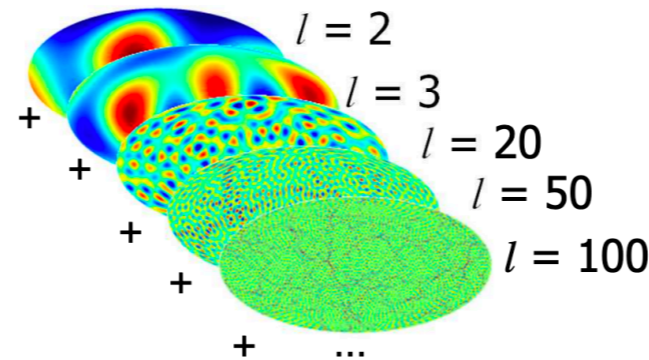
CMB fluctuations are zero mean (nearly) Gaussian random field on the sky

$$T(\hat{n}) = \sum_{lm} a_{T,lm} Y_{lm}(\hat{n})$$

$$(Q \pm iU)(\hat{n}) = \sum_{lm} a_{\pm 2,lm \pm 2} Y_{lm}(\hat{n})$$



=



$$a_{E,lm} = -(a_{2,lm} + a_{-2,lm})/2$$

$$a_{B,lm} = i(a_{2,lm} - a_{-2,lm})/2$$

$\ell \propto k$

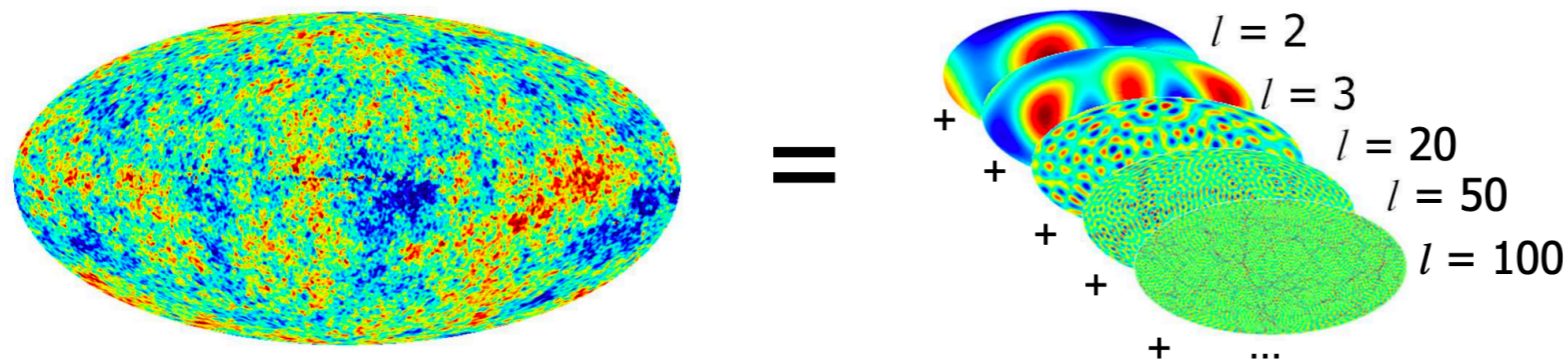
Angular Modes \longleftrightarrow Fourier Modes

OBSERVABLES OF GW PERTURBATIONS ON B-MODE

CMB fluctuations are zero mean (nearly) Gaussian random field on the sky

$$T(\hat{n}) = \sum_{lm} a_{T,lm} Y_{lm}(\hat{n})$$

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$$a_{B,lm} = i(a_{2,lm} - a_{-2,lm})/2$$

$$\ell \propto k$$

Angular Modes \longleftrightarrow Fourier Modes

Power spectra $C_{XX'l} = \frac{1}{2l+1} \sum_m \langle a_{X,lm}^* a_{X',lm} \rangle$

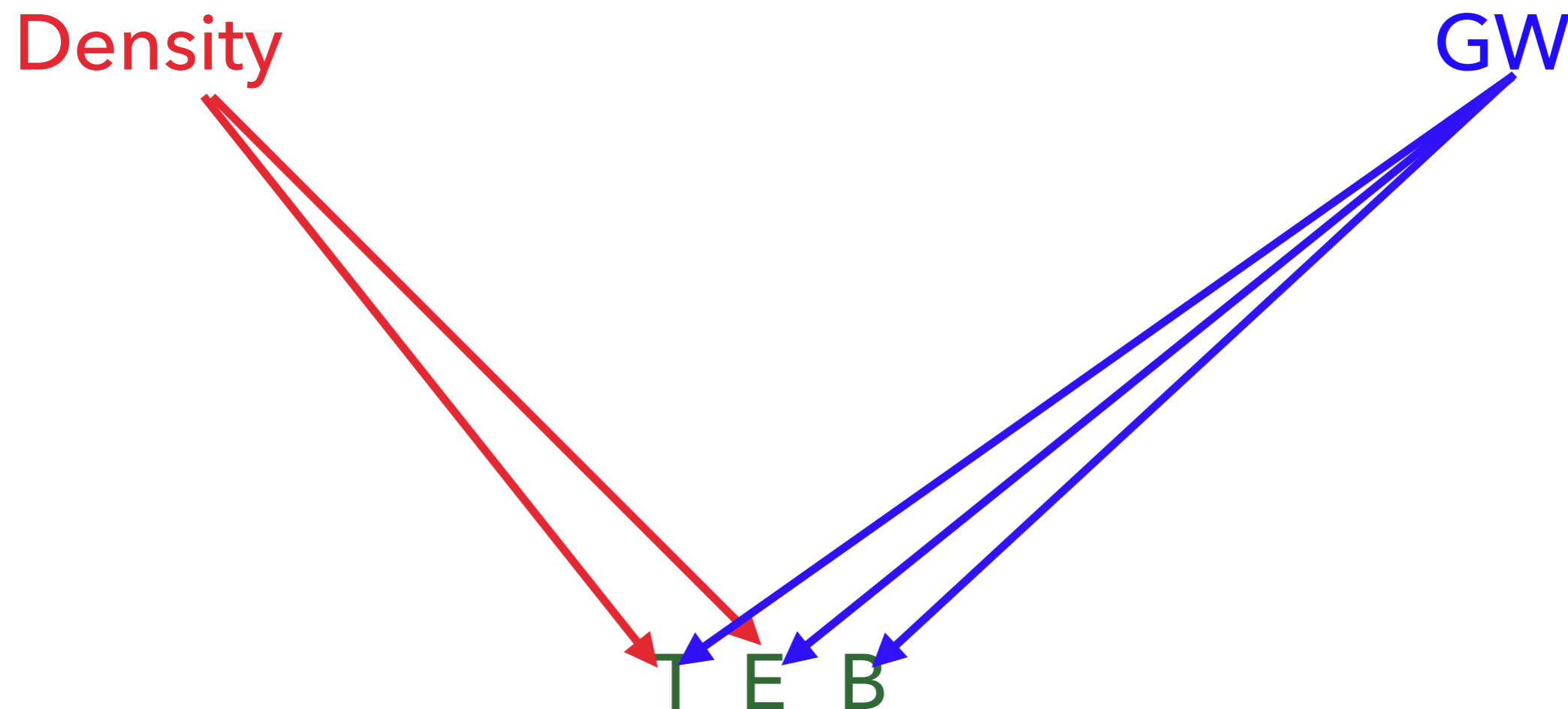
Harmonic coefficients

TT, EE, BB, TE are non-zero in absence of non-parity violating physics in early Universe

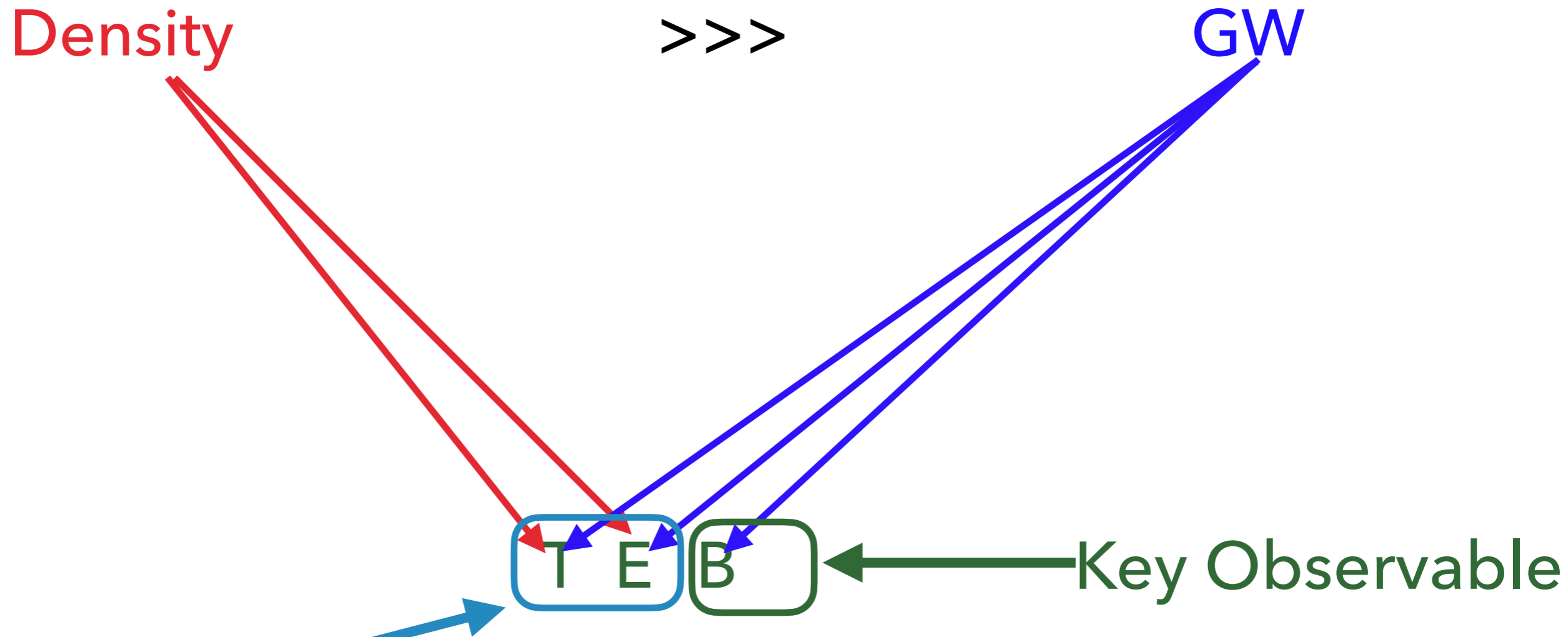
CMB as a Probe to Gravitational Waves

What is the expected strength of the observables?

OBSERVABLES OF DIFFERENT PERTURBATIONS



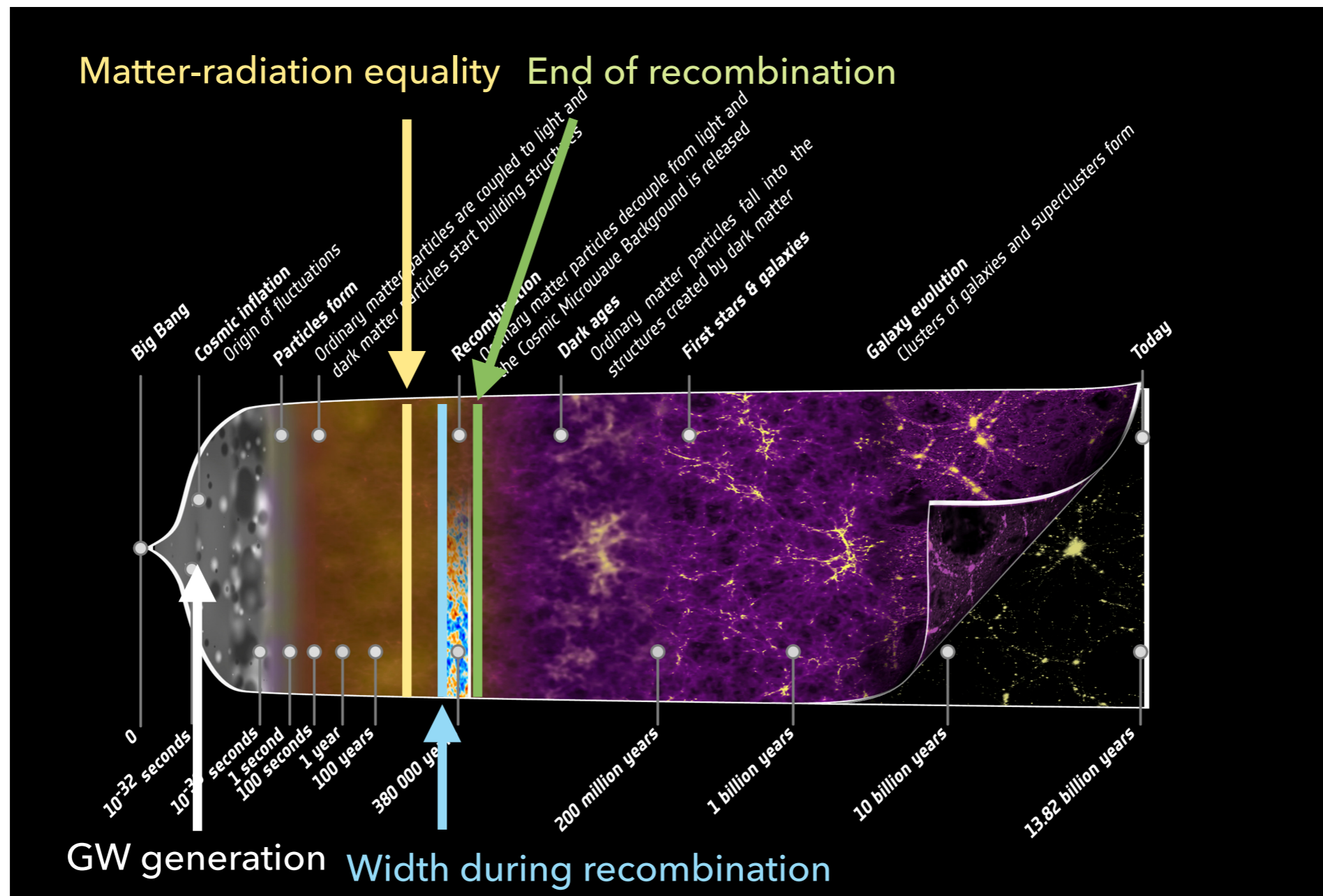
OBSERVABLES OF DIFFERENT PERTURBATIONS



Contribution from GW is not pronounced than density fluctuations

OBSERVABLES OF GW PERTURBATIONS ON B-MODE

Different Cosmic stages



OBSERVABLES OF GW PERTURBATIONS ON B-MODE

Evolution of GW signal in an expanding Universe

$$\ddot{h}_{ij} + 2\frac{\dot{a}}{a}\dot{h}_{ij} + k^2 h_{ij} = 0$$

OBSERVABLES OF GW PERTURBATIONS ON B-MODE

Evolution of GW signal in an expanding Universe

$$\ddot{h}_{ij} + 2\frac{\dot{a}}{a}\dot{h}_{ij} + k^2 h_{ij} = 0$$

For super-horizon modes: $k \ll \frac{\dot{a}}{a}$

$$h \sim \text{constant}$$

OBSERVABLES OF GW PERTURBATIONS ON B-MODE

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$$h \sim \text{constant}$$

For sub-horizon modes: $k \gg \frac{\dot{a}}{a}$

$$h \sim \frac{1}{a}$$

OBSERVABLES OF GW PERTURBATIONS ON B-MODE

Evolution of GW signal in an expanding Universe

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$$h \sim \text{constant}$$

For sub-horizon modes: $k \gg \frac{\dot{a}}{a}$

$$h \sim \frac{1}{a} \quad \longrightarrow \quad \frac{h_{\text{today}}}{h_{\text{entry}}} \sim \frac{a_{\text{entry}}}{a_{\text{today}}}$$

OBSERVABLES OF GW PERTURBATIONS ON B-MODE

Evolution of GW signal in an expanding Universe

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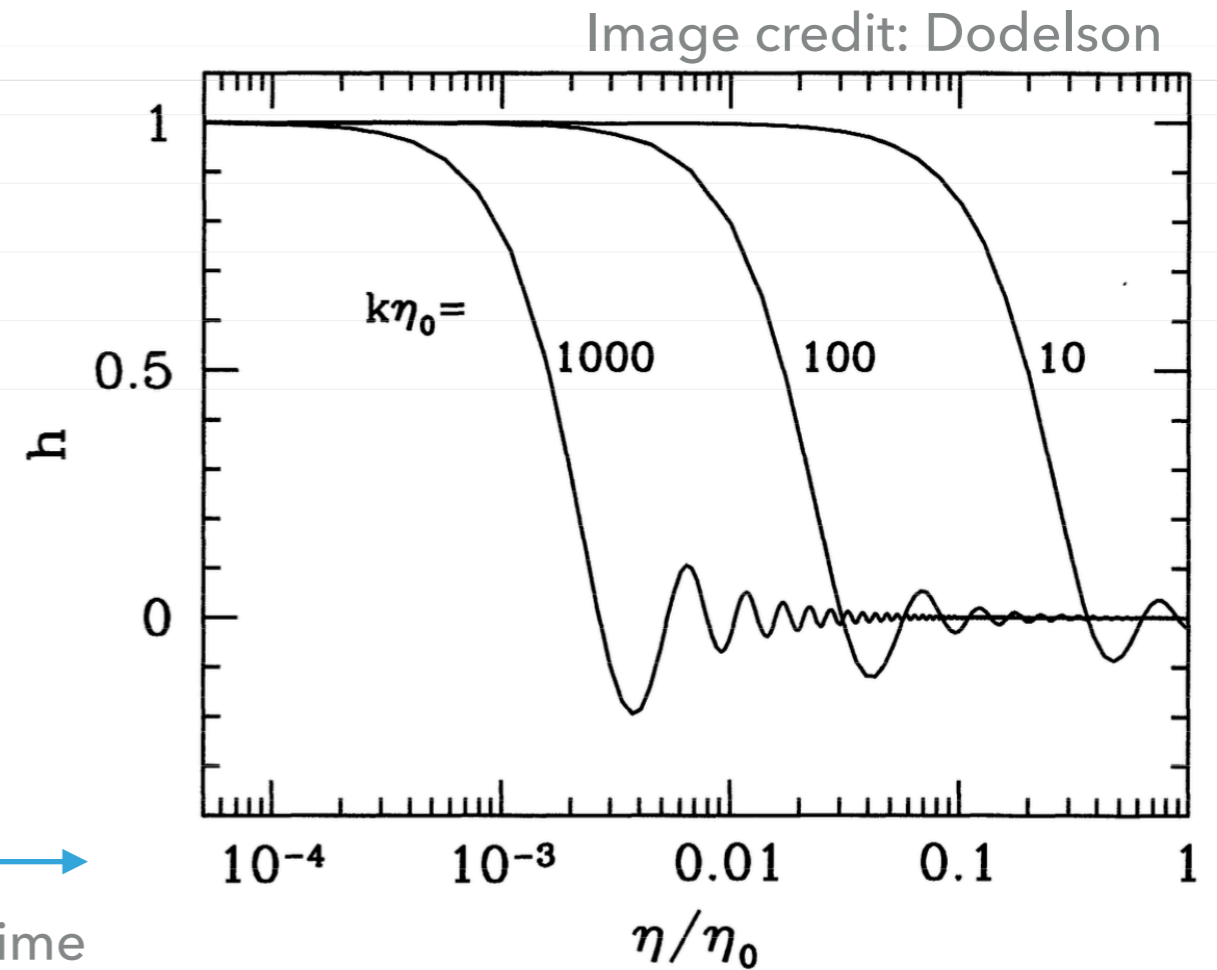
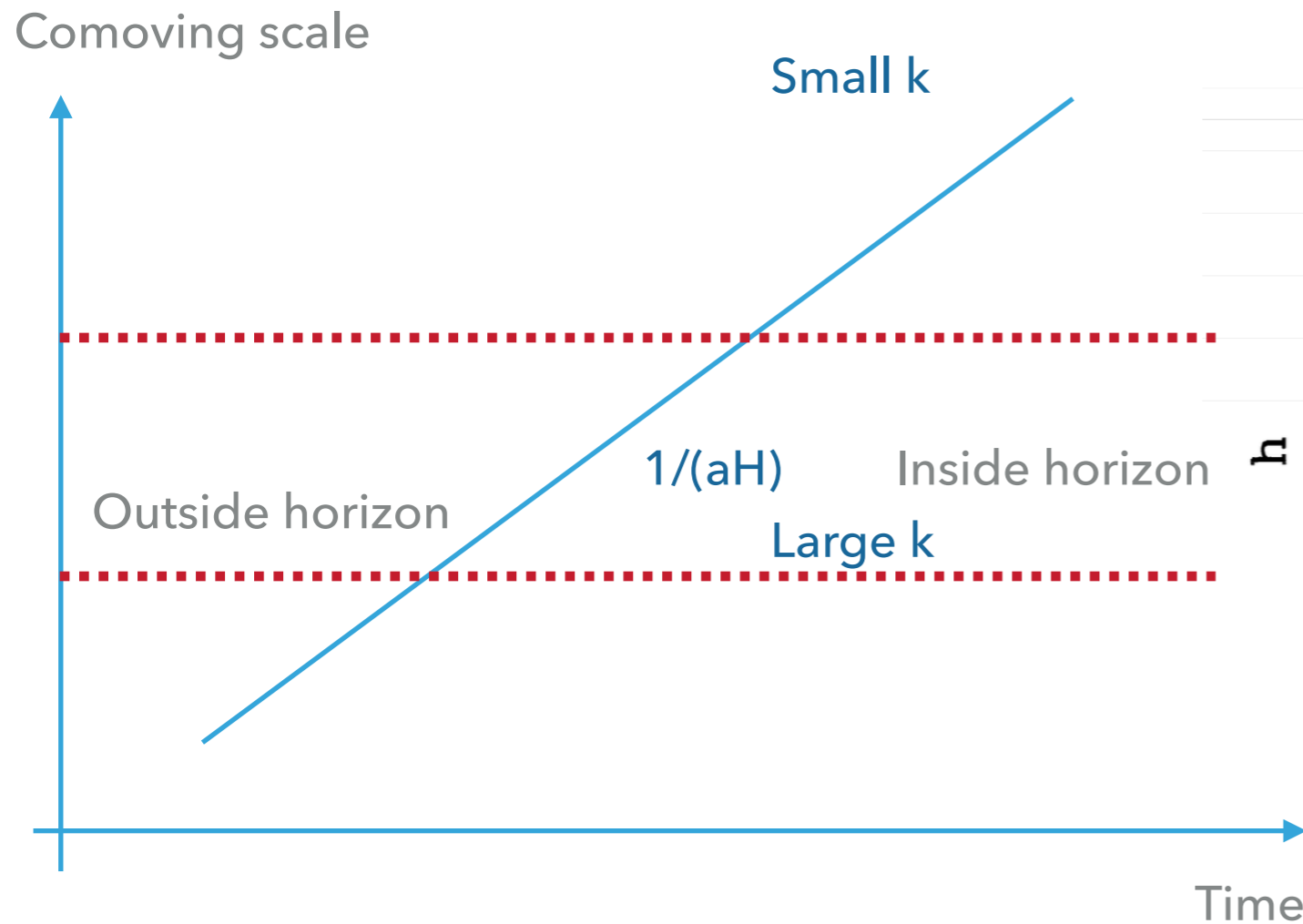
$$h \sim \frac{1}{a} \quad \longrightarrow \quad \frac{h_{\text{today}}}{h_{\text{entry}}} \sim \frac{a_{\text{entry}}}{a_{\text{today}}}$$

For modes at horizon entry:

$$a_{\text{enter}} H_{\text{enter}} \propto k \quad \longrightarrow \quad a_{\text{enter}} \propto k^{-2} \quad \text{for matter-dominated Universe}$$

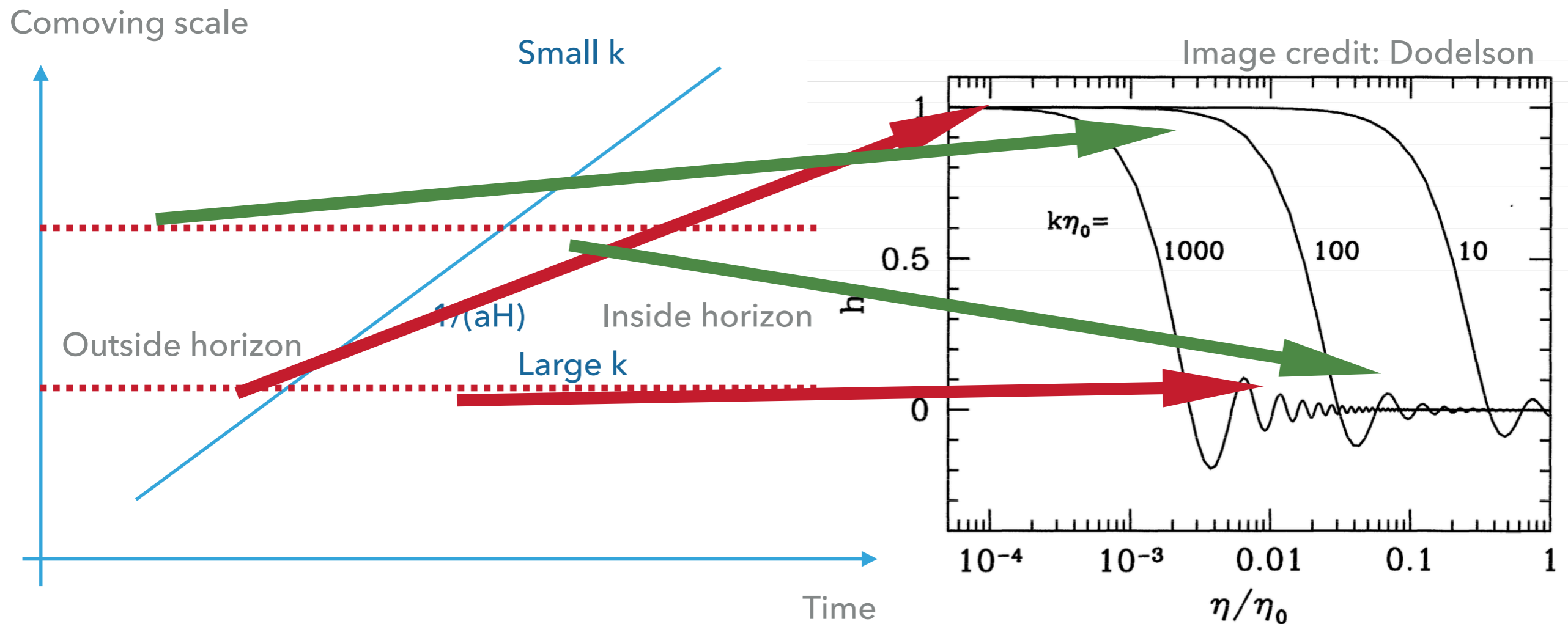
OBSERVABLES OF GW PERTURBATIONS ON B-MODE

Evolution of GW signal in an expanding Universe



OBSERVABLES OF GW PERTURBATIONS ON B-MODE

Evolution of GW signal in an expanding Universe



OBSERVABLES OF GW PERTURBATIONS ON B-MODE

Signature of evolution on the power spectrum

$$C_{XX'l} = (4\pi)^2 \int k^2 dk \underbrace{P_h(k)}_{\text{Initial GW fluctuation}} \underbrace{\Delta_{Xl}(k) \Delta_{X'l}(k)}_{\text{Evolution of the modes}}$$

Evolution of the modes

Initial GW fluctuation

$$\ell \propto k$$

Angular Modes \longleftrightarrow Fourier Modes

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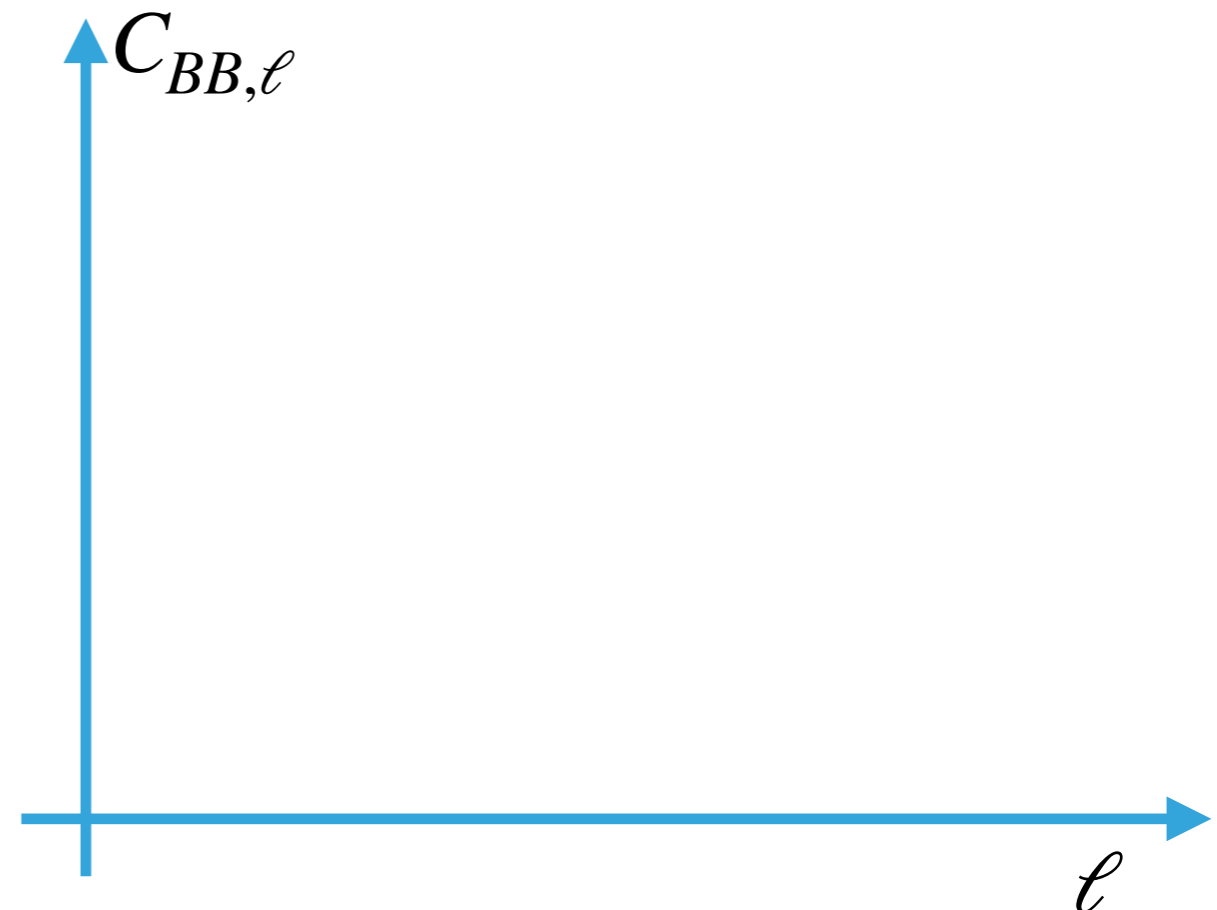
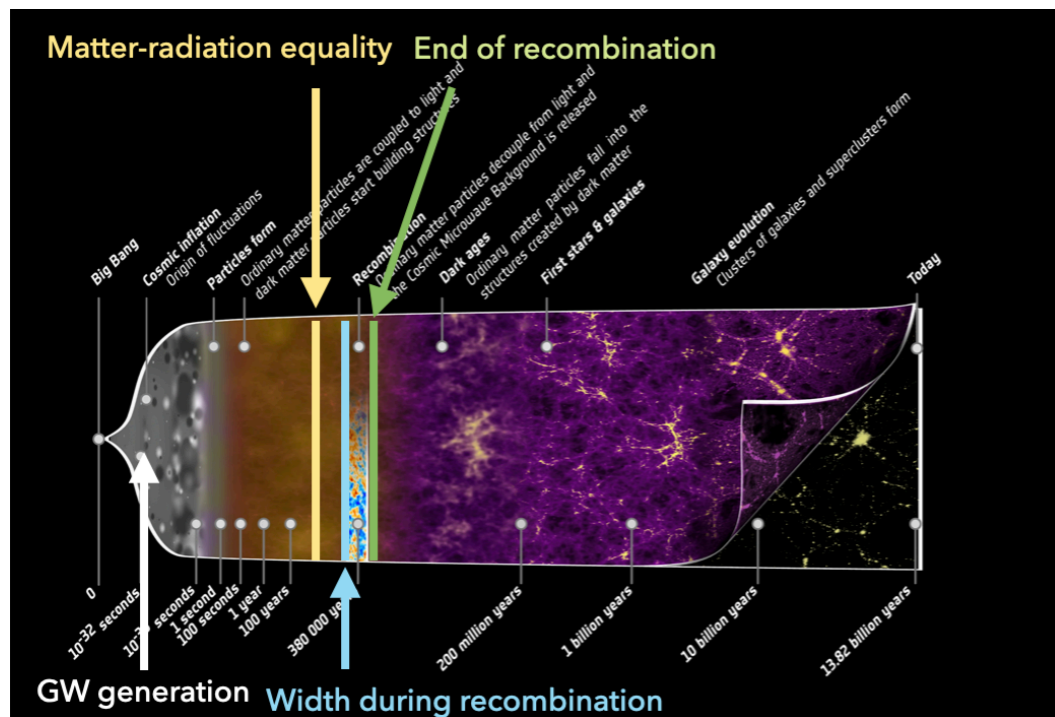
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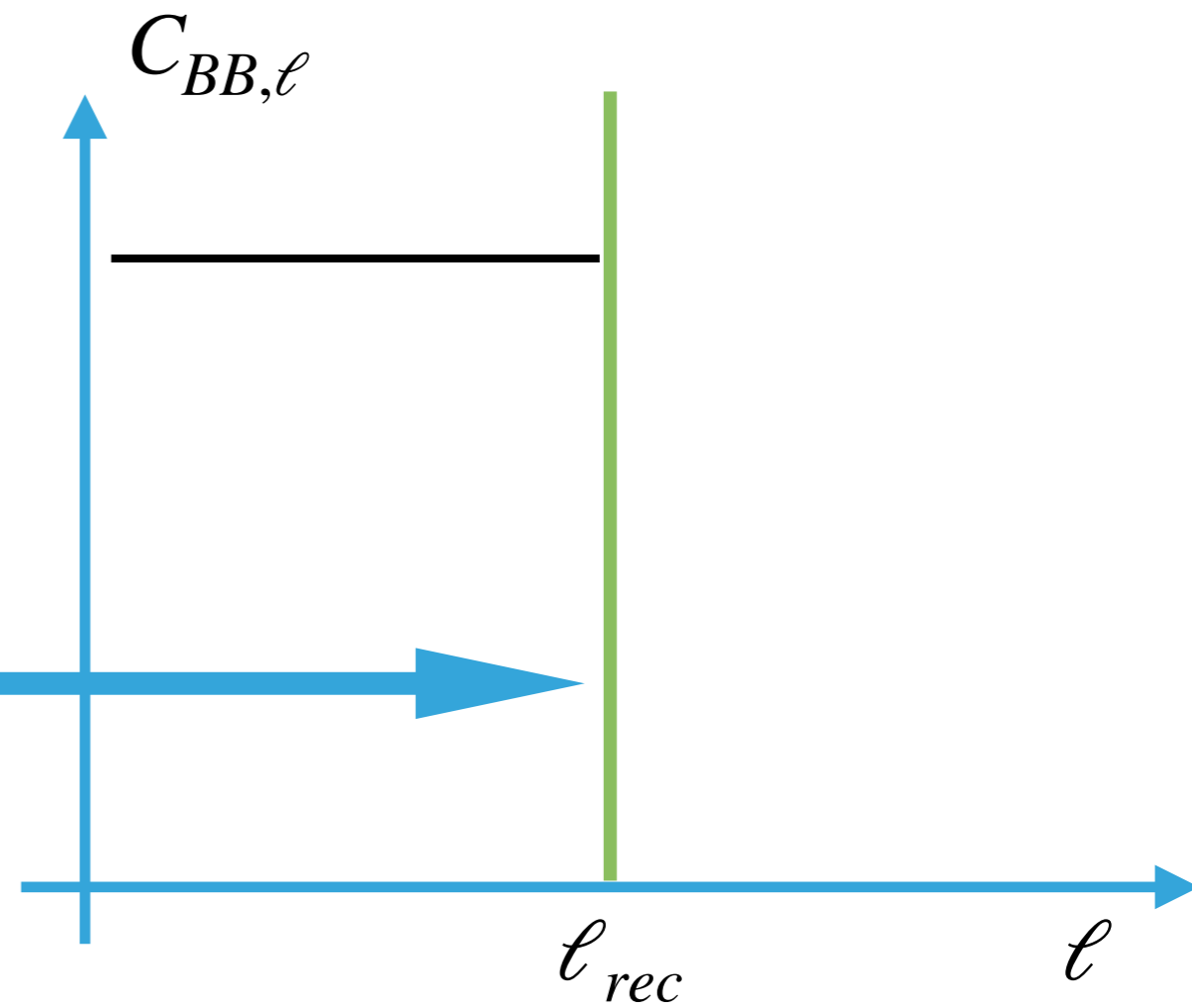
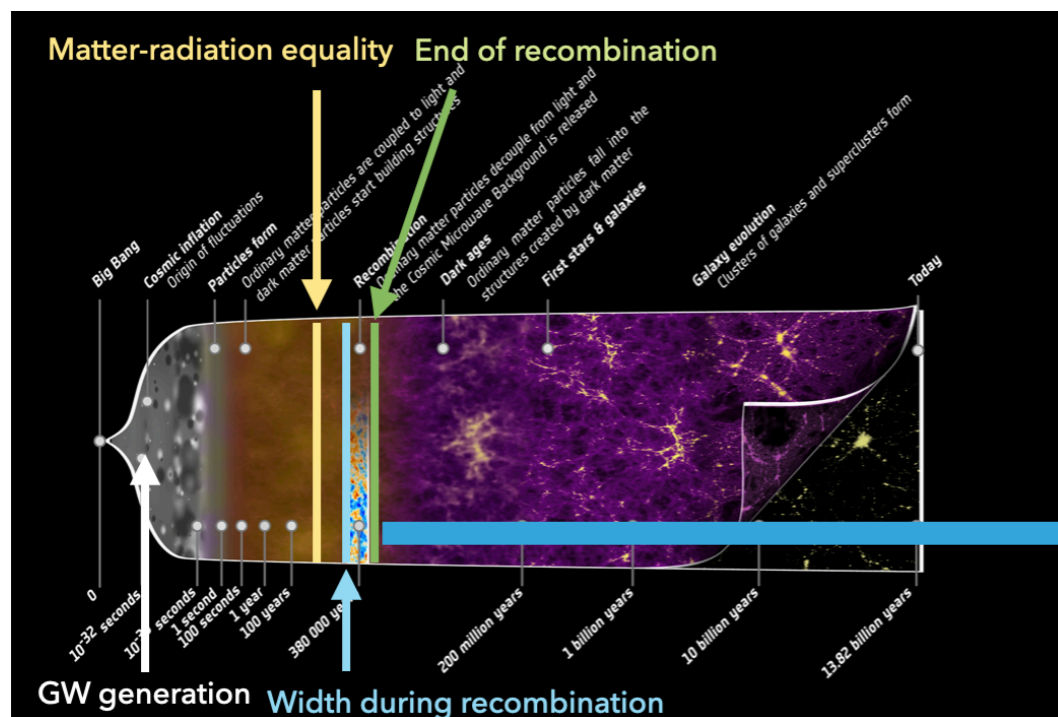
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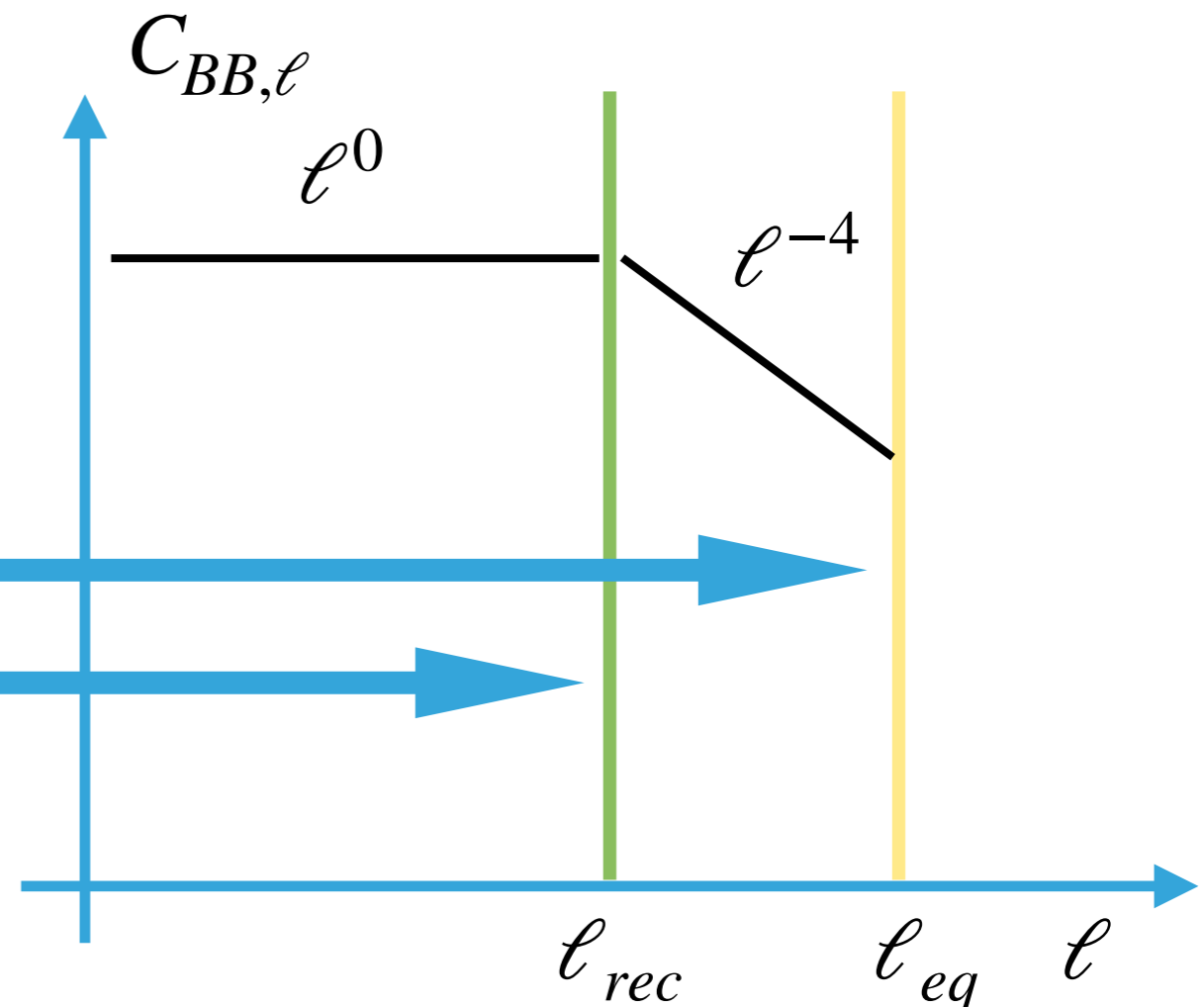
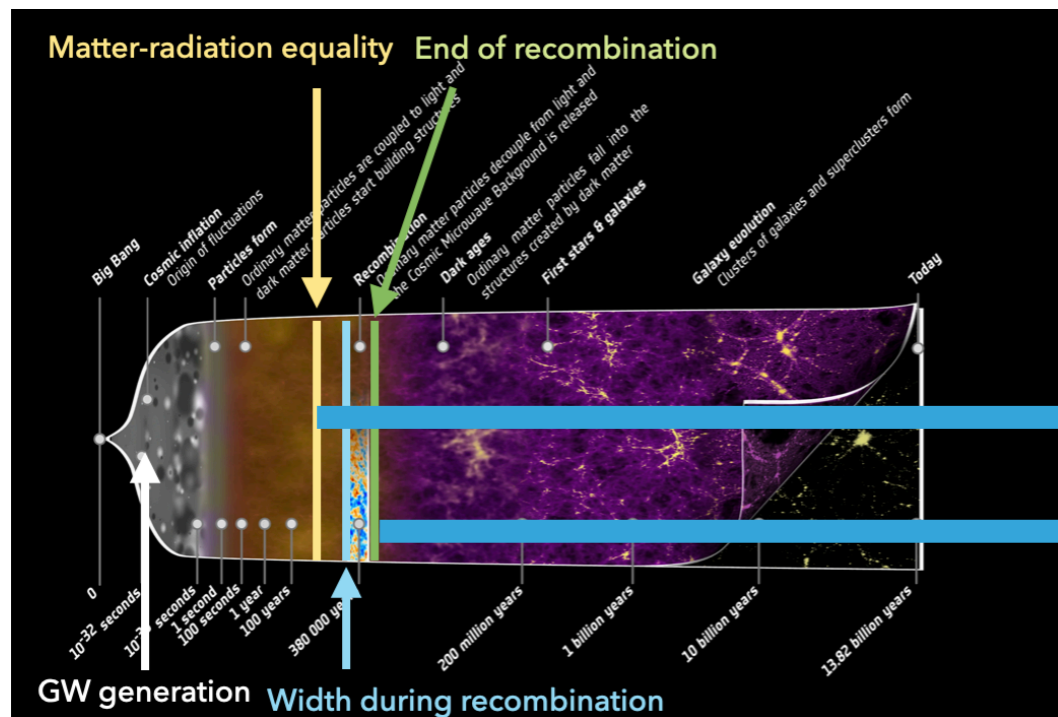
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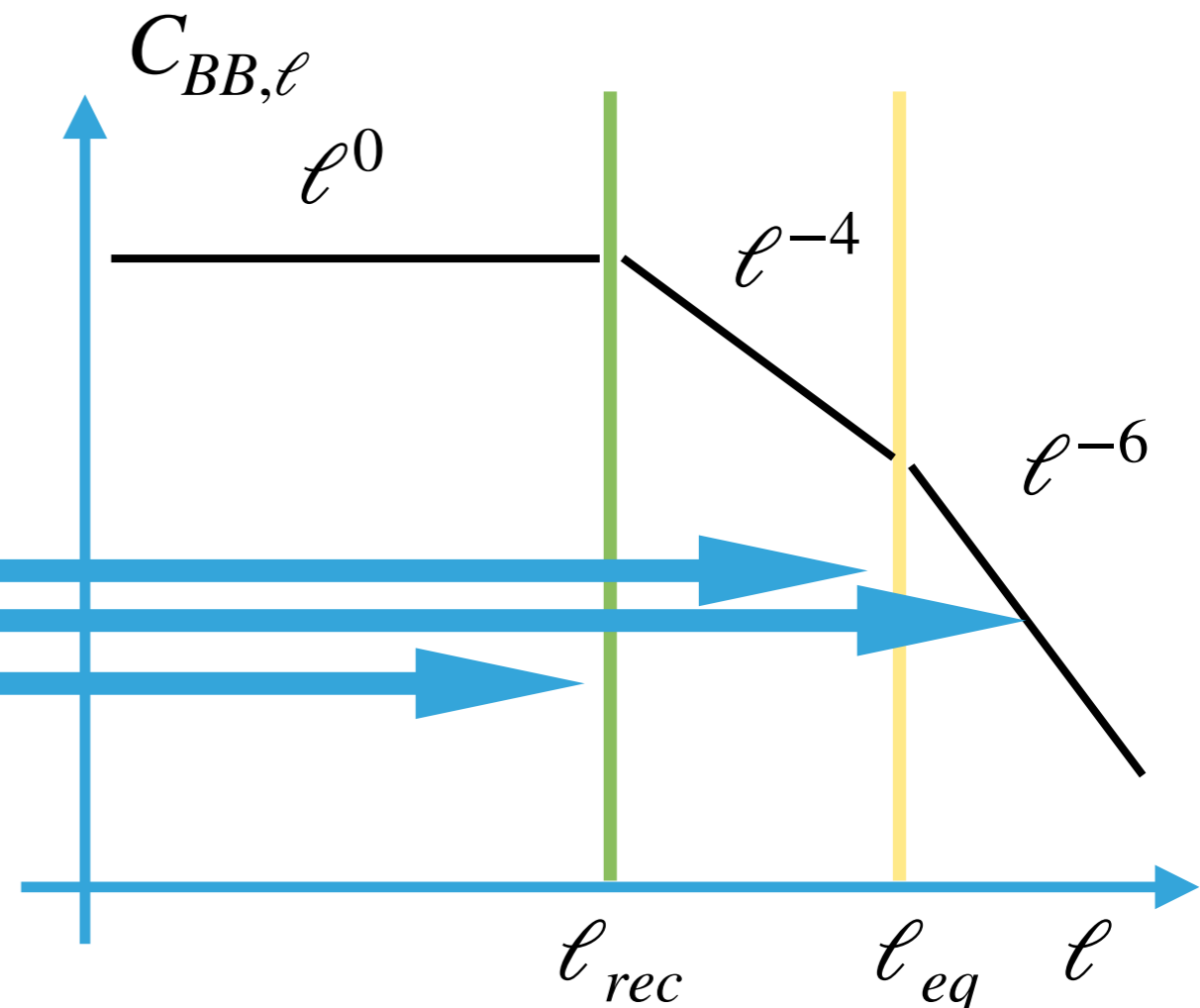
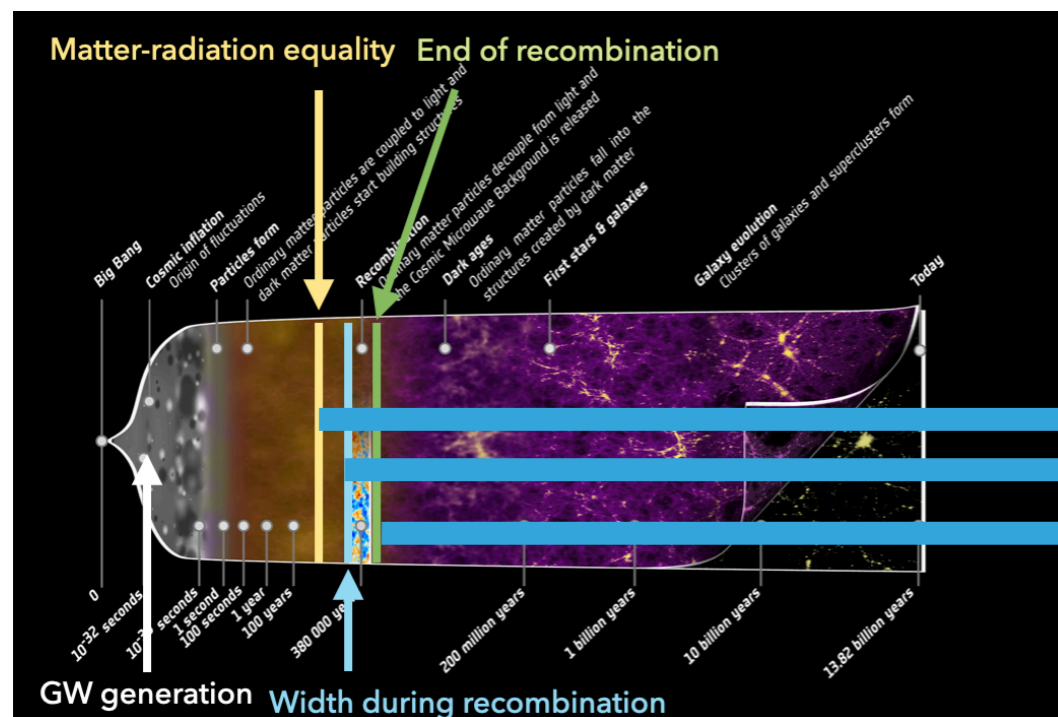
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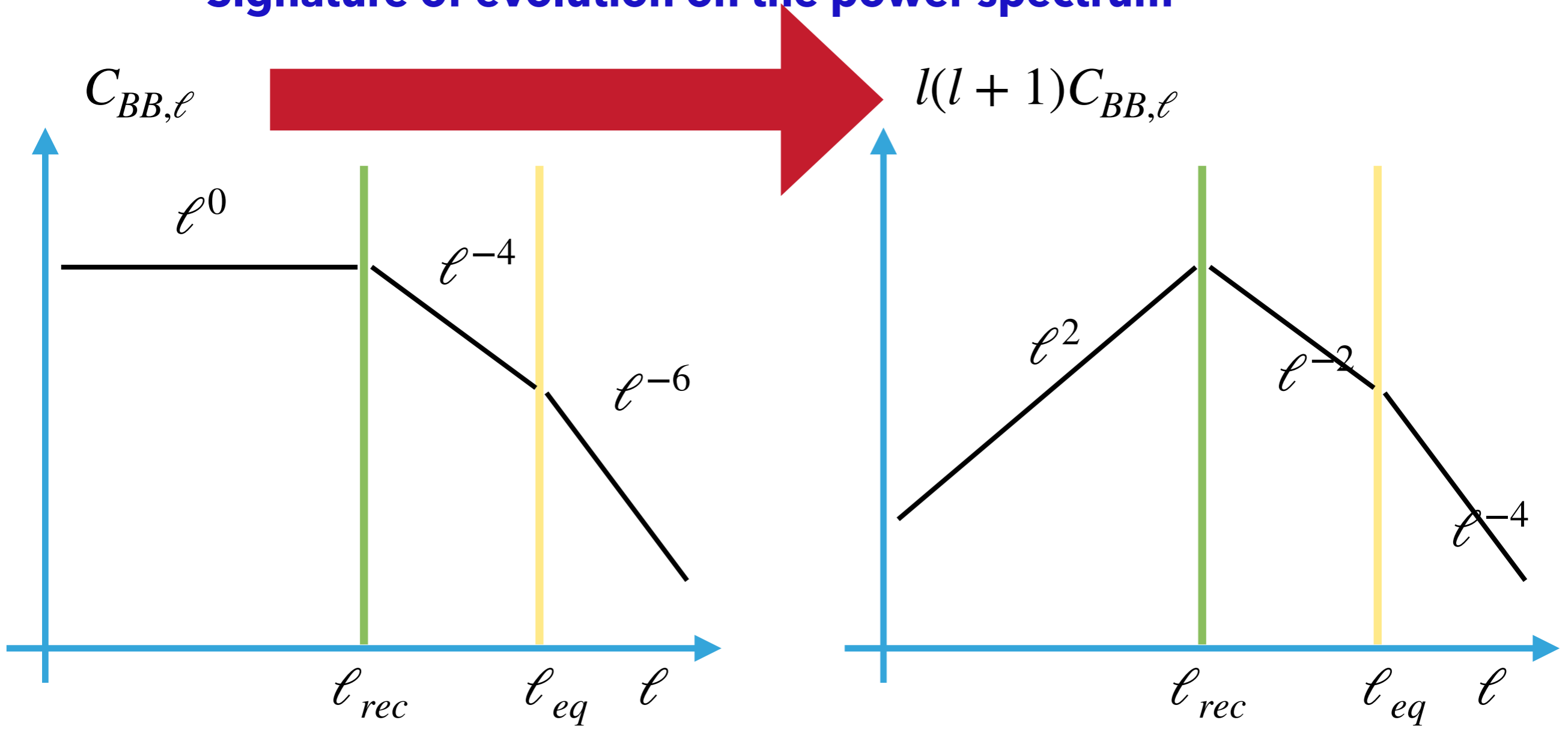
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OBSERVABLES OF GW PERTURBATIONS ON B-MODE

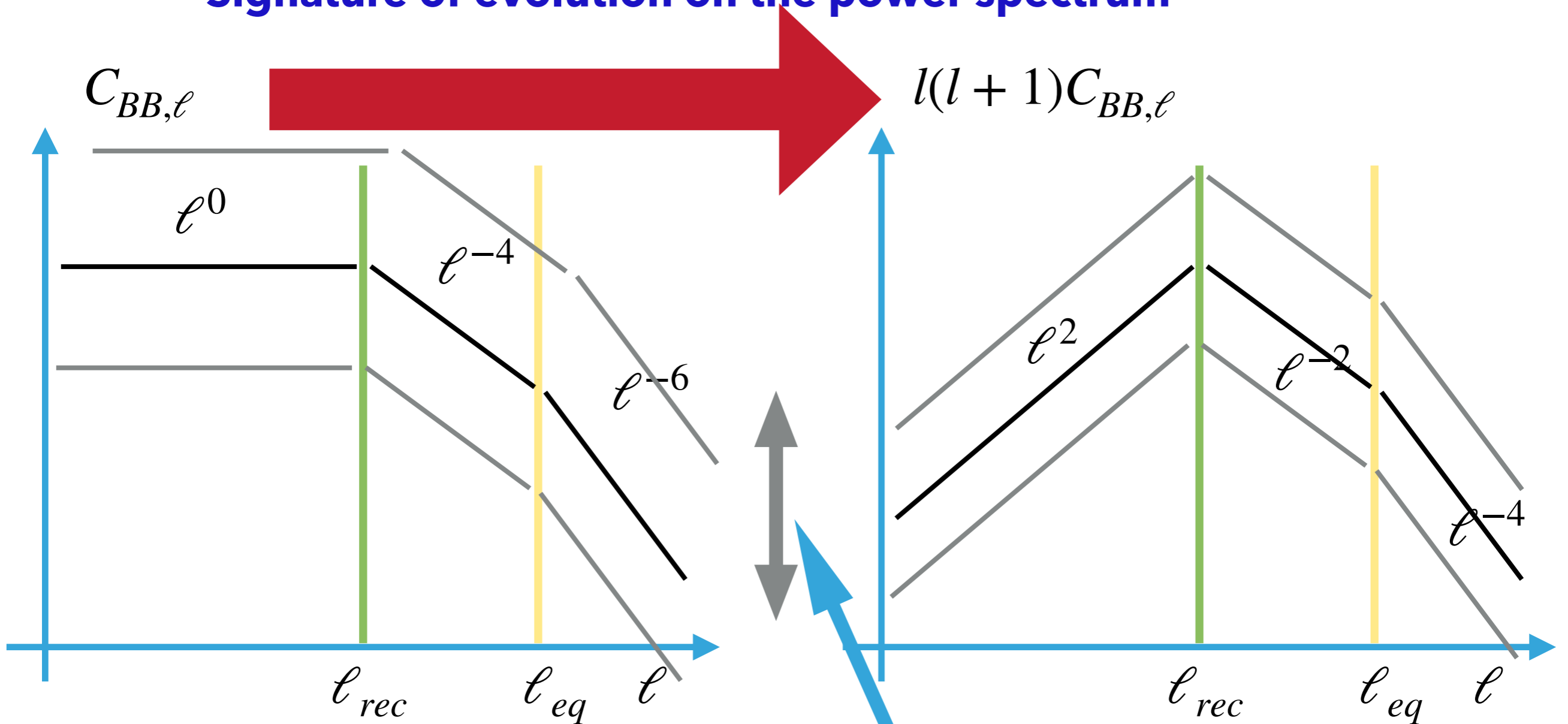
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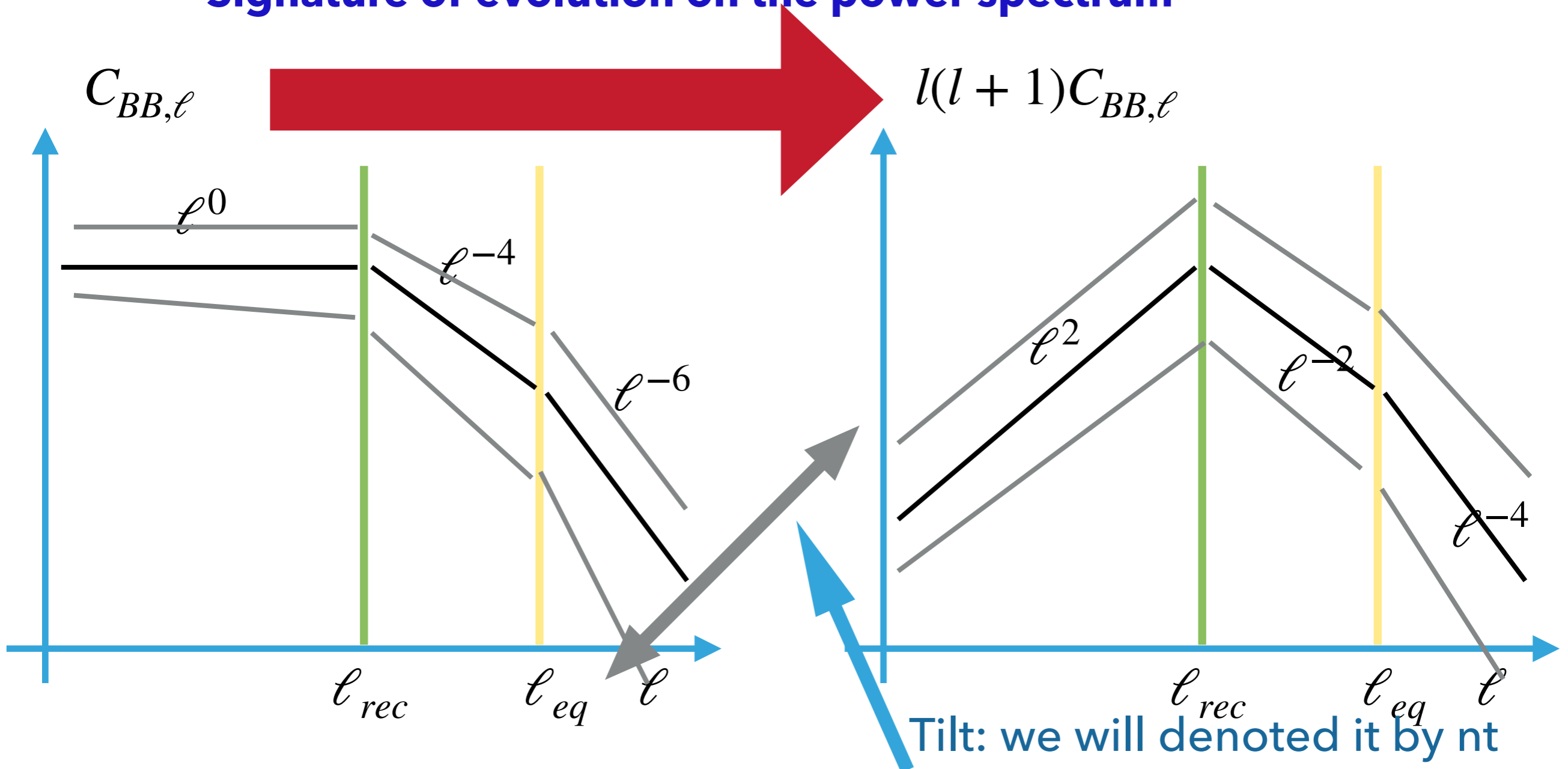
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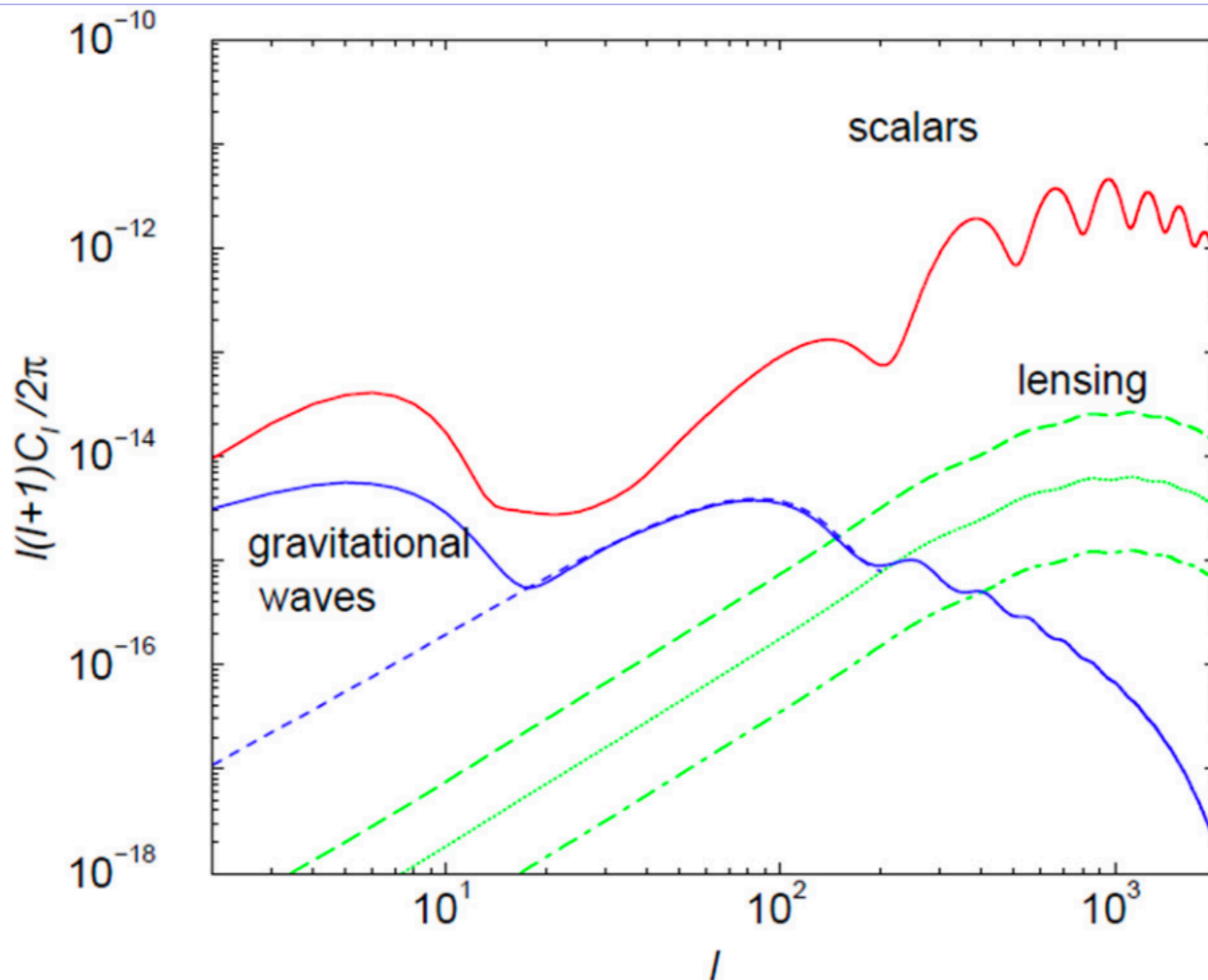


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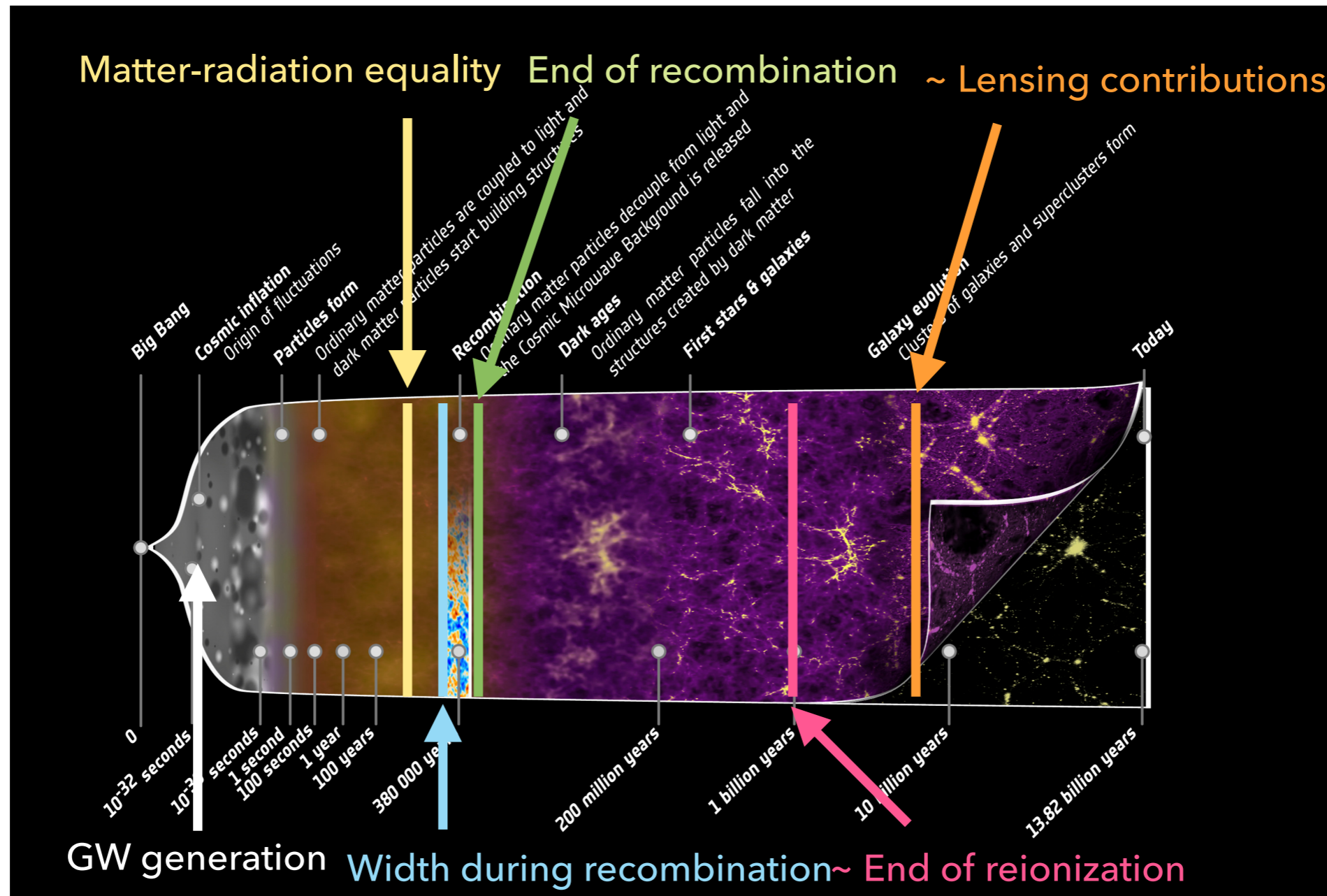
From numerical code

Lewis and Challinor 2006



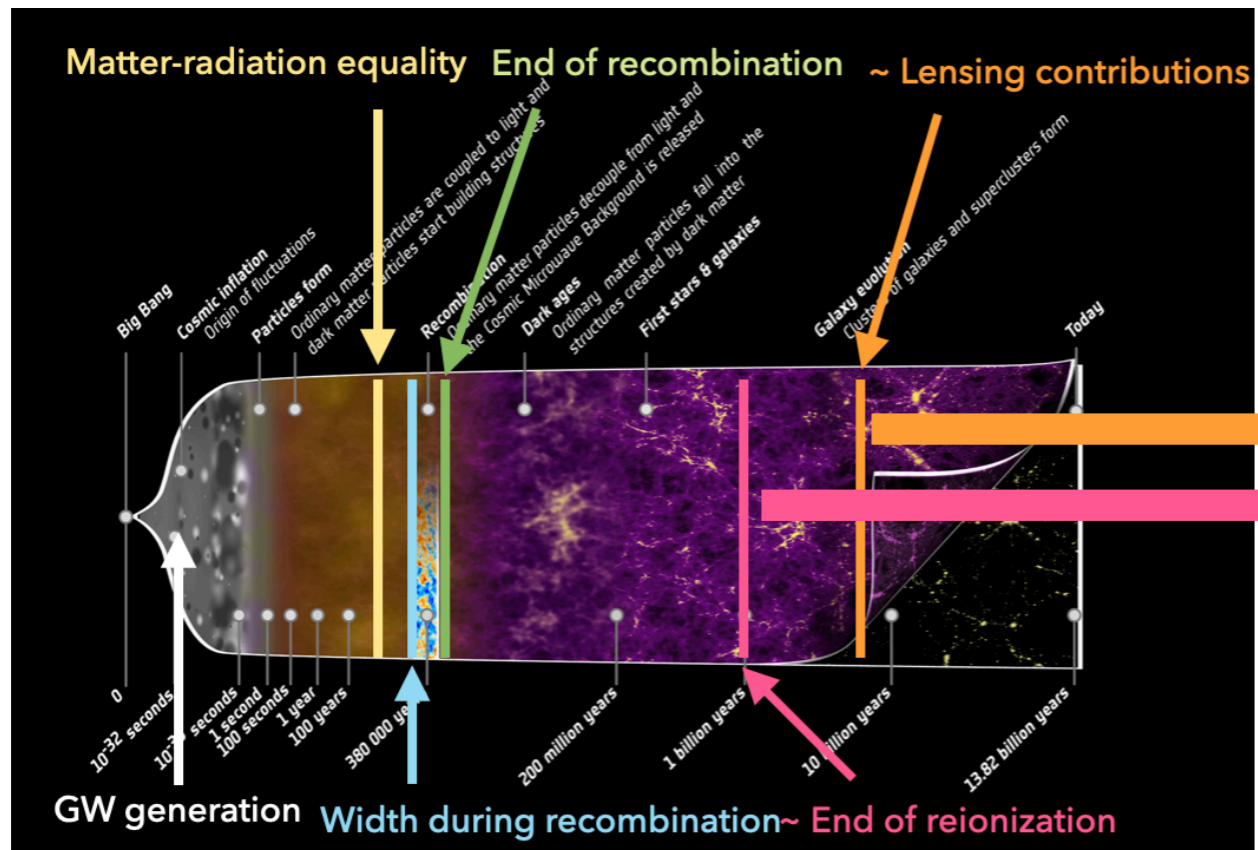
OBSERVABLES OF GW PERTURBATIONS ON B-MODE

Different Cosmic stages

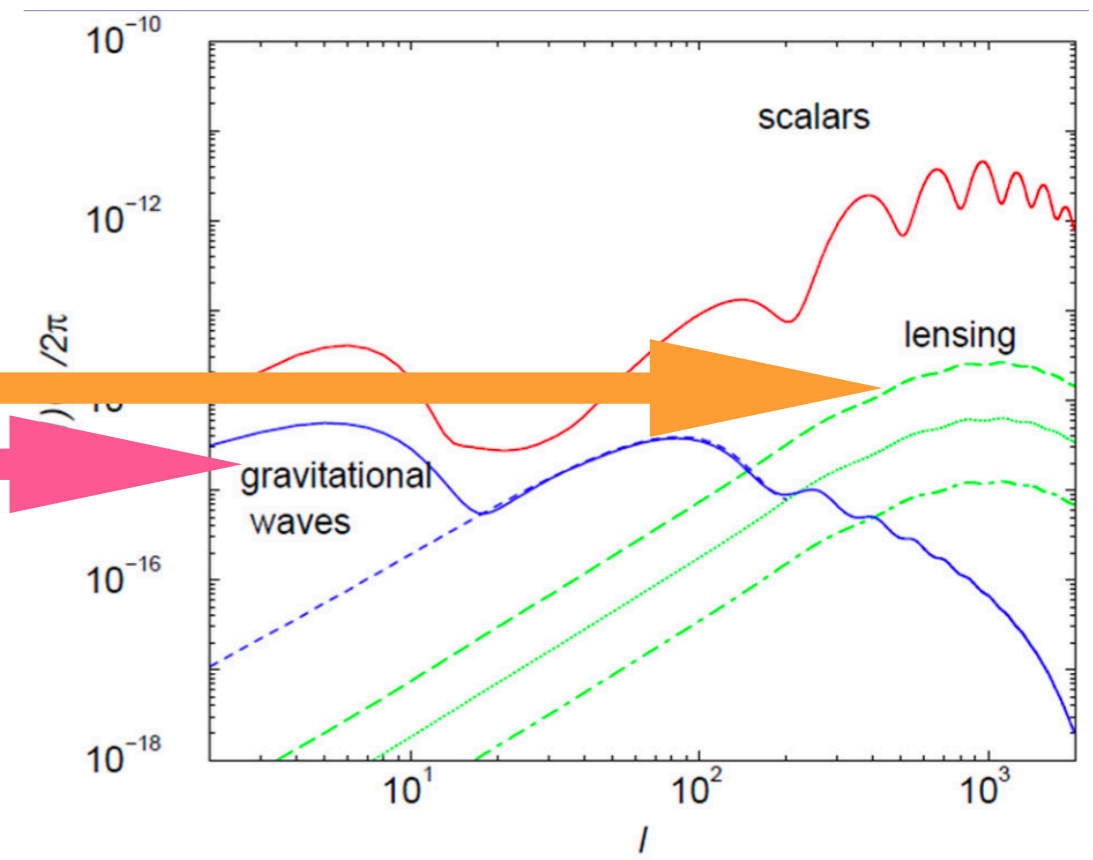


OBSERVABLES OF GW PERTURBATIONS ON B-MODE

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Lewis and Challinor 2006

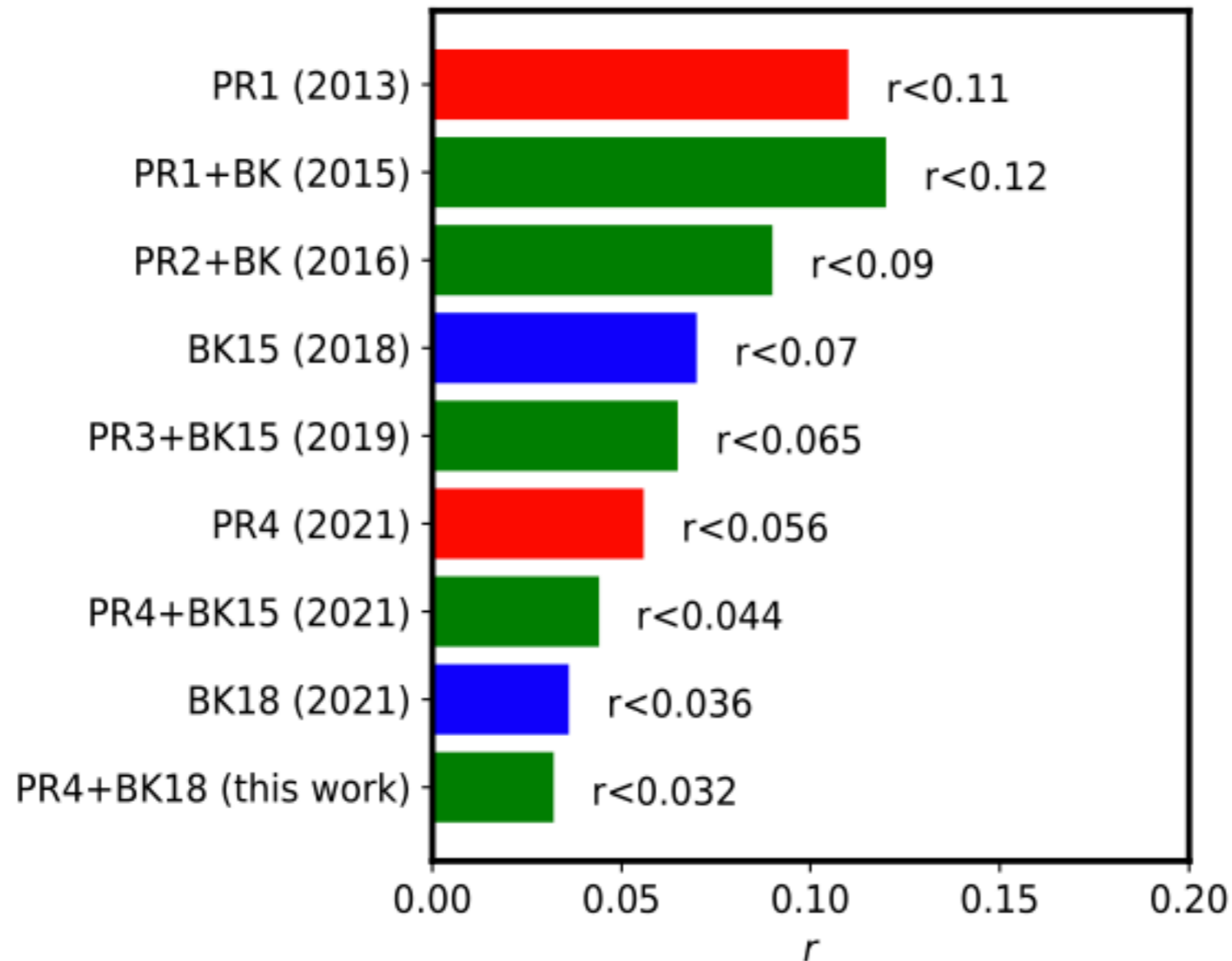


CMB as a Probe to Gravitational Waves

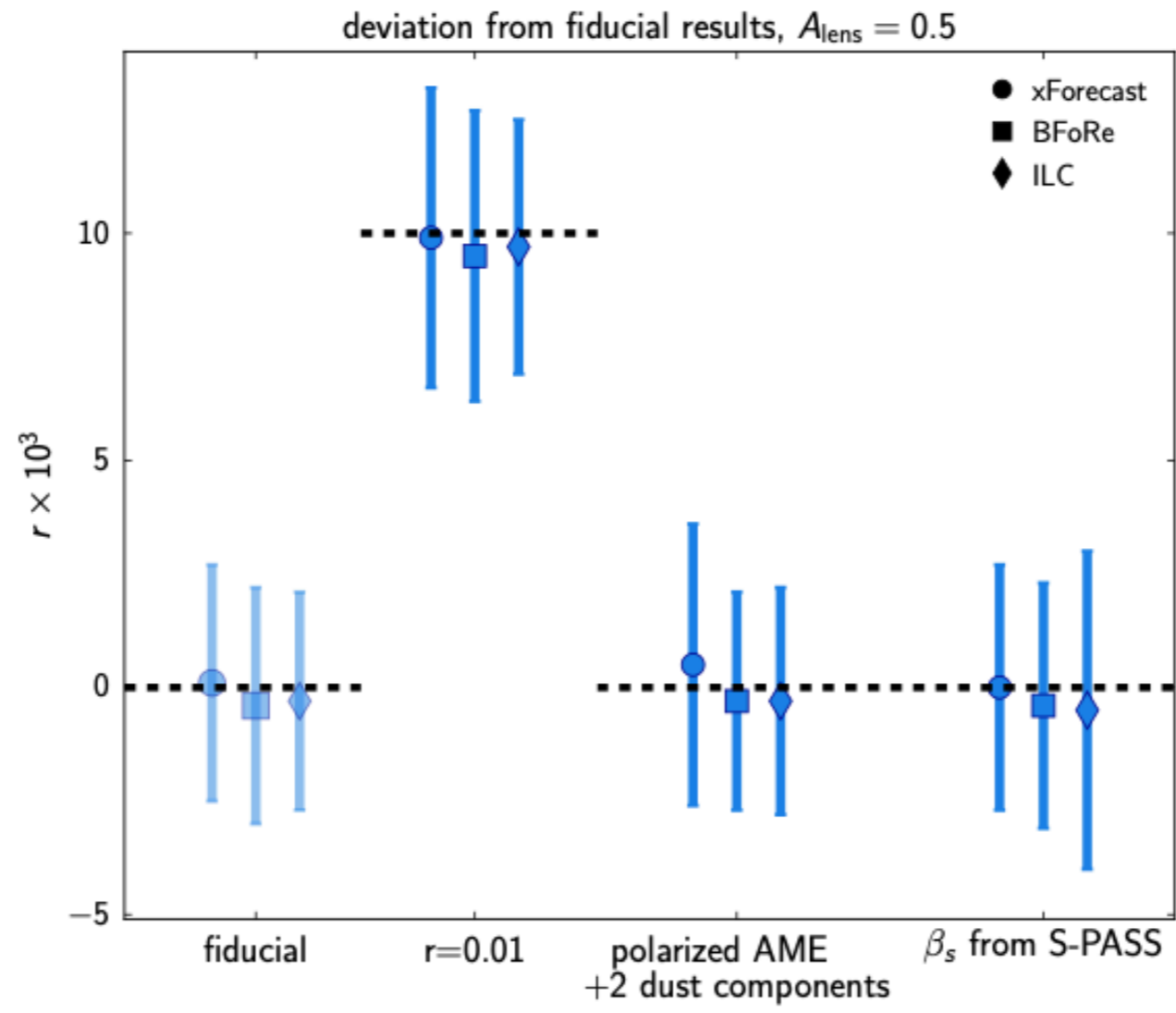
What is the prospect of detection?

What is the current status?: Upper Bound

Tristram (2022)

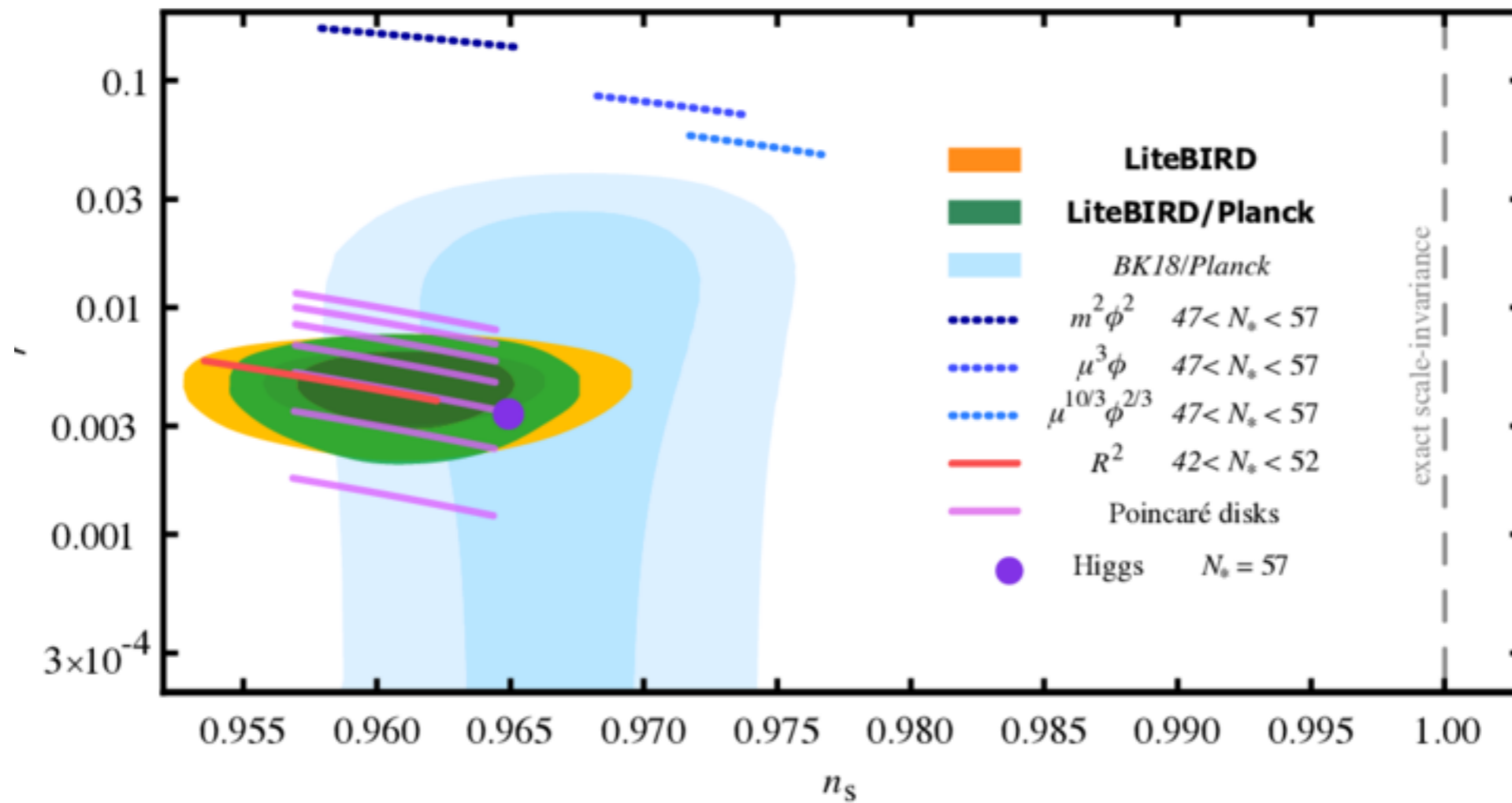


Future Prospects: Simons Observatory



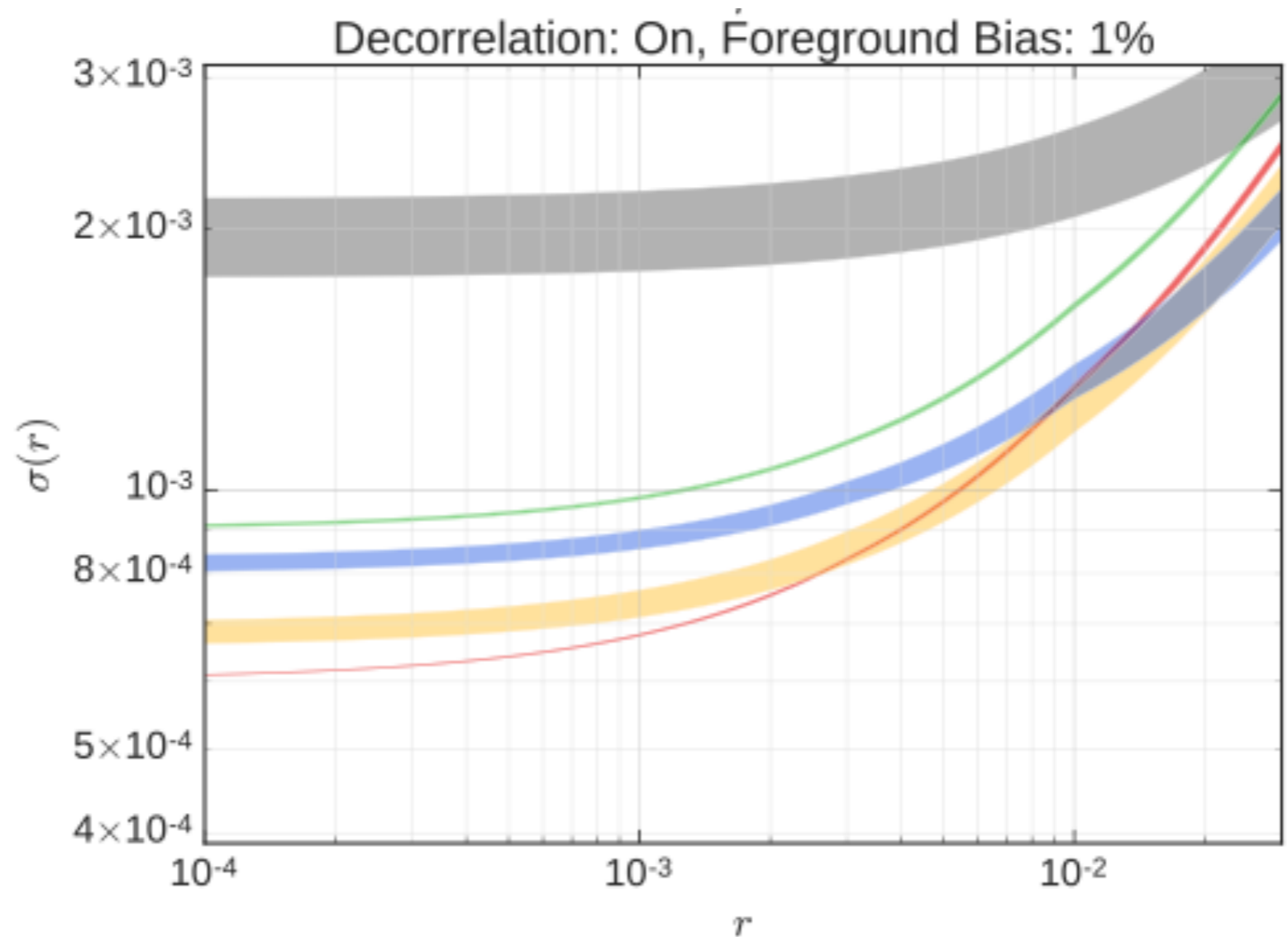
Simons Observatory Collaboration

Future Prospects: LiteBIRD



LiteBIRD Collaboration

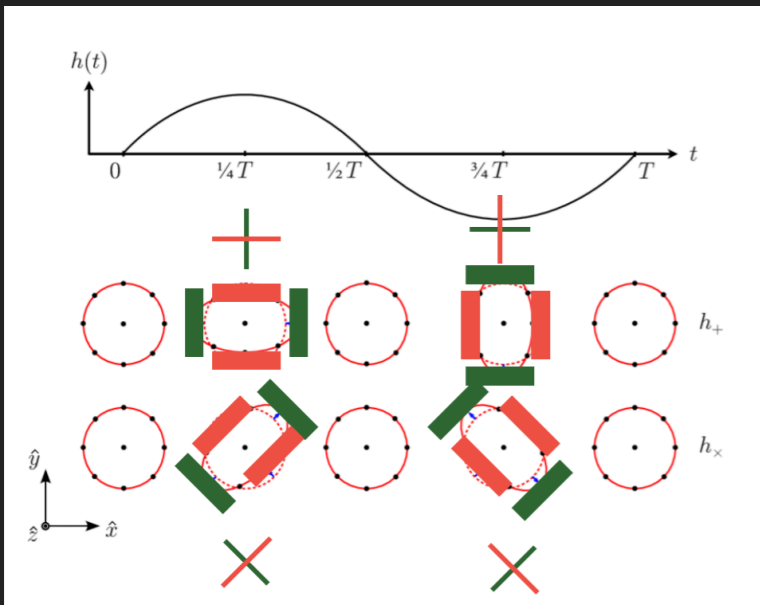
Future Prospects: CMB-S4



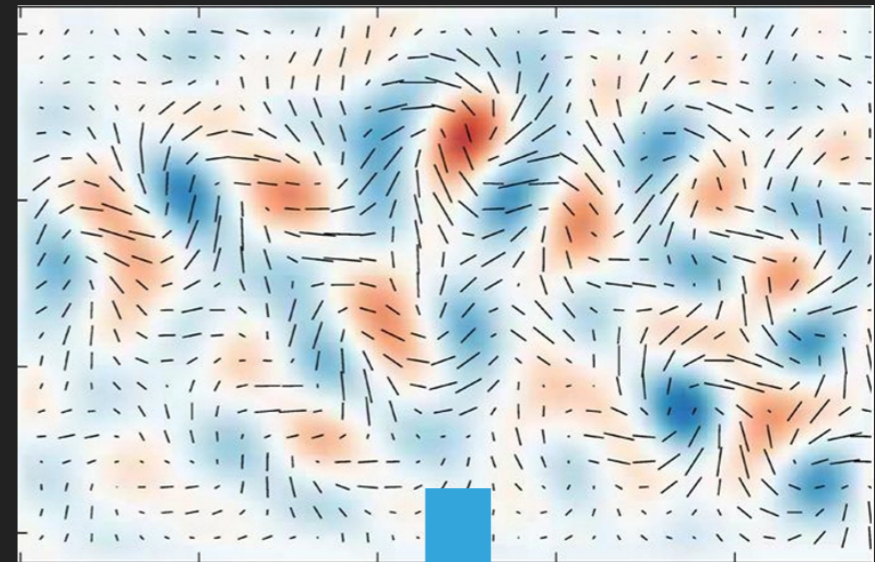
CMBS4 Collaboration

CMB B-MODE TO DETECT LOW FREQUENCY GW SIGNAL

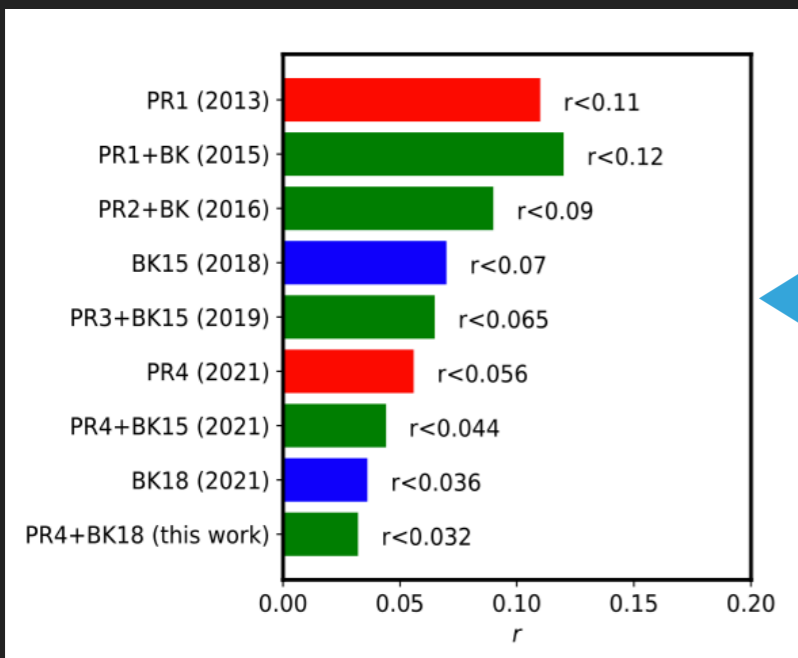
Interaction



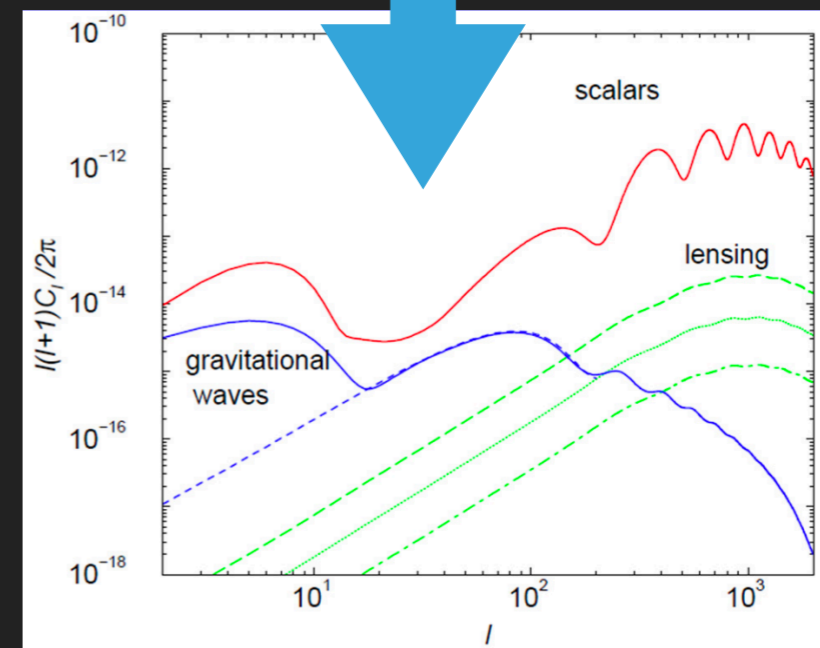
Observable



Prospects

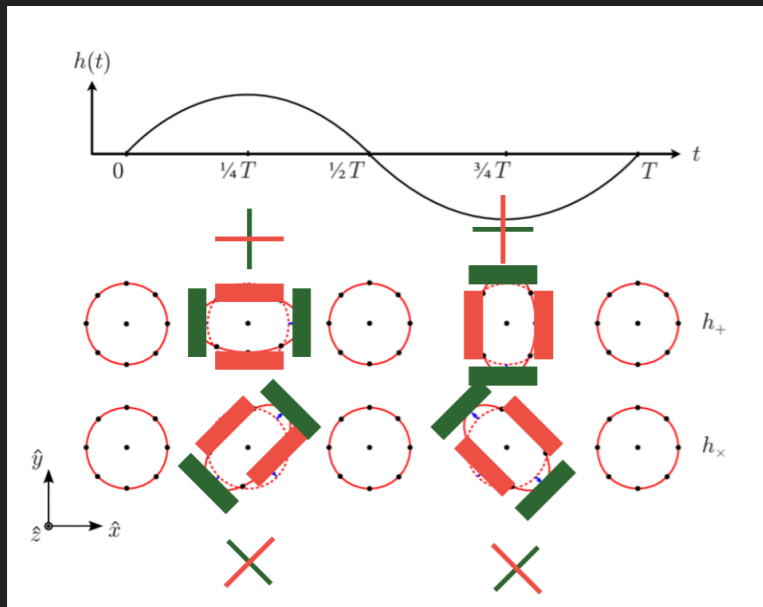


Expected strength

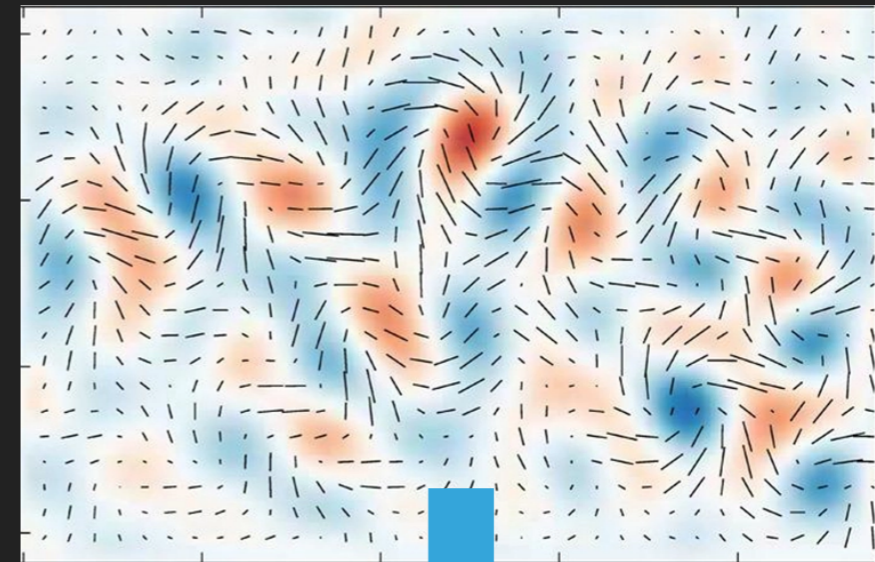


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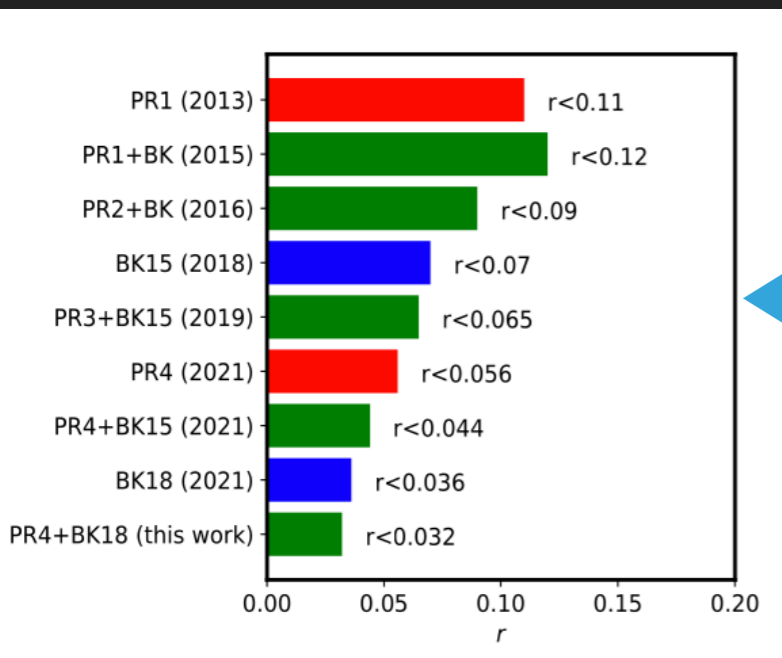
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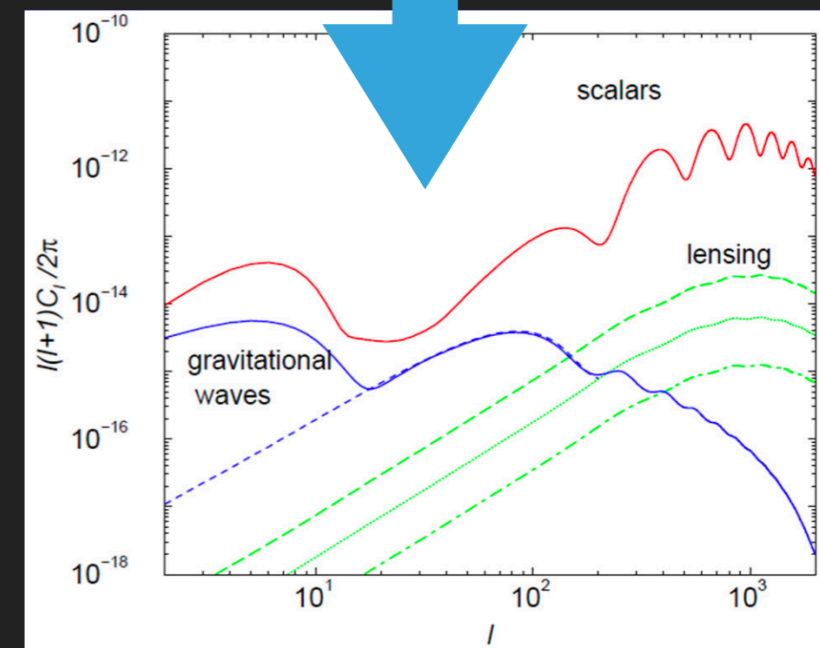


Prospects



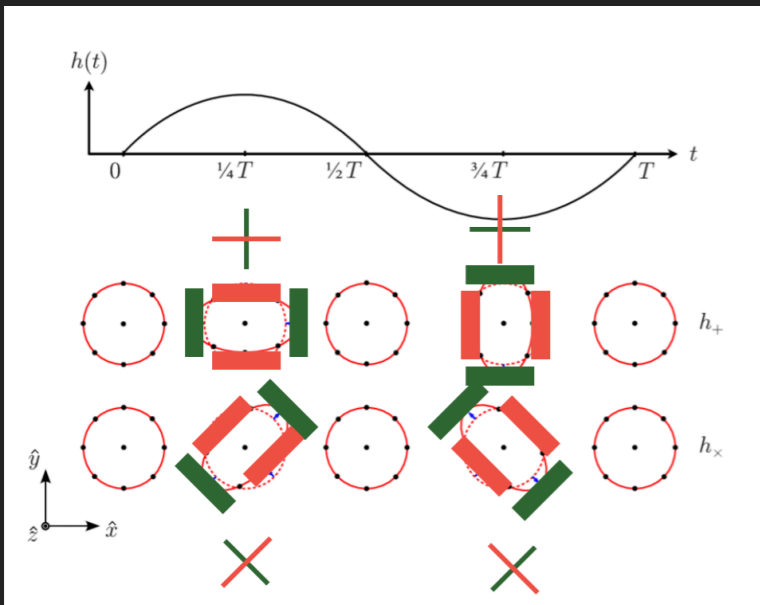
Challenges

Expected strength

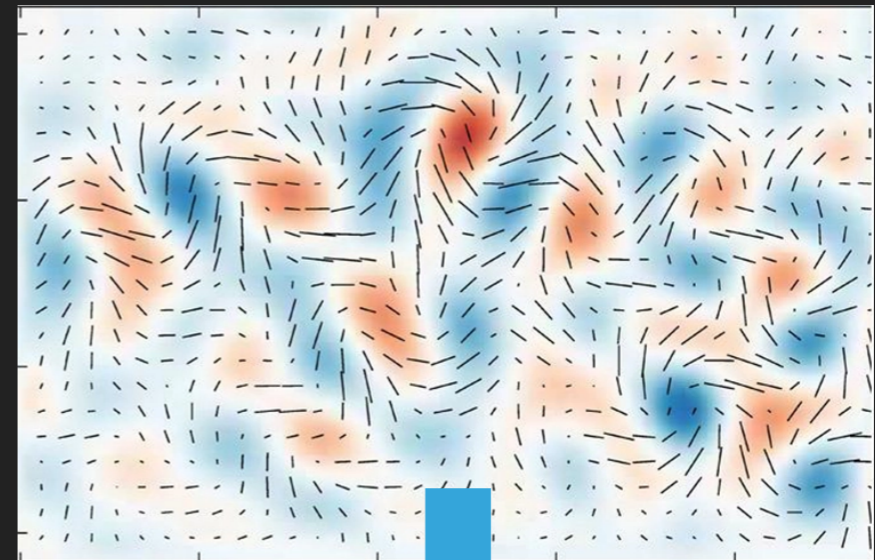


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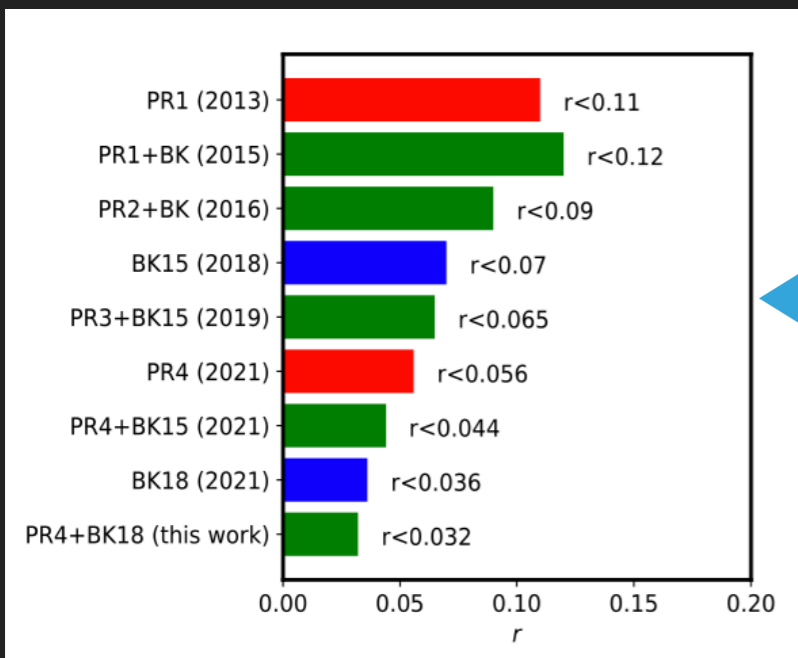
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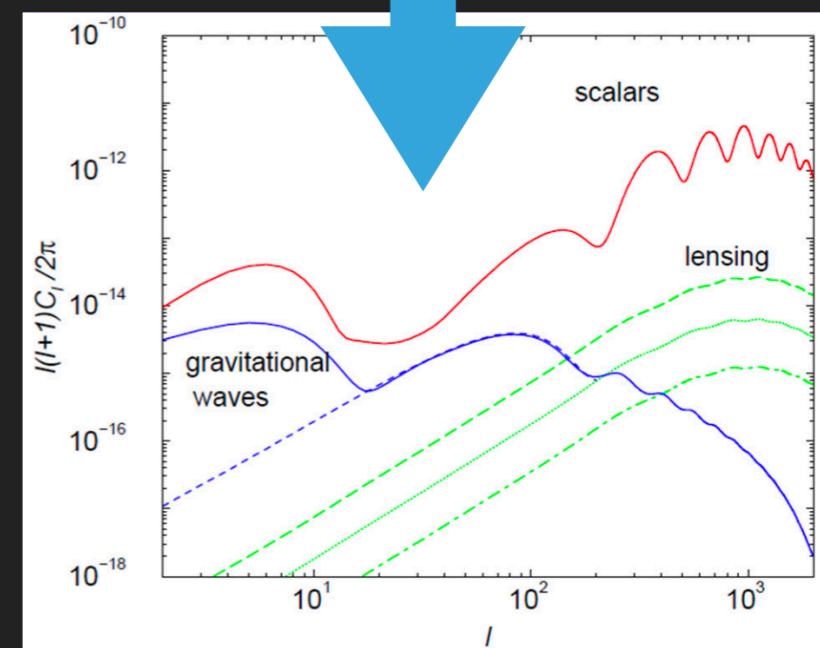


Prospects



Challenges

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REFERENCES (A PARTIAL LIST)

1. Starobinskii, 1985
2. Crittenden, Davis, Steinhardt, 1993
3. Atrio-Barandela and Silk, 1993
4. Pritchard and Kamionkowski, 2005
5. Modern Cosmology, Dodelson
6. Cosmology, Weinberg