Giant effective magnetic moments of chiral phonons

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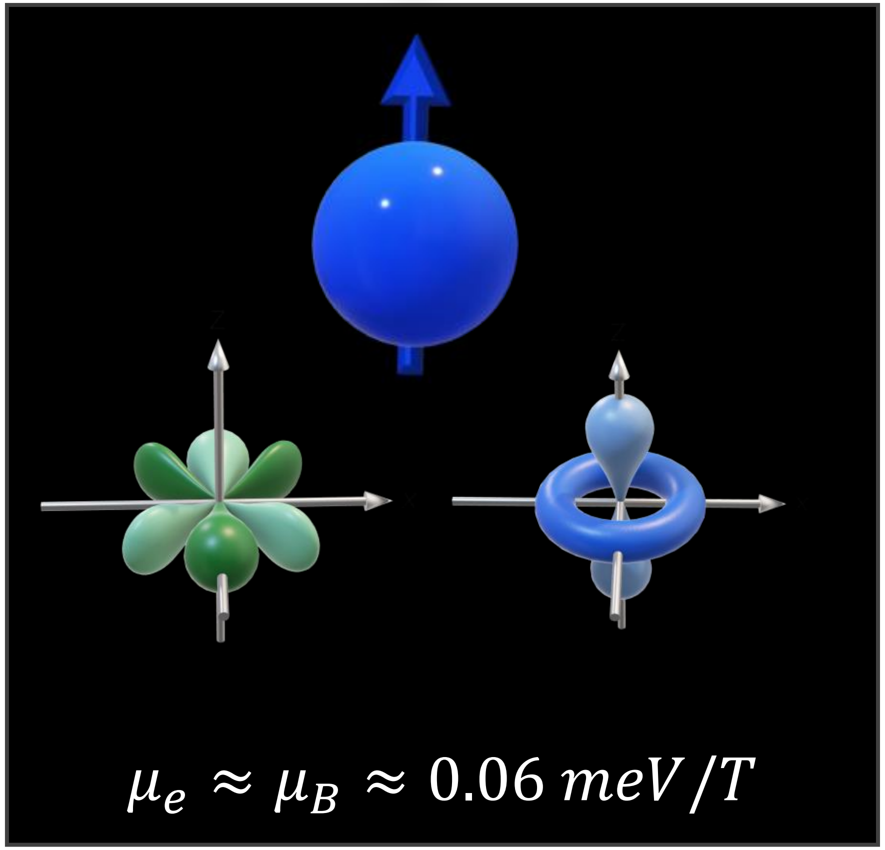
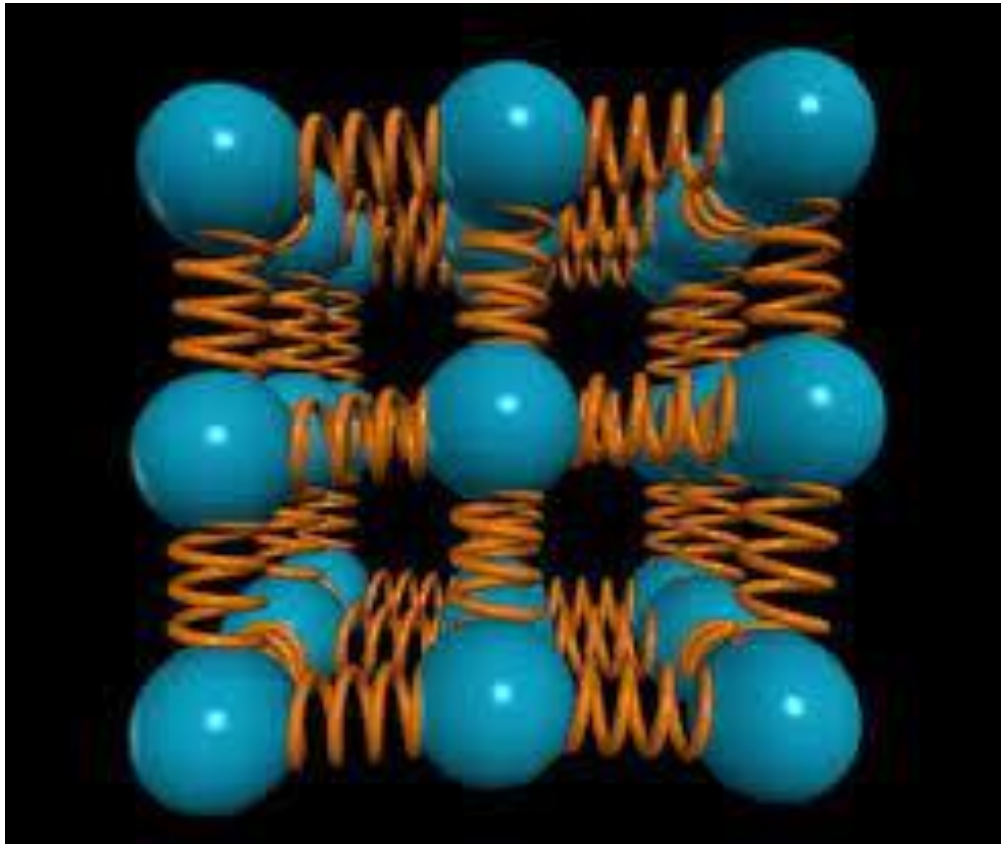
Electrons v/s phonons: Magnetic response

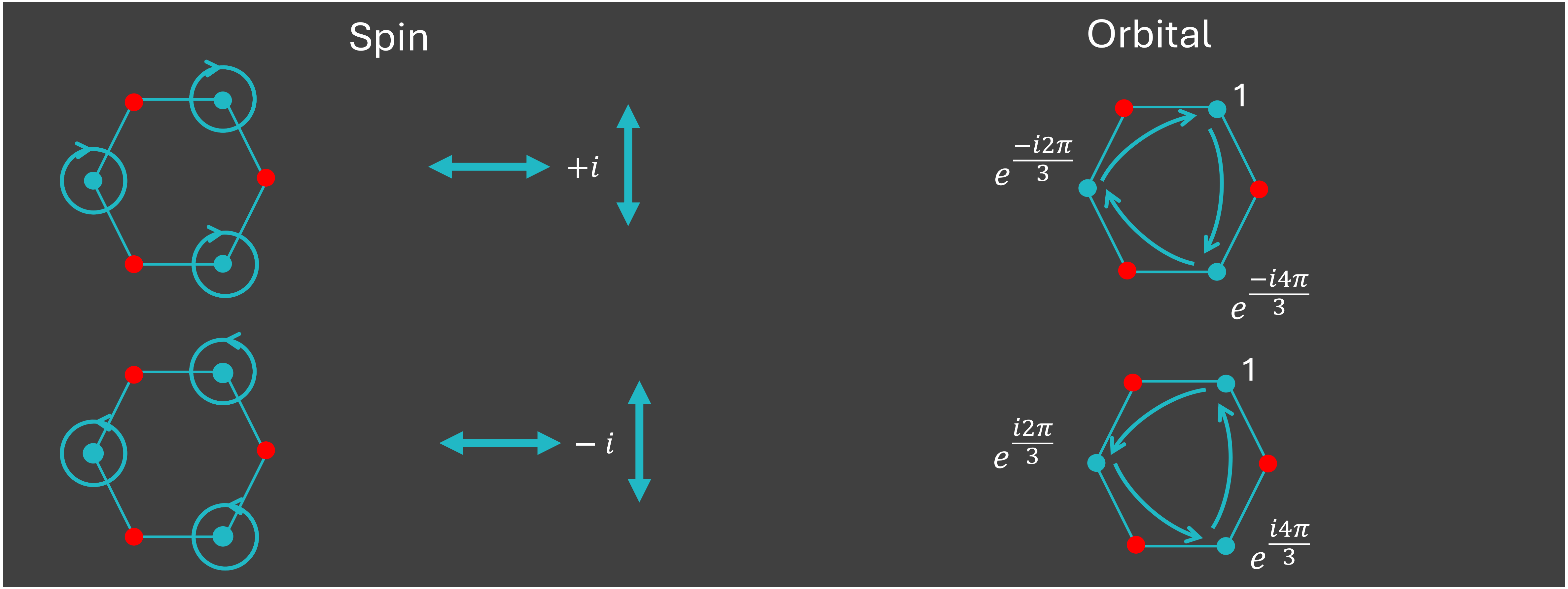
Electrons



Phonons



Chiral phonons can carry angular momentum!

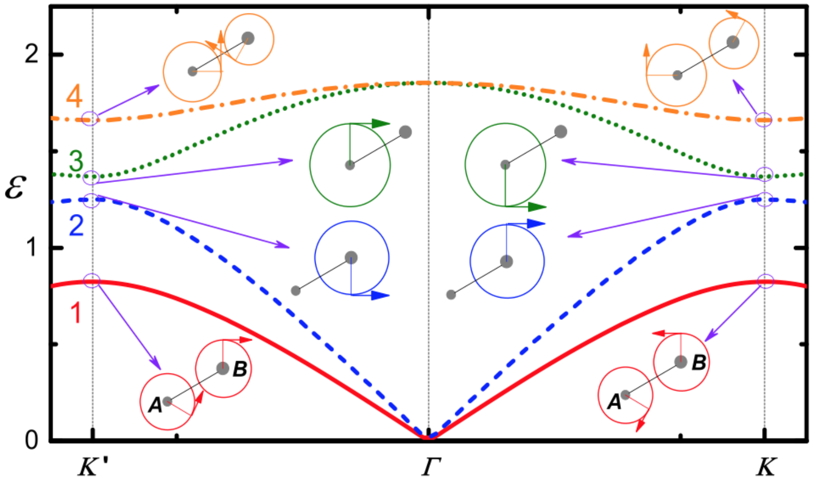


Lifa Zhang, Qian Niu, Phys. Rev. Lett. **112**, 085503 (2014)

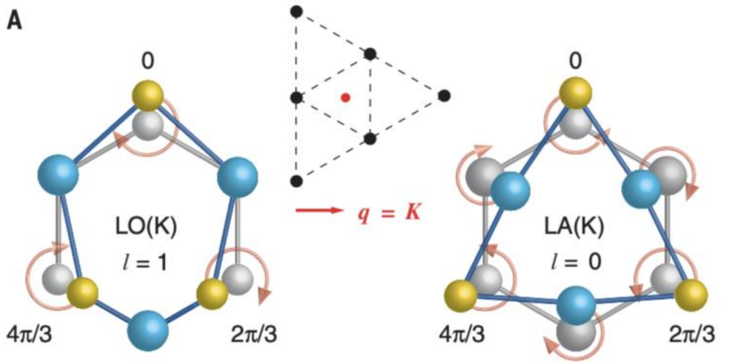
Lifa Zhang, Qian Niu, Phys. Rev. Lett. **115**, 115502 (2015)

Chiral phonons: recent examples

Broken inversion symmetry
(Valley chiral phonons)



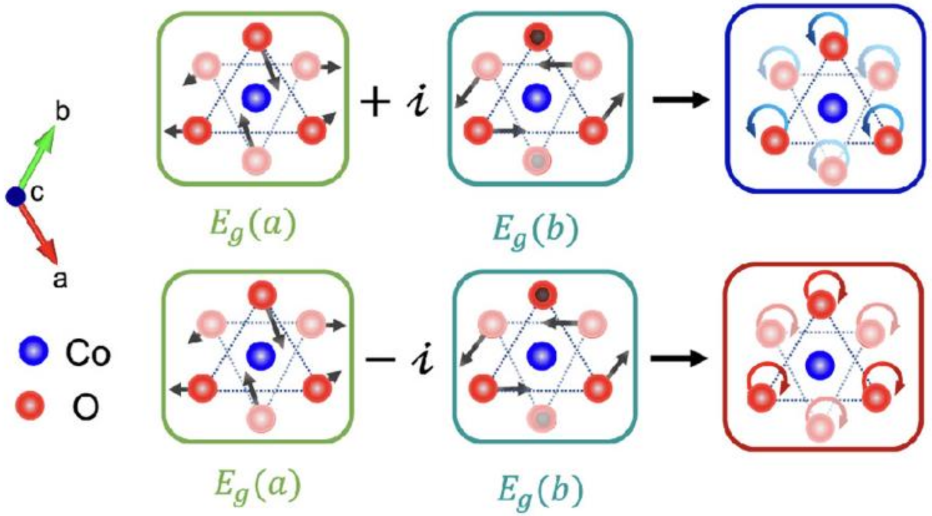
Zhang, Lifa, *et.al* Phys. Rev. Lett. **115**, 115502 (2015)



WSe_2

Zhu, Hanyu, et al. *Science* 359.6375 (2018): 579-582.

Broken time-reversal symmetry
(Zone-centered chiral phonons)



D. Lujan*, J. Choe*, **S. Chaudhary***, G. Fiete, & Xiaoqin Li *et al.*, PNAS, 121(11), e2304360121 (2024)

Other Works

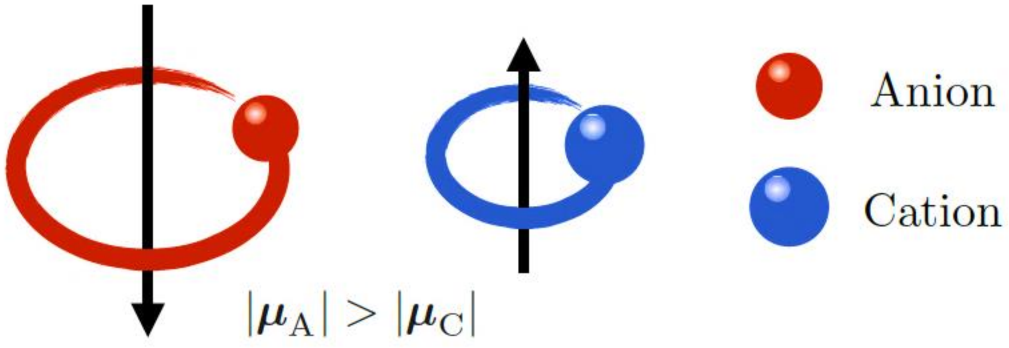
Bonini, John *et.al* Phys. Rev. Lett. **130**, 086701 (2023)

Zhang *et.al* Phys. Rev. Lett. **130**, 226302(2023)

Yin *et.al* *Advanced Materials* 33.36 (2021): 2101618.

Liu *et.al* Phys. Rev. Lett. **119**, 255901 (2017)

Magnetic response: classical picture



Material	μ_{ph} (μ_N)	Zeeman Splitting at 50 T	Zeeman Splitting at 1000 T
$BaHfO_3$	1.7	0.005 cm^{-1}	0.1 cm^{-1}
$KTaO_3$	3.0	0.01 cm^{-1}	0.2 cm^{-1}
$KNbO_3$	7	0.02 cm^{-1}	0.4 cm^{-1}
$SrTiO_3$	7.2	0.02 cm^{-1}	0.4 cm^{-1}

Angular Momentum $\boldsymbol{L} = \boldsymbol{Q} \times \partial_t \boldsymbol{Q}$

Phonon Magnetic moment $\mu_{ph} = \gamma \boldsymbol{L}$

Gyromagnetic ratio $\gamma = \sum_i \gamma_i (\mathbf{q}_{i,x} \times \mathbf{q}_{i,y}),$

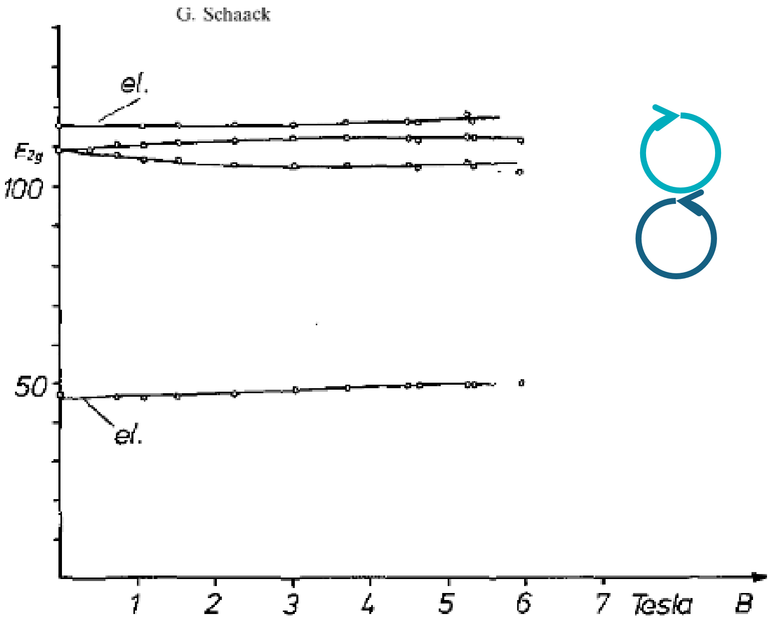
$$\gamma_i = \frac{eZ_i}{2M_i} \Rightarrow \mu_{ph} \approx \mu_N \approx 5 \text{ X } 10^{-4} \mu_B$$

Phonon Zeeman effect $\Delta\omega_{Zeeman} = \boldsymbol{2\mu_{ph}} B$

Juraschek, Dominik M., and Nicola A. Spaldin. Phys. Rev. Mat. **3** (6) (2019)

Observed Magnetic moment

Rare-Earth Paramagnet $CeCl_3$

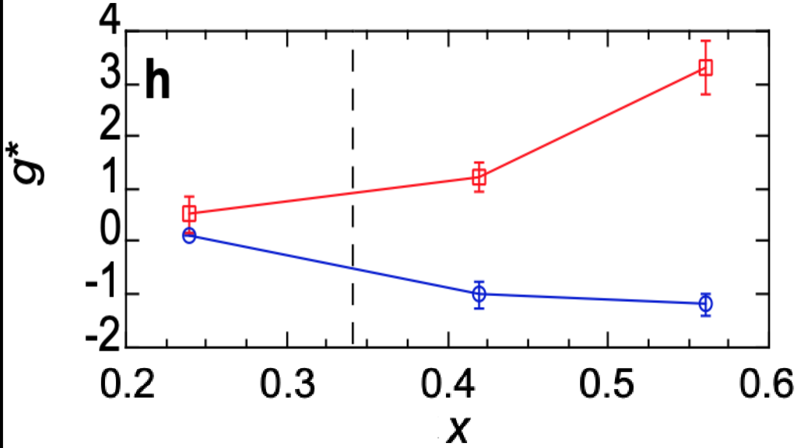


$\mu_{ph} \sim 3\ \mu_B$

G. Schaack *et al.* Z. Physik (1977)

$Pb_{1-x}Sn_xTe$

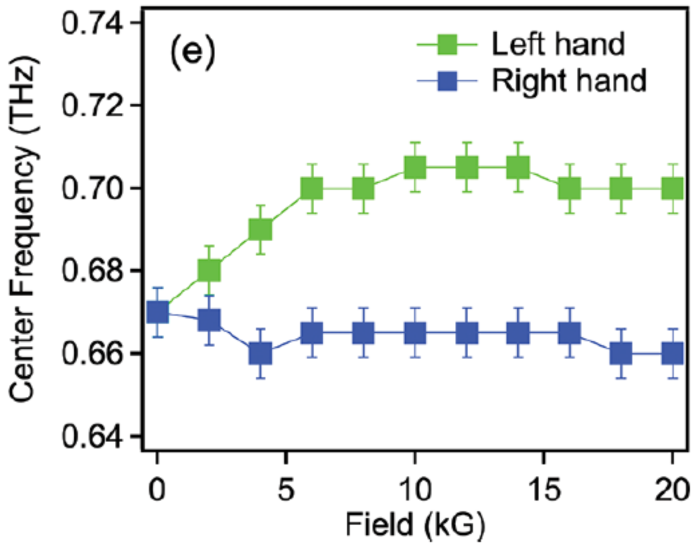
TCI(x>0.32)



$\mu_{ph} \sim \mu_B$

Hernandez, Baydin, Chaudhary, *et al*. Sci. Adv. **9**, eadj4074 (2023).

Dirac SM Cd_3As_2

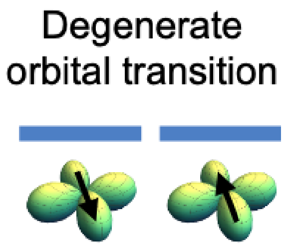


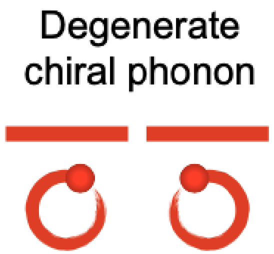
$\mu_{ph} \sim 3\ \mu_B$

B. Cheng. N.P. Armitage *et al.* Nano Lett. 2020, 20, 5991-5996

Possible mechanisms for phonon magnetic moment

Phonon magnetic moment from orbital lattice coupling



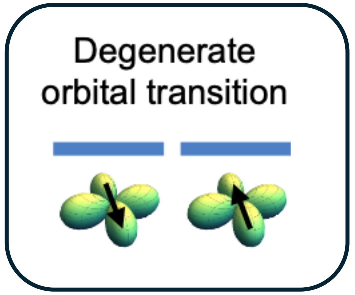


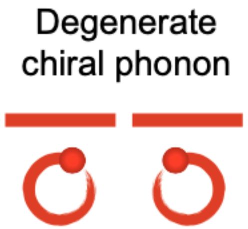
P. Thalmeier and P. Fulde, Zeitschrift fur Physik B Condensed Matter 26, 323 (1977)
S. Chaudhary, D. Juraschek, M. Rodriguez-Vega, & G. A. Fiete, *Giant effective magnetic moments of chiral phonons from orbit-lattice coupling*, arXiv:2306.11630

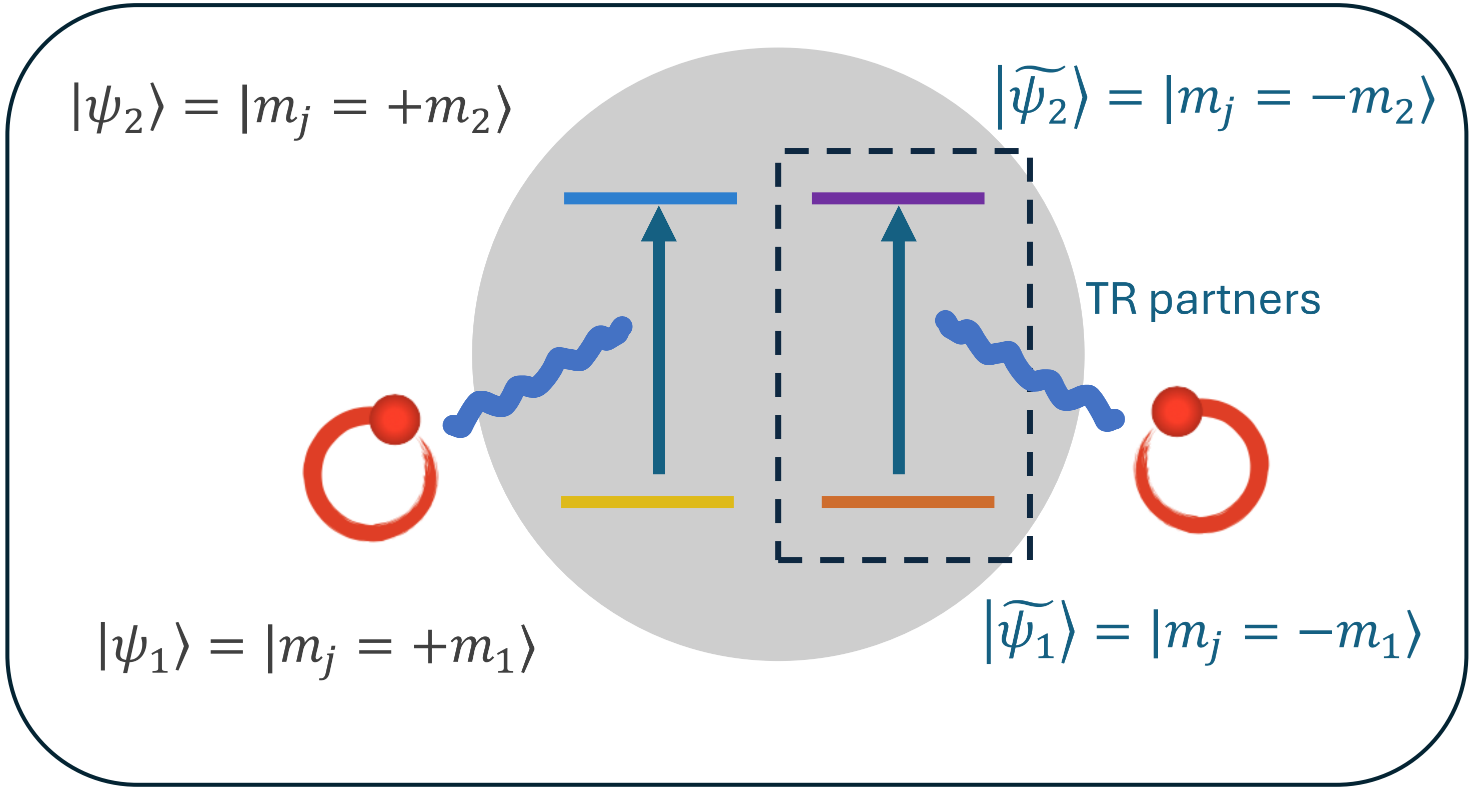
Phonon magnetic moment from orbital lattice coupling

1. Magnetic ion with two Kramers pair states
2. Chiral phonon coupling to excitation between Kramers pairs

$$H_{el-ph} = (u_x + iu_y)|\psi_1\rangle\langle\psi_2| + (u_x - iu_y)|\widetilde{\psi_1}\rangle\langle\widetilde{\psi_2}| + h.c$$







P. Thalmeier and P. Fulde, Zeitschrift fur Physik B Condensed Matter 26, 323 (1977)
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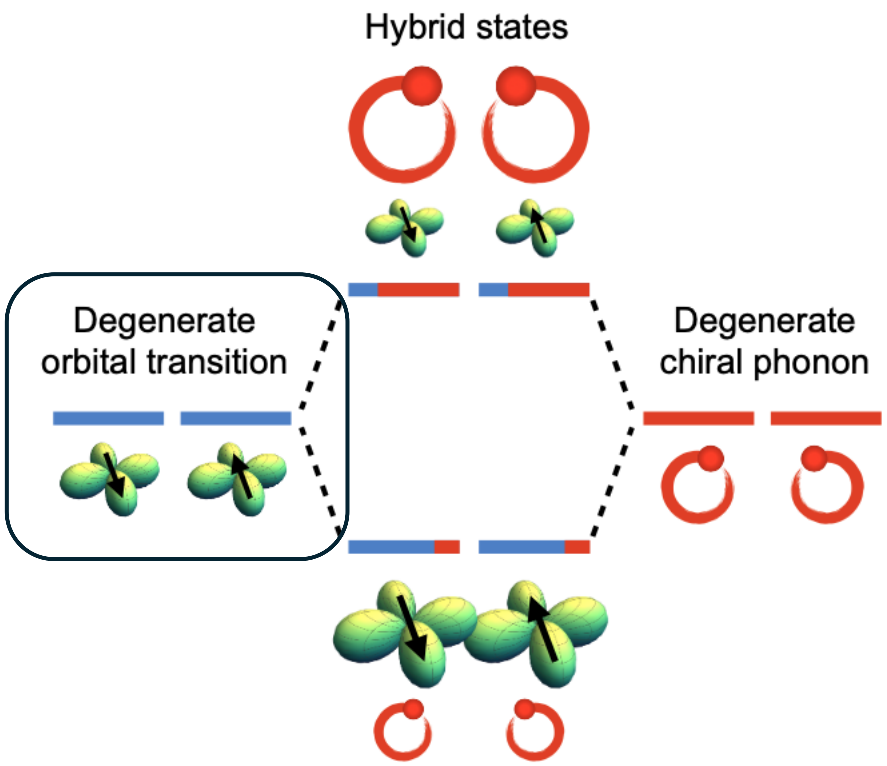

$B = 0$

1. Magnetic ion with two Kramers pair states
2. Chiral phonon coupling to excitation between Kramers pairs

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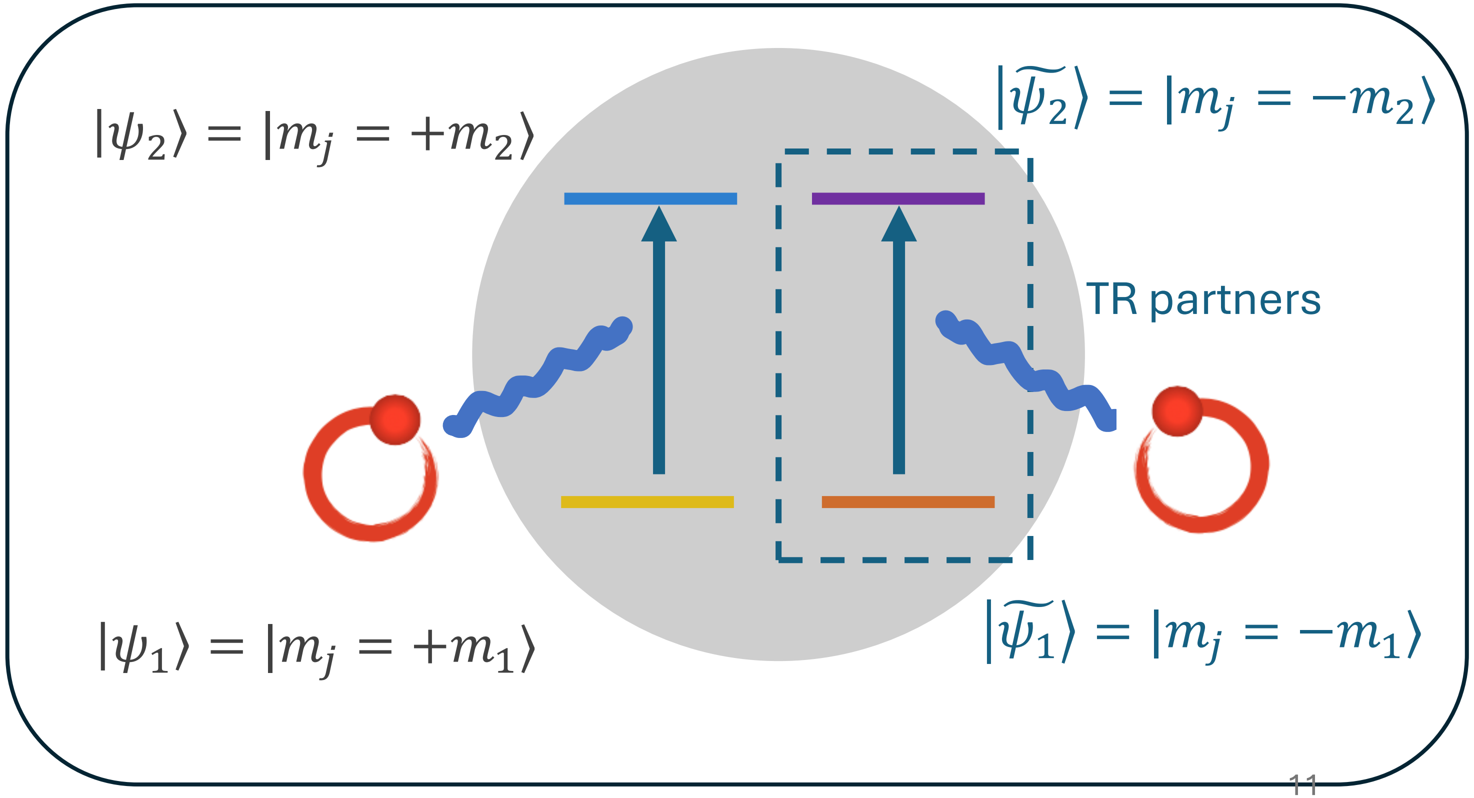

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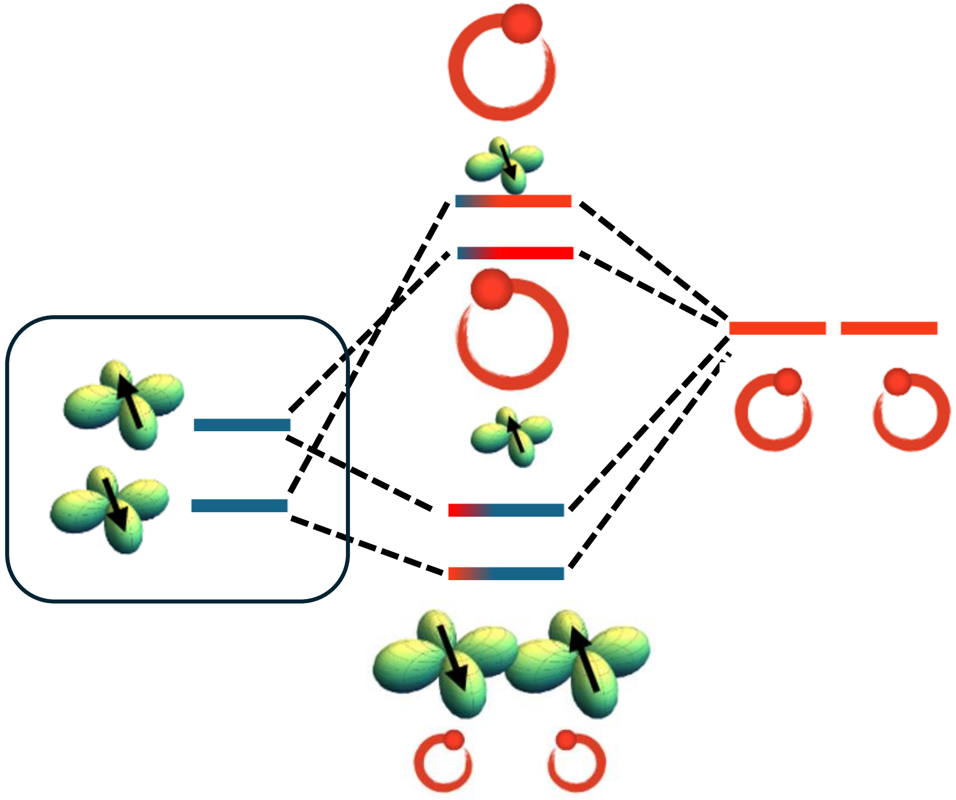

$B \neq 0$

1. Magnetic ion with two Kramers pair states
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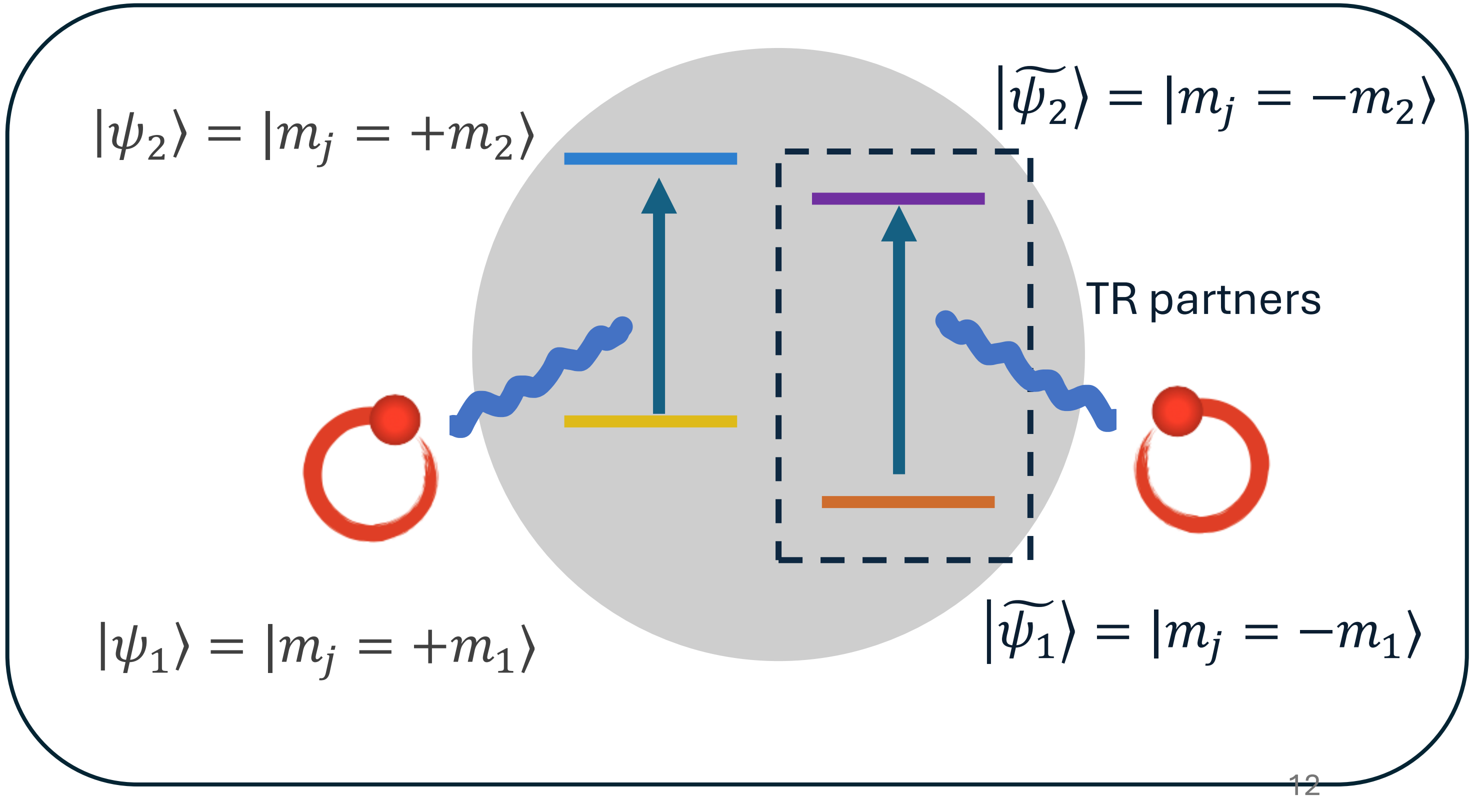
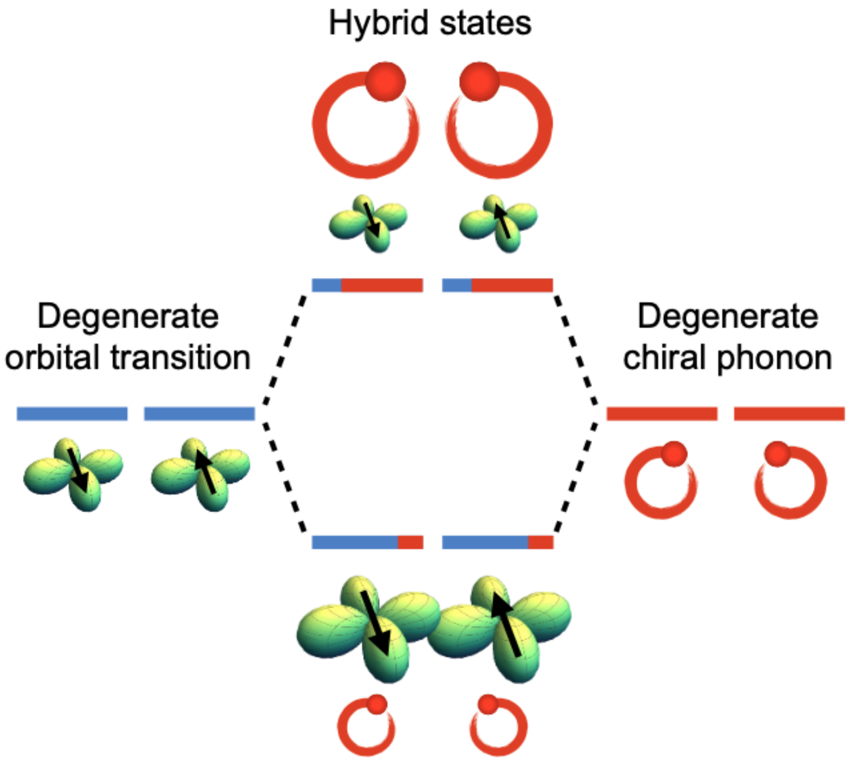
$$H_{el-ph} = (u_x + iu_y)|\psi_1\rangle\langle\psi_2| + (u_x - iu_y)|\widetilde{\psi_1}\rangle\langle\widetilde{\psi_2}| + h.c$$



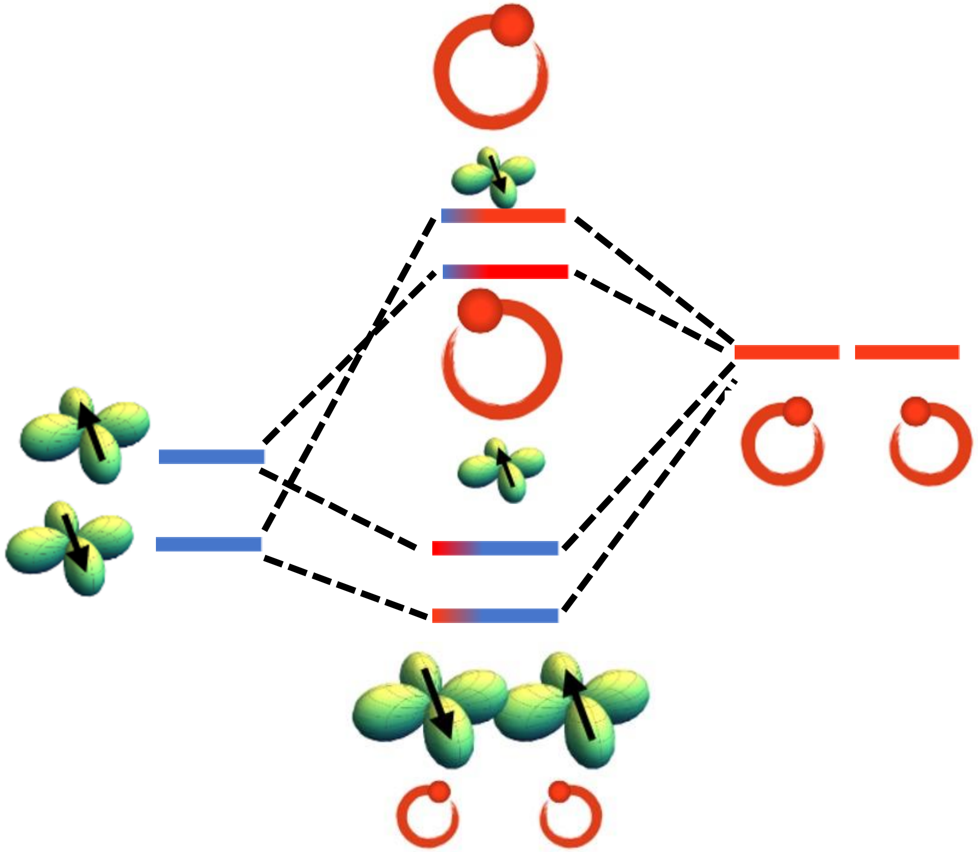
S. Chaudhary, D. Juraschek, M. Rodriguez-Vega, & G. A. Fiete arXiv:2306.11630

Splitting of chiral phonons

$B = 0$

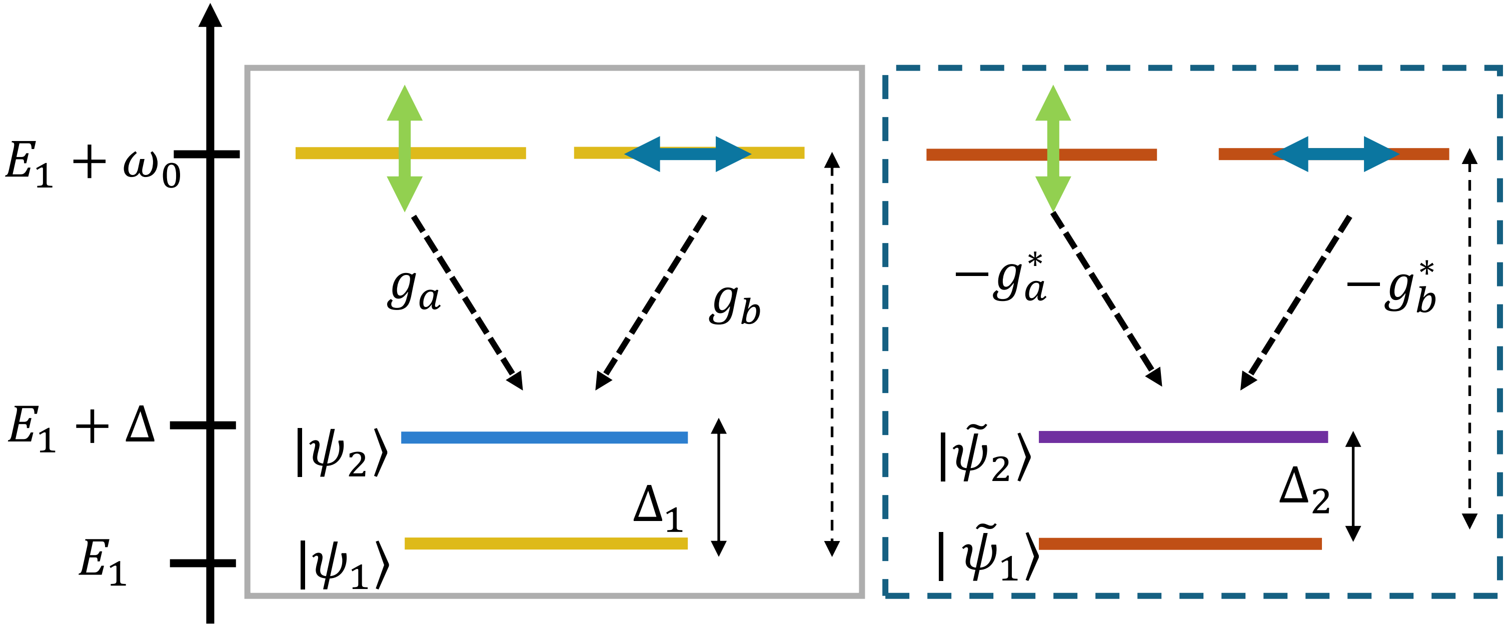


$B \neq 0$

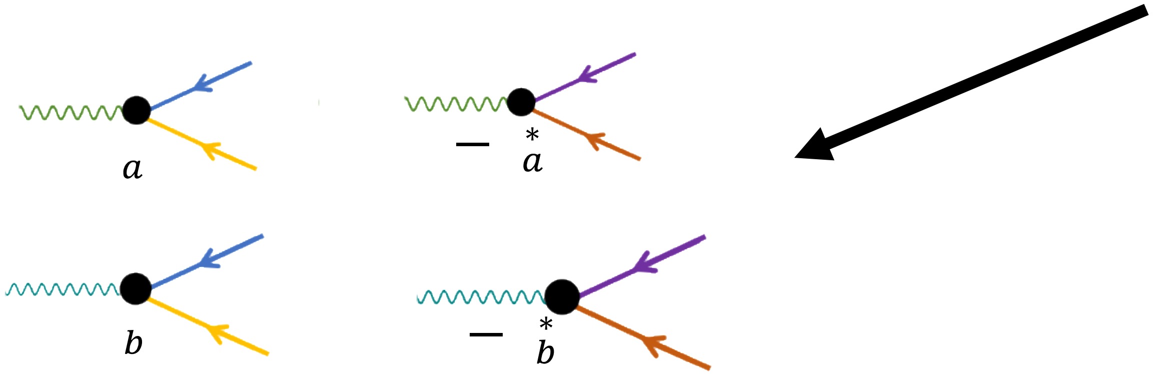


S. Chaudhary, D. Juraschek, M. Rodriguez-Vega, & G. A. Fiete arXiv:2306.11630

Details of model for phonon magnetic moment



Time-reversal copy



Phonons

$$H_{ph} = \omega_0 a^\dagger a + \omega_0 b^\dagger b$$

Electron-phonon interaction

$$H_{el-ph} = (a^\dagger + a)\widehat{\mathbf{O}}_a + (b^\dagger + b)\widehat{\mathbf{O}}_b$$

$$\hat{O}_a = g_a|\psi_1\rangle\langle\psi_2| - g_a^*|\psi_{\tilde{1}}\rangle\langle\psi_{\tilde{2}}| + \text{h.c}$$

$$\hat{O}_b = g_b|\psi_1\rangle\langle\psi_2| - g_b^*|\psi_{\tilde{1}}\rangle\langle\psi_{\tilde{2}}| + \text{h.c}$$

P. Thalmeier and P. Fulde, Zeitschrift fur Physik B Condensed Matter 26, 323 (1977)

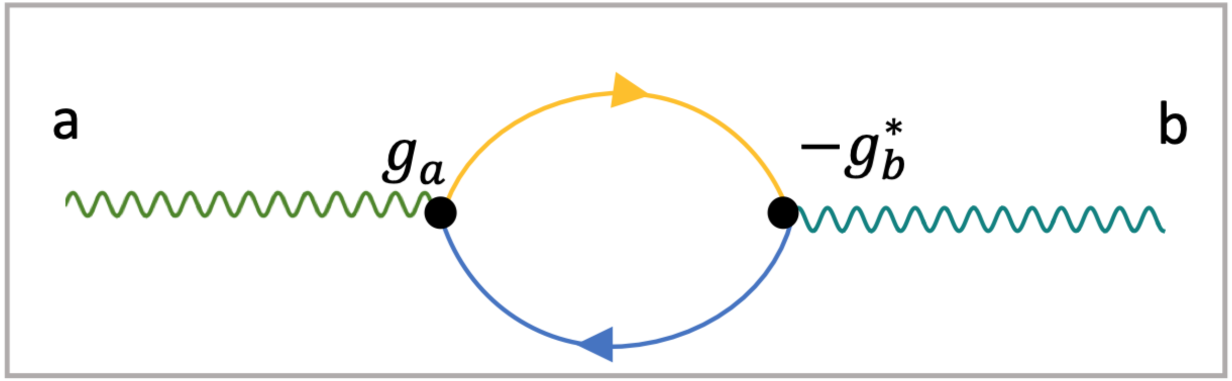
Phonon Green's function

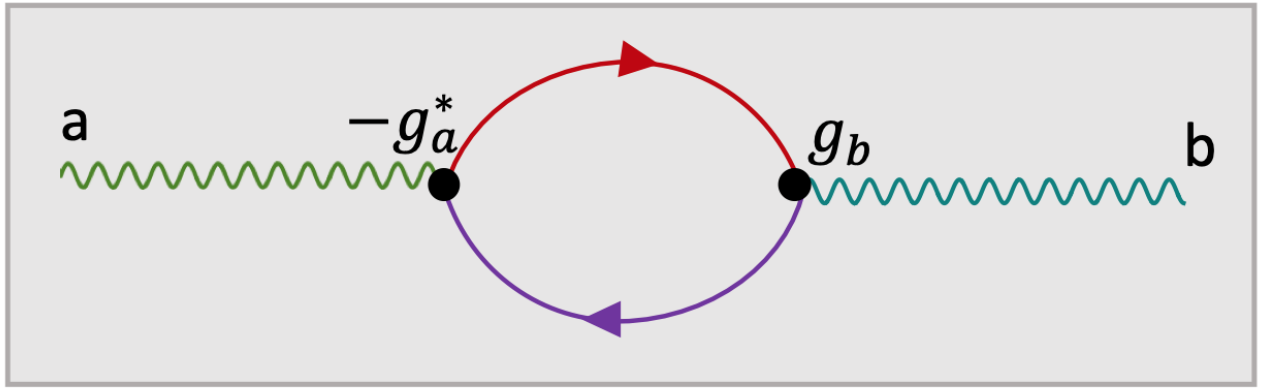
Non-interacting case

$$\widehat{\mathbf{D}}_{\mathbf{0}}(\omega) = \begin{pmatrix} D_0^{aa}(\omega) & 0 \\ 0 & D_0^{bb}(\omega) \end{pmatrix}$$

$$D_0^{aa}(\omega) = D_0^{bb}(\omega) = \frac{2\omega_0}{\omega^2 - \omega_0^2}$$

Corrections





P. Thalmeier and P. Fulde, Zeitschrift fur Physik B Condensed Matter 26, 323 (1977)

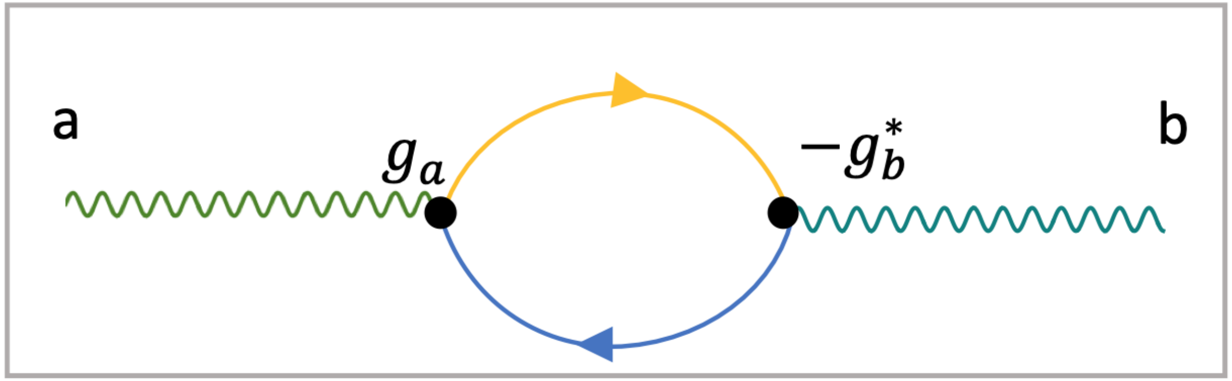
Phonon Green's function

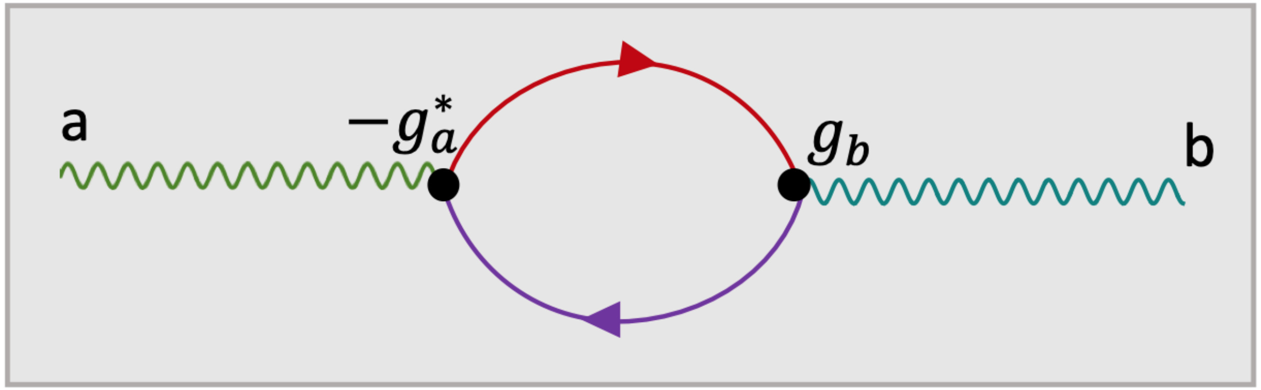
e-ph interactions included

$$\mathbf{D}^{-1} = \mathbf{D}_0^{-1} - \mathbf{\Pi}(q,\omega)$$

$$\Pi^{ab} \propto \frac{f_1 g_a g_b^*}{\omega - \Delta_1} - \frac{f_1 g_a^* g_b}{\omega + \Delta_1}$$

$$+ \frac{f_{\tilde{1}} g_a^* g_b}{\omega - \Delta_2} - \frac{f_{\tilde{1}} g_a g_b^*}{\omega + \Delta_2}$$





Splitting occurs if

1. $g_a g_b^* = i g^2$
2. $f_1 \neq f_{\tilde{1}}$ or $\Delta_1 \neq \Delta_2$

P. Thalmeier and P. Fulde, Zeitschrift fur Physik B Condensed Matter 26, 323 (1977)

Phonon energies and eigenmodes

Splitting occurs if

1. $g_a g_b^* = i g^2$
2. $f_1 \neq f_{\tilde{1}}$ or $\Delta_1 \neq \Delta_2$

Broken TRS

$$\Delta_1 = \Delta - \gamma B,$$
$$\Delta_2 = \Delta + \gamma B$$
$$f_1 - f_{\tilde{1}} = \beta B$$

Det($D^{-1}(\omega)$)=0

$$\frac{\omega_{ph}^{+} - \omega_{ph}^{-}}{\omega_{ph}(B=0)} = 2\frac{\gamma(\Omega_{+}^{2} - \omega_0^2)/\omega_0 + \tilde{g}\beta}{\sqrt{(\omega_0^2 - \Delta^2)^2 + 8\tilde{g} f_0 \omega_0 \Delta}} B + O(B^2)$$

New phonon modes

$$\phi_+ = \frac{1}{\sqrt{2}}(\phi_a - i\phi_b)$$
$$\phi_- = \frac{1}{\sqrt{2}}(\phi_a + i\phi_b)$$

Chiral

$$\tilde{g} = 4\pi g^2$$

P. Thalmeier and P. Fulde, Zeitschrift fur Physik B Condensed Matter 26, 323 (1977)

Phonon energies and eigenmodes

Splitting occurs if

1. $g_a g_b^* = i g^2$
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Broken TRS

$\Delta_1 = \Delta - \gamma B,$
$\Delta_2 = \Delta + \gamma B$
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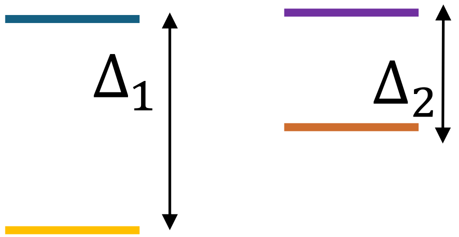


1. **Energy scales: Δ and ω_0**
2. **Electronic ‘g’ factor**
3. **Temperature**
4. **Electron -phonon coupling**

$\tilde{g} = 4\pi g^2$

P. Thalmeier and P. Fulde, Zeitschrift fur Physik B Condensed Matter 26, 323 (1977)

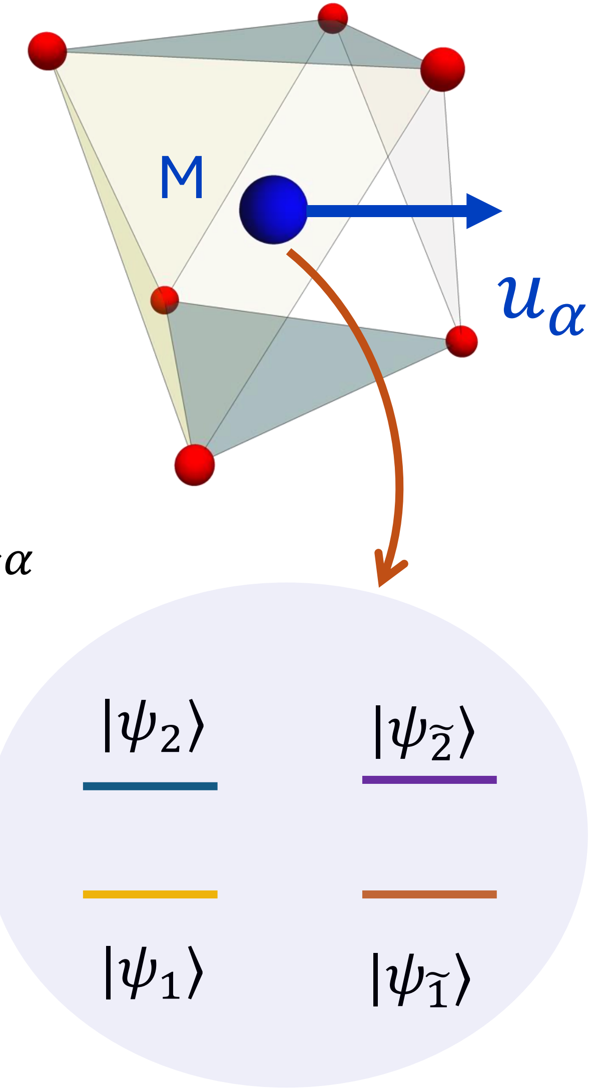
e-ph coupling strength using point charge model

$$H_{el-ph} = (a^\dagger + a)\hat{\mathbf{O}}_a + (b^\dagger + b)\hat{\mathbf{O}}_b$$

$$\hat{O}_a = g_a|\psi_1\rangle\langle\psi_2| - g_a^*|\psi_{\tilde{1}}\rangle\langle\psi_{\tilde{2}}|$$

$$\hat{O}_b = g_b|\psi_1\rangle\langle\psi_2| - g_b^*|\psi_{\tilde{1}}\rangle\langle\psi_{\tilde{2}}|$$

$$u_a = \frac{\hbar}{\sqrt{M\hbar\omega_{ph}}}(a + a^\dagger)$$



Modified crystal electric field due to phonon

$$V_M(u_\alpha) = [\alpha_1 xy + \alpha_2 yz + \alpha_3 xz + \alpha_4(x^2 - y^2) + \alpha_5(3z^2 - x^2 - y^2)]u_\alpha$$

$\langle\psi_i|r_\alpha r_\beta|\psi_j\rangle$ → $|\psi_i\rangle = |J_i, m_j^i\rangle$ → $\begin{cases} m_l & \text{Orbital} \\ m_s & \text{Spin} \end{cases}$

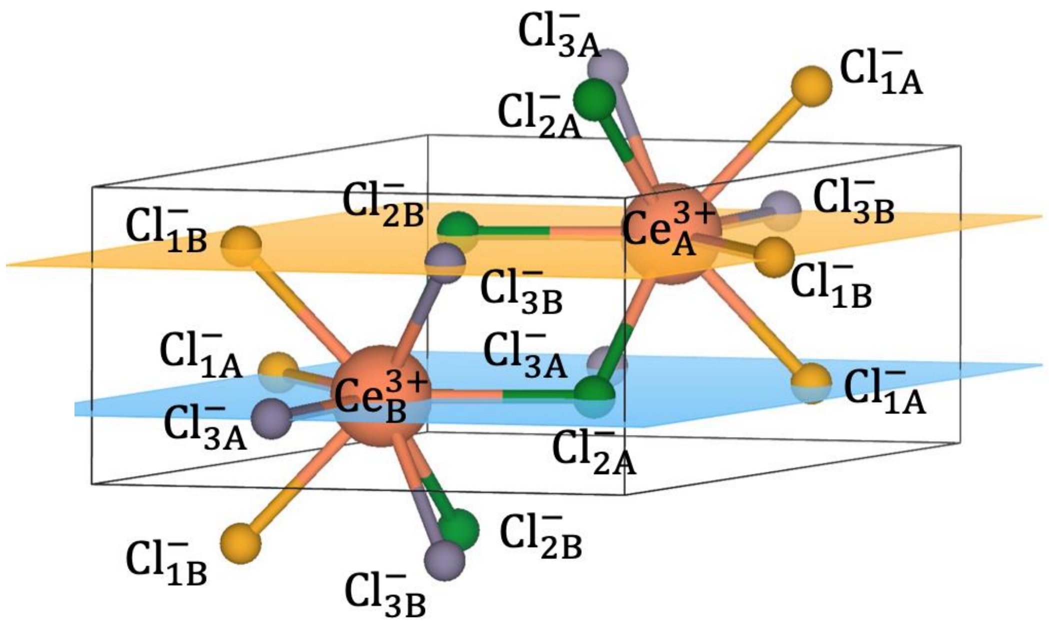
S. Chaudhary, D. Juraschek, M. Rodriguez-Vega, & G. A. Fiete arXiv:2306.11630

Model applied to Rare earth trihalide $CeCl_3$

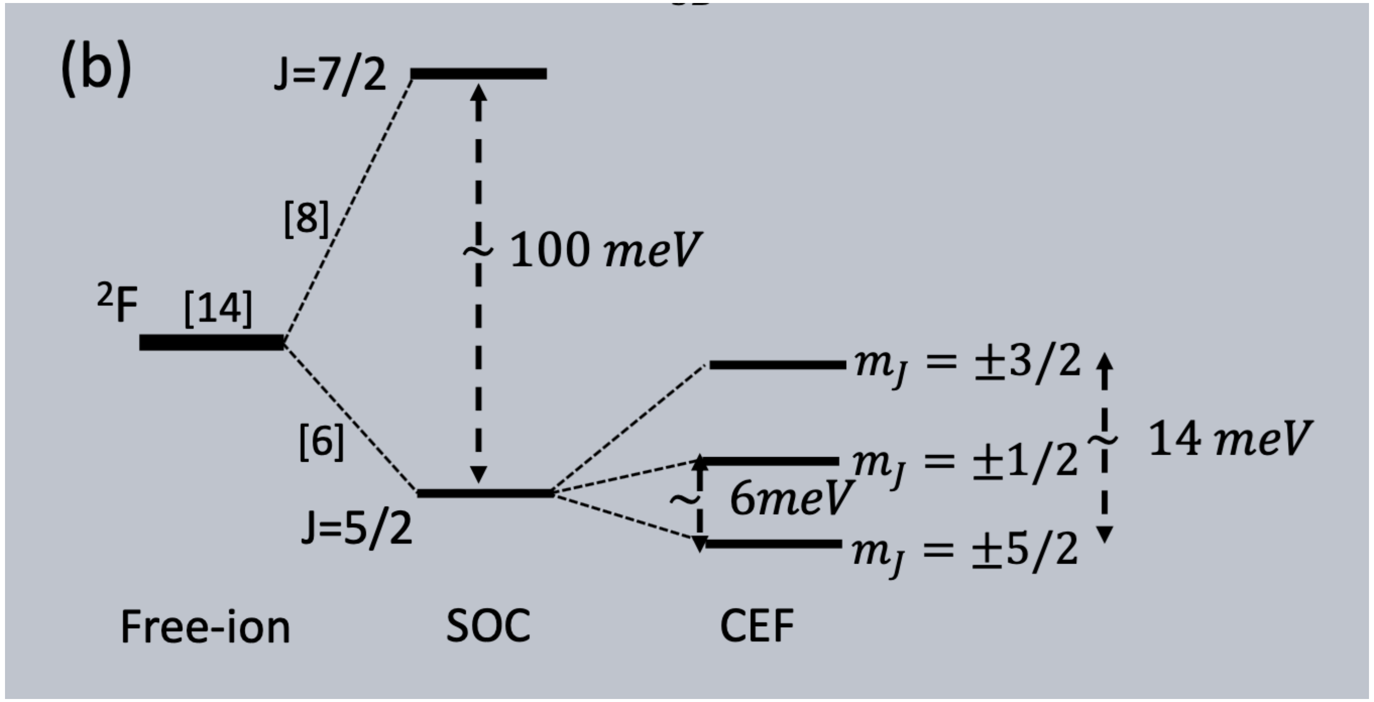
Properties of rare-earth paramagnet $CeCl_3$

$Ce^{3+}: 4f^1$

1. Strong SOC, $\lambda \approx 80\ meV$
2. Weak Crystal-electric fields (CEF)



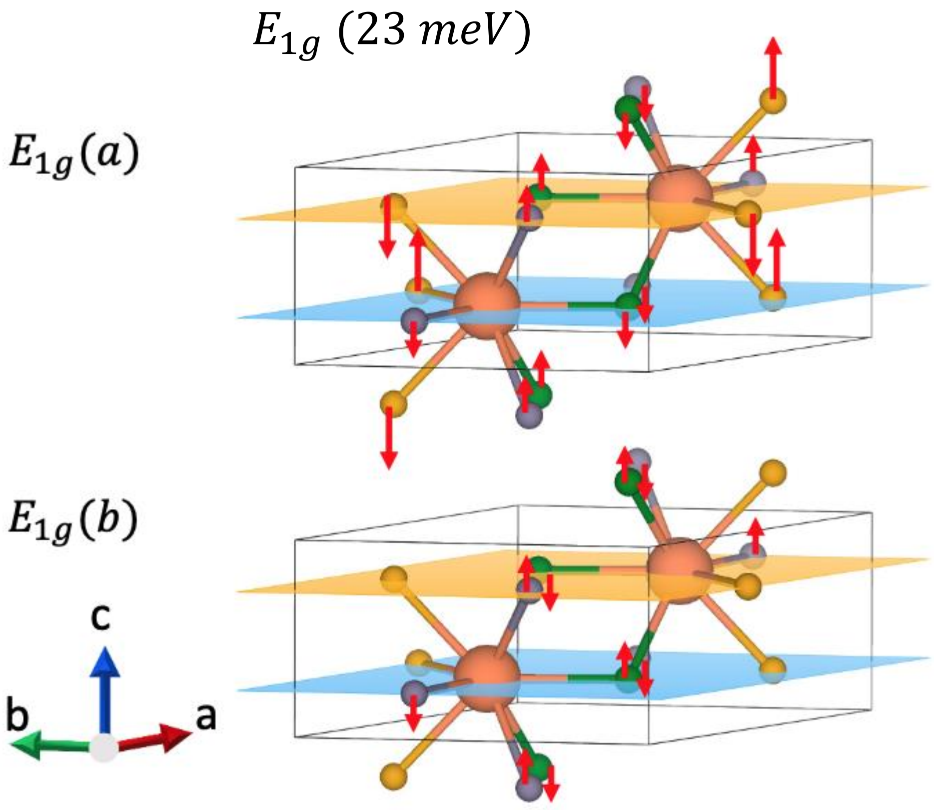
Crystal structure



Electronic states

S. Chaudhary, D. Juraschek, M. Rodriguez-Vega, & G. A. Fiete arXiv:2306.11630

Orbital lattice coupling in $CeCl_3$

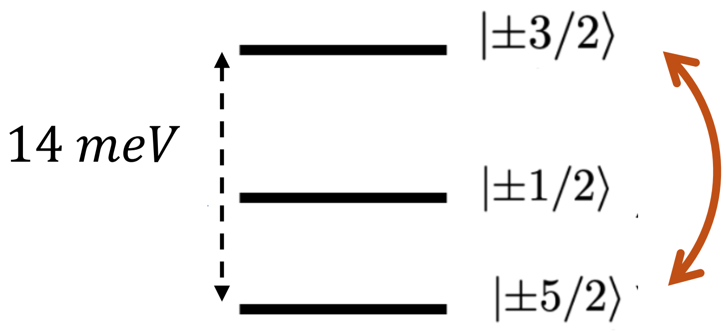


Perturbation to crystal electric field around Ce^{3+} ion

$$V(E_{1g}(a)) = [-0.06\,xz + 0.16\,yz]\;q$$
$$V(E_{1g}(b)) = [0.16\,xz + 0.06\,yz]\;q$$

$$H_1(xz) = -\frac{2}{7\sqrt{5}}\langle r^2\rangle \begin{pmatrix} & \left|\frac{5}{2},\pm\frac{5}{2}\right\rangle & \left|\frac{5}{2},\pm\frac{3}{2}\right\rangle \\ \left|\frac{5}{2}\pm\frac{5}{2}\right\rangle & 0 & \pm 1 \\ \left|\frac{5}{2},\pm\frac{3}{2}\right\rangle & \pm 1 & 0 \end{pmatrix}$$

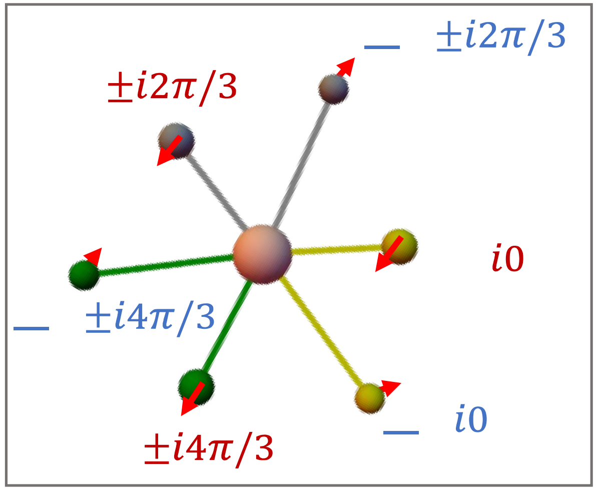
$$H_1(yz) = \frac{2}{7\sqrt{5}}\langle r^2\rangle \begin{pmatrix} & \left|\frac{5}{2},\pm\frac{5}{2}\right\rangle & \left|\frac{5}{2},\pm\frac{3}{2}\right\rangle \\ \left|\frac{5}{2},\pm\frac{5}{2}\right\rangle & 0 & i \\ \left|\frac{5}{2},\pm\frac{3}{2}\right\rangle & -i & 0 \end{pmatrix},$$



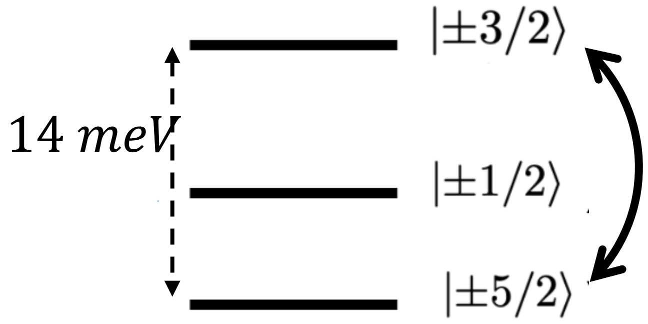
Perturbation to CEFs around Ce^{3+} from phonons in $CeCl_3$

$\square_{1g}$ $\square$23$\square$)

$_{1g}$( ) ± $\square_{1g}$( )

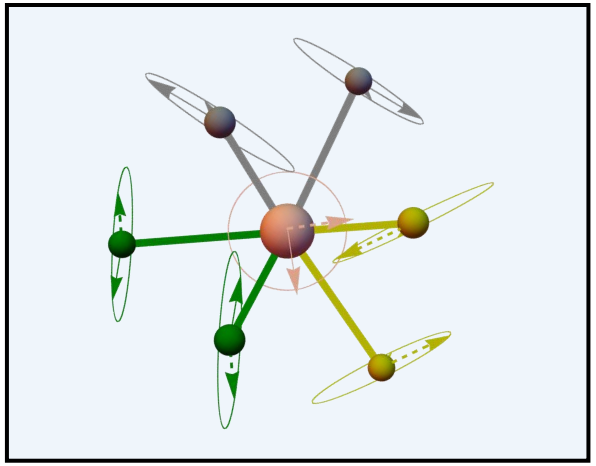


$$V(E_{1g}(a)) = [-0.06\,xz + 0.16\,yz]\; q$$
$$V(E_{1g}(b)) = [0.16\,xz + 0.06\,yz]\; q$$

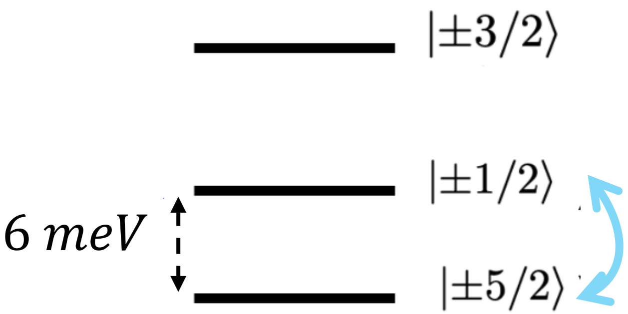


E^1_{2g} (12 meV)

$E^1_{2g}(a) \pm i\, E^1_{2g}(b)$

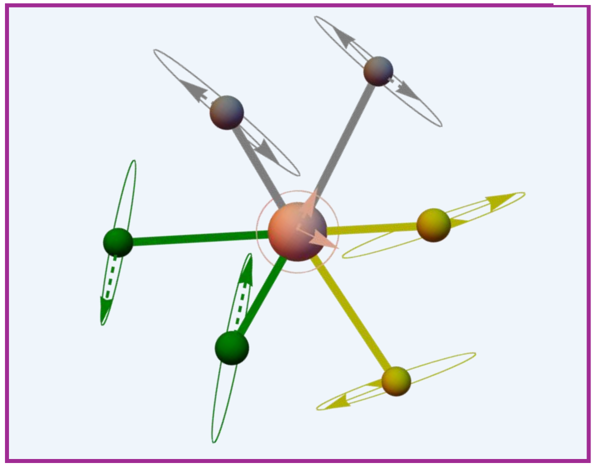


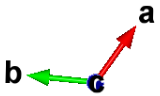
$$V(E^1_{2g}(a)) = [-0.05\,xy - 0.007\,(x^2 - y^2)]\; q$$
$$V(E^1_{2g}(b)) = [0.014\,xy - 0.025\,(x^2 - y^2)]\; q$$



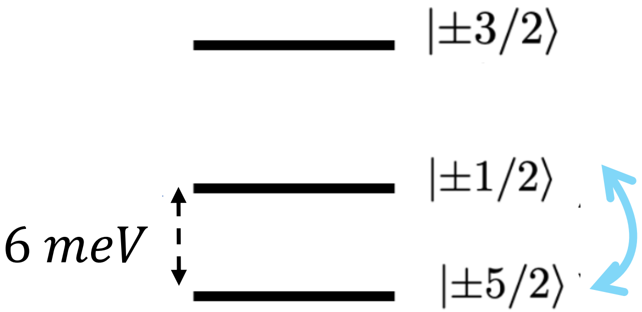
E^2_{2g} (21.5 meV)

$E^2_{2g}(a) \pm i\, E^2_{2g}(b)$

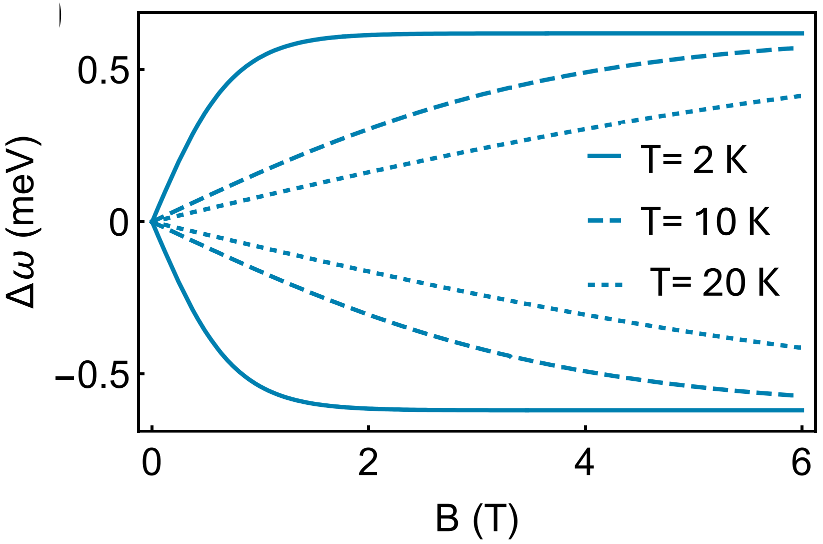




$$V(E^2_{2g}(a)) = [0.08\,xy + 0.01\,(x^2 - y^2)]\; q$$
$$V(E^2_{2g}(b)) = [-0.02\,xy + 0.04\,(x^2 - y^2)]\; q$$



Zeeman splitting and Phonon Magnetic moment in $CeCl_3$



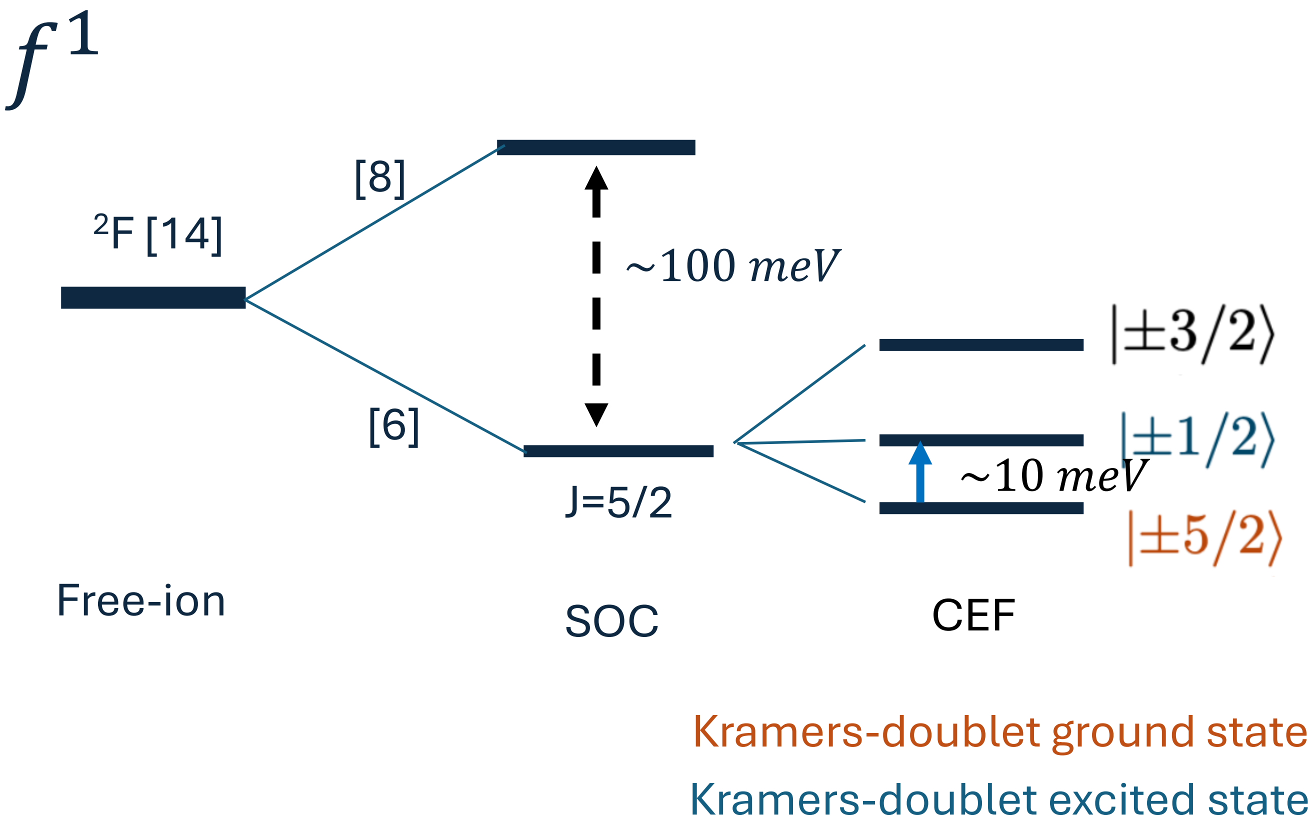
Phonon	μ_{ph} (μ_B) 10 K	$\mu_{ph}(\mu_B)$ 20 K
E_{1g} (22meV)	2	1
E_{2g}^{1}(12 meV)	0.5	0.3
E_{2g}^{2} (21.5 meV)	0.2	0.1

S. Chaudhary, D. Juraschek, M. Rodriguez-Vega, & G. A. Fiete arXiv:2306.11630

Electronic excitations on magnetic ions

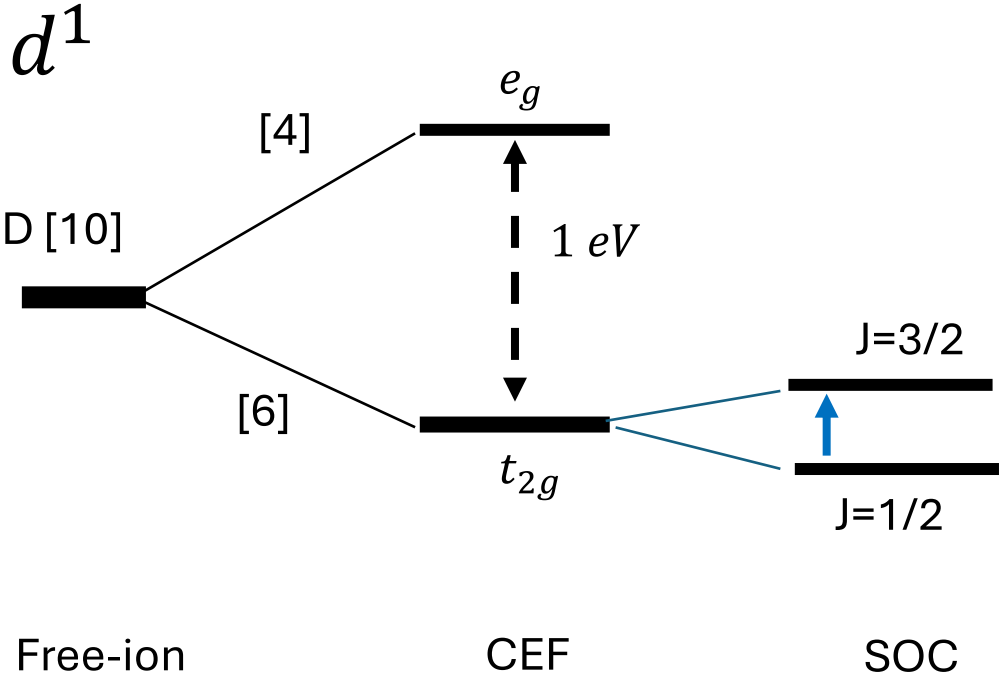
f - electrons

1. Strong SOC
2. Weak CEF effects



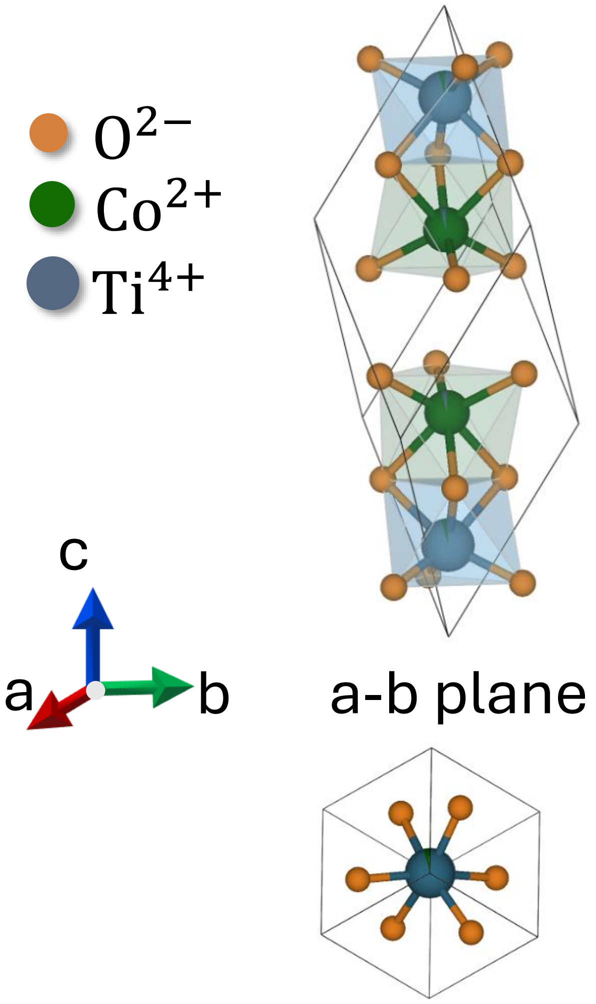
d - electrons

1. Weak SOC
2. Very strong. CEF effects (~1eV)

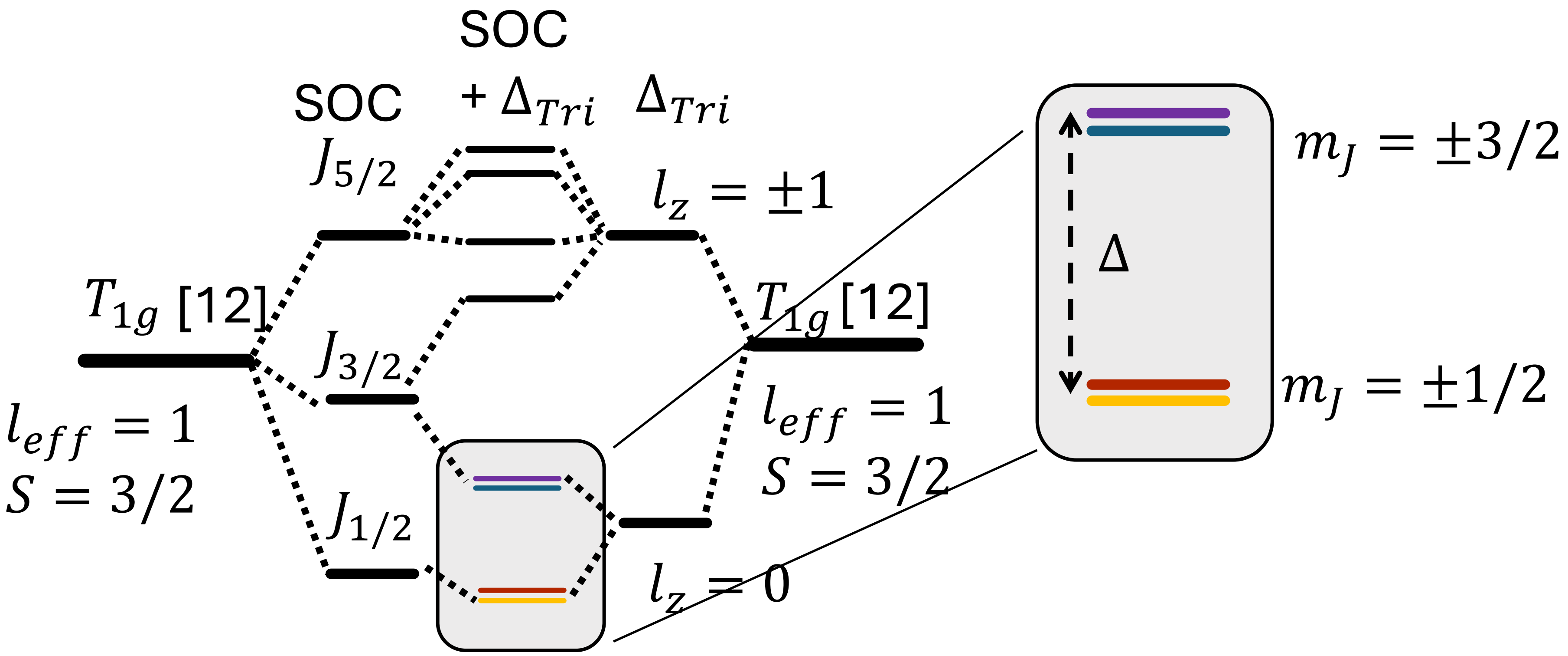


Model applied to a *d* orbital magnet

$CoTiO_3$ physical and electronic properties



Crystal Structure

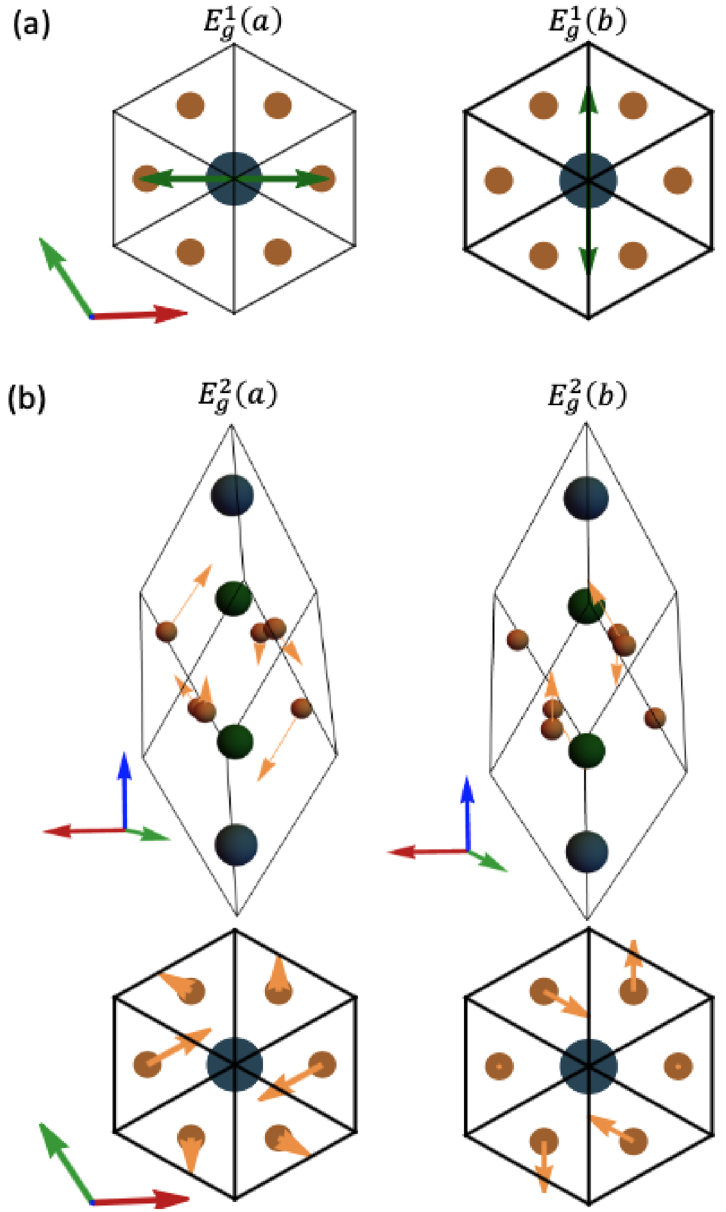


Co^{3+} electronic states (Three holes in 3d orbitals)

Low-energy Kramers doublets

Nature Communications 12.1 (2021): 3936.

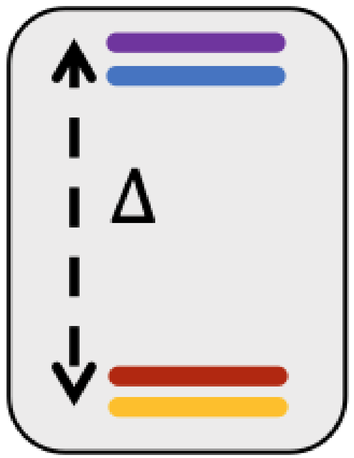
$CoTiO_3$ E_g phonon modes



26 meV

33 meV

$T > T_N$

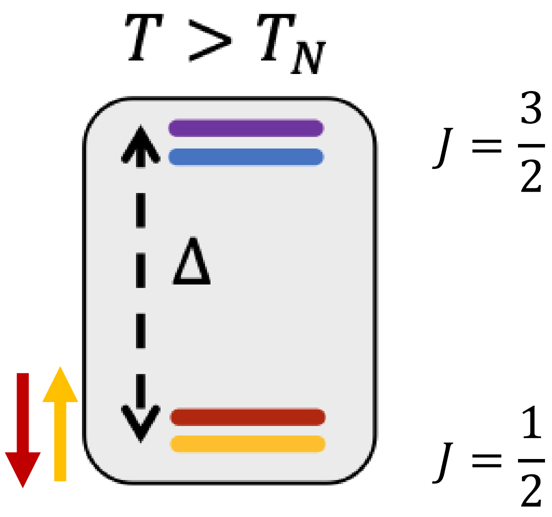


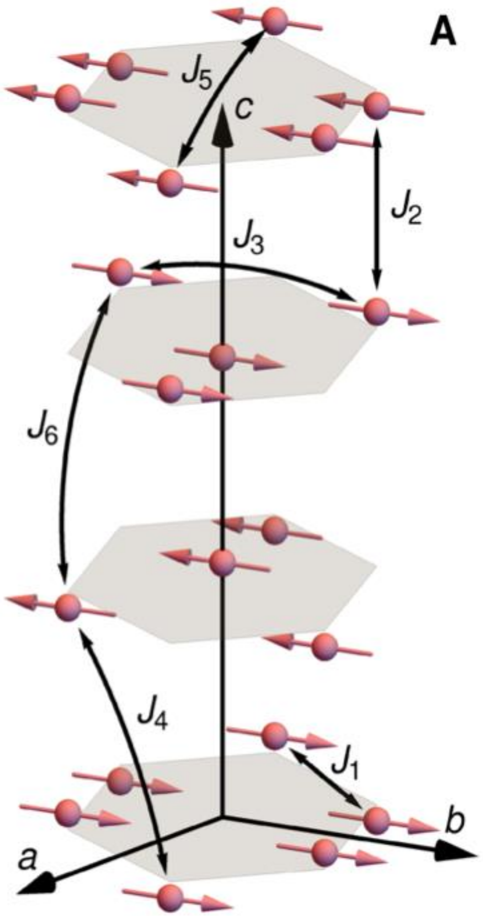
$\Delta = 23\ meV$

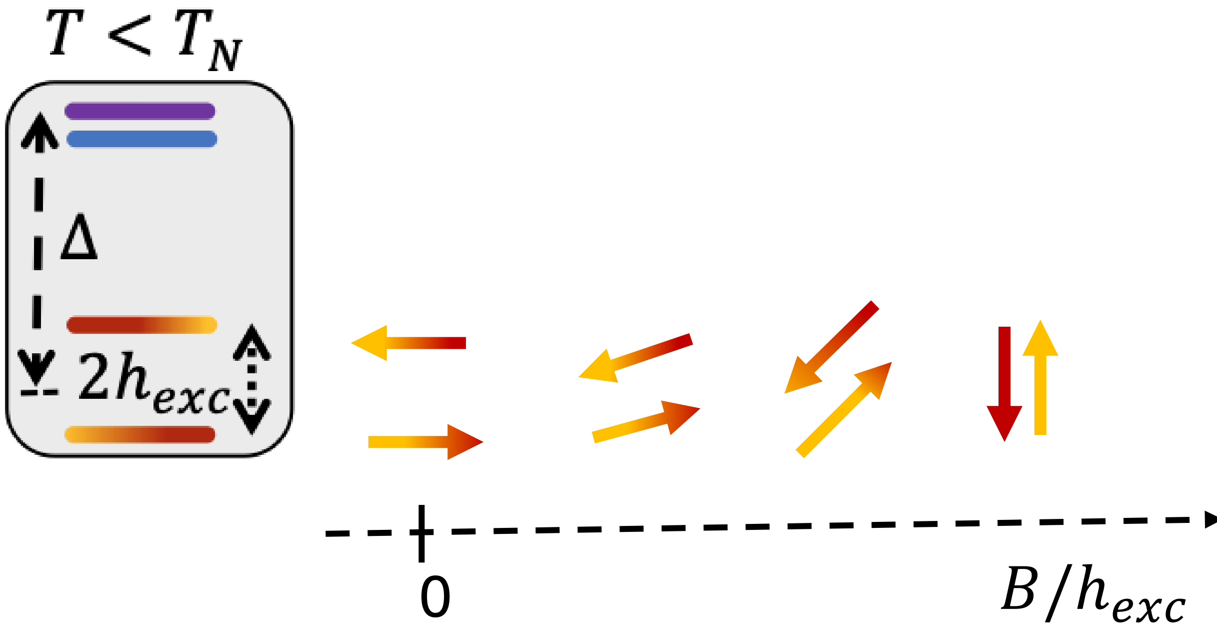
Phys. Rev. B **102**, 134404 (2020)

Nature Communications 12.1 (2021): 3936.

$CoTiO_3$ magnetic properties







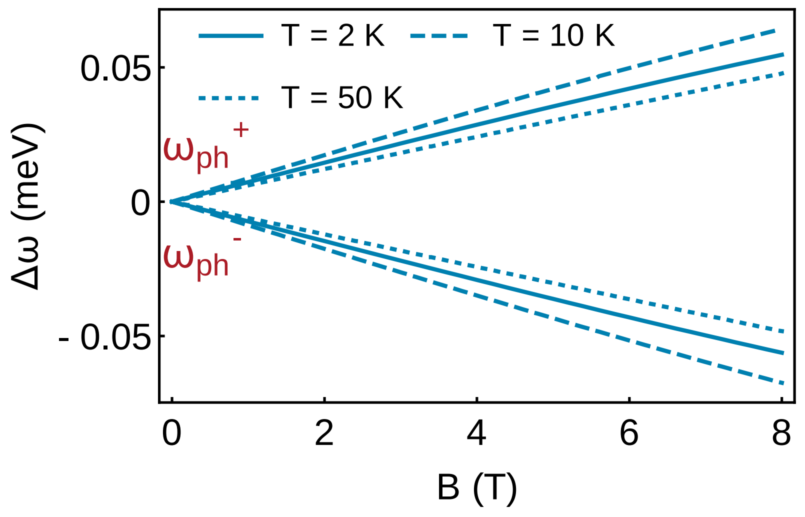
↑ $\left|J = \frac{1}{2}, m_j = -\frac{1}{2}\right\rangle$

↓ $\left|J = \frac{1}{2}, m_j = +\frac{1}{2}\right\rangle$

→ $\frac{1}{\sqrt{2}}\left|J = \frac{1}{2}, m_j = -\frac{1}{2}\right\rangle + \frac{1}{\sqrt{2}}\left|J = \frac{1}{2}, m_j = -\frac{1}{2}\right\rangle$

← $\frac{1}{\sqrt{2}}\left|J = \frac{1}{2}, m_j = -\frac{1}{2}\right\rangle - \frac{1}{\sqrt{2}}\left|J = \frac{1}{2}, m_j = -\frac{1}{2}\right\rangle$

$CoTiO_3$ E_g modes phonon magnetic moment

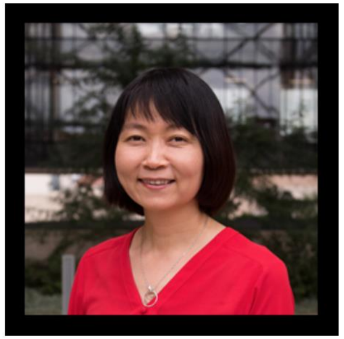


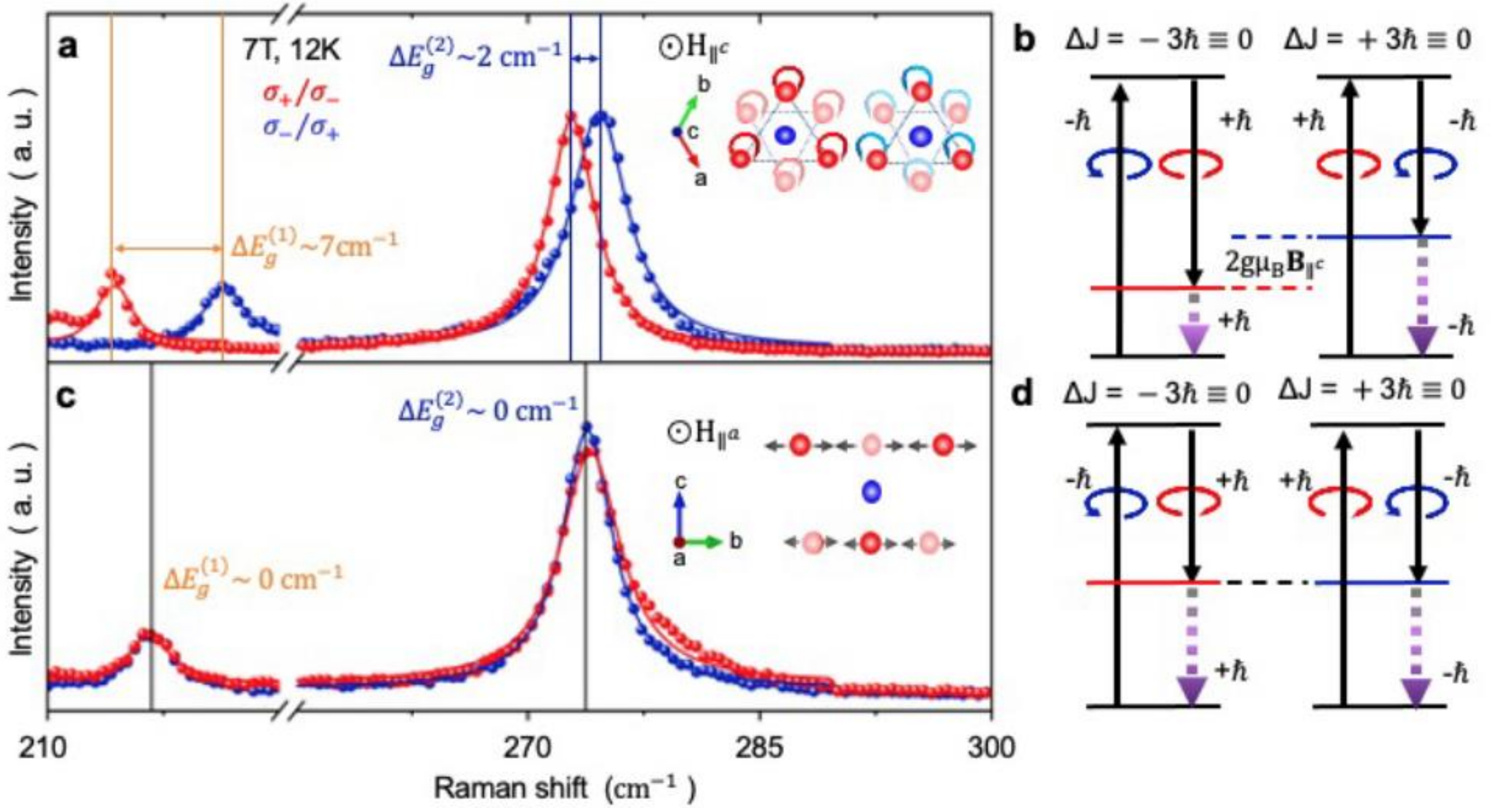
Phonon	μ_{ph} (μ_B) 50 K	$\mu_{ph}(\mu_B)$ 100 K
E_g^1 (22meV)	0.2	0.1
E_g^2 (21.5 meV)	0.12	0.6

- No saturation in the given B field limit
- T trend similar to magnetic susceptibility

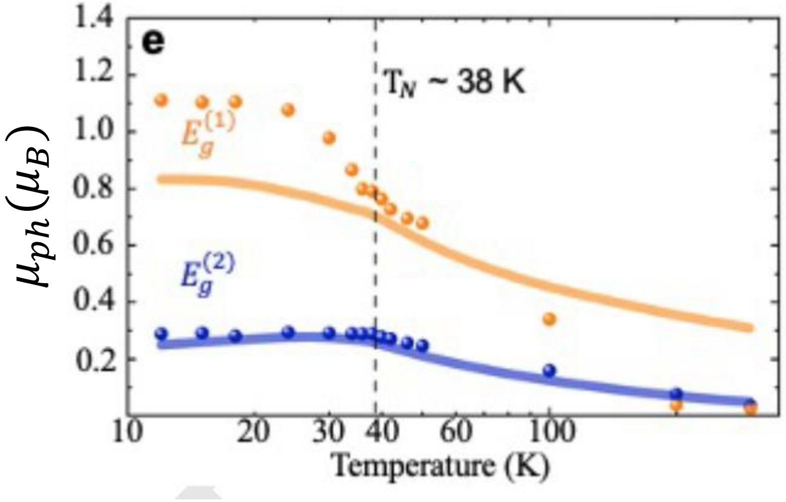
Results from helicity-resolved magneto-Raman

Prof. Xiaoqin (Elaine) Li
UT Austin





Cross-circular channel Raman spectra taken with a magnetic field applied along different crystalline axes

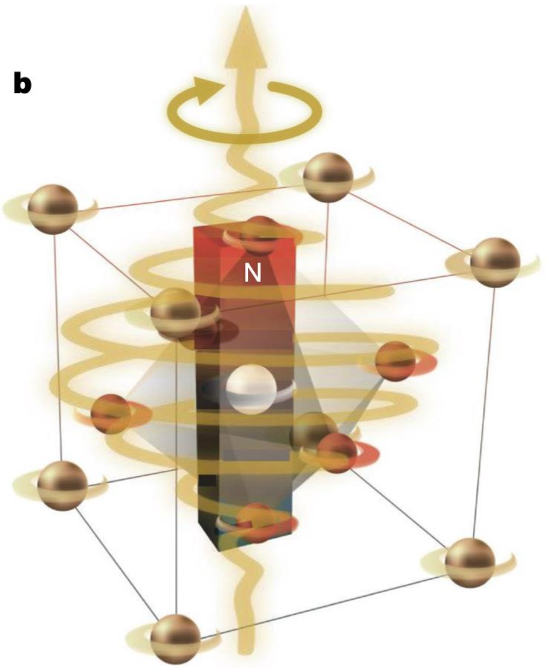


Phonon Magnetic moment as a function of temperature

D. Lujan*, J. Choe*, **S. Chaudhary***, G. Fiete, R. He, & Xiaoqin Li *et al., Spin-Orbit Exciton-Induced Phonon Chirality in a Quantum Magnet* PNAS, 121(11), e2304360121 (2024)

Applications: Mediator between light and magnetism

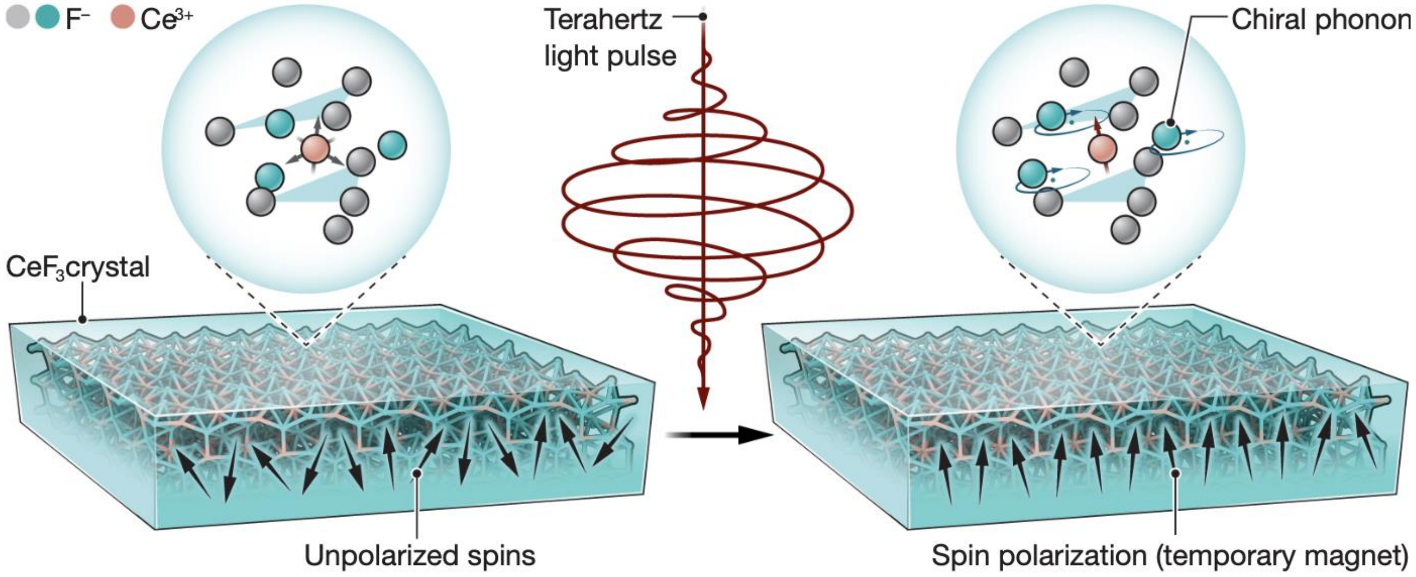
Phonon Barnett Effect



M. Basini et.al, Nature **628**, 534–539 (2024)

Davies *et.al*, Nature **628**, pages 540–544 (2024)

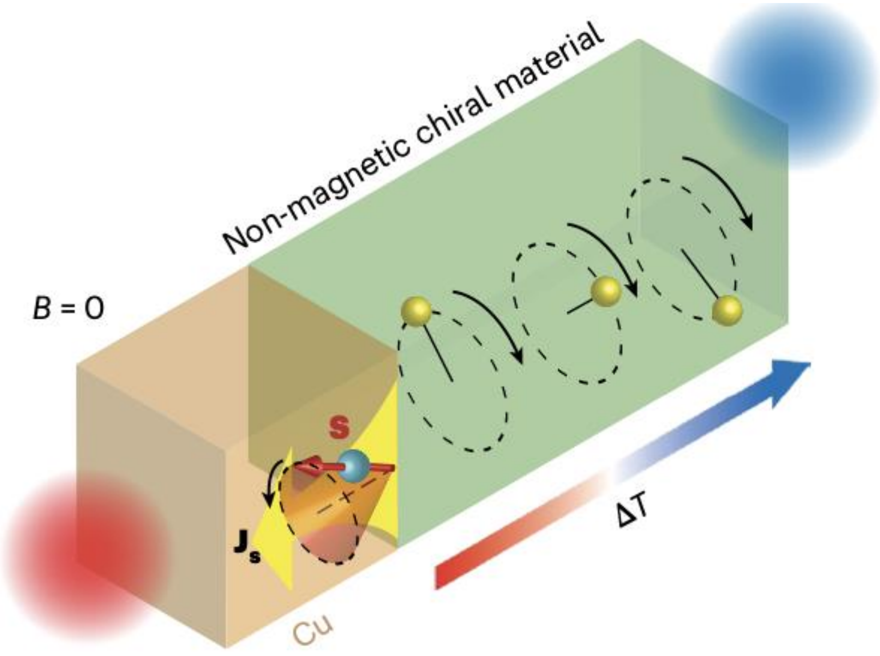
Giant effective magnetic fields from chiral phonons



Luo et al., Science 382, 698–702 (2023)

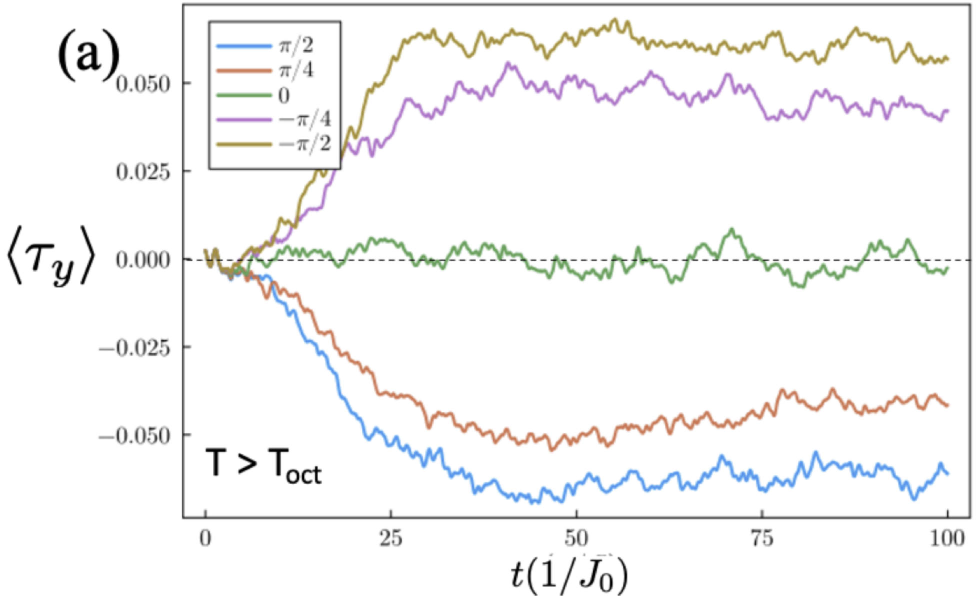
Applications: A new chapter in magnetism

Chiral phonon activated spin Seebeck effect



K. Kim, *et al*. Nature Materials **22**, 322–328 (2023)

Chiral phonon-trained octupolar order



K. Hart, A. Paramkanti, *et al*.
arXiv: 2404.17633

Summary and Outlook

1. Microscopic model for phonon magnetic moment based on orbital-lattice coupling
2. Estimate of phonon magnetic moments in different classes of materials

1. Other coupling mechanisms for phonon chirality:
   - Magnons?
   - Itinerant bands?

2. Beyond Gamma phonons - consequences for band topology?

3. Possible applications -Consequences for phonon linewidths?
   - Angular momentum conservation restricts scattering
   - Scattering rates tunable with magnetic field
   - Phonon thermal hall effect

Acknowledgement

Gregory A. Fiete

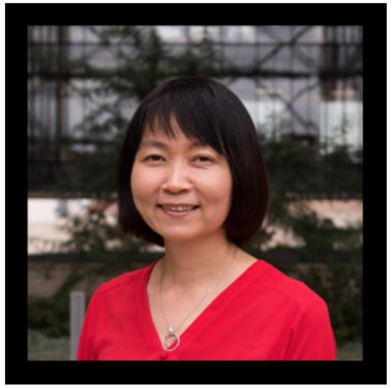
Northeastern University, Boston

Dominik Juraschek

Tel Aviv University

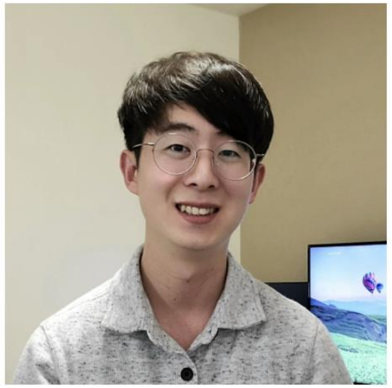
Martin Rodriguez-Vega

UT Austin



Xiaoqin (Elaine) Li

UT Austin



Jeongheon Choe

UT Austin

David Lujan

UT Austin

Thank you for your attention!

Center for Dynamics and Control of Materials: an NSF MRSEC

https://mrsec.utexas.edu

Group Theory of chiral phonons

1. E_g, E_u phonons

2. TRS breaking E irrep splits into two 1D irreps with complex basis functions

3. Axial vector along C_n axis allowed by symmetry

C_3 ✓

C_3	E	C_3	$(C_3)^2$	linear functions, rotations	quadratic functions
A	+1	+1	+1	z, R_z	x^2+y^2, z^2
E	+1 +1	+ε +ε*	+ε* +ε	x+iy; R_x+iR_y x-iy; R_x-iR_y	(x^2-y^2, xy) (yz, xz)

C_{3h} ✓

C_{3h}	E	C_3(z)	$(C_3)^2$	σ_h	S_3	$(S_3)^5$	linear functions, rotations	quadratic functions
A'	+1	+1	+1	+1	+1	+1	R_z	x^2+y^2, z^2
E'	+1 +1	+ε +ε*	+ε* +ε	+1 +1	+ε +ε*	+ε* +ε	x+iy x-iy	(x^2-y^2, xy)
A''	+1	+1	+1	-1	-1	-1	z	-
E''	+1 +1	+ε +ε*	+ε* +ε	-1 -1	-ε -ε*	-ε* -ε	R_x+iR_y R_x-iR_y	(xz, yz)

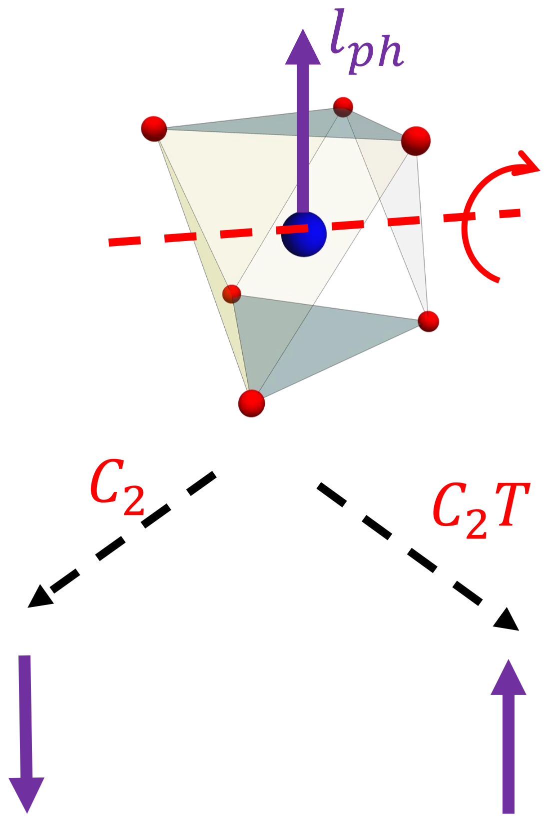
C_{3v} ✗

C_{3v}	E	$2C_3$ (z)	$3\sigma_v$	linear functions, rotations	quadratic functions
A_1	+1	+1	+1	z	x^2+y^2, z^2
A_2	+1	+1	-1	R_z	-
E	+2	-1	0	(x, y) (R_x, R_y)	(x^2-y^2, xy) (xz, yz)

Magnetic point groups for Zone-centered chiral phonons

Magnetic point group number	notation
9.1.29	4
10.1.32	$\bar{4}$
11.1.35	$4/m$
12.4.43	$42'2'$
14.5.52	$\bar{4}2'm'$
15.6.58	$4/mm'm'$
16.1.60	3
17.1.62	$\bar{3}$
18.3.67	$32'$
19.3.70	$3m'$
20.1.71	$\bar{3}m$

Magnetic point group number	notation
20.5.75	$\bar{3}'m'$
21.1.76	6
22.1.79	$\bar{6}$
23.1.82	$6/m$
24.4.90	$62'2'$
25.4.94	$6m'm'$
26.5.99	$\bar{6}m'2'$
27.5.104	$6'/m'mm'$
27.6.105	$6/mm'm'$
29.1.109	$m\bar{3}$
32.4.121	$m\bar{3}m'$



$CoTiO_3$

$$\begin{pmatrix} \mu_{el}^{gd} B_z^{\alpha} & h_{ex}(T) \\ h_{ex}(T) & -\mu_{el}^{gd} B_z^{\alpha} \end{pmatrix} |\psi_{\tilde{1}/\tilde{2}}^{\alpha}\rangle = E_{\tilde{1}/\tilde{2}} |\psi_{\tilde{1}/\tilde{2}}^{\alpha}\rangle$$

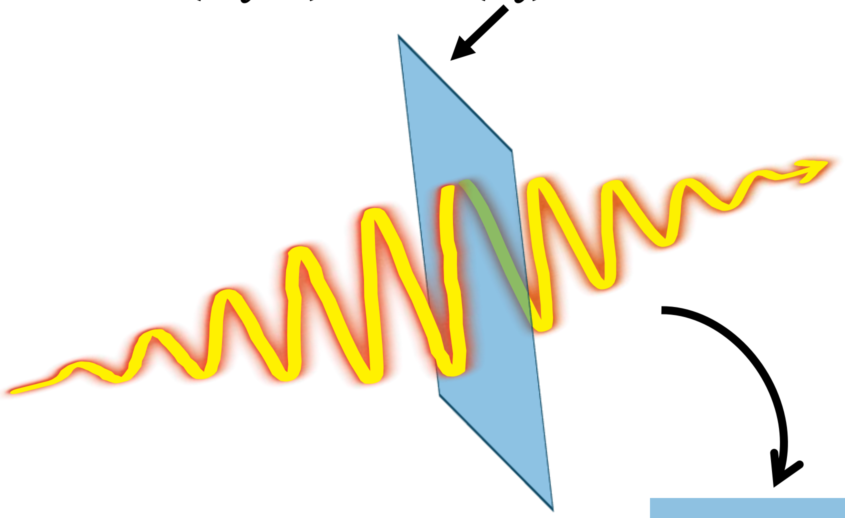
$$\begin{aligned} &\mathbf{D}^{-1}|_{\alpha\alpha} = \\ &\frac{\omega^2 - \omega_0^2}{2\omega_0} - 2\tilde{g} \left(\frac{f_{\tilde{1}} E_{\tilde{1}3} \left(\cos\frac{\theta}{2}\right)^2}{\omega^2 - E_{\tilde{1}3}^2} + \frac{f_{\tilde{1}} E_{\tilde{1}4} \left(\sin\frac{\theta}{2}\right)^2}{\omega^2 - E_{\tilde{1}4}^2} \right) \\ &- 2\tilde{g} \left(\frac{f_{\tilde{2}} E_{\tilde{2}3} \left(\sin\frac{\theta}{2}\right)^2}{\omega^2 - E_{\tilde{2}3}^2} + \frac{f_1 E_{24} \left(\cos\frac{\theta}{2}\right)^2}{\omega^2 - E_{\tilde{2}4}^2} \right), \end{aligned} \tag{69}$$

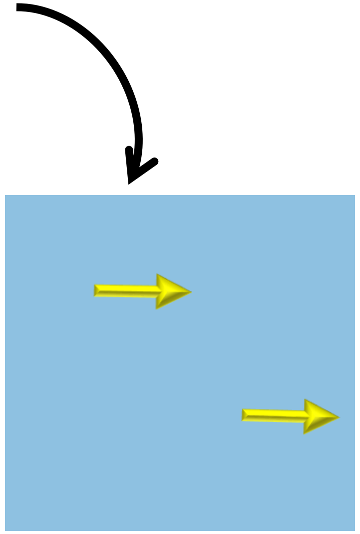
$$\begin{aligned} &\mathbf{D}^{-1}|_{ab} = -\mathbf{D}^{-1}|_{ba} = \\ &2i\tilde{g} \left(-\frac{f_{\tilde{1}}\omega \left(\cos\frac{\theta}{2}\right)^2}{\omega^2 - E_{\tilde{1}3}^2} + \frac{f_{\tilde{1}}\omega \left(\sin\frac{\theta}{2}\right)^2}{\omega^2 - E_{\tilde{1}4}^2} \right) \\ &+ 2i\tilde{g} \left(-\frac{f_{\tilde{2}}\omega \left(\sin\frac{\theta}{2}\right)^2}{\omega^2 - E_{\tilde{2}3}^2} + \frac{f_{\tilde{2}}\omega \left(\cos\frac{\theta}{2}\right)^2}{\omega^2 - E_{\tilde{2}4}^2} \right) \end{aligned}$$

Angular momentum of photons

Spin Angular momentum

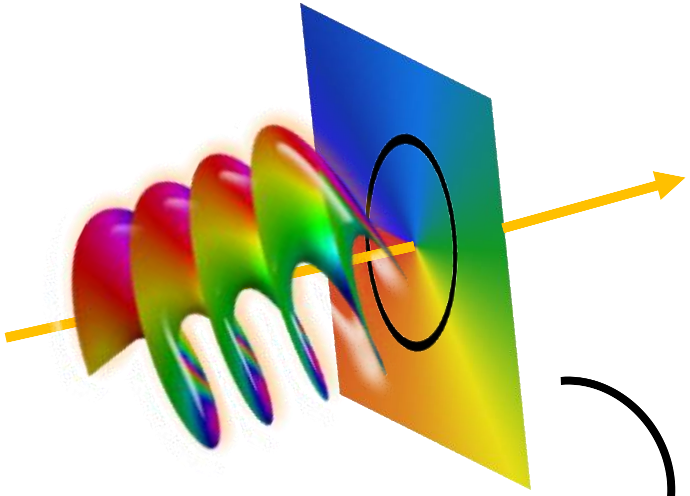
$$\boldsymbol{E}(\boldsymbol{r}_i, t) = E(r_i)e^{i\theta_i}e^{i\omega t}\ \hat{n}$$

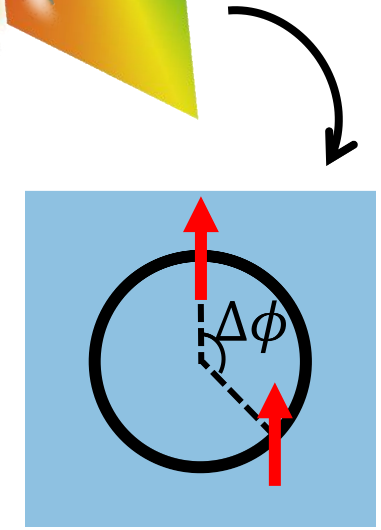




$$\theta_1 - \theta_2 = 0$$
$$\hat{n} = \hat{x} + i\ \hat{y}$$
$$l^{s}_{photon} = \pm\hbar$$

Orbital Angular momentum



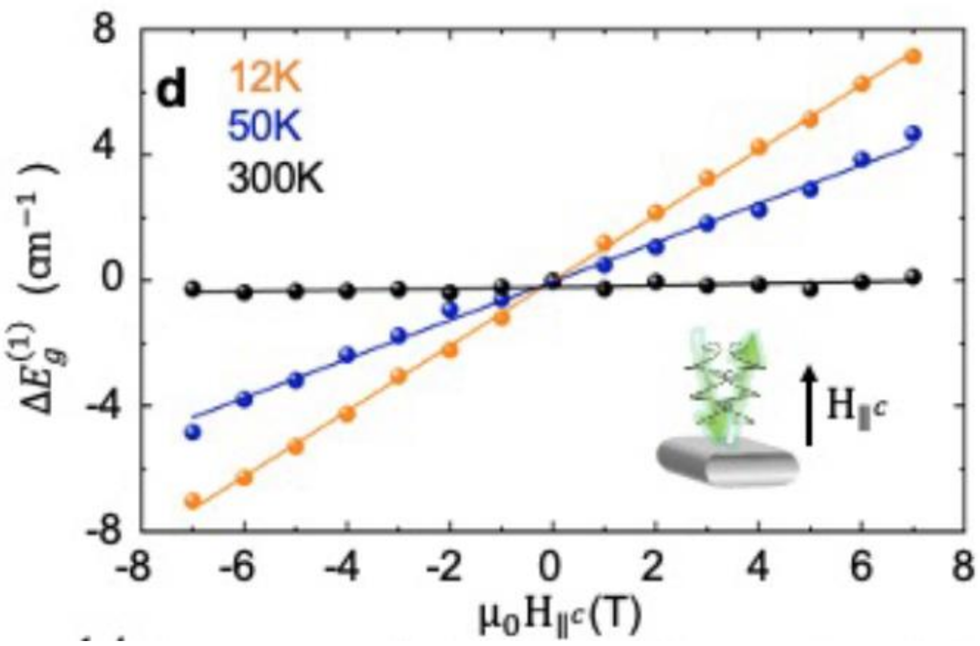


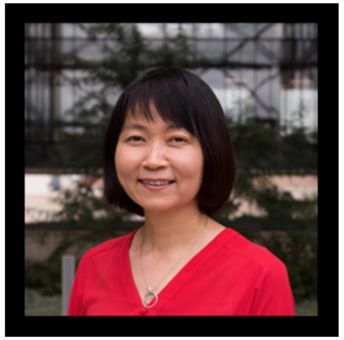
$$\theta_1 - \theta_2 = l\Delta\phi$$
$$\hat{n} = \hat{y}$$
$$l^{orb}_{photon} = \pm l\hbar$$
$$l = 1,2,3, \ldots .$$

Phys. Rev. A **45**, 8185 (1992)

Chiral phonons with giant magnetic moment in $CoTiO_3$

Zeeman splitting of E_g^1 phonon





Prof. Xiaoqin (Elaine) Li

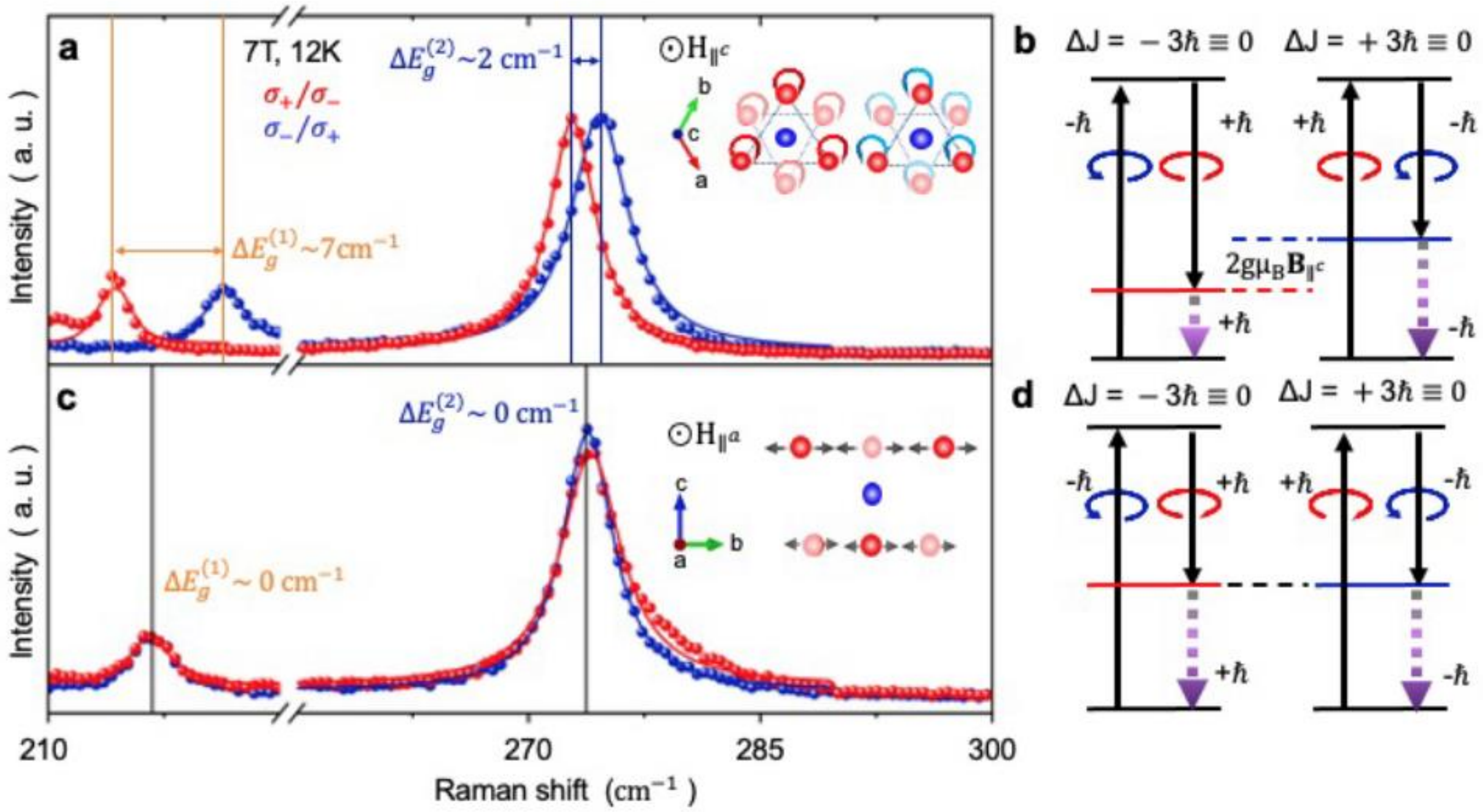
TEXAS
The University of Texas at Austin

1. Phonon magnetic moment $\boldsymbol{\mu_{ph} \approx \mu_B}$

2. First such example in a ‘**d**’ orbital quantum magnet

David Lujan, Jeongheon Choe, **Swati Chaudhary**, *et.al,* Spin-Orbit Exciton-Induced Phonon Chirality in a Quantum Magnet (under review)

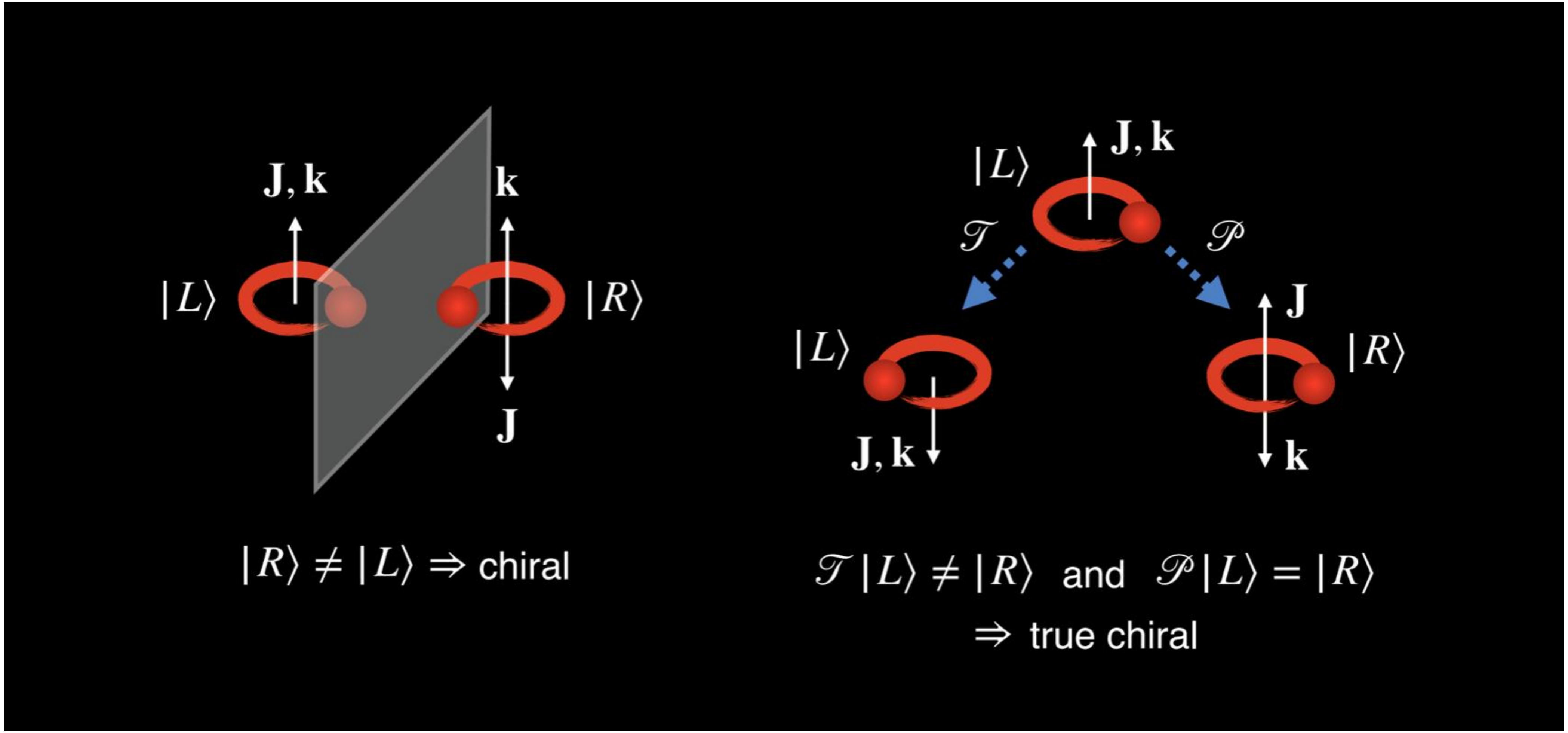
Results from helicity resolved magneto Raman spectroscopy



Exp. Data from Li Lab, UT Austin

Cross-circular channel Raman spectra taken with a magnetic field applied along different crystalline axes

Barron criteria: True chirality



Phonon angular momentum estimate: Quantization

Classical Picture

Phonon displacement :$\boldsymbol{u} = (\boldsymbol{u}^1, \boldsymbol{u}^2, \dots)$

Normal coordinates :

$$\mathbf{Q} = (\boldsymbol{u}^1\sqrt{m^1}, \boldsymbol{u}^2\sqrt{m^2}, \dots)$$

Circularly polarized phonons :

$$\boldsymbol{Q} = Q \sin \omega t \;\; \widehat{\boldsymbol{x}} + Q \cos \omega t \; \widehat{\boldsymbol{y}}$$

Angular Momentum:

$$\boldsymbol{L} = \boldsymbol{Q} \times \partial_t \boldsymbol{Q} \qquad \boldsymbol{L} = \omega Q^2 \;\; \hat{z}$$

Classical harmonic vibrational energy per unit cell :

$$E_{classical} = \frac{1}{2}\dot{Q}^2 + \frac{1}{2}\omega^2 Q^2 = \omega^2 Q^2$$

$$N_{phonon} = \frac{E_{classical}}{\hbar\omega} = \frac{\omega Q^2}{\hbar}$$

$$\boldsymbol{L} = \hbar N_{phonon} \; \hat{z}$$

Quantum Picture

$$\hat{Q}_{x/y} = \sqrt{\frac{\hbar}{\omega}}(a^+_{x/y} + a_{x/y})$$
$$\dot{\hat{Q}}_{x/y} = i\sqrt{\hbar\omega}(a^+_{x/y} - a_{x/y})$$

$$\widehat{\boldsymbol{L}} = \widehat{\boldsymbol{Q}} \times \dot{\widehat{\boldsymbol{Q}}} \equiv i\hbar(a^+_x a_y - a^+_y a_x)$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(a^+_x \pm i a^+_y)|0\rangle$$

$$\widehat{\boldsymbol{L}}|\pm\rangle = \pm\hbar|\pm\rangle$$

$$R^z\left(\frac{2\pi}{3}\right)\mathbf{q}_{\mathbf{k}\lambda} = e^{-i(2\pi/3)l^k_{ph}}\mathbf{q}_{\mathbf{k}\lambda}$$

D. Juraschek, N. Spaldin, Phys. Rev. Mat. 3, 064405 (2019)

Anharmonic effects

Temperature dependence of Optical phonon frequencies

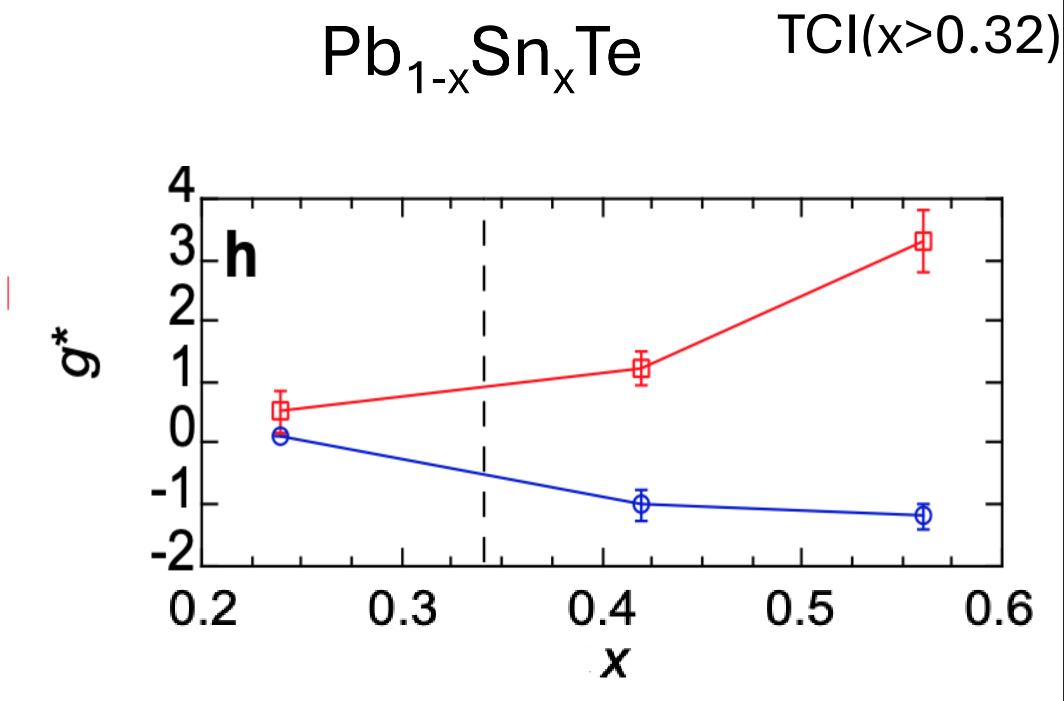
Effects dictated by three phonon scattering processes

$$\omega_p(T) = \omega_p(0) - A\left(1 + \frac{2}{\exp[x] - 1}\right)$$

Temperature dependence of Inverse Lifetime

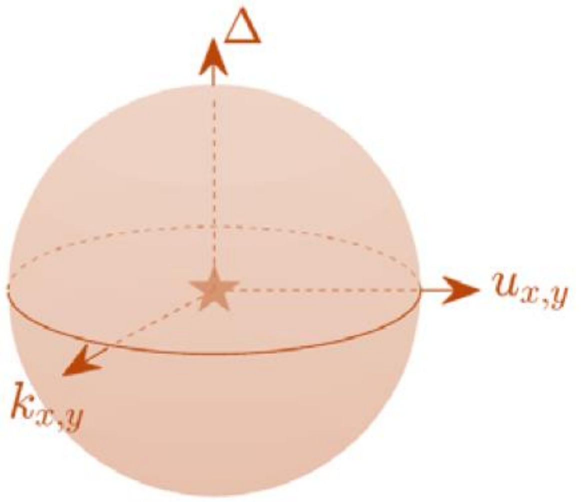
PHYSICAL REVIEW B **104**, L020401 (2021) , Martin's paper on Spin-phonon interaction in Yttrium Garnet

Phonon Magnetic moment from band topology in a TCI



$g_{ph} \sim 1$

arXiv: 2208.12235



$$M_z = \frac{e}{2m_I} L_I \int \frac{dk}{(2\pi)^2} \Omega_{k_\alpha k_\beta u_x u_y}$$

Second Chern form

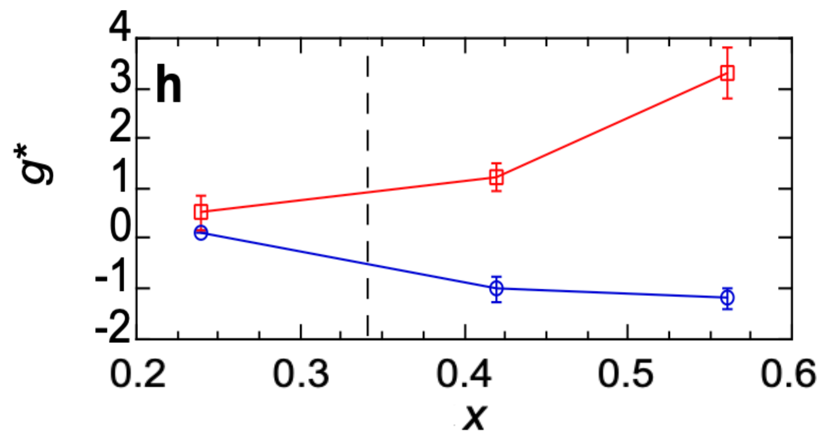
$$\Omega_{k_x k_y u_x u_y} = \Omega_{k_x u_y}\Omega_{k_y u_x} - \Omega_{k_x u_x}\Omega_{k_y u_y} + \Omega_{k_x k_y}\Omega_{u_x u_y}$$

F. G. G. Hernandez, A. Baydin**, S. Chaudhary**, F. Tay, & G. A. Fiete, et.al *Chiral Phonons with Giant Magnetic Moments in a Topological Crystalline Insulator*, arXiv:2208.12235 (under review in Science Advances)

Phy. Rev. Lett. **127**, 186403 (2021)

Phonon Magnetic moment from band topology in a TCI

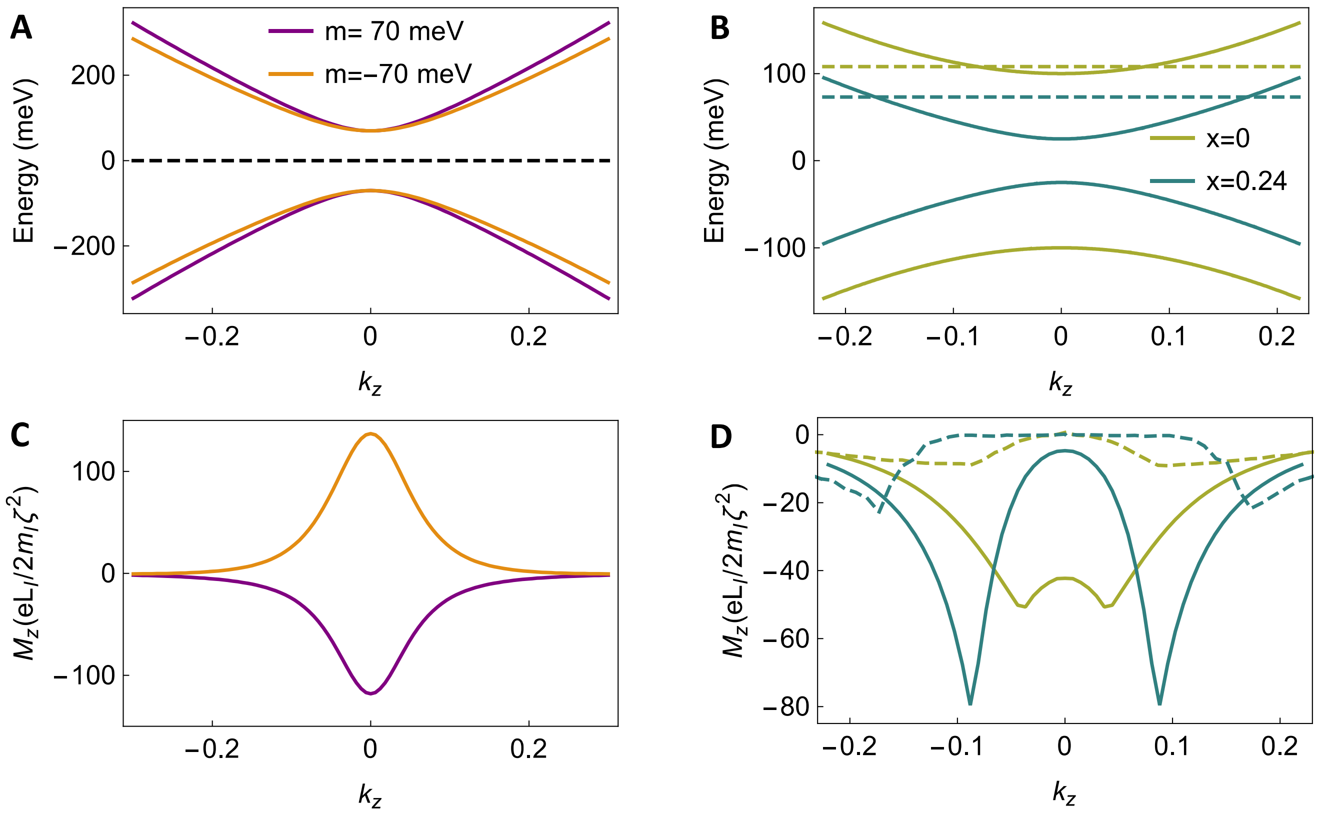
$Pb_{1-x}Sn_xTe$



$g_{ph} \sim 1$

$$H_0 = (m + ck_3^2)\sigma_z + v(k_1 s_y - k_2 s_x)\sigma_x + v_3 k_3 \sigma_y;$$

$$H_{ph} = \zeta\,(u_x s_0 \sigma_x - u_y s_z \sigma_y)$$



F. G. G. Hernandez, A. Baydin, **S. Chaudhary**, F. Tay, & G. A. Fiete, et.al *Chiral Phonons with Giant Magnetic Moments in a Topological Crystalline Insulator*, arXiv:2208.12235 (under review in Science Advances)