The Mystery and Magic of Component Tableaux.

Y. Fittouhi and A. Joseph

DMRT, Bengaluru, India ,2023

November 24, 2023

1. The Main Players

Take $G = GL(n)$, P a parabolic subgroup

1. The Main Players

Take $G = GL(n)$, P a parabolic subgroup and $\mathfrak{m}$ the nilradical of its Lie algebra $\mathfrak{p}$.

1. The Main Players

Take $G = GL(n)$, P a parabolic subgroup
and $\mathfrak{m}$ the nilradical of its Lie algebra $\mathfrak{p}$.
A theorem of Richardson implies that $\mathbb{C}[\mathfrak{m}]^{P'}$ is polynomial.

1. The Main Players

Take $G = GL(n)$, P a parabolic subgroup
and $\mathfrak{m}$ the nilradical of its Lie algebra $\mathfrak{p}$.
A theorem of Richardson implies that $\mathbb{C}[\mathfrak{m}]^{P'}$ is polynomial.
Generators were written down by Benlolo-Sanderson.

1. The Main Players

Take $G = GL(n)$, P a parabolic subgroup
and $\mathfrak{m}$ the nilradical of its Lie algebra $\mathfrak{p}$.
A theorem of Richardson implies that $\mathbb{C}[\mathfrak{m}]^{P'}$ is polynomial.
Generators were written down by Benlolo-Sanderson.
They are restrictions of truncations of the determinant.

1. The Main Players

Take $G = GL(n)$, P a parabolic subgroup
and $\mathfrak{m}$ the nilradical of its Lie algebra $\mathfrak{p}$.
A theorem of Richardson implies that $\mathbb{C}[\mathfrak{m}]^{P'}$ is polynomial.
Generators were written down by Benlolo-Sanderson.
They are restrictions of truncations of the determinant. Their simultaneous zeros in $\mathfrak{m}$ is called the nilfibre $\mathscr{N}$.

2. Weierstrass Sections

Algebraically a Weierstrass section $e + V$ gives a simultaneous linearisation of the generators by restriction.

2. Weierstrass Sections

Algebraically a Weierstrass section $e + V$ gives a simultaneous linearisation of the generators by restriction.

The classical method developed from the work of Kostant.

2. Weierstrass Sections

Algebraically a Weierstrass section $e + V$ gives a simultaneous linearisation of the generators by restriction.

The classical method developed from the work of Kostant.

is to construct an "oxymoron" (e, h).

2. Weierstrass Sections

Algebraically a Weierstrass section $e + V$ gives a simultaneous linearisation of the generators by restriction.

The classical method developed from the work of Kostant.

is to construct an "oxymoron" (e, h).

Thus e is simultaneously an h eigenvector (so special)

2. Weierstrass Sections

Algebraically a Weierstrass section $e + V$ gives a simultaneous linearisation of the generators by restriction.

The classical method developed from the work of Kostant.

is to construct an "oxymoron" (e, h).

Thus e is simultaneously an h eigenvector (so special)

and regular (so run of the mill).

2. Weierstrass Sections

Algebraically a Weierstrass section $e + V$ gives a simultaneous linearisation of the generators by restriction.

The classical method developed from the work of Kostant.

is to construct an "oxymoron" (e, h).

Thus e is simultaneously an h eigenvector (so special)

and regular (so run of the mill).

Then V is an h stable complement to $\mathfrak{p}.e$ in $\mathfrak{m}$.

2. Weierstrass Sections

Algebraically a Weierstrass section $e + V$ gives a simultaneous linearisation of the generators by restriction.

The classical method developed from the work of Kostant.

is to construct an "oxymoron" (e, h).

Thus e is simultaneously an h eigenvector (so special)

and regular (so run of the mill).

Then V is an h stable complement to $\mathfrak{p}.e$ in $\mathfrak{m}$.

The appellation is intended to suggest that oxymorons are hard to find.

2. Weierstrass Sections

Algebraically a Weierstrass section $e+V$ gives a simultaneous linearisation of the generators by restriction.

The classical method developed from the work of Kostant.

is to construct an "oxymoron" (e,h).

Thus e is simultaneously an h eigenvector (so special)

and regular (so run of the mill).

Then V is an h stable complement to $\mathfrak{p}.e$ in $\mathfrak{m}$.

The appellation is intended to suggest that oxymorons are hard to find.

In the present case a regular element need not exist.

2. Weierstrass Sections

Algebraically a Weierstrass section $e + V$ gives a simultaneous linearisation of the generators by restriction.

The classical method developed from the work of Kostant.

is to construct an "oxymoron" (e, h).

Thus e is simultaneously an h eigenvector (so special)

and regular (so run of the mill).

Then V is an h stable complement to $\mathfrak{p}.e$ in $\mathfrak{m}$.

The appellation is intended to suggest that oxymorons are hard to find.

In the present case a regular element need not exist.

and even then may not be an eigenvector.

3. A Richardson Element

Recall that a parabolic subalgebra may be specified by a composition $(c_1, c_2, \dots, c_k)$ of n.

Recall that a parabolic subalgebra may be specified by a composition $(c_1, c_2, \dots, c_k)$ of n.

A tableau $\mathscr{T}$ is drawn with columns C_i of height c_i with $1, 2, \dots, n$ inserted in the "boxes" obtained by first going down the columns and then across the rows.

3. A Richardson Element

Recall that a parabolic subalgebra may be specified by a composition $(c_1, c_2, \ldots, c_k)$ of n.

A tableau $\mathscr{T}$ is drawn with columns C_i of height c_i with $1, 2, \ldots, n$ inserted in the "boxes" obtained by first going down the columns and then across the rows.

Notably a right going line from any two boxes in different columns gives the co-ordinates of a vector in $\mathfrak{m}$ as its beginning and end-points.

3. A Richardson Element

Recall that a parabolic subalgebra may be specified by a composition $(c_1, c_2, \dots, c_k)$ of n.

A tableau $\mathscr{T}$ is drawn with columns C_i of height c_i with $1, 2, \dots, n$ inserted in the "boxes" obtained by first going down the columns and then across the rows.

Notably a right going line from any two boxes in different columns gives the co-ordinates of a vector in $\mathfrak{m}$ as its beginning and end-points.

Ringel et al drew all possible horizontal lines.

3. A Richardson Element

Recall that a parabolic subalgebra may be specified by a composition $(c_1, c_2, \ldots, c_k)$ of n.

A tableau $\mathscr{T}$ is drawn with columns C_i of height c_i with $1, 2, \ldots, n$ inserted in the "boxes" obtained by first going down the columns and then across the rows.

Notably a right going line from any two boxes in different columns gives the co-ordinates of a vector in $\mathfrak{m}$ as its beginning and end-points.

Ringel et al drew all possible horizontal lines.

They showed that the sum of the vectors defined by these lines was a Richardson element.

4. The Desired Modification of Ringel et al

To construct a Weierstrass section $e + V$, we modify the above lines.

To construct a Weierstrass section $e + V$, we modify the above lines. with a label 1 (resp $*$) for summands of e (resp. V).

4. The Desired Modification of Ringel et al

To construct a Weierstrass section $e+V$, we modify the above lines. with a label 1 (resp $*$) for summands of e (resp. V).

A composite line is a directed concatenation of such lines. Lines are called disjoint if they do not meet at any box,.

4. The Desired Modification of Ringel et al

To construct a Weierstrass section $e + V$, we modify the above lines. with a label 1 (resp $*$) for summands of e (resp. V).

A composite line is a directed concatenation of such lines. Lines are called disjoint if they do not meet at any box,.

Two columns of the same height are said to be neighbouring if there are no columns of that height between them. Each pair give rises to a Benlolo-Sanderson generating invariant.

4. The Desired Modification of Ringel et al

To construct a Weierstrass section $e + V$, we modify the above lines. with a label 1 (resp $*$) for summands of e (resp. V).

A composite line is a directed concatenation of such lines. Lines are called disjoint if they do not meet at any box,.

Two columns of the same height are said to be neighbouring if there are no columns of that height between them. Each pair give rises to a Benlolo-Sanderson generating invariant.

A Weierstrass section is obtained when between any two columns of a given height there is a disjoint union of composite lines.

4. The Desired Modification of Ringel et al

To construct a Weierstrass section $e + V$, we modify the above lines. with a label 1 (resp $*$) for summands of e (resp. V).

A composite line is a directed concatenation of such lines. Lines are called disjoint if they do not meet at any box,.

Two columns of the same height are said to be neighbouring if there are no columns of that height between them. Each pair give rises to a Benlolo-Sanderson generating invariant.

A Weierstrass section is obtained when between any two columns of a given height there is a disjoint union of composite lines.

With at most one $*$ (in an appropriate place) on the individual lines in the composite union.

4. The Desired Modification of Ringel et al

To construct a Weierstrass section $e + V$, we modify the above lines. with a label 1 (resp $*$) for summands of e (resp. V).

A composite line is a directed concatenation of such lines. Lines are called disjoint if they do not meet at any box,.

Two columns of the same height are said to be neighbouring if there are no columns of that height between them. Each pair give rises to a Benlolo-Sanderson generating invariant.

A Weierstrass section is obtained when between any two columns of a given height there is a disjoint union of composite lines.

With at most one $*$ (in an appropriate place) on the individual lines in the composite union.

To find such a presentation is quite a task! We achieved it through an arduous modification of the lines of Ringel et al.

5. The Composition Tableau

My colleague described to you how to modify $\mathscr{T}$ to obtain automatically (and mysteriously) the required assignment of lines.

5. The Composition Tableau

My colleague described to you how to modify $\mathscr{T}$ to obtain automatically (and mysteriously) the required assignment of lines.

We called this modification: the composition tableau. It determined a component $\mathscr{C}$ of $\mathscr{N}$ which we called the canonical component.

5. The Composition Tableau

My colleague described to you how to modify $\mathscr{T}$ to obtain automatically (and mysteriously) the required assignment of lines.

We called this modification: the composition tableau. It determined a component $\mathscr{C}$ of $\mathscr{N}$ which we called the canonical component.

Can we similarly describe the other components of $\mathscr{N}$?

5. The Composition Tableau

My colleague described to you how to modify $\mathscr{T}$ to obtain automatically (and mysteriously) the required assignment of lines.

We called this modification: the composition tableau. It determined a component $\mathscr{C}$ of $\mathscr{N}$ which we called the canonical component.

Can we similarly describe the other components of $\mathscr{N}$?

Let B be a fixed Borel subgroup which we assume is contained in P and $\mathfrak{n}$ the nilradical of its Lie algebra.

5. The Composition Tableau

My colleague described to you how to modify $\mathscr{T}$ to obtain automatically (and mysteriously) the required assignment of lines.

We called this modification: the composition tableau. It determined a component $\mathscr{C}$ of $\mathscr{N}$ which we called the canonical component.

Can we similarly describe the other components of $\mathscr{N}$?

Let B be a fixed Borel subgroup which we assume is contained in P and $\mathfrak{n}$ the nilradical of its Lie algebra.

We showed that $\mathscr{C} = \overline{B.\mathfrak{u}}$ for some subalgebra $\mathfrak{u}$ of $\mathfrak{n}$.

5. The Composition Tableau

My colleague described to you how to modify $\mathscr{T}$ to obtain automatically (and mysteriously) the required assignment of lines.

We called this modification: the composition tableau. It determined a component $\mathscr{C}$ of $\mathscr{N}$ which we called the canonical component.

Can we similarly describe the other components of $\mathscr{N}$?

Let B be a fixed Borel subgroup which we assume is contained in P and $\mathfrak{n}$ the nilradical of its Lie algebra.

We showed that $\mathscr{C} = \overline{B.\mathfrak{u}}$ for some subalgebra $\mathfrak{u}$ of $\mathfrak{n}$.

Trying to find those subalgebras which give further components appeared hopeless.

My colleague described to you how to modify $\mathscr{T}$ to obtain automatically (and mysteriously) the required assignment of lines.

We called this modification: the composition tableau. It determined a component $\mathscr{C}$ of $\mathscr{N}$ which we called the canonical component.

Can we similarly describe the other components of $\mathscr{N}$?

Let B be a fixed Borel subgroup which we assume is contained in P and $\mathfrak{n}$ the nilradical of its Lie algebra.

We showed that $\mathscr{C} = \overline{B.\mathfrak{u}}$ for some subalgebra $\mathfrak{u}$ of $\mathfrak{n}$.

Trying to find those subalgebras which give further components appeared hopeless.

Instead we generalize the (mysterious) construction of the composition tableau.

5. The Composition Tableau

My colleague described to you how to modify $\mathscr{T}$ to obtain automatically (and mysteriously) the required assignment of lines.

We called this modification: the composition tableau. It determined a component $\mathscr{C}$ of $\mathscr{N}$ which we called the canonical component.

Can we similarly describe the other components of $\mathscr{N}$?

Let B be a fixed Borel subgroup which we assume is contained in P and $\mathfrak{n}$ the nilradical of its Lie algebra.

We showed that $\mathscr{C} = \overline{B.\mathfrak{u}}$ for some subalgebra $\mathfrak{u}$ of $\mathfrak{n}$.

Trying to find those subalgebras which give further components appeared hopeless.

Instead we generalize the (mysterious) construction of the composition tableau.

They are called the component tableaux.

6. Recalling the Composition Tableau

My colleague explained that the composition tableau is obtained from $\mathscr{T}$ by successively filling empty boxes with further entries from $[1, n]$.

6. Recalling the Composition Tableau

My colleague explained that the composition tableau is obtained from $\mathscr{T}$ by successively filling empty boxes with further entries from $[1, n]$.

This involved moving an entry in row s, horizontally to the right from a column C to an adjacent column C' if the corresponding box is empty.

6. Recalling the Composition Tableau

My colleague explained that the composition tableau is obtained from $\mathscr{T}$ by successively filling empty boxes with further entries from $[1, n]$.

This involved moving an entry in row s, horizontally to the right from a column C to an adjacent column C' if the corresponding box is empty.
Or down by one row into C', again if the corresponding box was empty.

This last operation was called a “step”.

6. Recalling the Composition Tableau

My colleague explained that the composition tableau is obtained from $\mathscr{T}$ by successively filling empty boxes with further entries from $[1, n]$.

This involved moving an entry in row s, horizontally to the right from a column C to an adjacent column C' if the corresponding box is empty.

Or down by one row into C', again if the corresponding box was empty.

This last operation was called a "step".

It was only allowed if C, C' were surrounded by a pair of neighbouring columns of height s.

6. Recalling the Composition Tableau

My colleague explained that the composition tableau is obtained from $\mathscr{T}$ by successively filling empty boxes with further entries from $[1, n]$.

This involved moving an entry in row s, horizontally to the right from a column C to an adjacent column C' if the corresponding box is empty.

Or down by one row into C', again if the corresponding box was empty.

This last operation was called a "step".

It was only allowed if C, C' were surrounded by a pair of neighbouring columns of height s.

A step corresponds to the use of a Benlolo-Sanderson invariant.

6. Recalling the Composition Tableau

My colleague explained that the composition tableau is obtained from $\mathscr{T}$ by successively filling empty boxes with further entries from $[1, n]$.

This involved moving an entry in row s, horizontally to the right from a column C to an adjacent column C' if the corresponding box is empty.
Or down by one row into C', again if the corresponding box was empty.

This last operation was called a "step".

It was only allowed if C, C' were surrounded by a pair of neighbouring columns of height s.

A step corresponds to the use of a Benlolo-Sanderson invariant.

In this order relations are imposed so that no steps were omitted.
They are equivalent to this condition.

7. Examples

Let P be defined by the composition $(1, 2, 1)$

7. Examples

Let P be defined by the composition $(1, 2, 1)$

The corresponding tableau $\mathscr{T}$ has 1 in the first column C_1, $2, 3$ in the C_2 and 4 in C_3.

7. Examples

Let P be defined by the composition $(1, 2, 1)$

The corresponding tableau $\mathscr{T}$ has 1 in the first column C_1, $2, 3$ in the C_2 and 4 in C_3.
Since C_1, C_3 are neighbouring of height 1, we can make a step by putting (an extra) 2 below $4 \in C_3$. This gives the composition tableau $\mathscr{T}(\infty)$.

7. Examples

Let P be defined by the composition $(1, 2, 1)$

The corresponding tableau $\mathscr{T}$ has 1 in the first column C_1, $2, 3$ in the C_2 and 4 in C_3.
Since C_1, C_3 are neighbouring of height 1, we can make a step by putting (an extra) 2 below $4 \in C_3$. This gives the composition tableau $\mathscr{T}(\infty)$.

Or we push (an extra) 3 across into C_3. Yet this would block the step. Thus it is forbidden.

7. Examples

Let P be defined by the composition $(1, 2, 1)$

The corresponding tableau $\mathscr{T}$ has 1 in the first column C_1, $2, 3$ in the C_2 and 4 in C_3.
Since C_1, C_3 are neighbouring of height 1, we can make a step by putting (an extra) 2 below $4 \in C_3$. This gives the composition tableau $\mathscr{T}(\infty)$.

Or we push (an extra) 3 across into C_3. Yet this would block the step. Thus it is forbidden.

Consider the composition $(2, 1, 1, 2)$. We shall create the composition tableau row by row.

7. Examples

Let P be defined by the composition $(1, 2, 1)$

The corresponding tableau $\mathscr{T}$ has 1 in the first column C_1, $2, 3$ in the C_2 and 4 in C_3.
Since C_1, C_3 are neighbouring of height 1, we can make a step by putting (an extra) 2 below $4 \in C_3$. This gives the composition tableau $\mathscr{T}(\infty)$.

Or we push (an extra) 3 across into C_3. Yet this would block the step. Thus it is forbidden.

Consider the composition $(2, 1, 1, 2)$. We shall create the composition tableau row by row.

First we create a step by putting an (extra) 3 below 4 in C_3 and put an extra 2 into C_2 to fill the gap.

7. Examples

Let P be defined by the composition $(1, 2, 1)$

The corresponding tableau $\mathscr{T}$ has 1 in the first column C_1, $2, 3$ in the C_2 and 4 in C_3.
Since C_1, C_3 are neighbouring of height 1, we can make a step by putting (an extra) 2 below $4 \in C_3$. This gives the composition tableau $\mathscr{T}(\infty)$.

Or we push (an extra) 3 across into C_3. Yet this would block the step. Thus it is forbidden.

Consider the composition $(2, 1, 1, 2)$. We shall create the composition tableau row by row.

First we create a step by putting an (extra) 3 below 4 in C_3 and put an extra 2 into C_2 to fill the gap.

Finally we create a further step by putting an extra 3 below 6. This gives $\mathscr{T}(\infty)$.

8. Labelled lines in $\mathscr{T}(\infty)$

We draw labelled lines in $\mathscr{T}(\infty)$

8. Labelled lines in $\mathscr{T}(\infty)$

We draw labelled lines in $\mathscr{T}(\infty)$

Every step acquires a vertical line which we label by a $*$.

8. Labelled lines in $\mathscr{T}(\infty)$

We draw labelled lines in $\mathscr{T}(\infty)$

Every step acquires a vertical line which we label by a $*$.
Notice that entries in $\mathscr{T}(\infty)$ form (connected) strings with a given label.

8. Labelled lines in $\mathscr{T}(\infty)$

We draw labelled lines in $\mathscr{T}(\infty)$

Every step acquires a vertical line which we label by a $*$.
Notice that entries in $\mathscr{T}(\infty)$ form (connected) strings with a given label.

Yet an entry i in C_j may fail to appear in C_{j+1}. Then we say that it is stopped at C_j.

8. Labelled lines in $\mathscr{T}(\infty)$

We draw labelled lines in $\mathscr{T}(\infty)$

Every step acquires a vertical line which we label by a $*$.
Notice that entries in $\mathscr{T}(\infty)$ form (connected) strings with a given label.

Yet an entry i in C_j may fail to appear in C_{j+1}. Then we say that it is stopped at C_j.

In this case we connect i to an entry r in C_j to the highest available entry in C_{j+1}.

8. Labelled lines in $\mathscr{T}(\infty)$

We draw labelled lines in $\mathscr{T}(\infty)$

Every step acquires a vertical line which we label by a $*$.
Notice that entries in $\mathscr{T}(\infty)$ form (connected) strings with a given label.

Yet an entry i in C_j may fail to appear in C_{j+1}. Then we say that it is stopped at C_j.

In this case we connect i to an entry r in C_j to the highest available entry in C_{j+1}.

This gives a line with label 1 in $\mathscr{T}(\infty)$.

9. Labelled lines in $\mathscr{T}$

One end-point of any of the above lines lies in $\mathscr{T}$.

9. Labelled lines in $\mathscr{T}$

One end-point of any of the above lines lies in $\mathscr{T}$.

The second may not but determines a unique box in $\mathscr{T}$.

9. Labelled lines in $\mathscr{T}$

One end-point of any of the above lines lies in $\mathscr{T}$.

The second may not but determines a unique box in $\mathscr{T}$.

Thus we can translate our lines uniquely back to $\mathscr{T}$.

9. Labelled lines in $\mathscr{T}$

One end-point of any of the above lines lies in $\mathscr{T}$.

The second may not but determines a unique box in $\mathscr{T}$.

Thus we can translate our lines uniquely back to $\mathscr{T}$.

The result always satisfies our criteria for a Weierstrass section.

9. Labelled lines in $\mathscr{T}$

One end-point of any of the above lines lies in $\mathscr{T}$.

The second may not but determines a unique box in $\mathscr{T}$.

Thus we can translate our lines uniquely back to $\mathscr{T}$.

The result always satisfies our criteria for a Weierstrass section.

This may be checked in our examples.

Thus a seemingly almost impossibly challenging problem was readily solved!

10. Excluded Root Vectors

Let $\mathbf{M}$ be the full $n \times n$ matrix. We can translate our labelling of lines in $\mathscr{T}$ to labelled entries in $\mathbf{M}$.

10. Excluded Root Vectors

Let $\mathbf{M}$ be the full $n \times n$ matrix. We can translate our labelling of lines in $\mathscr{T}$ to labelled entries in $\mathbf{M}$.

Let $\mathbf{B}_i$ denote the column block above the Levi block $\mathbf{C}_i$.

.

.

10. Excluded Root Vectors

Let **M** be the full $n \times n$ matrix. We can translate our labelling of lines in $\mathscr{T}$ to labelled entries in **M**.

Let $\mathbf{B}_i$ denote the column block above the Levi block $\mathbf{C}_i$.

We encircle vectors in **M** by a certain procedure which ensures that every $*$ is encircled and possibly more.

.

.

10. Excluded Root Vectors

Let **M** be the full $n \times n$ matrix. We can translate our labelling of lines in $\mathscr{T}$ to labelled entries in **M**.

Let $\mathbf{B}_i$ denote the column block above the Levi block $\mathbf{C}_i$.

We encircle vectors in **M** by a certain procedure which ensures that every $*$ is encircled and possibly more.

These vectors are called the excluded root vector spaces. Let $\mathfrak{u}$ be their root vector space complement in $\mathfrak{n}$.

.

.

10. Excluded Root Vectors

Let $\mathbf{M}$ be the full $n \times n$ matrix. We can translate our labelling of lines in $\mathscr{T}$ to labelled entries in $\mathbf{M}$.

Let $\mathbf{B}_i$ denote the column block above the Levi block $\mathbf{C}_i$.

We encircle vectors in $\mathbf{M}$ by a certain procedure which ensures that every $*$ is encircled and possibly more.

These vectors are called the excluded root vector spaces. Let $\mathfrak{u}$ be their root vector space complement in $\mathfrak{n}$.

Our procedure does not encircle a 1, so $e \in \mathfrak{u}$.

.

.

10. Excluded Root Vectors

Let **M** be the full $n \times n$ matrix. We can translate our labelling of lines in $\mathscr{T}$ to labelled entries in **M**.

Let $\mathbf{B}_i$ denote the column block above the Levi block $\mathbf{C}_i$.

We encircle vectors in **M** by a certain procedure which ensures that every $*$ is encircled and possibly more.

These vectors are called the excluded root vector spaces. Let $\mathfrak{u}$ be their root vector space complement in $\mathfrak{n}$.

Our procedure does not encircle a 1, so $e \in \mathfrak{u}$.

When the encircled vectors are all set equal to 0. Every generating invariant vanishes. Let g be their number.

.

.

10. Excluded Root Vectors

Let **M** be the full $n \times n$ matrix. We can translate our labelling of lines in $\mathscr{T}$ to labelled entries in **M**.

Let $\mathbf{B}_i$ denote the column block above the Levi block $\mathbf{C}_i$.

We encircle vectors in **M** by a certain procedure which ensures that every $*$ is encircled and possibly more.

These vectors are called the excluded root vector spaces. Let $\mathfrak{u}$ be their root vector space complement in $\mathfrak{n}$.

Our procedure does not encircle a 1, so $e \in \mathfrak{u}$.

When the encircled vectors are all set equal to 0. Every generating invariant vanishes. Let g be their number.

Set $\mathscr{C} = \overline{B.\mathfrak{u}}$. From the previous observation one checks that $\mathscr{C}$ has codimension g in $\mathfrak{m}$.

.

10. Excluded Root Vectors

Let **M** be the full $n \times n$ matrix. We can translate our labelling of lines in $\mathscr{T}$ to labelled entries in **M**.

Let $\mathbf{B}_i$ denote the column block above the Levi block $\mathbf{C}_i$.

We encircle vectors in **M** by a certain procedure which ensures that every $*$ is encircled and possibly more.

These vectors are called the excluded root vector spaces. Let $\mathfrak{u}$ be their root vector space complement in $\mathfrak{n}$.

Our procedure does not encircle a 1, so $e \in \mathfrak{u}$.

When the encircled vectors are all set equal to 0. Every generating invariant vanishes. Let g be their number.

Set $\mathscr{C} = \overline{B.\mathfrak{u}}$. From the previous observation one checks that $\mathscr{C}$ has codimension g in $\mathfrak{m}$.

Through the existence of a Weierstrass section this implies that $\mathscr{C}$ is a component of $\mathscr{N}$.

11. Component Tableaux

From our second example above we could have placed 2 in C_3 creating a double step not previously allowed.

11. Component Tableaux

From our second example above we could have placed 2 in C_3 creating a double step not previously allowed.
More complicated multiple steps are possible which we shall only sketch, heeding the warning of the Rolling Stones.

11. Component Tableaux

From our second example above we could have placed 2 in C_3 creating a double step not previously allowed.
More complicated multiple steps are possible which we shall only sketch, heeding the warning of the Rolling Stones.

My head's fit to bust
'Cause she's so complicated.

11. Component Tableaux

From our second example above we could have placed 2 in C_3 creating a double step not previously allowed.
More complicated multiple steps are possible which we shall only sketch, heeding the warning of the Rolling Stones.

My head's fit to bust
'Cause she's so complicated.

Starting from the composition tableau $\mathscr{T}$ empty boxes are successively filled with further entries from $[1, n]$ to form $\mathscr{T}(t)$ by going down the rows.

11. Component Tableaux

From our second example above we could have placed 2 in C_3 creating a double step not previously allowed.
More complicated multiple steps are possible which we shall only sketch, heeding the warning of the Rolling Stones.

My head's fit to bust
'Cause she's so complicated.

Starting from the composition tableau $\mathscr{T}$ empty boxes are successively filled with further entries from $[1, n]$ to form $\mathscr{T}(t)$ by going down the rows.

Let R_t denote the t^{th} row of $\mathscr{T}(t)$.

11. Component Tableaux

From our second example above we could have placed 2 in C_3 creating a double step not previously allowed.
More complicated multiple steps are possible which we shall only sketch, heeding the warning of the Rolling Stones.

My head's fit to bust
'Cause she's so complicated.

Starting from the composition tableau $\mathscr{T}$ empty boxes are successively filled with further entries from $[1, n]$ to form $\mathscr{T}(t)$ by going down the rows.

Let R_t denote the t^{th} row of $\mathscr{T}(t)$.

For all $t \in \mathbb{N}^+$, let $C_1^t, C_2^t, \ldots, C_{r_t}^t$ be the columns of height t in $\mathscr{T}$.

11. Component Tableaux

From our second example above we could have placed 2 in C_3 creating a double step not previously allowed.
More complicated multiple steps are possible which we shall only sketch, heeding the warning of the Rolling Stones.

My head's fit to bust
'Cause she's so complicated.

Starting from the composition tableau $\mathscr{T}$ empty boxes are successively filled with further entries from $[1, n]$ to form $\mathscr{T}(t)$ by going down the rows.

Let R_t denote the t^{th} row of $\mathscr{T}(t)$.

For all $t \in \mathbb{N}^+$, let $C_1^t, C_2^t, \ldots, C_{r_t}^t$ be the columns of height t in $\mathscr{T}$.

The columns of $\mathscr{T}(t)$ will have distinct entries but the rows may have a string of entries of the same value in the above mutually adjacent columns.

12. Batches

The batches $\mathscr{B}_i^t : i \in [1, r_{t-1}]$. are defined to consist of the rightmost entries having a given value $r \in R_t \cap C : C \in [C_i^t, C_{i+1}^t[$ with the following property.

12. Batches

The batches $\mathscr{B}_i^t : i \in [1, r_{t-1}]$. are defined to consist of the rightmost entries having a given value $r \in R_t \cap C : C \in [C_i^t, C_{i+1}^t[$ with the following property.

If C' is the right adjacent column to C with the height of $C'(t)$ equal to $t' \geq t$. Then the pair C, C' is surrounded by sets of neighbouring columns of heights $s \in [t, t']$.

12. Batches

The batches $\mathscr{B}_i^t : i \in [1, r_{t-1}]$. are defined to consist of the rightmost entries having a given value $r \in R_t \cap C : C \in [C_i^t, C_{i+1}^t[$ with the following property.

If C' is the right adjacent column to C with the height of $C'(t)$ equal to $t' \geq t$. Then the pair C, C' is surrounded by sets of neighbouring columns of heights $s \in [t, t']$.

The batches are ordered from right to left but may consist of several entries which are not ordered.

12. Batches

The batches $\mathscr{B}_i^t : i \in [1, r_{t-1}]$. are defined to consist of the rightmost entries having a given value $r \in R_t \cap C : C \in [C_i^t, C_{i+1}^t[$ with the following property.

If C' is the right adjacent column to C with the height of $C'(t)$ equal to $t' \geq t$. Then the pair C, C' is surrounded by sets of neighbouring columns of heights $s \in [t, t']$.

The batches are ordered from right to left but may consist of several entries which are not ordered.

As before this ordering is equivalent to no $*$ suppression.

12. Batches

The batches $\mathscr{B}_i^t : i \in [1, r_{t-1}]$. are defined to consist of the rightmost entries having a given value $r \in R_t \cap C : C \in [C_i^t, C_{i+1}^t[$ with the following property.

If C' is the right adjacent column to C with the height of $C'(t)$ equal to $t' \geq t$. Then the pair C, C' is surrounded by sets of neighbouring columns of heights $s \in [t, t']$.

The batches are ordered from right to left but may consist of several entries which are not ordered.

As before this ordering is equivalent to no $*$ suppression.

This leads to a multitude of component tableau $\mathscr{T}^{\mathscr{C}}$ each labelled by some batch data $\mathscr{C}$.

12. Batches

The batches $\mathscr{B}_i^t : i \in [1, r_{t-1}]$. are defined to consist of the rightmost entries having a given value $r \in R_t \cap C : C \in [C_i^t, C_{i+1}^t[$ with the following property.

If C' is the right adjacent column to C with the height of $C'(t)$ equal to $t' \geq t$. Then the pair C, C' is surrounded by sets of neighbouring columns of heights $s \in [t, t']$.

The batches are ordered from right to left but may consist of several entries which are not ordered.

As before this ordering is equivalent to no $*$ suppression.

This leads to a multitude of component tableau $\mathscr{T}^{\mathscr{C}}$ each labelled by some batch data $\mathscr{C}$.
The rules for writing down $\mathscr{T}^{\mathscr{C}}$ with its labels and the excluded roots are much the same as before.

13. Example

Let P be defined by the composition $(2, 1, 2, 1)$

13. Example

Let P be defined by the composition $(2, 1, 2, 1)$

The composition tableau is obtained by pushing an additional 4 into C_4 and then an additional 2 into C_2 and then lowering 2 into C_3 and across into C_4.

13. Example

Let P be defined by the composition $(2, 1, 2, 1)$

The composition tableau is obtained by pushing an additional 4 into C_4 and then an additional 2 into C_2 and then lowering 2 into C_3 and across into C_4.
However unlike the $(2, 1, 1, 2)$ example, if we cannot get a further tableau if we only allow one step at a time.

13. Example

Let P be defined by the composition $(2, 1, 2, 1)$

The composition tableau is obtained by pushing an additional 4 into C_4 and then an additional 2 into C_2 and then lowering 2 into C_3 and across into C_4.
However unlike the $(2, 1, 1, 2)$ example, if we cannot get a further tableau if we only allow one step at a time.

However we can get a new component tableau by moving 3 down *two* steps into C_3.

14. Results.

Every component tableau $\mathscr{T}^{\mathscr{C}}$ gives rise to a Weierstrass section $e + V$.

14. Results.

Every component tableau $\mathscr{T}^{\mathscr{C}}$ gives rise to a Weierstrass section $e + V$.

Every component tableau $\mathscr{T}^{\mathscr{C}}$ gives a set of excluded roots hence a subalgebra $\mathfrak{u}^{\mathscr{C}}$ of $\mathfrak{m}$.

14. Results.

Every component tableau $\mathscr{T}^{\mathscr{C}}$ gives rise to a Weierstrass section $e+V$.

Every component tableau $\mathscr{T}^{\mathscr{C}}$ gives a set of excluded roots hence a subalgebra $\mathfrak{u}^{\mathscr{C}}$ of $\mathfrak{m}$.

The B saturation set $\mathscr{C} := \overline{B.\mathfrak{u}^{\mathscr{C}}}$ has codimension the number of Benlolo-Sanderson generators.

14. Results.

Every component tableau $\mathscr{T}^{\mathscr{C}}$ gives rise to a Weierstrass section $e + V$.

Every component tableau $\mathscr{T}^{\mathscr{C}}$ gives a set of excluded roots hence a subalgebra $\mathfrak{u}^{\mathscr{C}}$ of $\mathfrak{m}$.

The B saturation set $\mathscr{C} := \overline{B.\mathfrak{u}^{\mathscr{C}}}$ has codimension the number of Benlolo-Sanderson generators.

Using the Weierstrass section one concludes it is a component of $\mathscr{N}$.

14. Results.

Every component tableau $\mathscr{T}^{\mathscr{C}}$ gives rise to a Weierstrass section $e + V$.

Every component tableau $\mathscr{T}^{\mathscr{C}}$ gives a set of excluded roots hence a subalgebra $\mathfrak{u}^{\mathscr{C}}$ of $\mathfrak{m}$.

The B saturation set $\mathscr{C} := \overline{B.\mathfrak{u}^{\mathscr{C}}}$ has codimension the number of Benlolo-Sanderson generators.

Using the Weierstrass section one concludes it is a component of $\mathscr{N}$.

The map $\mathscr{T}^{\mathscr{C}} \mapsto \mathscr{C}$ of component tableaux to components is injective.

14. Results.

Every component tableau $\mathscr{T}^{\mathscr{C}}$ gives rise to a Weierstrass section $e + V$.

Every component tableau $\mathscr{T}^{\mathscr{C}}$ gives a set of excluded roots hence a subalgebra $\mathfrak{u}^{\mathscr{C}}$ of $\mathfrak{m}$.

The B saturation set $\mathscr{C} := \overline{B.\mathfrak{u}^{\mathscr{C}}}$ has codimension the number of Benlolo-Sanderson generators.

Using the Weierstrass section one concludes it is a component of $\mathscr{N}$.

The map $\mathscr{T}^{\mathscr{C}} \mapsto \mathscr{C}$ of component tableaux to components is injective.

This is proved using a "swapping lemma" which asserts that for any two distinct component tableaux admit a pair of lines, one with a 1 and one with a $*$ in one component tableau and vice-versa in the second component tableau.

14. Results.

Every component tableau $\mathscr{T}^{\mathscr{C}}$ gives rise to a Weierstrass section $e + V$.

Every component tableau $\mathscr{T}^{\mathscr{C}}$ gives a set of excluded roots hence a subalgebra $\mathfrak{u}^{\mathscr{C}}$ of $\mathfrak{m}$.

The B saturation set $\mathscr{C} := \overline{B.\mathfrak{u}^{\mathscr{C}}}$ has codimension the number of Benlolo-Sanderson generators.

Using the Weierstrass section one concludes it is a component of $\mathscr{N}$.

The map $\mathscr{T}^{\mathscr{C}} \mapsto \mathscr{C}$ of component tableaux to components is injective.

This is proved using a "swapping lemma" which asserts that for any two distinct component tableaux admit a pair of lines, one with a 1 and one with a $*$ in one component tableau and vice-versa in the second component tableau.

One concludes using the existence of a Weierstrass section.

15. A Further Example and Remarks.

Consider the composition $(2, 1, 2, 1, 2, 1)$.

15. A Further Example and Remarks.

Consider the composition $(2, 1, 2, 1, 2, 1)$.

This gives rise to 5 component tableaux.

15. A Further Example and Remarks.

Consider the composition $(2, 1, 2, 1, 2, 1)$.

This gives rise to 5 component tableaux.

Here the existence of a Weierstrass section may be verified.

15. A Further Example and Remarks.

Consider the composition $(2, 1, 2, 1, 2, 1)$.

This gives rise to 5 component tableaux.

Here the existence of a Weierstrass section may be verified.

Again the swapping lemma may be checked for every pair.

15. A Further Example and Remarks.

Consider the composition $(2, 1, 2, 1, 2, 1)$.

This gives rise to 5 component tableaux.

Here the existence of a Weierstrass section may be verified.

Again the swapping lemma may be checked for every pair.

The full matrix $\mathbf{M}$ is drawn with its entries.

15. A Further Example and Remarks.

Consider the composition $(2, 1, 2, 1, 2, 1)$.

This gives rise to 5 component tableaux.

Here the existence of a Weierstrass section may be verified.

Again the swapping lemma may be checked for every pair.

The full matrix **M** is drawn with its entries.

The canonical component is characterized by $*$ only appearing in the right hand column of a column block and appearing at most once in each, as verifiable in this case. We called it “canonical” as it exists for all parabolics.

Batches 1

The example $(2, 1, 2, 1, 2, 1)$

	C_1	C_2	C_3	C_4	C_5	C_6
R_1	1	3	4	6	7	9
R_2	2		5		8	

Representation of $(2, 1, 2, 1, 2, 1)$

Batches 2

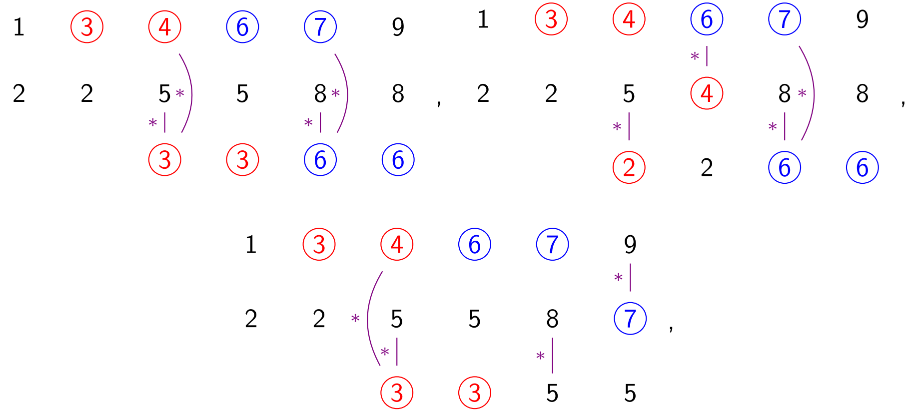
The example $(2,1,2,1,2,1)$

	C_1	C_2	C_3	C_4	C_5	C_6
R_1	1	3	4	6	7	9
R_2	2		5		8	

Batches $\mathscr{B}_1^1$ and $\mathscr{B}_1^2$

Component Tableaux $1 - 3$

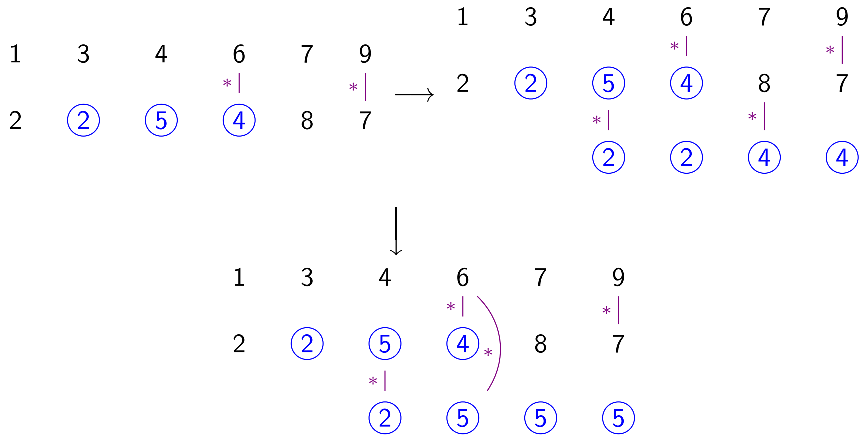
The example $(2, 1, 2, 1, 2, 1)$



Batches $\mathscr{B}_1^1$ and $\mathscr{B}_1^2$

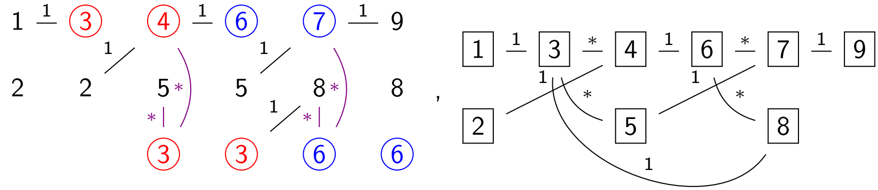
Component Tableaux 4, 5

The example $(2, 1, 2, 1, 2, 1)$



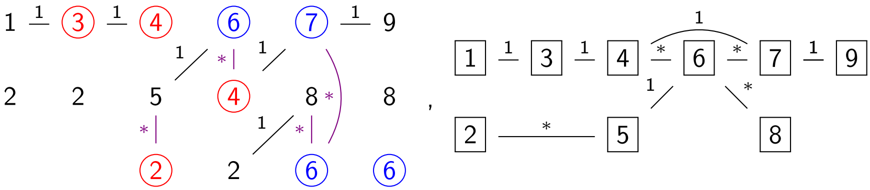
Component Tableau and Matrix 1

The example $(2, 1, 2, 1, 2, 1)$



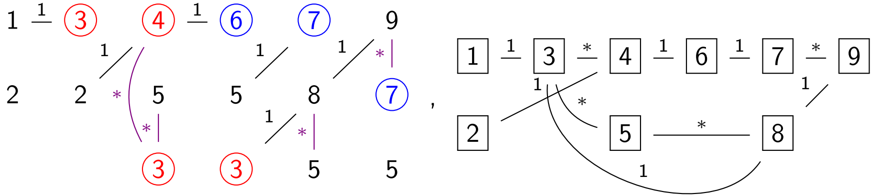
$$
\begin{pmatrix}
1 & 0 & 1 & & & & & & \\
0 & 1 & & 1 & & & & & \\
 & & 1 & \circledast & \circledast & & & 1 & \\
 & & & 1 & 0 & 1 & & & \\
 & & & 0 & 1 & & 1 & & \\
 & & & & & 1 & \circledast & \circledast & \\
 & & & & & & 1 & 0 & 1 \\
 & & & & & & 0 & 1 & \\
 & & & & & & & & 1
\end{pmatrix}
$$

Component Tableau and Matrix 2



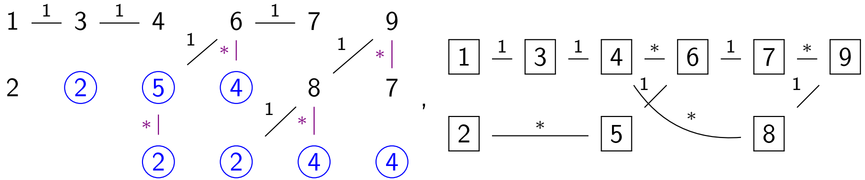
$$
\begin{pmatrix}
1 & 0 & 1 & & & & & & \\
0 & 1 & & & \circledast & & & & \\
 & & 1 & 1 & \bigcirc & & & & \\
 & & & 1 & 0 & \circledast & 1 & & \\
 & & & 0 & 1 & & & & \\
 & & & & & 1 & \circledast & \circledast & \\
 & & & & & & 1 & 0 & 1 \\
 & & & & & & 0 & 1 & \\
 & & & & & & & & 1
\end{pmatrix}
$$

Component Tableau and Matrix 3



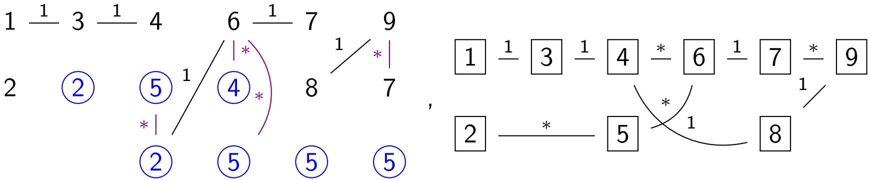
$$\begin{pmatrix}
\boxed{\begin{matrix}1 & 0\\ 0 & 1\end{matrix}} & & 1 & & & & & & \\
 & & & 1 & & & & & \\
 & & \boxed{1} & \circledast & \circledast & & & & 1 \\
 & & & \boxed{\begin{matrix}1 & 0\\ 0 & 1\end{matrix}} & & 1 & & \bigcirc & \\
 & & & & & & & \circledast & \circledast \\
 & & & & & \boxed{1} & 1 & & \\
 & & & & & & \boxed{\begin{matrix}1 & 0\\ 0 & 1\end{matrix}} & & \circledast \\
 & & & & & & & & \\
 & & & & & & & & \boxed{1}
\end{pmatrix}$$

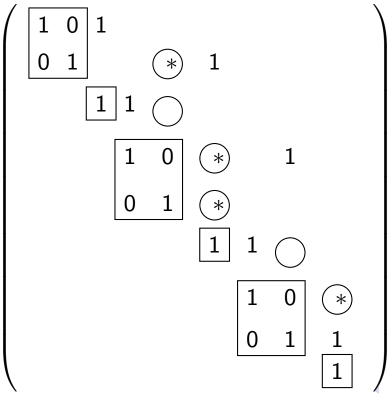
Component Tableau and Matrix 4



$$\begin{pmatrix}
1 & 0 & 1 & & \bigcirc & & & & \\
0 & 1 & & & \circledast & & & & \\
 & & \boxed{1} & 1 & \bigcirc & & & & \\
 & & & 1 & 0 & \circledast & & \circledast & \\
 & & & 0 & 1 & 1 & & & \\
 & & & & & \boxed{1} & 1 & \bigcirc & \\
 & & & & & & 1 & 0 & \circledast \\
 & & & & & & 0 & 1 & 1 \\
 & & & & & & & & \boxed{1}
\end{pmatrix}$$

Component Tableau and Matrix 5





16. Acknowledgement and Thanks.

16. Acknowledgement and Thanks.

These results are proving to be some of the best of my career and *due in no small part to a very fruitful and inspiring collaboration with Yasmine Fittouhi*.

16. Acknowledgement and Thanks.

These results are proving to be some of the best of my career and *due in no small part to a very fruitful and inspiring collaboration with Yasmine Fittouhi*.

I was born in Bombay and this is the first time I have been in India for 80 years.

16. Acknowledgement and Thanks.

These results are proving to be some of the best of my career and *due in no small part to a very fruitful and inspiring collaboration with Yasmine Fittouhi*.

I was born in Bombay and this is the first time I have been in India for 80 years.

Thus it is a particular pleasure for me to thank the organizers for this wonderful conference.