Classical & Quantum dynamics in out of equilibrium systems
Bangalore, December 16th 2024
Valentina Ros, @ LPTMS Orsay

Out-of equilibrium, activated dynamics in glassy systems: landscape approach

Based on work with Alessandro Pacco & Alberto Rosso, arXiv:2410.18010

Laboratoire de Physique Théorique et Modèles Statistiques
cnrs
université PARIS-SACLAY GRADUATE SCHOOL Physique
PhOM Physique des Ondes et de la Matière

The setting: high-d dynamics in rugged landscapes

Classical stochastic dynamics:

$$\frac{d\mathbf{s}(t)}{dt} = -\nabla \mathscr{E}[\mathbf{s}(t)] + \sqrt{2T}\,\eta(t) \qquad \langle \eta_i(t)\eta_j(t')\rangle = \delta_{ij}\delta(t-t') \qquad \mathbf{s} = (s_1, \cdots, s_N) \in \mathscr{C}_N$$

configuration space

High dimension $N \gg 1$, **weak noise** $T \ll 1$

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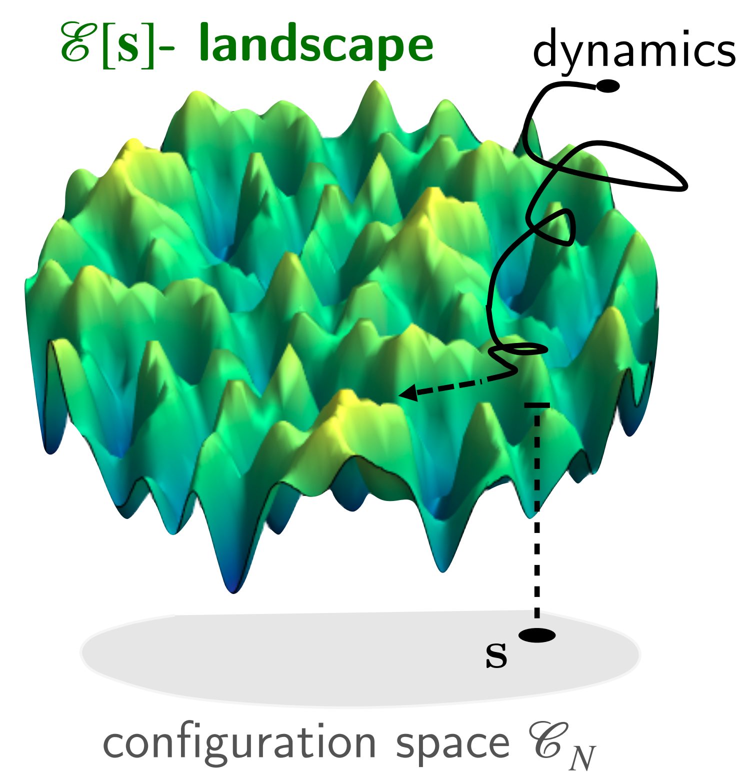
High dimension $N \gg 1$, **weak noise** $T \ll 1$

- Rugged energy landscape $\mathscr{E}[\mathbf{s}(t)]$ with $\mathcal{N} \sim e^{N\Sigma}$ local minima (→ **metastable states**), maxima, saddles such that $\nabla\mathscr{E}[\mathbf{s}] = 0$

  Σ= **"landscape complexity"**

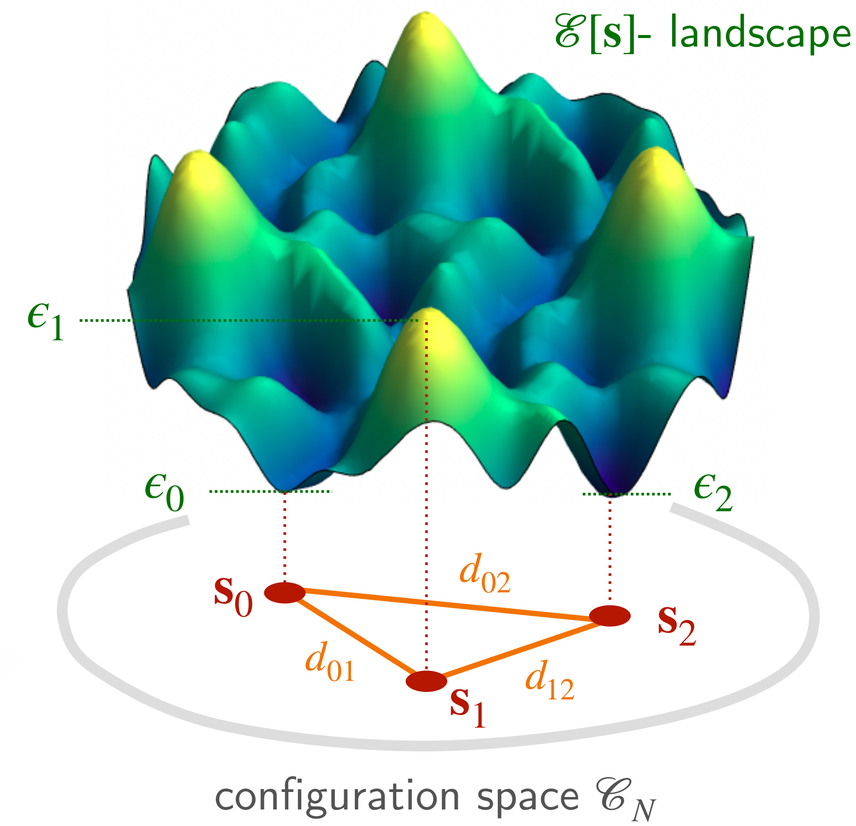
- $\mathscr{E}[\mathbf{s}(t)]$ **random landscape** on $\mathscr{C}_N$, Gaussian statistics.



In this work [arXiv:2410.18010]

We compute the **(conditional) entropy of triplets of metastable states** (local minima), more generally of stationary points $\nabla \mathscr{E}[\mathbf{s}] = 0$, as a function of;

(i) their energy density $\epsilon = \mathscr{E}/N$
(ii) their distances in configuration space d_{ab}



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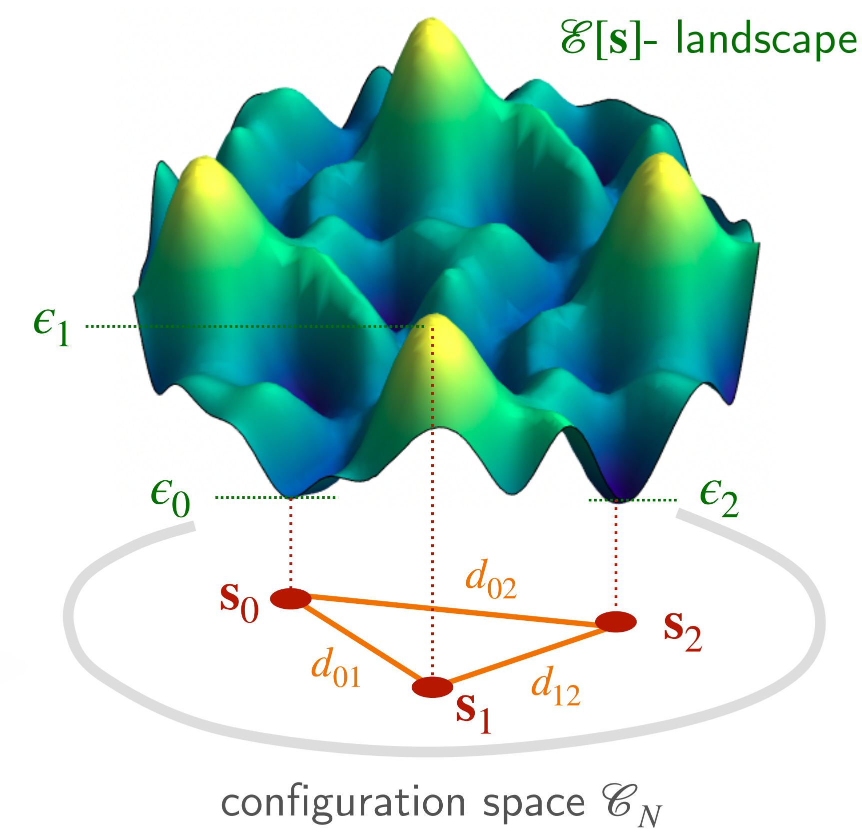

- Given the random variable

$\mathscr{N}_{\mathbf{s}_0,\mathbf{s}_1}(\epsilon_2, d_{02}, d_{12})$ = number of stationary points $\mathbf{s}_2$ at energy $\mathscr{E}[\mathbf{s}_2] = N\epsilon_2$ and conditioned to fixed distances d_{12}, d_{02} from two other stationary points $\mathbf{s}_0, \mathbf{s}_1$ of energies $N\epsilon_1, N\epsilon_0$ and distance d_{01}

we compute

$$\Sigma^{(3)}(\epsilon_2, d_{02}, d_{01} \,|\, \epsilon_0, \epsilon_1, d_{01}) = \lim_{N\to\infty} \left\langle \frac{1}{N} \log[\mathscr{N}_{\mathbf{s}_0,\mathbf{s}_1}(\epsilon_2, d_{02}, d_{12})] \right\rangle_{0,1}$$

"three-point complexity"

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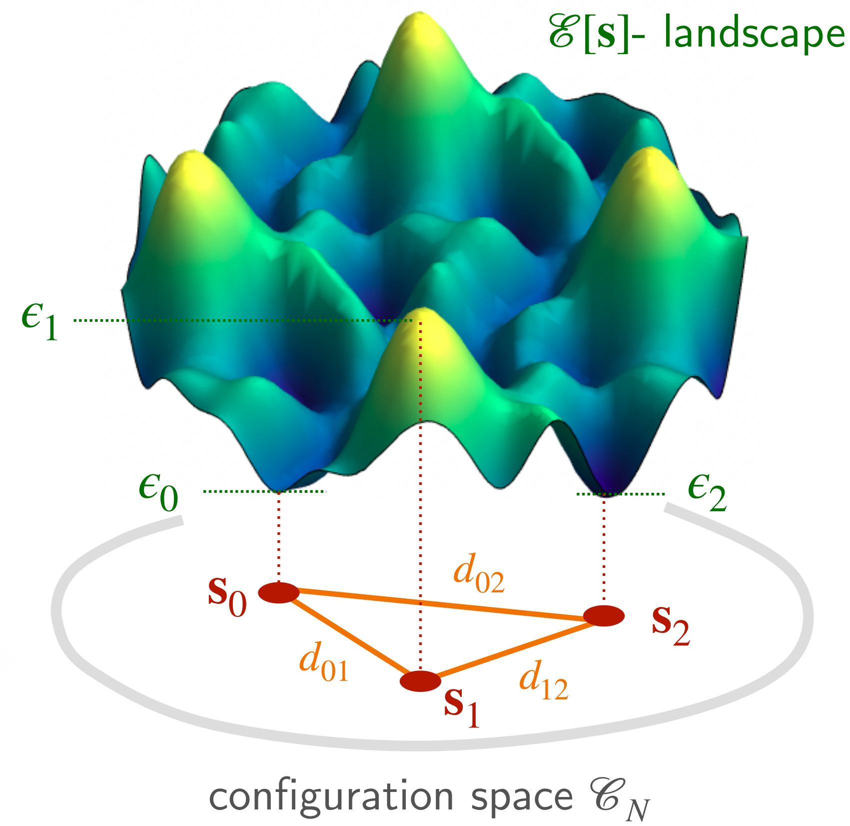

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"three-point complexity"

- "Replicated Kac-Rice formalism": replica theory + random matrix theory Review: VR, Fyodorov 2023

In this talk: from dynamics to landscape, and back

Part I

A good mean-field model of glasses

The known and the unknown

The unknown: activated dynamics

Part II

From dynamics to landscape's geometry

The local landscape's geometry

Hints on activated dynamics

Part I

A good mean-field model for glasses

The spherical p-spin model: the “baby glass”

$$\mathscr{E}(\mathbf{s}) = \sqrt{\frac{p!}{2N^{p-1}}} \sum_{i_1,\cdots,i_p} J_{i_1,\cdots,i_p} s_{i_1} \cdots s_{i_p} \qquad \sum_{i=1}^{N} s_i^2 = N$$

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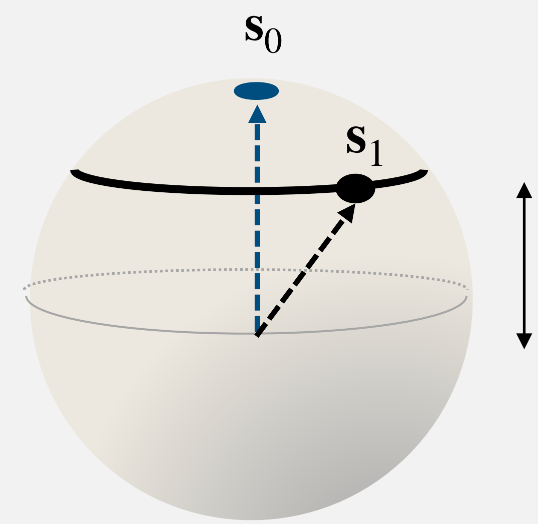
“**Pure spherical p-spin model**”, aka isotropic Gaussian landscapes on high-dimensional hypersphere



“overlap” $q(\mathbf{s}_0, \mathbf{s}_1) = \frac{\mathbf{s}_0 \cdot \mathbf{s}_1}{N}$

Configuration space $\mathscr{C}_N$

THE SIMPLEST SPIN GLASS

D J GROSS[1] and M MEZARD

Laboratoire de Physique Théorique de l'Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris Cedex 05, France*

Received 7 May 1984

We study a system of Ising spins with quenched random infinite ranged p-spin interactions

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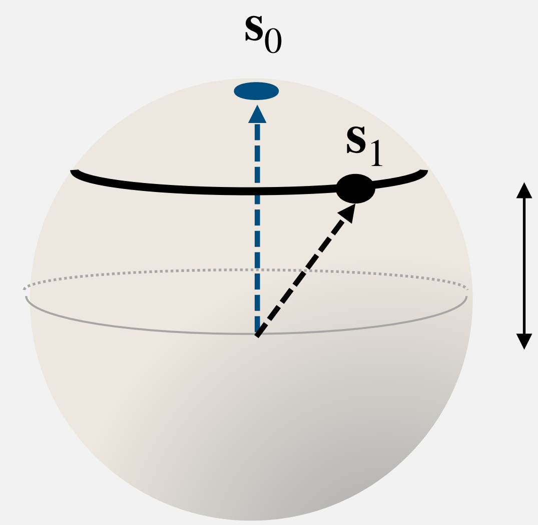

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High-d inference & denoising

Complex Energy Landscapes in Spiked-Tensor and Simple Glassy Models: Ruggedness, Arrangements of Local Minima, and Phase Transitions

Valentina Ros
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Gerard Ben Arous
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(Received 24 April 2018; revised manuscript received 31 October 2018; published 4 January 2019)

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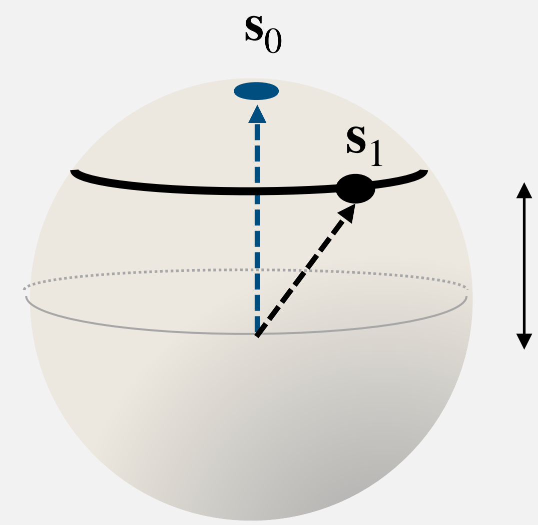

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Quantum ergodicity (ETH) breaking

Clustering of non-ergodic eigenstates in quantum spin glasses

C. L. Baldwin,[1,2] C. R. Laumann,[1] A. Pal,[3] and A. Scardicchio[4,5]

[1] *Department of Physics, Boston University, Boston, MA 02215, USA*
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[3] *Rudolf Peierls Centre for Theoretical Physics, Oxford University, Oxford OX1 3NP, UK*
[4] *Abdus Salam ICTP Trieste, Strada Costiera 11, 34151 Trieste, Italy*
[5] *INFN, Sezione di Trieste, Via Valerio 2, 34127 Trieste, Italy*

(Dated: February 1, 2017)

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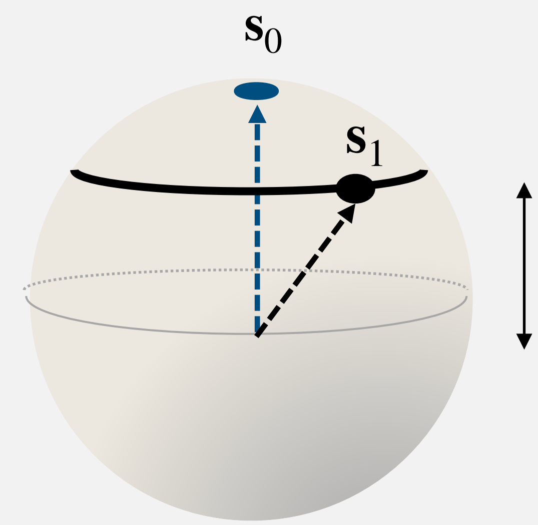

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Holography

The quantum p-spin glass model: a user manual for holographers

Tarek Anous[3] and Felix M Haehl

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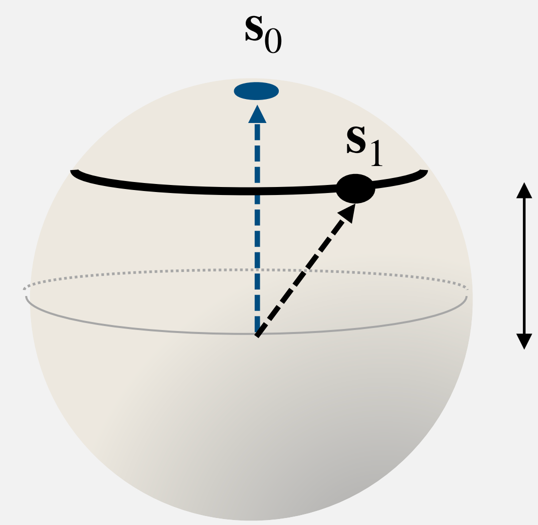

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Theory of learning

Statistical physics of learning in high-dimensional chaotic systems

Samantha J Fournier[2] and Pierfrancesco Urbani

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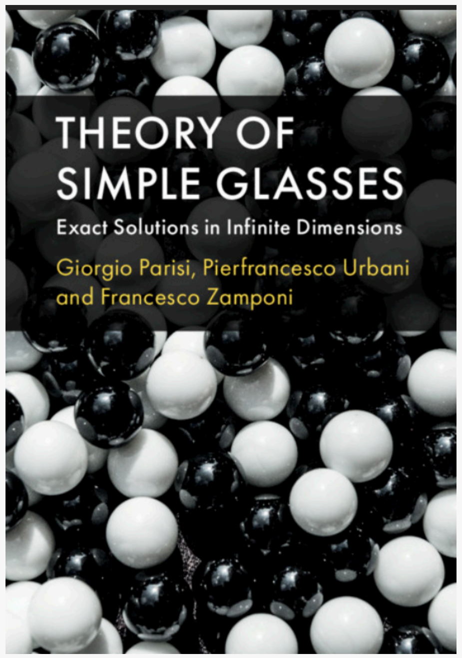
Journal of Statistical Mechanics: Theory and Experiment, Volume 2023, November 2023

A good mean-field model of glasses



Parisi, Urbani, Zamponi 2020

- Mean-field theory of interacting particles developed in last ~ 10 years. Non trivial theoretical construct.

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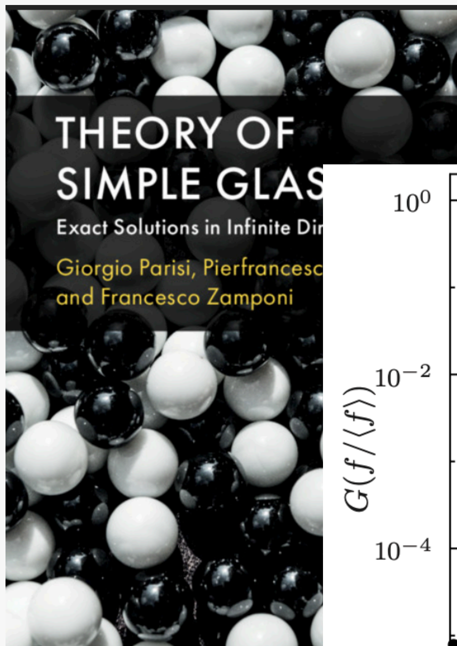

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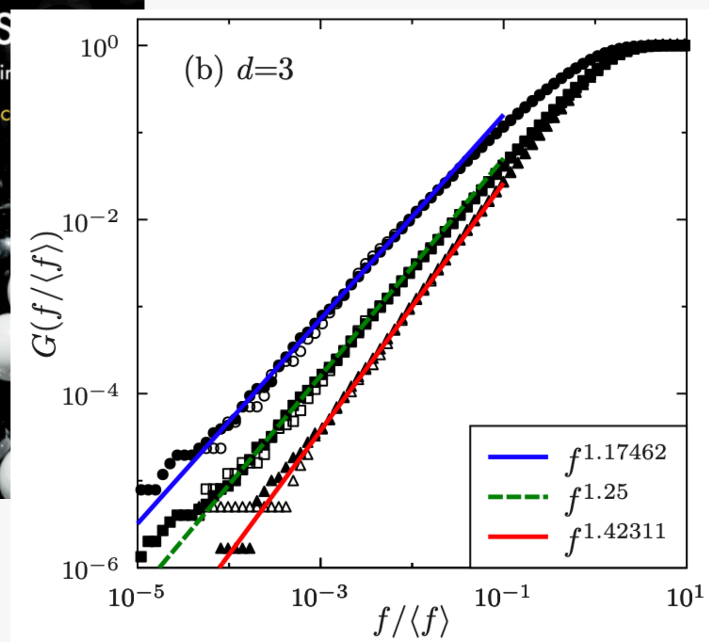

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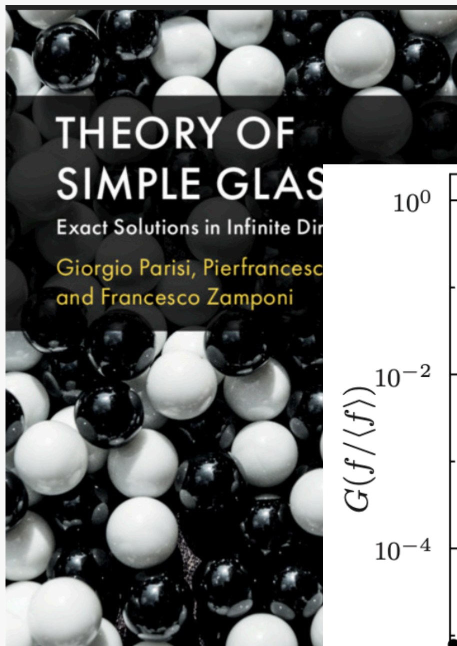

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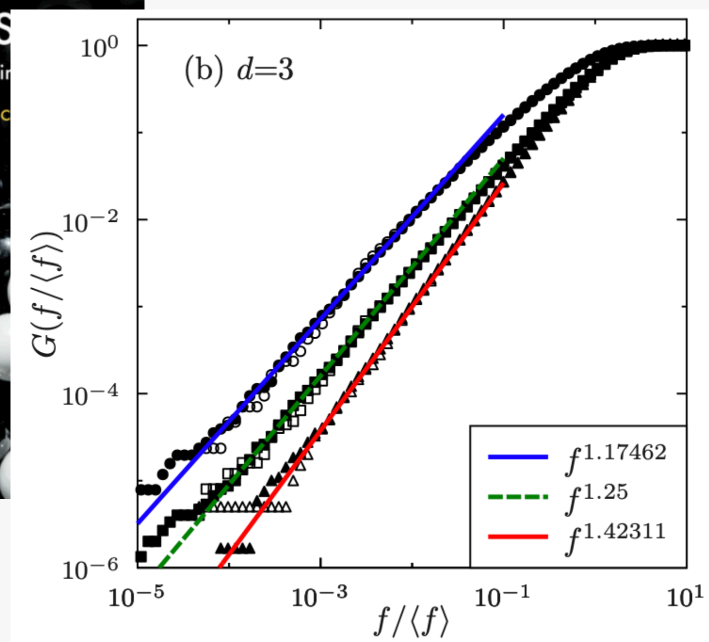

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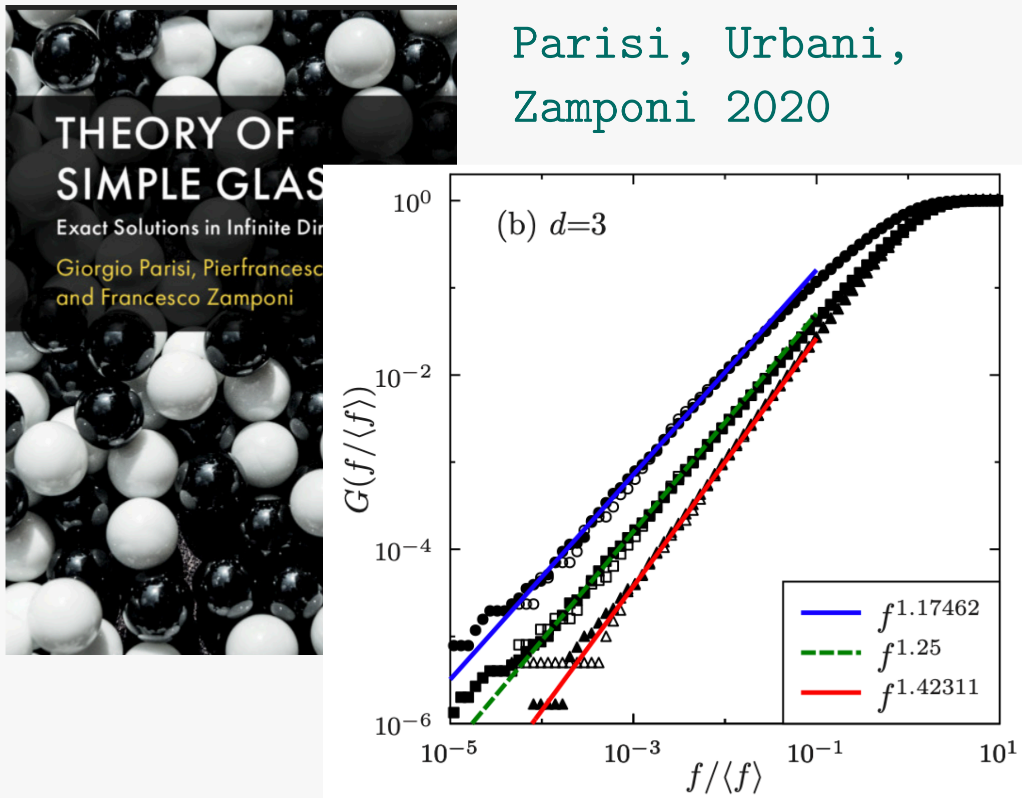

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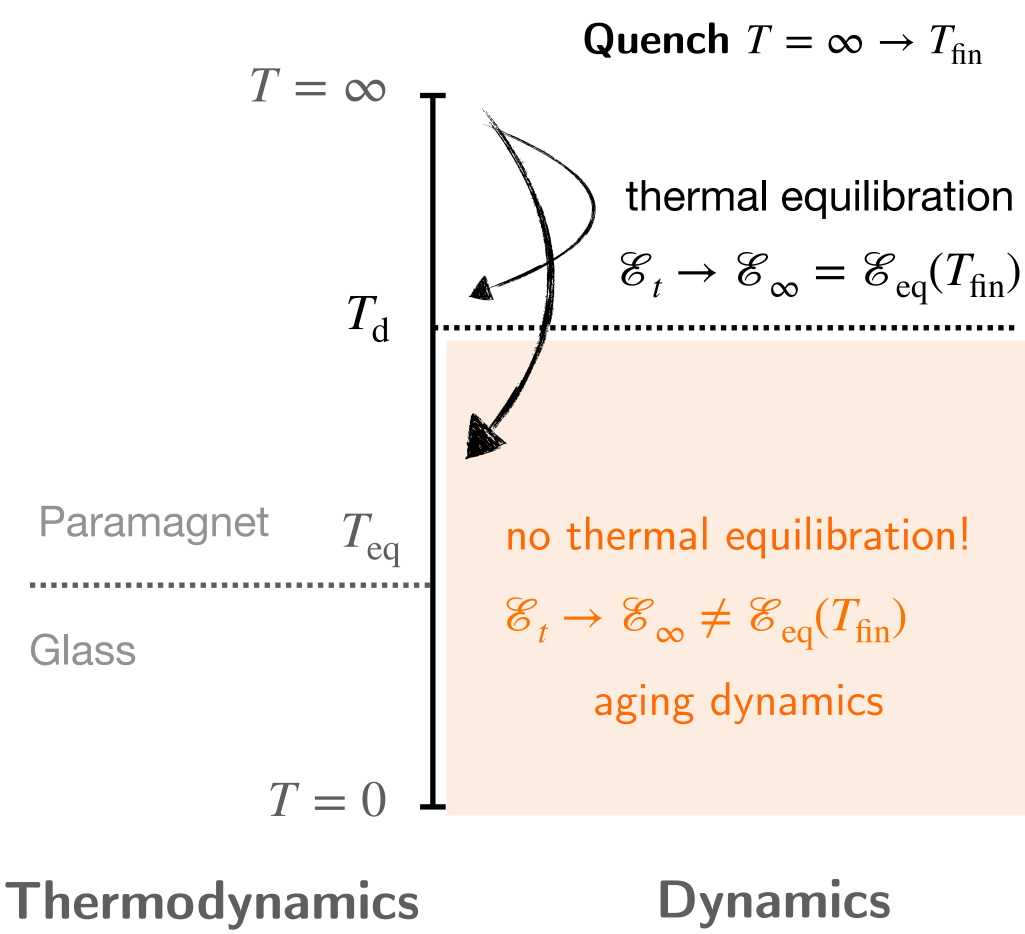

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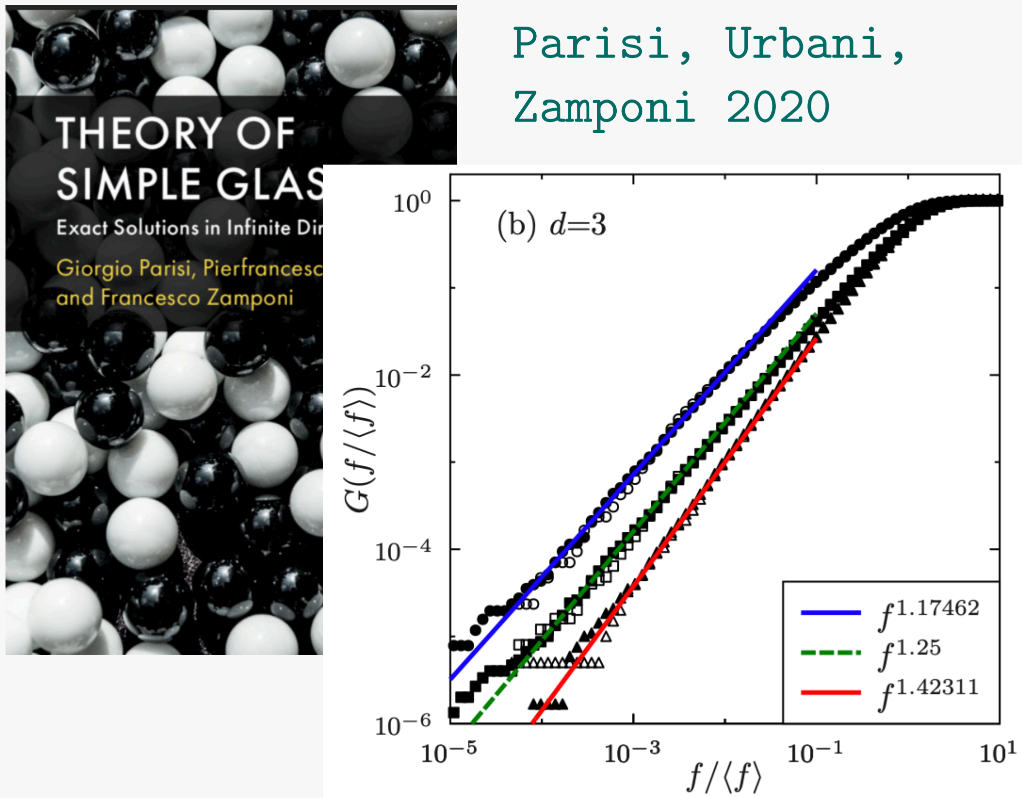

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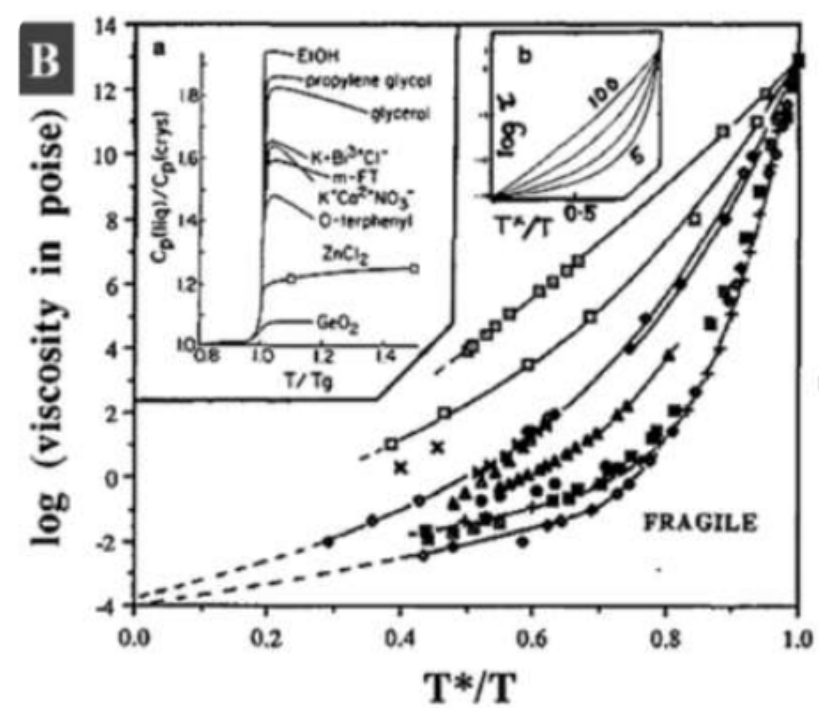
$T_d \to$ dynamical slowdown supercooled liquids



Angell 1995

Part I

The known & the unknown

The known: relaxational dynamics (1/2)

"A solvable mean-field dynamics..."

- Can derive exact equations (DMFT) for dynamical quantities when $N \to \infty$

$$\{s_i(t)\}_{i=1}^N \to \quad C(t,t') = \frac{\mathbf{s}(t)\cdot\mathbf{s}(t')}{N}, \qquad \epsilon(t) = \frac{\mathscr{E}(\mathbf{s}(t))}{N}, \qquad \cdots$$

Sompolinsky, Zippelius 1981
Crisanti, Horner, Sommers 1993

$$\frac{\partial C(t,t')}{\partial t} = -\mu(t)C(t,t') + \int_0^{t'} du D(t,u)R(t',u) + \int_0^t du \Sigma(t,u)C(u,t')$$

$$\frac{\partial R(t,t')}{\partial t} = -\mu(t)R(t,t') + \delta(t-t') + \int_{t'}^t du \Sigma(t,u)R(u,t')$$

$$\Sigma(t,t') = \frac{g^2}{2}C^{p-1}(t,t')R(t,t')$$

$$D(t,t') = 2T\delta(t-t') + \frac{g^2}{6}[C(t,t')]^p$$

- Can find ansatz solving the equations asymptotically, when $t, t' \to \infty$ Cugliandolo, Kurchan 1993

The known: relaxational dynamics (1/2)

"A solvable mean-field dynamics..."

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"A non-trivial mean-field dynamics..."

- Ansatz informative on **relaxational, out-of-equilibrium dynamics**: *separation of timescales, weak ergodicity breaking scenario, aging, effective temperatures, violation of fluctuation-dissipation, "quasi equilibrium" dynamics* Review: Bouchaud, Cugliandolo, Kurchan, Mezard 1998

The known: relaxational dynamics (2/2)

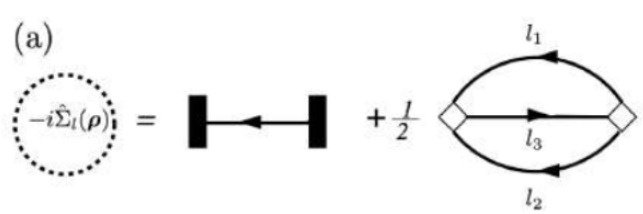
"... capturing structure of mean-field theory of particles"

p-spin DMFT equations are equivalent to **mode coupling equations** for supercooled liquids:
(i) perturbative, diagrammatic expansion of Langevin, (ii) keep only line (vs vertex) corrections

Kirkpatrick, Thirumalai, Wolynes 1989

Incidentally....

Similar diagrams retained in BAA scheme for MBL, or integrals of motion contruction within "forward approximation"



Basko, Aleiner, Altshuler 2005

Mode-coupling approximations, glass theory and disordered systems

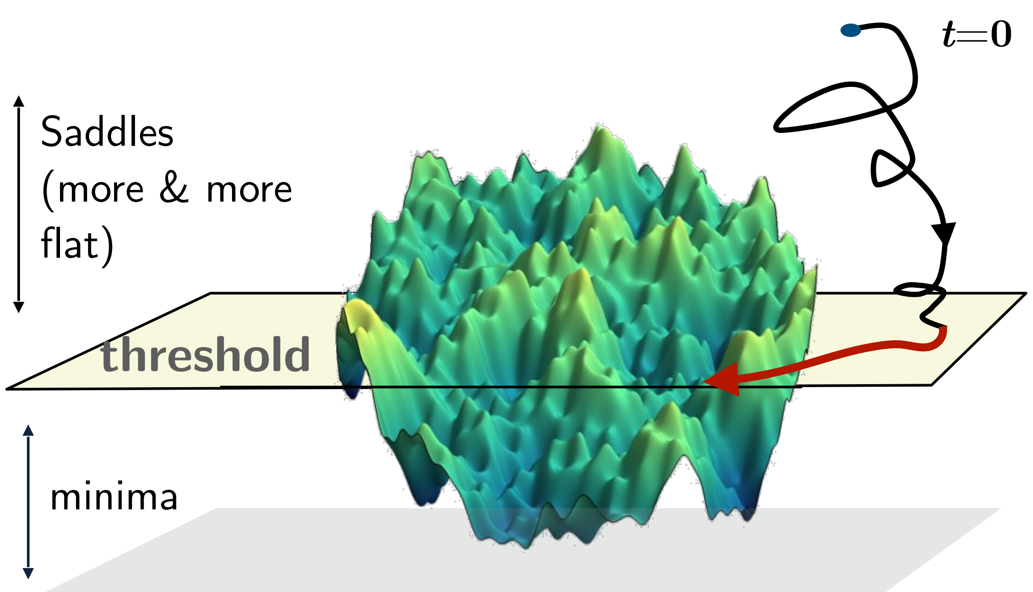
Jean-Philippe Bouchaud [a,1], Leticia Cugliandolo [a,2], Jorge Kurchan [b,3], Marc Mézard [b,4]

Let us face it: there are not so many techniques to deal with the score of strongly non-linear problems that Nature perversely offers, to the theoretical physicist's dismay.

The known: dynamical transition and metastability

$\mathcal{N}(\epsilon)$= number of stationary points $\nabla\mathcal{E}(\mathbf{s}) = 0, \quad \mathcal{E}(\mathbf{s}) = N\epsilon$

$\Sigma^{(1)}(\epsilon) = \lim_{N\to\infty} \frac{1}{N} \log \mathcal{N}(\epsilon)$ "(one-state) complexity" Cavagna, Giardina, Parisi 1997, 1998



A "threshold" energy $\epsilon_{\text{th}} = \epsilon_{\text{eq}}(T_d)$:

■ $\epsilon > \epsilon_{\text{th}}$: saddles with less and less downhill directions decreasing $\epsilon \searrow \epsilon_{\text{th}}$ Not trapping!

■ $\epsilon < \epsilon_{\text{th}}$: isolated local minima, separated by energy barriers $\Delta\mathcal{E} \sim N$

Trapping metastable states!

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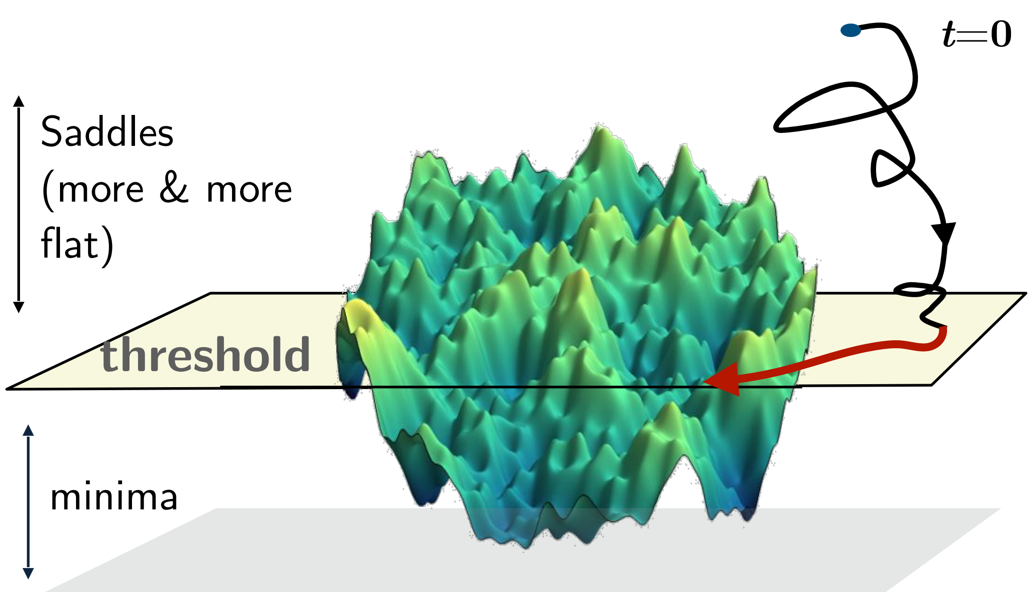

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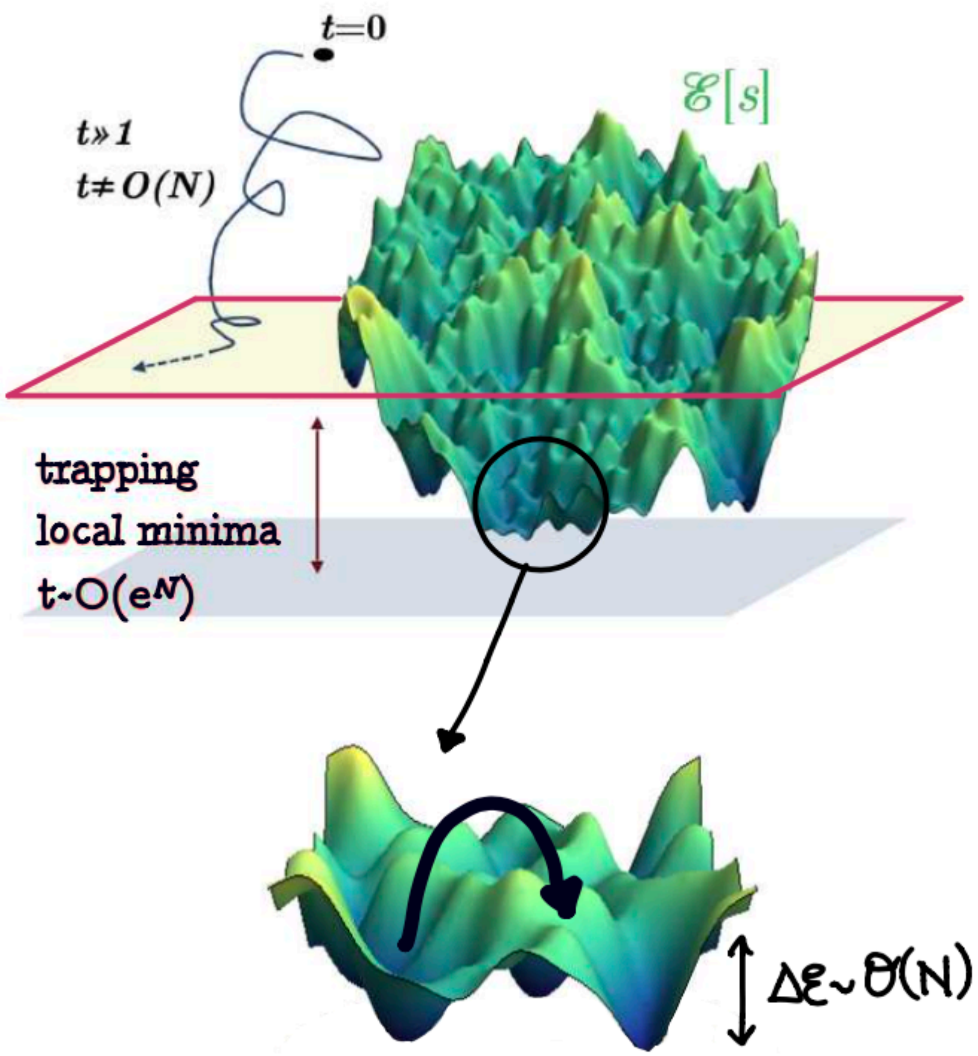
Trapping metastable states!

Dynamics is relaxing until it reaches ϵ_{th}, where metastable states start to appear.
Slowing down & aging because of landscape's geometry: the more descends, the more finding downhill directions gets harder.

The unknown: activated dynamics



$N \gg 1$ but finite: the dynamical transition at T_d **becomes a crossover** due to activated processes

■ Mean-field $N \to \infty$: local minima (metastable states) separated by barriers $\Delta\mathscr{E} \sim N \to \infty$: truly trapping.

■ $N \gg 1$ but finite: escape processes are possible, at times $\tau \sim e^{\beta\Delta\mathscr{E}} \sim e^N$.

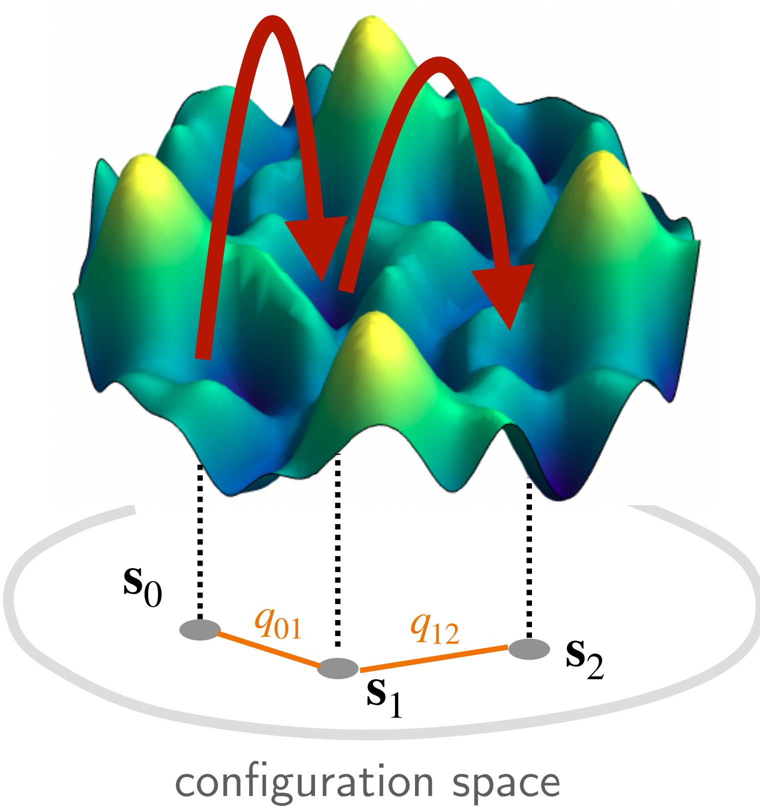
Rare activated processes drive dynamical exploration of bottom of landscape.
How to capture this physics?

Activated dynamics not captured by dynamical mean-field theory (that takes $t \to \infty$ after $N \to \infty$), nor by perturbation in N... Instantons of the dynamical theory.

Part II

From dynamics to landscape’s geometry

A random jump process among minima



Coarse grained dynamics: sequence of jumps between metastable states.

Transition rate: $\mathscr{T} = t^{(1)}(\mathbf{s}_1 \,|\, \mathbf{s}_0)\; t^{(2)}(\mathbf{s}_2 \,|\, \mathbf{s}_1, \mathbf{s}_0) \cdots$

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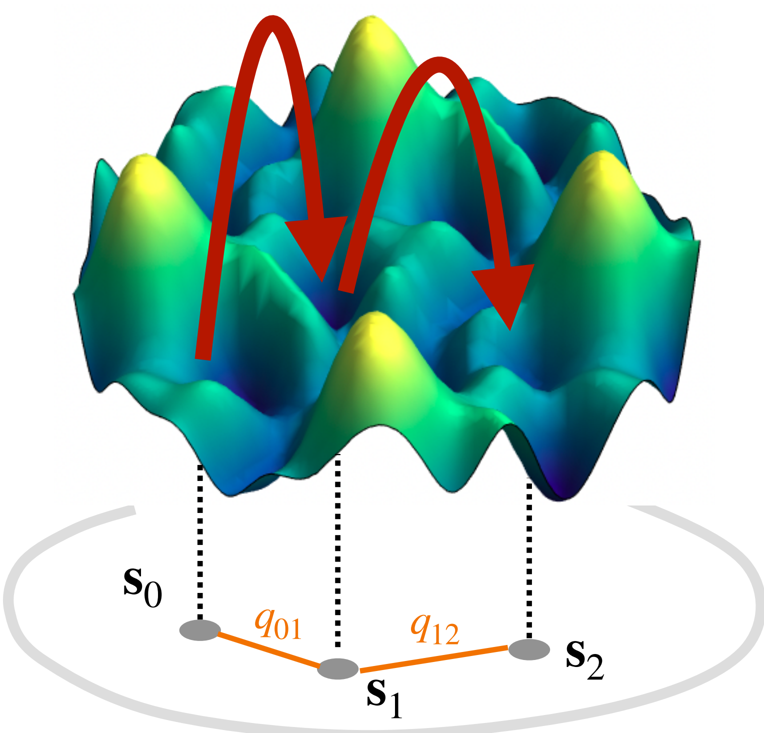

configuration space

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Assumptions:

■ rates depend only on energies $\epsilon_0, \epsilon_1, \epsilon_2, \cdots$ and overlaps $q_{ab} = \dfrac{\mathbf{s_a} \cdot \mathbf{s}_b}{N}$

(isotropy): $t^{(1)}(\mathbf{s}_1 \,|\, \mathbf{s}_0) \to t^{(1)}(\epsilon_1, q_{01} \,|\, \epsilon_0), \cdots$

■ for fixed energy: assume jumps always to **closest minima at that energy**. Why? Energy barriers decreasing with q

$q_{01} \to q^{(1)}_{\max}(\epsilon_1 \,|\, \epsilon_0)$

$q_{12} \to q^{(2)}_{\max}(\epsilon_2 \,|\, \epsilon_1, \epsilon_0)$

⟹

$t^{(1)}(\mathbf{s}_1 \,|\, \mathbf{s}_0) \to t^{(1)}(\epsilon_1, q^{(1)}_{\max} \,|\, \epsilon_0)$

$t^{(2)}(\mathbf{s}_2 \,|\, \mathbf{s}_1, \mathbf{s}_0) \to t^{(2)}(\epsilon_2, q^{(2)}_{\max} \,|\, \epsilon_1, \epsilon_0, q^{(1)}_{\max})$

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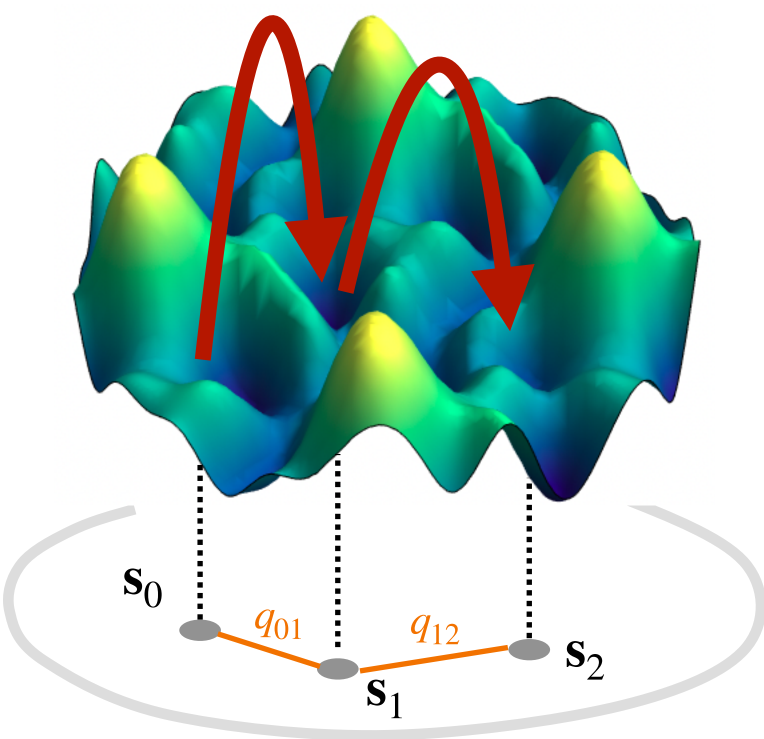

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Our question: How much correlations propagate along path?

Memoryless or avalanche-like?

Two scenarios. **"Avalanche-like" dynamics:** big activated jumps facilitate subsequent rearrangements

Observed in finite-d models like elastic manifolds

$$t^{(2)}(\epsilon_2 \,|\, \epsilon_1, \epsilon_0) > t^{(1)}(\epsilon_2 \,|\, \epsilon_1)$$

Ferrero, Foini, Giamarchi, Kolton, Rosso 2021

"Memoryless" dynamics: subsequent jumps uncorrelated to first

$$t^{(2)}(\epsilon_2 \,|\, \epsilon_1, \epsilon_0) = t^{(1)}(\epsilon_2 \,|\, \epsilon_1)$$

Dyre 1987, Bouchaud 1992, Gayrard 2019

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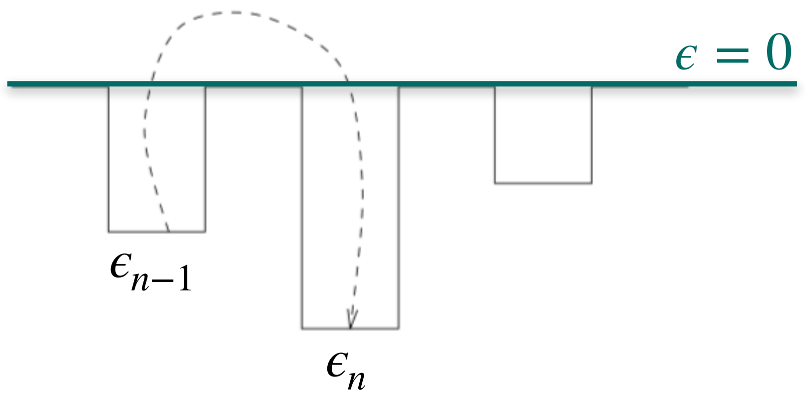
Memoryless dynamics assumed in "**trap models**": effective, solvable models of activated dynamics in very non-convex landscapes. Assumes "uniform" energy barriers.

$$t^{(n)}(\epsilon_n \,|\, \epsilon_{n-1} \cdots \epsilon_0) = t(\epsilon_{n-1}) \propto e^{\beta \epsilon_{n-1}}$$



Correct description for Random Energy Model (no correlations in energy landscape) Gayrard 2019

For p-spin models, unclear Stariolo, Cugliandolo 2019, 2020

Mapping into landscape geometry

Two scenarios, rephrased.

"Avalanche-like" dynamics: big activated jumps facilitate subsequent rearrangements

Observed in finite-d models like elastic manifolds

$$t^{(2)}(\epsilon_2 | \epsilon_1, \epsilon_0) > t^{(1)}(\epsilon_2 | \epsilon_1) \longrightarrow q^{(2)}_{\max}(\epsilon_2 | \epsilon_1, \epsilon_0) > q^{(1)}_{\max}(\epsilon_2 | \epsilon_1)$$

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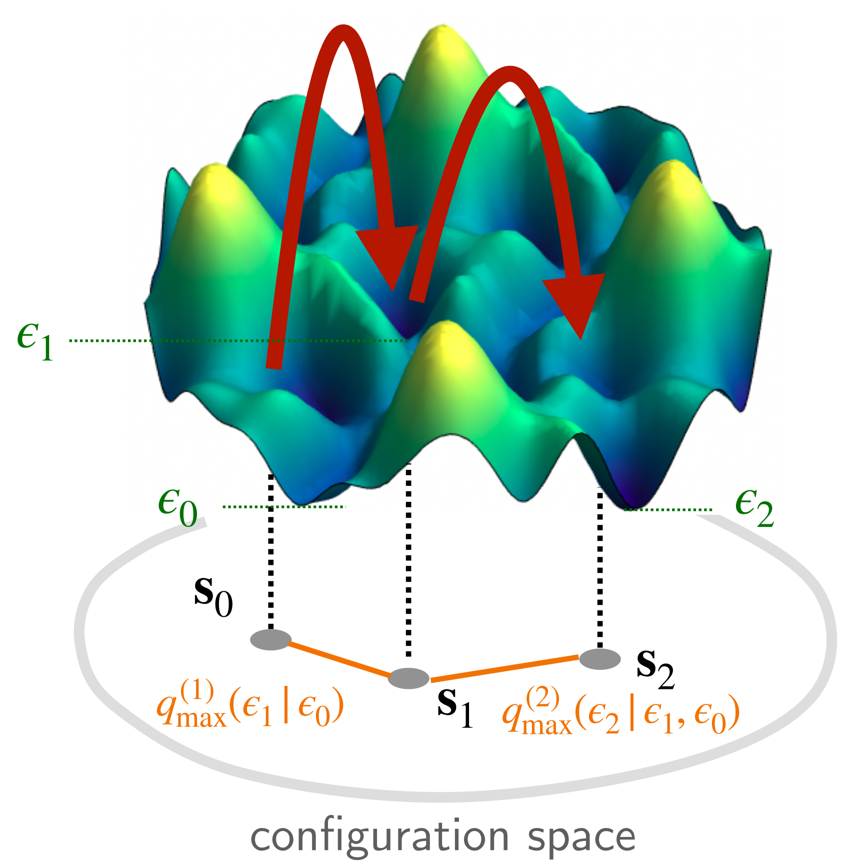


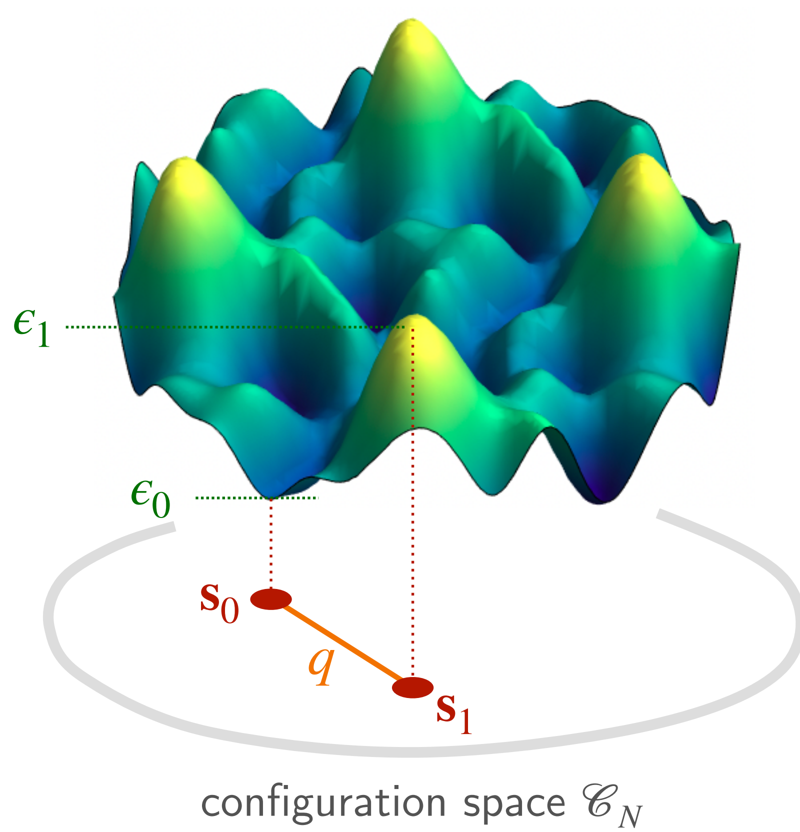
Part II

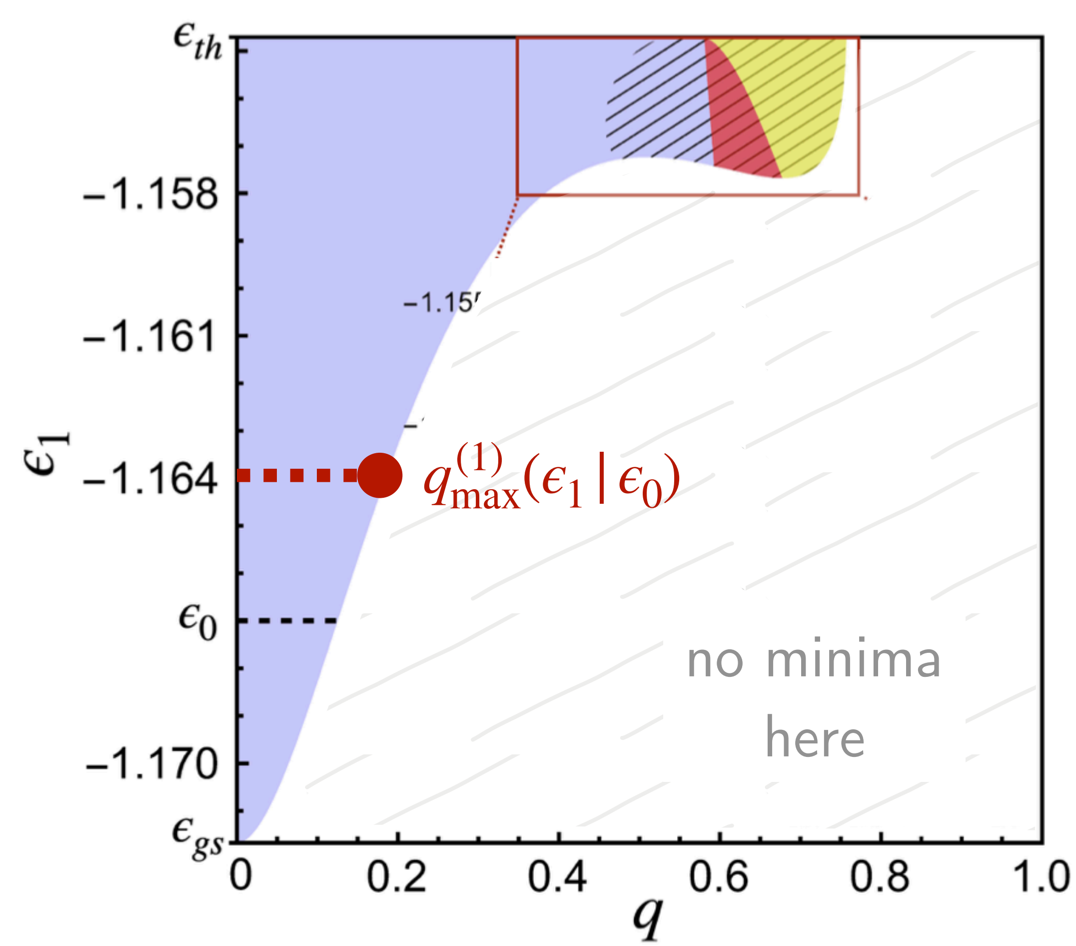
Landscape's geometry & hints on activation

Local geometry: two-point complexity

- Select minimum $\mathbf{s}_0$ with energy ϵ_0 - flat measure
- Count number $\mathcal{N}(\epsilon_1 | \epsilon_0, q)$ of minima $\mathbf{s}_1$ at energy ϵ_1 and $q = \frac{\mathbf{s}_0 \cdot \mathbf{s}_1}{N}$
- Two-point complexity $\Sigma^{(2)}(\epsilon_1 | \epsilon_0, q) = \lim_{N\to\infty} \left\langle \frac{1}{N} \log \mathcal{N}(\epsilon_1 | \epsilon_0, q) \right\rangle_0$ → flat average over minima $\mathbf{s}_0$

VR, Biroli, Cammarota 2019



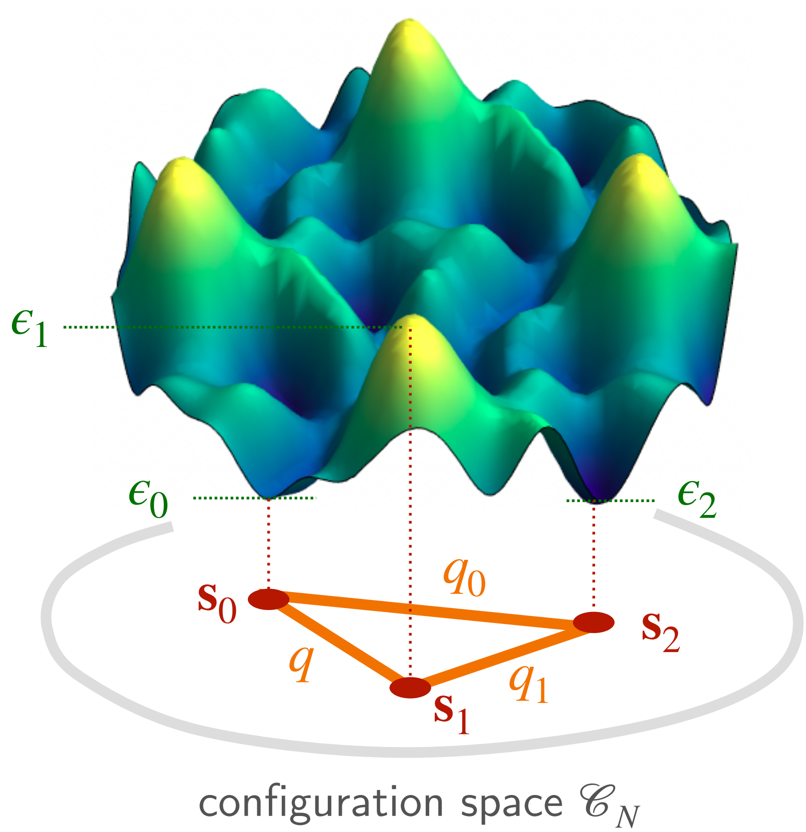


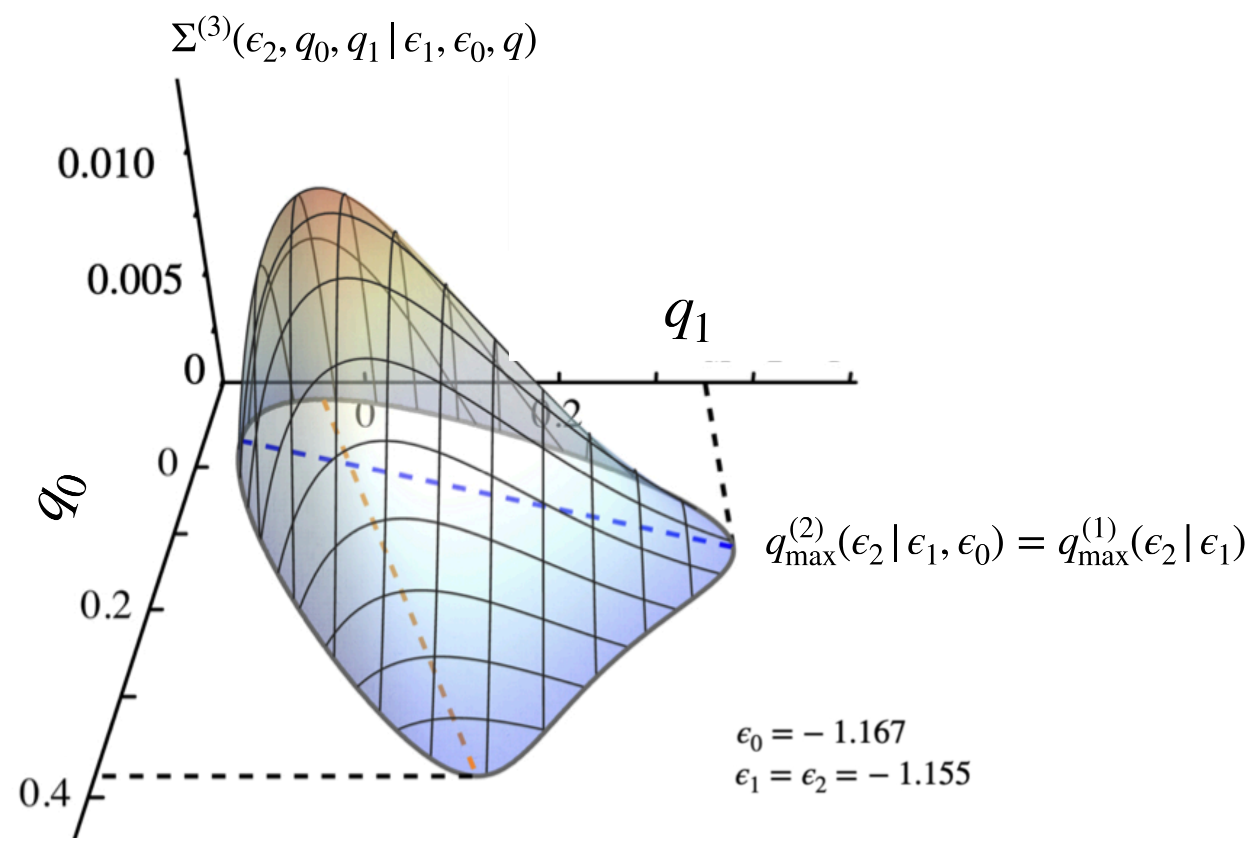
Local geometry: three-point complexity

- Select minimum $\mathbf{s}_0$ with energy ϵ_0 - flat measure
- Select minimum $\mathbf{s}_1$ with energy ϵ_1 and $q = \frac{\mathbf{s}_0 \cdot \mathbf{s}_1}{N}$- flat measure
- Count number $\mathcal{N}(\epsilon_2, q_0, q_1 \,|\, \epsilon_1, \epsilon_0, q)$ of minima $\mathbf{s}_2$ at energy ϵ_2 and given overlaps
- Three-point complexity $\Sigma^{(3)}(\epsilon_2, q_0, q_1 \,|\, \epsilon_1, \epsilon_0, q) = \lim_{N\to\infty} \left\langle \frac{1}{N} \log \mathcal{N}(\epsilon_2, q_0, q_1 \,|\, \epsilon_1, \epsilon_0, q) \right\rangle_{0,1}$

flat average over minima $\mathbf{s}_0$ and $\mathbf{s}_1$

Pacco, Rosso, VR 2024





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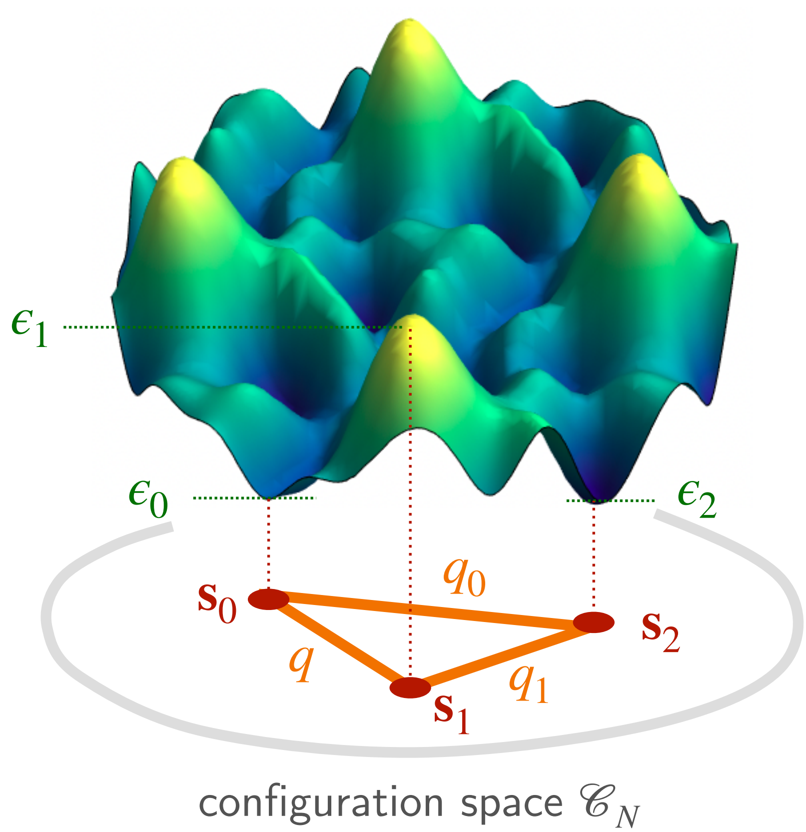
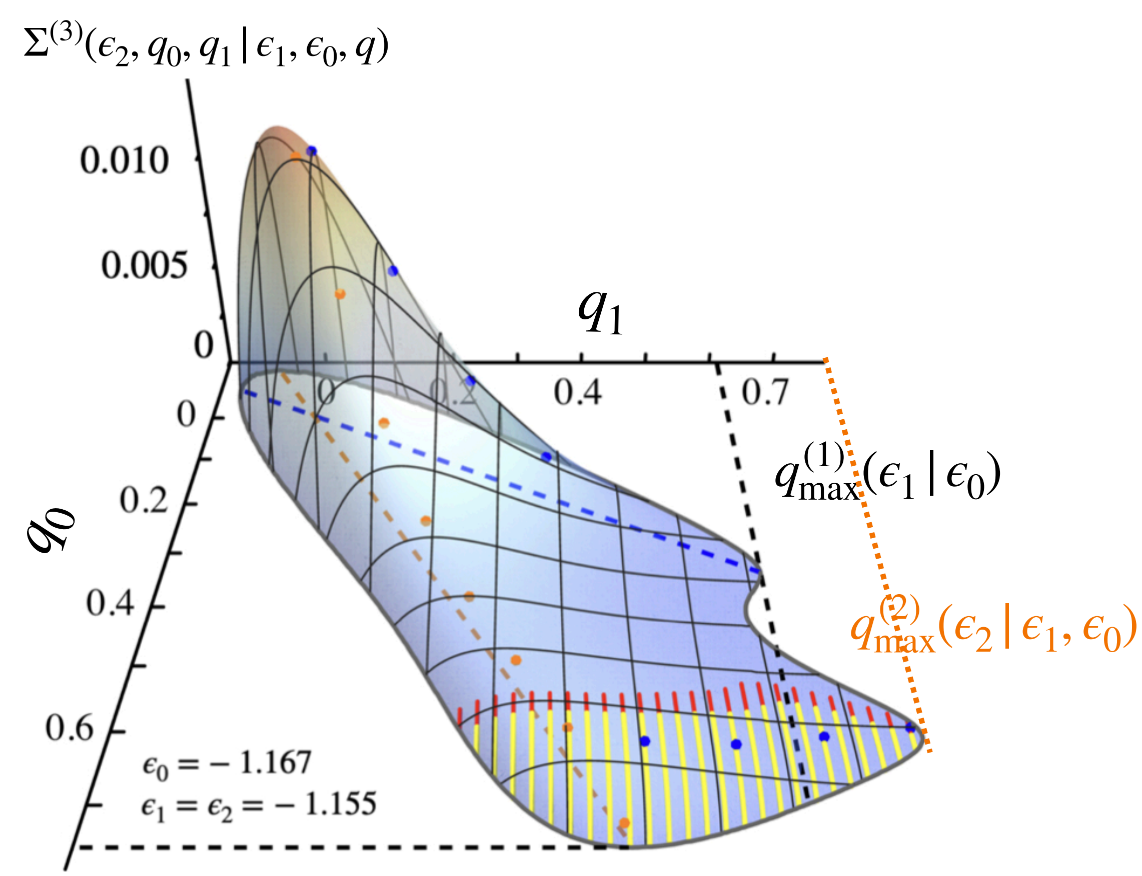




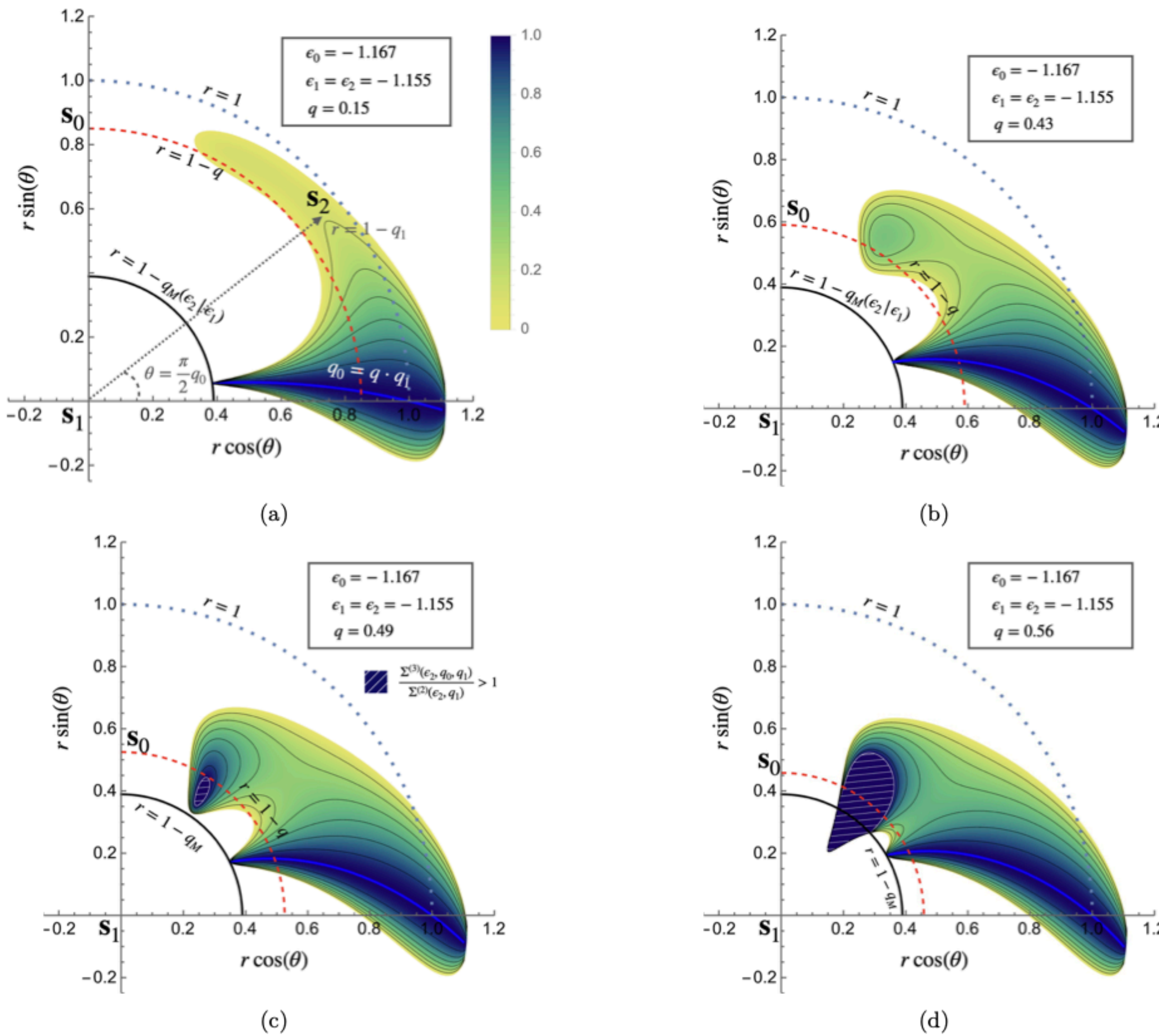
Landscape's transitions

Depletion

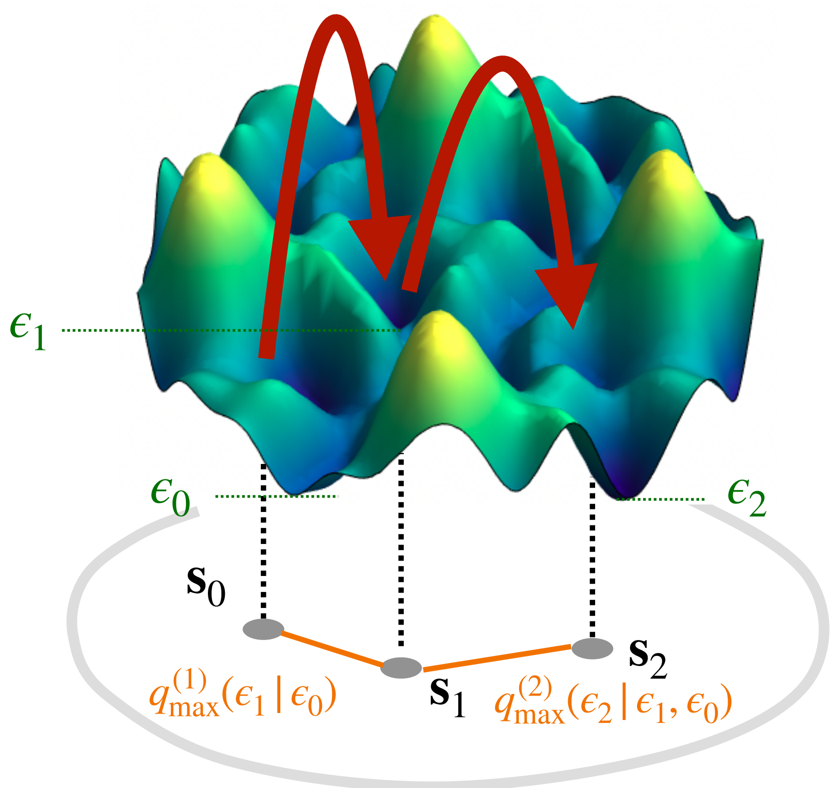
Non-monotonicity

Accumulation

Clustering



Hints on activated dynamics



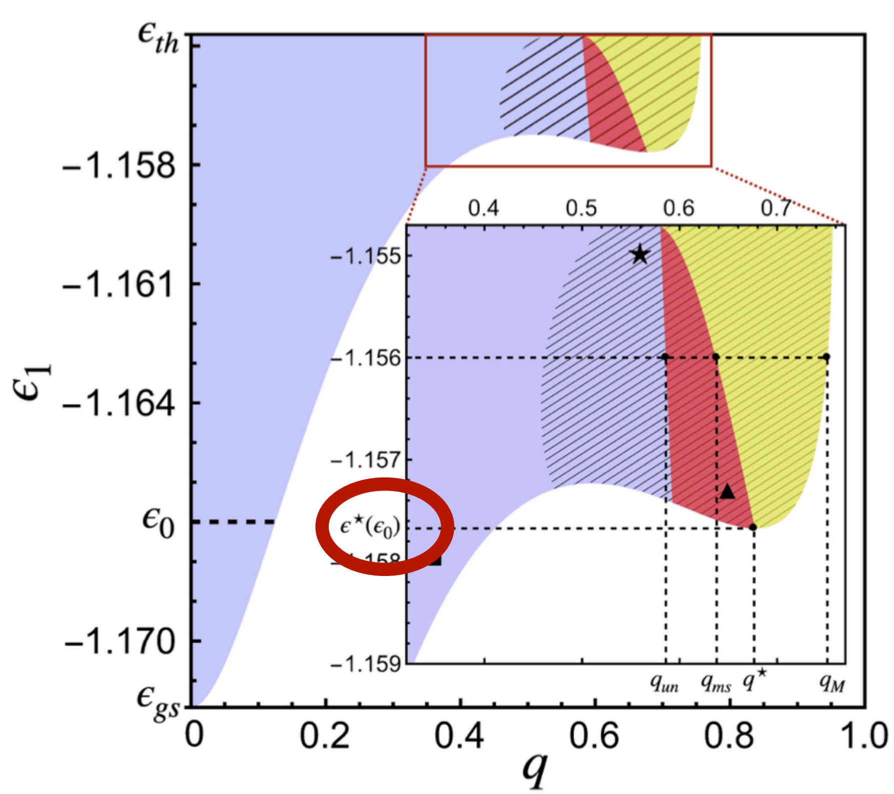
We identify a **"critical" energy curve $\epsilon^*(\epsilon)$:**

- Minima at $\epsilon_1 > \epsilon^*(\epsilon_0)$ and $\epsilon_2 < \epsilon^*(\epsilon_1)$ are such that $q^{(2)}_{\min}(\epsilon_2 | \epsilon_1, \epsilon_0) > q^{(1)}_{\min}(\epsilon_2 | \epsilon_1)$

  → **"avalanche-like" scenario**

- In all other cases, $q^{(2)}_{\min}(\epsilon_2 | \epsilon_1, \epsilon_0) = q^{(1)}_{\min}(\epsilon_2 | \epsilon_1)$

  → **"memoryless" scenario**



Jumps to equal-energy or lower-energy minima are memoryless: no traces of thermal avalanches, unlike finite-dimensional systems.

For details:

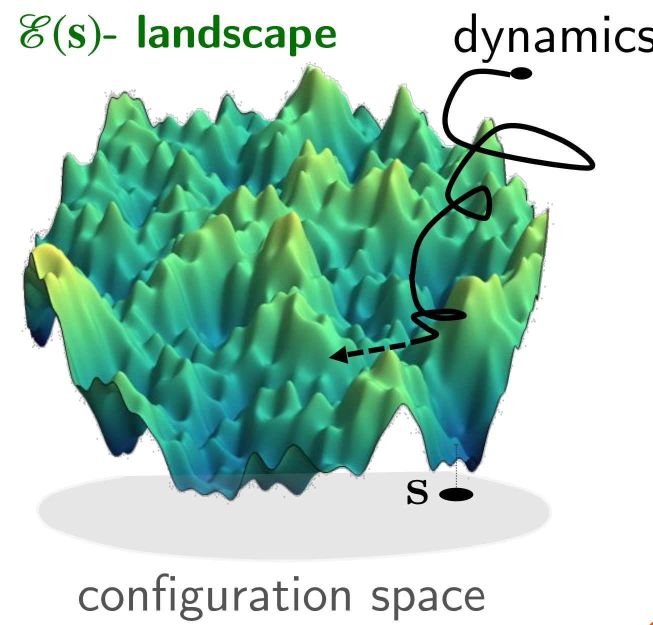
`A. Pacco, A. Rosso, VR, arXiv:2410.18010`

Summary.

Spherical p-spin is a good mean-field model for glasses.
Out of-equilibrium, relaxational dynamics is understood via DMFT.
Out-of-equilibrium, activated dynamics is open theory problem.

Topology

entropy of metastable states at arbitrary energy; minima-saddles transition at threshold



Geometry

entropy of triplets of metastable states at arbitrary energy and distance in configuration space

Dynamics

Relaxational:
dynamics stuck where first metastable states appear; aging due to flatness of landscape

Activated:
jumps to minima at equal/lower energy are "memoryless"; correlations when jumping at higher energy