Criticality and marginal stability of frictionless shear jammed spheres.

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Outline

- The jamming transition.
- Criticality of the jamming transition.
- Marginal stability at jamming transition.
- Jamming through shear
- Results

Granular packings are a familiar class of material.

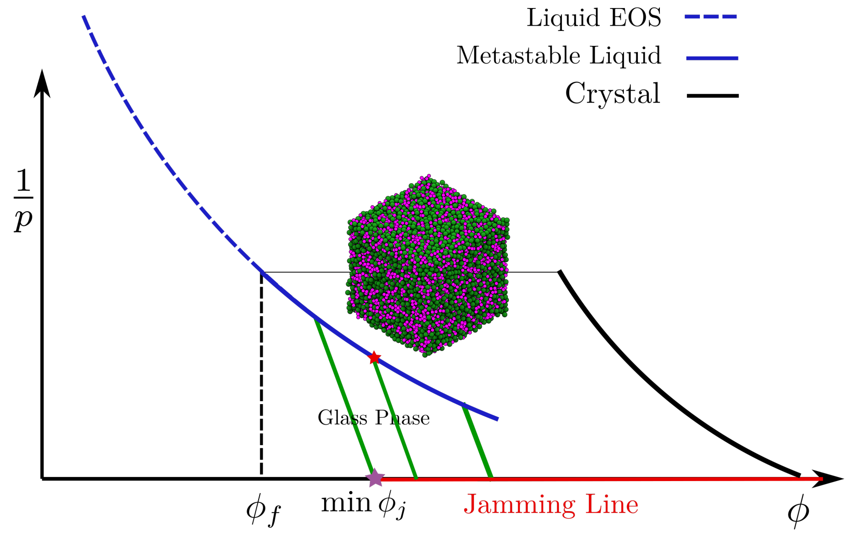
- Granular materials are comprised of particles which are large enough that they are not subject to thermal fluctuations.
- They defy conventional classification into liquids or solids.
- The transition of granular materials between a flowing state and a rigid state is called the **jamming transition.**

Hard-spheres are the simplest model for studying the jamming transition

$$U(|\vec{r}_{ij}|) = \begin{cases} \infty & |\vec{r}_{ij}| < \frac{\sigma_i + \sigma_j}{2} \\ 0 & otherwise \end{cases}$$

Inter-particle Potential

- A Hard-sphere system is a collection of non-overlapping spheres in a given volume.
- Despite the simplicity of the model hard spheres can exist as liquid, metastable liquid, glass or a crystal.
- Compression of hard-sphere liquids will result in a transition from flowing to a rigid state at the jamming density.
- The jamming density is not unique and depends on the density of the hard-sphere liquid to which the compression is done[1].



The density at which jamming happens is identified by ϕ_j

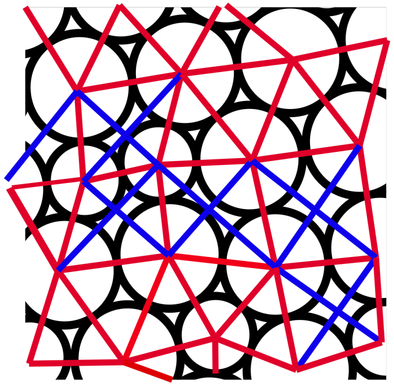
[1]Chaudhuri, Pinaki, Ludovic Berthier, and Srikanth Sastry. "Jamming transitions in amorphous packings of frictionless spheres occur over a continuous range of volume fractions." Physical review letters 104.16 (2010): 165701.

The jamming density shows many features of a critical point.

- Configurations at the jamming point are isostatic, the number of contacts in the system exactly balances the number of degrees of freedom. $z \approx 2d$ where z is the average co-ordination number.
- Pressure, excess contact number scales with distance from the jamming point [1]

$$P \sim (\phi - \phi_j); \delta z \sim (\phi - \phi_j)^{\frac{1}{2}}$$

- The vibrational density of states of systems close to the jamming transition have excess low energy modes compared to Debye solid.
- Interparticle force and gap distributions are described by powerlaws. $P(f) \sim f^{\theta}; P(h) \sim h^{-\gamma}$



Blue bonds represent the gaps between particles and the red bonds represent the contacts.

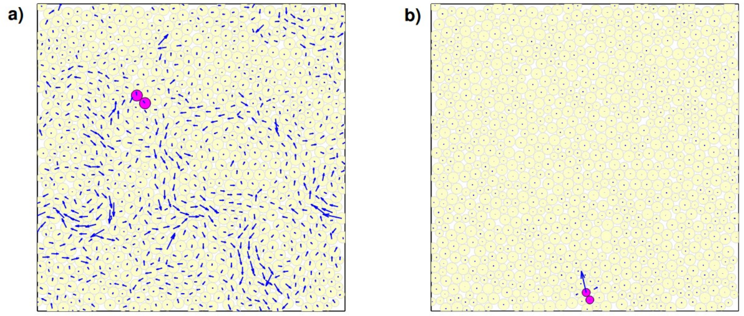
[1]O'hern, Corey S., et al. "Jamming at zero temperature and zero applied stress: The epitome of disorder." Physical Review E 68.1 (2003): 011306.

Stability of the iso-static system against compression constraints the force and the gap distribution exponents.

- A contact can only be opened till another contact is formed elsewhere in the system.
- One can work out the condition such that there is no contact in the system that can be opened to increase the density of the system[1].
- This condition is a constraint between the gap exponent and the force exponents[2].

$$\gamma \geq \frac{1}{2+\theta_e};\gamma \geq \frac{1-\theta_l}{2}$$

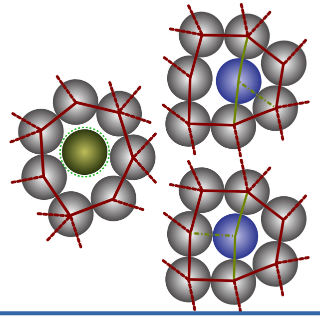
- θ_e is the exponent for forces corresponding to strongly coupled (extended) contact.
- θ_l is the exponent for forces corresponding to weakly coupled (localized) contacts



Extended contacts

Localized contacts

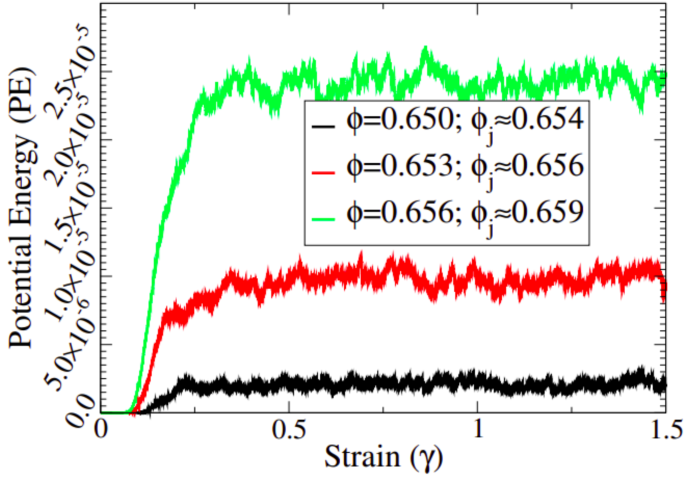
Localized contacts can be identified as those belonging to “bucklers” or particles with d+1 contacts[3].



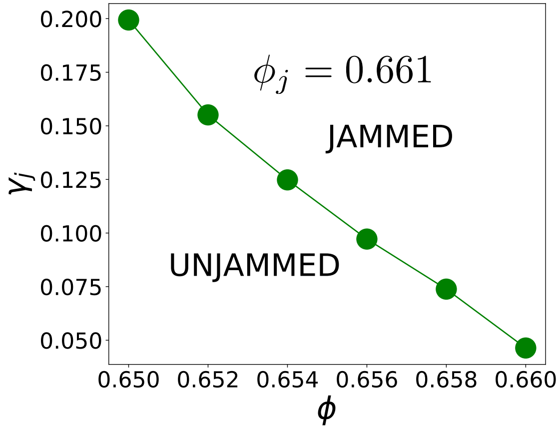
[1] Wyart, Matthieu. "Marginal stability constrains force and pair distributions at random close packing." Physical review letters 109.12 (2012): 125502.
[2] Lerner, Edan, Gustavo Düring, and Matthieu Wyart. "Low-energy non-linear excitations in sphere packings." Soft Matter 9.34 (2013): 8252-8263.
[3] Charbonneau, Patrick, et al. "Jamming criticality revealed by removing localized buckling excitations." Physical review letters 114.12 (2015): 125504.

Jamming through shear

- Originally observed in experimental frictional systems.
- Numerically shear jammed configurations can be observed for frictionless systems in deeply annealed sphere packings[1].
- Unjammed configurations generated by decompressing configurations at jamming density undergo shear jamming when subjected to AQS shear deformation.
- A configuration with density ϕ such that $\min\{\phi_i\} < \phi < \phi_j$ will undergo shear jamming at γ_j
- The locus of (ϕ, γ_j) divides the strain-density space into jammed and unjammed regions.

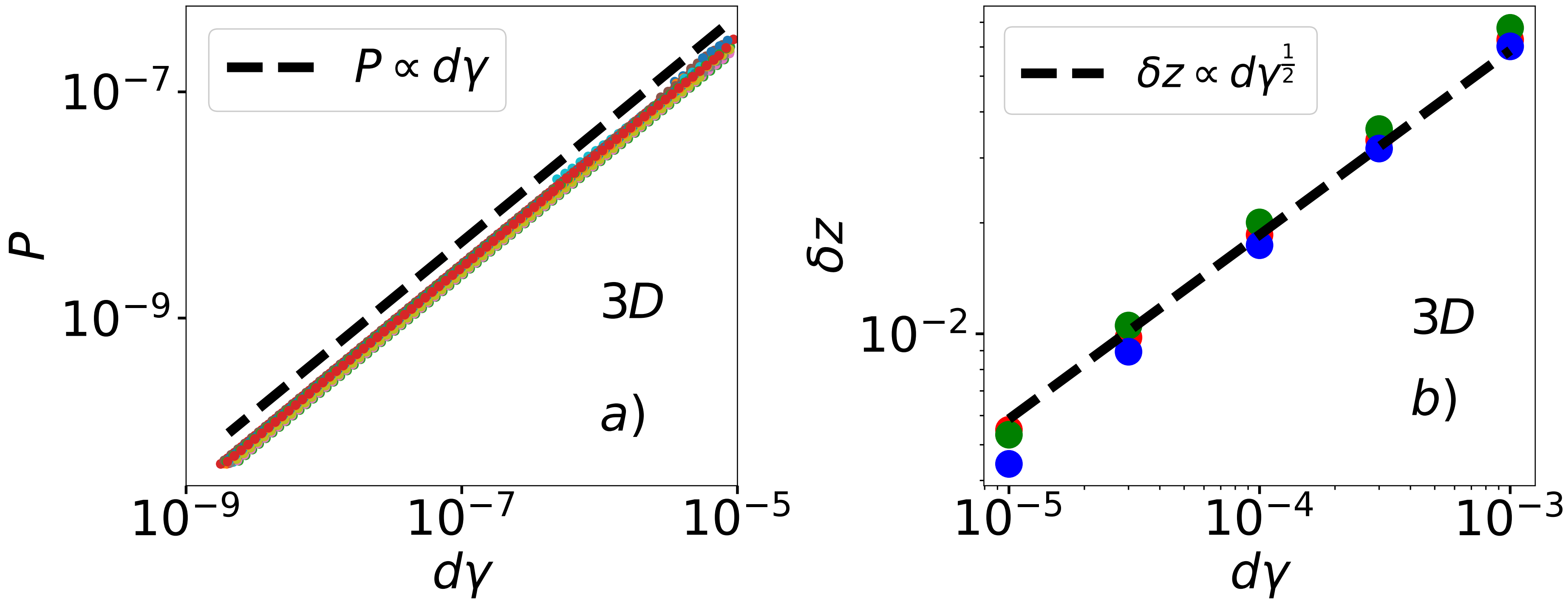


How does the shear jamming point compare with isotropic jamming point?



[1]Babu, Varghese, et al. "Dilatancy, shear jamming, and a generalized jamming phase diagram of frictionless sphere packings." Soft Matter 17.11 (2021): 3121-3127.

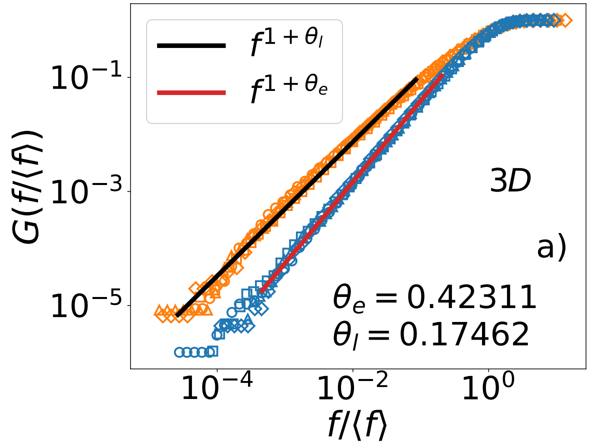
Criticality of the shear jamming transition



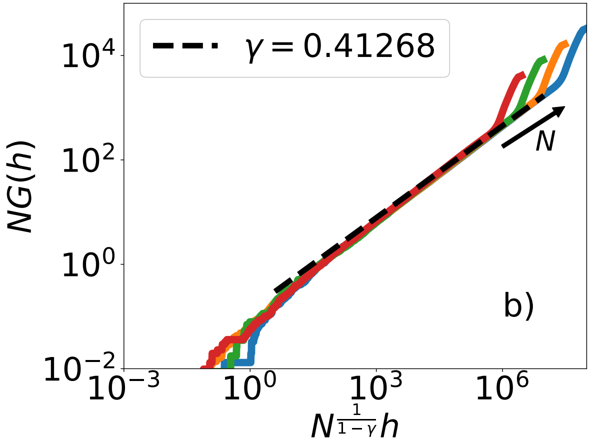
$P \sim d\gamma$ and $\delta z \sim d\gamma^{\frac{1}{2}}$ just as in isotropic jamming with $d\gamma$ replacing $d\phi$.

Marginal Stability of shear jammed packing

$$G(x) = \int_0^x P(x')dx'$$



The exponents measured for shear jammed configurations from the force distribution after excluding bucklers shows good agreement with the mean field exponent.



The gap distribution at shear jamming is described by the mean field exponent.

$$\gamma \geq \frac{1}{2+\theta_e}; \gamma \geq \frac{1-\theta_l}{2}$$

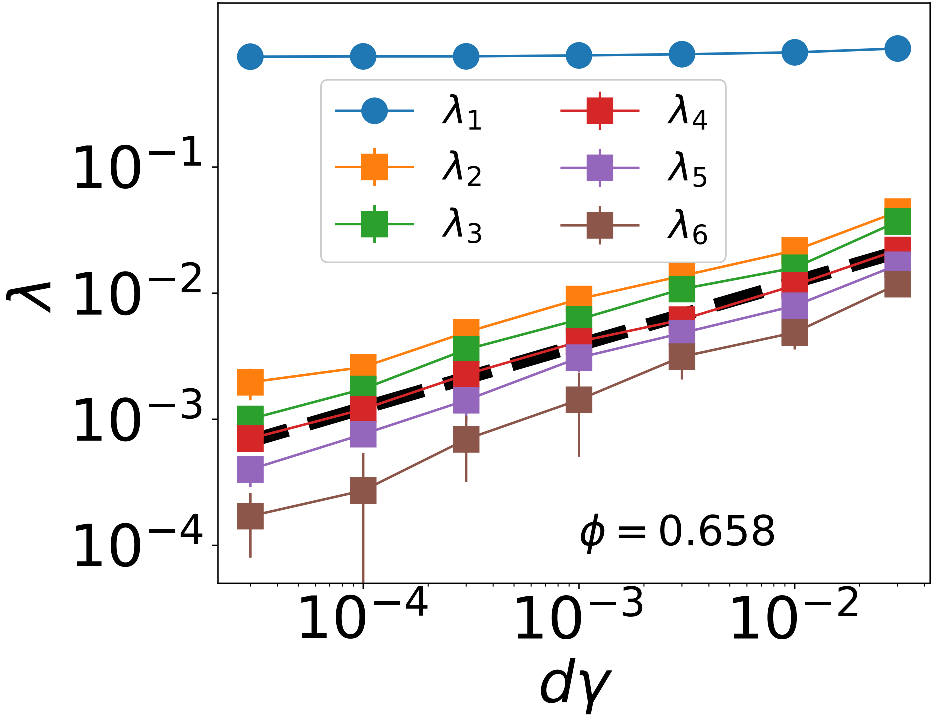
Shear jammed configurations are marginally stable.

Calculated for isostatic configurations (single self stressed state).

Elastic properties of shear jammed configurations.

The stiffness matrix is given by $C_{\alpha\beta\kappa\lambda} = \frac{dU}{d\eta_{\alpha\beta} d\eta_{\kappa\lambda}}$ where U is the potential energy of the system.

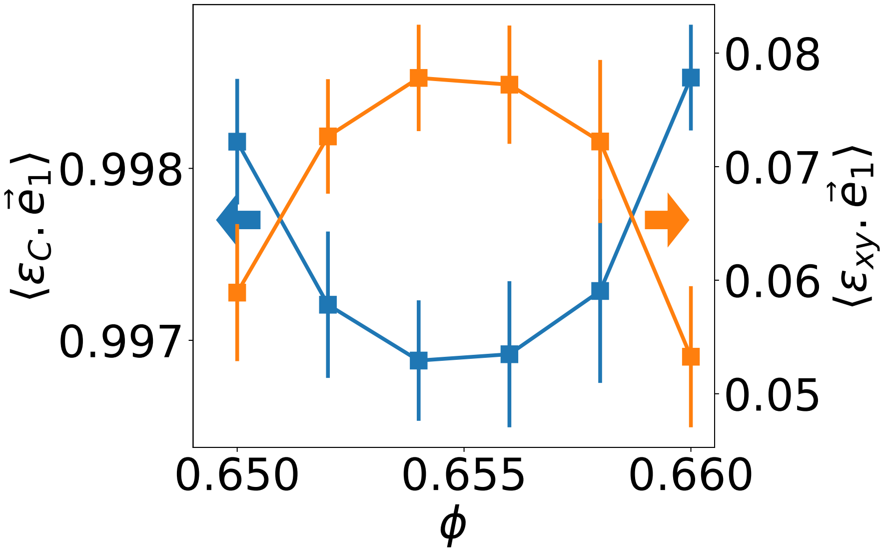
- At shear jamming, both bulk and shear modulus is non-zero, whereas in isotropic jamming only bulk modulus is non-zero at jamming.
- However for both isotropic and shear jamming transition, only the largest eigenvalue of the elastic matrix remains non-zero as we approach the jamming point.[1]
- The eigenstrain that corresponds to the largest eigenvalue has both shear and compressive components at shear jamming, while it has only compressive component at isotropic jamming.



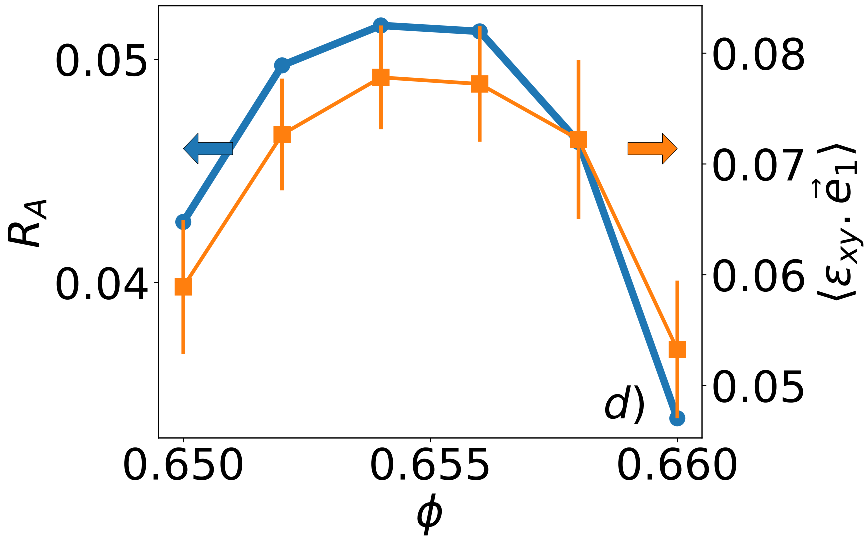
[1]Baity-Jesi, Marco, et al. "Emergent SO (3) symmetry of the frictionless shear jamming transition." Journal of Statistical Physics 167.3 (2017): 735-748.

The projection of shear component on the stiffest eigenvector is related to the anisotropy of the shear jammed configurations.

- The anisotropy of the shear jammed configurations at different densities when measured using fabric anisotropy R_A shows non-monotonic behavior.



Projection of compressive and shear strain on the dominant eigenstrain at different densities.



The projection of shear strain on the dominant eigenstrain of the stiffness matrix is proportional to the anisotropy in the system.

Conclusions.

- Jammed packings can be generated through compression and shear deformation.
- Shear jamming transition shows the same criticality as compression driven jamming transition.
- Shear jammed packings are marginally stable.
- The dominant eigenstrain of the stiffness matrix of isotropically jammed configurations and that of shear jammed configurations are related by a rotation.

Reference: **Varghese Babu, and Srikanth Sastry. "Criticality and marginal stability of the shear jamming transition of frictionless soft spheres." arXiv preprint arXiv:2201.09726 (2022).**