Entanglement in Quantum Fields

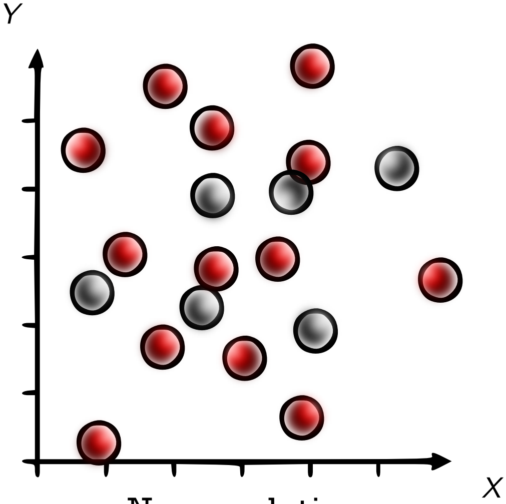
From the Early Universe to the Lab

Patricia Ribes Metidieri
(1851 Research Fellow)

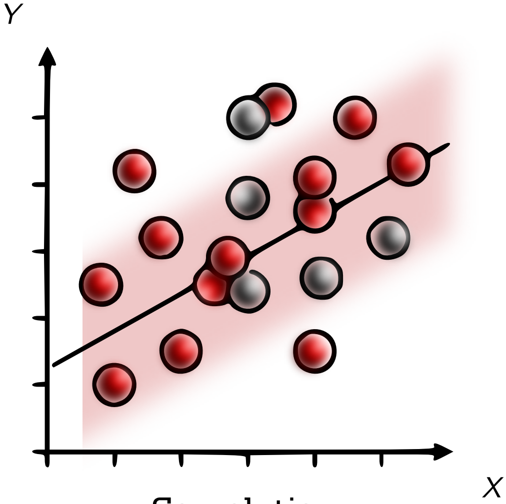
Plan

1. MOTIVATION

What this talk is about: Correlations

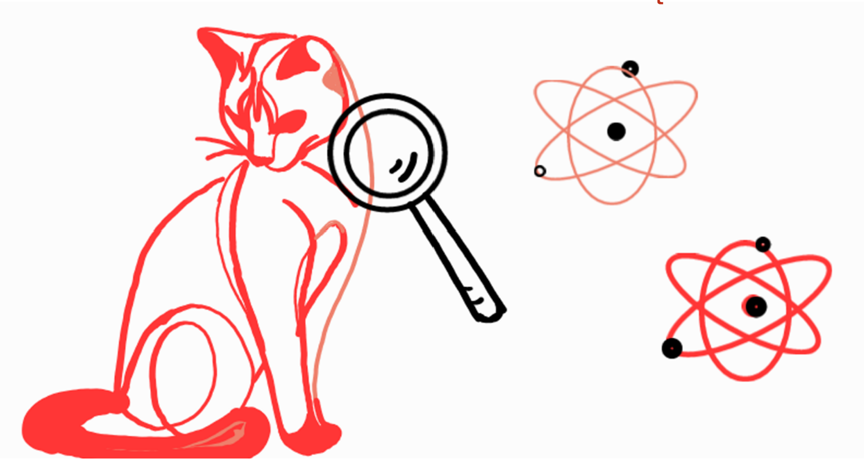


No correlations



Correlations

Classical	Quantum

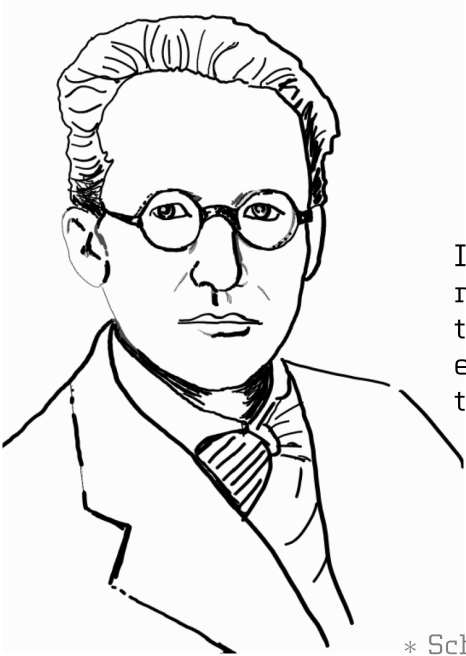


- Trajectories
- Deterministic
- Correlations (as before)

- Uncertainty principle
- Probabilistic
- Additional correlations

ENTANGLEMENT

A type of correlation in quantum systems
with no classical counterpart



I would not call [**entanglement**] one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.*

E. Schrödinger

∗ Schrödinger (1935)

Why Does Entanglement Matter?

Quantum Technologies

Fundamental Physics

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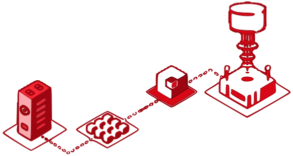
Quantum Technologies

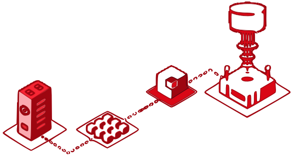


Quantum
computation

Fundamental Physics

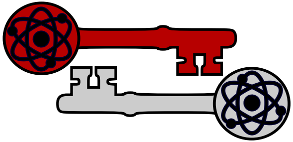
Why Does Entanglement Matter?

Quantum Technologies



Quantum computation

Quantum key distribution



Fundamental Physics

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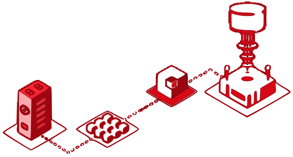

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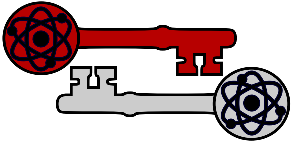

Quantum teleportation

Fundamental Physics

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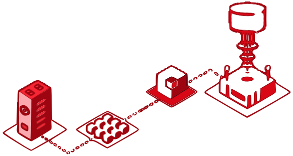

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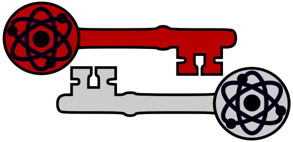

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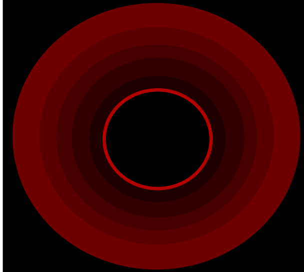

Black hole information loss

Why Does Entanglement Matter?

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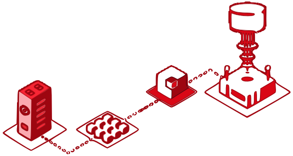

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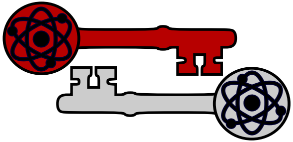

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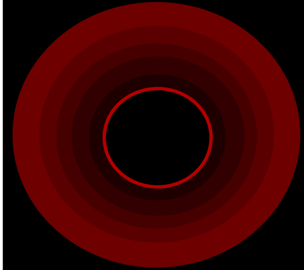

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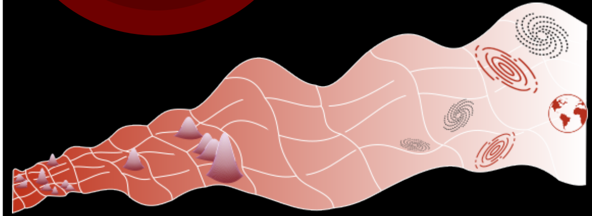
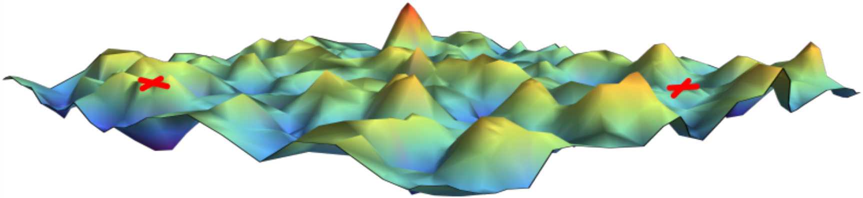


Quantum signals from the early universe

A Surprising Property of QFT

Consider a quantum field in the vacuum



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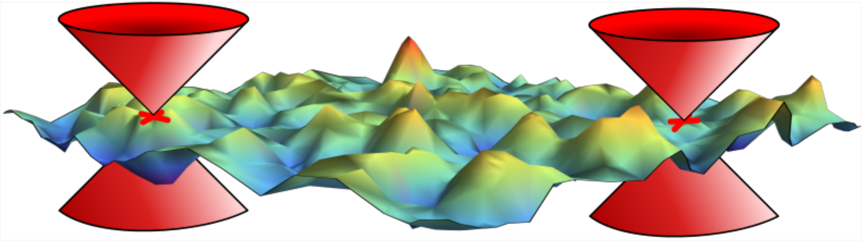

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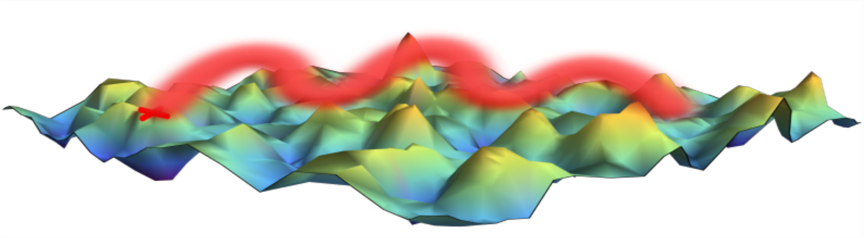

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No signal can travel between these points.
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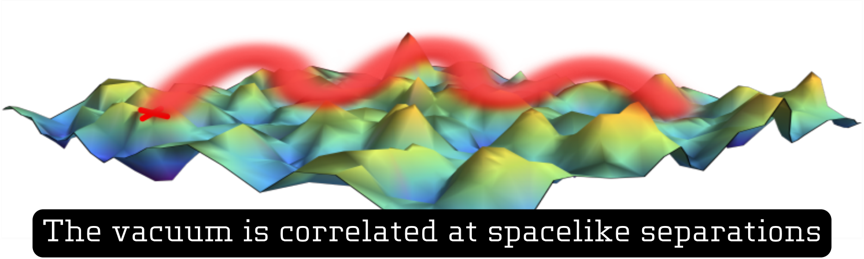

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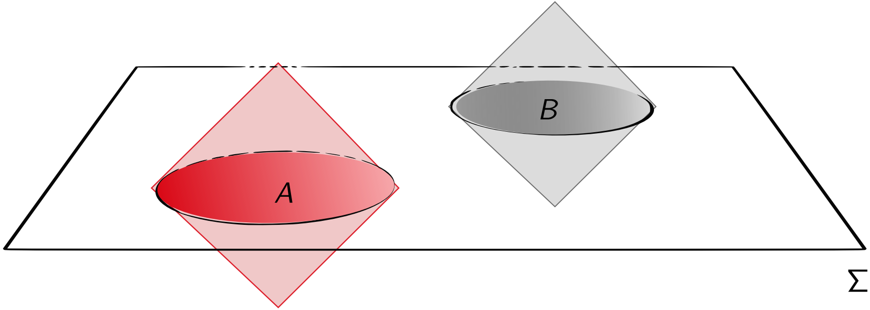
No signal can travel between these points.
The vacuum two-point function is nonzero.

Entanglement is built into the very structure of the vacuum in QFT

The Reeh-Schlieder Theorem implies that the vacuum is so entangled that local operations on it can approximate any global state in the theory.*



* Reeh, Schlieder (1961); See Witten (2018) for the interpretation.

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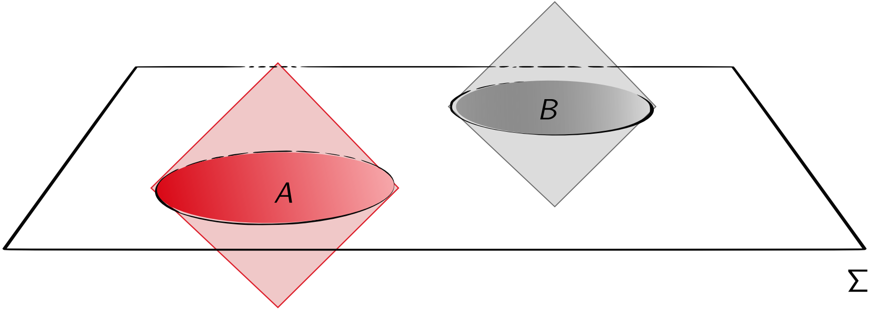
How much entanglement is there and how is it distributed?



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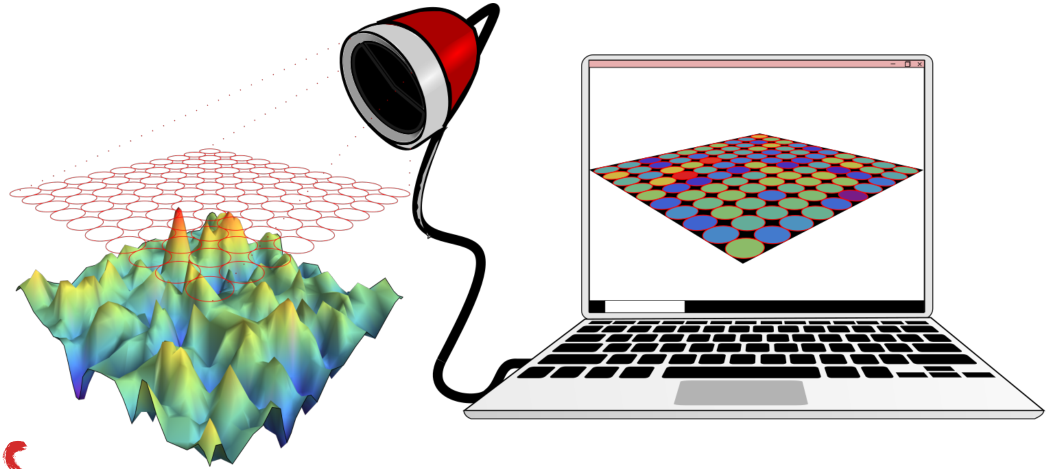
GOAL I:

Present a complementary approach for analyzing entanglement in QFT *

* Bianchi and Satz (2019); Martin and Vennin (2021); Hackl and Bianchi (2021); Agullo, Bonga, and Ribes-Metidieri (2021)

Physical Motivation



Framework Overview

Features:

Truncation of degrees of freedom + **Gaussian formalism (Gaussian states + free evolution)**

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Truncation of degrees of freedom	+	Gaussian formalism (Gaussian states + free evolution)

⬇

(1) We avoid UV divergencies (Type I algebras).

(2) We can compute reduced states.

(3) We can use results from quantum information.

(4) We retain features of the full QFT.

Framework Overview

Features:

Truncation of degrees of freedom	+	Gaussian formalism (Gaussian states + free evolution)

Limitations:

- Limited to free theories and Gaussian states
- We focus on a finite subset of degrees of freedom (infinitely many of them!)

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Truncation of degrees of freedom + Gaussian formalism (Gaussian states + free evolution)

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- Limited to free theories and Gaussian states
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Focus on universal properties!

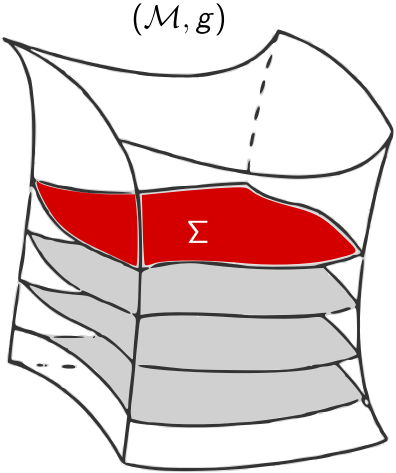
GOAL II:

Discuss two interesting applications *

*Agullo, Bonga, Ribes-Metidieri (2024); Ribes-Metidieri, Agullo, Bonga (2025); Agullo, Martín-Martínez, Nadal-Gisbert, Ribes-Metidieri, Yamaguchi (2025); Agullo, Delhom, Parra, Ribes-Metidieri (In preparation).

2. FRAMEWORK

Physical Setup

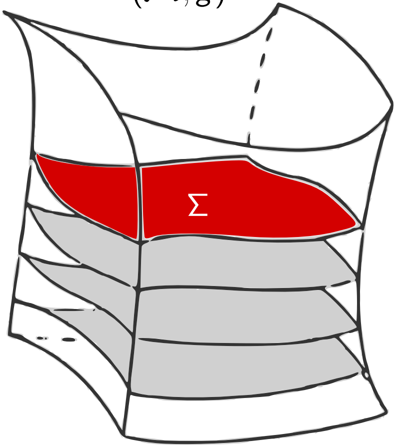


$\hbar = G = c = 1$

Free quantum scalar field $(\hat{\phi}(\vec{x}), \hat{\pi}(\vec{x}))$ propagating on a **globally hyperbolic** spacetime $(\mathcal{M}, g)$**:**

Physical Setup

$(\mathcal{M}, g)$



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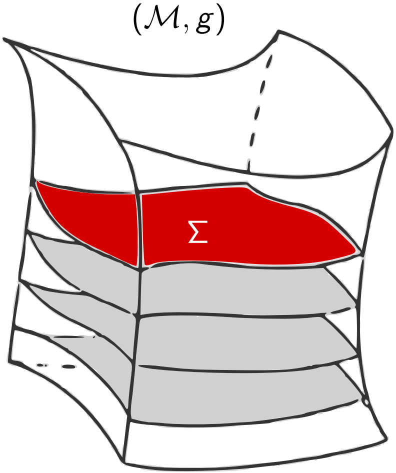

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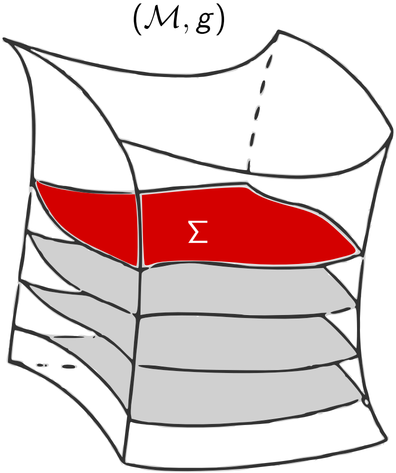

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- Phase space $\Gamma \equiv$ vector space.
- Symplectic form on phase space:

$$\omega(\gamma_1, \gamma_2) = \int_\Sigma \mathrm{d}\Sigma \left(f_1(\vec{x}) g_2(\vec{x}) - f_2(\vec{x}) g_1(\vec{x}) \right),$$

where $\gamma_1 = (g_1, f_1) \in \Gamma$ and $\gamma_2 = (g_2, f_2) \in \Gamma$.

Physical Setup

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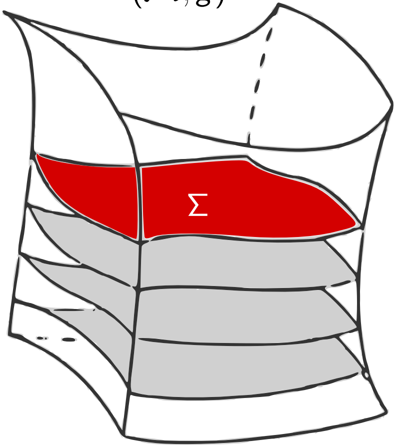

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- Phase space $\Gamma \equiv$ vector space.
- Klein-Gordon product:

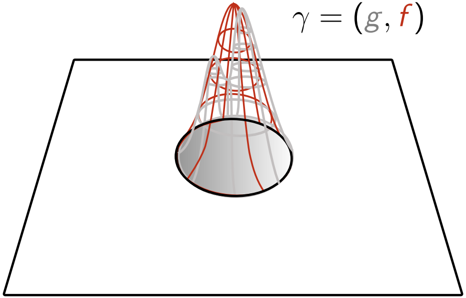
$$\langle \gamma_1, \gamma_2 \rangle = -i\,\omega(\gamma_1^*, \gamma_2), \text{ where } \gamma_1, \gamma_2 \in \Gamma_{\mathbb{C}}.$$

Notation

Convenient way of organizing linear operators

$$\gamma \in \Gamma_{\mathbb{C}} \to \hat{O}_\gamma = i \langle \gamma, \hat{R} \rangle \,,$$

where $\hat{R} = (\hat{\phi}(\vec{x}), \hat{\pi}(\vec{x}))$.



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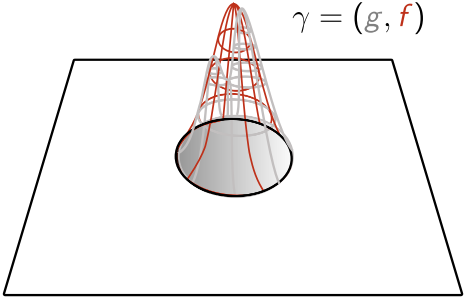

Local single mode: Algebra generated by $(\hat{O}_\gamma, \hat{O}_{\gamma'})$, with $[\hat{O}_\gamma, \hat{O}_{\gamma'}] = -i\omega(\gamma^\star, \gamma'^\star)\, \hat{1} \neq 0\,.$

Subsystems

Classical subsystem

↓

Symplectic subspaces

Quantum subsystem

↓

Subalgebra of observables

Subsystems

Classical subsystem → Symplectic subspaces

Quantum subsystem → Subalgebra of observables

Symplectic subspaces ⟷ Subalgebra of observables

$$\gamma \in A \subset \Gamma_{\mathbb{C}} \to \hat{O}_\gamma = i \langle \gamma, \hat{R} \rangle$$

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Classical subsystem → Symplectic subspaces

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Symplectic subspaces ↔ Subalgebra of observables

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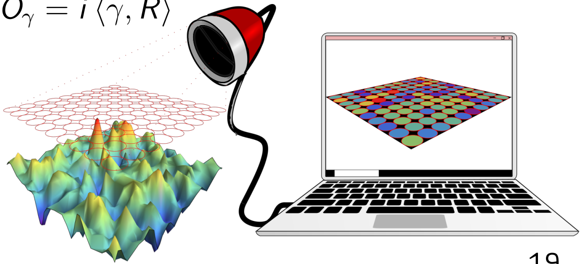
Focus on finite-dimensional subsystems



Gaussian States

Gaussian states are fully characterized by:

- $\mu(\gamma) := \langle \hat{O}^{\dagger}_{\gamma} \rangle$, $\gamma \in \Gamma_{\mathbb{C}}$, defines a **co-vector** in $\Gamma_{\mathbb{C}}$.

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covariance matrix (two-covariant tensor in $\Gamma_{\mathbb{C}}$)

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  covariance matrix (two-covariant tensor in $\Gamma_{\mathbb{C}}$)
- Higher correlations follow from Wick's theorem

Gaussian states and Kähler Structures

For **pure** Gaussian states, define

$$J := \omega^{-1}\sigma : \Gamma_{\mathbb{C}} \to \Gamma_{\mathbb{C}}$$

J defines a **complex structure**, i.e., $J^2 = -\mathbb{I}$, making (σ, ω, J) a Kähler space.*

* See, e.g. Bianchi and Hackl (2024) for a recent review.

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The spectrum of J encodes all information
about correlations and entanglement.

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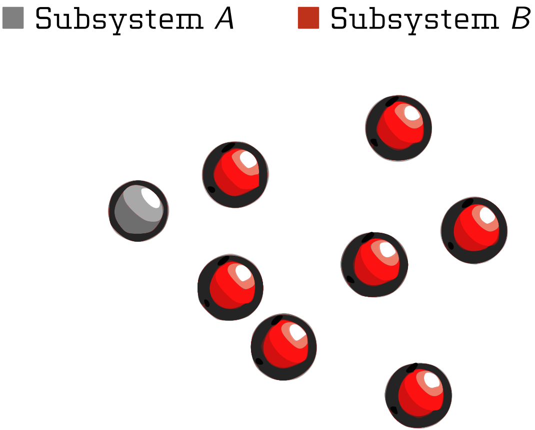
How is entanglement distributed?



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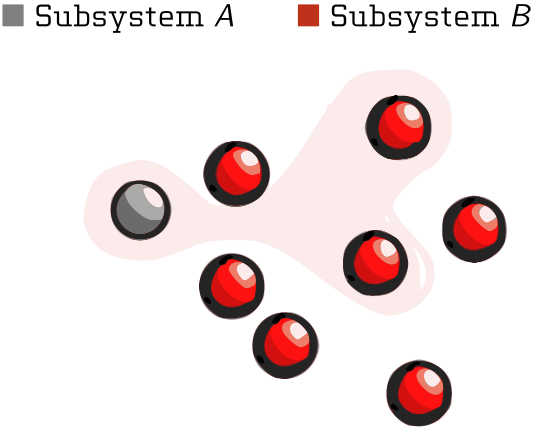
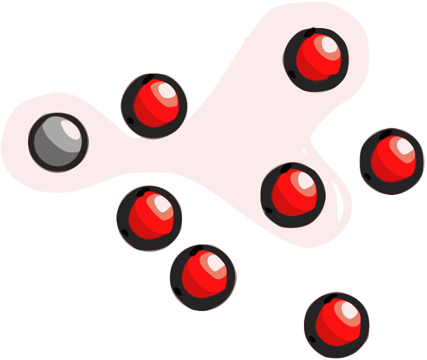


How is entanglement distributed?

The partner of A in the state J is the subsystem A_p that encodes all correlations and entanglement with A.$*$

Subsystem A Subsystem B



*Wald (1975); Botero and Reznik (2003); Hotta, Schützhold, and Unruh (2015); Trevison, Yamaguchi, and Hotta (2019); Agullo, Martin-Martinez, Nadal-Gisbert, Ribes-Metidieri, and Yamaguchi (2025)

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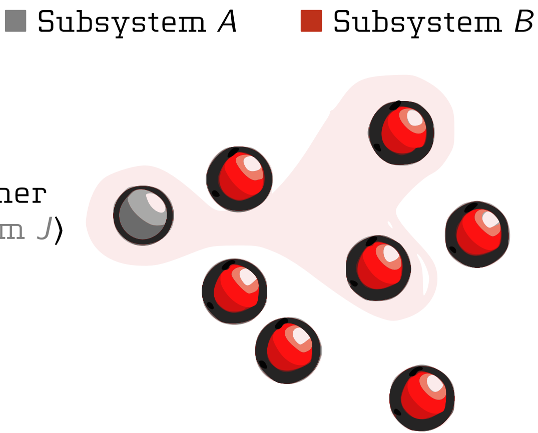

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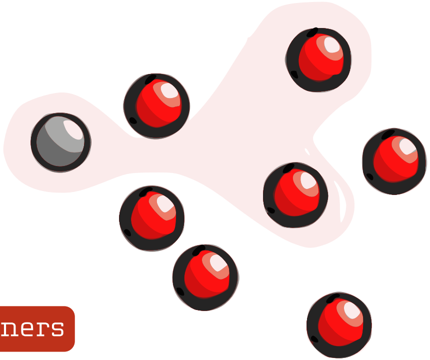
■ Subsystem A ■ Subsystem B



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■ For **mixed** Gaussian states, we have

Entanglement partners   Correlations partners

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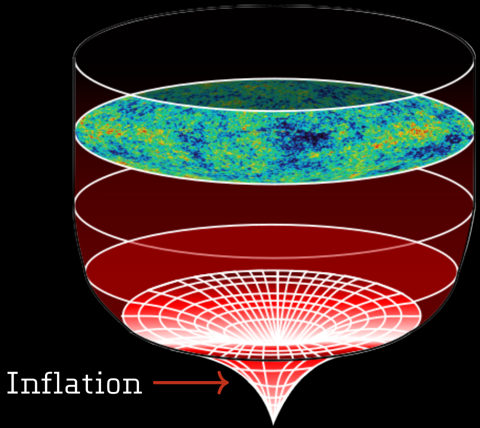
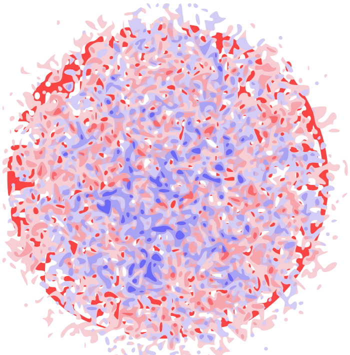
3. APPLICATIONS

Summary

Entanglement generation during inflation



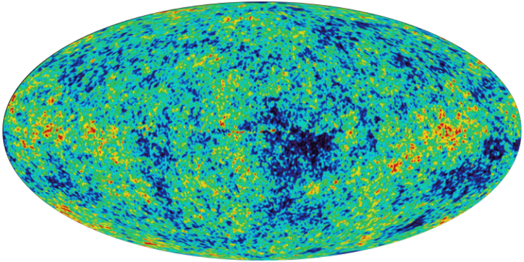
A proposal to probe QFT vacuum entanglement



Density profile of a 2-dimensional Bose-Einstein condensate.∗

∗ Adapted from Viermann et al. (2022)

3.1. Entanglement generation during inflation

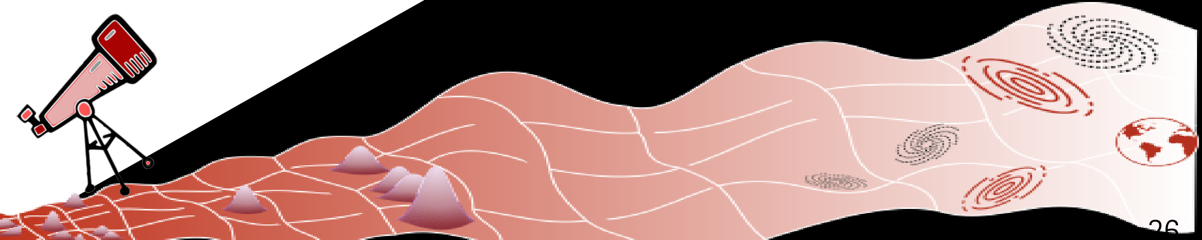


All we need is
classical physics...

Observations

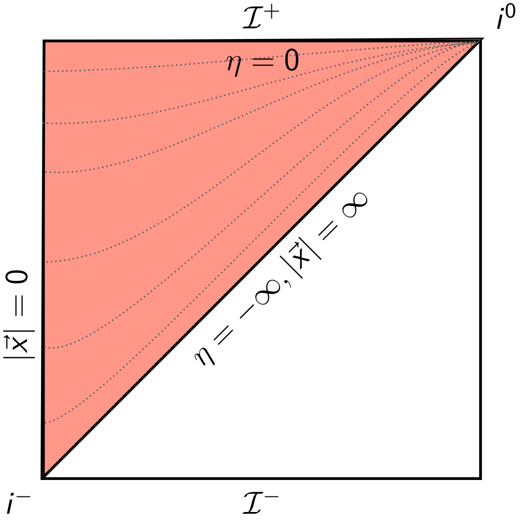
Theory

Cosmic structures
from QUANTUM
fluctuations



de Sitter spacetime ⟨Cosmological patch⟩

$$ds^2 = a(\eta)^2(-d\eta^2 + d\vec{x}^2),\ a(\eta) = -\frac{1}{H\eta},\ \text{and } \eta \in (-\infty, 0).$$



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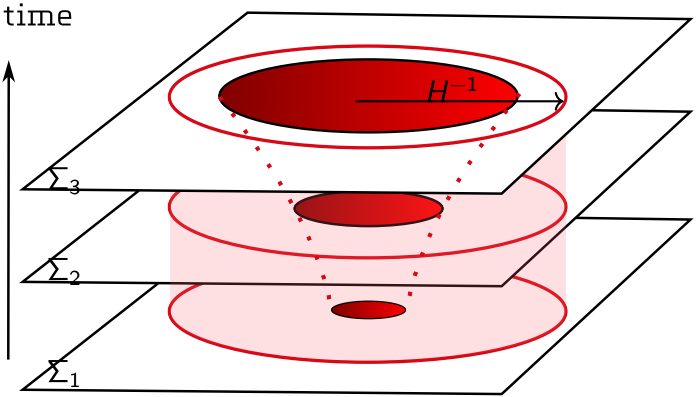
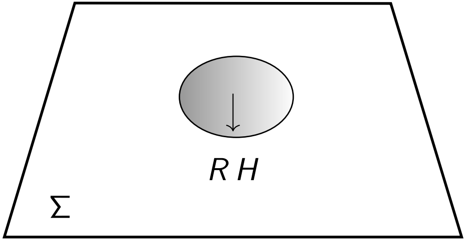


Setup: Local observables

Free, scalar field in de Sitter spacetime in the Bunch-Davies vacuum



H^{-1}: Hubble radius

Setup: Local observables

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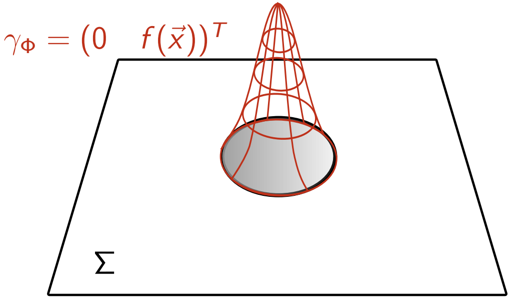

$$\hat{\Phi}[f] := \int_\Sigma d^3x \, f(\vec{x}) \hat{\phi}(\vec{x}) \,,$$

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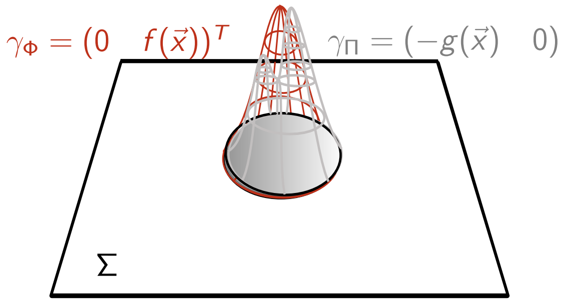

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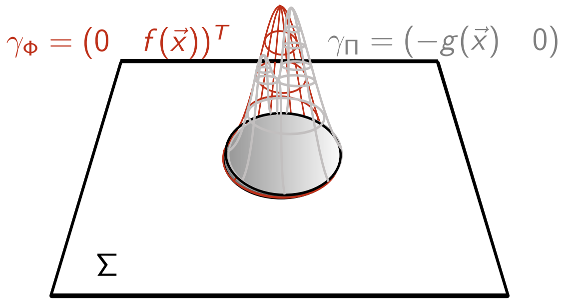

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Setup: Local observables

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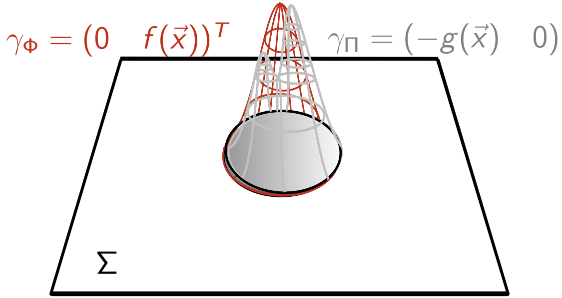

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$[\hat{\Phi}[f], \hat{\Pi}[g]] = i \rightarrow (\hat{\Phi}[f], \hat{\Pi}[g])$ is a single-mode

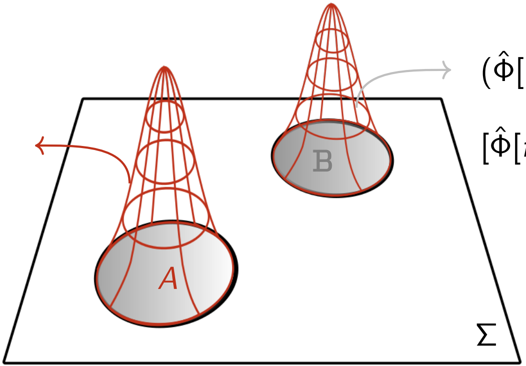
Setup: Local observables

Free, scalar field in de Sitter spacetime in the Bunch-Davies vacuum

Similarly for 2-mode subsystem

$(\hat{\Phi}[f_A], \hat{\Pi}[g_A])$

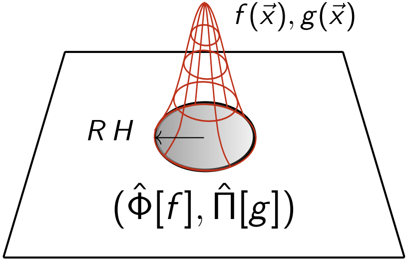
$[\hat{\Phi}[f_A], \hat{\Pi}[g_A]] = i$

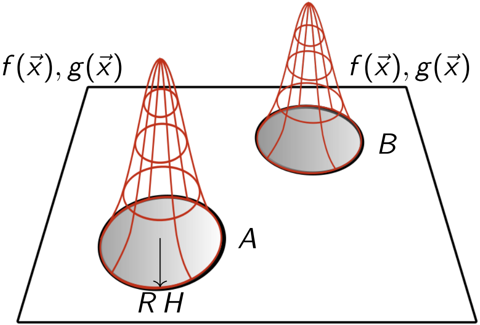


$(\hat{\Phi}[f_B], \hat{\Pi}[g_B])$

$[\hat{\Phi}[f_B], \hat{\Pi}[g_B]] = i$

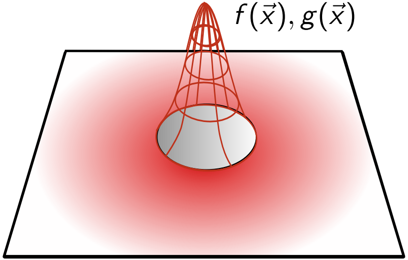
1



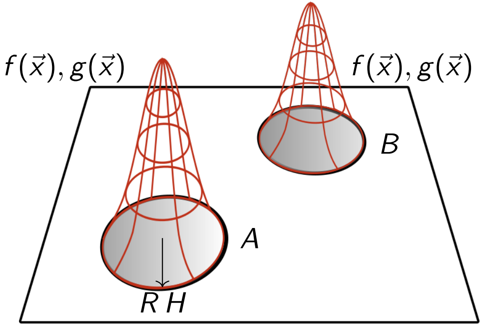


2

1

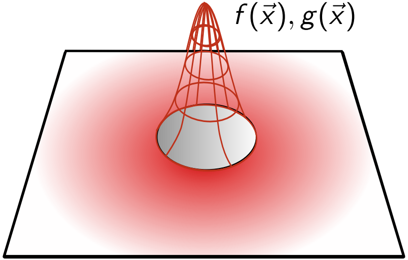


Entanglement between $(\hat{\Phi}[f], \hat{\Pi}[g])$ and the rest?



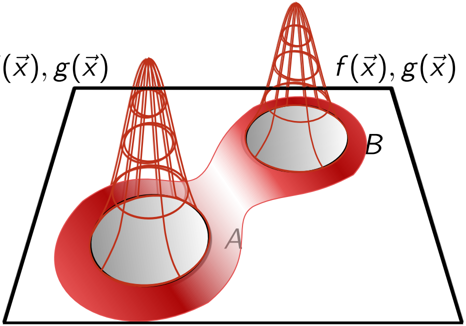
2

1



Entanglement between $(\hat{\Phi}[f], \hat{\Pi}[g])$ and the rest?

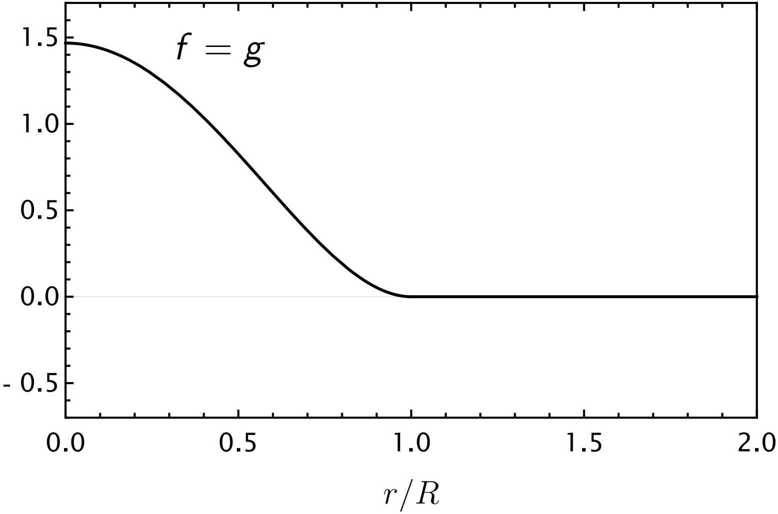
Entanglement between $(\hat{\Phi}_A[f], \hat{\Pi}_A[g])$ and $(\hat{\Phi}_B[f], \hat{\Pi}_B[g])$?



2

1 Entanglement between $(\hat{\Phi}[f], \hat{\Pi}[g])$ and the rest?

Idea: Find $\langle \bar{f}, \bar{g} \rangle$ such that $(\hat{\Phi}[\bar{f}], \hat{\Pi}[\bar{g}])$ purifies $(\hat{\Phi}[f], \hat{\Pi}[g])$



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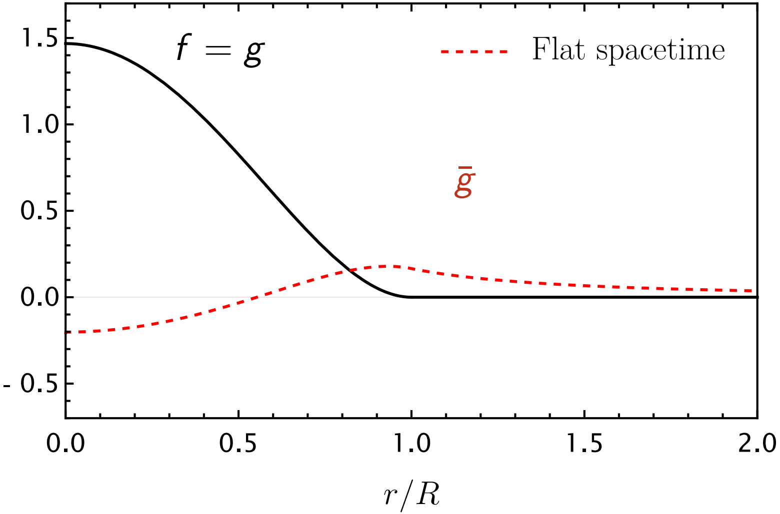

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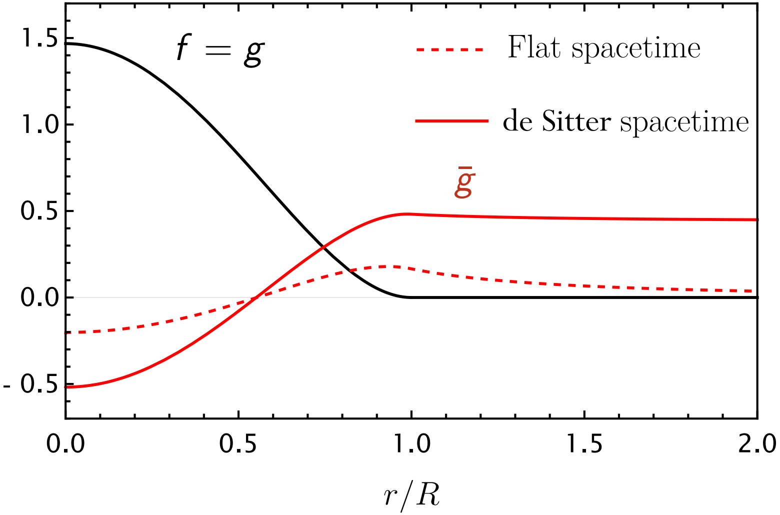
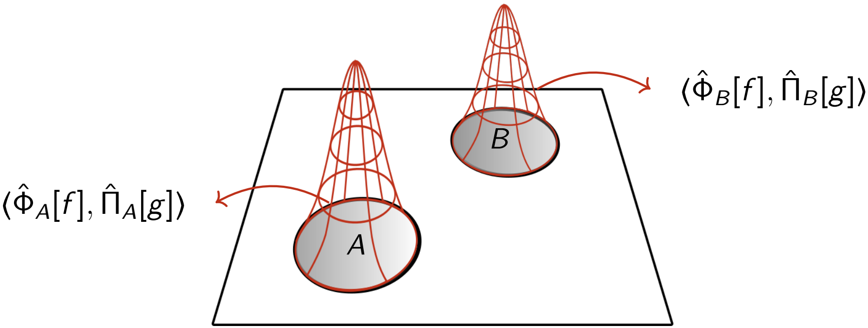


Entanglement is spread out at long distances

2 Entanglement $(\hat{\Phi}_A[f], \hat{\Pi}_A[g])$ and $(\hat{\Phi}_B[f], \hat{\Pi}_B[g])$?



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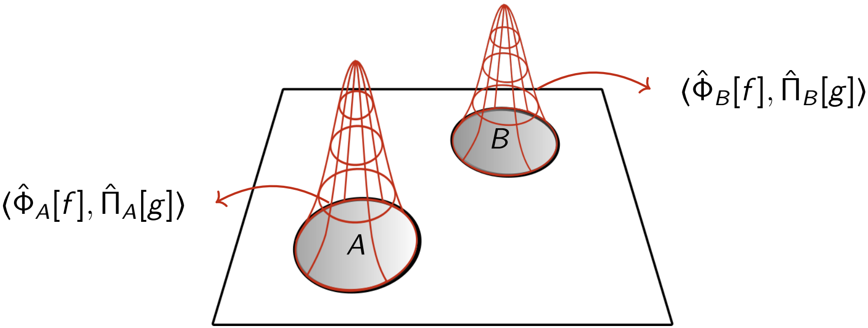

$$E_{\mathcal{N}}(A, B) = E_{\mathcal{N}}(A, B)^{\text{Mink}} + \text{extra contribution}$$

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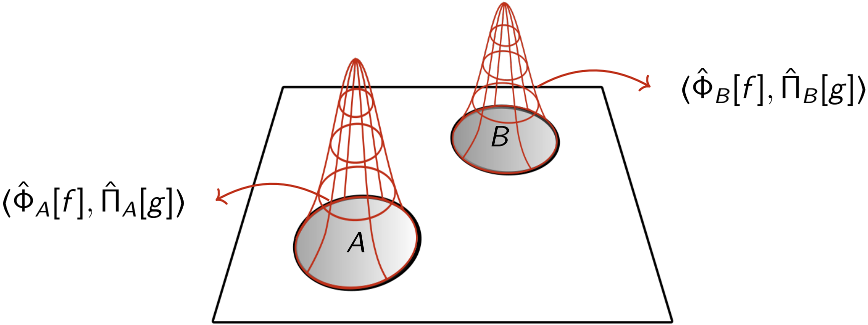


$$E_{\mathcal{N}}(A, B) = E_{\mathcal{N}}(A, B)^{\mathrm{Mink}} + \text{extra contribution}$$

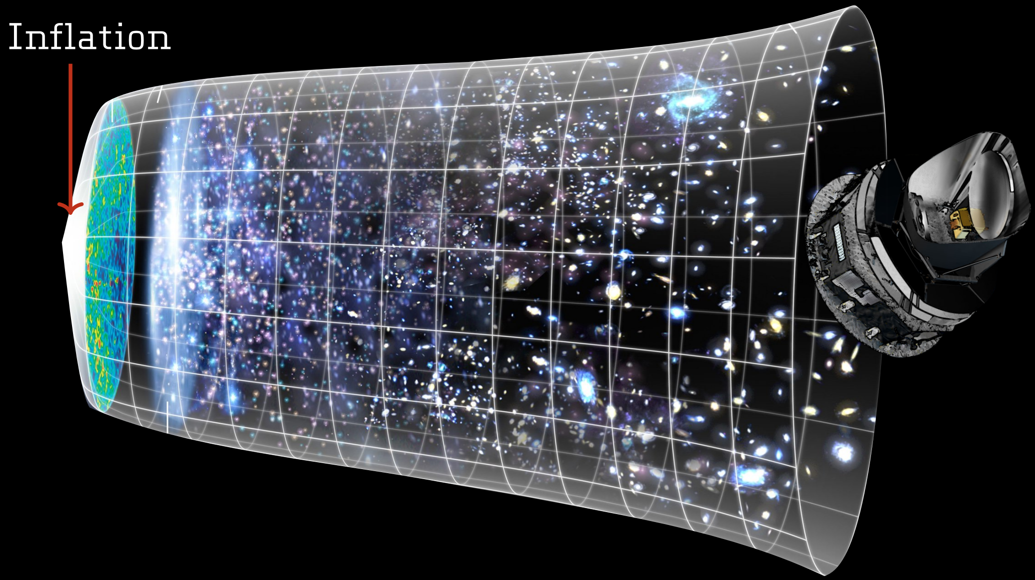
$$\text{extra contribution} < 0$$

Entanglement
with the partner acts as
noise
for local observables

Inflation does not create entanglement in local observables*

* Ribes-Metidieri, Agullo, and Bonga (2025); Agullo, Bonga, Ribes-Metidieri (2024).

Implications for cosmology



3.2. A proposal to probe QFT vacuum entanglement

Entanglement between localized modes...

- ...becomes sparser in increasing spatial dimensions...
  Agullo, Bonga, Ribes-Metidieri, Kranas, and Nadal-Gisbert (2023)
- ...is generic in multimode systems
  Agullo, Bonga, Martín-Martínez, Nadal-Gisbert, Perche, Polo-Gómez, Ribes-Metidieri, Torres (2023)
- ...is affected by the spacetime curvature
  Nambu, Yamaguchi (2023) ; Agullo, Bonga, Ribes-Metidieri (2024); Ribes-Metidieri, Agullo, Bonga (2025)

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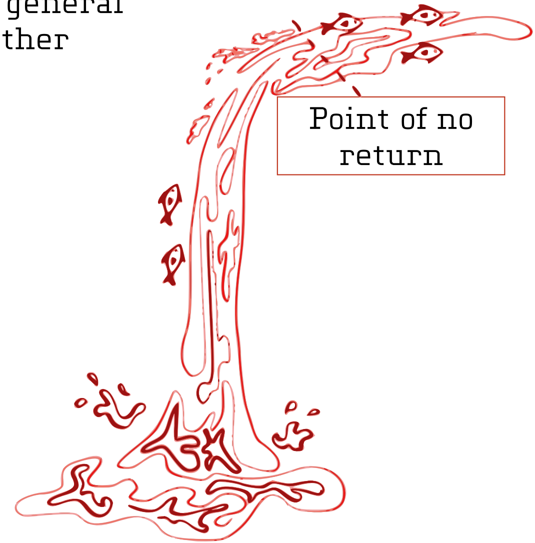
Can we experimentally test QFT vacuum entanglement and its properties?

Quantum Fluid Simulators

Analogue gravity explores analogues of general relativistic gravitational fields within other physical systems.*



Point of no return

* Carlos Barceló, Liberati, Visser (2024); ** Unruh (1980); Visser (1993)

Quantum Fluid Simulators

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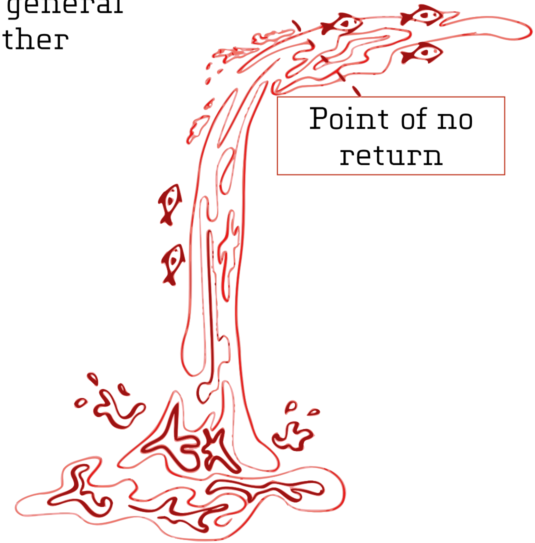


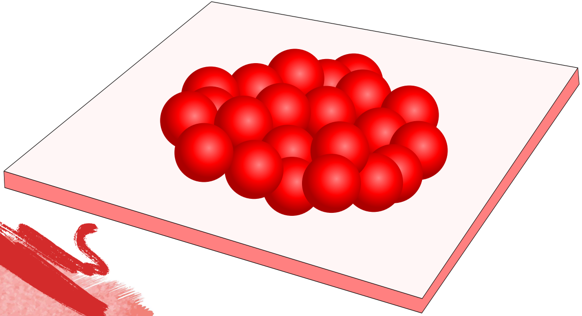
Theorem: Sound waves in inviscid, barotropic, irrotational fluids satisfy a curved Klein-Gordon equation.∗∗

Can simulate Black holes, expanding universes, ergorregions, ...

∗ Carlos Barceló, Liberati, Visser (2024); ∗∗ Unruh (1980); Visser (1993)

Analogue Experiments in Bose-Einstein Condensates (BEC)

BEC: atoms are cooled down until they "condense" in their lowest energy state



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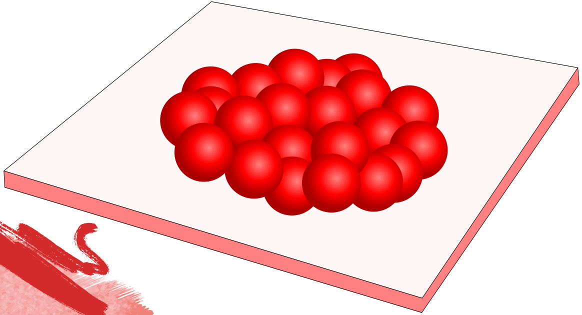

Linearized collective excitations in BECs, φ, satisfy

Healing length

$$\left(1-\frac{\xi^2}{2}\nabla^2\right)^{-1}(\partial_t^2 - c_s^2\nabla^2)\varphi = 0$$

Speed of sound

Analogue Experiments in Bose-Einstein Condensates (BEC)

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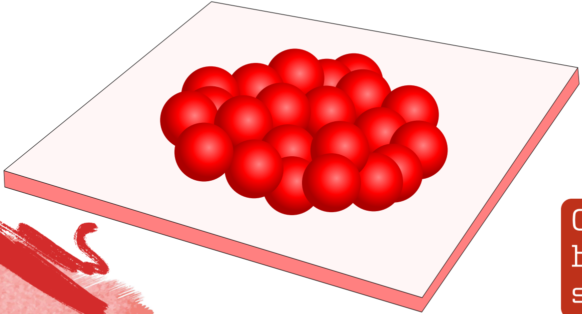
Linearized collective excitations in BECs, φ, satisfy



$$\left(1-\frac{\xi^2}{2}\nabla^2\right)^{-1}(\partial_t^2-c_s^2\nabla^2)\varphi=0$$

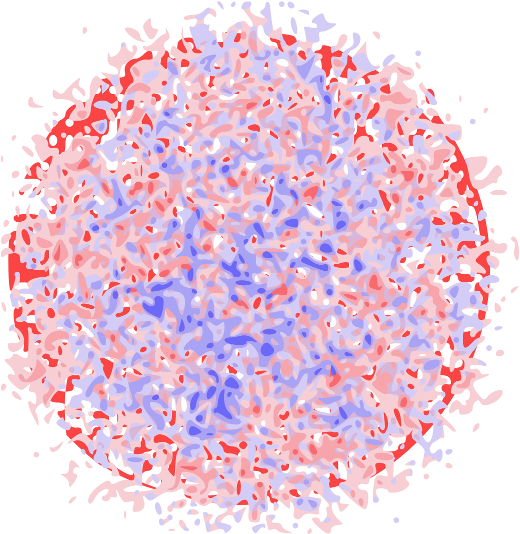
Collective linear excitations in BECs behave as a scalar field in a thermal state when $\xi^2\nabla^2\varphi \ll \varphi$!

Existing experimental platforms

- Experimental platform*: $(1+2)$-dimensional BEC

- Quantum state reconstruction

- Temperatures ~ 10 nK



*Viermann, Sparn, Liebster, Hans, Kath, Parra-López, Tolosa-Simeón, Sánchez-Kuntz, Haas, Strobel, Floerchinger, Oberthaler (2022)

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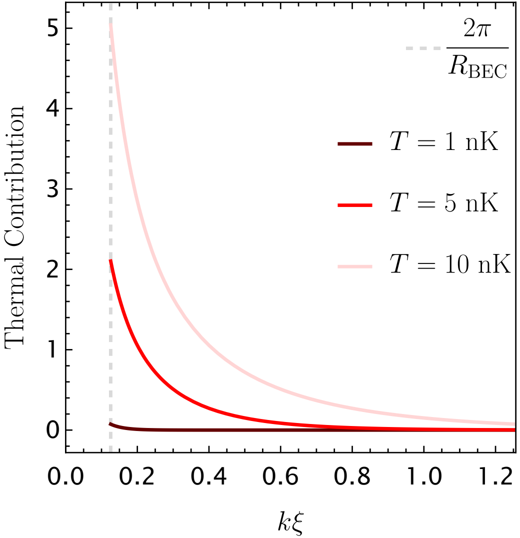

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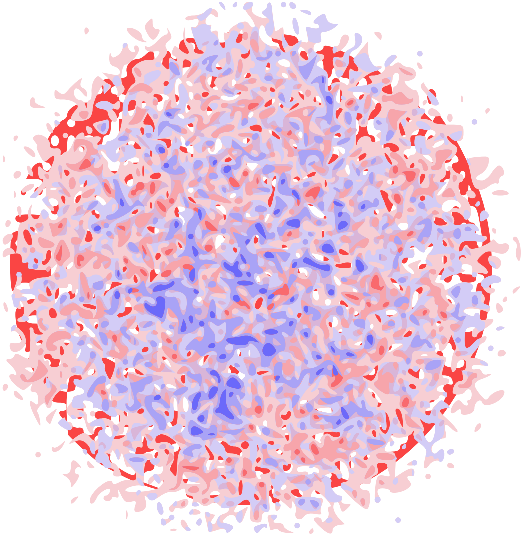
Local degrees of freedom in BECs



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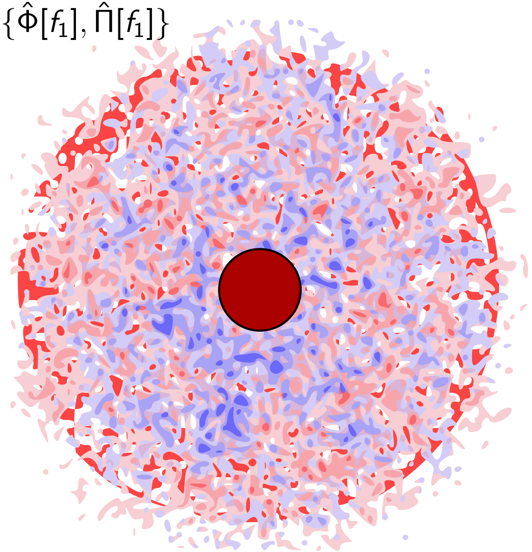
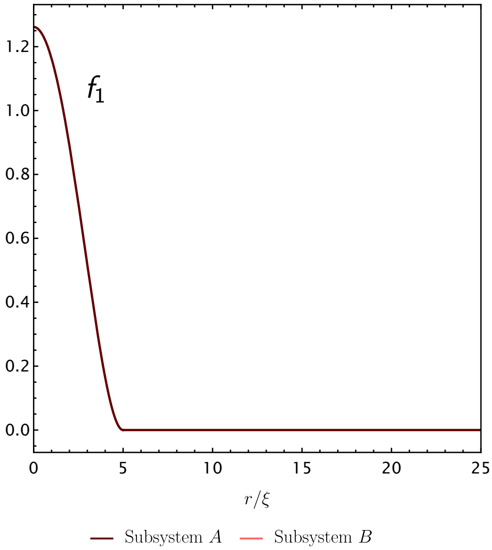
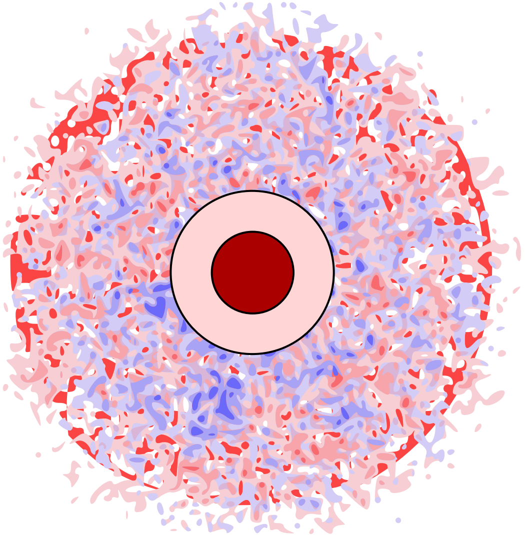
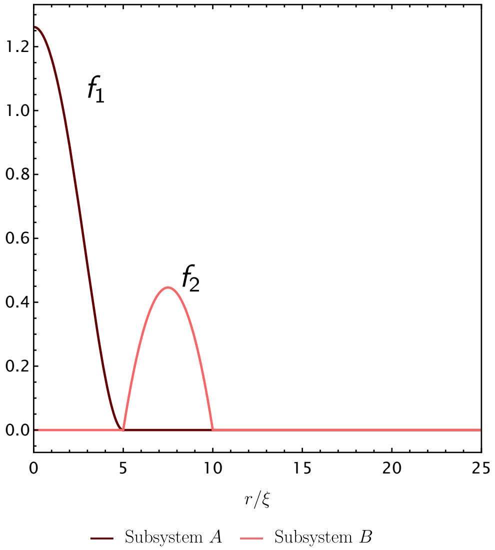




Local degrees of freedom in BECs

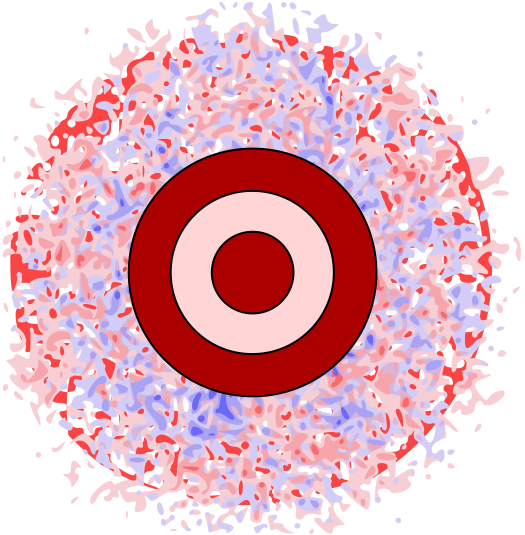
$$\{\hat{\Phi}[f_1], \hat{\Pi}[f_1], \hat{\Phi}[f_2], \hat{\Pi}[f_2]\}$$

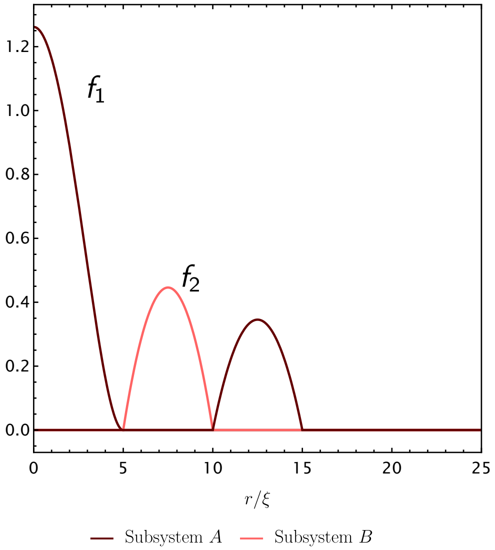




Local degrees of freedom in BECs

$$\{\hat{\Phi}[f_1], \hat{\Pi}[f_1], \hat{\Phi}[f_2], \hat{\Pi}[f_2], \dots\}$$





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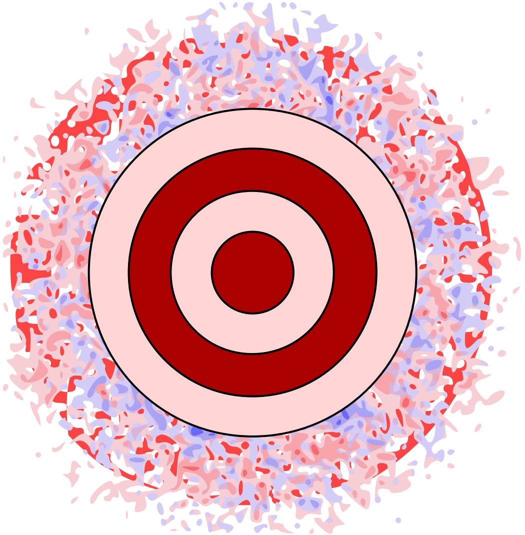
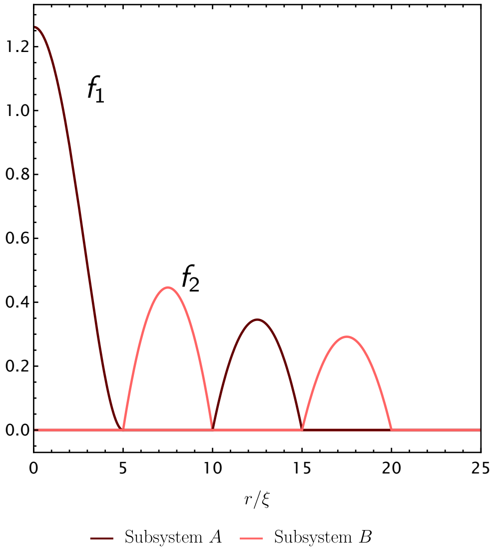
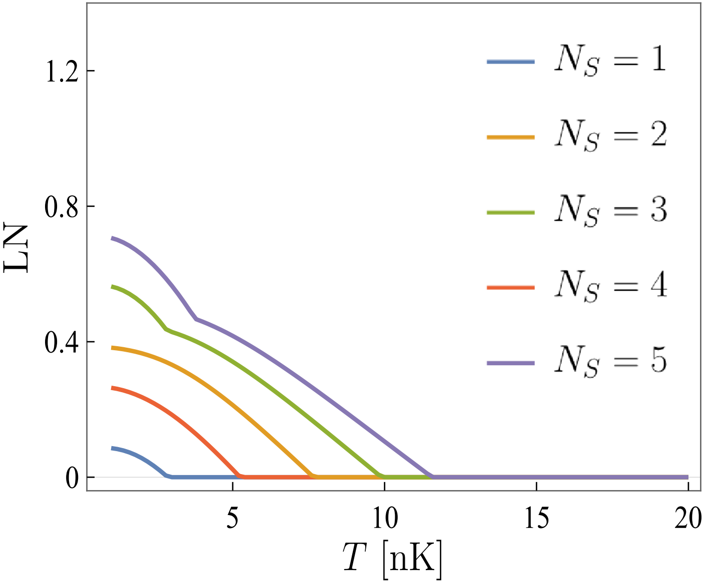
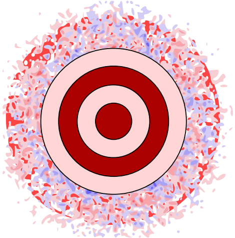




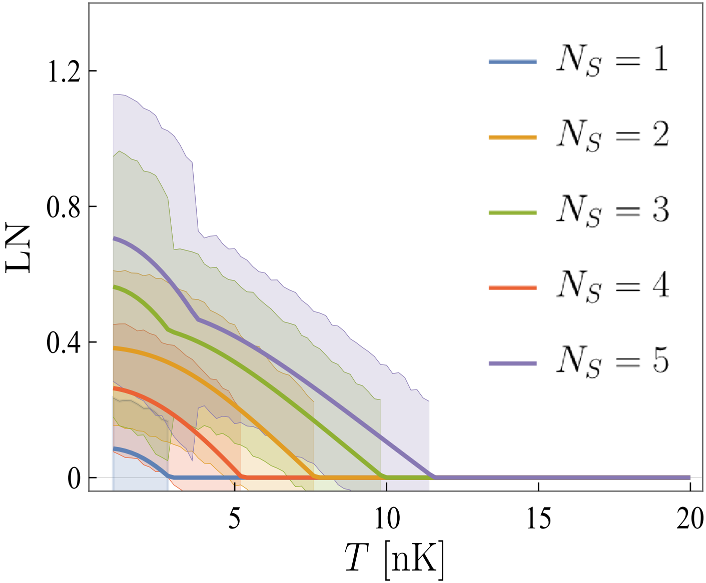
QFT vacuum entanglement with BECs

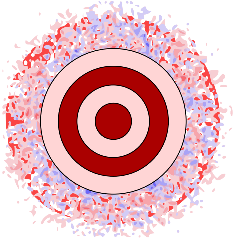




*Agullo, Delhom, Parra-López, Ribes-Metidieri (In preparation)

QFT vacuum entanglement with BECs





2D
5% relative errors
3σ contours

*Agullo, Delhom, Parra-López, Ribes-Metidieri (In preparation)

Quantum fluid platforms are promising as probes of QFT vacuum entanglement

4. SUMMARY AND CONCLUSIONS

Summary

QFT vacuum has a surprisingly rich entanglement structure

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How is entanglement distributed?

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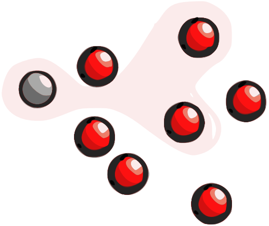

Partner systems

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QFT vacuum has a surprisingly rich entanglement structure

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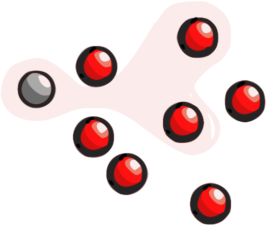
What part of this entanglement can we access?



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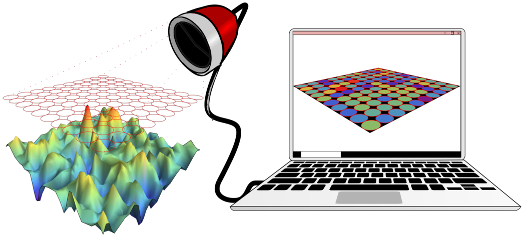
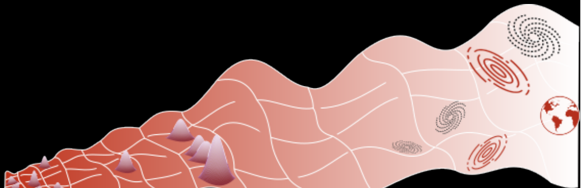


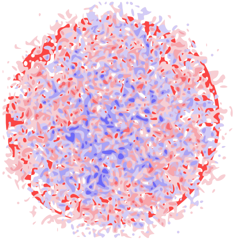
Entanglement between a finite number of local degrees of freedom

Conclusions

Entanglement generation during inflation



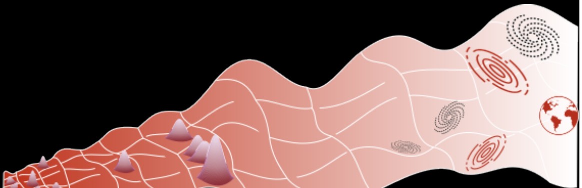
A proposal to probe QFT vacuum entanglement



Conclusions

Entanglement generation during inflation

- Entanglement with the partner spreads over large distances



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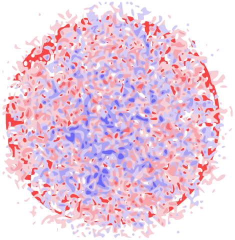

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Entanglement generation during inflation

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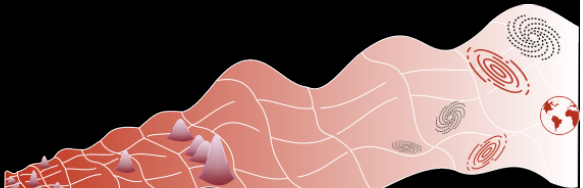

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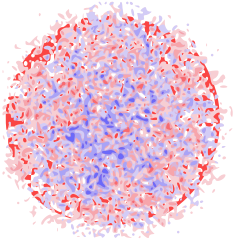

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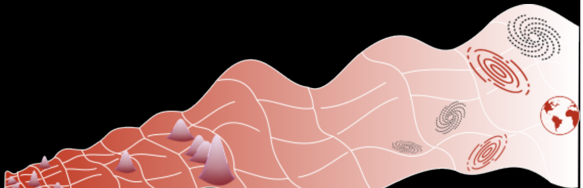

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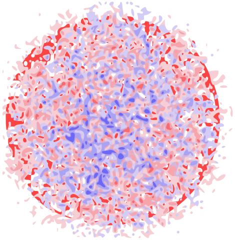

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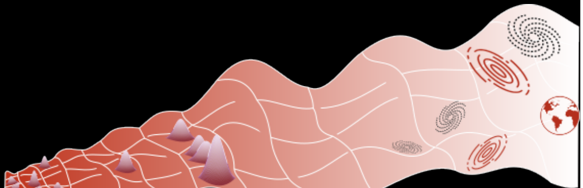
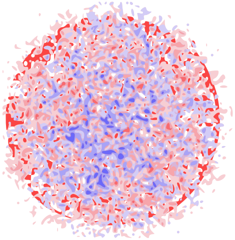


A proposal to probe QFT vacuum entanglement

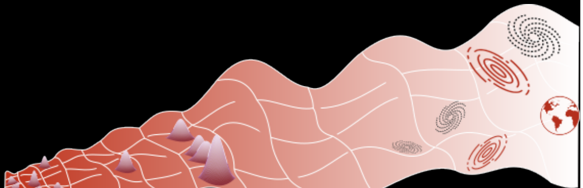


- Extremely low temperatures needed—Finally feasible!

Conclusions

Entanglement generation during inflation

- Entanglement with the partner spreads over large distances
- This "spread out" entanglement acts as noise for local observables
- **Inflation does not create entanglement in local observables**



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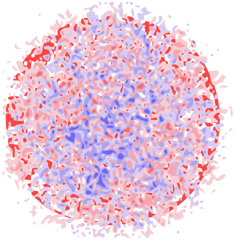

- Extremely low temperatures needed—Finally feasible!
- Quantum fluid platforms are promising as probes of QFT vacuum entanglement

Thank you for your attention!

Questions?

Contact information:

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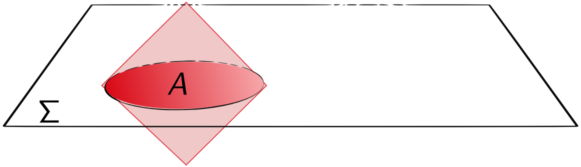
BACKUP

The Reeh-Schlieder Theorem

Consider a quantum free scalar field $\hat{\phi}$ in Minkowski space



- Let $|0\rangle$ denote the vacuum.
- Define $\hat{\Phi}_F := \int \mathrm{d}V F(x)\hat{\phi}(x)$ such that $\mathrm{supp} F \subset A$.

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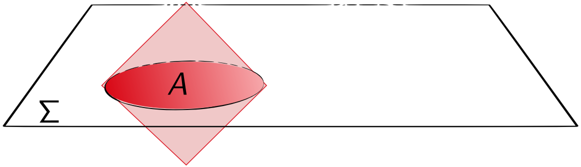


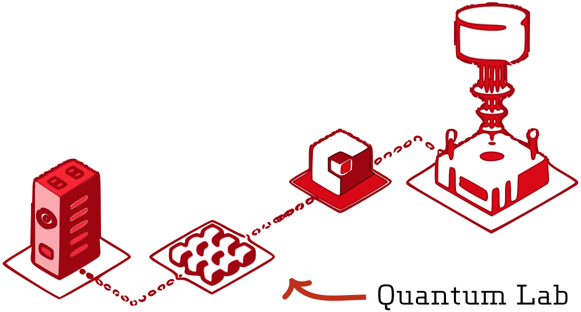
- Let $|0\rangle$ denote the vacuum.
- Define $\hat{\Phi}_F := \int \mathrm{d}V F(x)\hat{\phi}(x)$ such that $\mathrm{supp} F \subset A$.

Theorem: Acting on $|0\rangle$ with operators $\hat{\Phi}_{F_1} \cdots \hat{\Phi}_{F_N}$ localized in any open spacetime region A, we can approximate any state in the Hilbert space of the theory arbitrarily well.$*$

$*$ Reeh, Schlieder (1961). See Witten (2018) for a pedagogical proof.

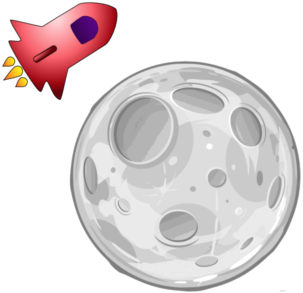
The Reeh-Schlieder Theorem II

Why surprising?



Quantum Lab

The Reeh-Schlieder Theorem II



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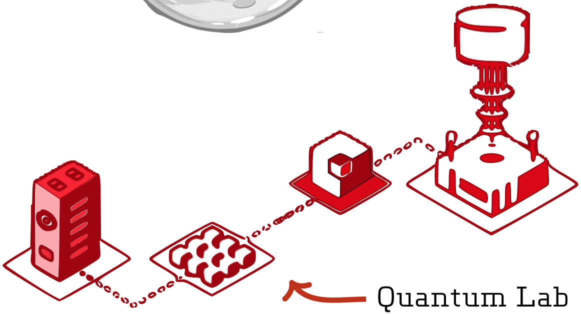

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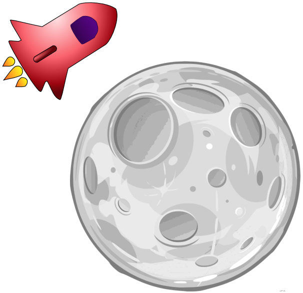
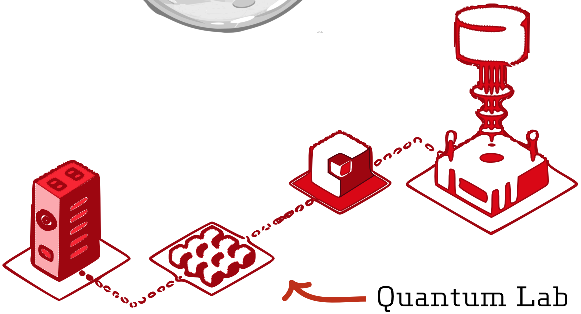


Why surprising?

Entanglement is
ubiquitous in the vacuum



Quantum Lab

Intuitive explanation: Entanglement

This is reminiscent of something well-known in Quantum Mechanics

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Let $\mathcal{H}_A$ and $\mathcal{H}_B$ be Hilbert spaces of two systems, both of dimension n. Any state in $\mathcal{H}_A \otimes \mathcal{H}_B$ can be written as

$$|\Psi\rangle = \sum_{i=1}^{n} c_i \, |i\rangle_A \otimes |i\rangle_B \ , \qquad \text{(Schmidt form)}$$

with $\{|i\rangle_A\}_{i=1}^n$ and $\{|i\rangle_B\}_{i=1}^n$ bases in $\mathcal{H}_A$ and $\mathcal{H}_B$

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Any state in $\mathcal{H}_A \otimes \mathcal{H}_B$ can be written as $\hat{O}_A \otimes \hat{I}_B \, |\Psi\rangle$

Quantifying Entanglement

Entanglement monotones: Functions of a quantum state that

- vanish on **separable** states
- do not increase under LOCC

Example: **Logarithmic negativity**

- Measures distillable entanglement
- Valid for mixed Gaussian states
- Faithful for system of 1 vs N modes

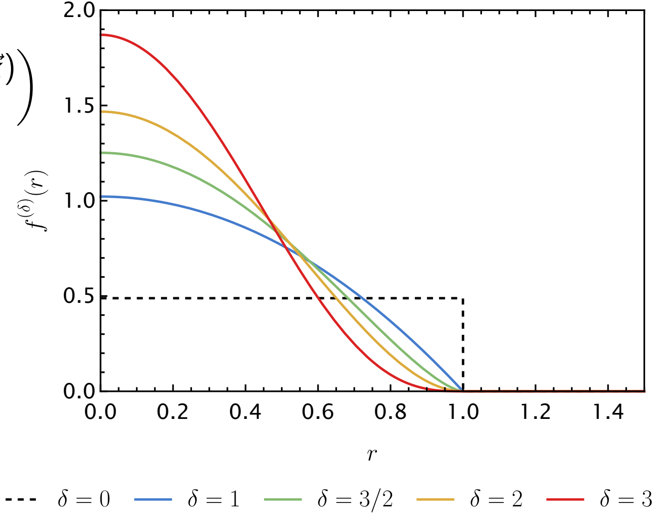
Single-Mode Subsystem in a Massless Scalar Theory

- Free, massless scalar field in Minkowski spacetime

$$\gamma_1(\vec{x}) = \begin{pmatrix} 0 \\ f^{(\delta)}(\vec{x}) \end{pmatrix}, \quad \gamma_2(\vec{x}) = \begin{pmatrix} -f^{(\delta)}(\vec{x}) \\ 0 \end{pmatrix}$$

$$f^{(\delta)}(\vec{x}) = A_\delta \left(1 - \frac{|\vec{x}|^2}{R^2}\right)^\delta \Theta(R-r)\,,$$

$$\Gamma_A = \mathrm{span}(\gamma_1, \gamma_2) \subset \bar{\Gamma}_{\mathbb{C}}$$



Single-Mode Subsystem in a Massless Scalar Theory

The corresponding smeared operators are:

$$\hat{\Phi}_\delta := \hat{O}_{\gamma_1} = \int_\Sigma \mathrm{d}^3x\, f^{(\delta)}(\vec{x})\hat{\phi}(\vec{x})\,, \quad \hat{\Pi}_\delta := \hat{O}_{\gamma_2} = \int_\Sigma \mathrm{d}^3x\, f^{(\delta)}(\vec{x})\hat{\pi}(\vec{x})\,,$$

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$$[\hat{\Phi}_\delta, \hat{\Pi}_\delta] = -i\omega_A^{12} = i$$

$$\omega_A = \begin{pmatrix} \omega(\gamma_1, \gamma_1) & \omega(\gamma_1, \gamma_2) \\ \omega(\gamma_2, \gamma_1) & \omega(\gamma_2, \gamma_2) \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

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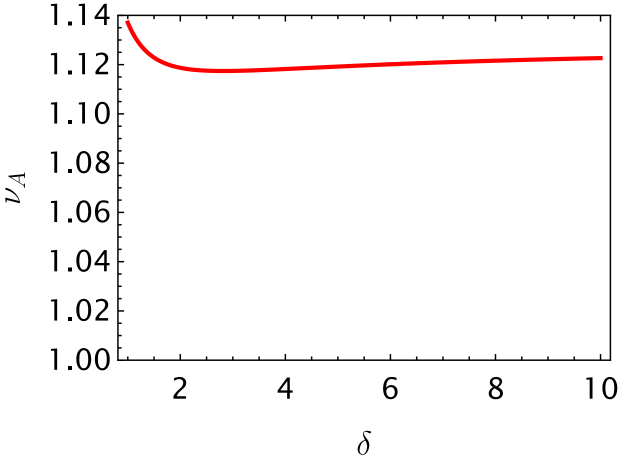
$$(f|g)_s := \int \frac{\mathrm{d}^3k}{(2\pi)^3} |\vec{k}|^{2s} \tilde{f}(\vec{k})^\star \tilde{g}(\vec{k})$$ Sobolev inner product

Single-Mode Subsystem in a Massless Scalar Theory

All symplectic invariant information is contained in the symplectic eigenvalue!

$$||f^{(\delta)}||^2_{1/2} = \frac{4\Gamma(2\delta)\Gamma(\delta+1)^2\Gamma(2\delta+5/2)}{\sqrt{\pi}\Gamma(\delta+1/2)^2\Gamma(2\delta+1)\Gamma(2\delta+2)}, \quad ||f^{(\delta)}||^2_{-1/2} = \frac{2\Gamma(\delta+1)^2\Gamma(2\delta+5/2)}{\sqrt{\pi}\Gamma(\delta+1/2)\Gamma(\delta+3/2)\Gamma(2\delta+3)}.$$



$\nu_A^2 = \mathrm{Det}\sigma_A = ||f^{(\delta)}||^2_{-1/2}||f^{(\delta)}||^2_{1/2} > 1$ for all $\delta \geq 1$

Entropy of a single mode

$$\nu_A^2 - (\nu_A^{\mathrm{Mink}})^2 \approx \mathcal{F}_A + \dots$$

Proposition: $\mathcal{F}_A \geq 0$.

Proof: There are three cases to consider:

1. $f_A^{(1)} \neq f_A^{(2)}$, with both functions different from zero;
2. $f_A^{(1)} = f_A^{(2)} \neq 0$;
3. $f_A^{(1)} = 0$, $f_A^{(2)} \neq 0$ (or vice-versa).

Case 1:

$$\nu_A^2 - (\nu_A^{\mathrm{Mink}})^2 = \mathfrak{a}_A\,(RH)^{4-4\mu^2}(1+\mathcal{O}(\mu^2)) + \mathcal{O}\big((RH)^{3-3\mu^2}\big)\,,$$

with

$$\mathfrak{a}_I = ||f_A^{(1)}||^2_{-\frac{3}{2}+\mu^2}||f_A^{(2)}||^2_{-\frac{3}{2}+\mu^2} - \mathrm{Re}(f_A^{(1)}|f_A^{(2)})^2_{-\frac{3}{2}+\mu^2}\,.$$

Cauchy-Schwarz inequality

Case 2:

$$\nu_A^2 - (\nu_A^{\mathrm{Mink}})^2 = \mathfrak{b}_A\,(RH)^{2-2\mu^2}(1+\mathcal{O}(\mu^2)) + \mathcal{O}\big((RH)^{1-\mu^2}\big)\,,$$

where

$$\mathfrak{b}_A = ||f_A||^2_{-\frac{3}{2}+\mu^2}||g_A^{(1)} - g_A^{(2)}||^2_{\frac{1}{2}} - \mathrm{Re}(f_A|g_A^{(1)} - g_A^{(2)})^2_{-\frac{1-\mu^2}{2}}\,.$$

Applying Hölder's inequality for $s' \in [-1,1]$ and $s = -(1-\mu^2)/2$, we find $\mathfrak{b}_A \geq 0$.

Case 3:

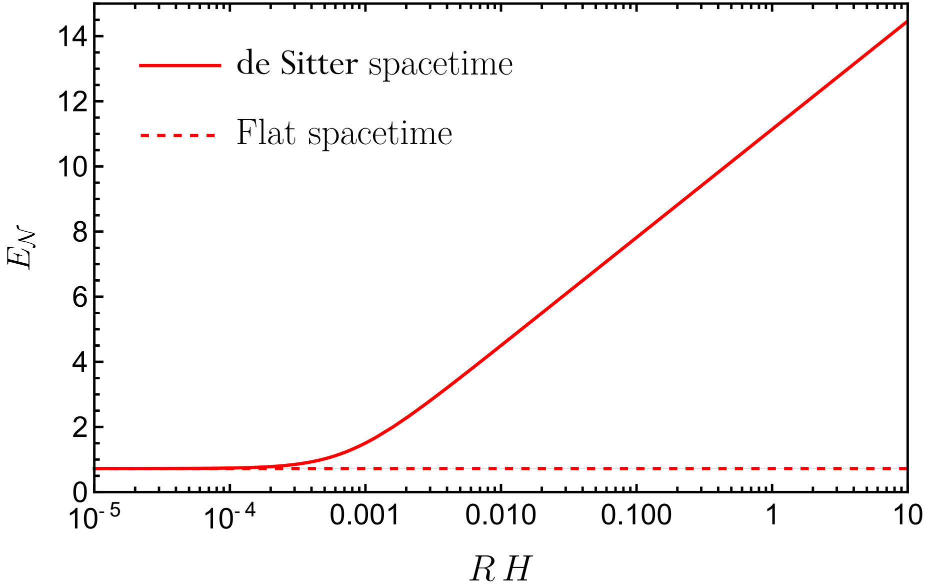
$$\nu_A^2 - (\nu_A^{\mathrm{Mink}})^2 = \mathfrak{c}_A\,(RH)^{2-2\mu^2}\left(1+\mathcal{O}(\mu^2)\right) + \mathcal{O}\big((RH)^{1-\mu^2}\big)$$

with

$$\mathfrak{c}_I = ||f_A^{(2)}||^2_{-\frac{3}{2}+\mu^2}||g_A^{(1)}||^2_{\frac{1}{2}} - \mathrm{Re}(f_A^{(2)}|g_A^{(1)})^2_{-\frac{1-\mu^2}{2}}\,.$$

Following the same argument as in case $\langle 2 \rangle$, we find $\mathfrak{c}_I \geq 0$.

1 Entanglement between $(\hat{\Phi}[f], \hat{\Pi}[g])$ and the rest?



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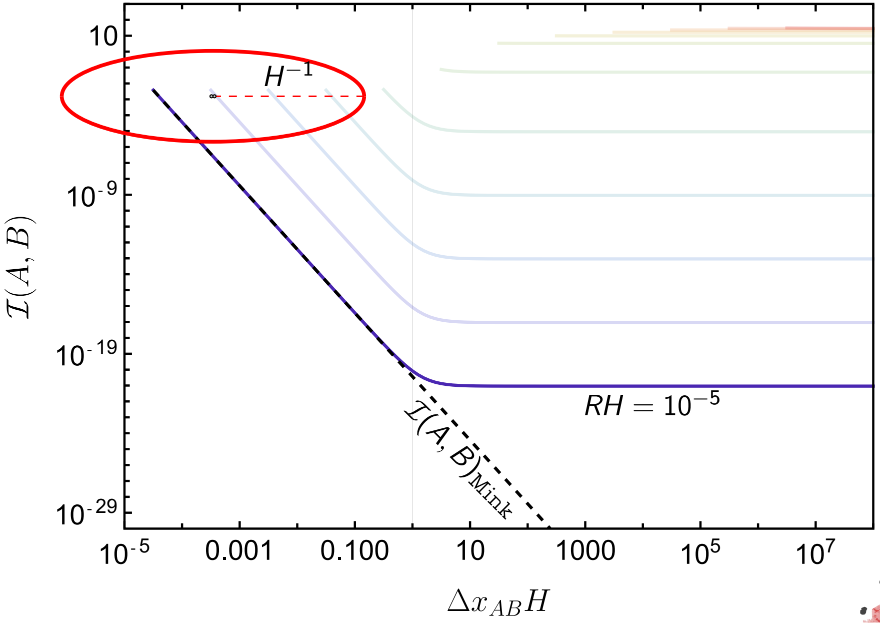

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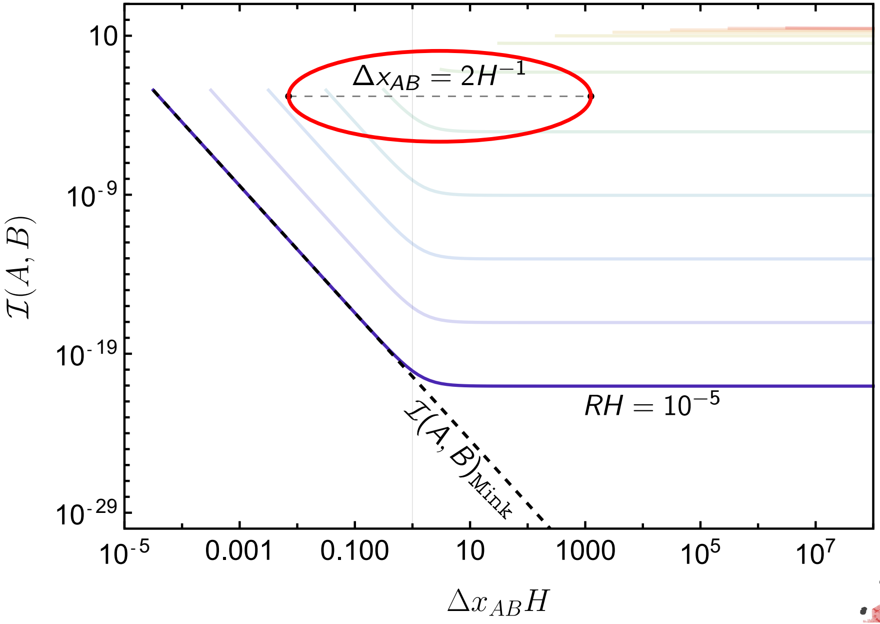

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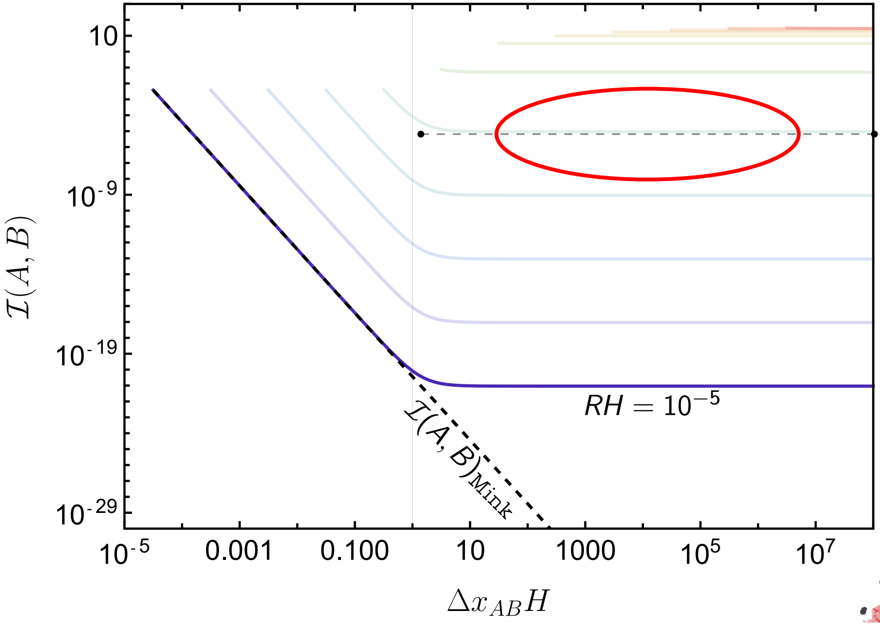

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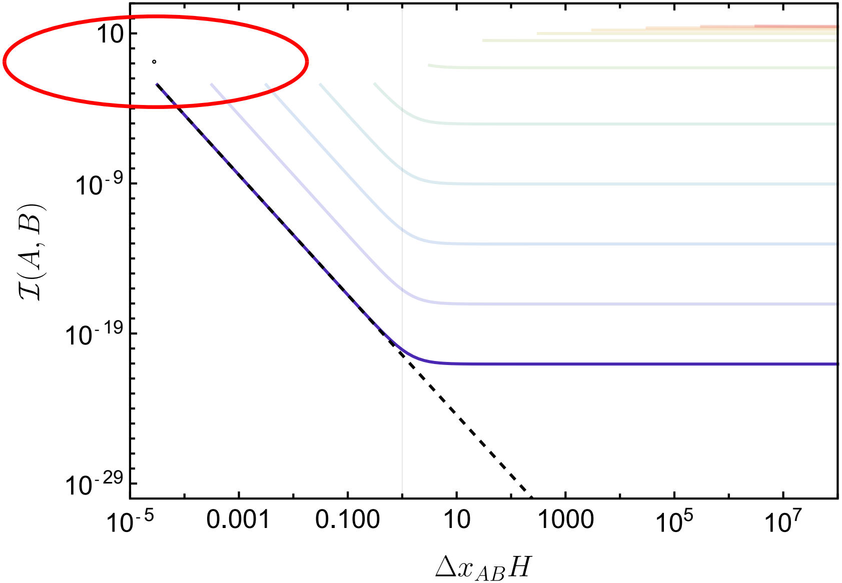

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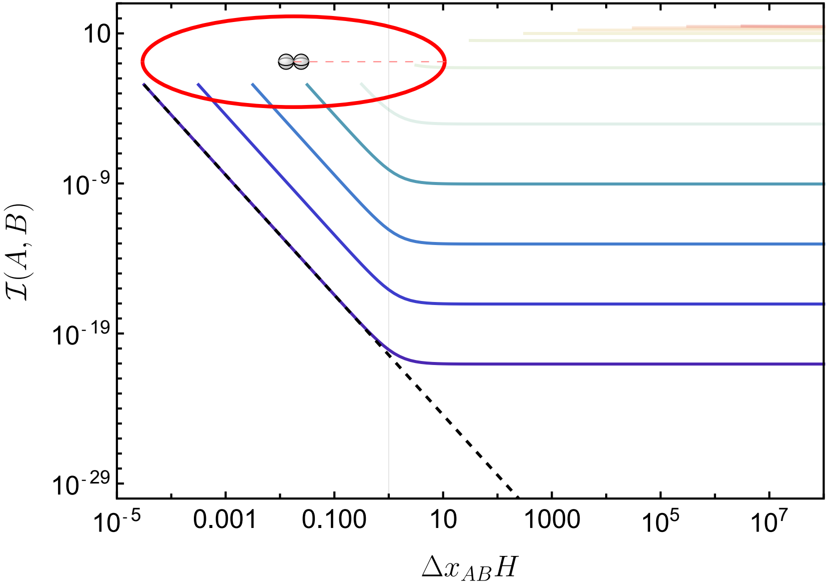

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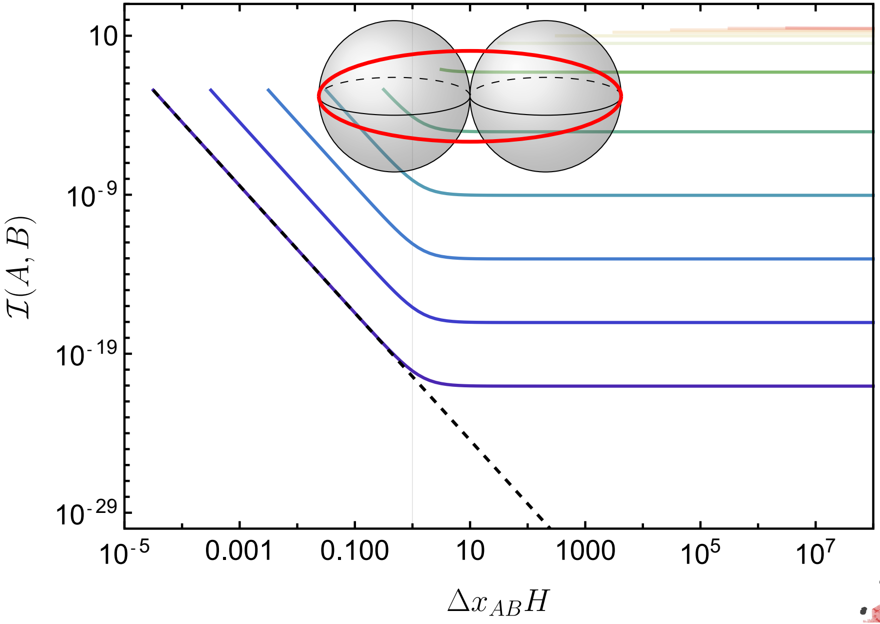

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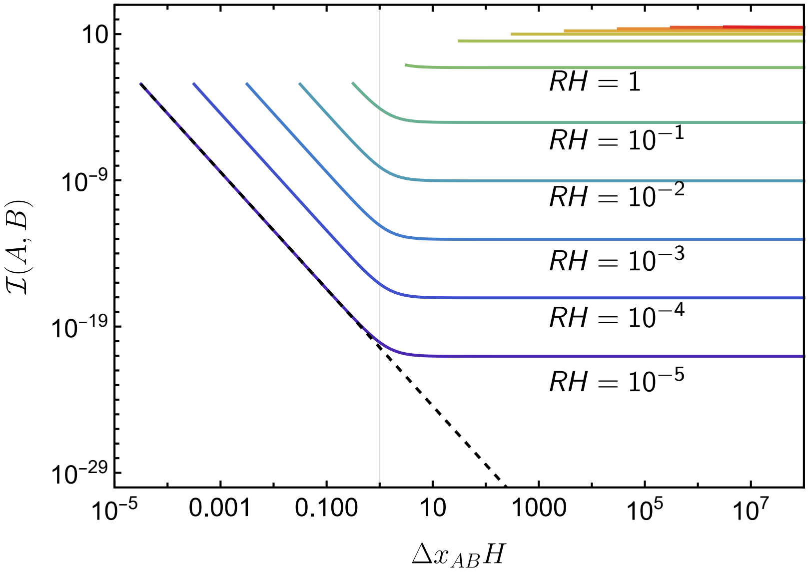

Correlated Subsystems

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Definition: A is uncorrelated in $\hat{\rho}$ if

$$\mathrm{Tr}\left[\hat{\rho}\,\hat{O}_\gamma \hat{O}_{\gamma'}\right] - \mathrm{Tr}\left[\hat{\rho}\hat{O}_\gamma\right]\mathrm{Tr}\left[\hat{\rho}\hat{O}_{\gamma'}\right] = 0\,, \quad \forall\, \gamma \in A \;\text{ and }\; \gamma' \in A_\perp.$$

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Proposition: A is uncorrelated in $\hat{\rho}$ if and only if

$$[\Pi_A, J] = 0.$$

Partner of a System in a Pure State

Proposition: Let A be an N-mode subsystem correlated in the pure Gaussian state J. Then, the single-mode subsystem

$$A_P = \Pi_A^{\perp}(JA),$$

is the partner of A in J. *

* Agullo, Martín-Martínez, Nadal-Gisbert, RM, and Yamaguchi (In preparation)

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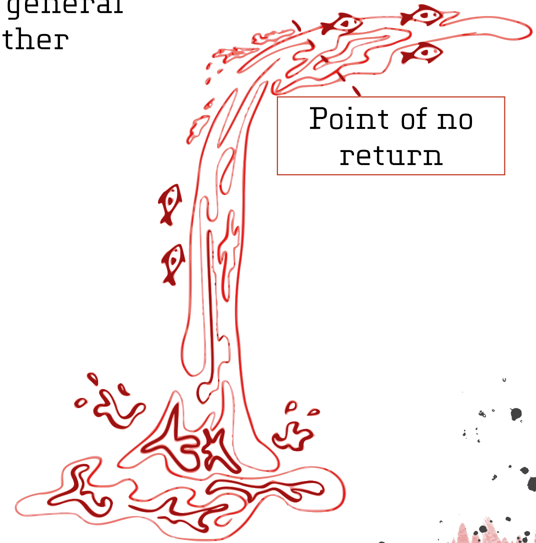
Quantum Fluid Simulators

Analogue gravity explores analogues of general relativistic gravitational fields within other physical systems.*



* Carlos Barceló, Liberati, Visser (2024); ** Unruh (1980); Visser (1993)

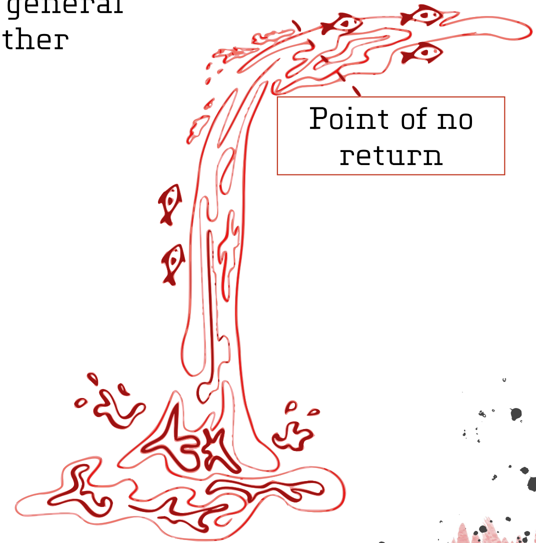
Quantum Fluid Simulators

Analogue gravity explores analogues of general relativistic gravitational fields within other physical systems.*

Theorem: Sound waves in inviscid, barotropic, irrotational fluids satisfy a curved Klein-Gordon equation.**

$$g \propto -(c_s^2 - v^2)dt^2 - 2\vec{v} \cdot d\vec{x}dt + d\vec{x} \cdot d\vec{x}$$

v = fluid velocity and
c_s = speed of sound



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