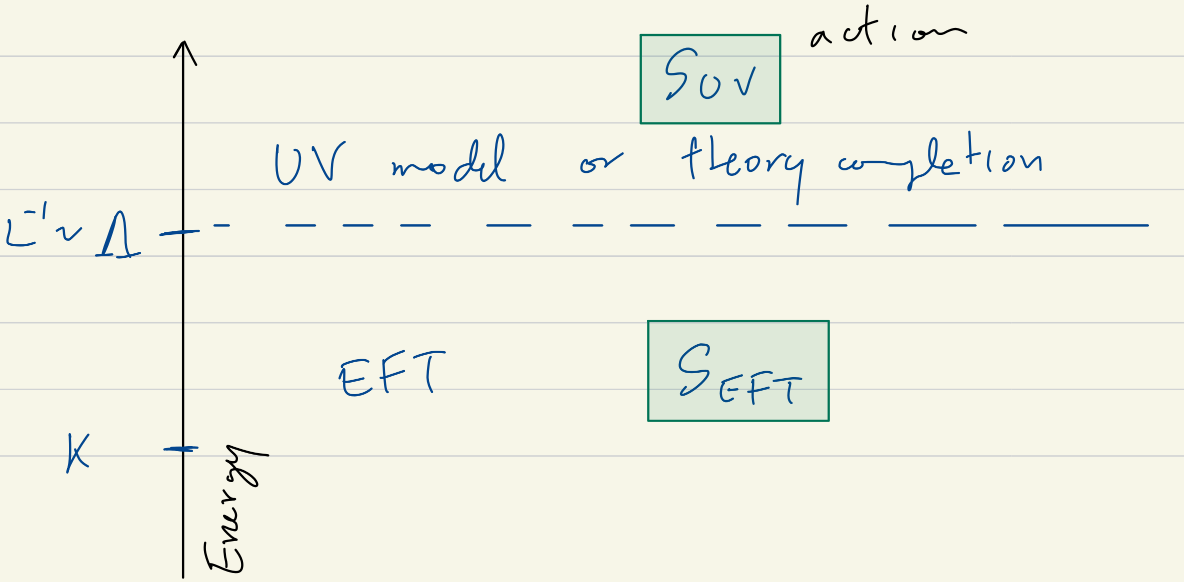
L2 Model dependence and independence

You might have heard EFT offers a model-independent approach in a bottom-up setup. This lecture will aim at understanding & sharpening this statement.

In this road our first stop will in fact be to look at model dependence, in the form of matching for which purpose we need the full theory, valid beyond the EFT regime.

To set notation



A) Fermi Theory

It is perhaps the most used example but it'll serve us to get started even if some of you did this before.

The UV model's action ($W_+ = (W_-)^\dagger$)

$$S_{UV} = \int d^4x \Big[W_+^\nu (\partial_\mu \partial^\mu + M_W^2) W_\nu^- - \frac{g}{\sqrt{2}} W_\mu^+ J_-^\mu + h.c. \Big]$$

where $J_\mu^- = \bar{\nu} \gamma^\mu P_L e + \bar{u} \gamma^\mu P_L d$ and $\alpha_{em} = 0$. for a single fermion generation.

We will connect, i.e. match this theory to the EFT

$$S_{EFT} = S_{QED, QCD} + \int d^4x \left(- \frac{4}{\sqrt{2}} G_F \bar{u} \gamma^\mu P_L d \, \bar{e} \gamma_\mu \nu_L + h.c. \right)$$

For that purpose the Feynman Rules for each

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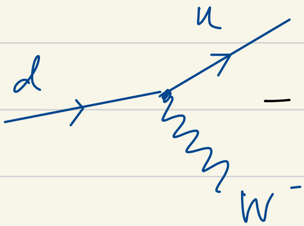
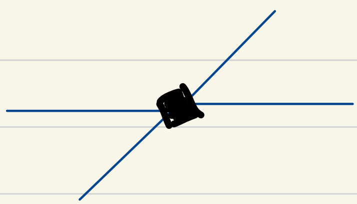


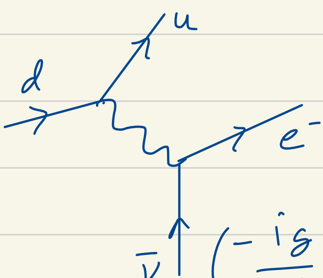
$$-\frac{ig}{\sqrt{2}}\gamma_\mu P_L \qquad \frac{-i\eta^{\mu\nu}}{k^2-M_W^2}$$

EFT



$$-\frac{4i}{\sqrt{2}}G_F(\gamma^\mu P_L)\otimes(\gamma_\mu P_L)$$

So we can turn the diagram-equation



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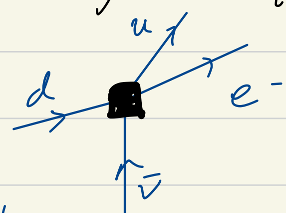

$$\left(-\frac{ig}{\sqrt{2}}\right)^2\frac{(-i\eta_{\mu\nu})\gamma^\mu P_L\otimes\gamma^\nu P_L}{k^2-M_W^2} = -\frac{4G_F}{2}i\,\gamma_\mu P_L\otimes\gamma^\mu P_L$$

expanding on k^2/M_W^2 to get $\quad M_W = \frac{gv}{2}$

$$\downarrow$$

$$\frac{-ig^2}{2M_W^2} = -2\sqrt{2}G_F i\ , \quad G_F = \frac{g^2}{4\sqrt{2}M_W^2} = \frac{1}{\sqrt{2}v^2}.$$

The diagrams also help look at the expansion in position space; if one does not have energy enough to produce W's $k \ll M_W \sim L^{-1}$ so your microscope does not have resolution ($\sim k^{-1}$) small enough to "see" the W boson & the interaction looks point-like = contact.

In fact the position space effects can be made explicit and what one obtains is a Yukawa-like potential induced by the W

$$V_{weak} \sim \frac{g^2}{4\pi} \frac{e^{-M_W r}}{r}$$

It is interesting, after our toy QM examples with simplistic potentials, to set eyes on an actual short distance fundamental potential in nature.

Some elements (γ_5) came along for the ride but weren't needed for matching. One can re-do the same matching without them using the path integral formulation

$$e^{iS_{EFT}} \equiv \int DW^+ DW^- \, e^{iS_{UV}}$$

$$= e^{iS_{UV}[W_{EOM}]} \left(1 + \mathcal{O}(\hbar)\right)$$

The intuitive connection is we integrate over the field we can't see directly to be left with the effects on other particles & in a quantum expansion the first term follows the classical path i.e. the E.o.M.

(II.a) You can do this yourselves, take W's EoM

$$(\partial^2 + M^2) W^-_{EoM} - \frac{g}{\sqrt{2}} J^- = 0$$

& putting it back on the action, then expanding on ∂^2/M^2 will produce

$$S_{EFT} = \int d^4x \, C_{EoM} J^+_\mu J^{-\mu}$$

which contains Fermi's operator

$$+ \int d^4x \left(-\frac{4}{\sqrt{2}} G_F \bar{u} \gamma^\mu P_L d \, \bar{e} \gamma_\mu \nu_L + h.c. \right)$$

and by equating the two & finding C_{EoM} you can reproduce the diagram result for G_F as a function of g, M_W.

B) - A complex light scalar

Consider now a complex scalar ϕ being the only particle with mass m below a scale Λ. This will be the central scenario for the rest of the lectures so better get familiar

$$\phi \equiv \frac{1}{\sqrt{2}} (\varphi_1 + i \varphi_2)$$

there is a $U(1)$ global symmetry $\phi \to e^{i\theta} \phi$ which we assume is conserved.

It's action to order $1/\Lambda^2$ $\left(\phi^* \overleftrightarrow{\partial_\mu} \phi \equiv \phi^* \partial_\mu \phi - (\partial_\mu \phi^*) \phi \right)$

$$\equiv S_4 \equiv \int d^4x \, \mathcal{L}_4$$

$$S_{EFT} \equiv \boxed{\int d^4x \Big(\partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \frac{\lambda}{4} (\phi \phi^*)^4} + \frac{c_6}{\Lambda^2} (\phi^* \phi)^3 + \frac{c_5}{\Lambda^2} \phi^* \phi \, \partial^2 \phi^* \phi + \frac{c_7}{\Lambda^2} \left(\phi^* \overleftrightarrow{\partial_\mu} \phi \right) \left(\phi^* \overleftrightarrow{\partial^\mu} \phi \right) + \mathcal{O}\left(\frac{1}{\Lambda^4} \right) \Big)$$

If you don't know or have forgotten dimensional analysis gives $[\phi] = [\partial_\mu] = 1$ and determines in turn powers of Λ.

For the remainder of this lecture we will work on matching to illustrate model (in)dependence

B.1 Heavy Singlet

Consider a real scalar S with action in the UV

$$\mathcal{L}_4^{UV} = \mathcal{L}_4 \left(m \to m_{UV},\ \lambda \to \lambda_{UV} \right)$$

$$S_{UV} = \int d^4x \left(\mathcal{L}_4^{UV} - \frac{1}{2} S \left(\partial^2 + M_S^2 \right) S - \kappa M_S S \phi^* \phi \right)$$

where we take S heavy $M_S \gg m$ so we can integrate it out as we did with the W to get

You can do this yourselves

$$S_{EFT}^{S} = \int d^4x \left(\mathcal{L}_4^{UV} + \frac{1}{2} \kappa^2 \left(\phi^* \phi \right)^2 - \frac{\kappa^2}{2} \frac{1}{M_S^2} \left(\phi^* \phi \right) \partial^2 \left(\phi^* \phi \right) + \mathcal{O}\left(M_S^{-4} \right) \right)$$

and matching with the original $S_{EFT}^{S} = S_{EFT}$

$$\lambda = \lambda_{UV} - 2\kappa^2, \quad m_{UV}^2 = m^2, \quad \frac{c_S}{\Lambda^2} = -\frac{\kappa^2}{2 M_S^2}, \quad c_6 = c_g = 0.$$

B.2 SU(2)/U(1) Goldstones

Consider the case in which ϕ is a pseudo goldstone boson and so naturally light although we won't get into how it obtained its mass.

The goldstone could have come from a strongly interacting sector of scale $\Lambda_S = 4\pi f$ but regardless the action is largely determined by the group structure that defines the Goldstones. Some of this has been covered but let me give the basics

Any group element can be factorised into a broken & unbroken part

Element: $G_T = G_X G_t$

Broken exponential of | Unbroken exponential of

Generators: $T = \{X, t\}$

Broken | unbroken

Consider the field dependent group element ξ which is of the form G_X.

How does ξ transform?

As groups elements do, if I do a transformation G_1 & then another G_2 the resulting is $G_2 G_1$. Now take $G_1 = \xi$, $G_2 = G$

$$\xi \longrightarrow G\xi = \xi' h \qquad \xi' = G \xi h^{-1}$$

field independent

for the RHS to be in G_X $h(\xi)$

Now to the case at hand $SU(2)$ broken to $U(1)$

$$\xi = e^{i\sigma\cdot\varphi/f} \qquad \varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

scale $[f] = 1$

$$\sigma\cdot\varphi = \varphi_1 \sigma_1 + \varphi_2 \sigma_2$$

where σ_1, σ_2 are the broken generators (X) while σ_3 is unbroken (t). They satisfy:

- Generators $t = \{\sigma_3\}$, $X = \{\sigma_1, \sigma_2\}$,
- Lie Algebra $[t, t] \to t$, $[t, X] \to X$,
- Trace $\mathrm{Tr}(X t) = 0$.

as follows from the unbroken group being closed & fully antisymmetric structure constants.

The fields φ transform non-linearly

$$\xi \to G\,\xi\, h(\xi)\,; \qquad h = e^{i\sigma_3 g(\xi)}$$

but the combination

$$\xi^\dagger \partial_\mu \xi \to \underbrace{h^\dagger(\xi)\,\partial_\mu h(\xi)}_{\text{in } t} + \underbrace{h^\dagger(\xi)\left(\xi^\dagger \partial_\mu \xi\right) h(\xi)}_{\text{in } X}$$

when projected into the broken piece

$$u^a_\mu \equiv \mathrm{Tr}\left(\frac{\sigma^a}{2}\,\xi^\dagger \partial_\mu \xi\right) \qquad a = 1,2$$

transform as $\quad u^a_\mu \sigma_a \to h^\dagger u^a_\mu \sigma_a h$

Now we can write invariant (the symmetry is non-linear but it is still there!) terms in our action, the first:

$$S^g_{UV} = \int d^4x \left(-\frac{f^2}{2} \sum_{a=1,2} u^a_\mu u^{a\mu} \right)$$

One can find u_μ in exact form but for us it's enough to expand on $1/f$

(IIb) One can use the formula

$$e^{-A}\partial e^{A} = \sum_n \frac{1}{n!} \underbrace{[-A,[-A,\dots[\partial A]\dots]}_{n \text{ times}}$$

to obtain

$$u_\mu^a = i\left[\frac{\partial_\mu \varphi^a}{f} + \frac{2}{3f^3}\varphi_1 \overleftrightarrow{\partial}\varphi_2 \varepsilon_{b3a}\varphi^b + \mathcal{O}\left(\frac{\varphi^5}{f^5}\right)\right]$$

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This yields an action

$$S_{EFT}^{\varphi} = \int d^4x \left(\frac{\partial\varphi\partial\varphi}{2} - \frac{2}{3f^2}(\varphi_1 \overleftrightarrow{\partial}\varphi_2)^2 + \mathcal{O}\left(\frac{1}{f^4}\right)\right)$$

where it is useful to note / define

$$\varepsilon_{ab3}\varphi^a\partial_\mu\varphi^b = \varphi^1\partial_\mu\varphi^2 - \varphi^2\partial_\mu\varphi^1 = \varphi^1 \overleftrightarrow{\partial}_\mu \varphi^2$$
$$\equiv \varphi^T T \partial_\mu \varphi = \phi^* \overleftrightarrow{\partial}_\mu \phi$$

with $T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $\phi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$, where the last relation allows connection with the original EFT action as

$$c_6 = c_5 = 0 \,, \quad \frac{c_g}{\Lambda^2} = \frac{2}{3f^2} \,.$$

Extra Problem You asked about it so here's a perturbative UV completion of S_{UV}^{g}:

$$S_{(UV)^2}^{g} = \int d^4x \left(\frac{1}{4} \mathrm{Tr}\left(\partial_\mu \Delta \partial^\mu \Delta^\dagger\right) - \frac{\lambda}{4}\left[\tilde{f}^2 - \frac{\mathrm{Tr}(\Delta \Delta^\dagger)}{2}\right]\right)$$

where

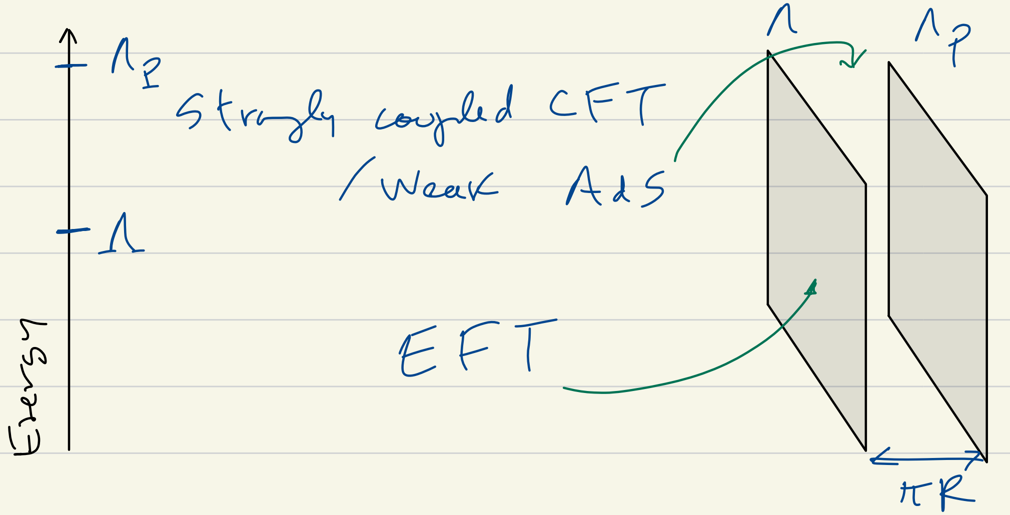
$$\Delta = \begin{pmatrix} \Delta_3 & \Delta_1 - i\Delta_2 \\ \Delta_1 + i\Delta_2 & -\Delta_3 \end{pmatrix} = \xi S \sigma_3 \xi^\dagger$$

with Δ_i, S real scalars & ξ as given and $\Delta \to G \Delta G^\dagger$ under $SU(2)$

- What is the v.e.v. of S, $\langle 0 | S | 0 \rangle$?
- Is there any unbroken symmetry?
- Do you obtain S_{UV}^{g} if you substitute S by its vev in the action?

B.3 A slice of AdS/CFT

Conformal field theory has no scale but one can introduce scales above and below a conformal range (Λ, Λ_P). In the AdS dual this corresponds to a "slice" i.e. two branes each associated with Λ, Λ_P where $\Lambda \sim \Lambda_P e^{-\Lambda_P \pi R}$ with Λ_P the AdS curvature & πR the slice length.



And below Λ we have that EET works.

To cut it short the AdS side allows us to compute correlation functions, for a scalar coupled to a CFT operator of dimension $\nu+3$

$$S_{UV} = \int d^4x \left(\mathcal{L}_4^{UV}(\underline{\varphi}) + \frac{\omega}{\Lambda_P^{\nu}} \underline{\varphi}\, \mathcal{O}_{CFT} + \mathcal{L}_{CFT} \right)$$

with the self energy (Gherghetta TASI Lectures) being

$$S_{UV} = \int \frac{d^4p}{(2\pi)^4} \frac{1}{2} \underline{\tilde{\varphi}}(-p) \Sigma_{CFT}(p) \underline{\tilde{\varphi}}(p) + S_4$$

$$\Sigma_{CFT}(p) = \beta \frac{q_0 \left[I_\nu(q_0) K_\nu(q_1) - I_\nu(q_1) K_\nu(q_0) \right]}{I_{\nu+1}(q_0) K_\nu(q_1) + I_\nu(q_1) K_{\nu+1}(q_0)}$$

$$q_1 = \frac{p}{\Lambda}, \quad q_0 = \frac{p}{\Lambda_P},$$

and I_ν, K_ν are the first & second modified Bessel functions of the first & second kind with expansion

$$I_a(z) = \left(\frac{z}{2}\right)^a \sum_{n=0}^{\infty} \frac{(z^2/4)^n}{n!\,\Gamma(n+a+1)}$$

$$K_a(z) = \frac{\pi}{2}\left(\frac{I_{-a}(z) - I_a(z)}{\sin(\pi a)}\right).$$

(II.c) with this much information one can expand on $p/\Lambda \ll 1$ to obtain $(\alpha \equiv \Lambda/\Lambda_p)$

$$\Sigma_{CFT} = \beta\left(\frac{p^2}{2\nu}\left(1-\alpha^{-2\nu}\right) + a_4(\alpha)\frac{p^4}{\Lambda^2}\right)$$

$$a_4 = \frac{(1-\nu)\,\alpha^{-4\nu-2} + (1+\nu)\alpha^{-2} - \alpha^{-2\nu}\left(2\alpha^{-2} + (\nu-1)\nu\left(1-\alpha^{-2}\right)\right)}{8\nu^2(\nu^2-1)}$$

which can be taken back to position rep to obtain

$$S^{AdS}_{EFT} = \int d^4x \left(\frac{1}{2} \underline{\varphi}^T \left[-\partial^2 \left(1 + \beta \frac{1-\alpha^{-2\nu}}{2\nu} \right) + \frac{a_4 \beta}{\Lambda^2} (\partial^2)^2 \right] \underline{\varphi} + \dots \right.$$

which we can turn into the form of S_{EFT} by first renormalising the fields

$$\underline{\varphi} = \left(1 + \frac{\beta (1-\alpha^{-2\nu})}{2\nu} \right)^{-1/2} \varphi \equiv Z^{-1/2} \varphi$$

using EoM for φ

$$S_4^{UV}(\varphi) = \int d^4x \left(\frac{1}{2} \varphi^T \left(-\partial^2 - \frac{m_{UV}}{Z} \right) \varphi - \frac{\lambda_{UV}}{4Z^2} \frac{(\varphi^T \varphi)^2}{4} \right)$$

$$\hookrightarrow^{\text{EoM}} \quad -\left(\partial^2 + \frac{m^2_{UV}}{Z^2} \right) \varphi - \frac{4 \lambda_{UV}}{16 Z^2} \varphi \varphi^2 = \mathcal{O}\left(\frac{1}{\Lambda^2} \right)$$

$$S^{AdS}_{EFT} \subset \int d^4x \, \frac{a_4}{2\Lambda^2 Z} \left(\frac{m^2_{UV} \varphi}{Z} + \frac{\lambda_{UV}}{4} \frac{\varphi \varphi^2}{Z^2} \right)^2$$

to obtain

$$m^2 = \frac{m^2_{UV}}{Z} + ? \qquad \lambda = \frac{\lambda_{UV}}{Z^2} + ?$$

$$\frac{c_6}{\Lambda^2} = \frac{a_4 \lambda^2_{UV}}{4 \Lambda^2 Z^5} \simeq \frac{a_4 \lambda^2}{4 \Lambda^2 Z^5} + \mathcal{O}(\Lambda^{-4})$$

Summary

We have seen that different UV models will give rise to different coefficients c_i & some might be zero. However if one keeps all coefficients general and computes observables there's always the possibility to set whichever coefficient vanishing in the predictions.

That is the sense in which EFTs are model independent, one doesn't need the c's as a function of UV parameters just to know that they are there to work in an EFT.

That's what we'll do in the next lecture.