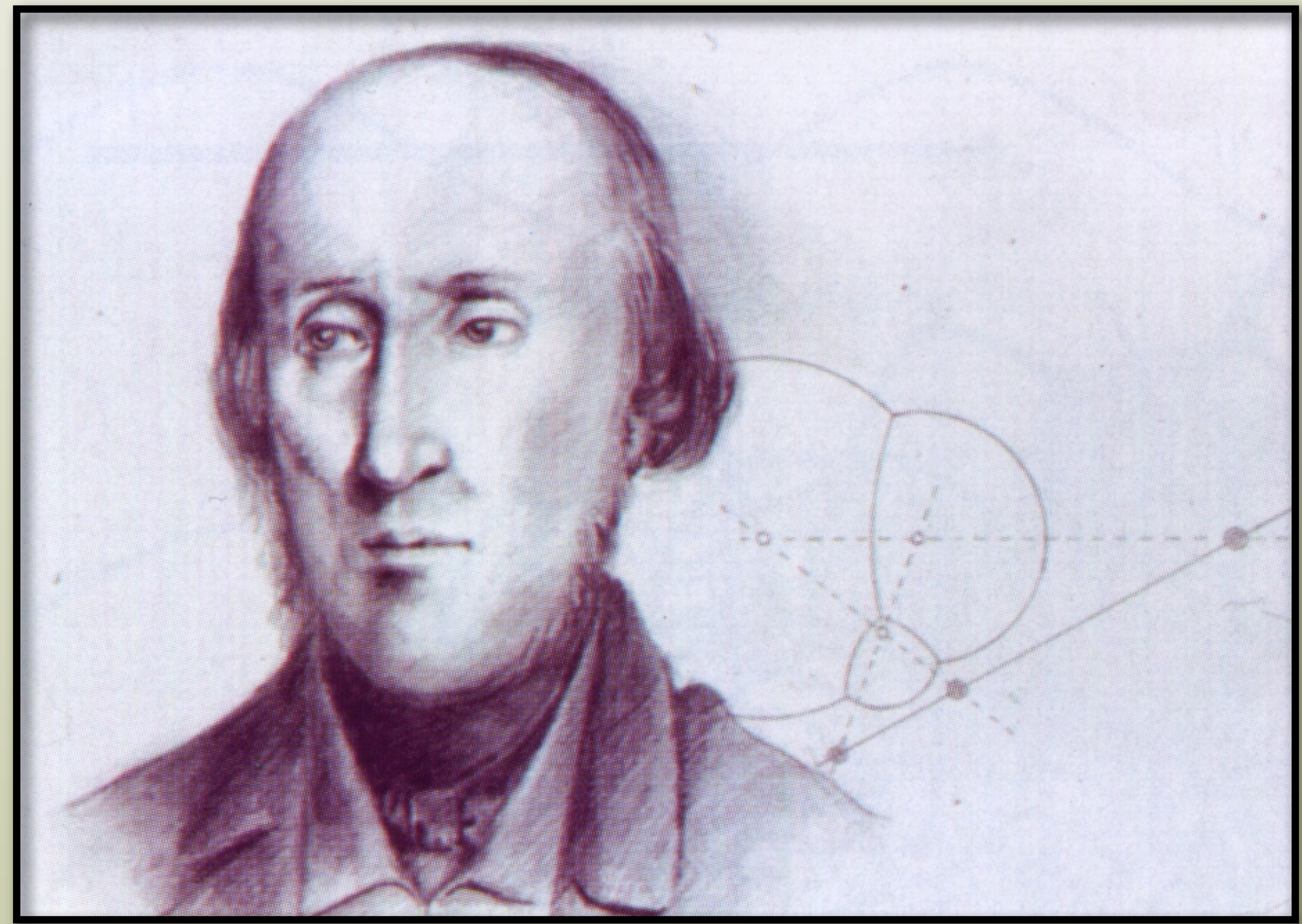
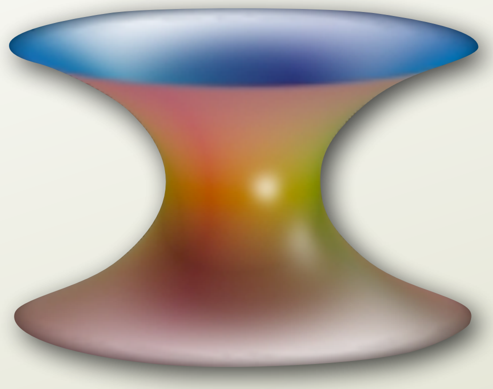
Minimal surfaces: what are they good for?

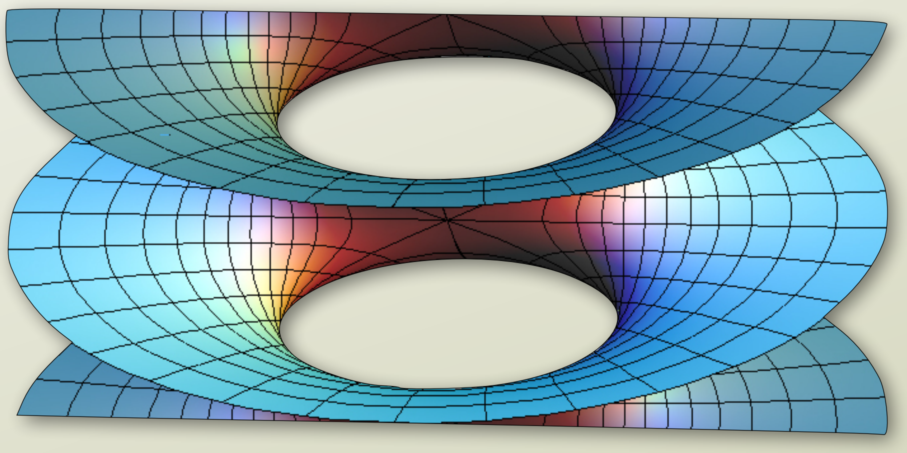


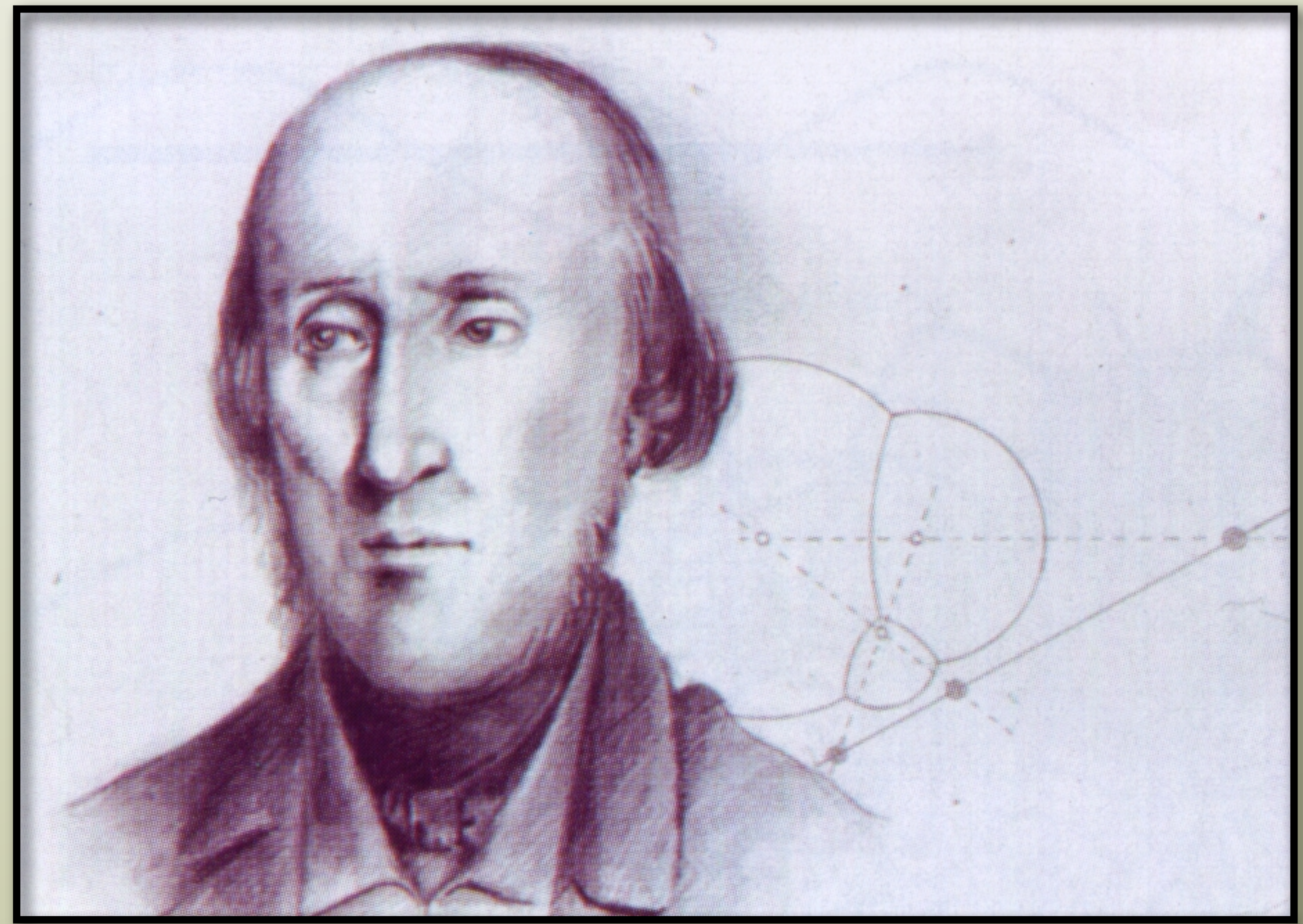
Joseph Antoine Ferdinand Plateau (1801-1883)

Randall D. Kamien
Physics & Astronomy
Penn

Minimal surfaces: what are they good for?







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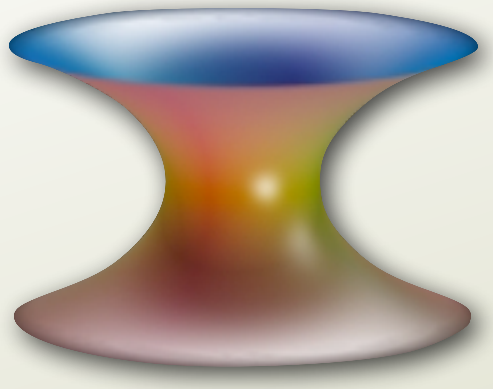
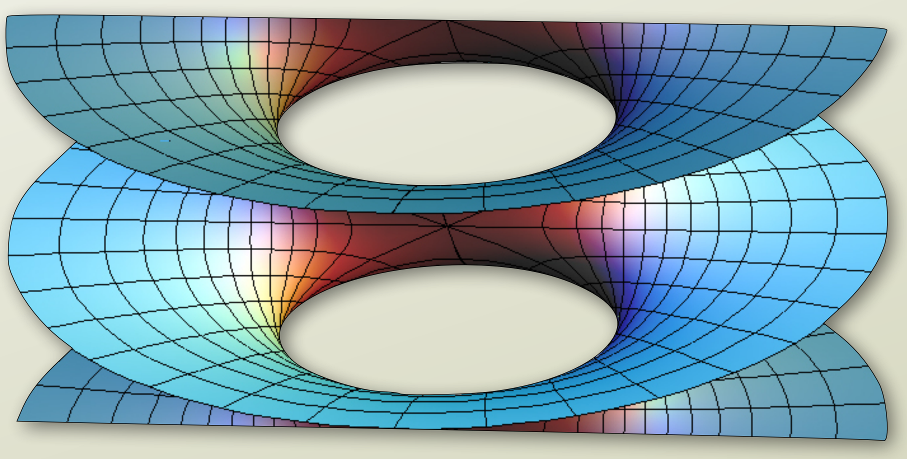
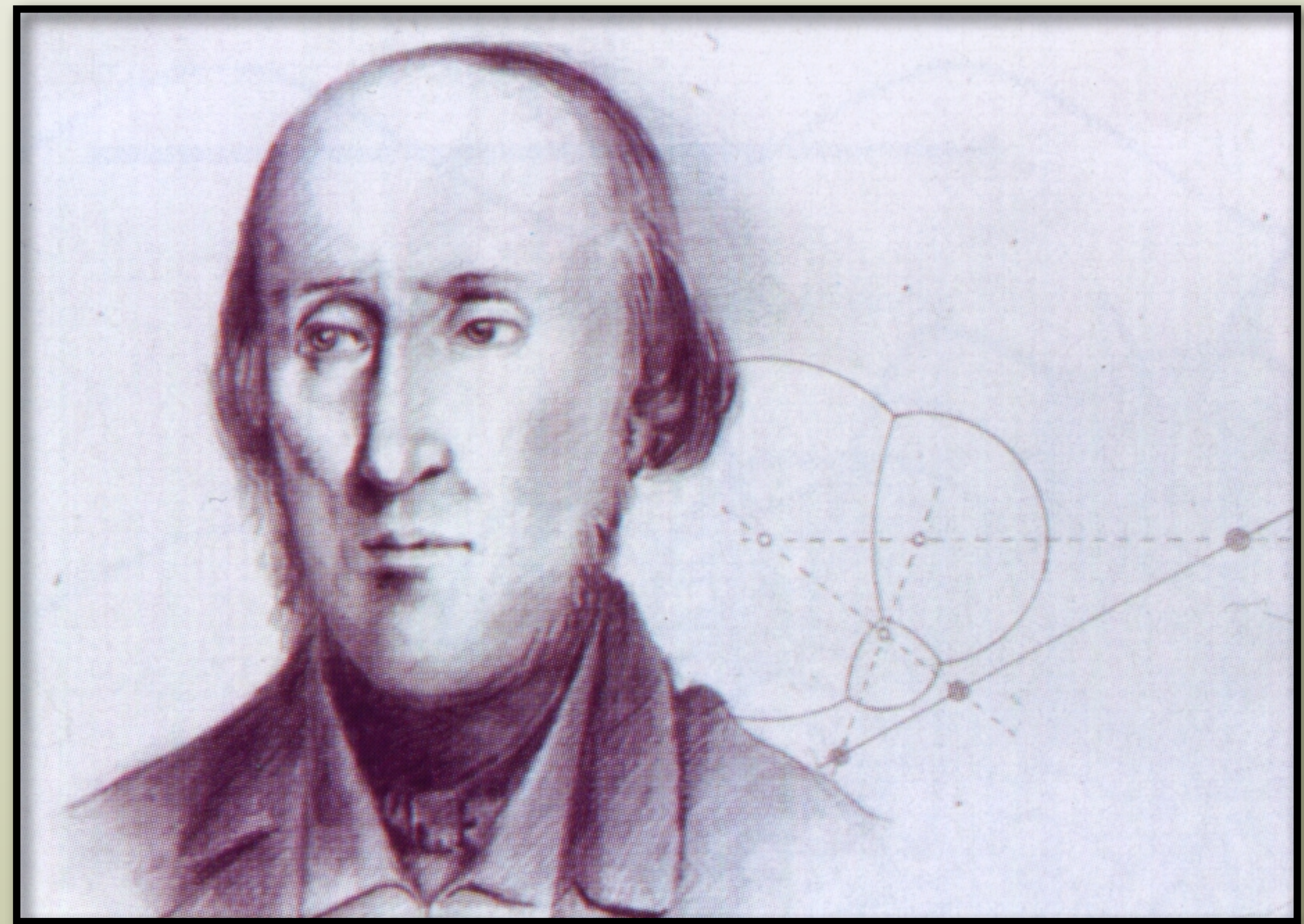




https://heat-exchanger-world.com/revolutionizing-heat-exchangers-with-additively-manufactured-gyroids/



Joseph Antoine Ferdinand Plateau (1801-1883)

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The *Dramatis Personæ*







The *Dramatis Personæ*

Dido

Hero

Fermat

Ja. Bernoulli

Jo. Bernoulli

Euler

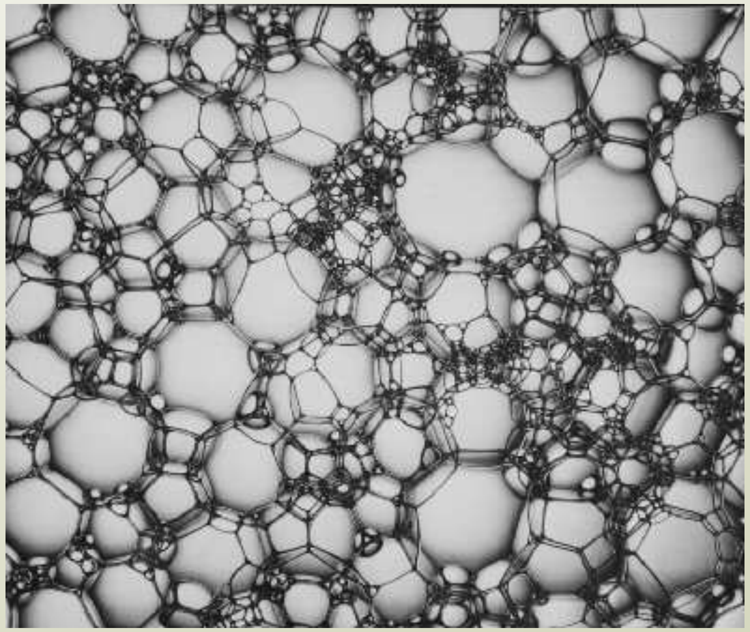
Lagrange

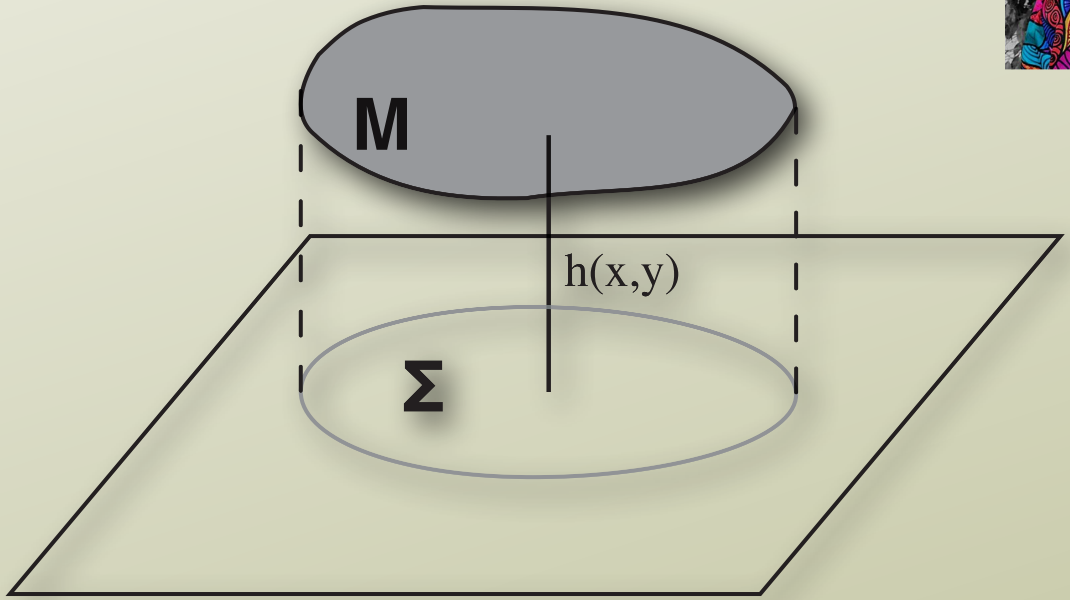
Minimal Surfaces and Foams

Plateau's Rules:

- each edge must border three faces which meet at 120°
- each vertex must join four edges which meet at 109.5°

Plateau (1873)
Taylor (1976)





$$E = \gamma A - PV = \gamma \int_\Sigma dxdy \sqrt{1 + h_x^2 + h_y^2} - P \int_\Sigma dxdy\, h(x,y)$$

Minimal Surfaces and Foams

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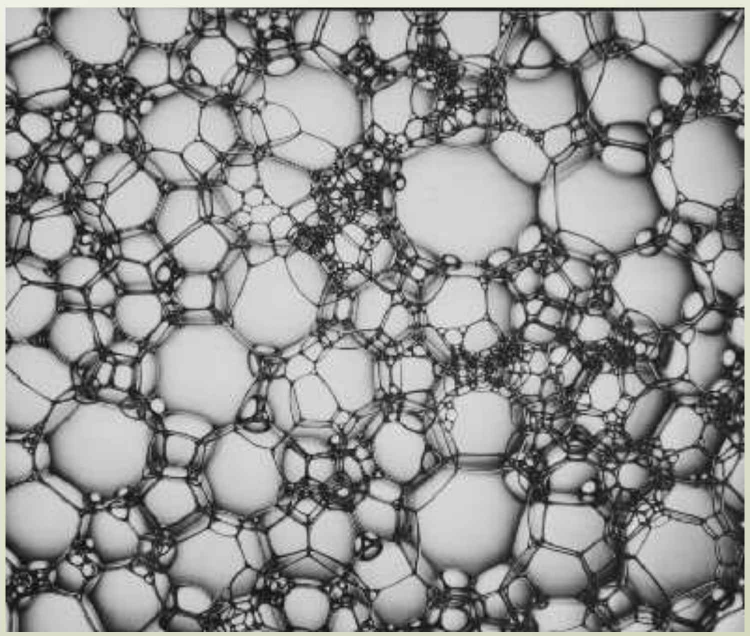

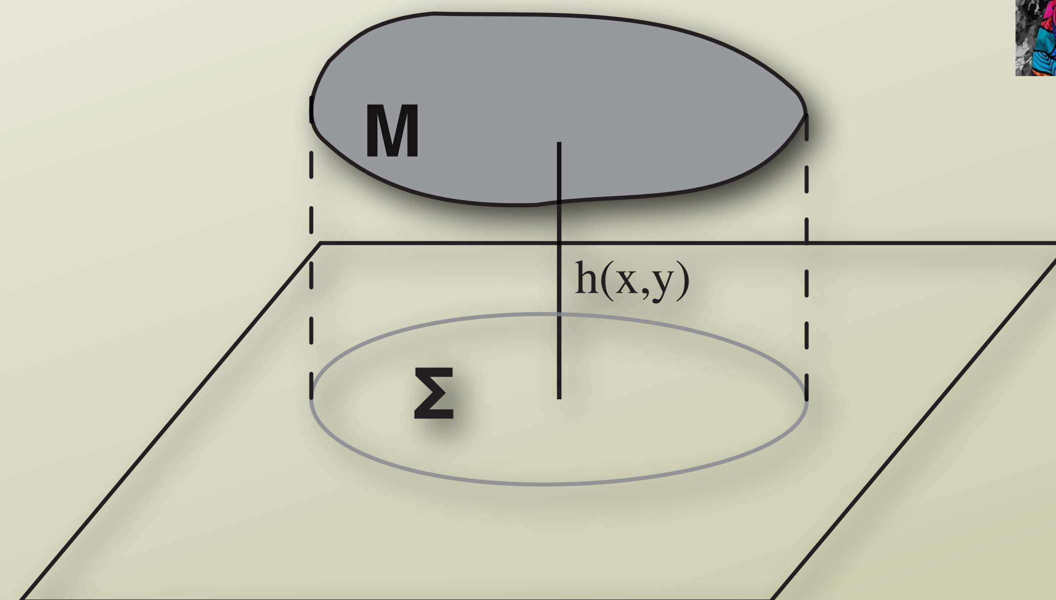

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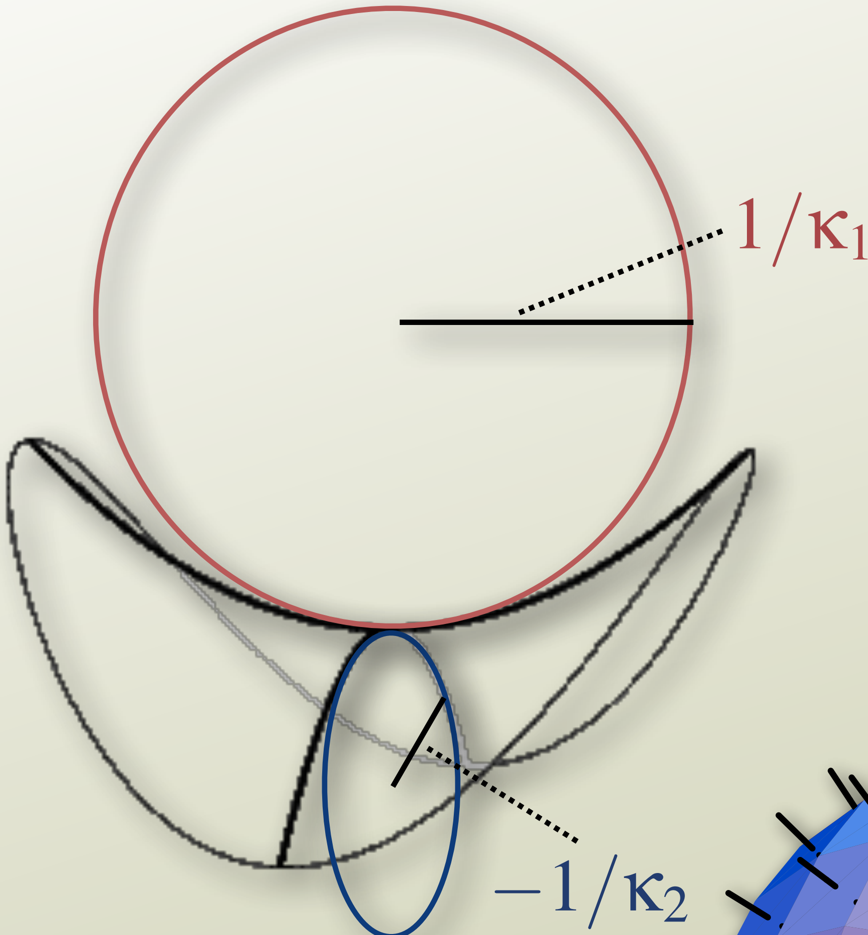
The Laplace-Young Law:

$$P = 2\gamma H$$

pressure (Lagrange multiplier)
mean curvature
surface tension

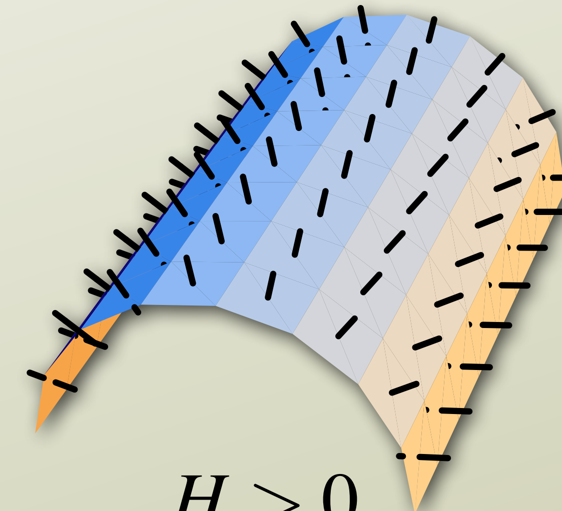
Penn

Two Kinds of Curvature

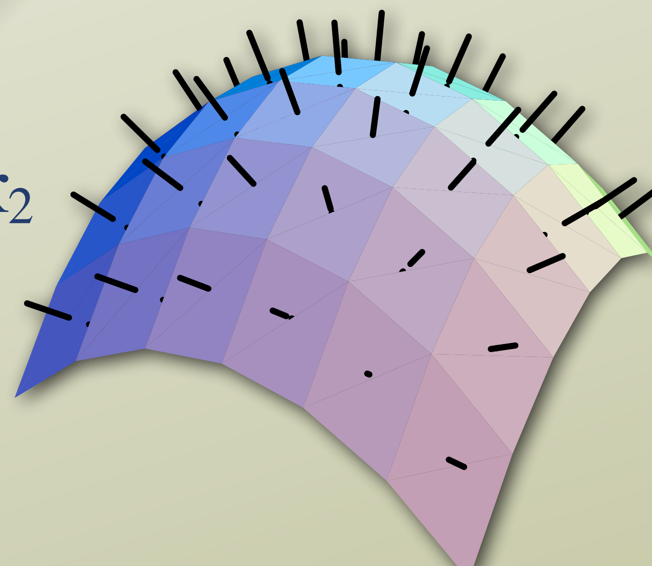


Mean or Extrinsic Curvature

$$H = \frac{1}{2}(\kappa_1 + \kappa_2)$$



$H > 0$
$K = 0$



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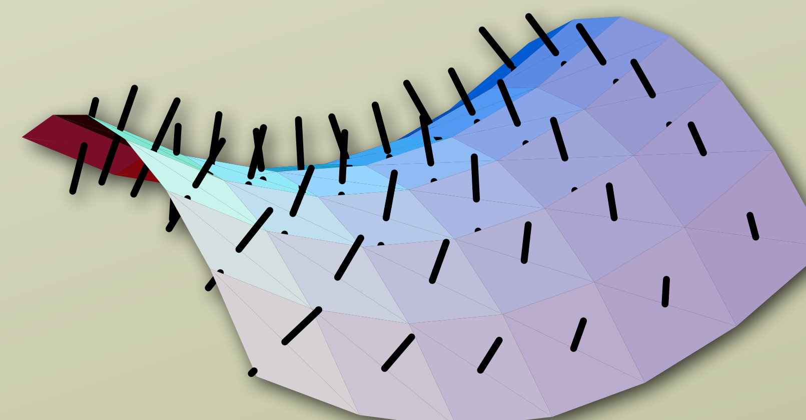


$H = 0$
$K < 0$

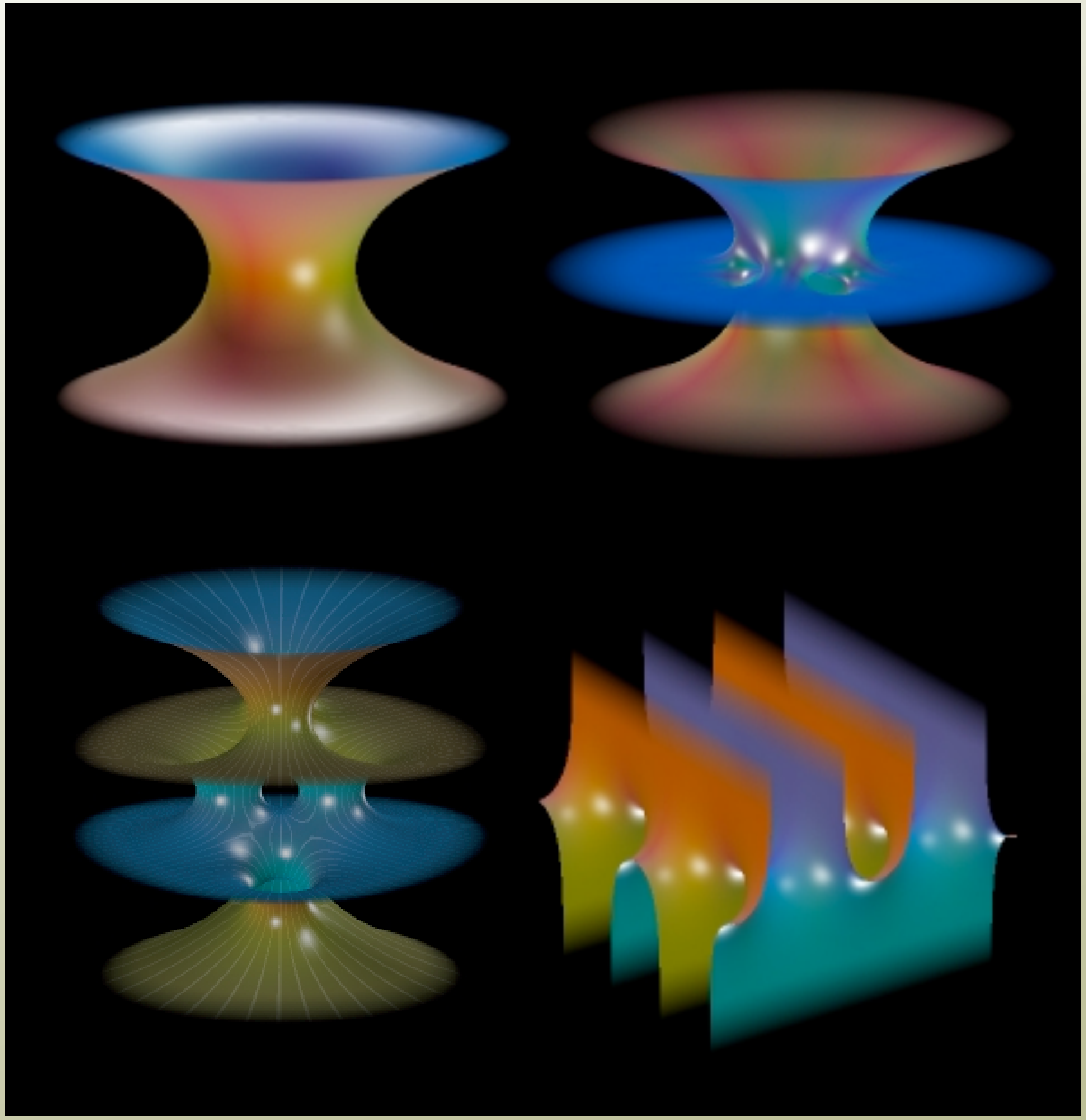
Gaussian or Intrinsic Curvature

$$K = \kappa_1 \kappa_2$$

Minimal Surfaces

Catenoid

Costa-Hoffman-Meeks



Four-End Handled

Scherk's First Surface

Graphics from MSRI - http://www.msri.org/publications/sgp/

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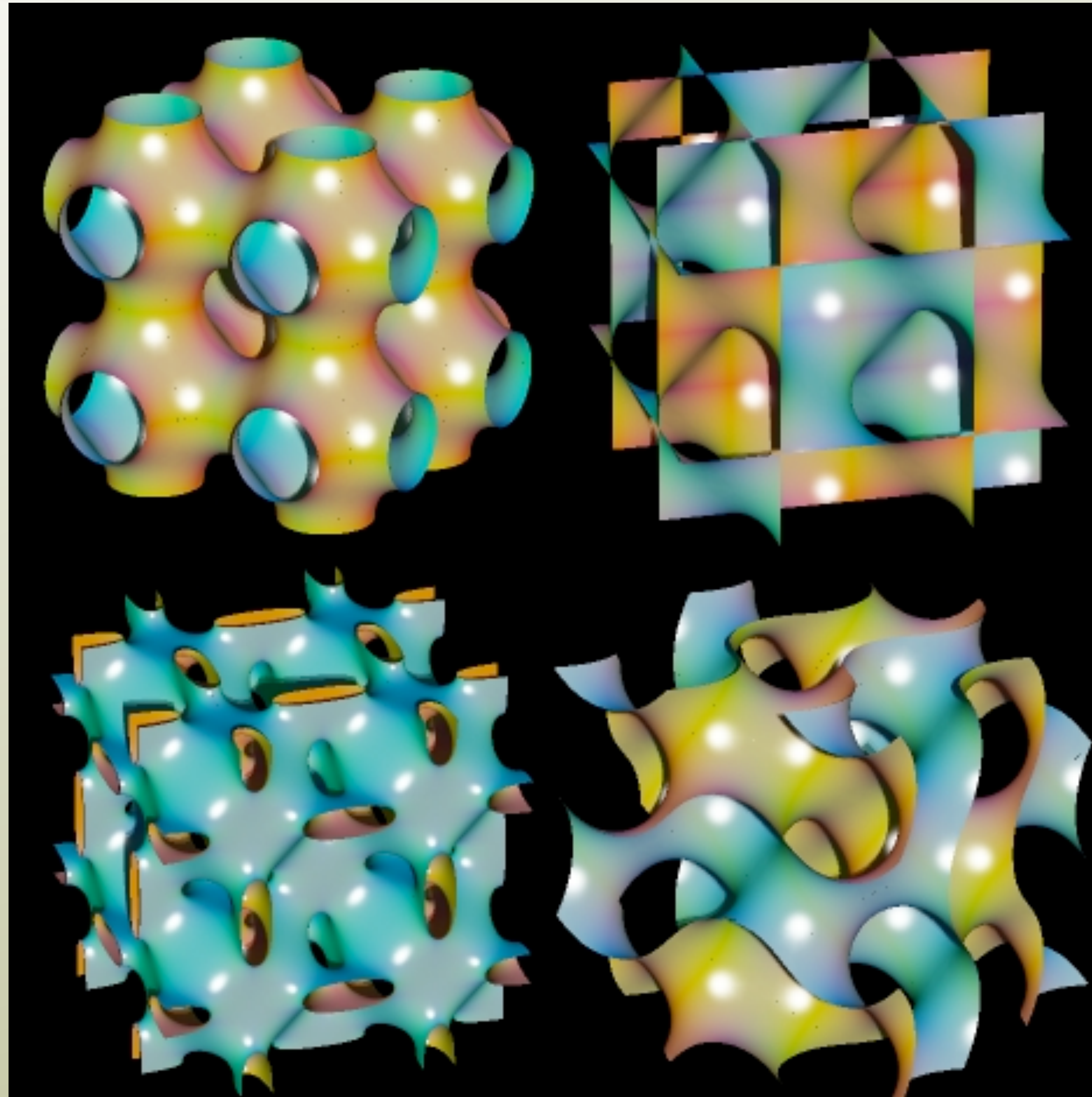
Minimal Surfaces

Schwartz P

Diamond



Neovius

Gyroid

Graphics from MSRI - http://www.msri.org/publications/sgp/

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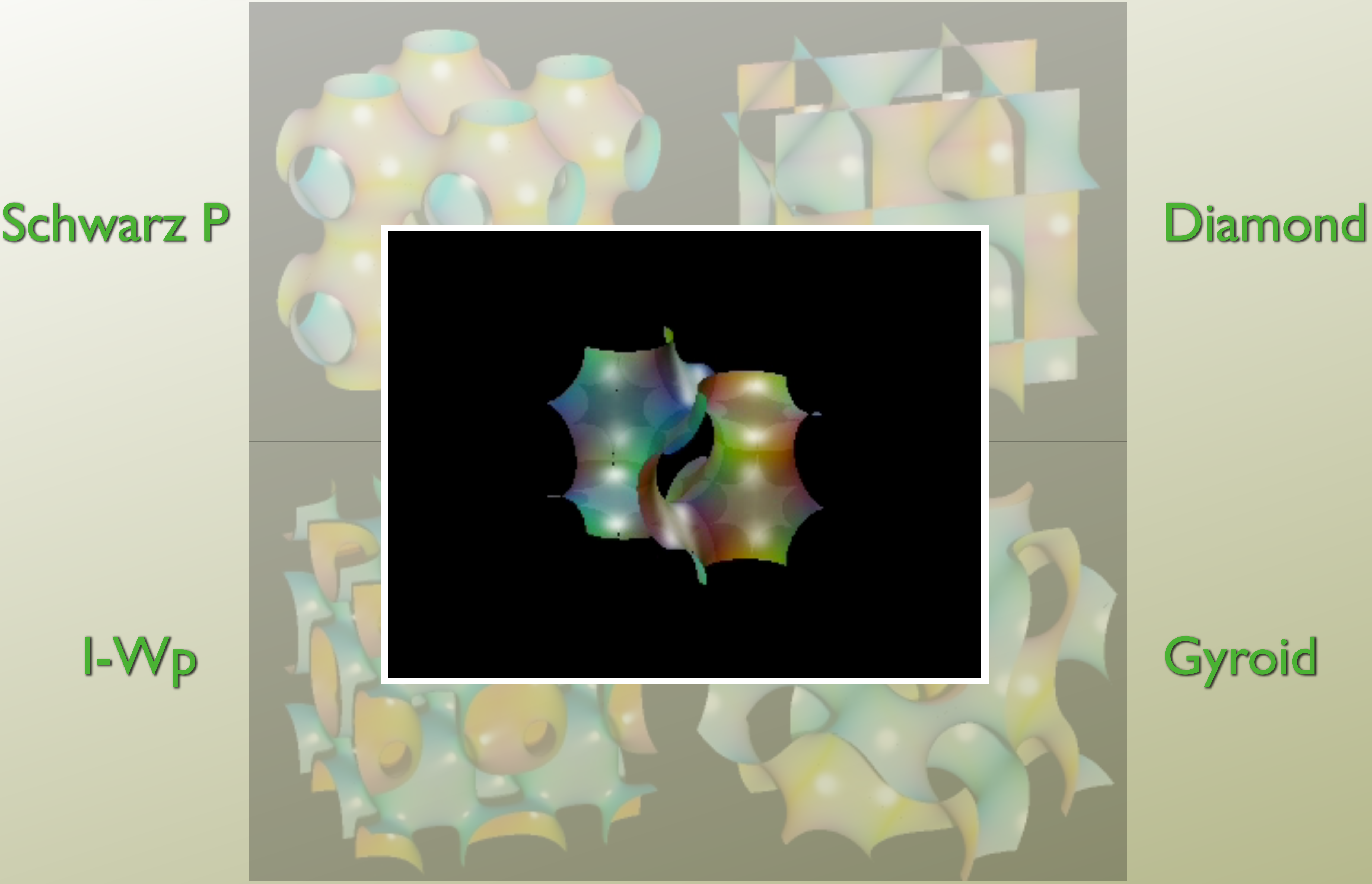
Minimal Surfaces

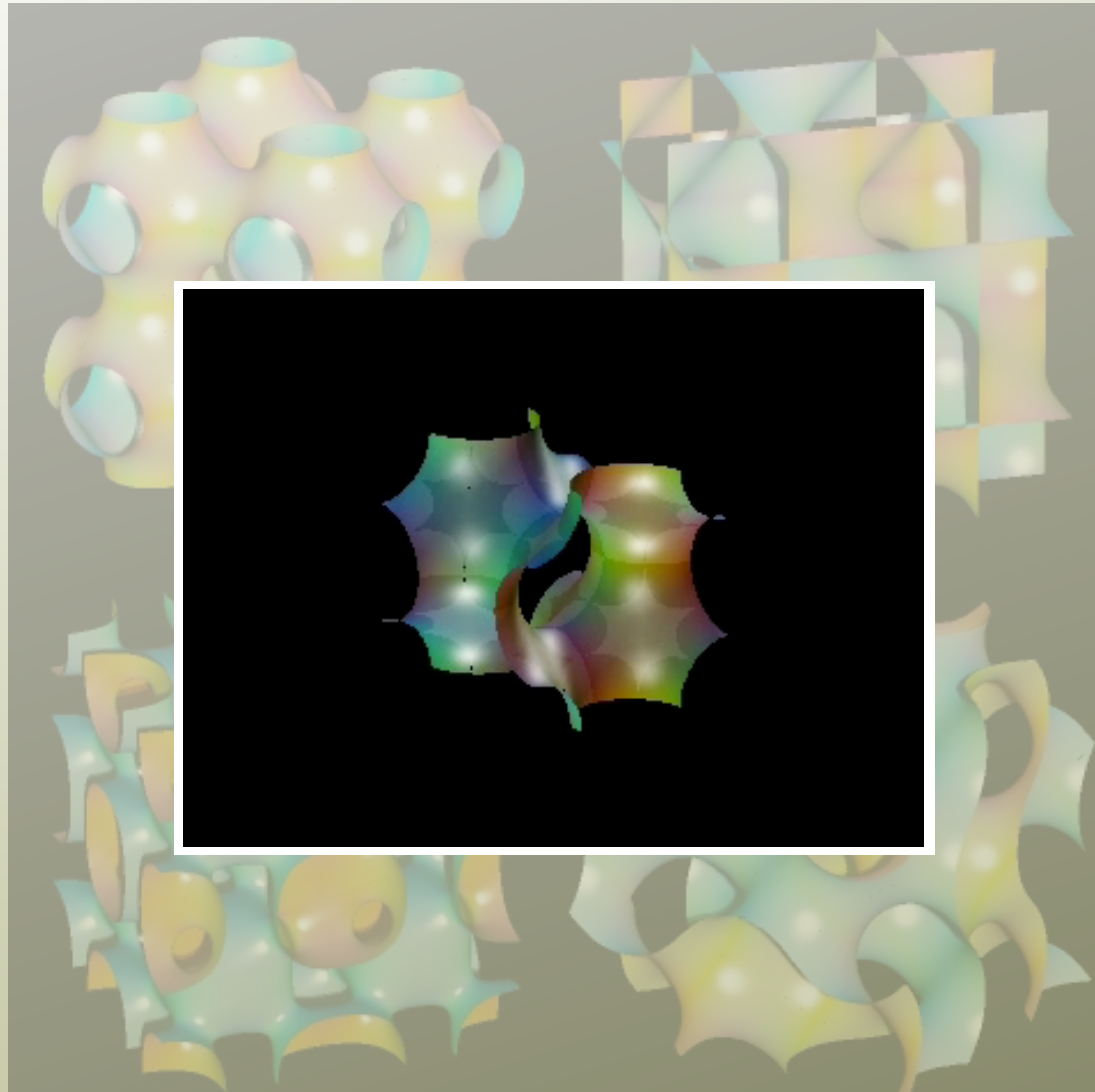


Graphics from MSRI - http://www.msri.org/about/sgp/SGP

Minimal Surfaces

Schwarz P

Diamond



I-Wp

Gyroid

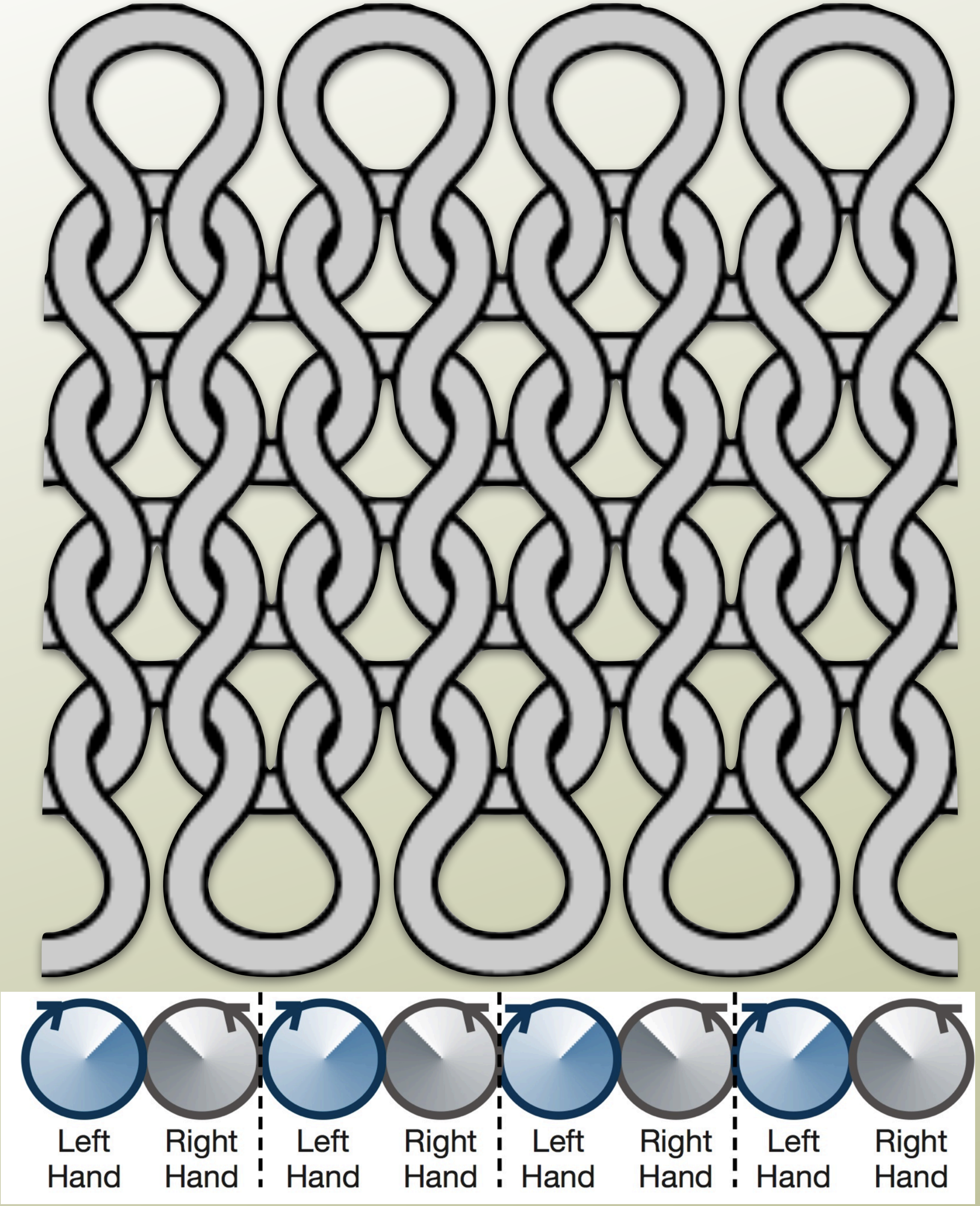
Graphics from MSRI - http://www.msri.org/about/sgp/SGP

Revenge of the Stitch

Revenge of the Stitch

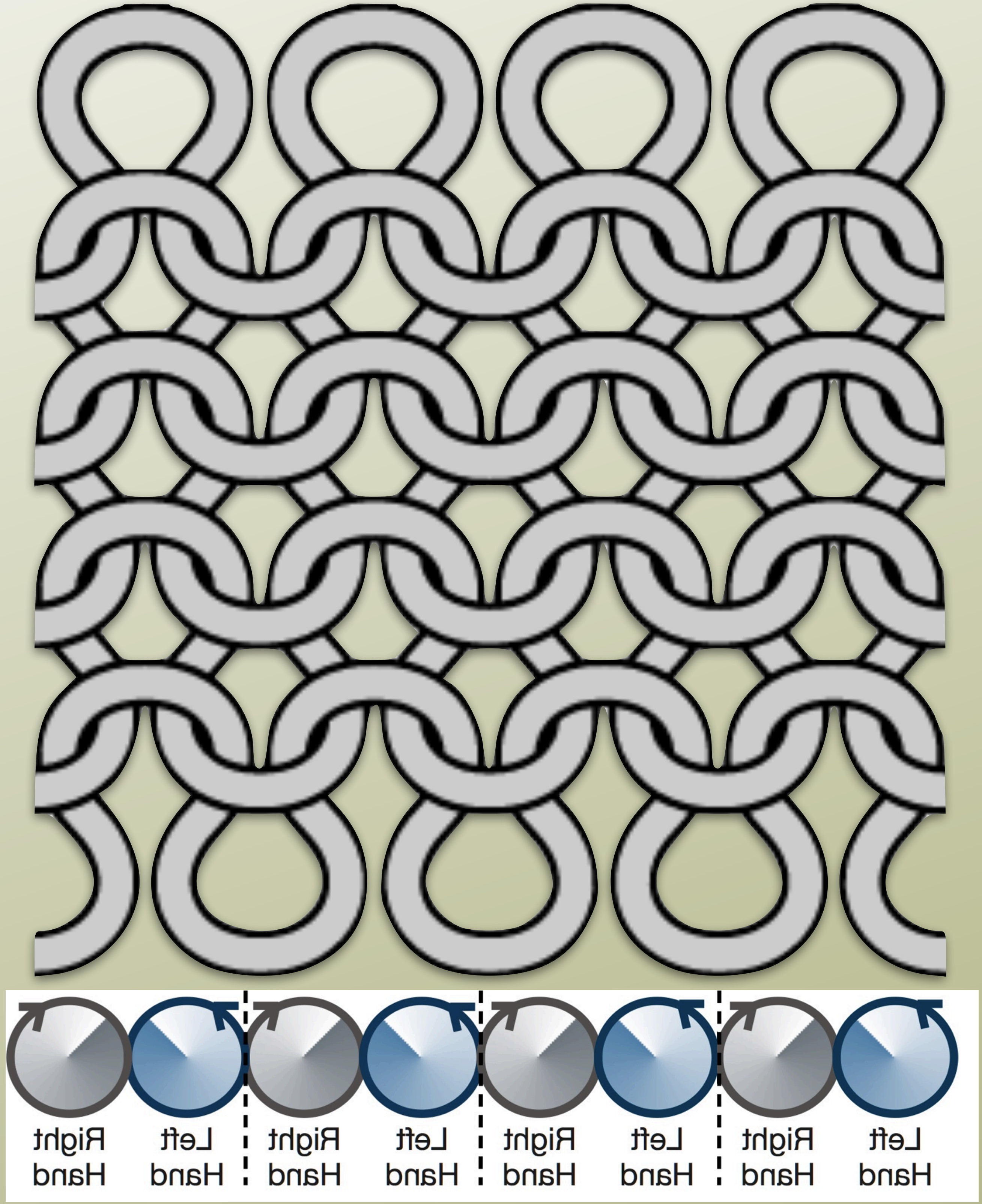
Penn

KNIT



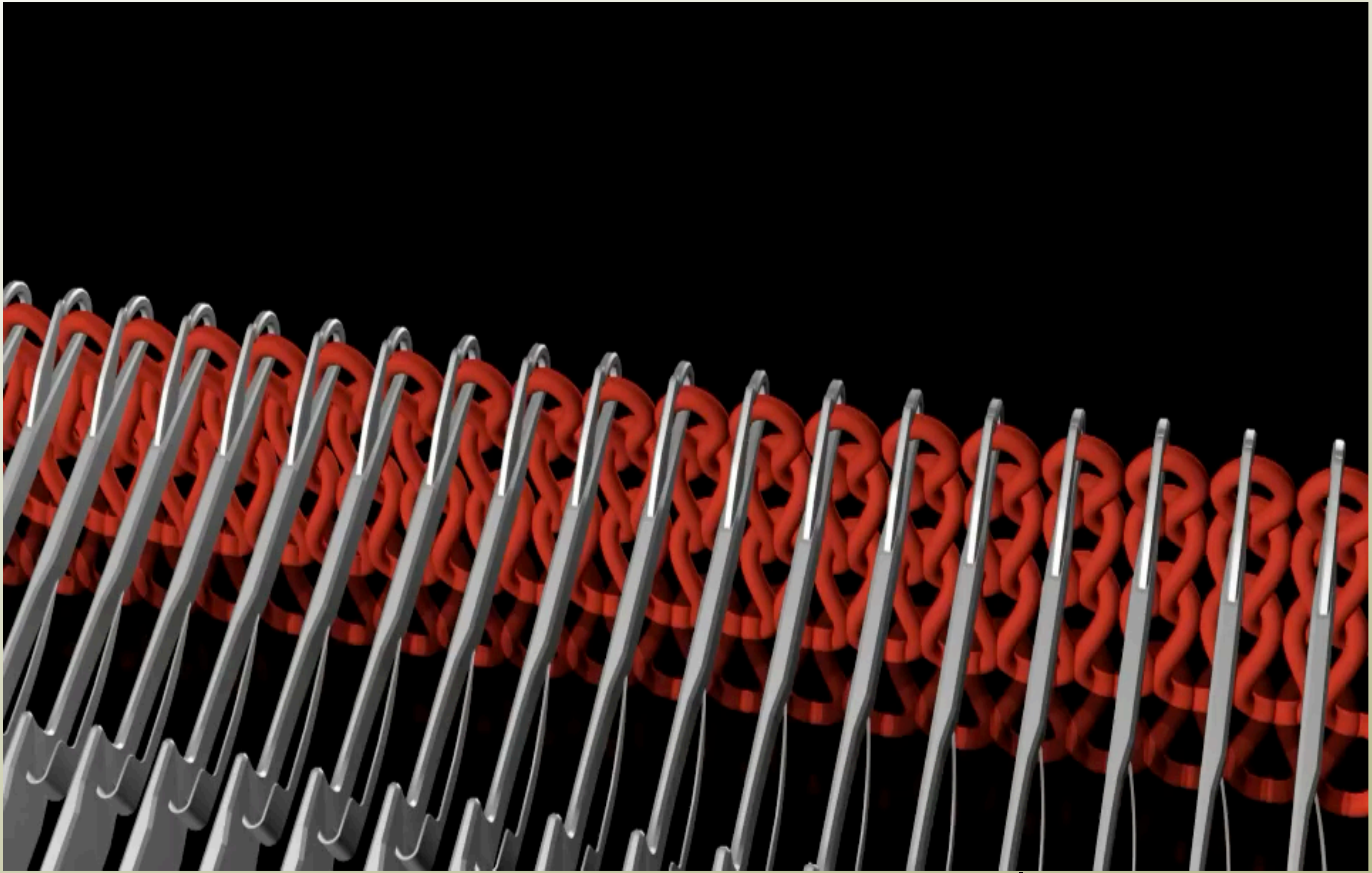
Left Hand
Right Hand
Left Hand
Right Hand
Left Hand
Right Hand
Left Hand
Right Hand

PURL

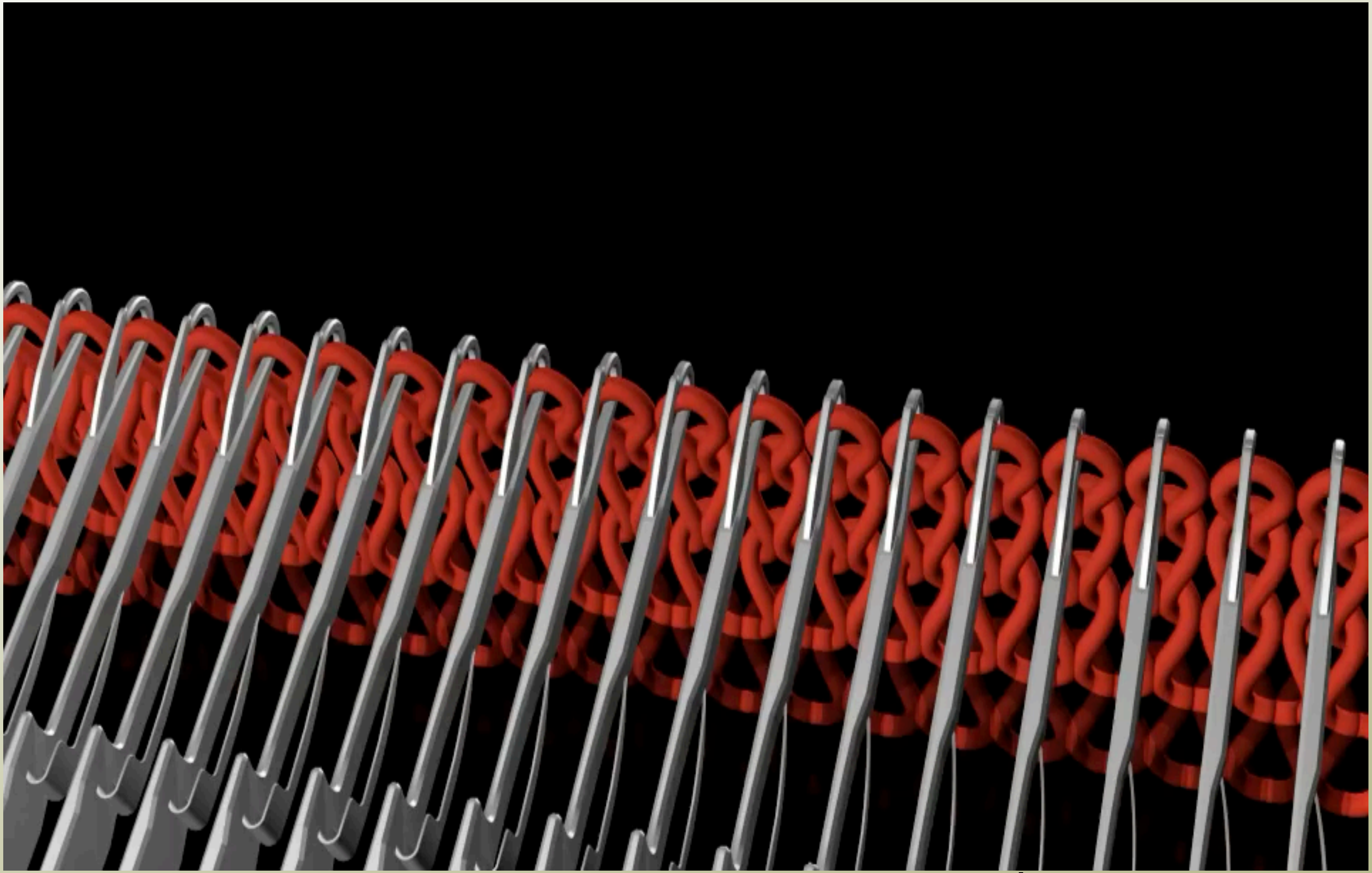


Right Hand
Left Hand
Right Hand
Left Hand
Right Hand
Left Hand
Right Hand
Left Hand

Knitogami



Knitogami



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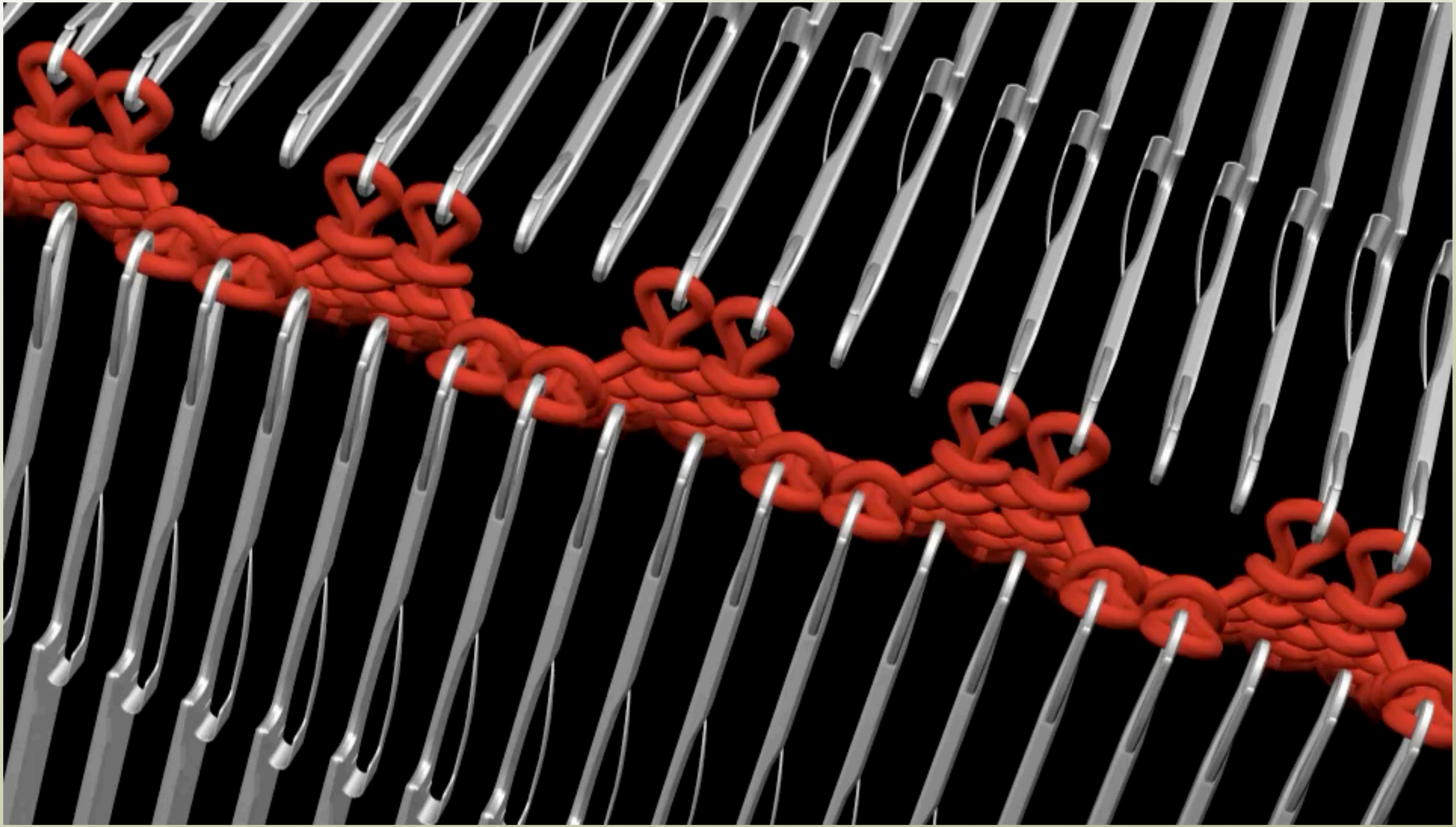
Knitogami



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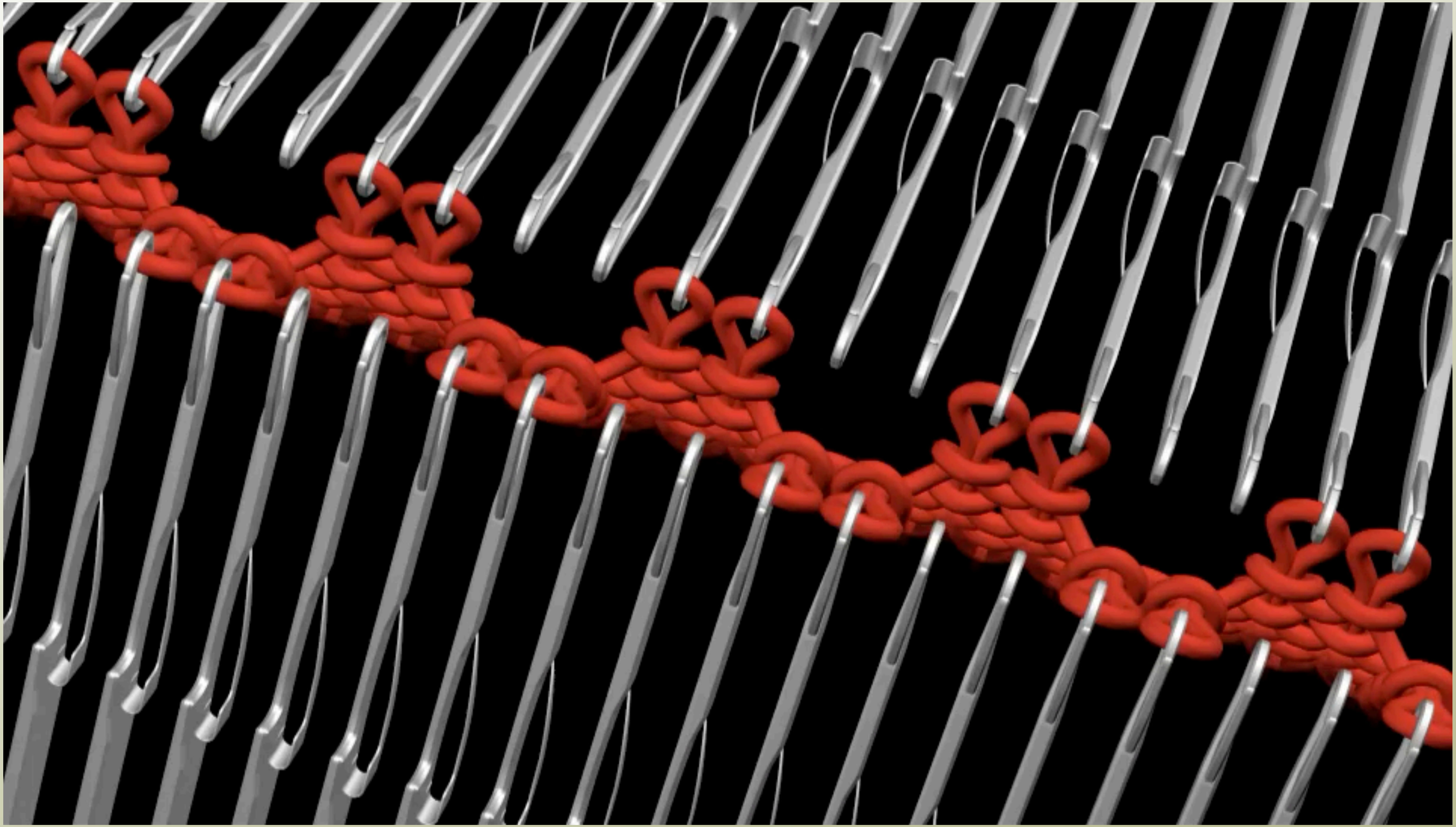
Knitogami



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Knitogami

Knitogami

Stolen from Jim McCann (https://www.cs.cmu.edu/~jmccann/)

Knitogami

https://www.youtube.com/watch?v=27XyAvFXbhg

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Additive Manufacturing!

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Additive Manufacturing!

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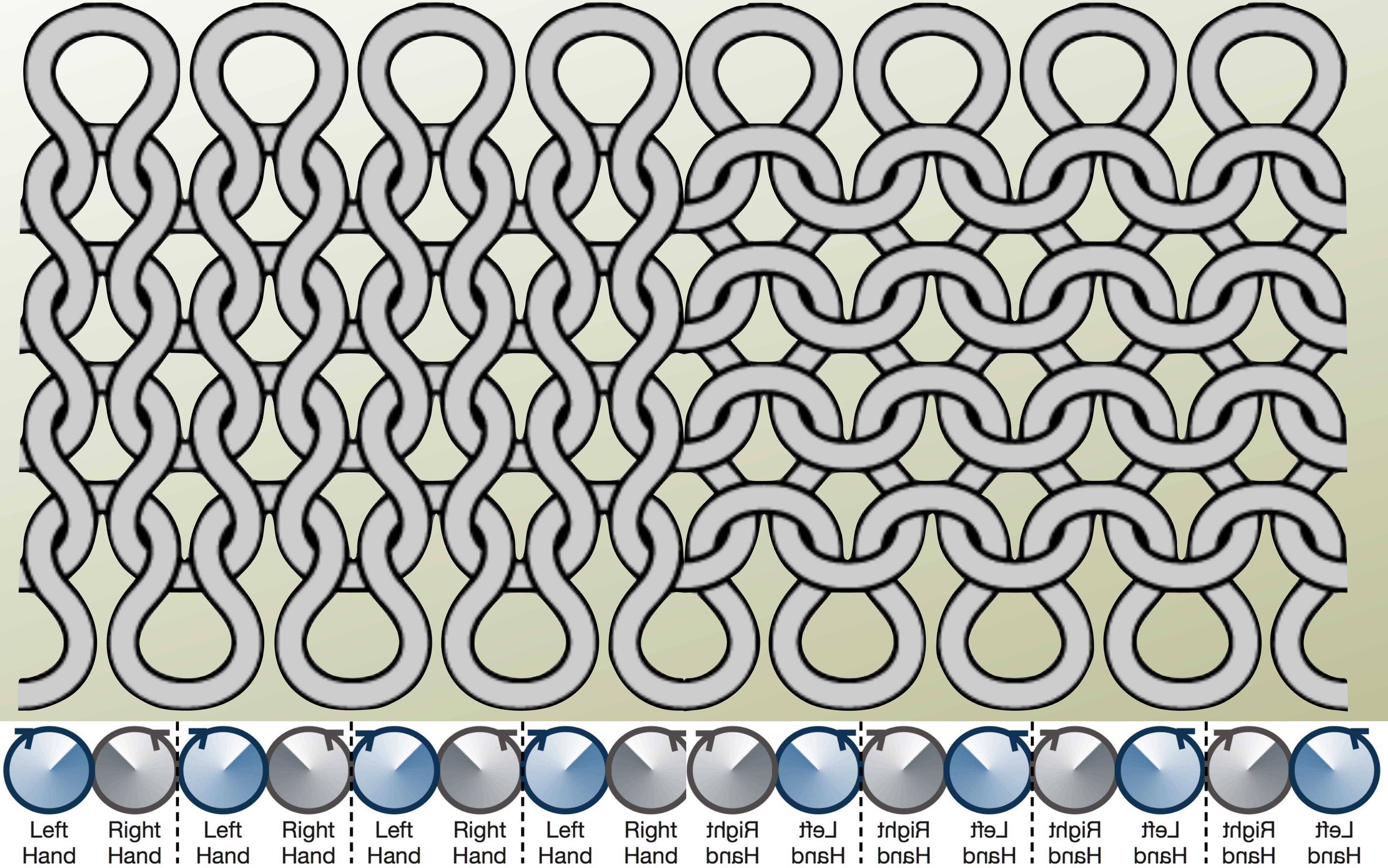
KNIT PURL



Left Hand
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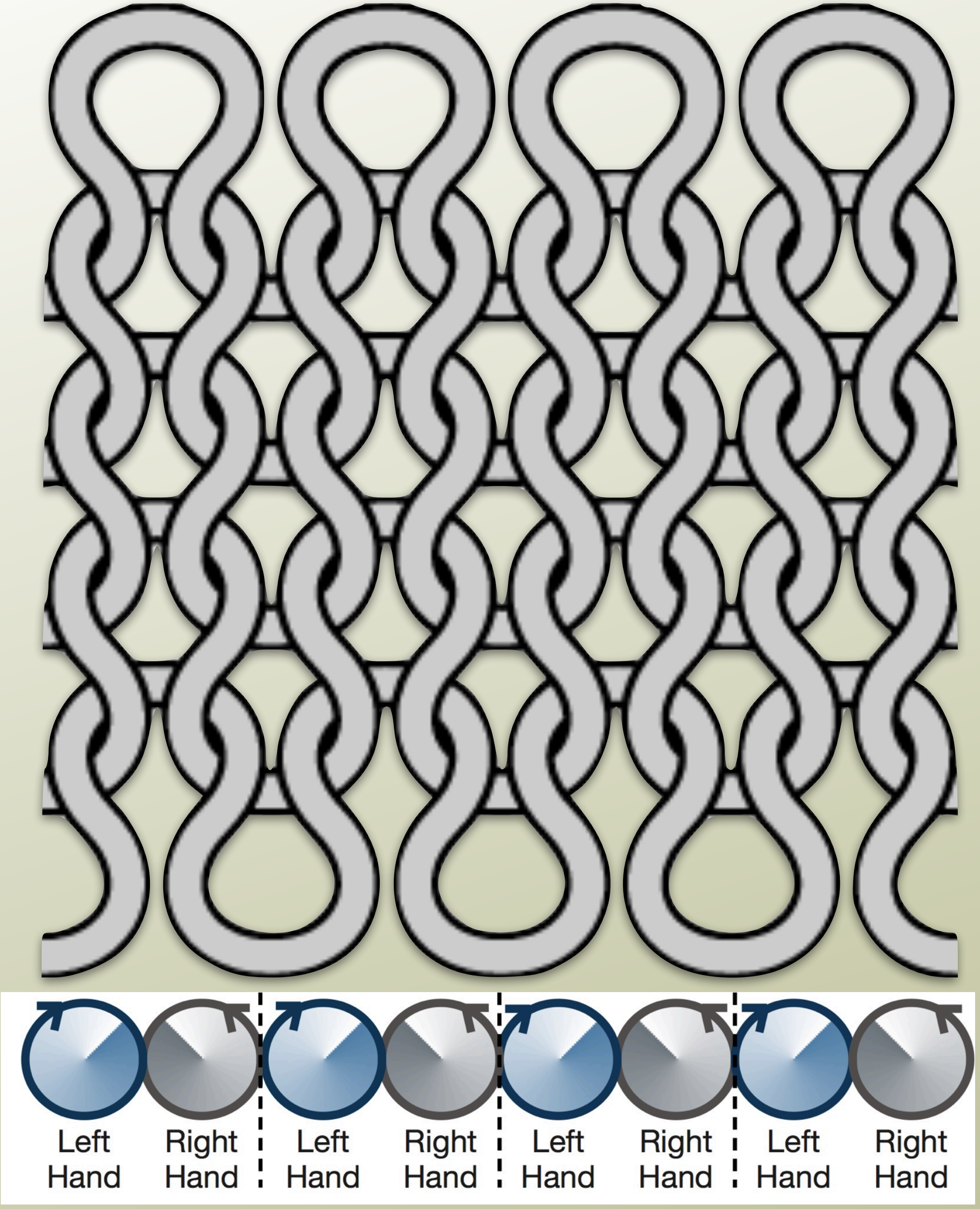
Right Hand
Left Hand

The Knit Fits



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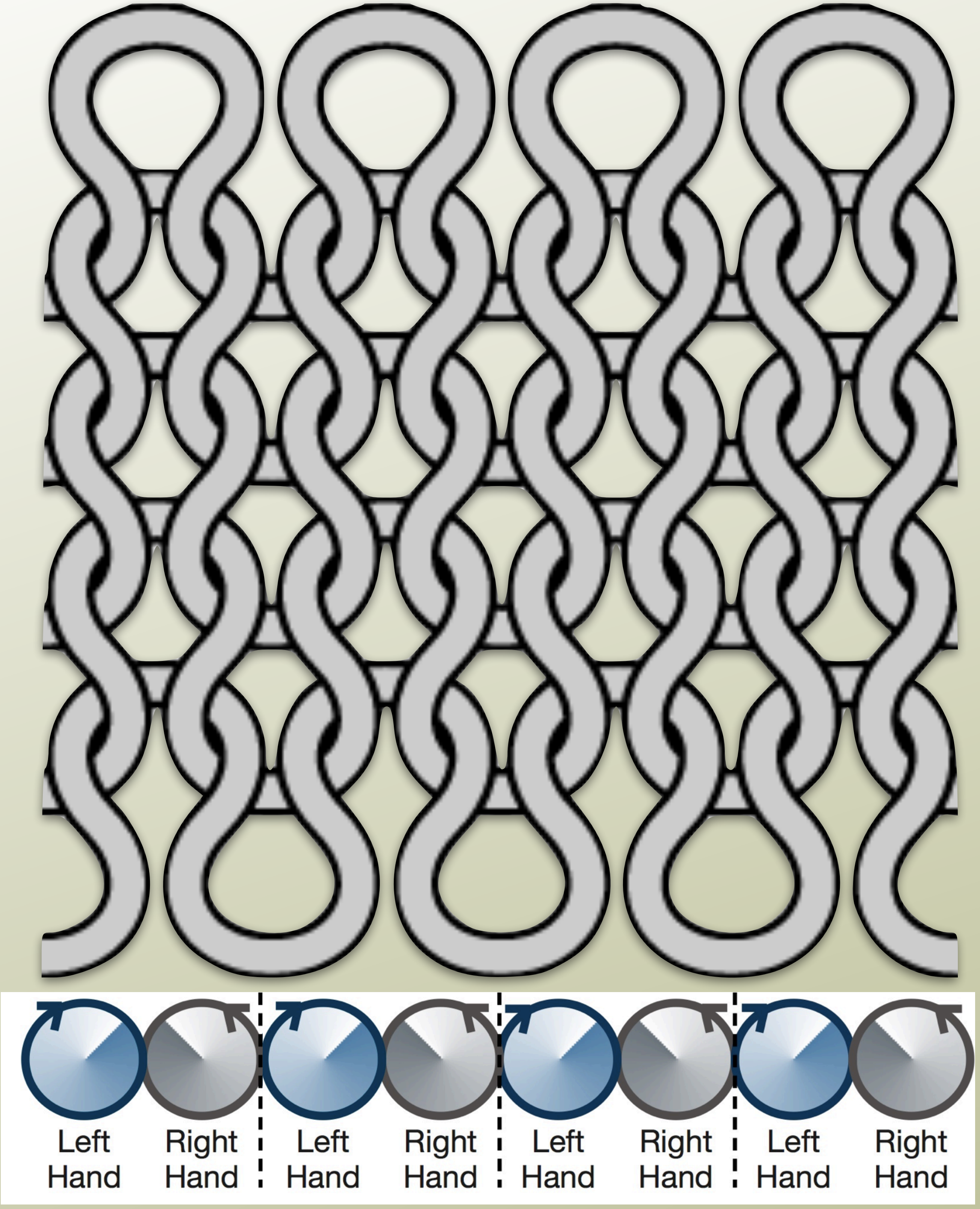
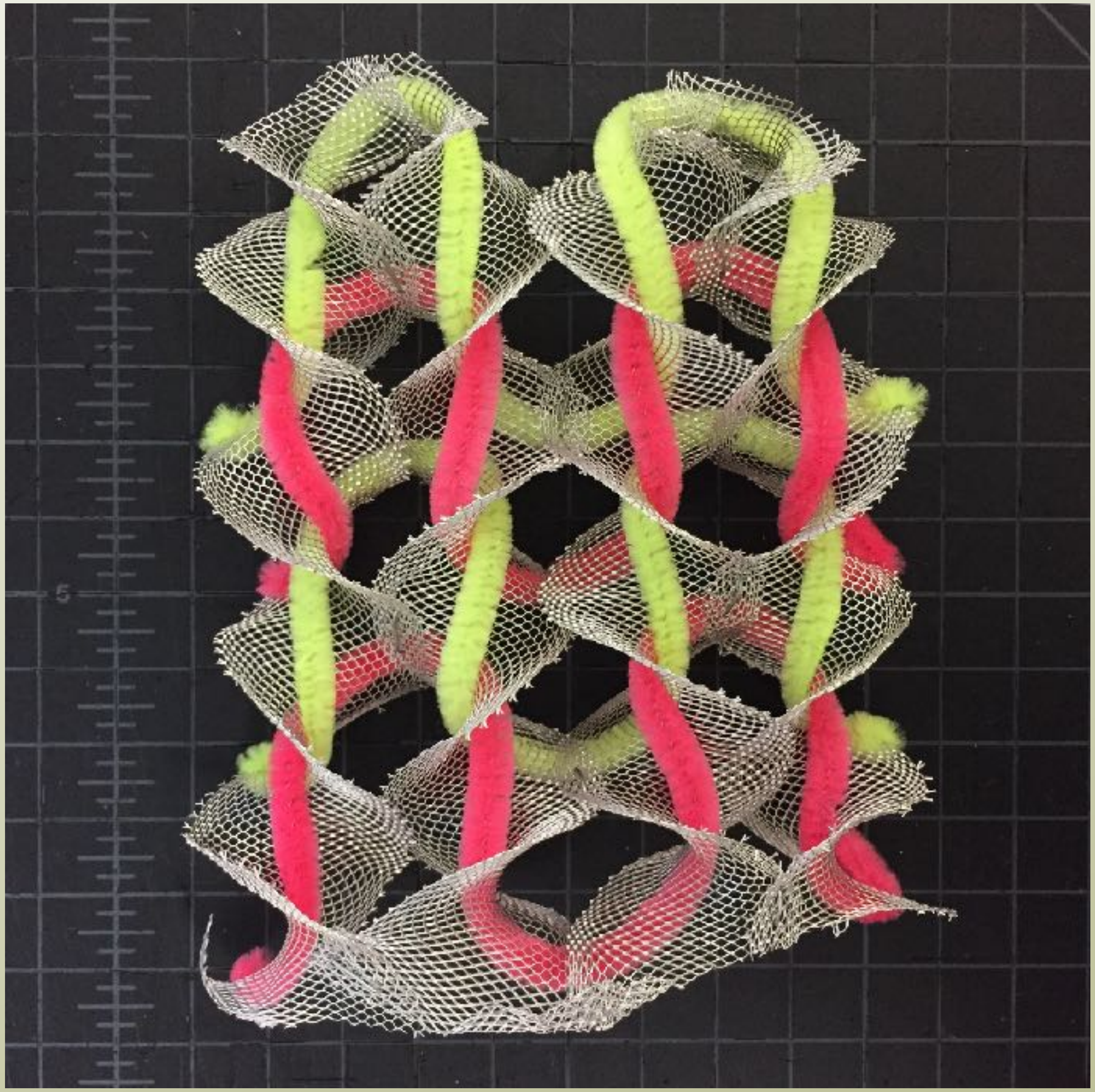
Knittel, Tanis, Stoltzfus, Castle, RDK, and Dion (2018)

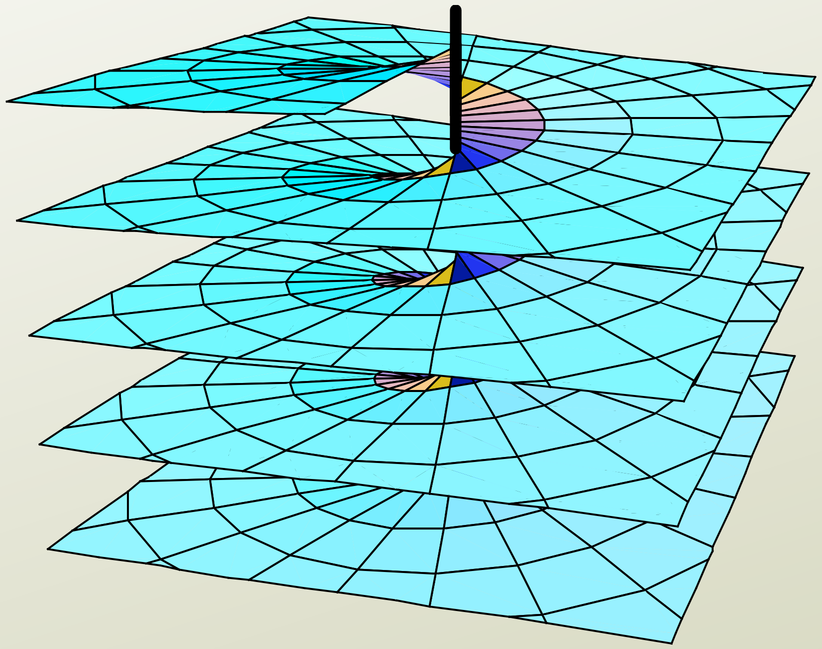
The Knit Fits





Knittel, Tanis, Stoltzfus, Castle, RDK, and Dion (2018)

A Twist Grain Boundary



$$\Phi = z - \frac{b}{2\pi}\tan^{-1}\left(\frac{y}{x}\right)$$

$$\Phi = z - \frac{b}{2\pi}\tan^{-1}\left(\frac{y}{x-\ell_d}\right)$$

$$\Phi = z - \frac{b}{2\pi}\tan^{-1}\left(\frac{y}{x-2\ell_d}\right)$$

$\vdots$

RDK and T. Lubensky, *PRL* **82** (1999) 2892
RDK, *Appl. Math. Lett.* **14** (2001) 797

Penn

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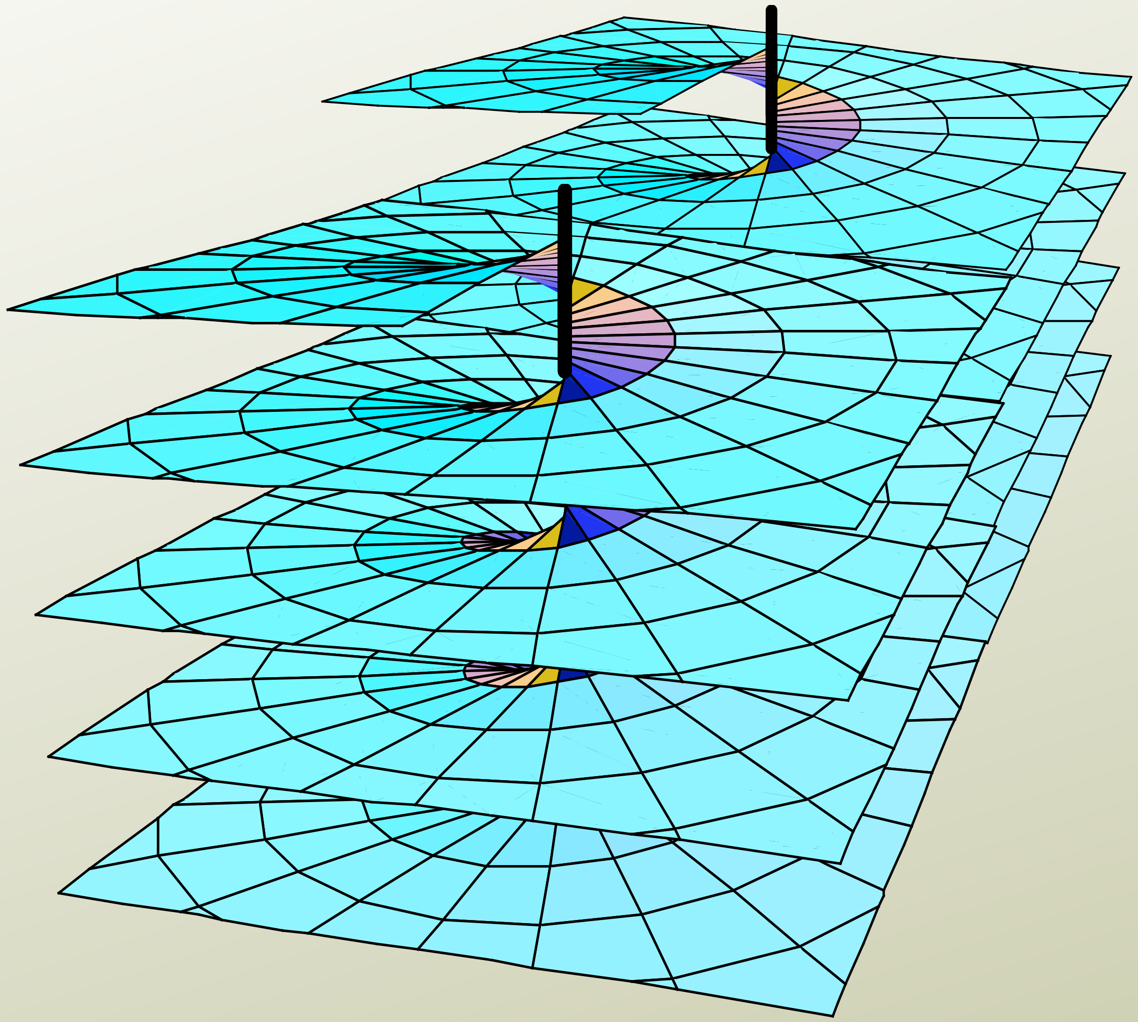

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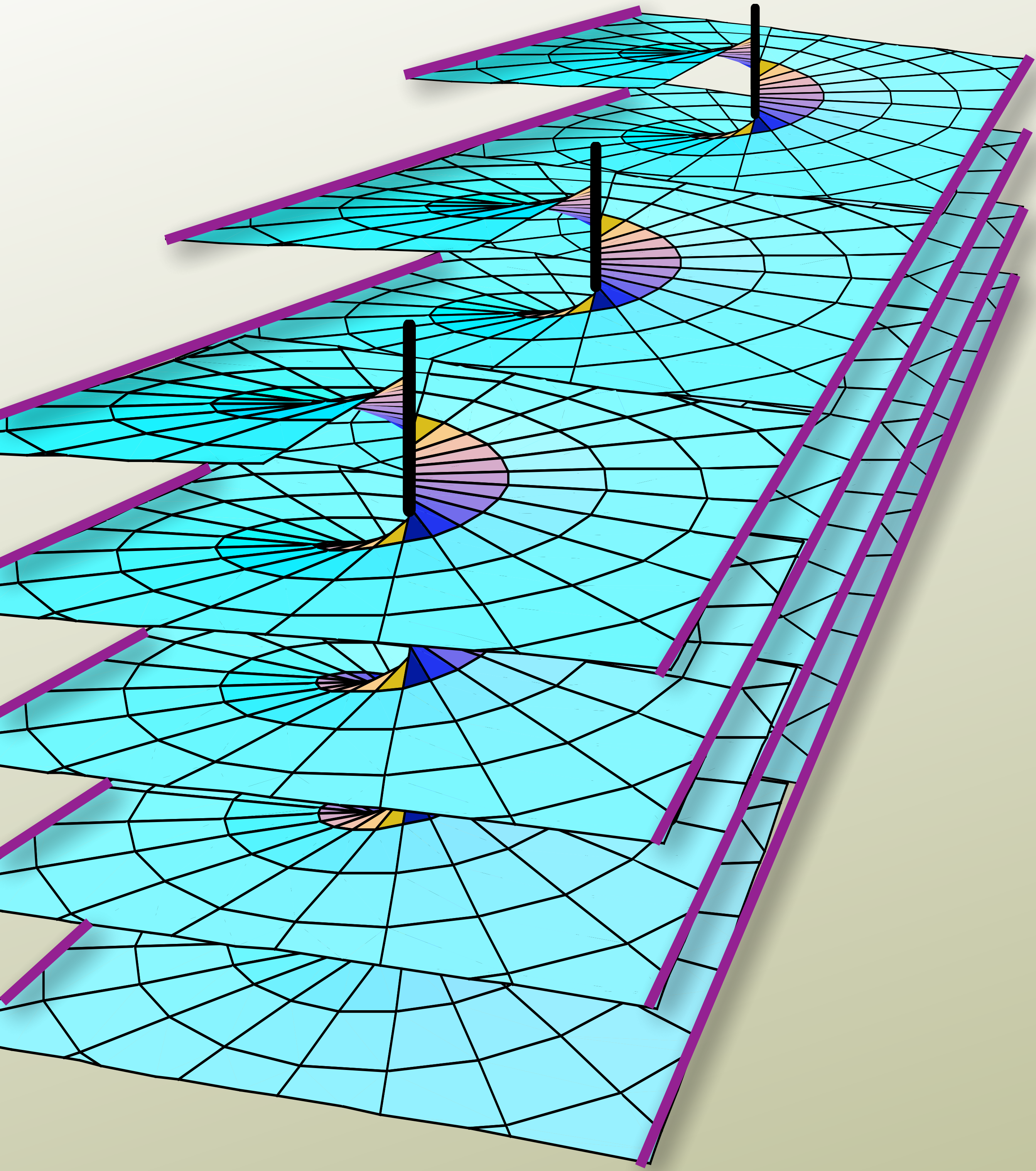

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A Twist Grain Boundary

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Penn

RDK, *Appl. Math. Lett.* **14** (2001) 797
Santangelo and RDK, *PRL* **96** (2006) 137801

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$$\sin w = w\prod_{n=1}^{\infty}\left[1 - \frac{w^2}{n^2\pi^2}\right]$$

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boundary conditions: no strain at infinity

$$\lim_{y\pm\infty}\left(\nabla\Phi\right)^2 = 1 \quad\Rightarrow\quad \gamma^2 = 1-\left(\frac{b}{2\ell_d}\right)^2$$

RDK, *Appl. Math. Lett.* **14** (2001) 797
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level sets of Φ

$$\Phi = ma \quad \Rightarrow \quad z(x, \pm\infty) = \mp\frac{b}{2\ell_d\sqrt{1-(b/2\ell_d)^2}}\,x - \frac{ma}{\sqrt{1-(b/2\ell_d)^2}} + \text{constant}$$

RDK, *Appl. Math. Lett.* **14** (2001) 797
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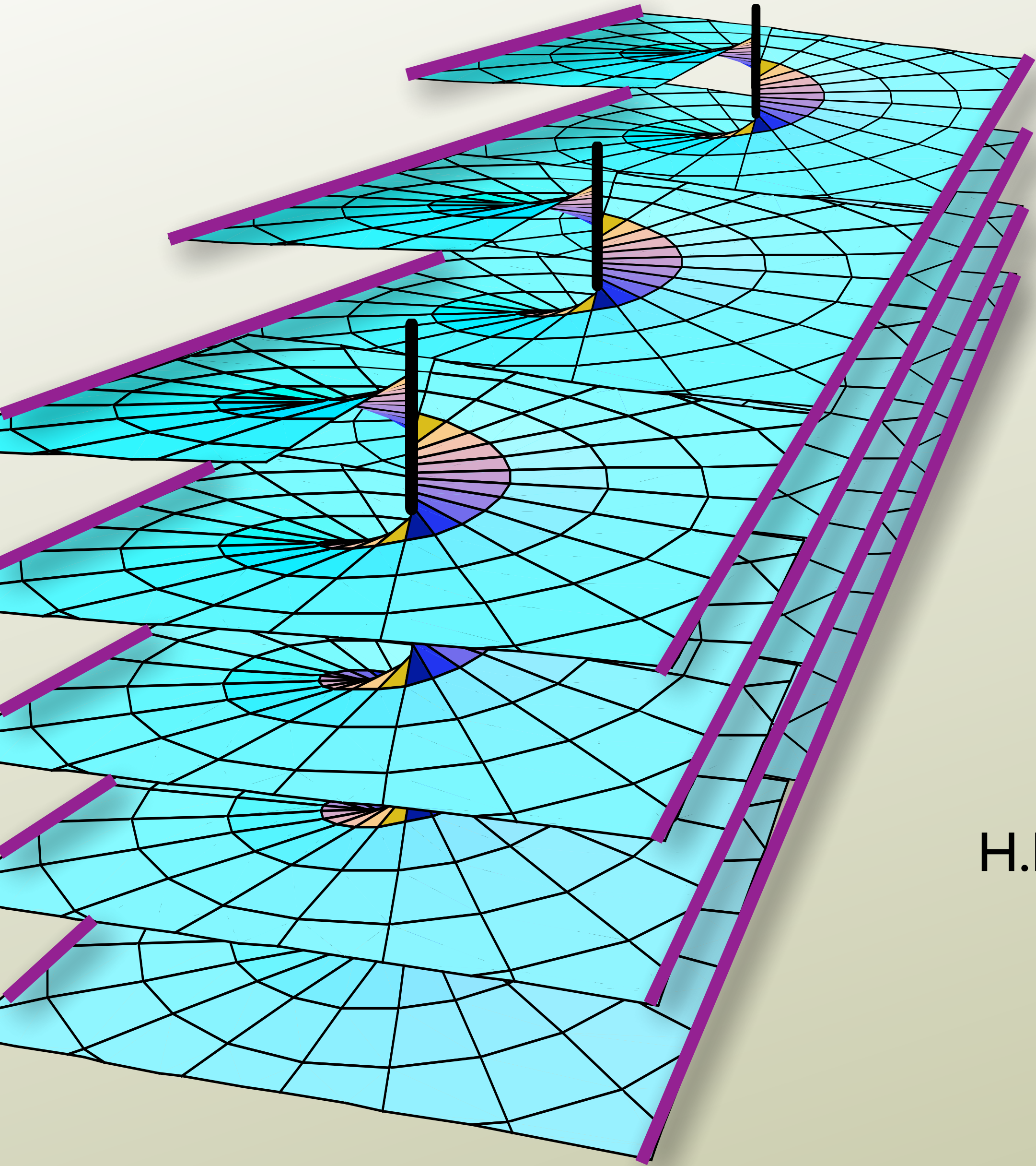
$$z(x,\pm\infty) = \mp x\tan\alpha - ma\sec\alpha + \text{constant}$$

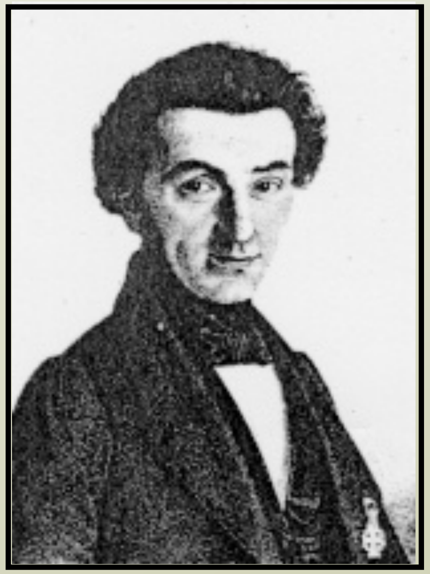
parallel planes a apart rotatated by 2α

$$\sin\alpha = \frac{b}{2\ell_d}$$

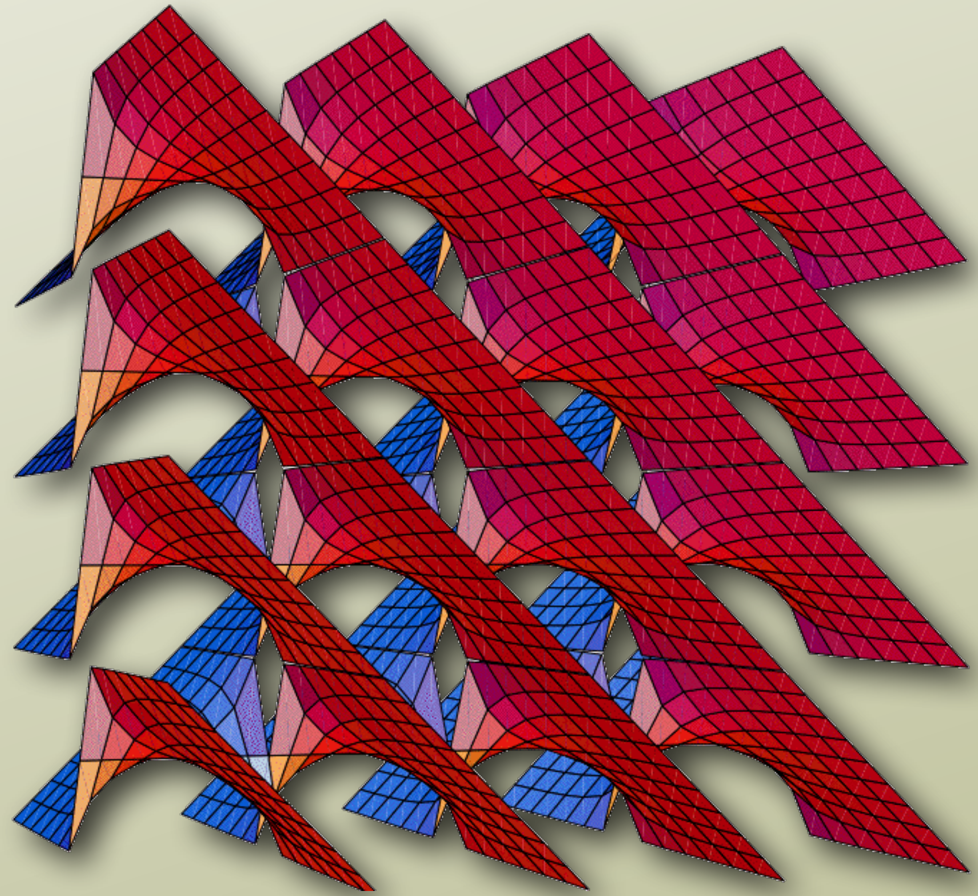
RDK, *Appl. Math. Lett.* **14** (2001) 797
Santangelo and RDK, *PRL* **96** (2006) 137801

Scherk's First Surface - A Twist Grain Boundary





H.F. Scherk (1798-1885)



$$z(x,y) = \frac{b}{2\pi}\sec\alpha\tan^{-1}\left\{\frac{\tanh\left[(2\pi y/b)\sin\alpha\right]}{\tan\left[(2\pi x/b)\sin\alpha\right]}\right\}$$

$$z(x,y) = \sec\alpha\tan^{-1}\left\{\frac{\tanh\left[y\sin\alpha\cos\alpha\right]}{\tan\left[x\sin\alpha\right]}\right\}$$

RDK and T. Lubensky, *PRL* **82** (1999) 2892

RDK, *Appl. Math. Lett.* **14** (2001) 797

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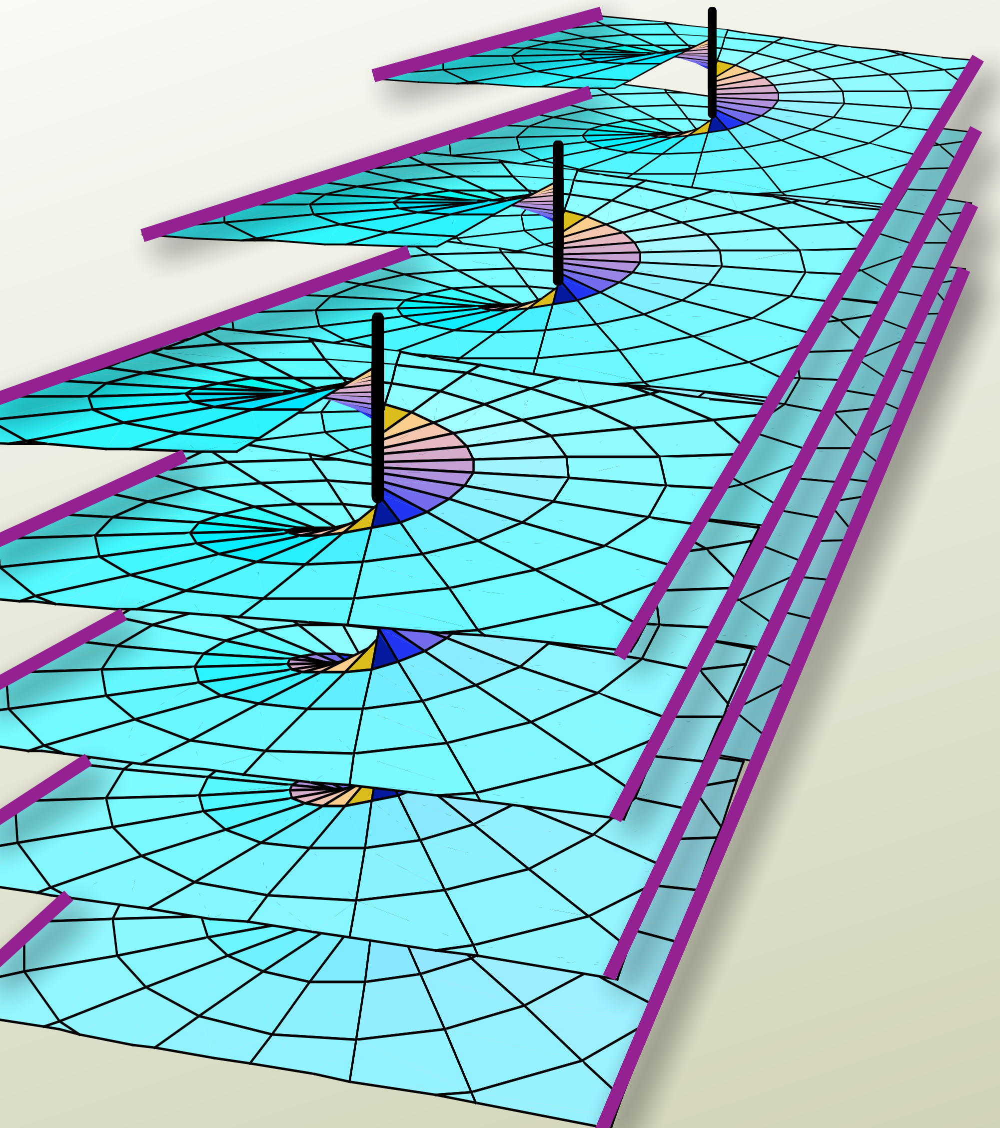
Twist Grain Boundaries - A Useful Duality



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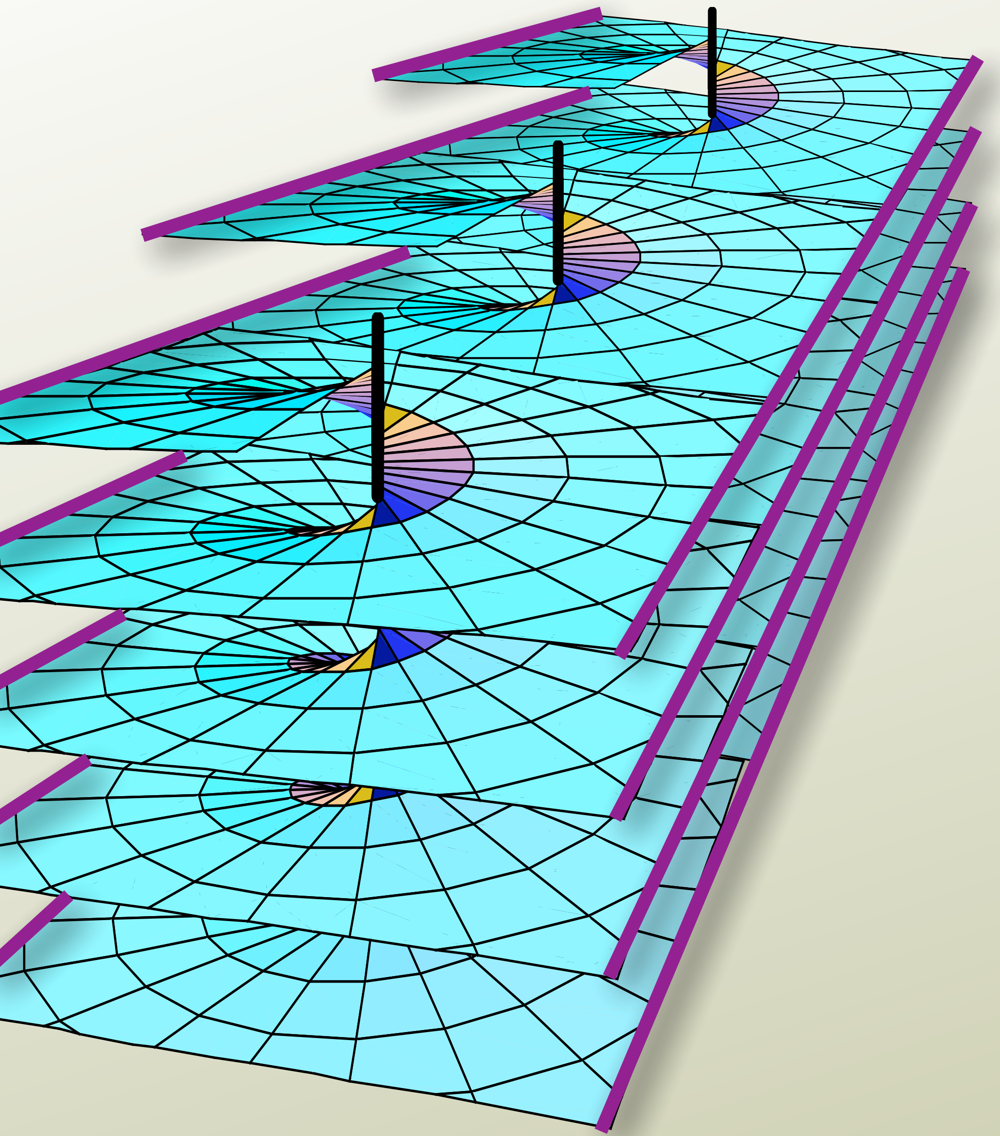

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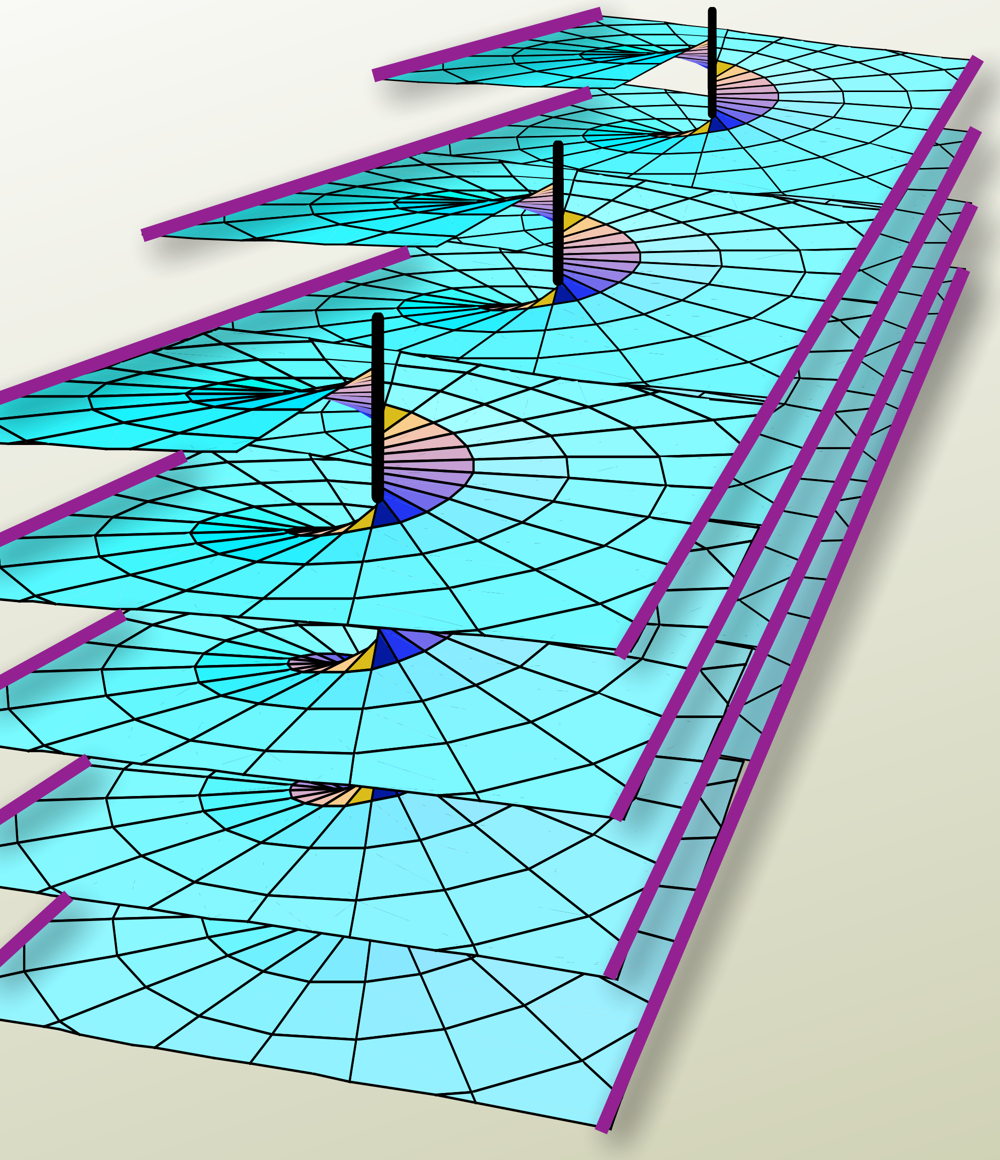
$$\Rightarrow \tan\left[(2\pi z/b)\cos\alpha\right]\tan\left[(2\pi x/b)\sin\alpha\right] = \tanh\left[(2\pi y/b)\sin\alpha\right]$$

RDK and T. Lubensky, *PRL* **82** (1999) 2892

RDK, *Appl. Math. Lett.* **14** (2001) 797

Penn

Twist Grain Boundaries - A Useful Duality



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$$\alpha \to \frac{\pi}{2} - \alpha$$
$$(x,z) \to (z,-x)$$
$$b \to -b$$
$$y \to y\tan\alpha$$

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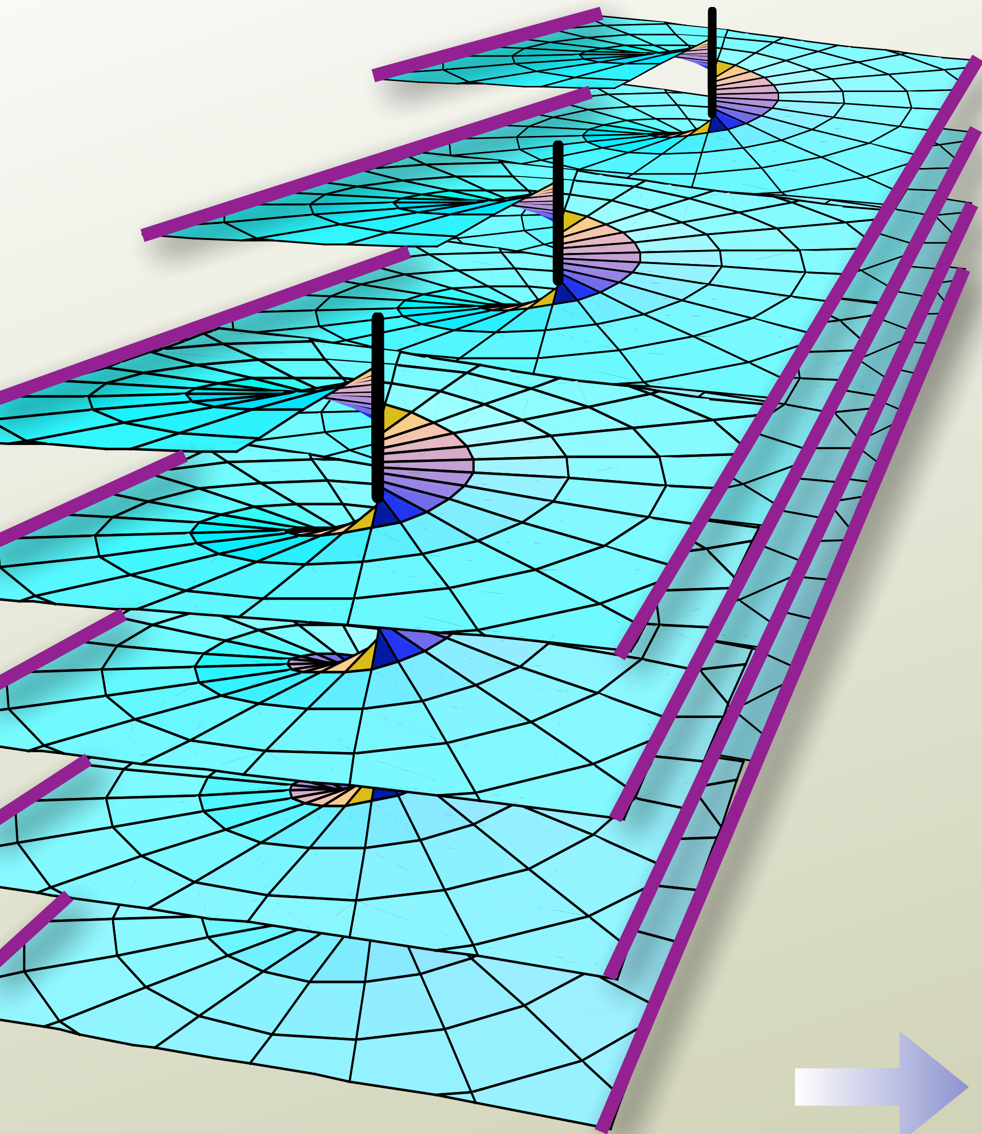
Twist Grain Boundaries - A Useful Duality



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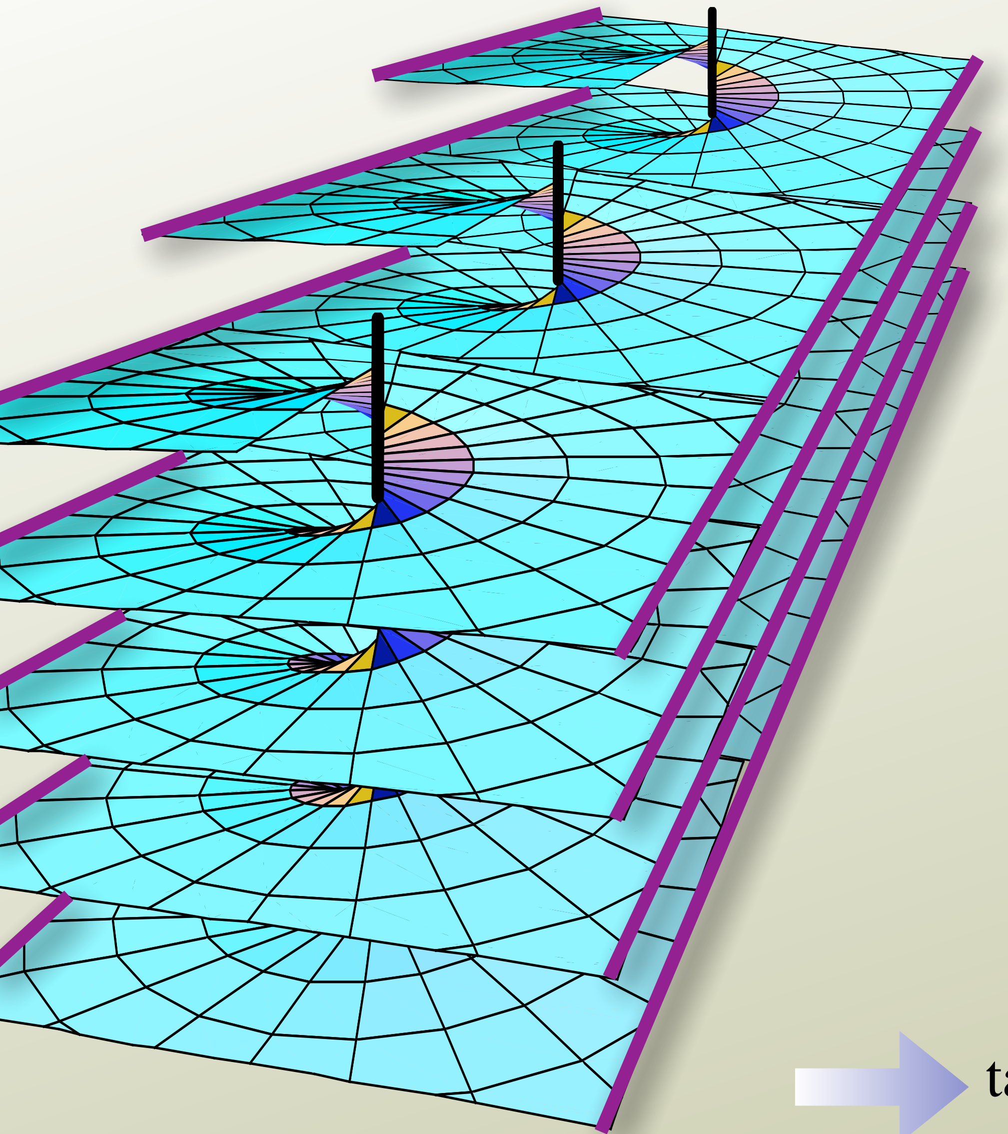
RDK and T. Lubensky, *PRL* **82** (1999) 2892

RDK, *Appl. Math. Lett.* **14** (2001) 797

Twist Grain Boundaries - A Useful Duality



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dual description of a single grain boundary

RDK and T. Lubensky, *PRL* **82** (1999) 2892

RDK, *Appl. Math. Lett.* **14** (2001) 797

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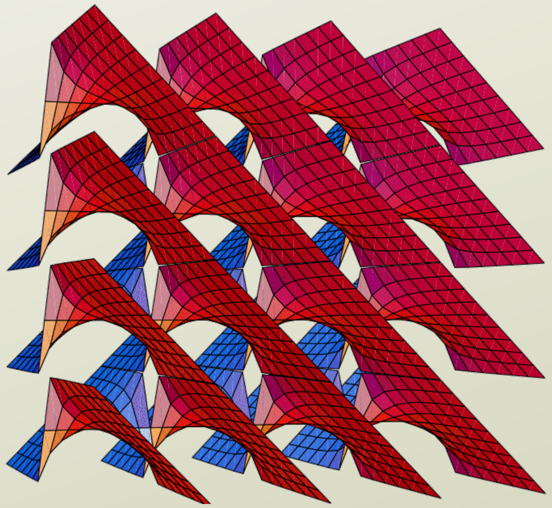
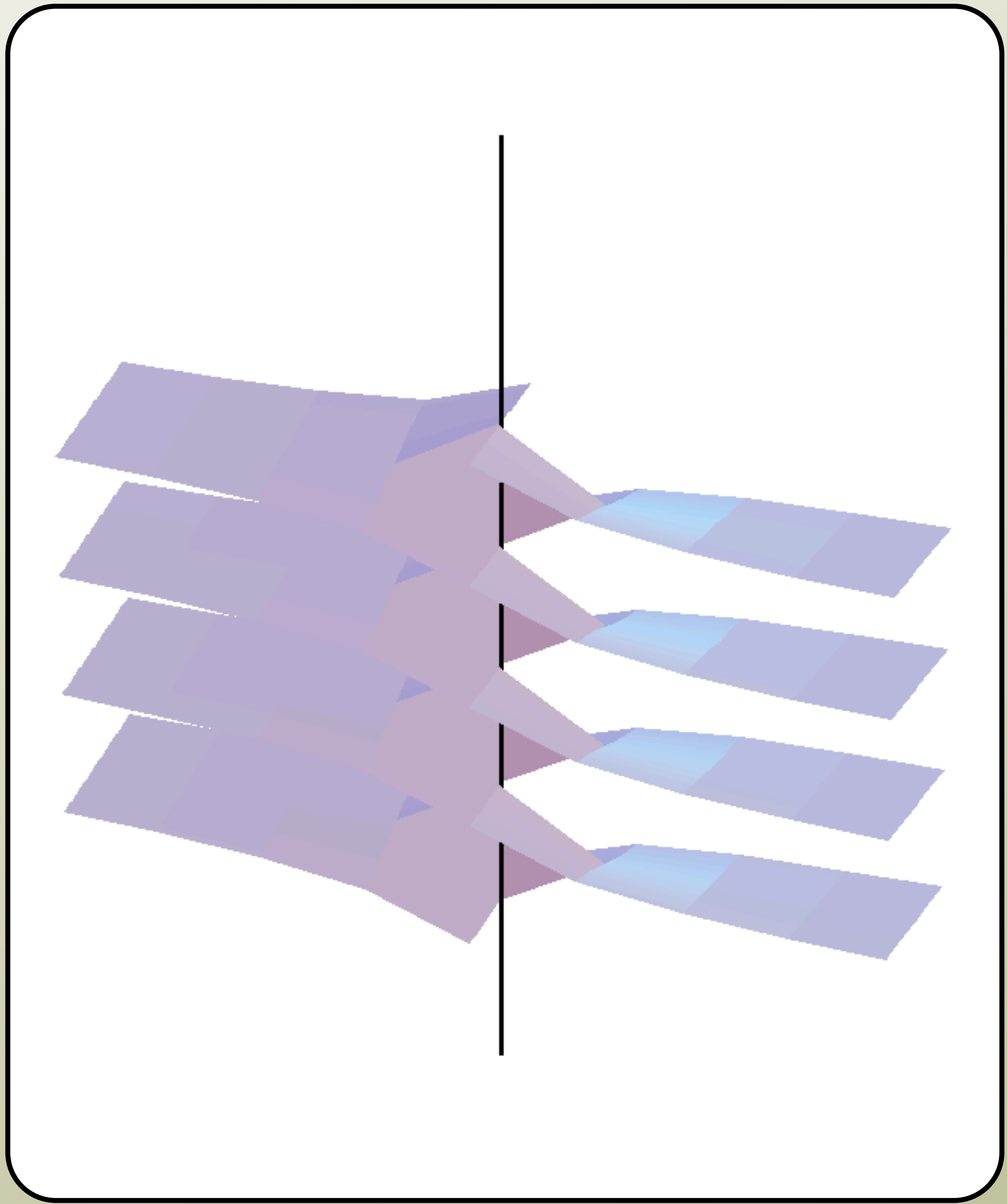
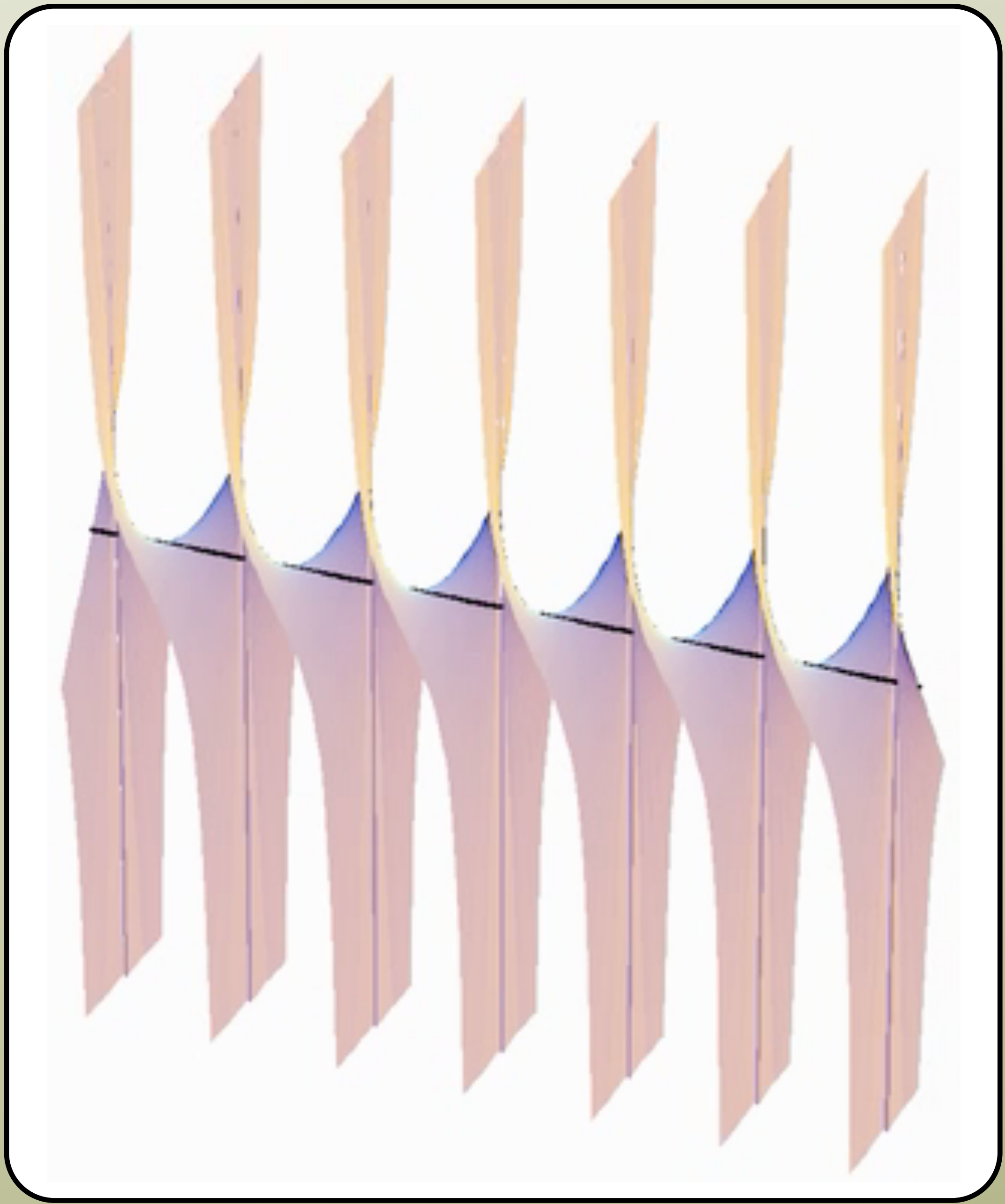
Scherk's First Surface - Duality







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RDK and T. Lubensky, *PRL* **82** (1999) 2892
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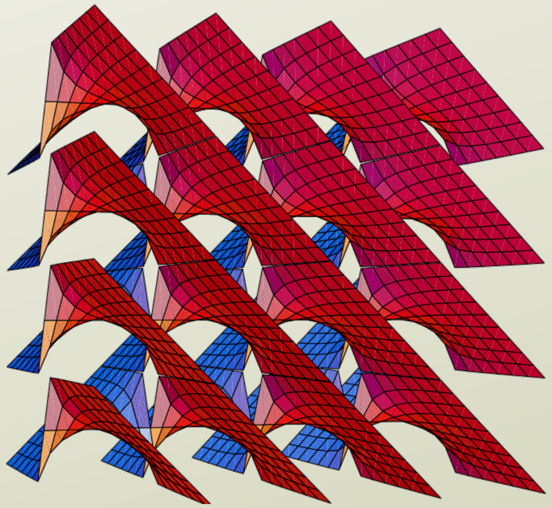
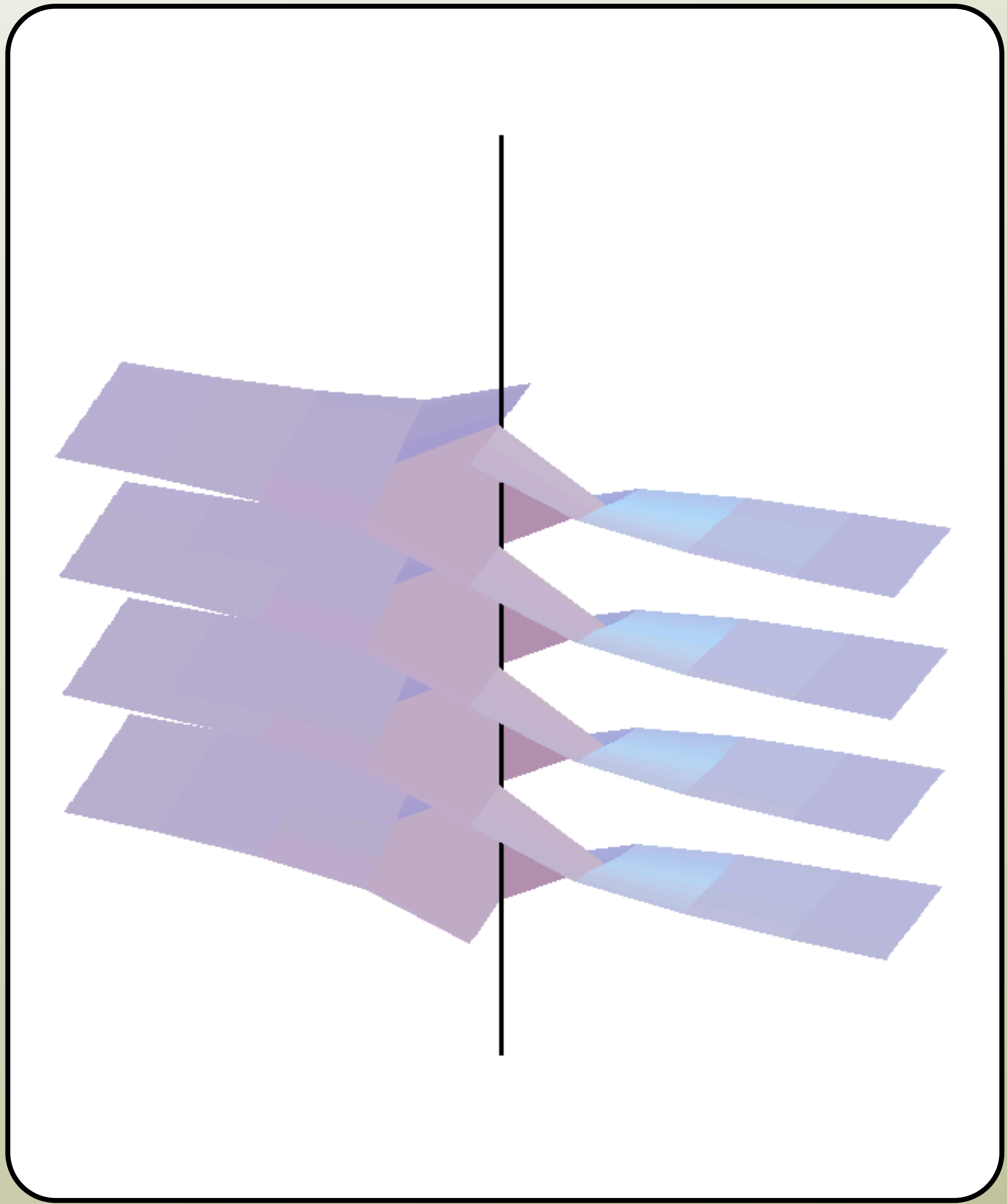
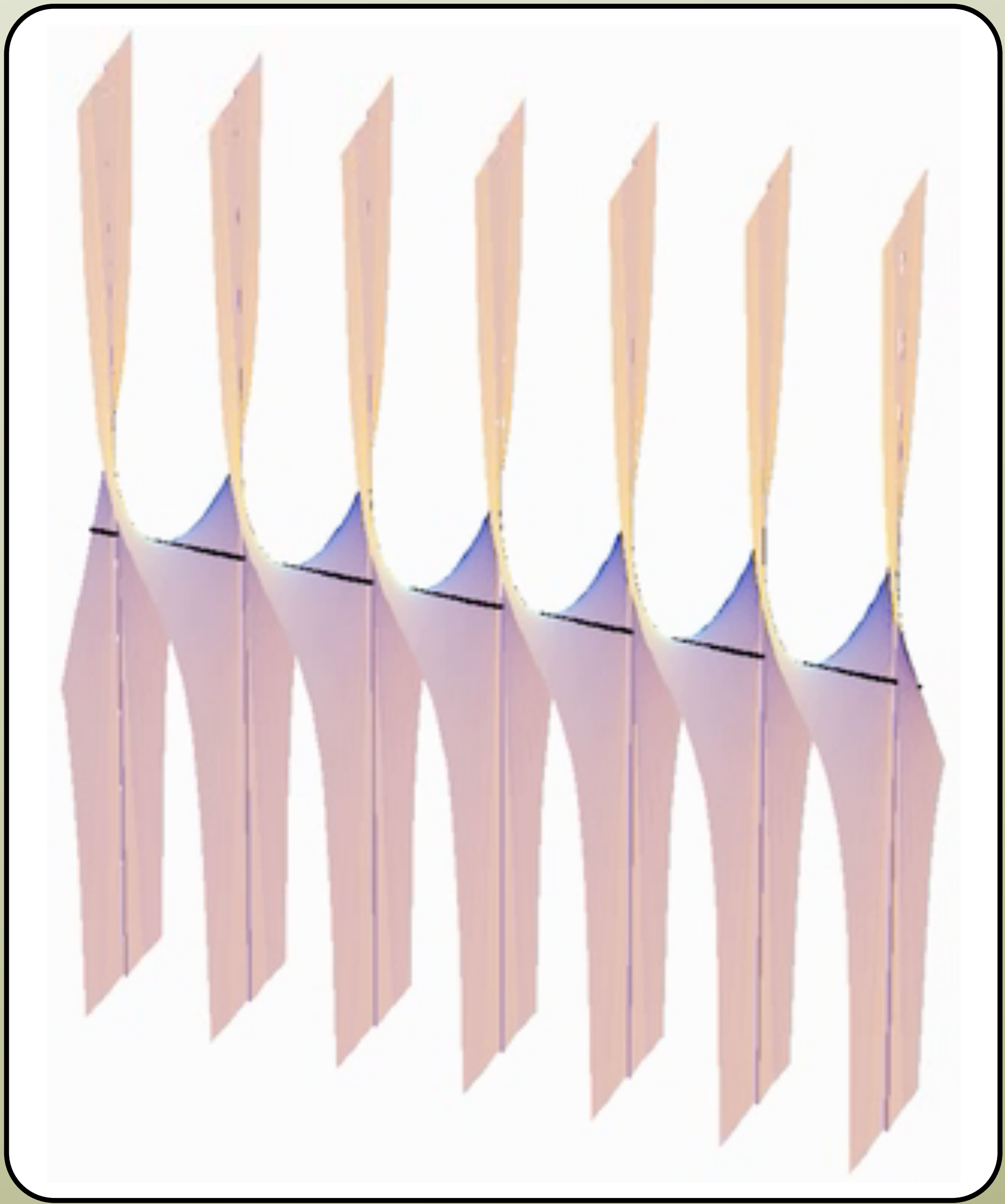
Scherk's First Surface - Duality







RDK and T. Lubensky, *PRL* **82** (1999) 2892
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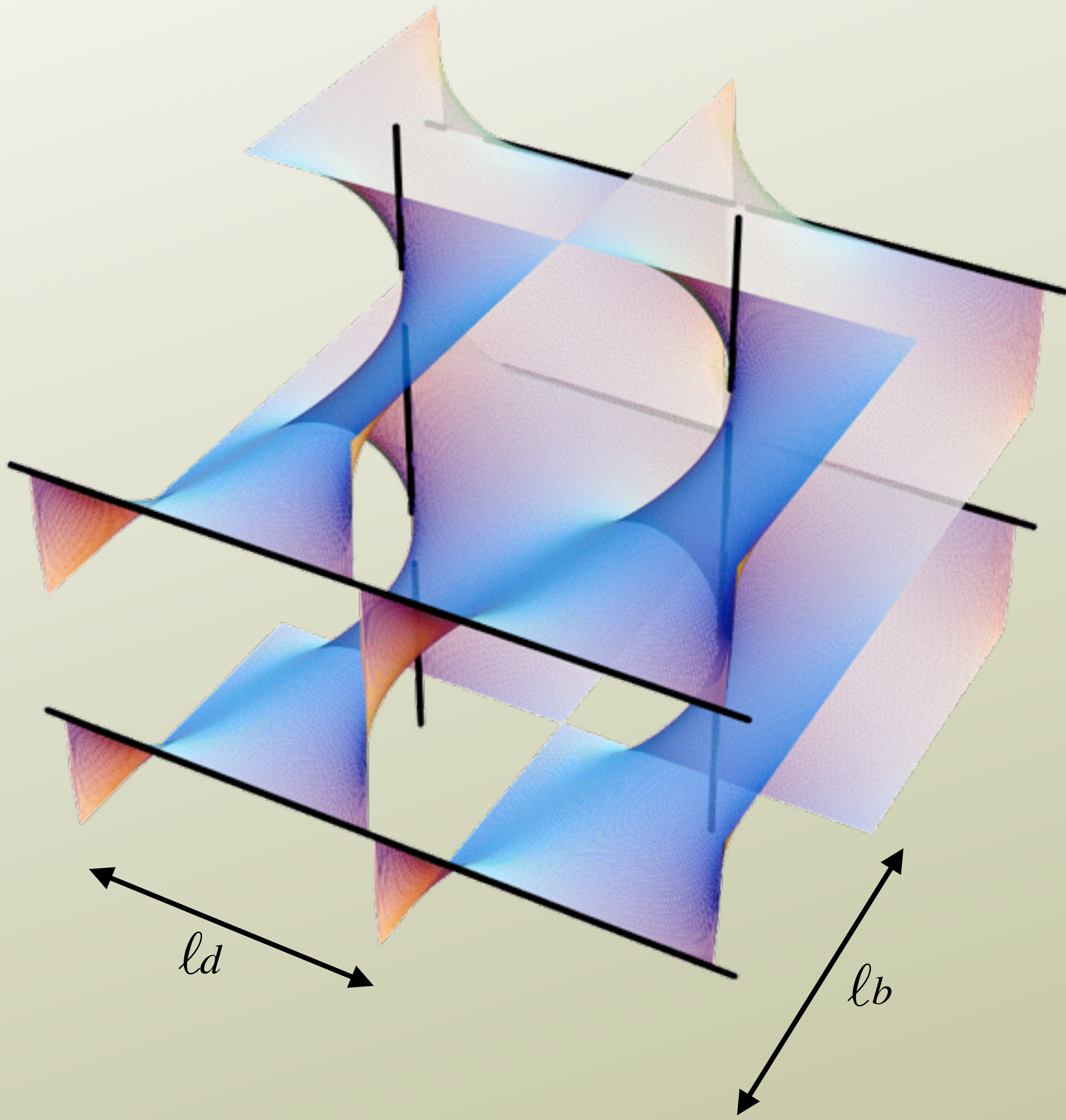
A Large Angle Twist Grain Boundary Phase



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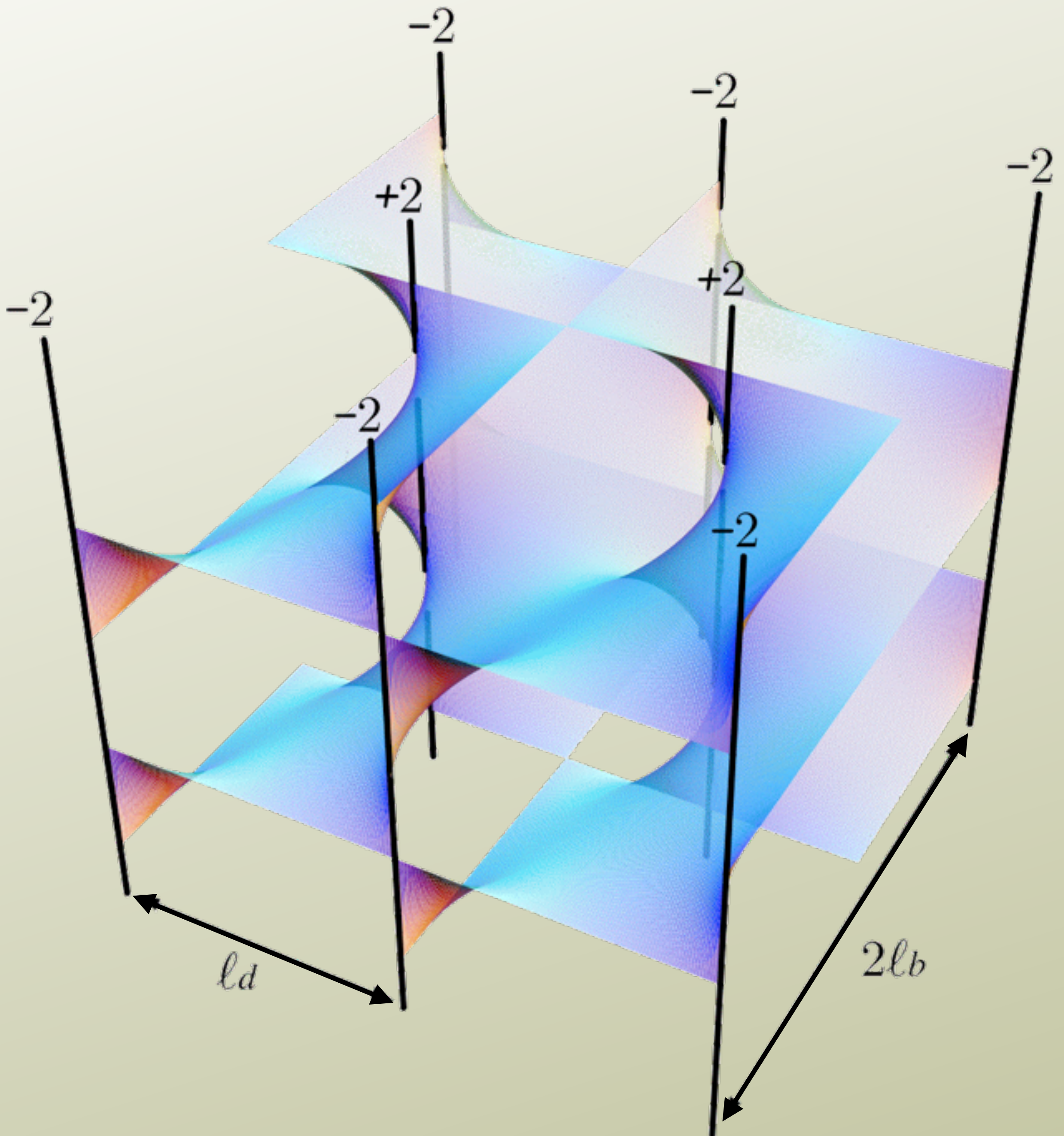
Santangelo and RDK, *PRL* **96** (2006) 137801

A Large Angle Twist Grain Boundary Phase



use duality to get parallel screw dislocations with alternating Burgers vectors

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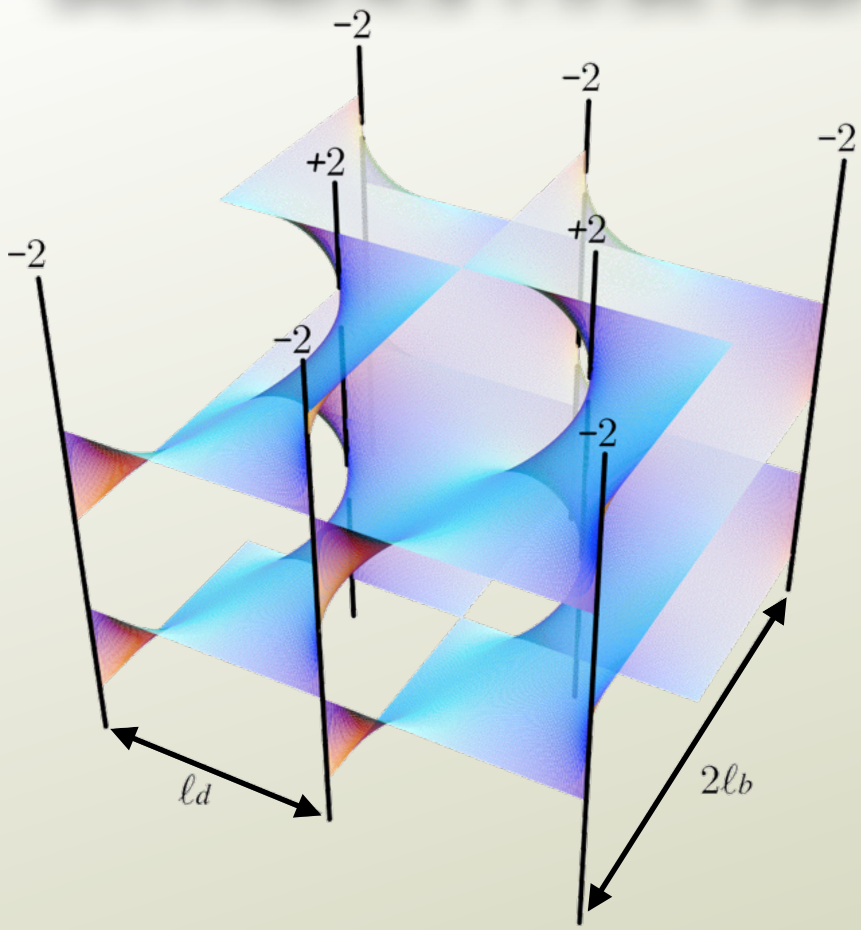
Santangelo and RDK, *PRL* **96** (2006) 137801

Schnerk's First Surface - A Twist Grain Boundary Phase



start with

$$\Phi = \gamma z - \frac{b}{2\pi}\mathrm{Im}\ln\sin\left[\frac{\pi(x+iy)}{\ell_d}\right]$$

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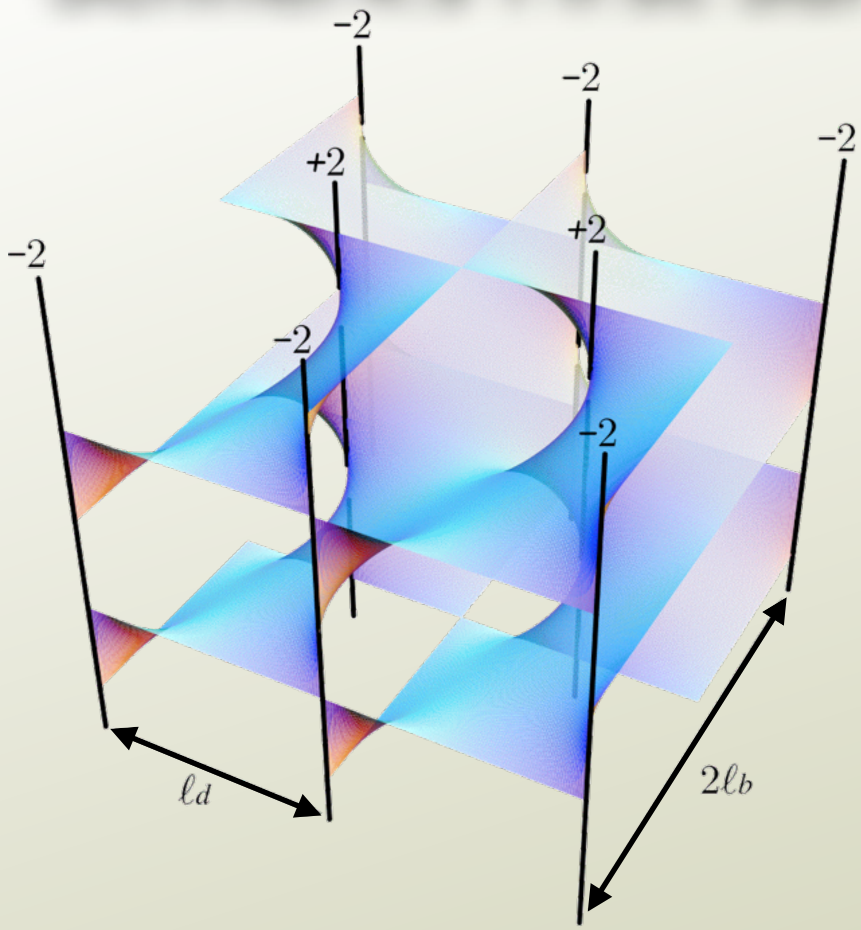
Santangelo and RDK, *PRL* **96** (2006) 137801

Schnerk's First Surface - A Twist Grain Boundary Phase



start with

$$\Phi = \gamma z - \frac{b}{2\pi}\mathrm{Im}\ln\sin\left[\frac{\pi\left(x+iy\right)}{\ell_d}\right]$$

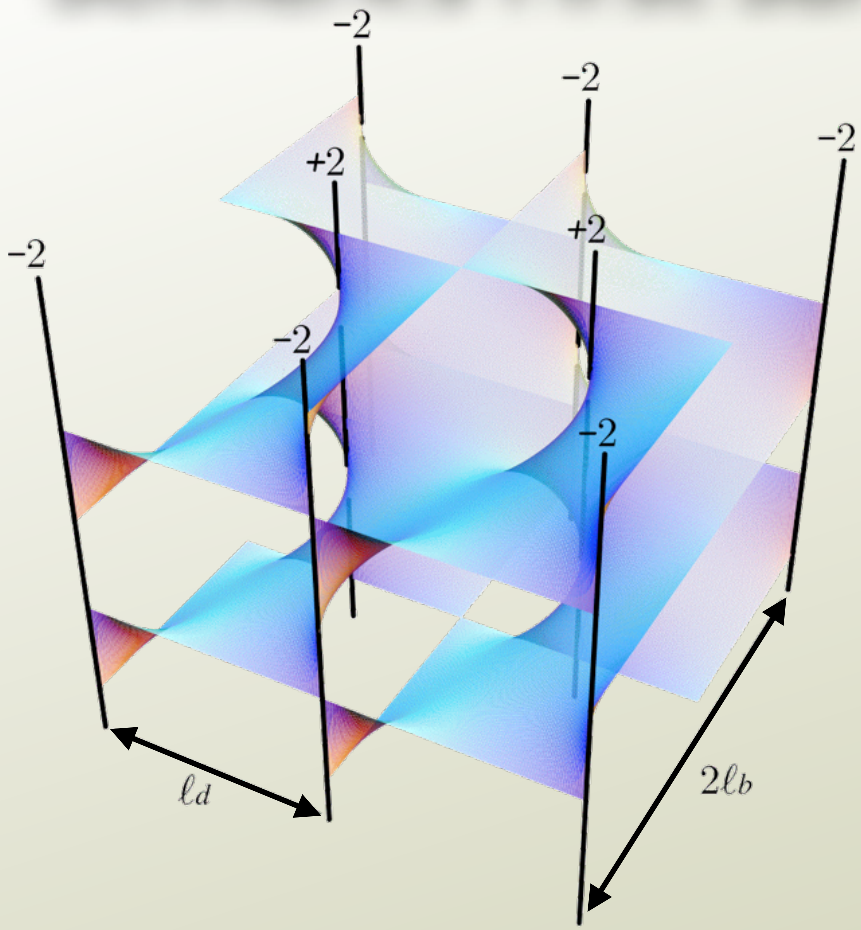
add

alternating charges

$$\Phi = \gamma z - \frac{b}{2\pi}\mathrm{Im}\sum_{m=-\infty}^{\infty}(-1)^m\ln\sin\left(\frac{\pi\left[\left(x+\frac{1}{2}m\right)+i\left(y+m\ell_b\right)\right]}{\ell_d}\right)$$

Santangelo and RDK, *PRL* **96** (2006) 137801

Schnerk's First Surface - A Twist Grain Boundary Phase



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or, in other symbols

$$\Phi = \gamma z - \frac{b}{2\pi}\mathrm{Im}\sum_{m=-\infty}^{\infty}(-1)^m\ln\sin\left(\frac{\pi(x+iy)}{\ell_d}+m\frac{\pi\tau}{2}\right)$$

$$\tau = 1 + \frac{2i\ell_b}{\ell_d}$$

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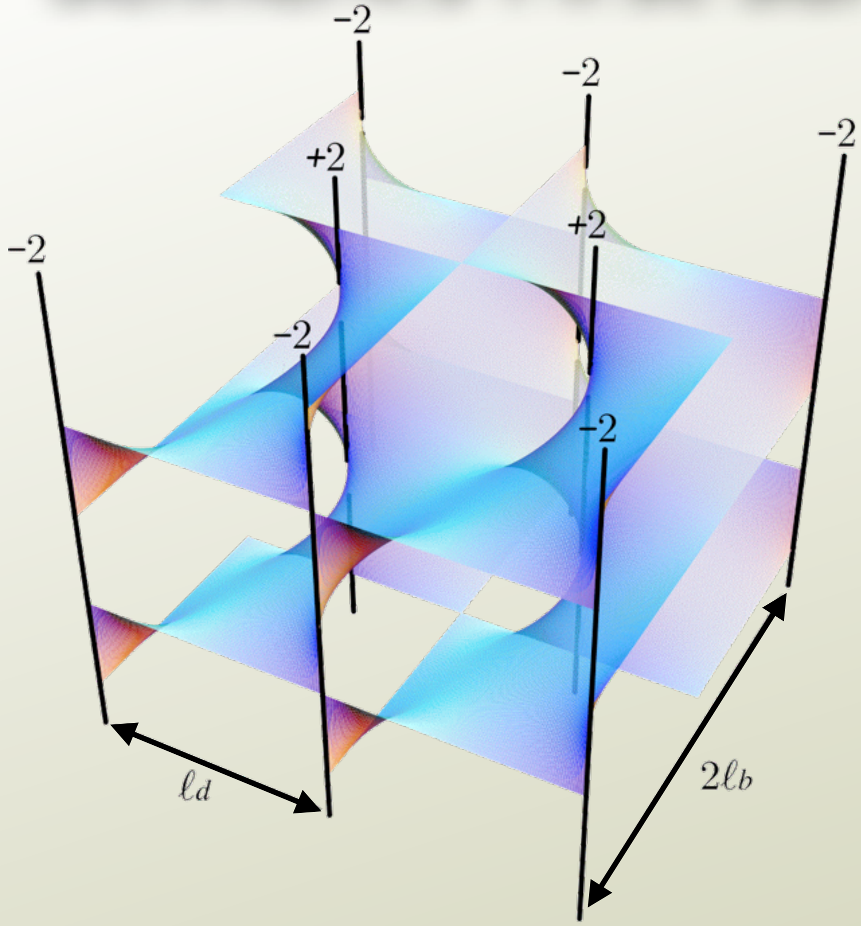
Santangelo and RDK, *PRL* **96** (2006) 137801

Schnerk's First Surface - A Twist Grain Boundary Phase



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$$\tau = 1 + \frac{2i\ell_b}{\ell_d}$$

match poles and zeroes of a doubly periodic function

$$\Phi = \gamma z - \frac{b}{2\pi}\mathrm{Im}\ln\mathrm{sn}\left[\frac{2\mathbf{K}(k)x}{\ell_d}+i\frac{\mathrm{Re}\mathbf{K}'(k)y}{\ell_b},k\right]$$

Jacobi Elliptic Function (Generalizes sin)

$$\mathbf{K}(k) = \int_0^1 dx\,\frac{1}{\sqrt{(1-x^2)(1-k^2x^2)}} \qquad \mathbf{K}'(k) = \mathbf{K}\left(\sqrt{1-k^2}\right) \qquad \tau = i\frac{\mathbf{K}'(k)}{\mathbf{K}(k)}$$

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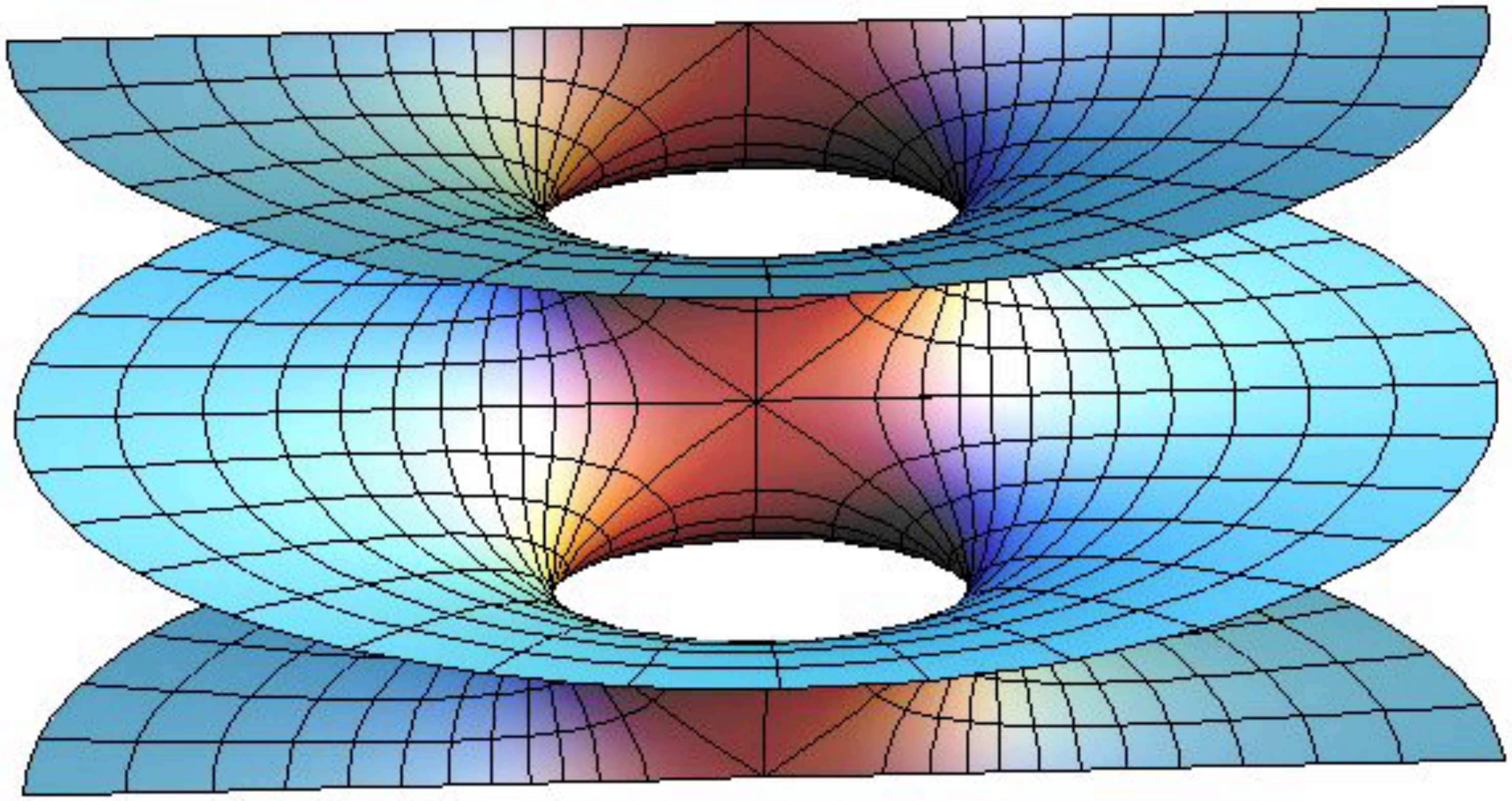
Santangelo and RDK, *PRL* **96** (2006) 137801

Riemann's Minimal Surface - A Set of Dislocations

$$\Phi = (x-\alpha(z))^2 + (y-\beta(z))^2 - R^2(z)$$



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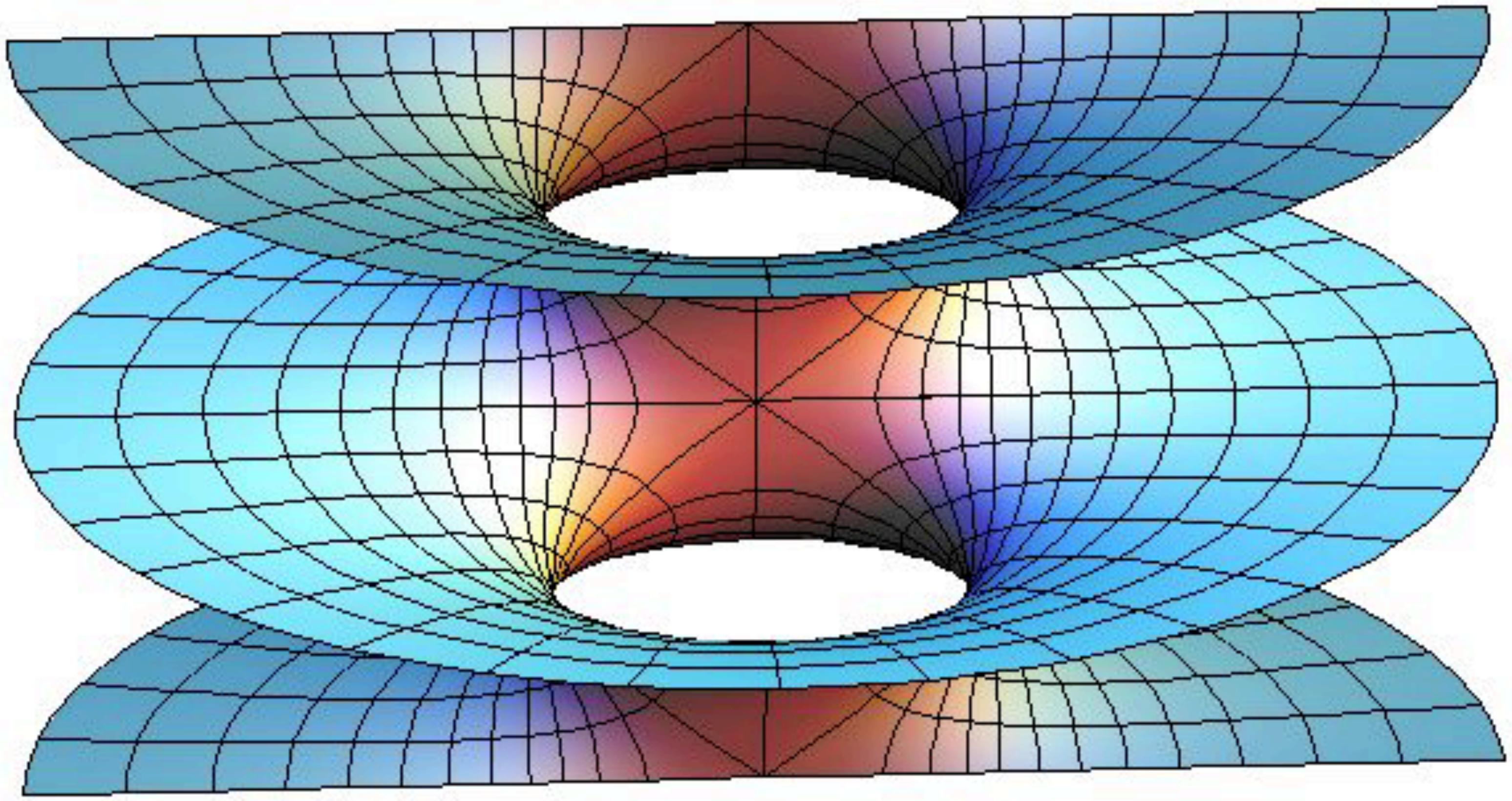
Riemann's Minimal Surface - A Set of Dislocations

$$\Phi = (x - \alpha(z))^2 + (y - \beta(z))^2 - R^2(z)$$



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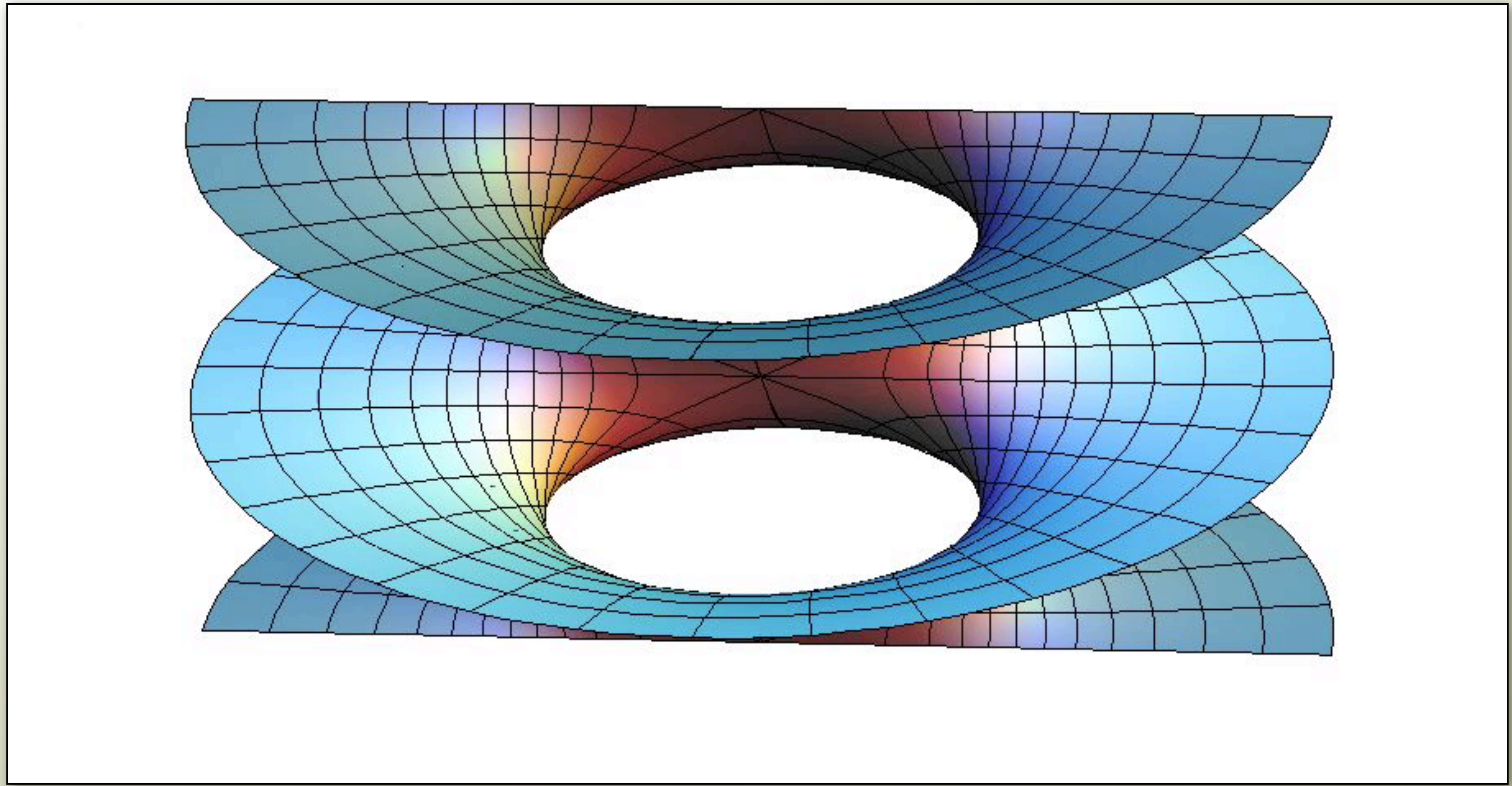

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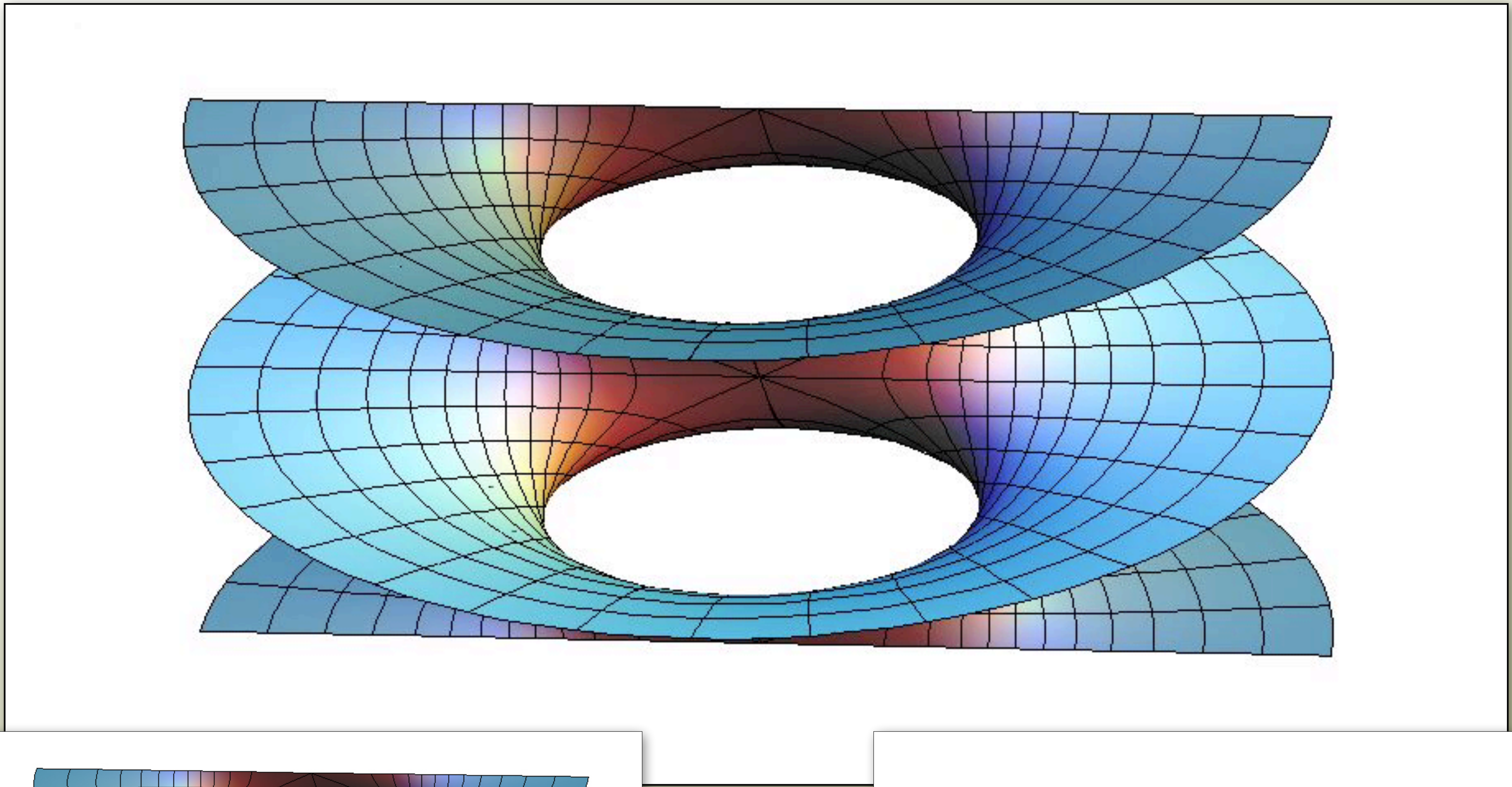
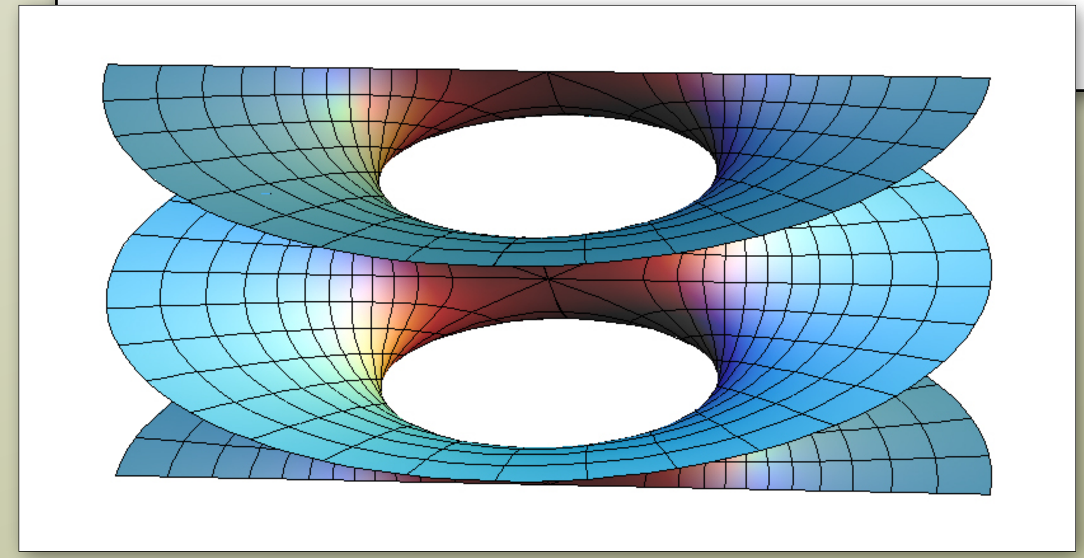
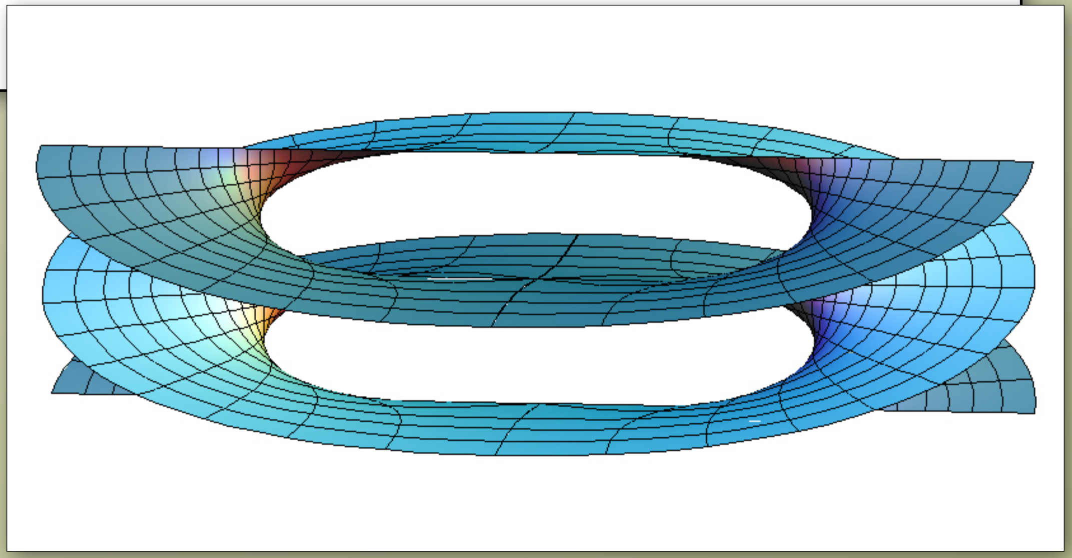
Riemann's Minimal Surface







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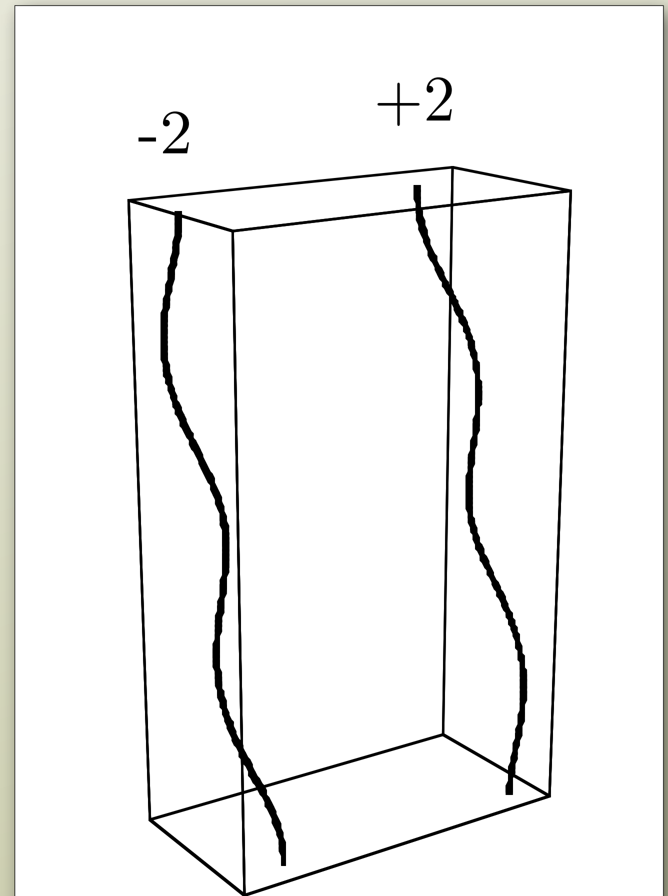
Decomposition: Two Screw Dislocations

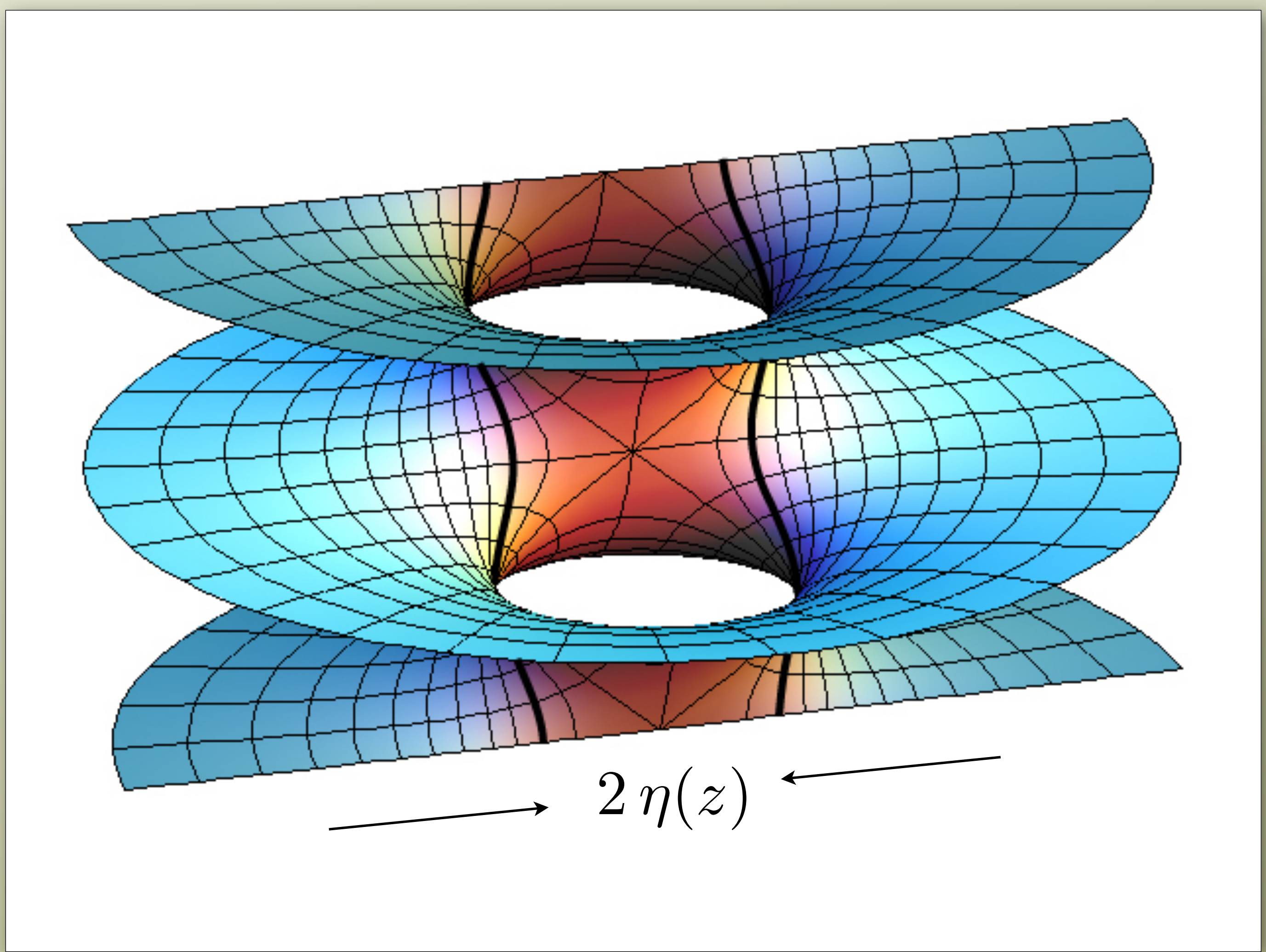
Phase field: $\tan^{-1}\left[\frac{y-\zeta(z)}{x+\eta(z)}\right] - \tan^{-1}\left[\frac{y-\zeta(z)}{x-\eta(z)}\right] = \tan^{-1}\left[\frac{\eta(z)}{\sigma}\frac{\mathrm{sc}(z)}{\mathrm{dn}(z)}\right]$

$$\zeta(z) = z\,\gamma - \sigma\, E\left(\mathrm{am}(z,k),k\right), \quad \eta(z) = \sqrt{\frac{\gamma}{b} + \sigma^2\, k^2\, \mathrm{cn}^2(z)}$$



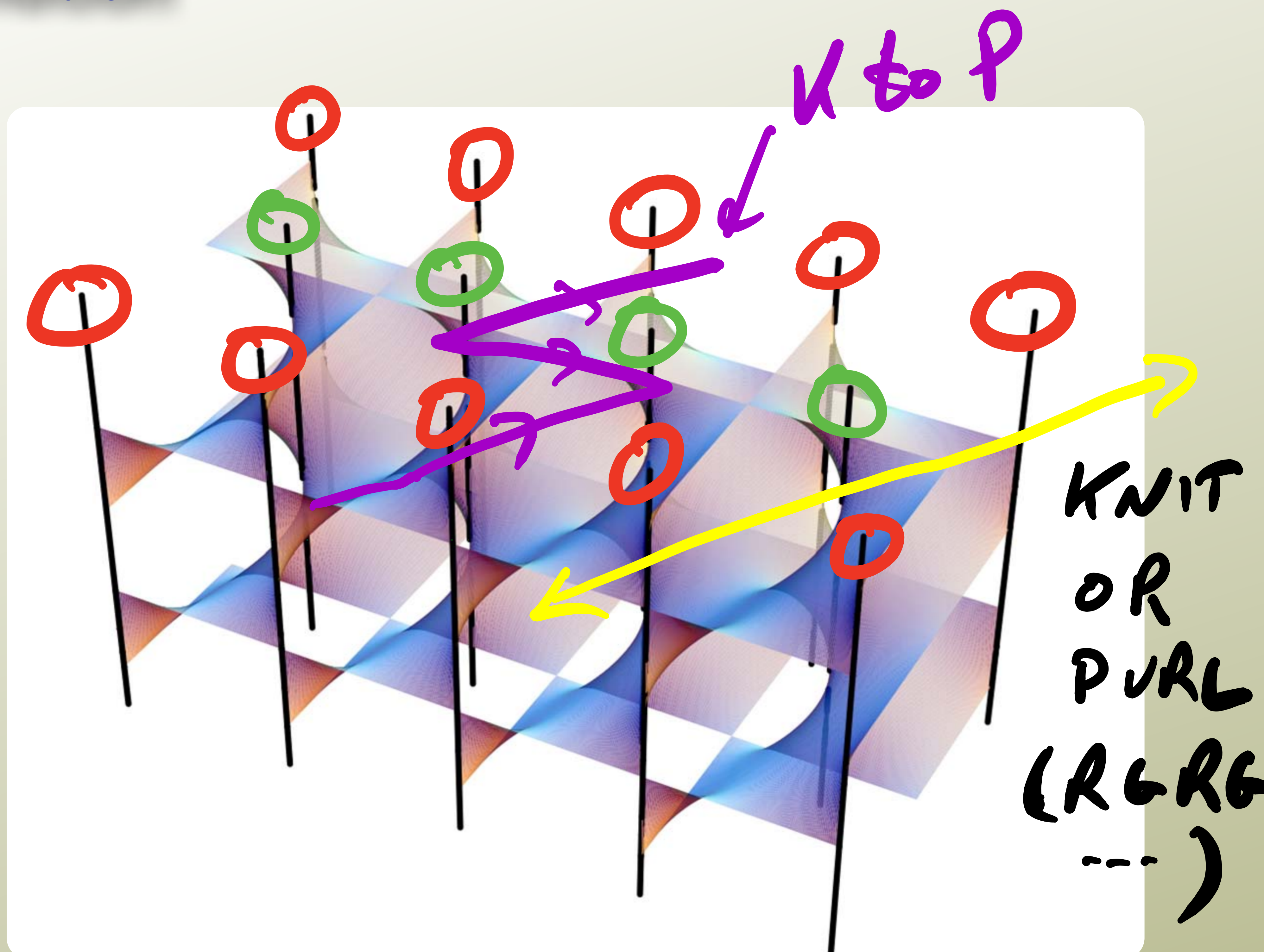
Dislocation cores

$\Rightarrow$



Matsumoto (2006)

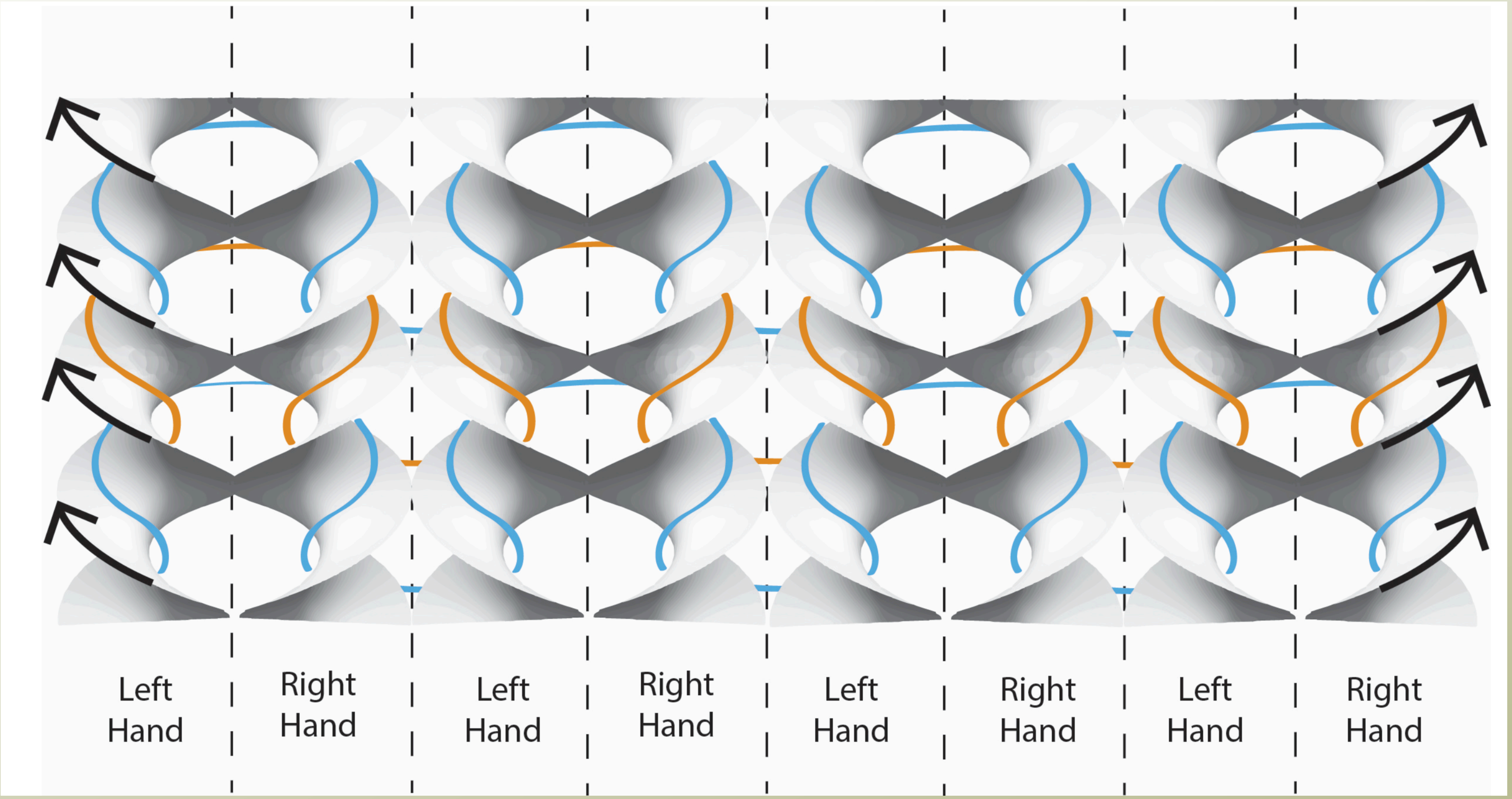
My Rendition



Red is left-handed

Green is right-handed

Geometry to Mechanics



Knittel, Aharoni, RDK, and Dion (2019)

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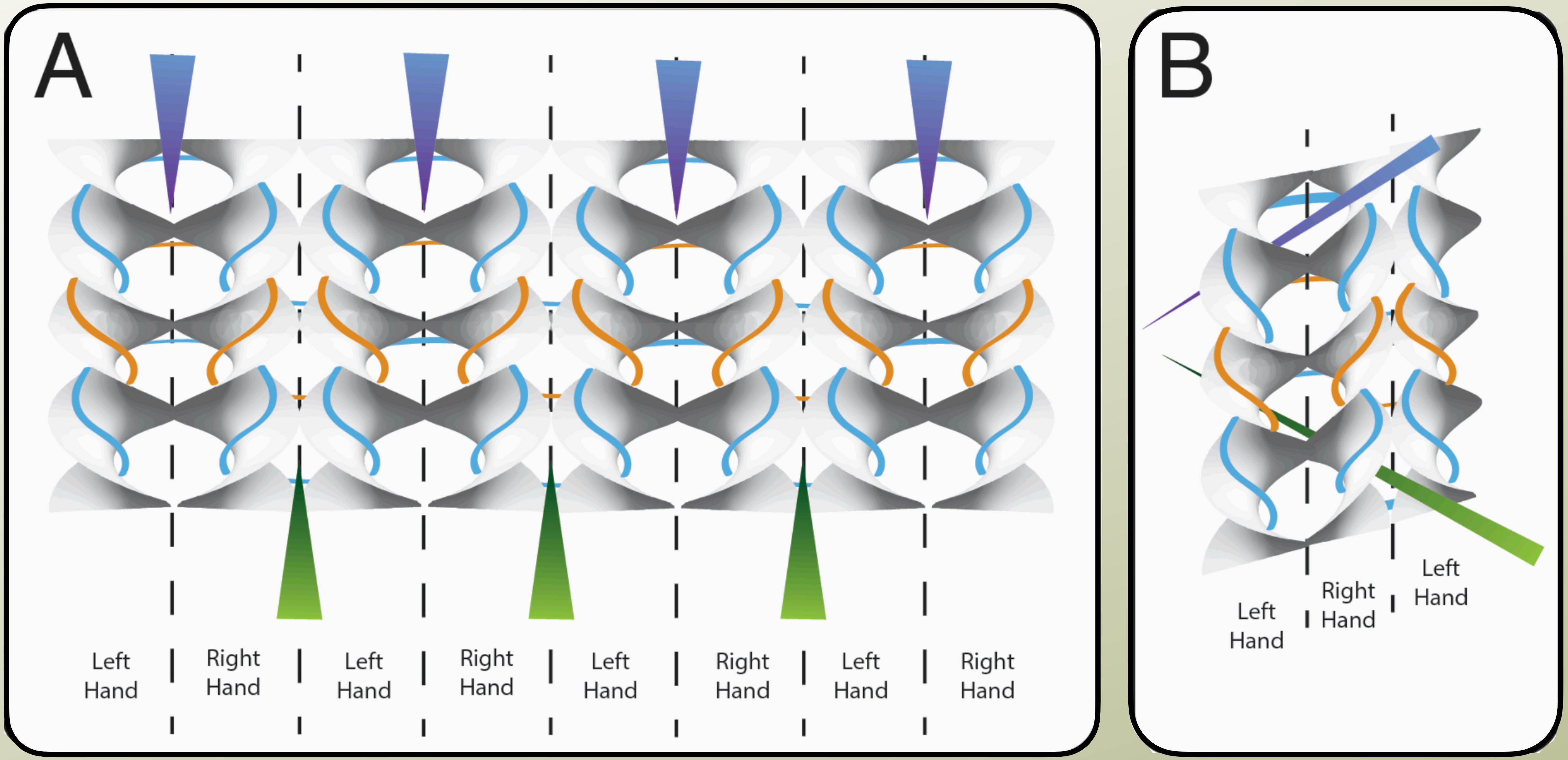
Geometry to Mechanics



Knittel, Aharoni, RDK, and Dion (2019)

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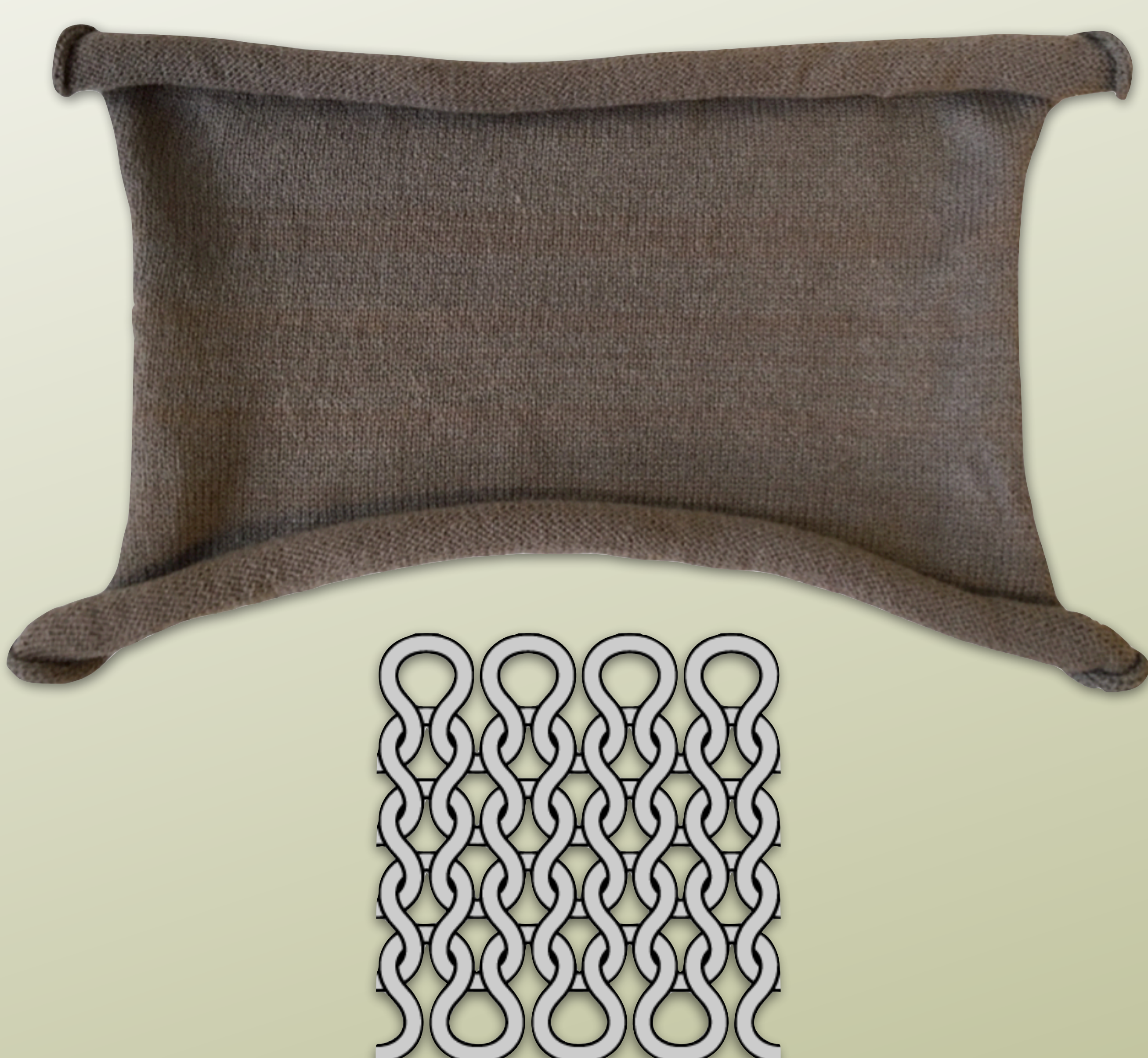
Geometry to Mechanics

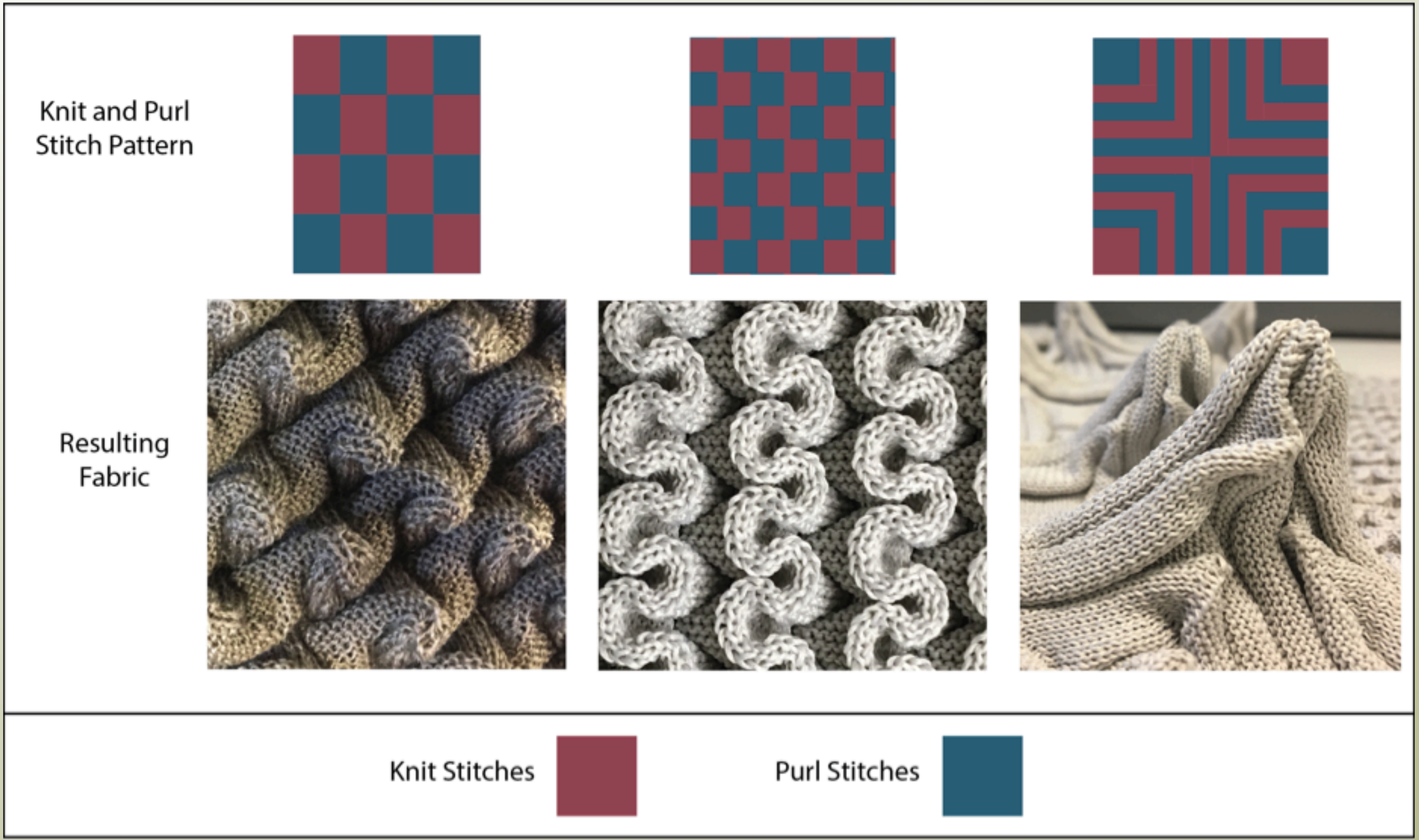
KNIT



PURL

Knittel, Aharoni, RDK, and Dion (2019)

Non-Isometric Origami



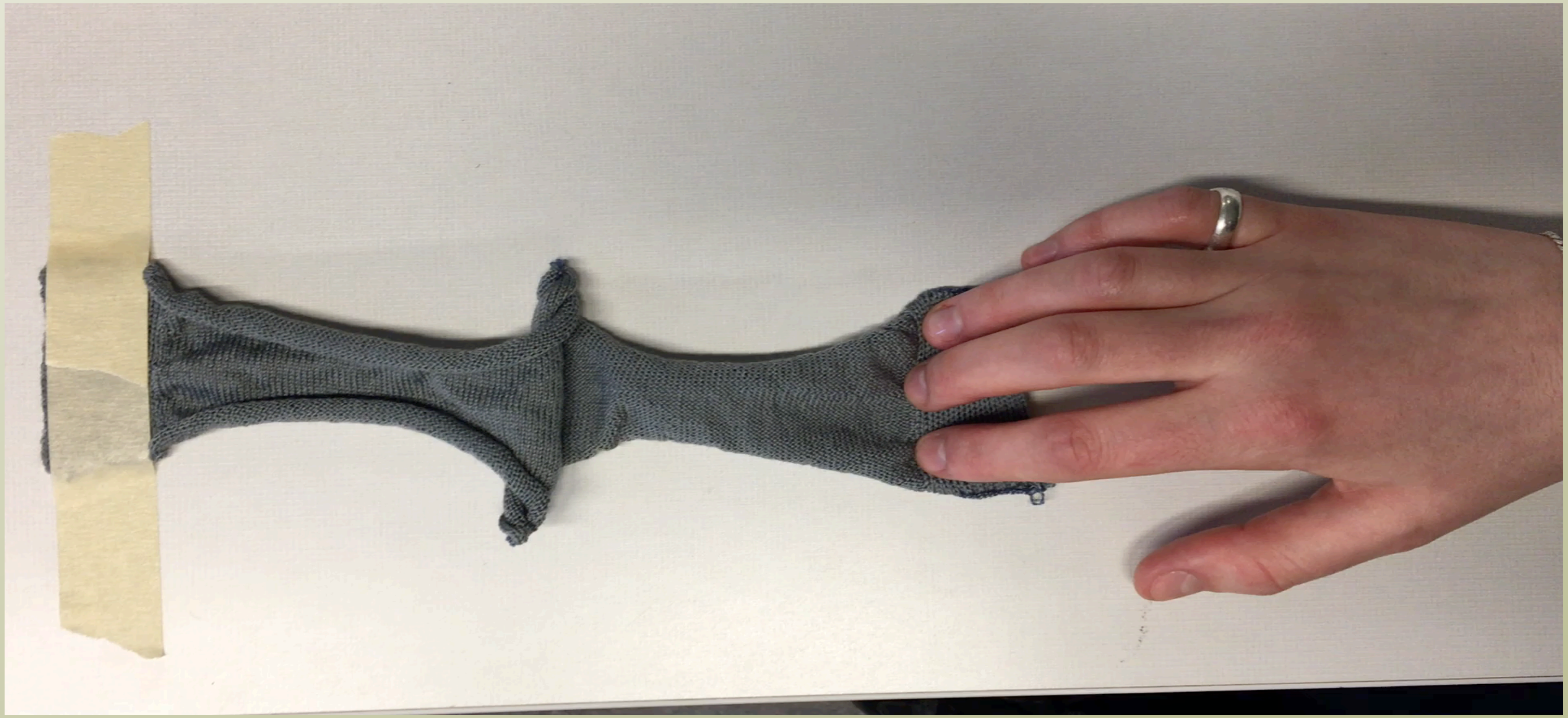
Knittel, Aharoni, RDK, and Dion (2019)

Non-Isometric Origami

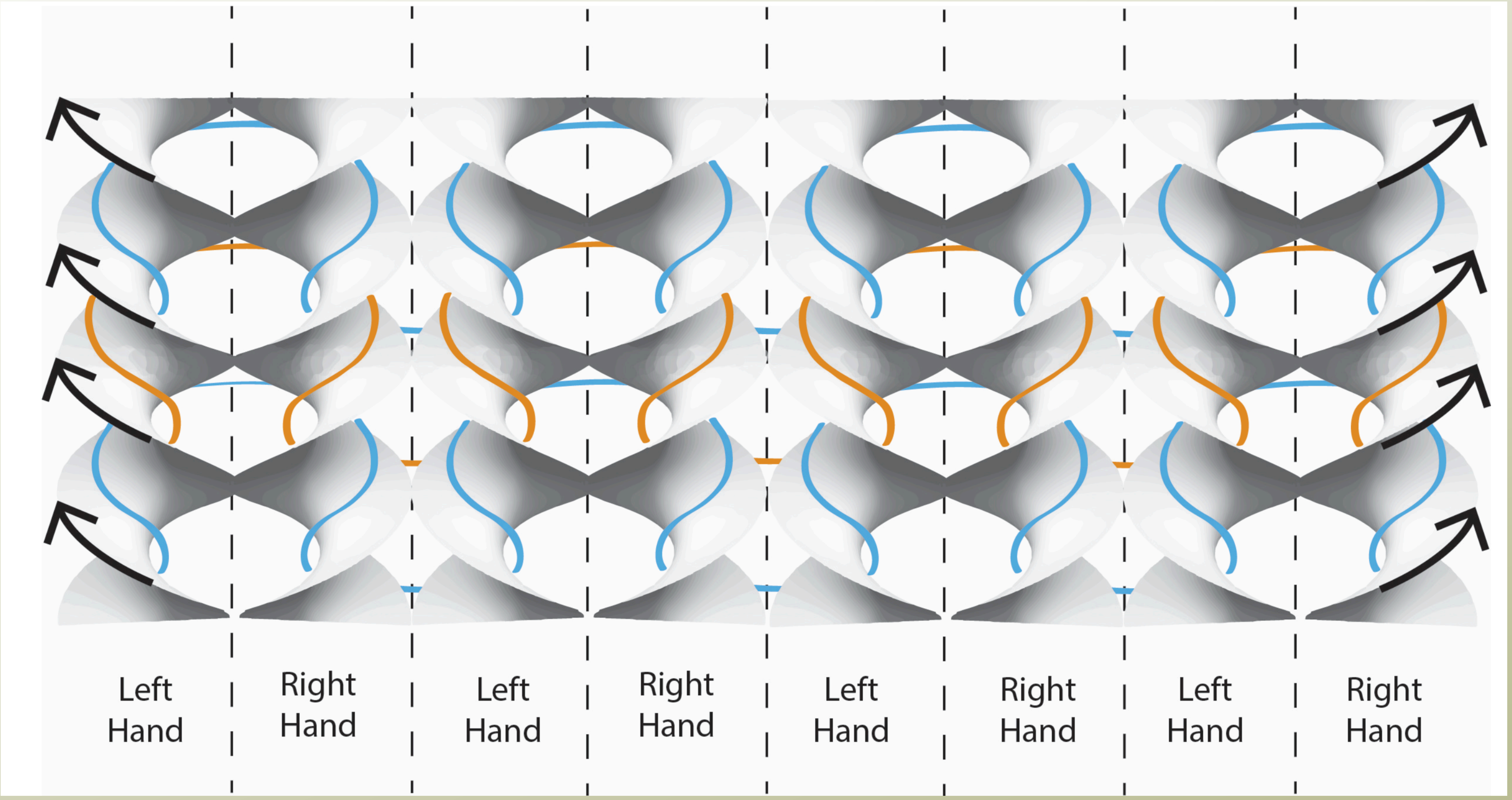
Knit Side	Purl Side

Non-Isometric Origami

Knit Side	Purl Side



Geometry to Mechanics



Knittel, Aharoni, RDK, and Dion (2019)

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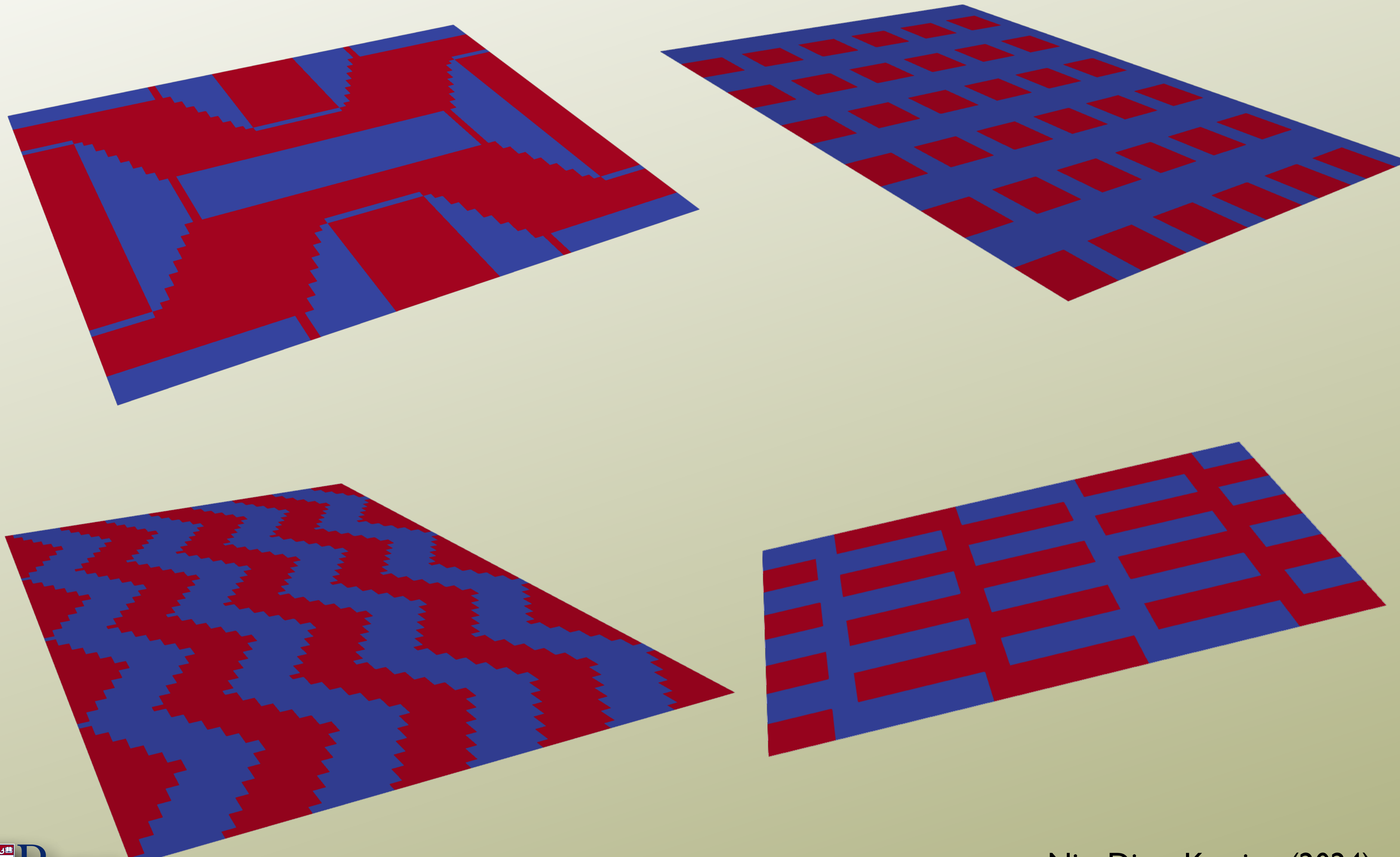
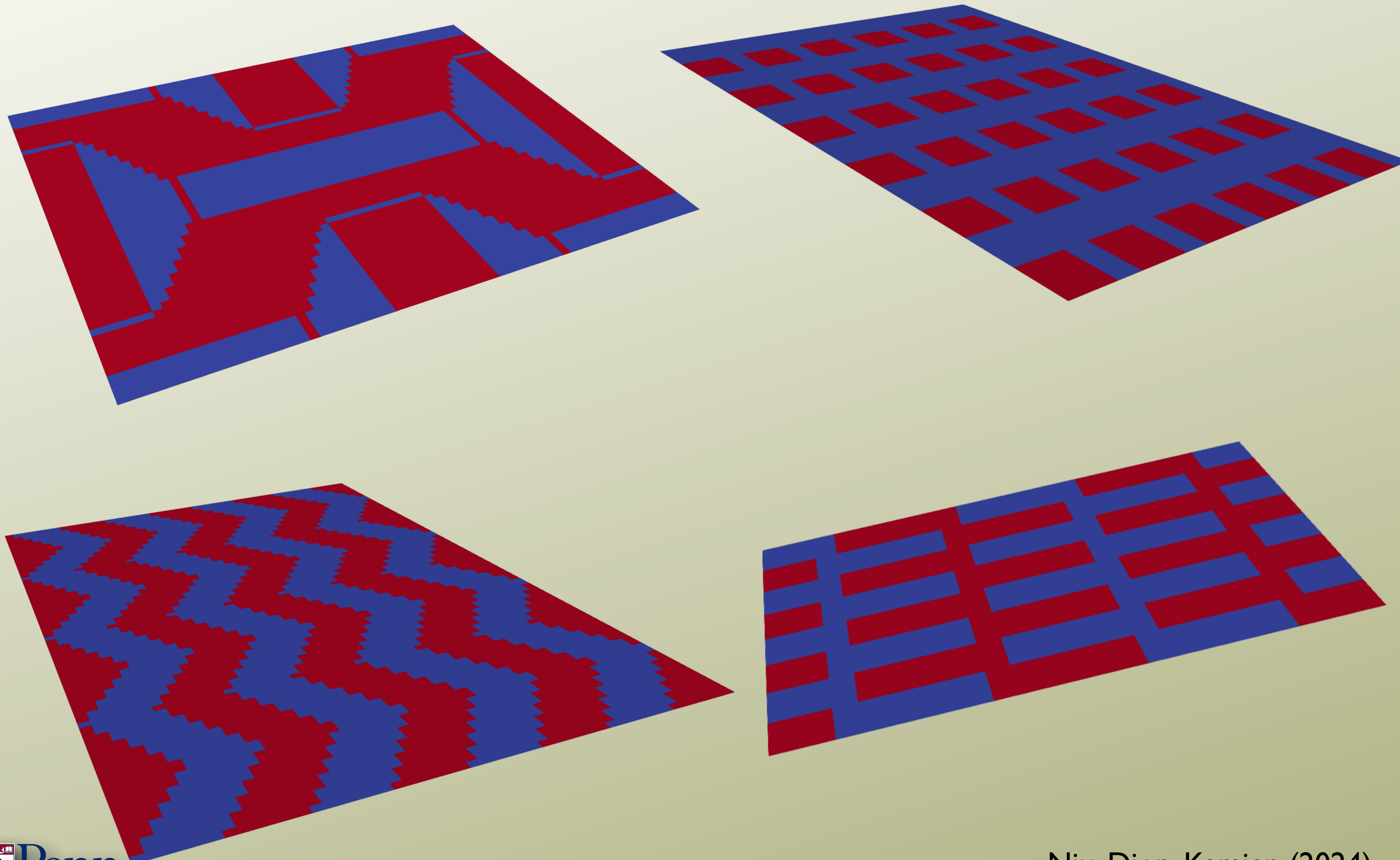
Folding (*à la* Lauren Niu)



Niu, Dion, Kamien (2024)

Folding (*à la* Lauren Niu)



Niu, Dion, Kamien (2024)

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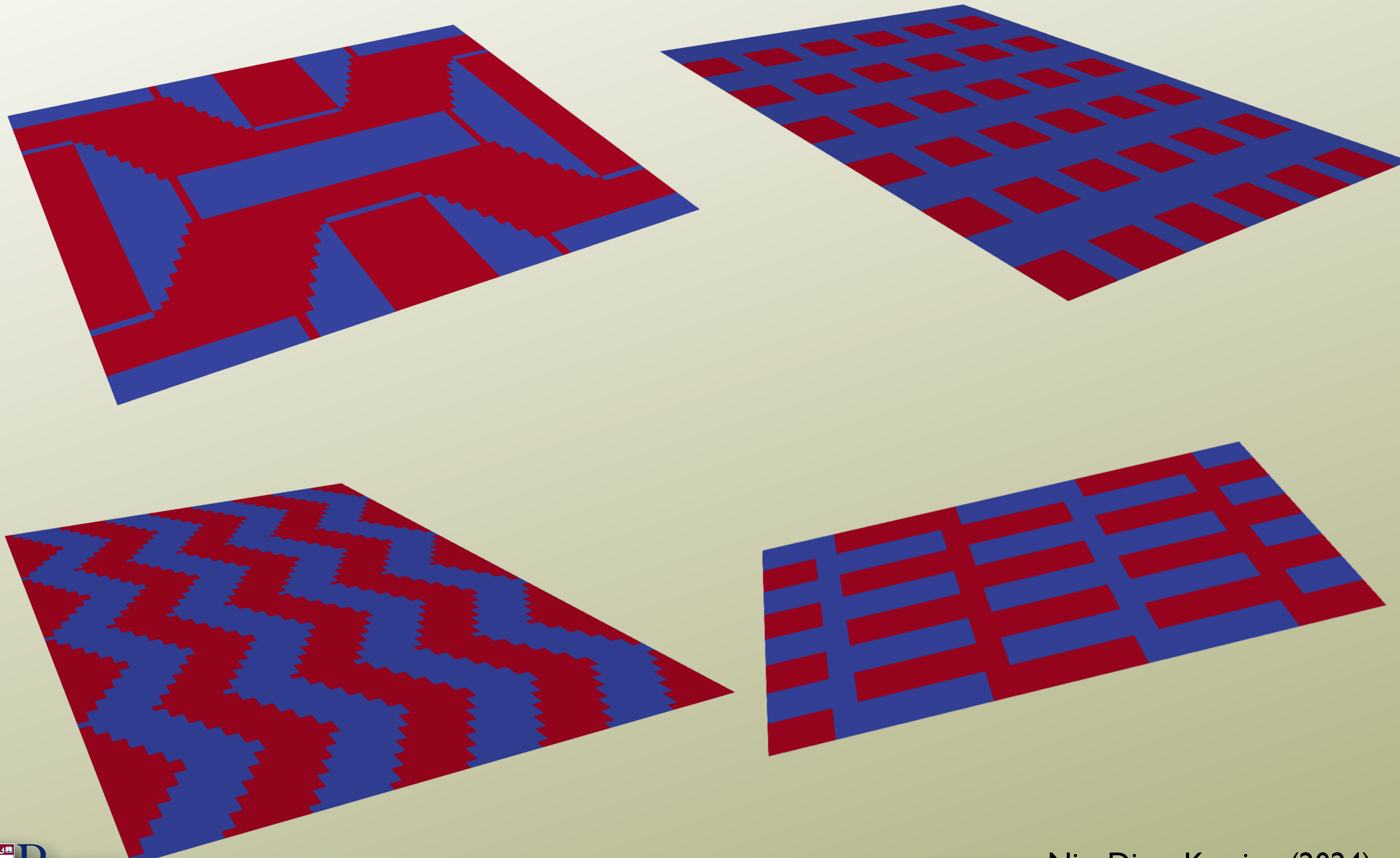

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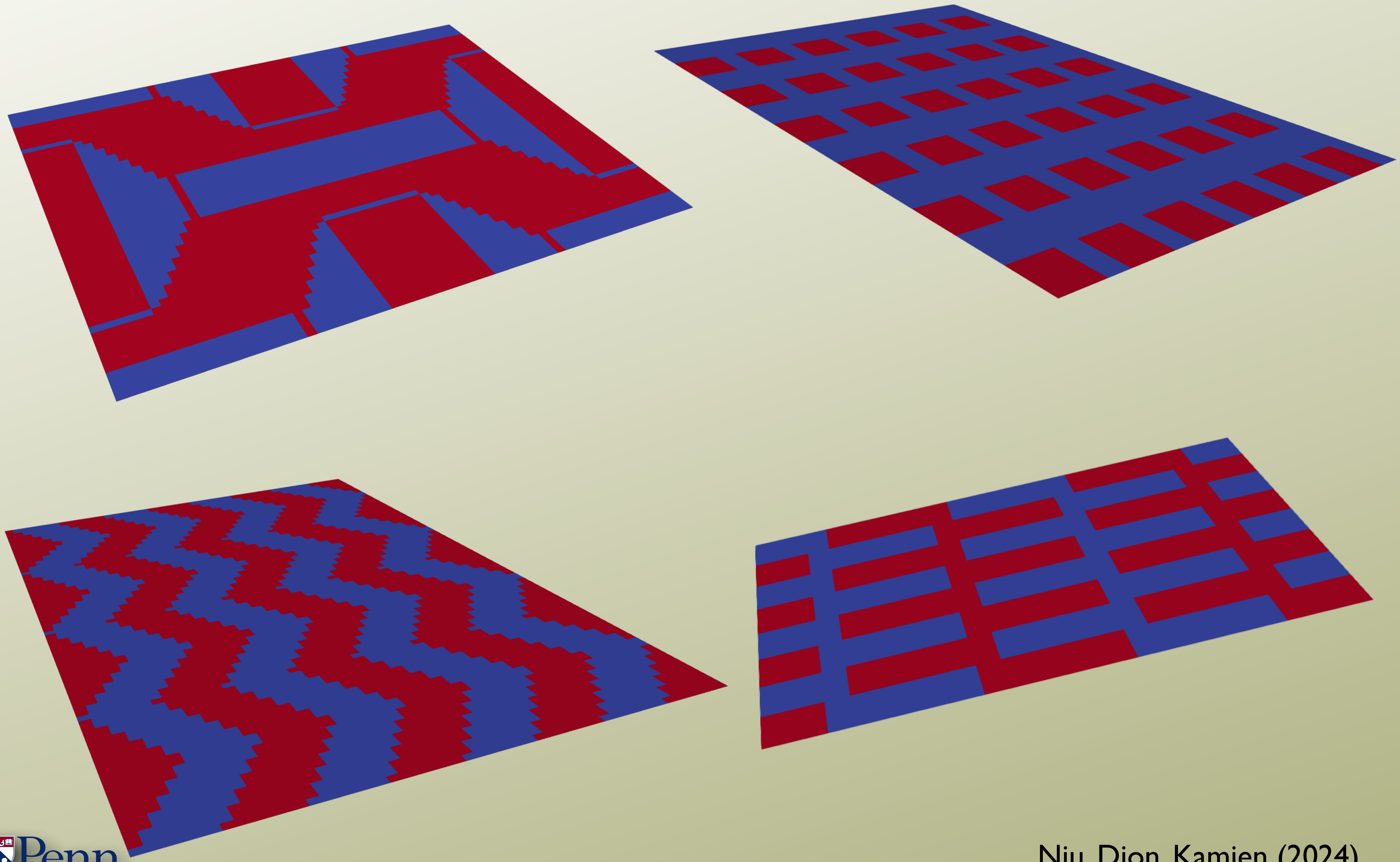

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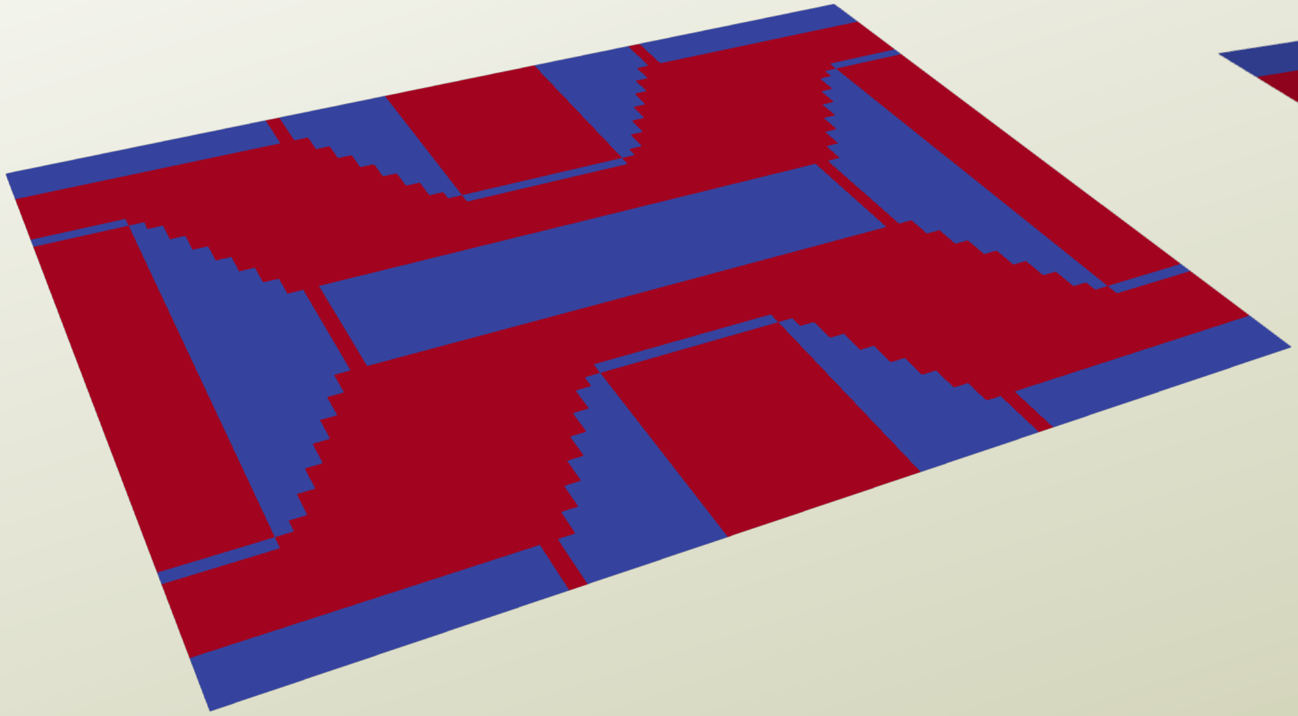

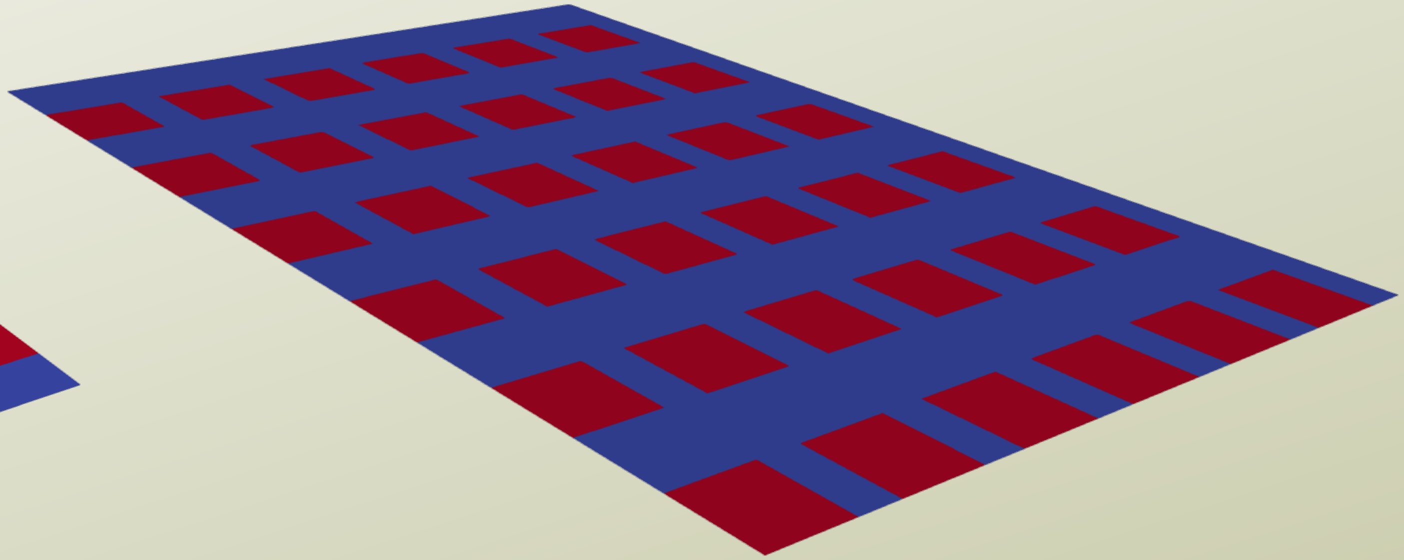

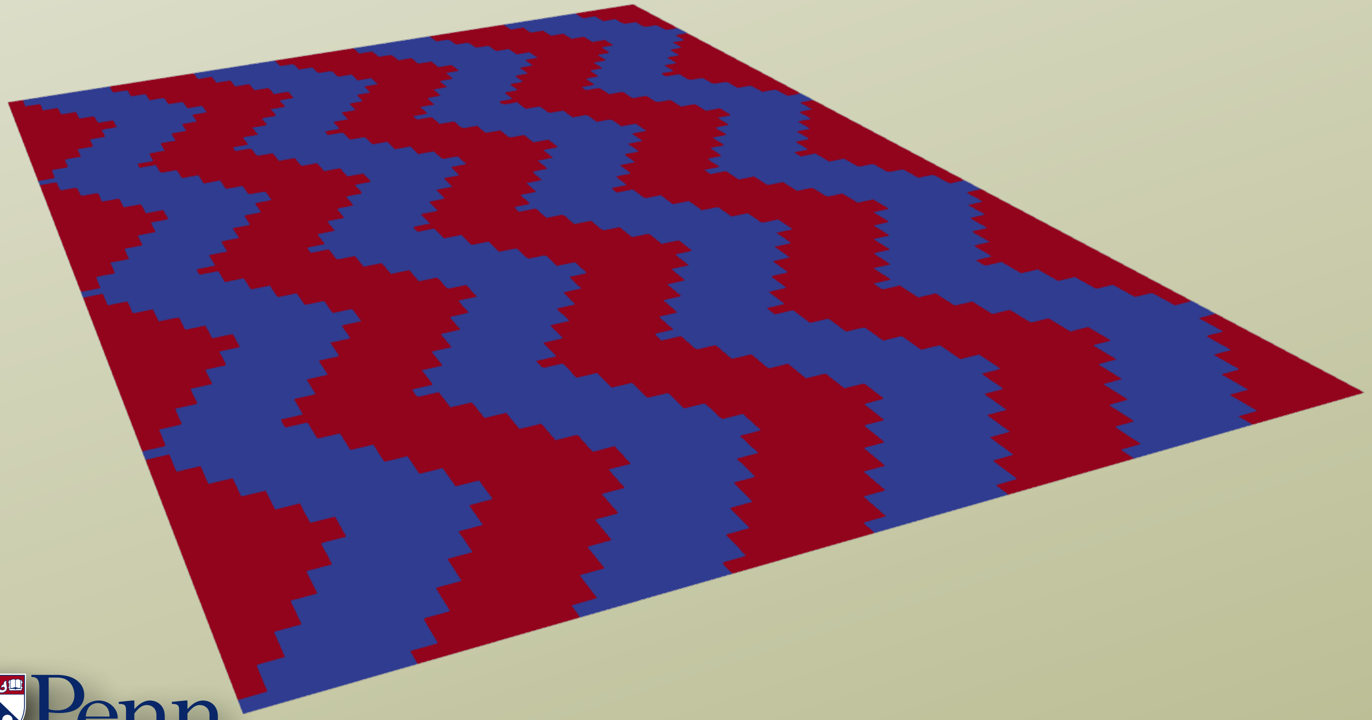

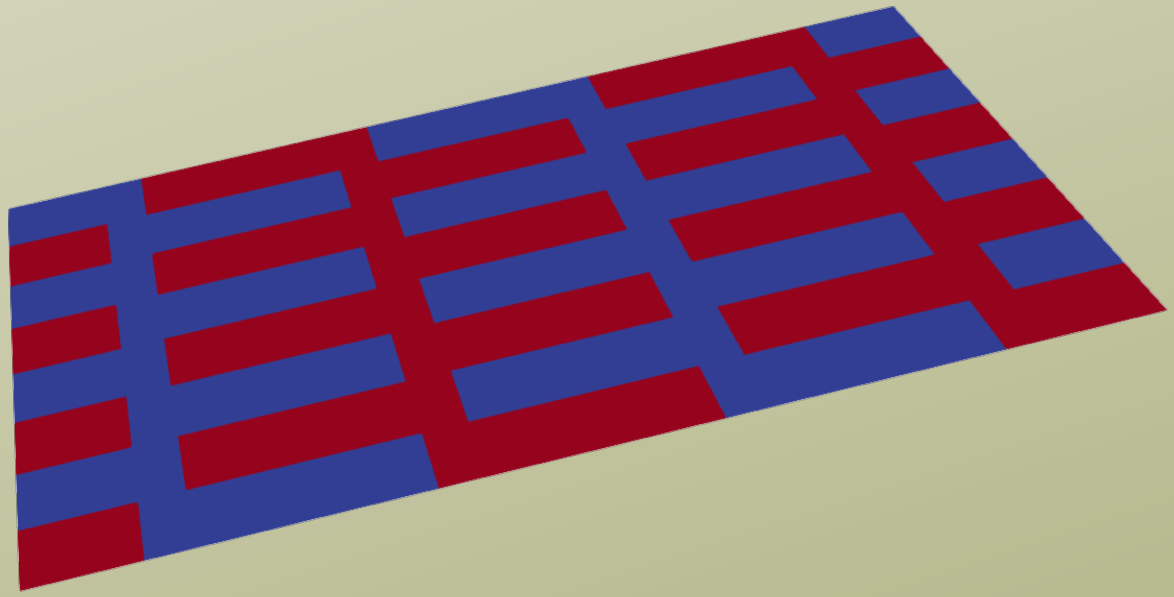

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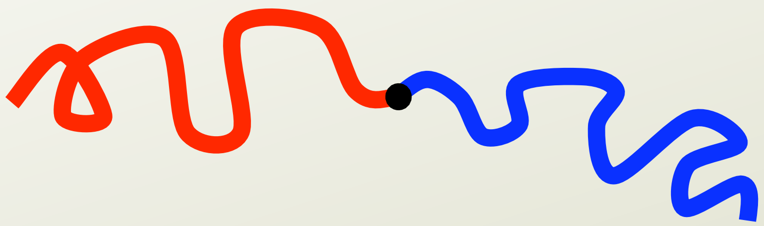
Niu, Dion, Kamien (2024)

Diblock Copolymers



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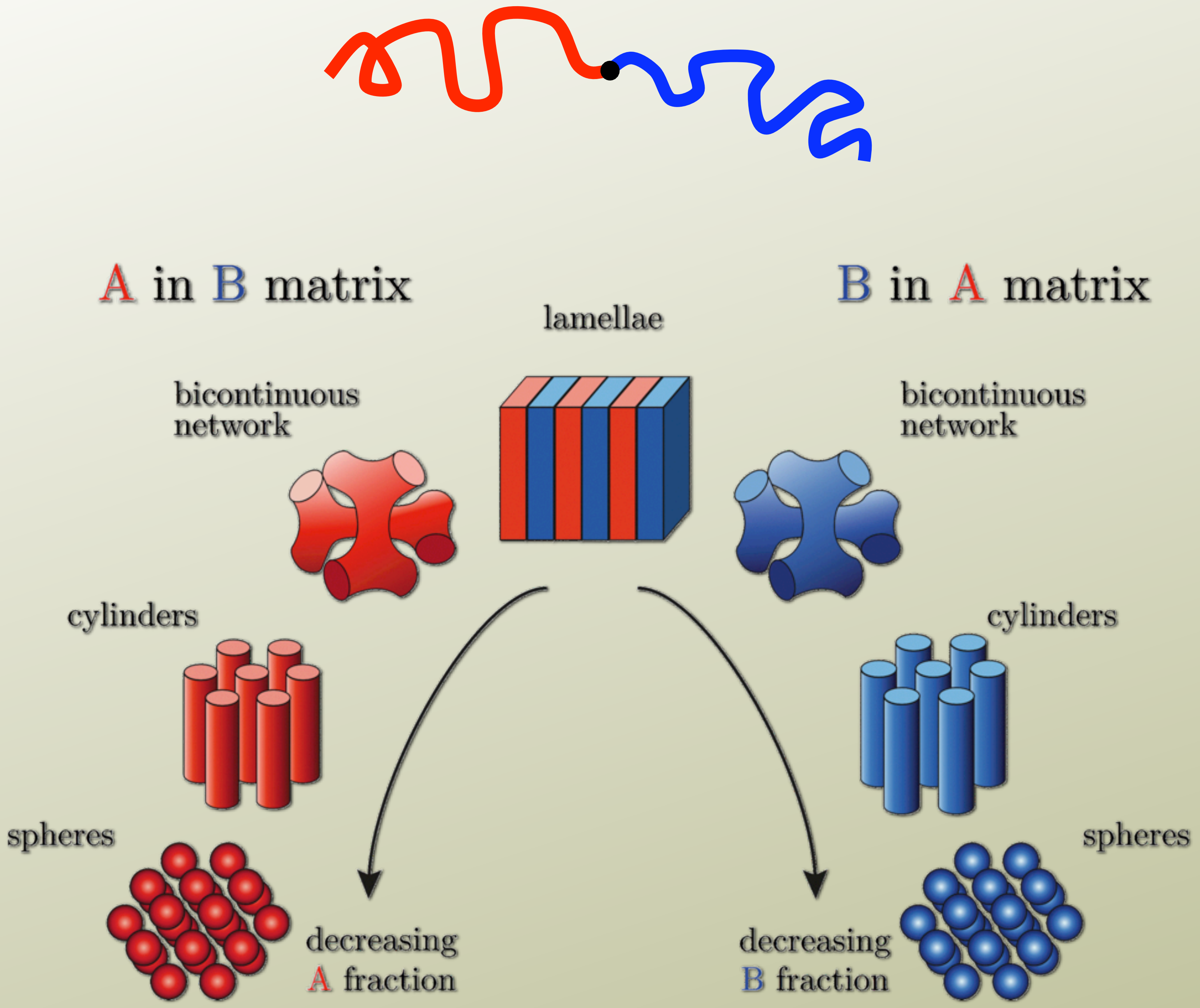
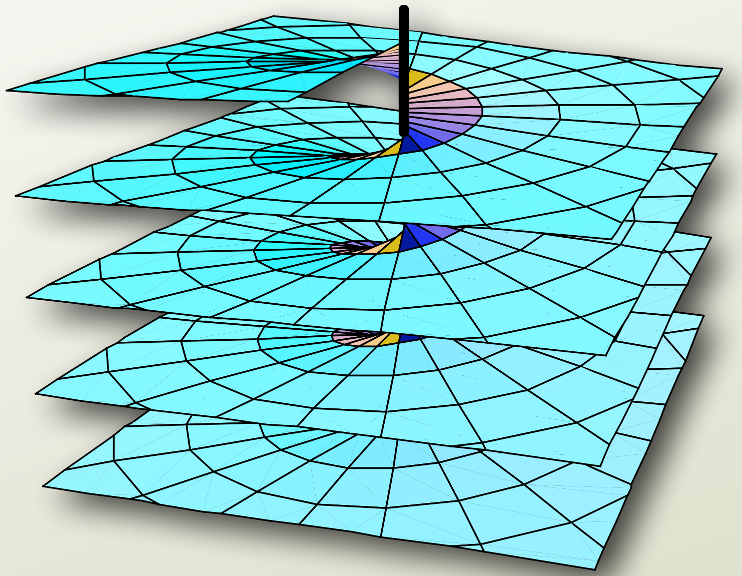
Diblock Copolymers



Twist Grain Boundary Phases



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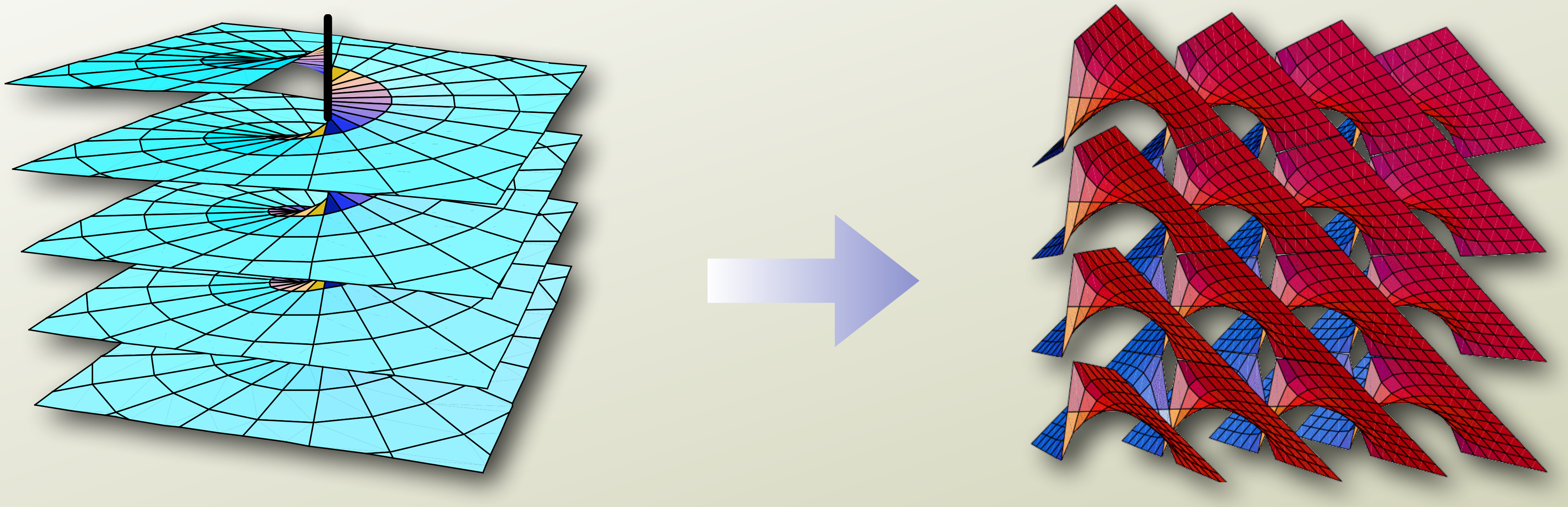

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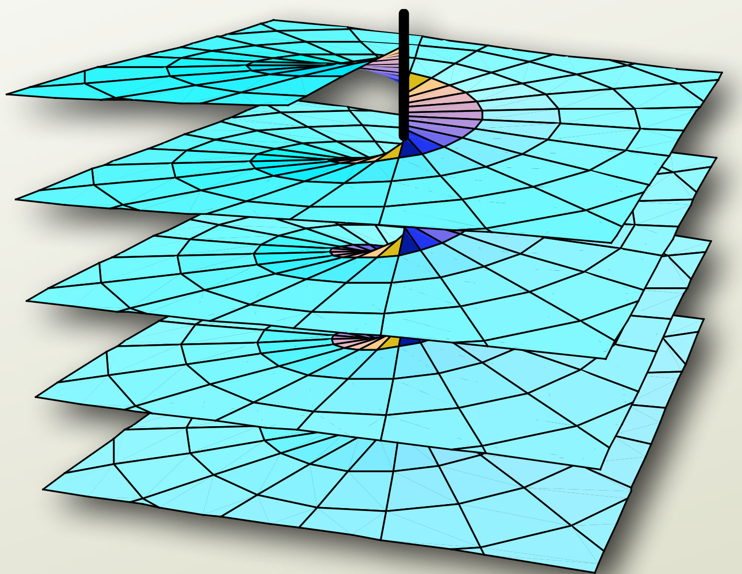

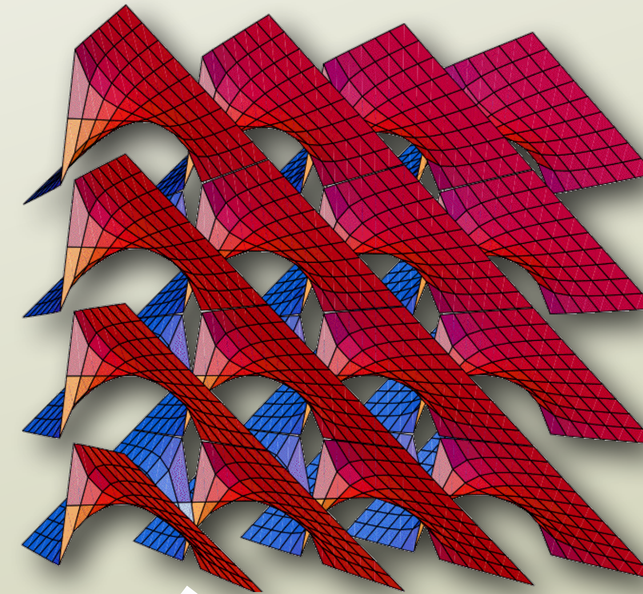

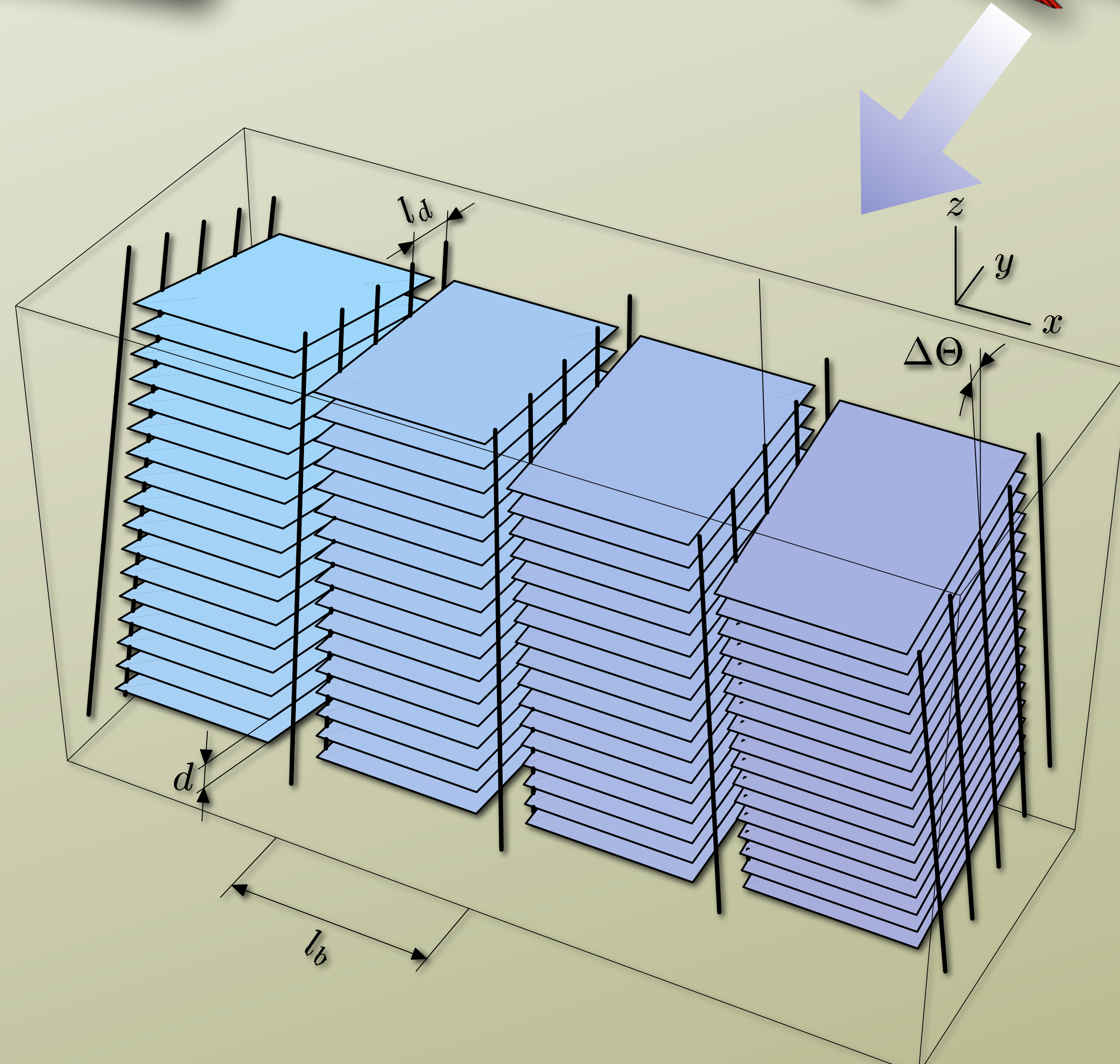

smectic analog of the flux line lattice

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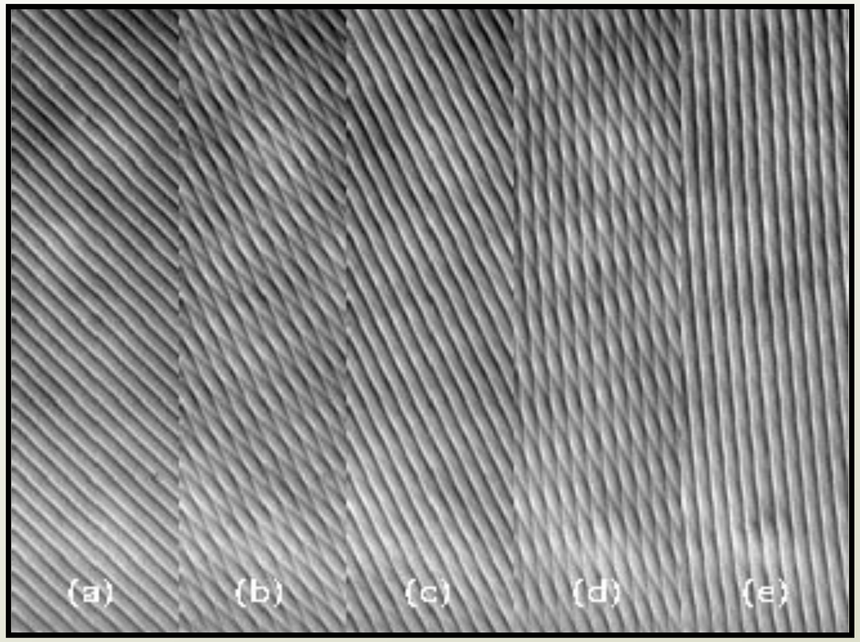
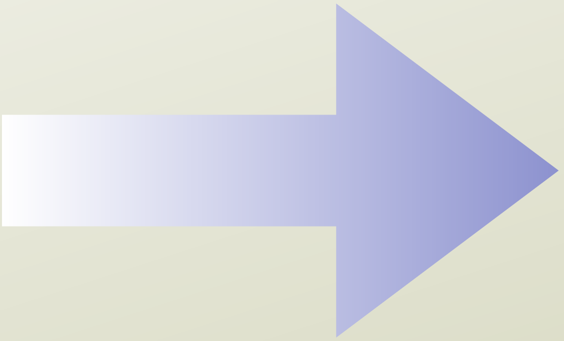
Renn and T. Lubensky, *PRA* **38** (1990) 4392

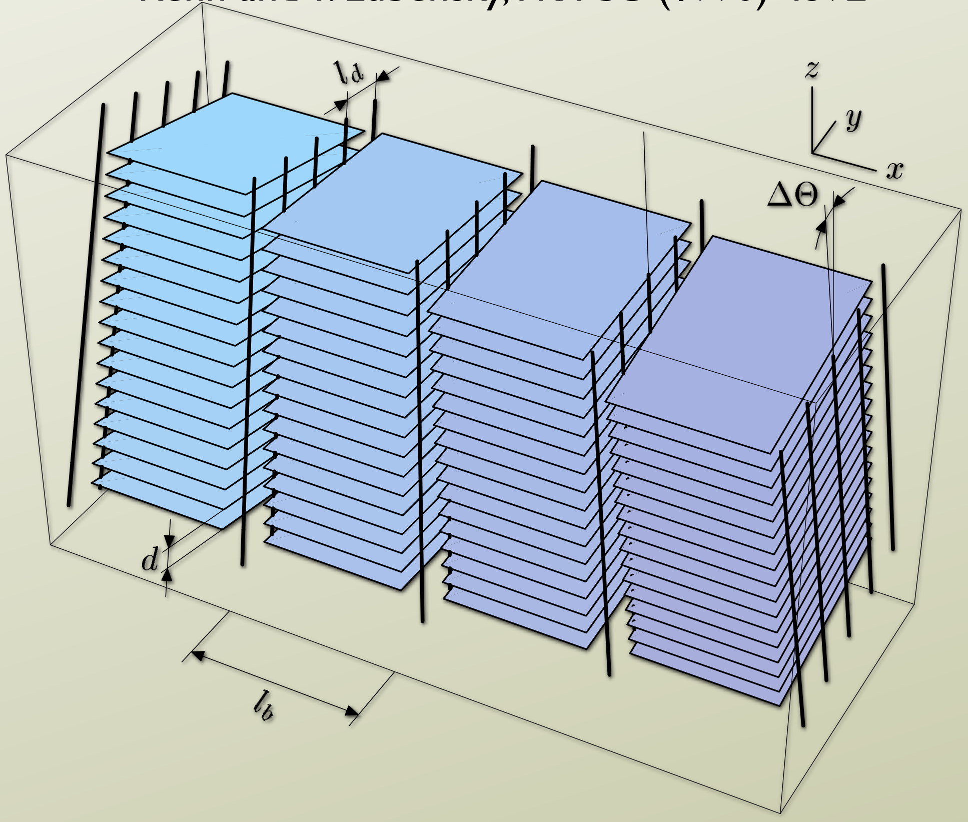
Twist Grain Boundary Phases



Fraden, In "Observation, Prediction, and Simulation of Phase Transitions in Complex Fluids" (Kluwer, 1995)



Renn and T. Lubensky, *PRA* **38** (1990) 4392



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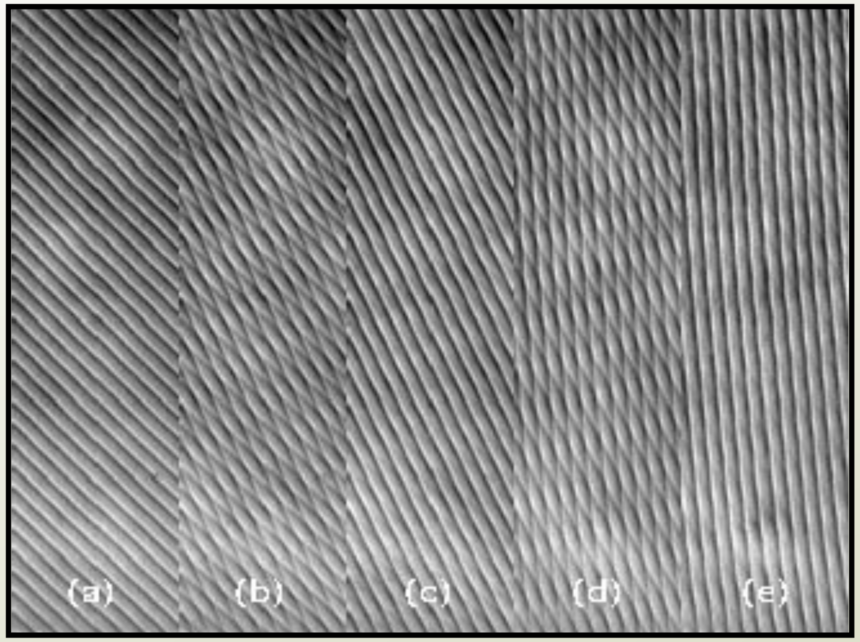
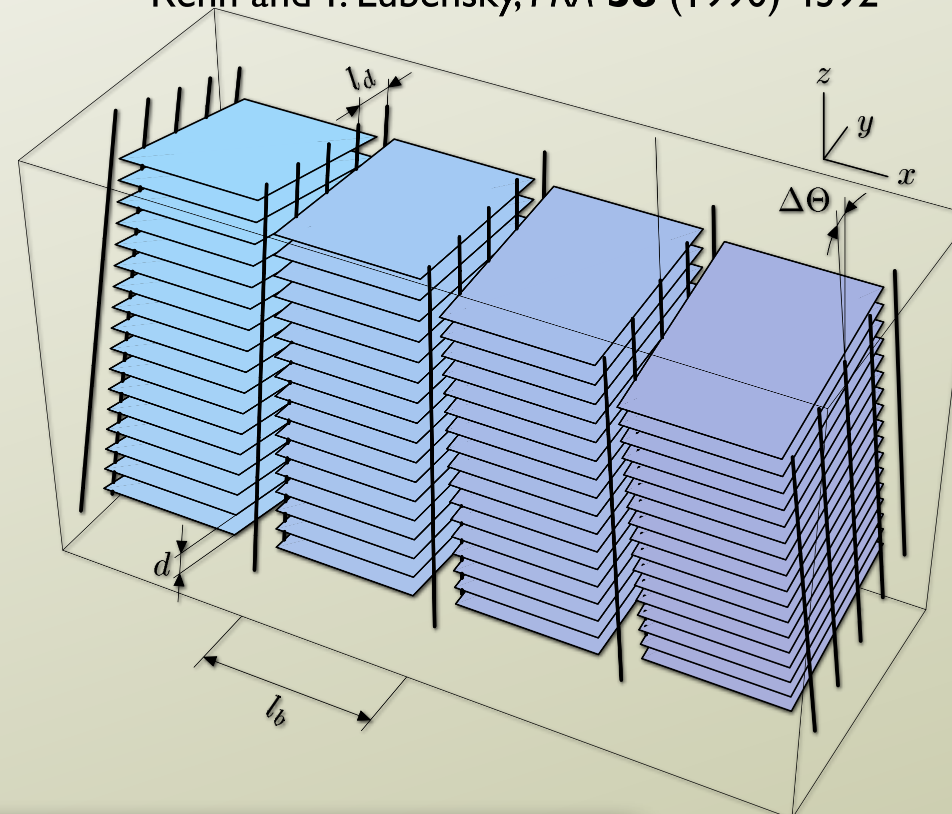
Twist Grain Boundary Phases

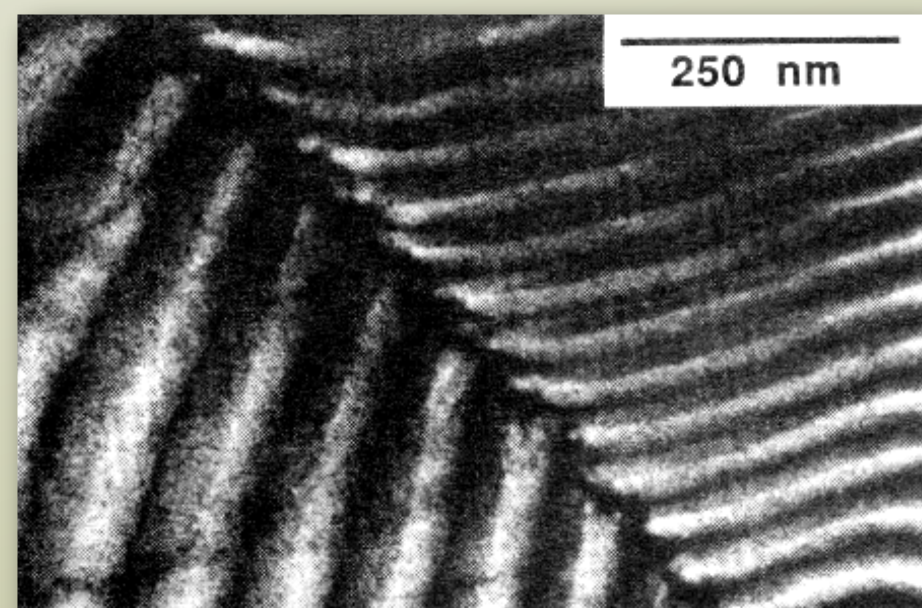


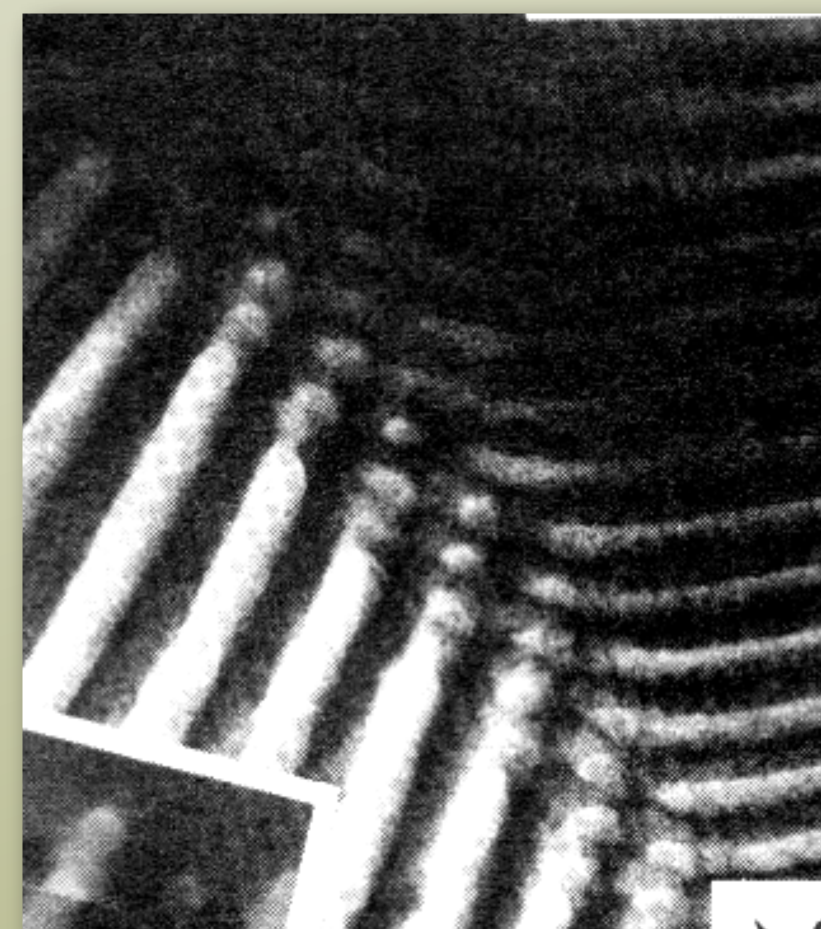
From http://www.elsie.brandeis.edu/

Fraden, In "Observation, Prediction, and Simulation of Phase Transitions in Complex Fluids" (Kluwer, 1995)

Renn and T. Lubensky, *PRA* **38** (1990) 4392



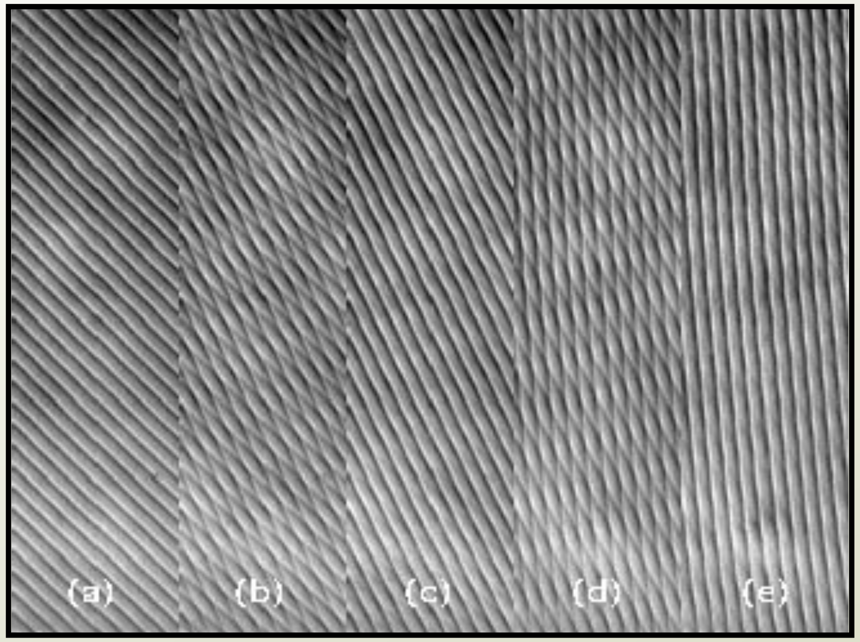




Gido, Gunther, Thomas, & Hoffman, *Macromolecules* **26** (1993)

Penn

Twist Grain Boundary Phases



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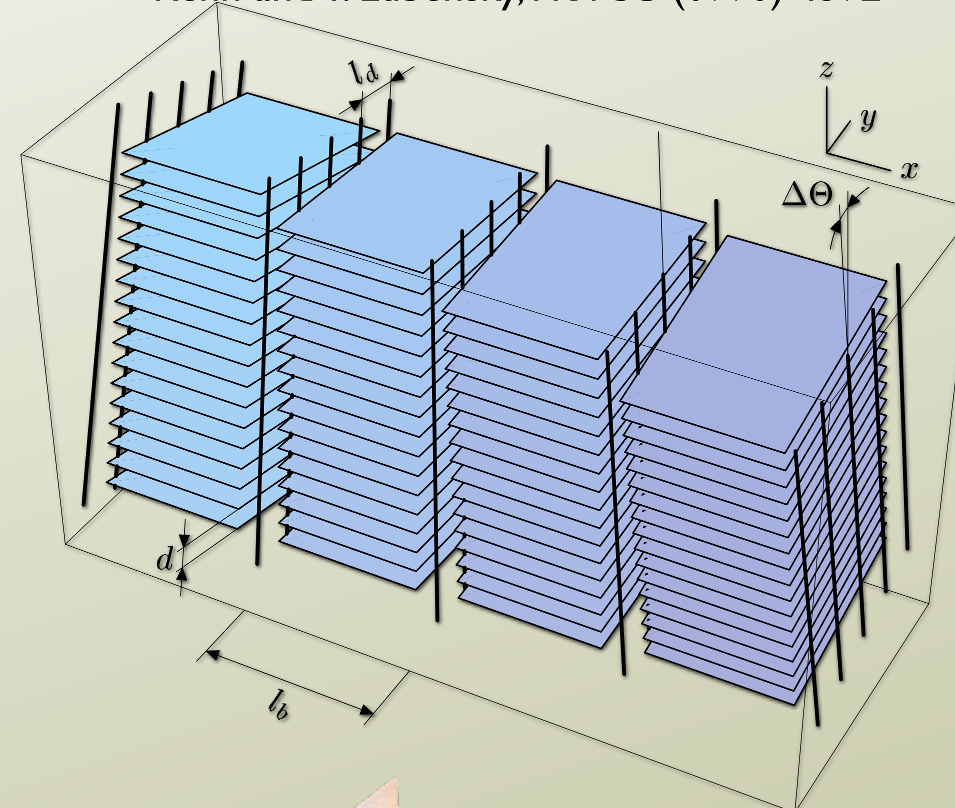
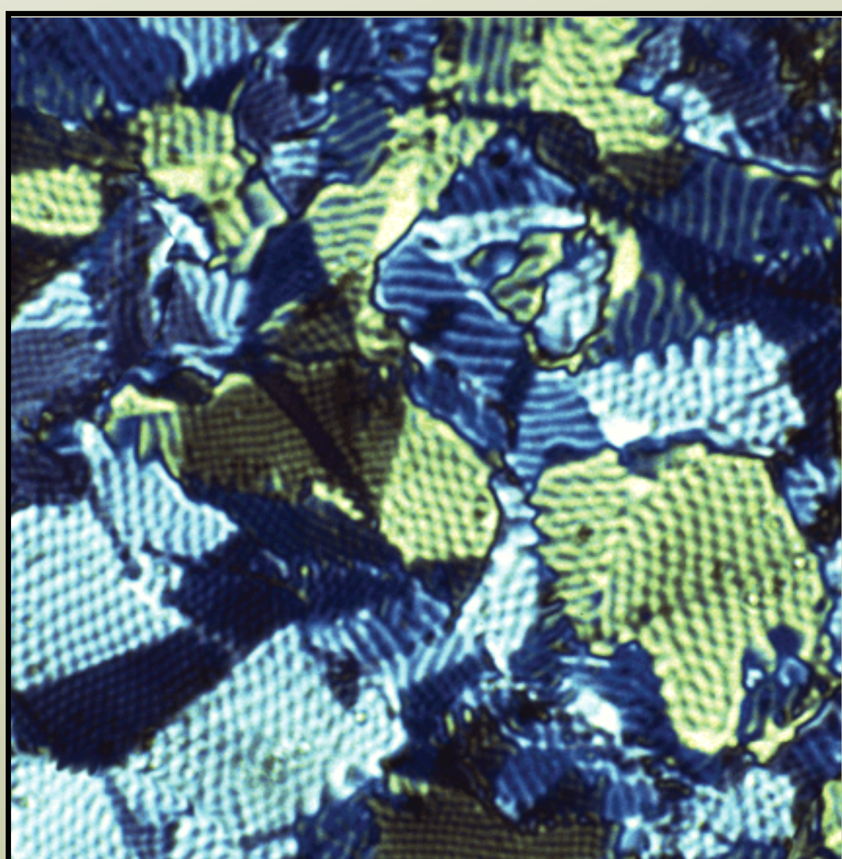
Fraden, In "Observation, Prediction, and Simulation of Phase Transitions in Complex Fluids" (Kluwer, 1995)

Low Angles

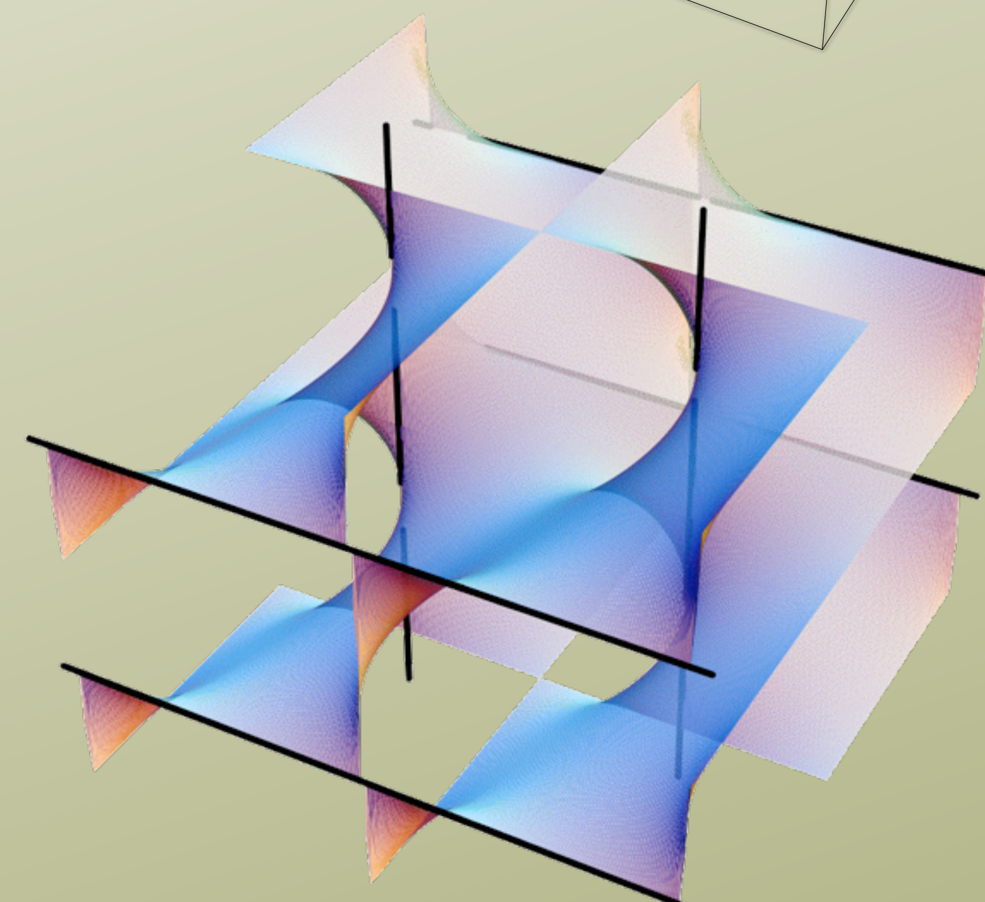
Renn and T. Lubensky, *PRA* **38** (1990) 4392





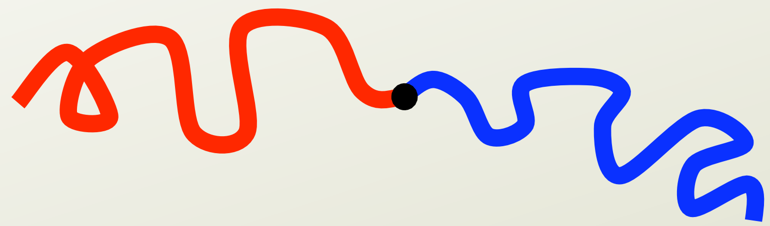
Fernsler *et al.*, *PNAS* **102** (2005) 14191

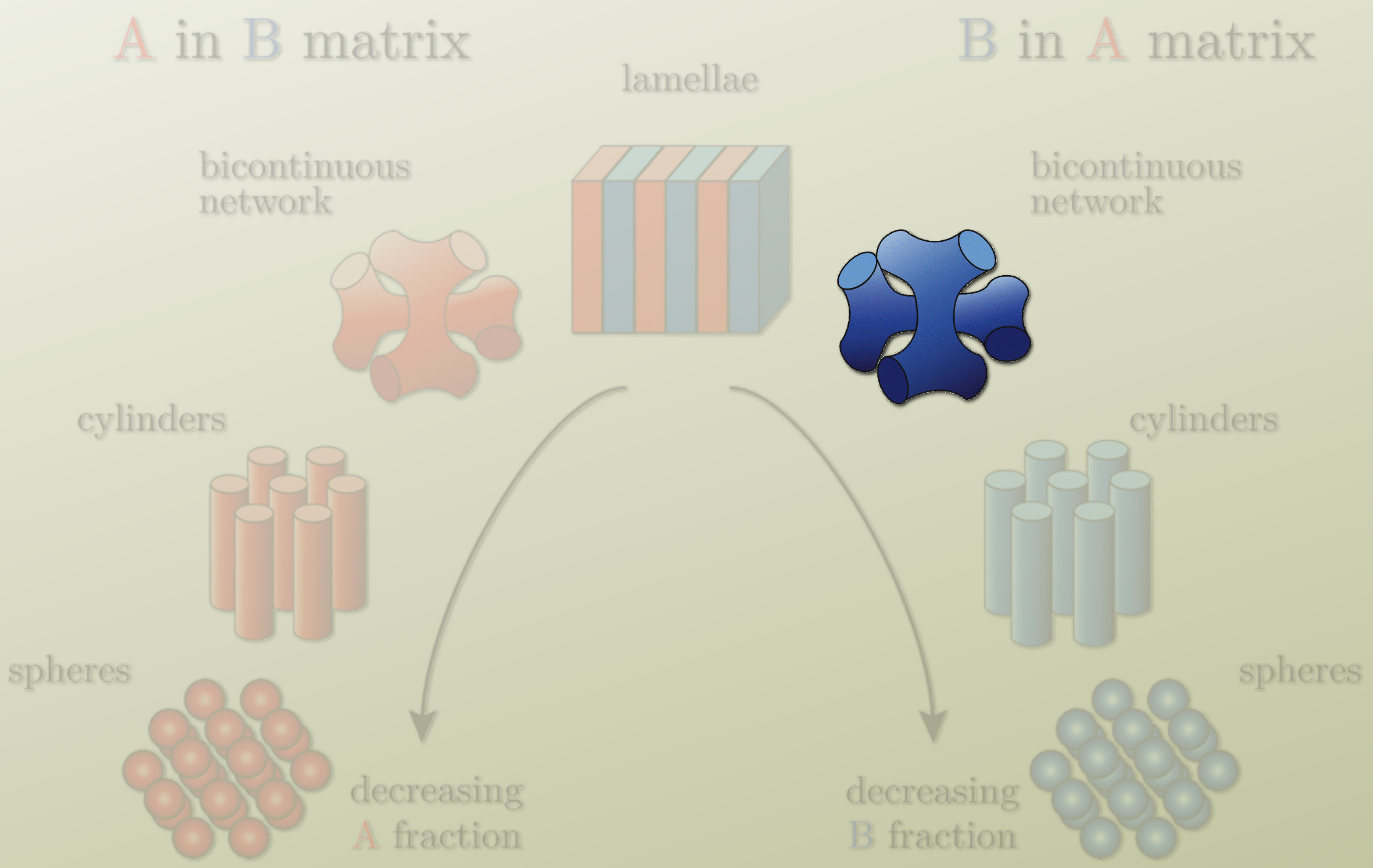
Large Angles



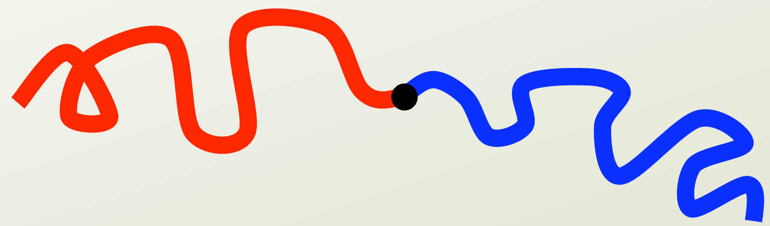
Penn

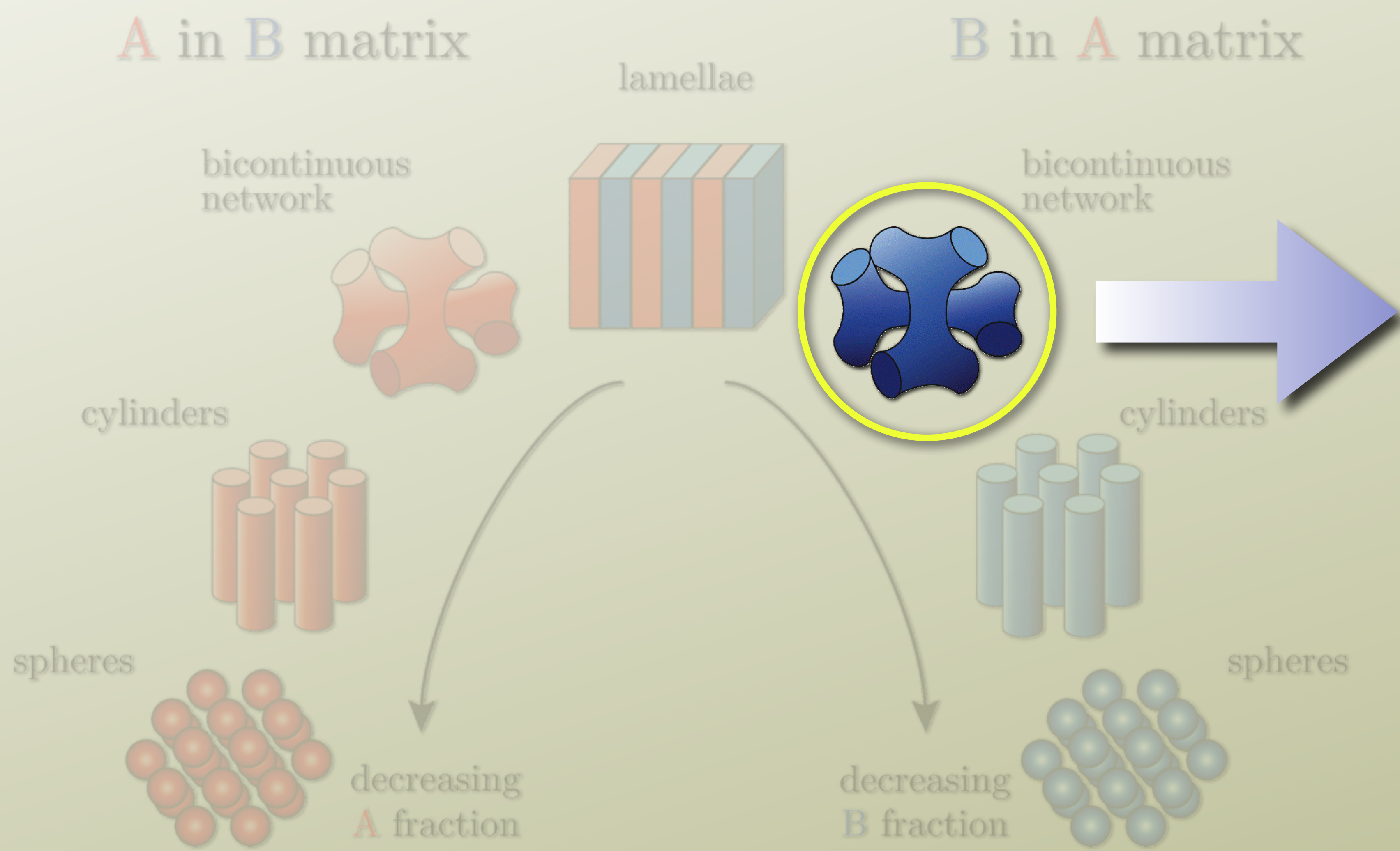
Diblock Copolymers

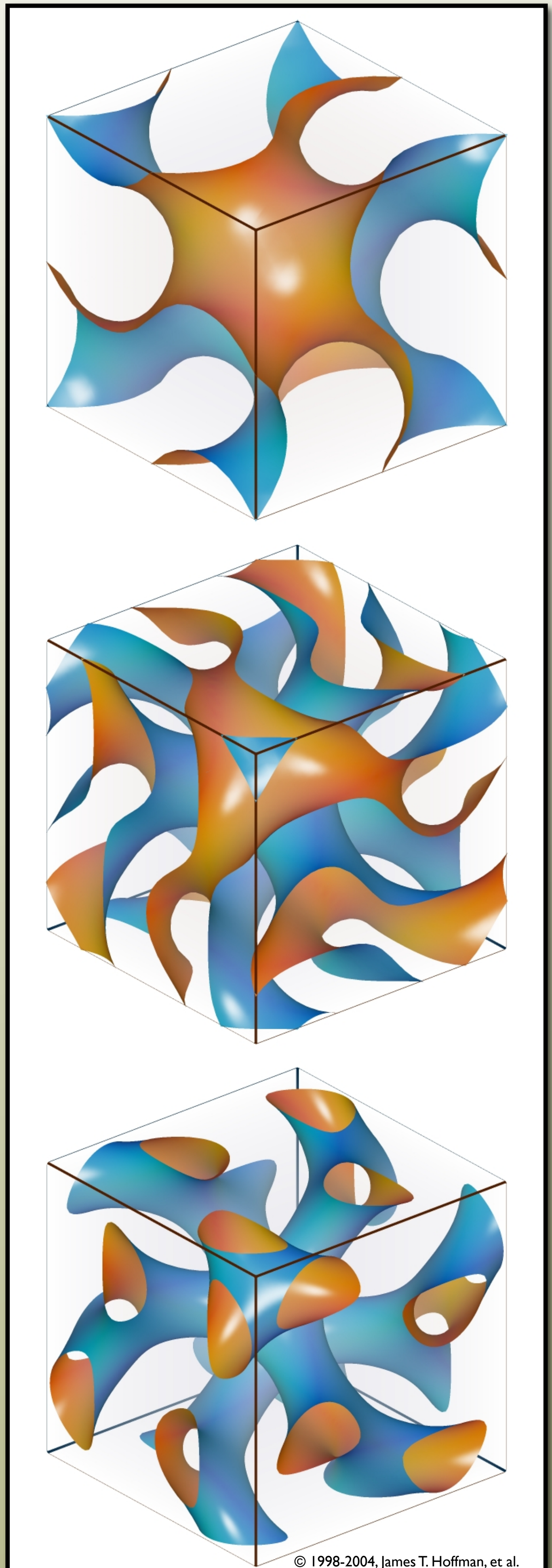




Diblock Copolymers

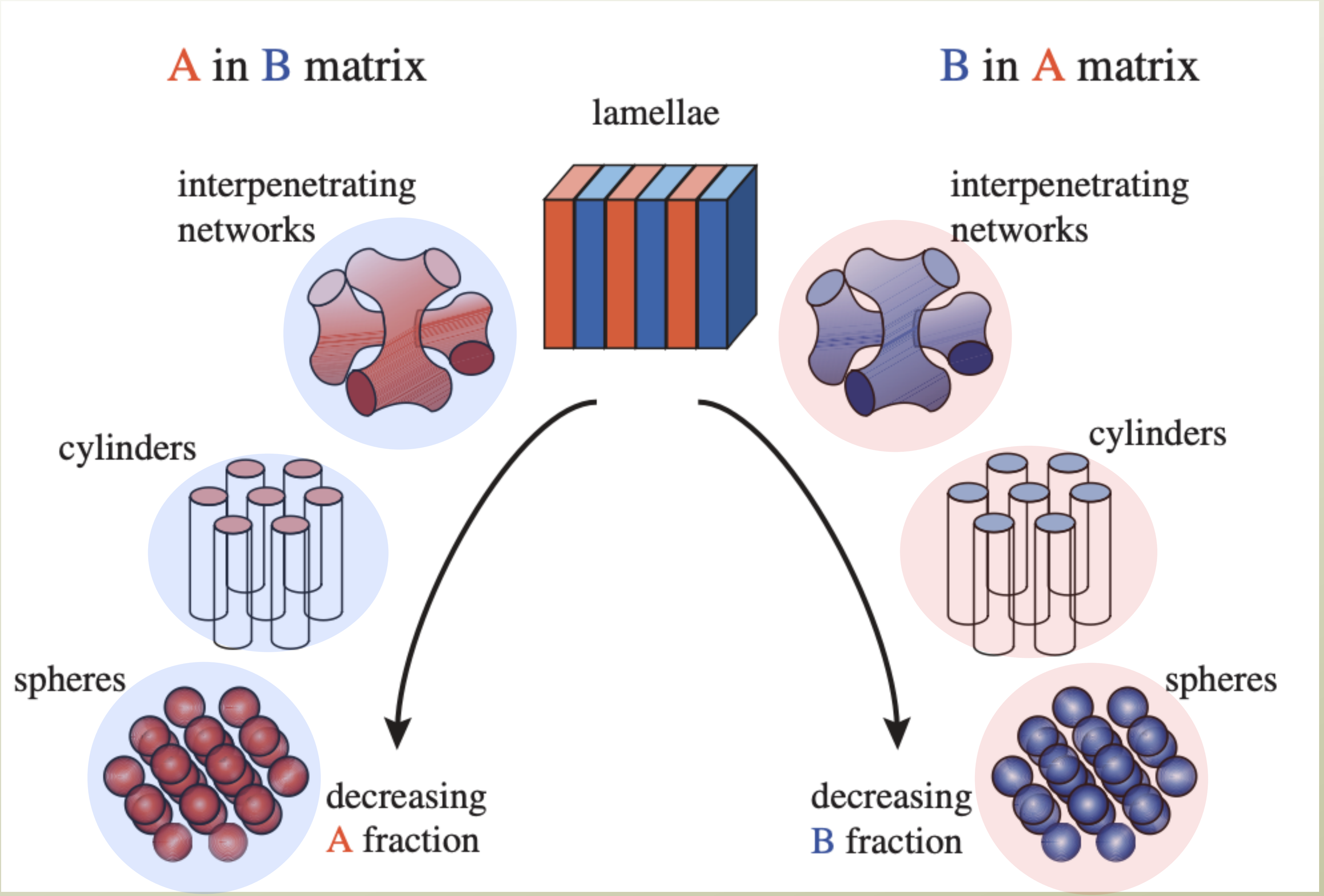




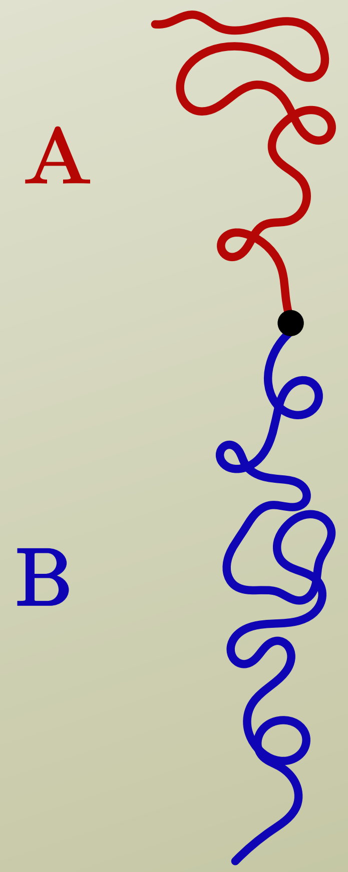


http://www.msri.org/about/sgp/jim/papers/morphbysymmetry/table/main.html

Recap: Diblock copolymers

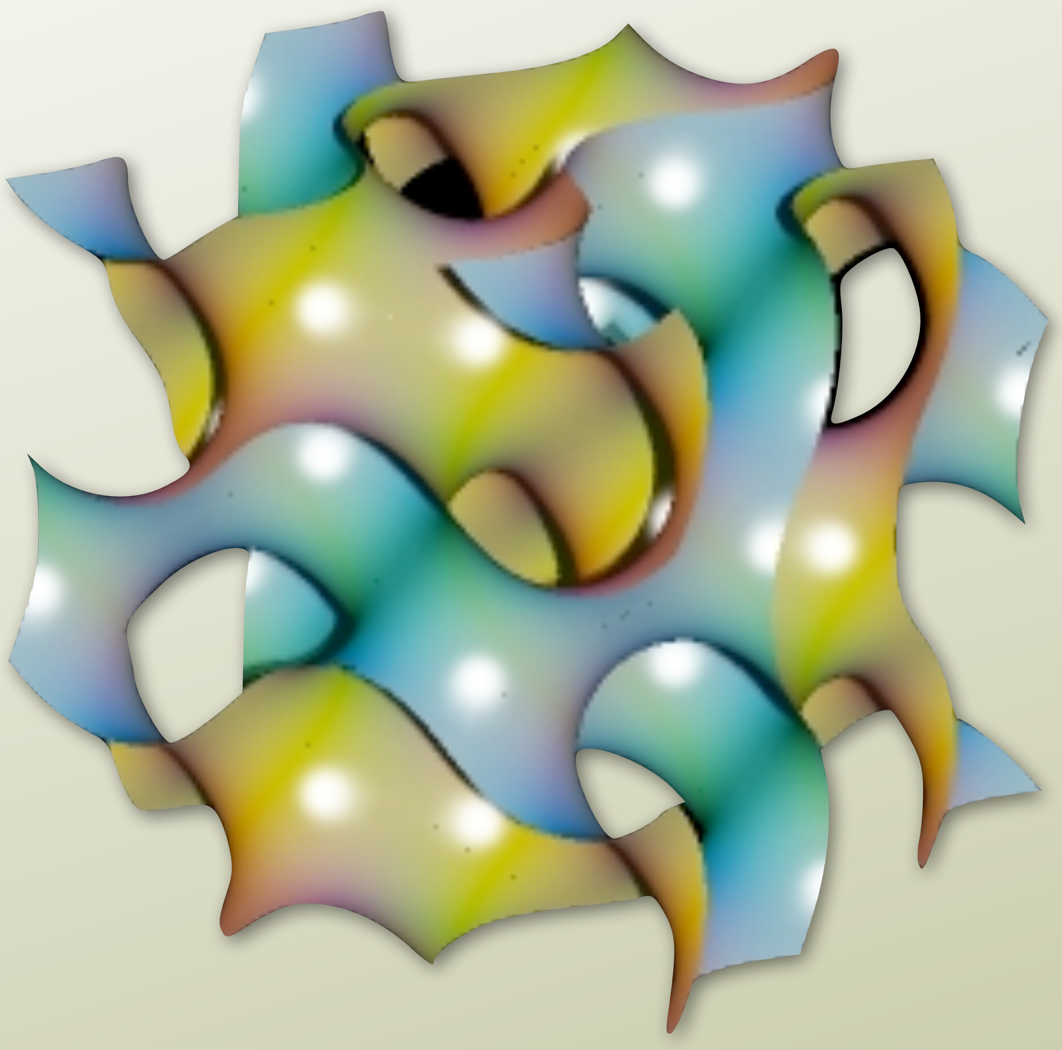


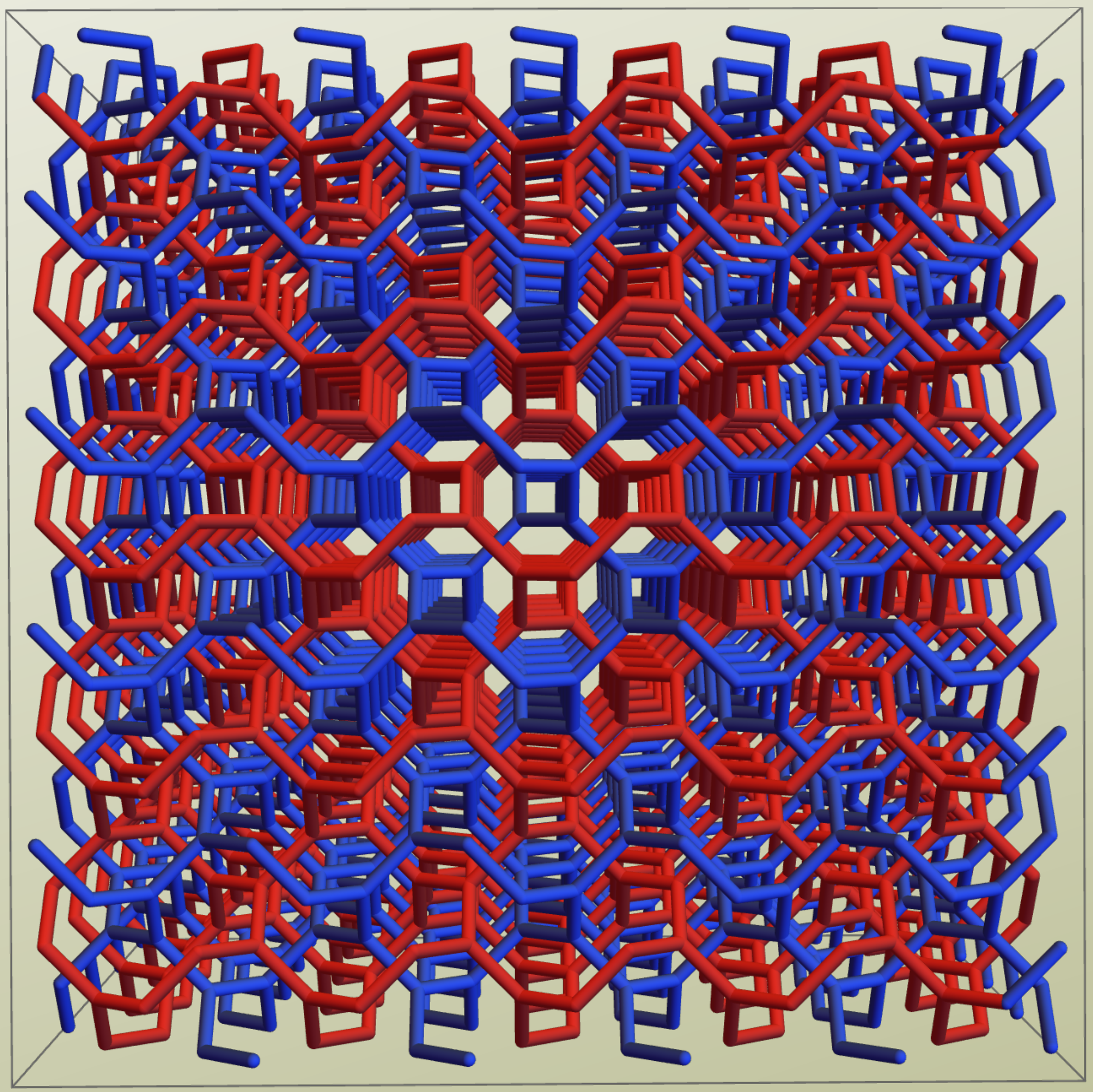
G. M. Grason, Physics Reports (2006)



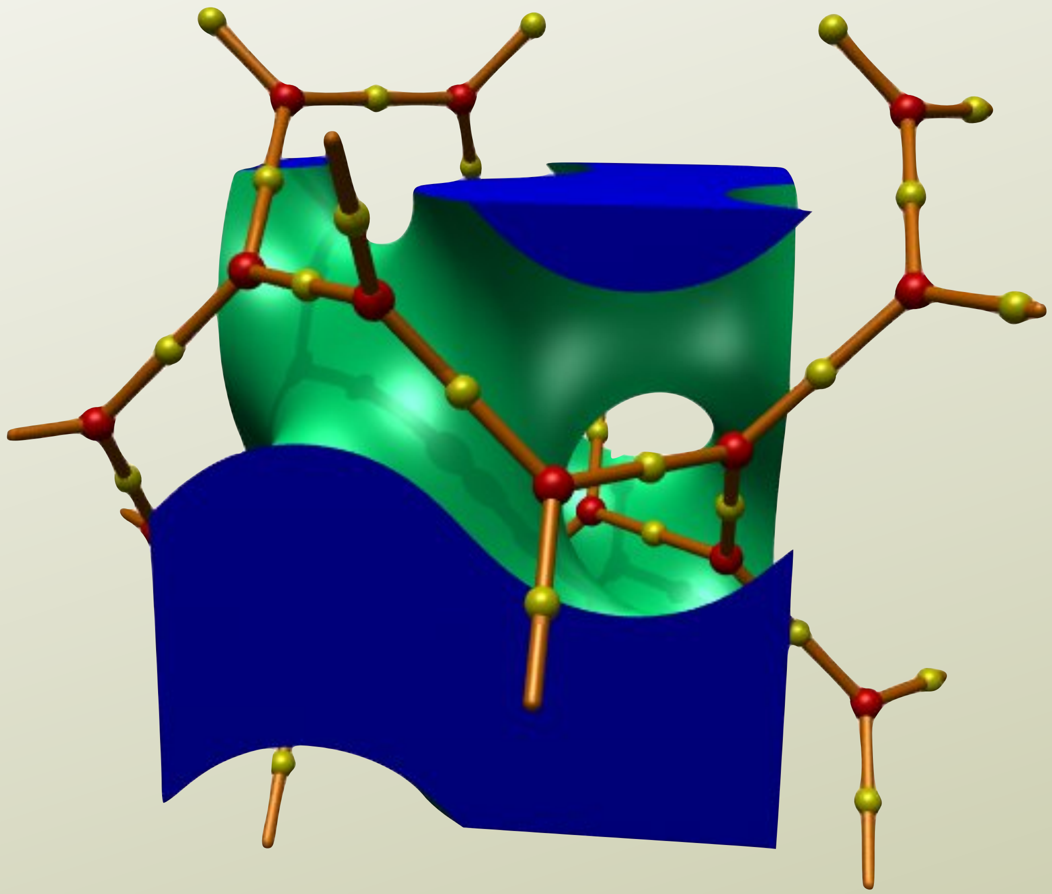
Distinct chains of incompatible monomers linked together via a covalent bond.

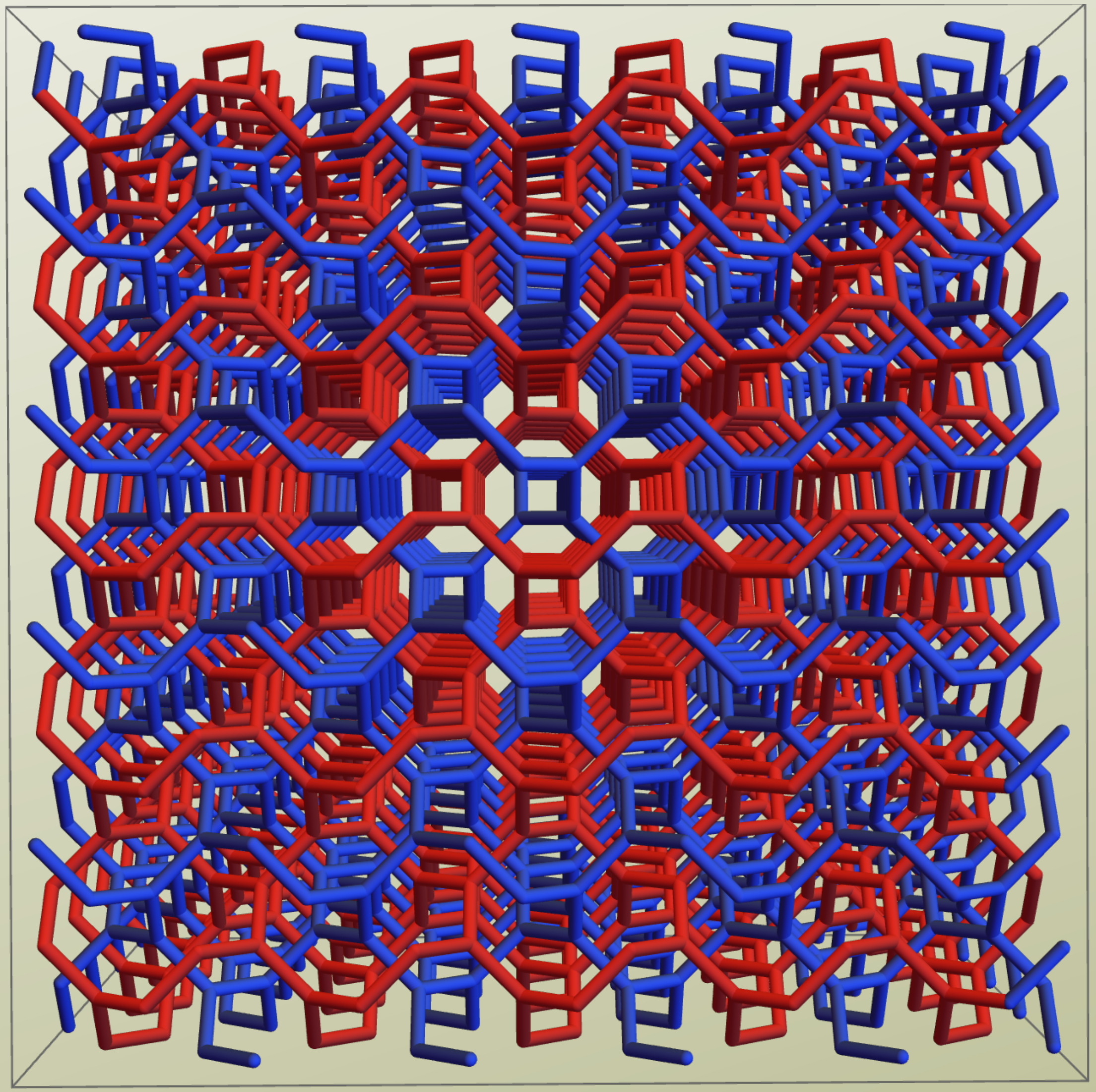
Diblock copolymers: Double Gyroid phase





Diblock copolymers: Double Gyroid phase





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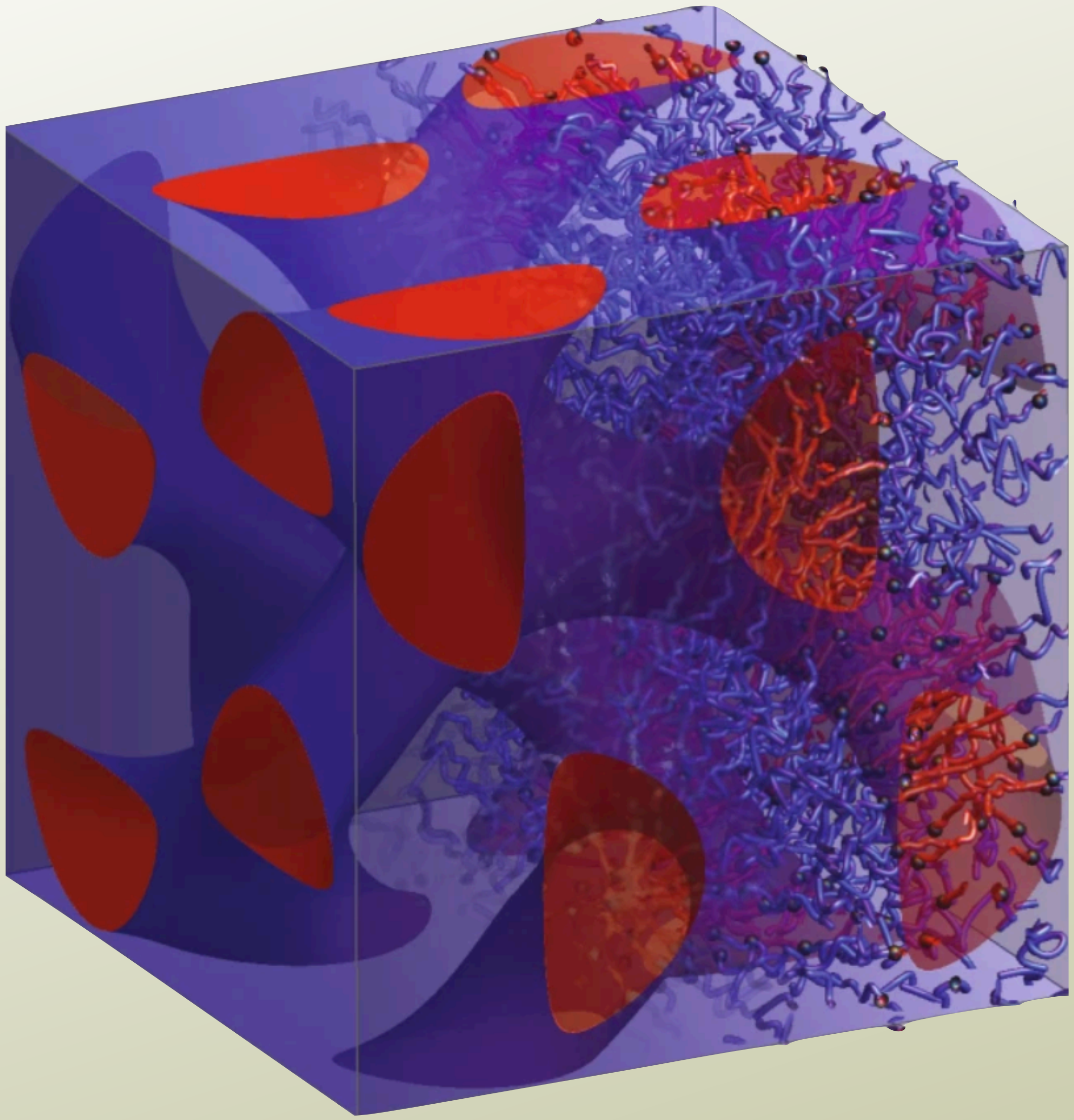
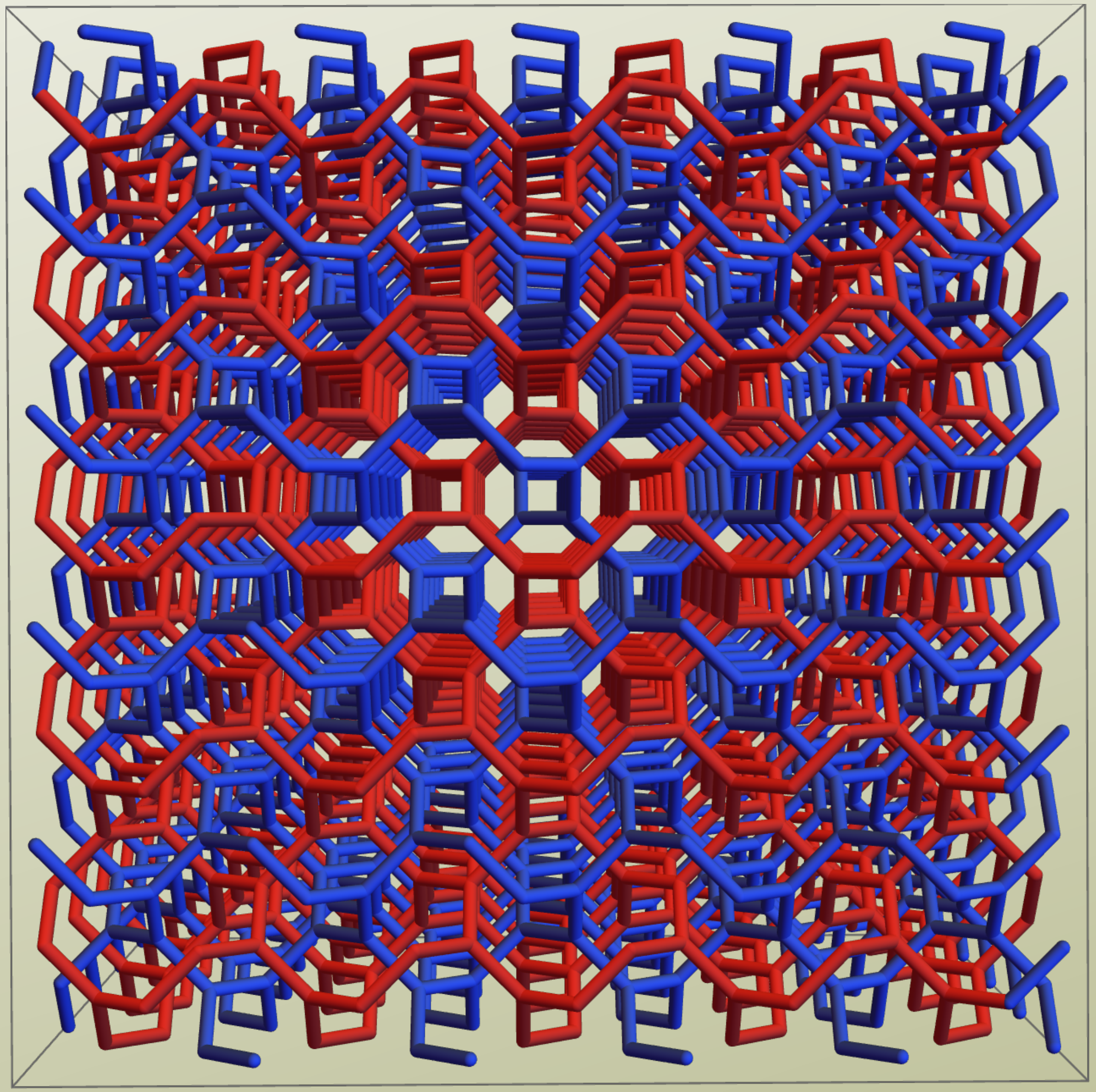
Diblock copolymers: Double Gyroid phase



Dimitriyev, *et al.* PRL (2024)



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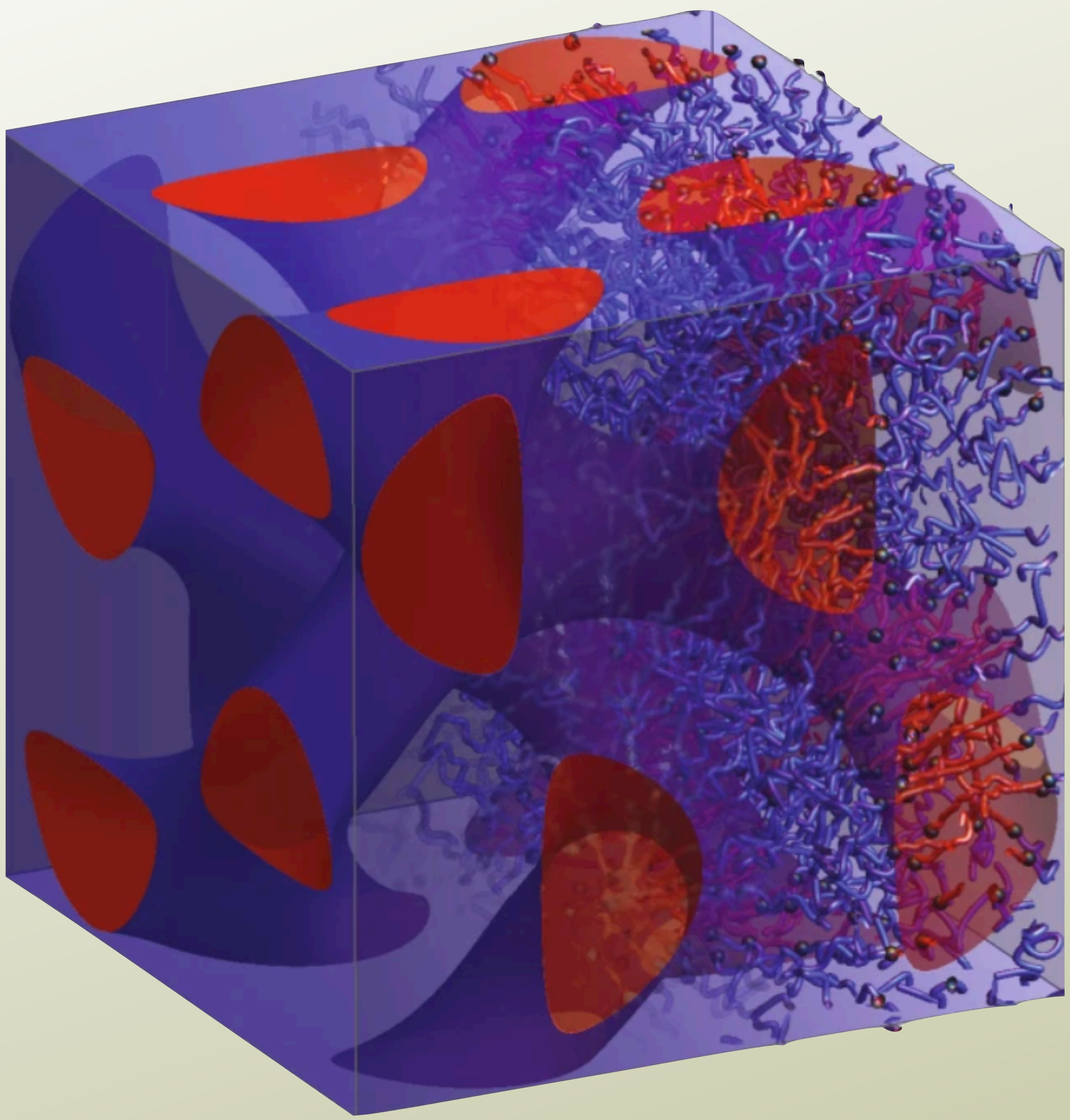
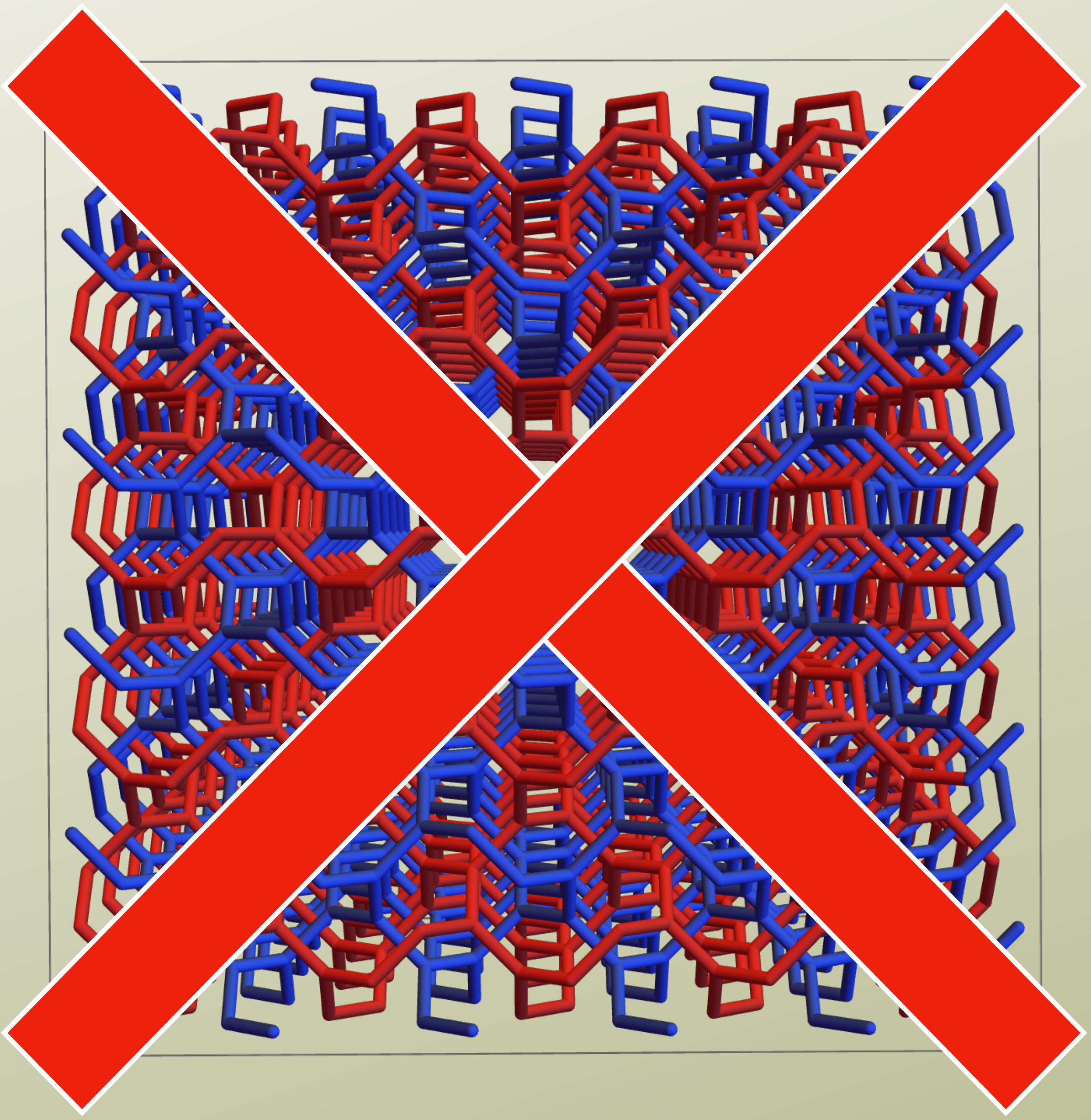
Diblock copolymers: Double Gyroid phase



Dimitriyev, *et al.* PRL (2024)



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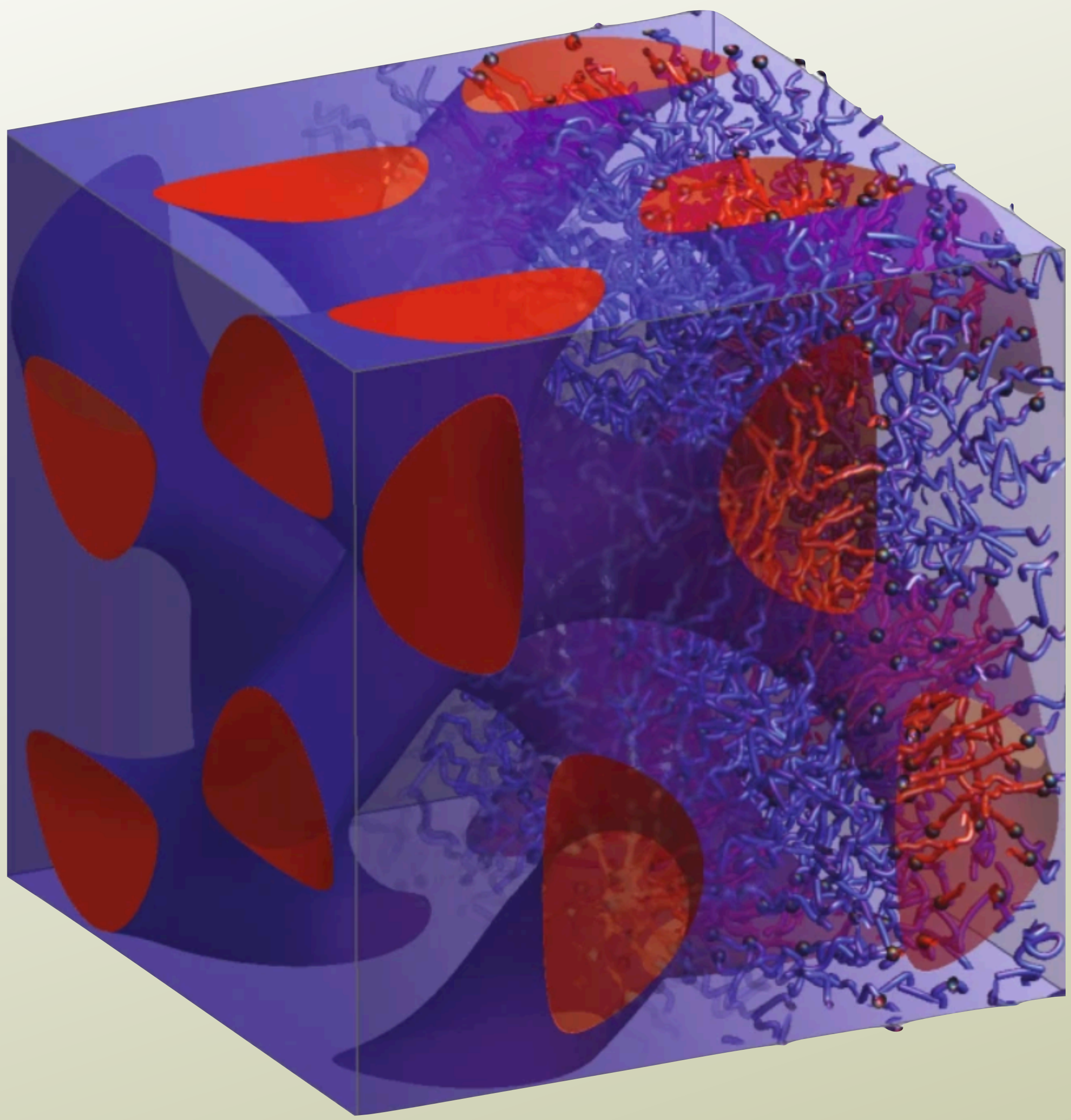
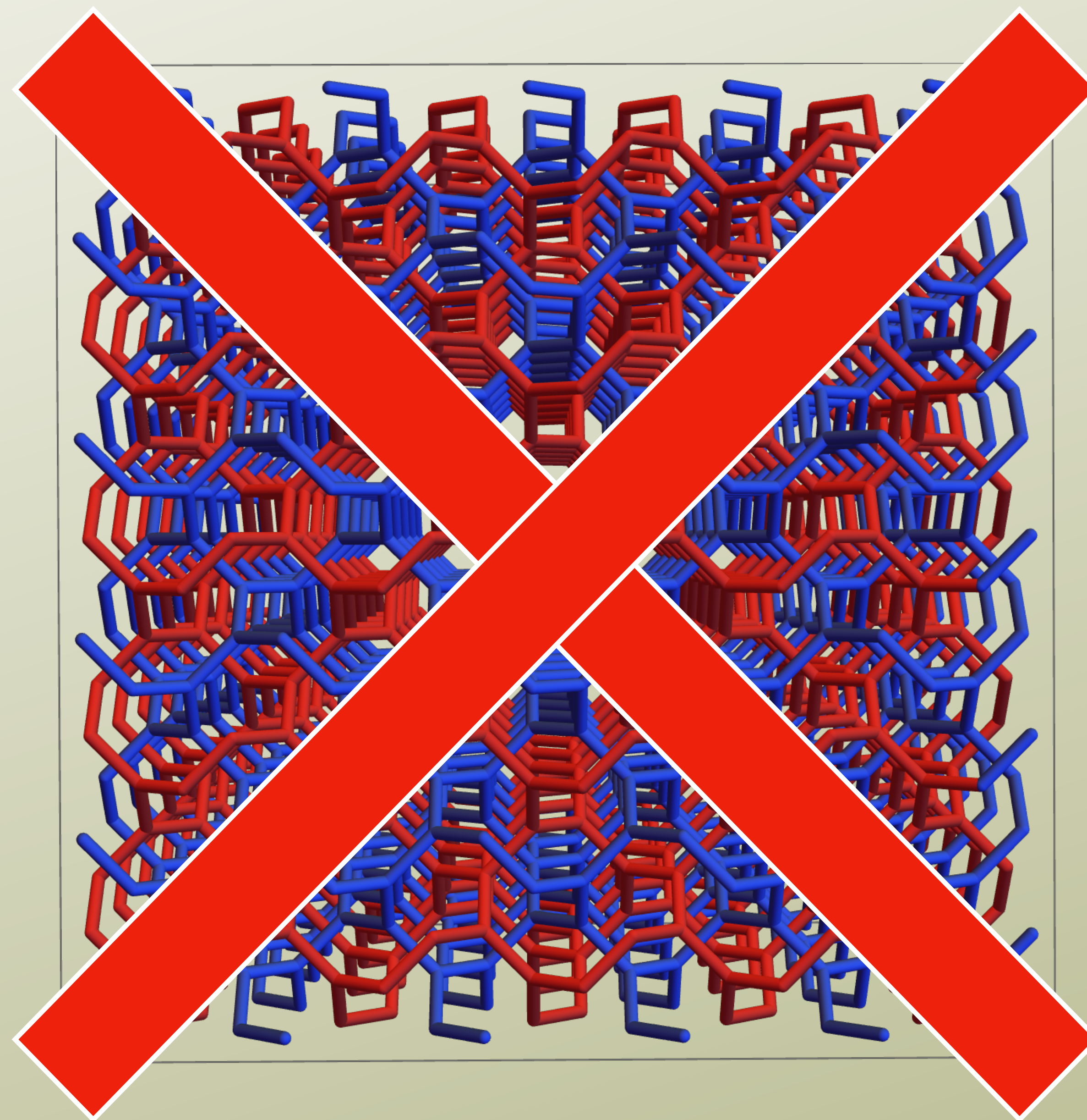
Diblock copolymers: Double Gyroid phase

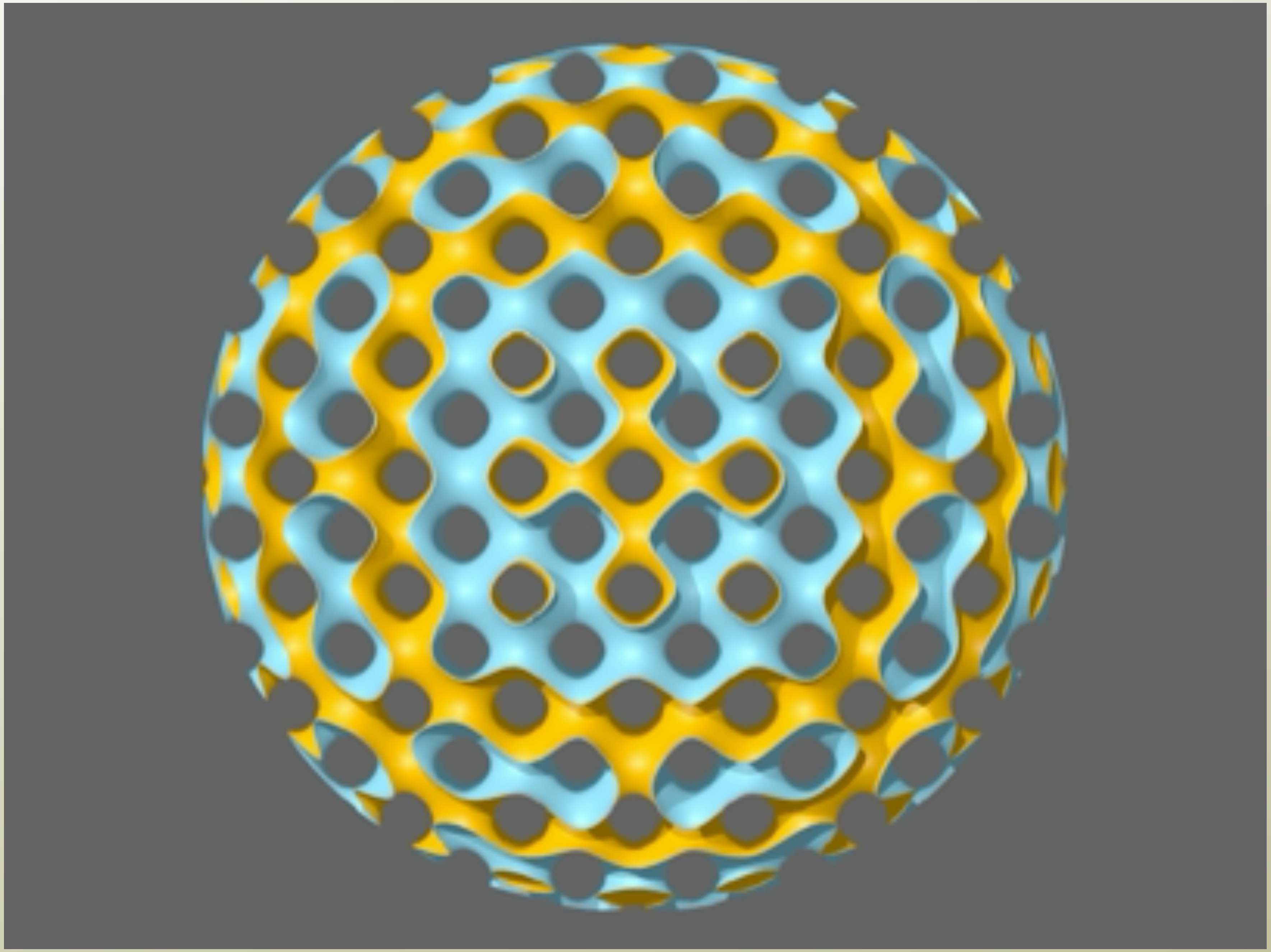


Dimitriyev, *et al.* PRL (2024)

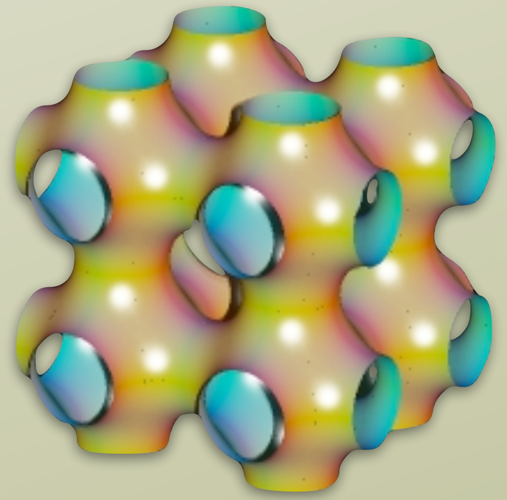


It is made of the same stuff!

Defects in Bicontinuous Phases

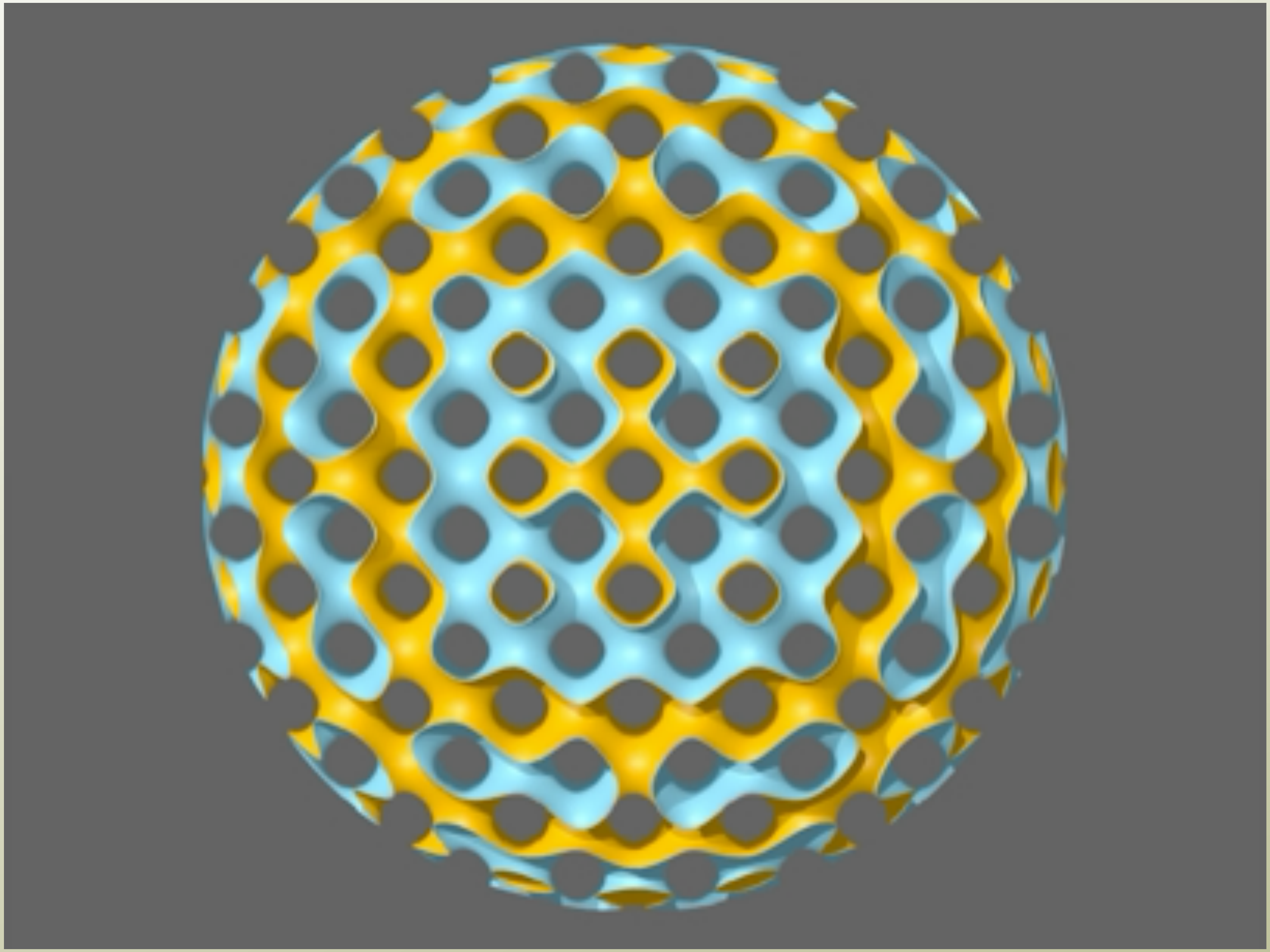


Pawel Pieranski

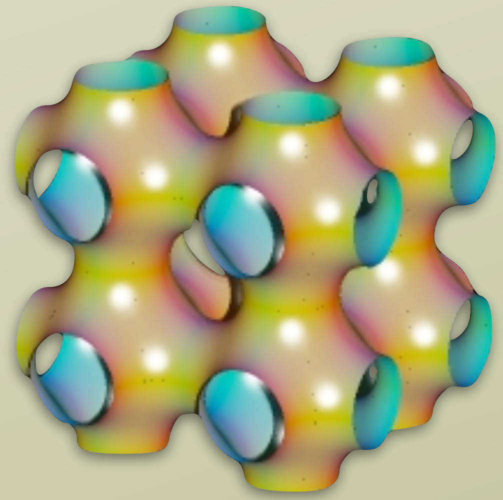


Penn

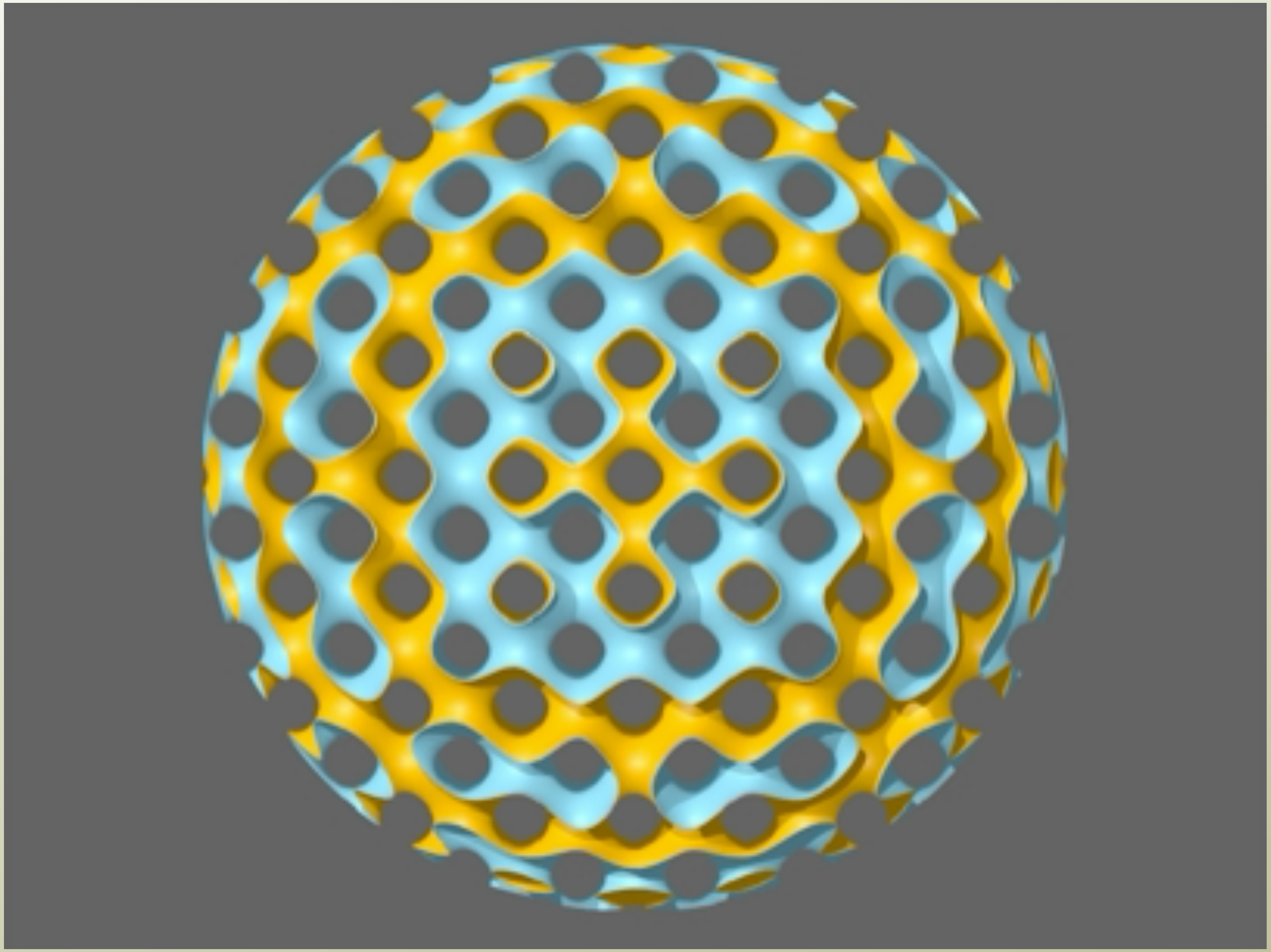
Defects in Bicontinuous Phases



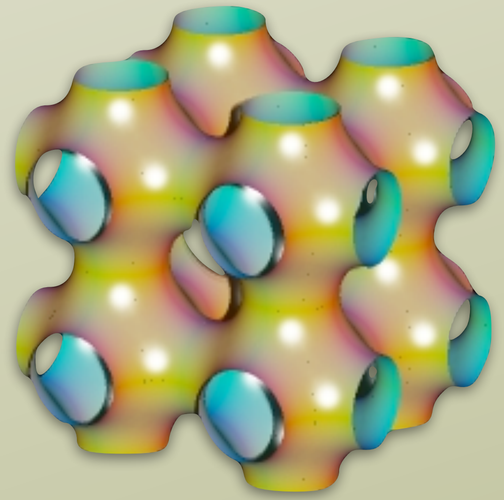
Pawel Pieranski

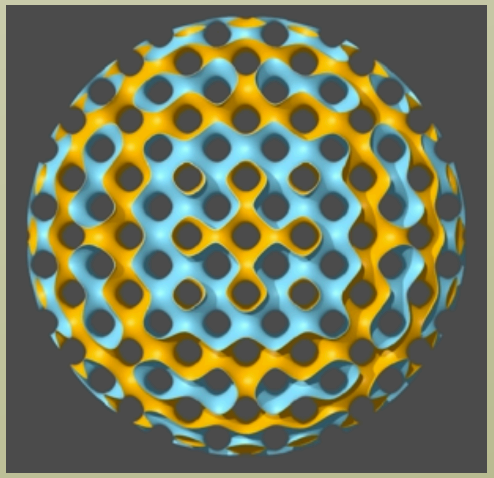


Defects in Bicontinuous Phases



Pawel Pieranski



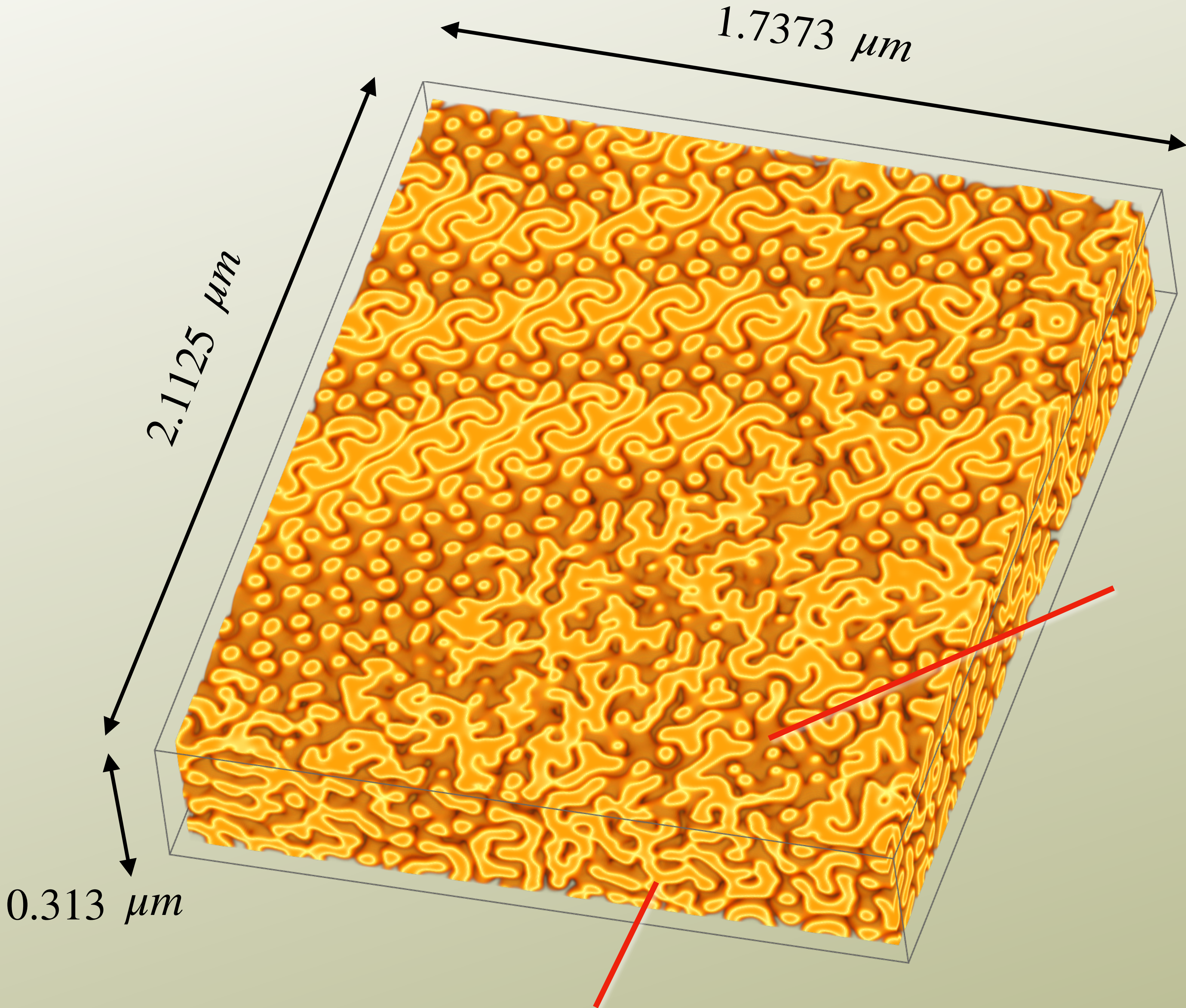


A Möbius defect

Penn

Raw data sample - the Mac & Cheese

Suman Kulkarni
Ned Thomas
Greg Grason
Mike Dimitryev



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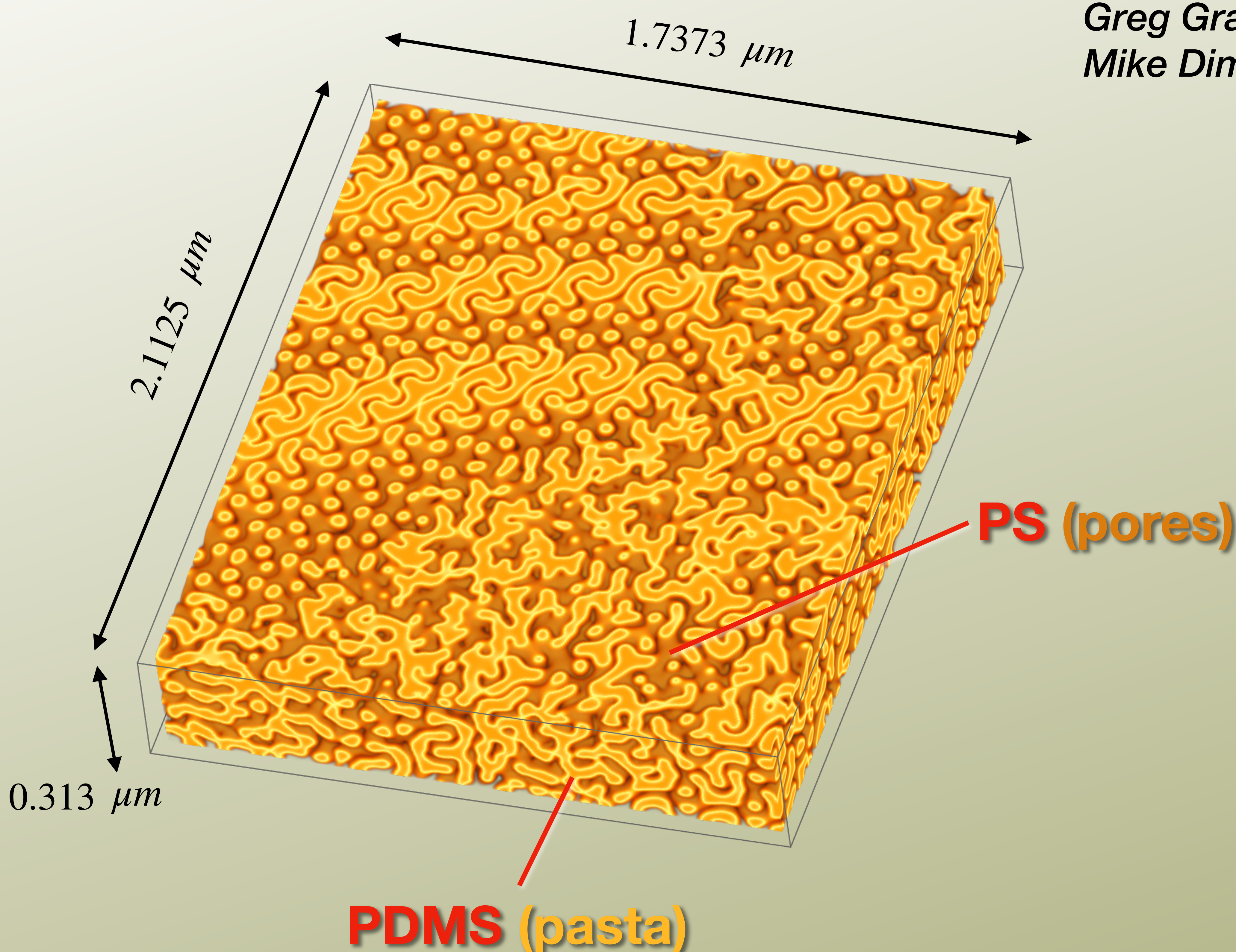

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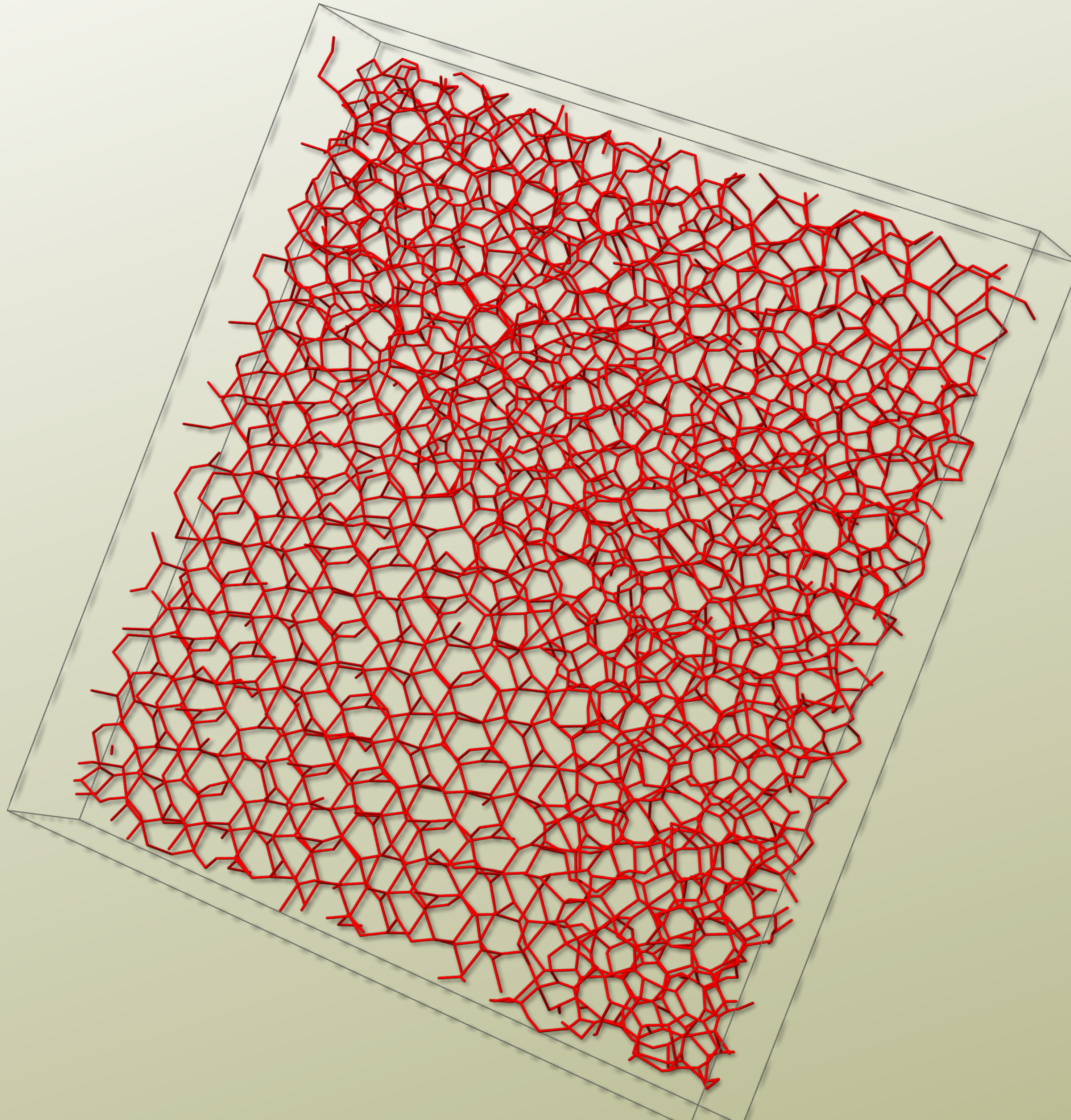
Skeletonization



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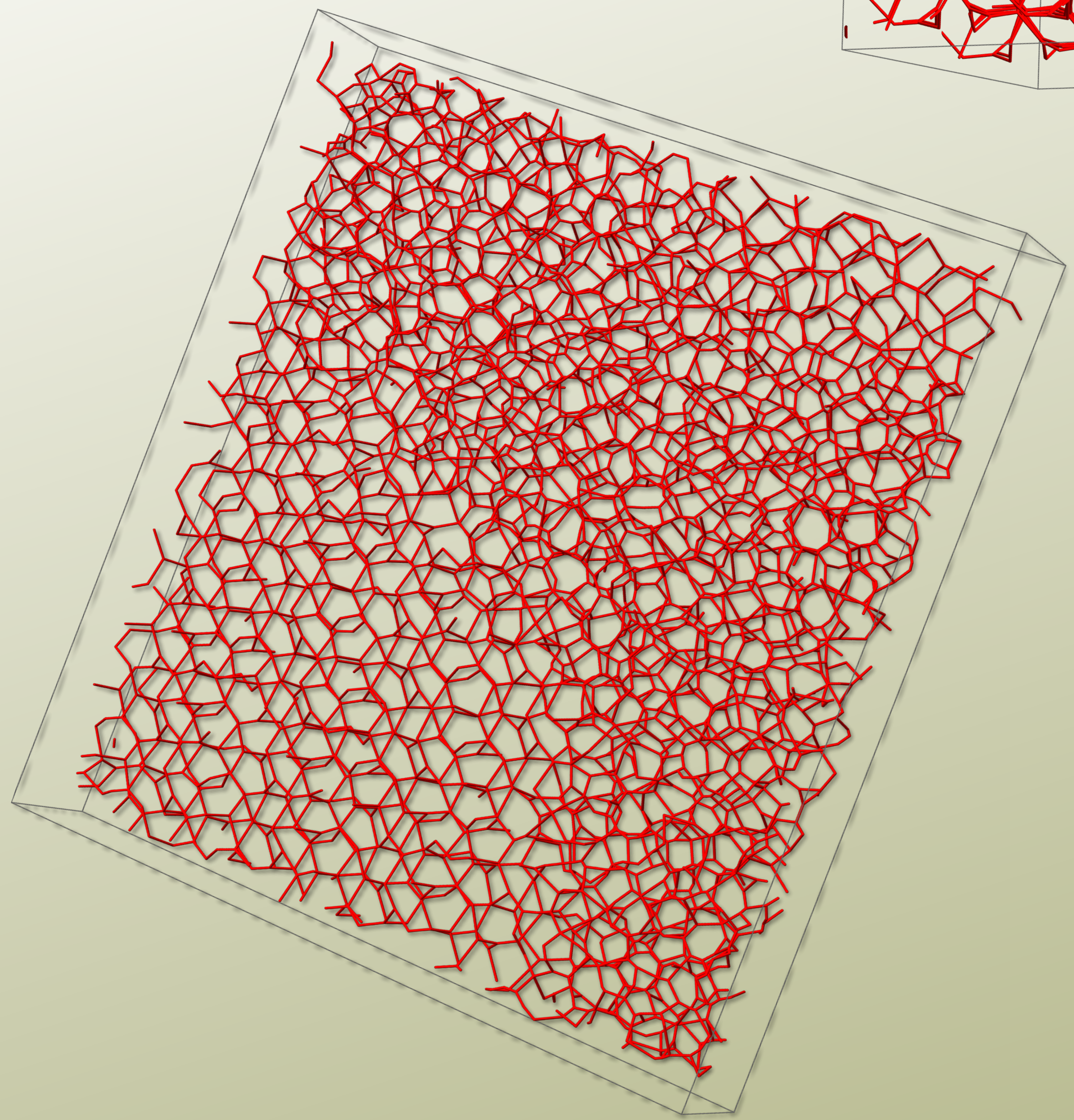
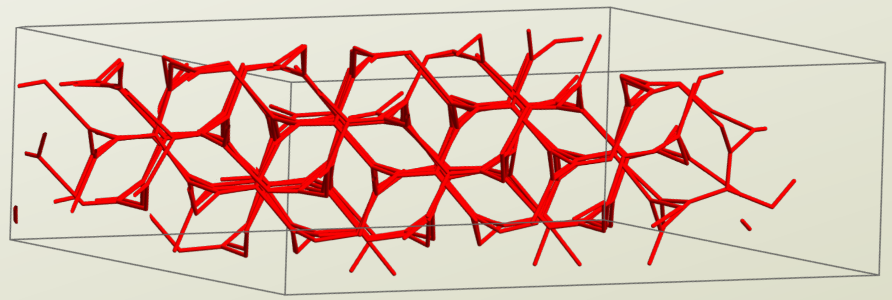
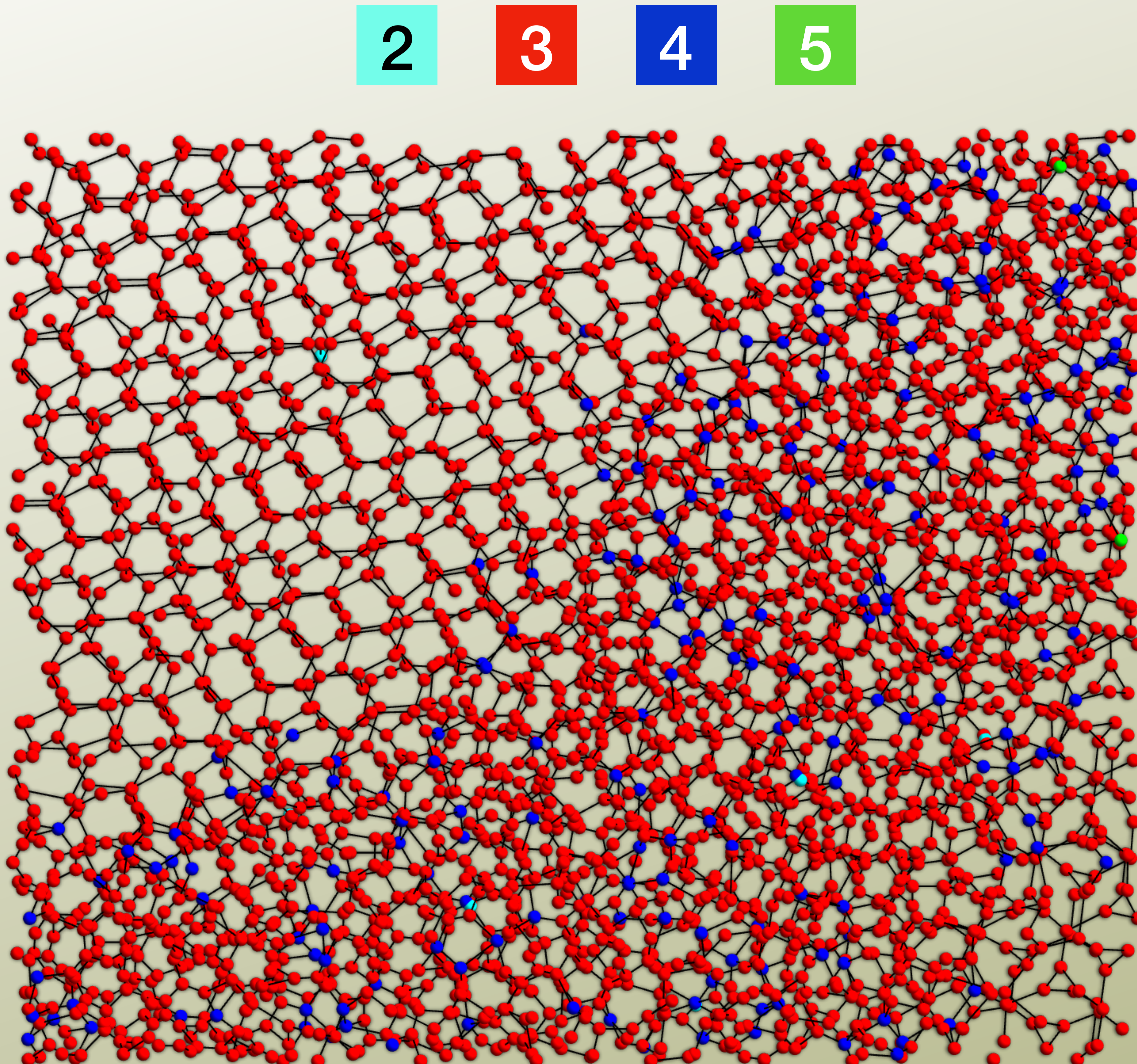
Skeletonization





side view

Coördination number



In the aperiodic region, about 90% of nodes have degree 3

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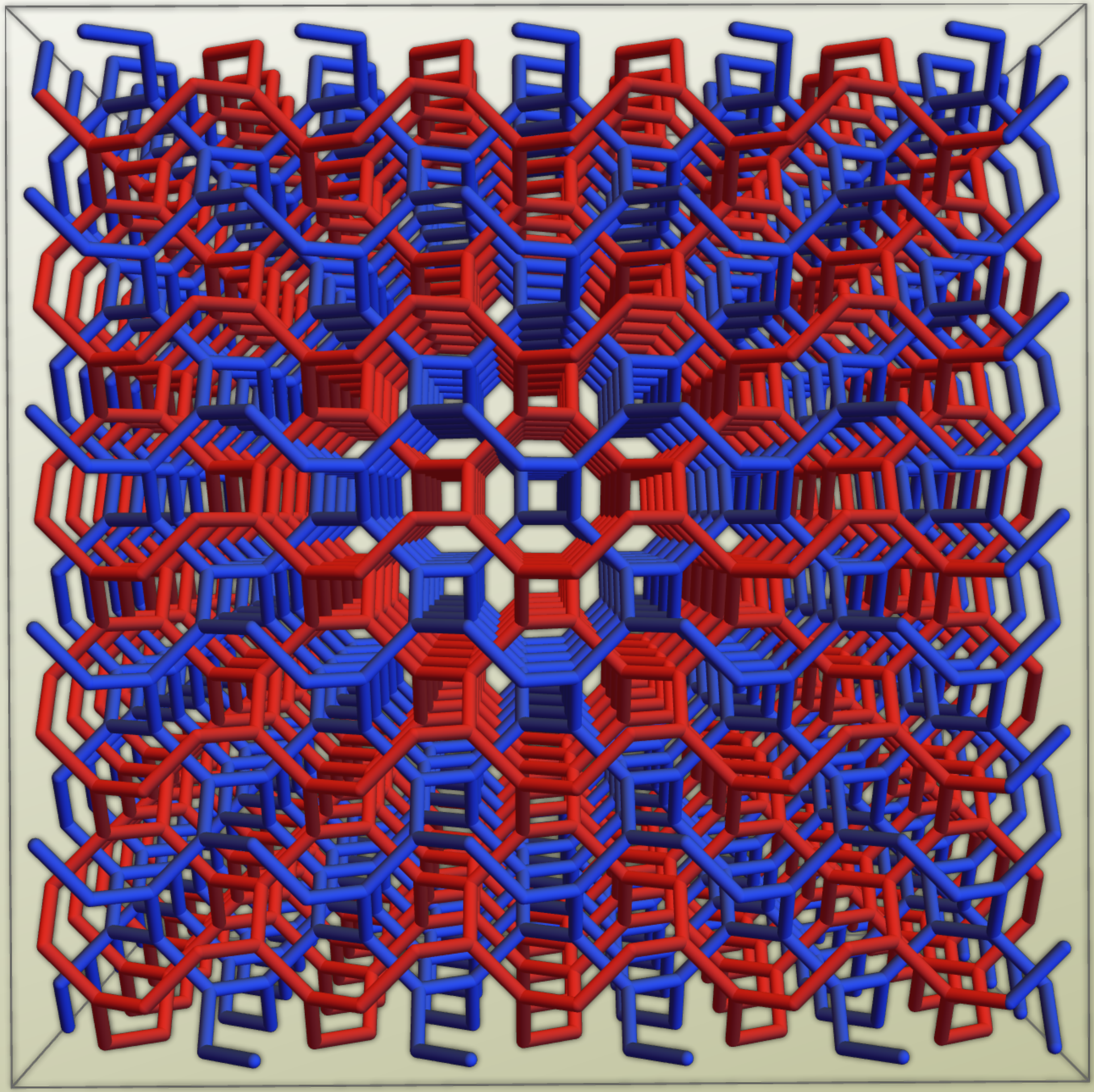
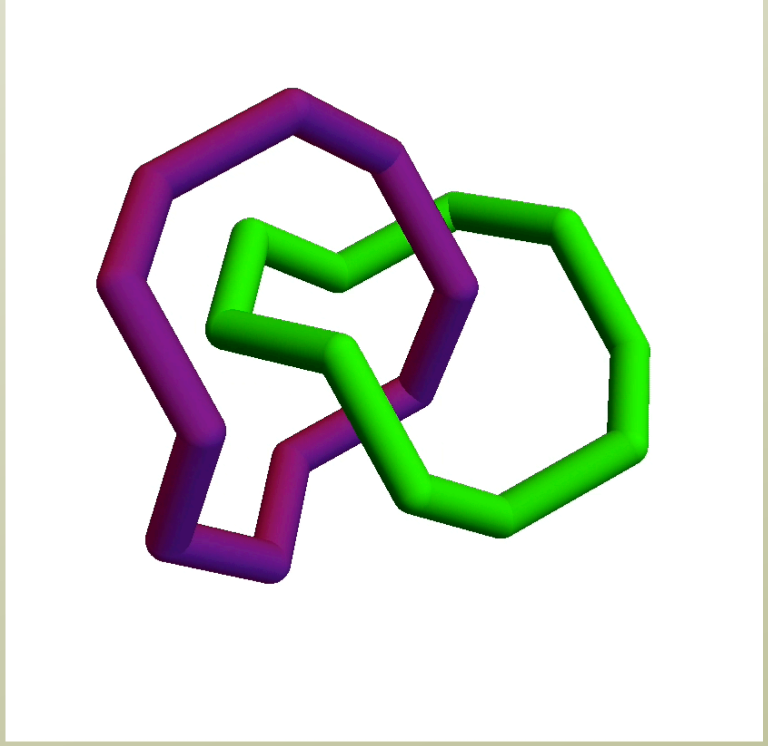
Loops in a double gyroid





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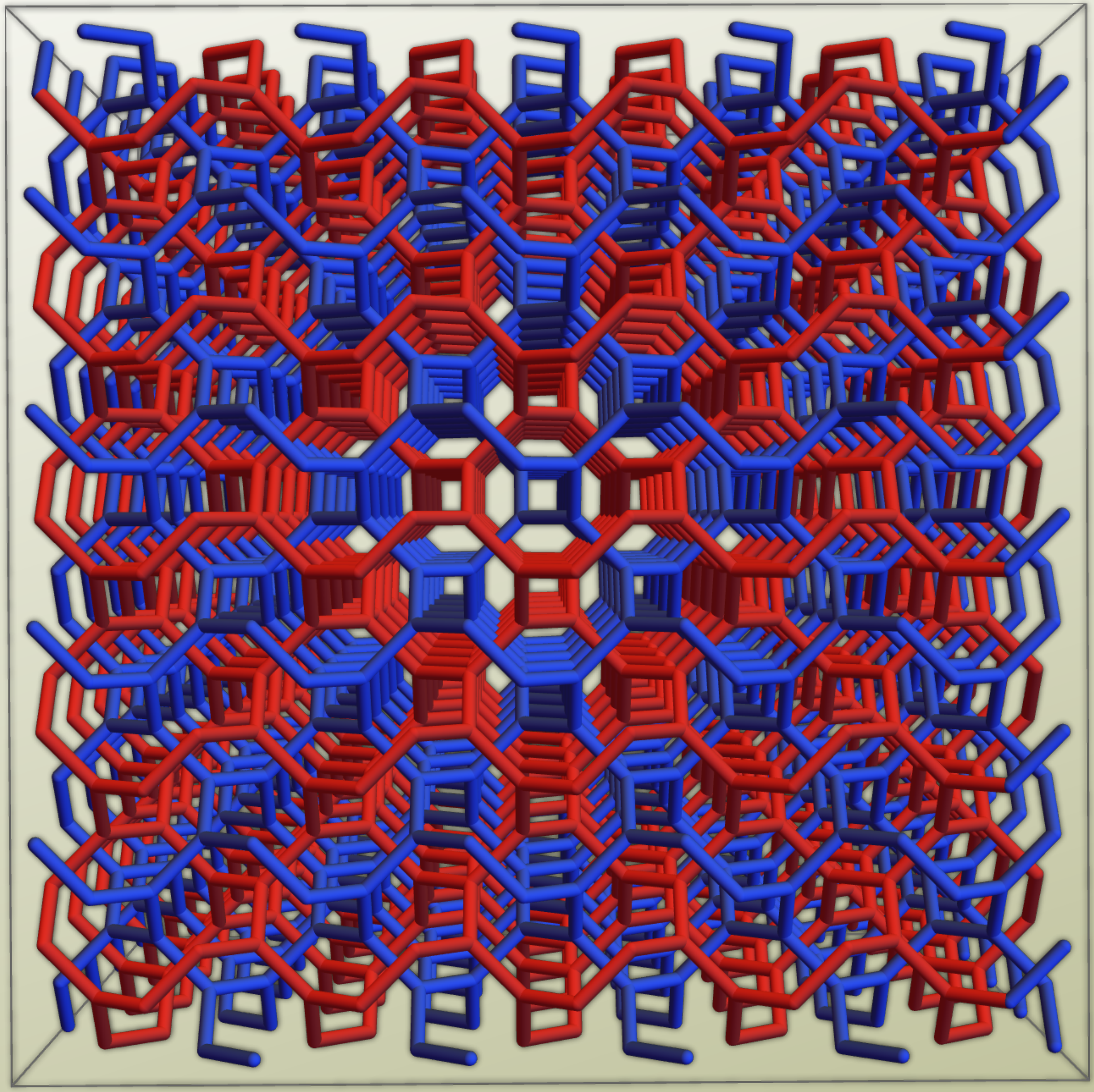
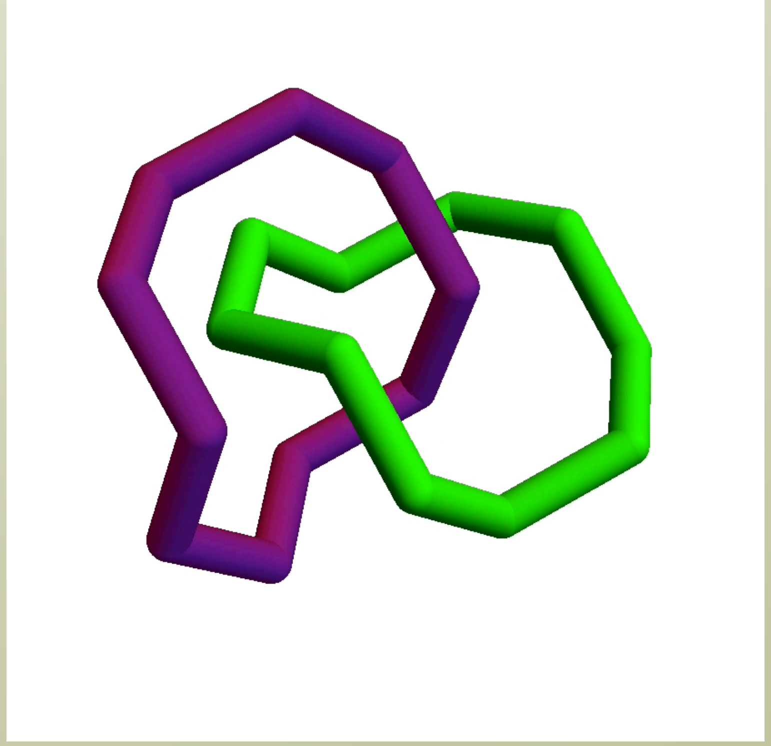
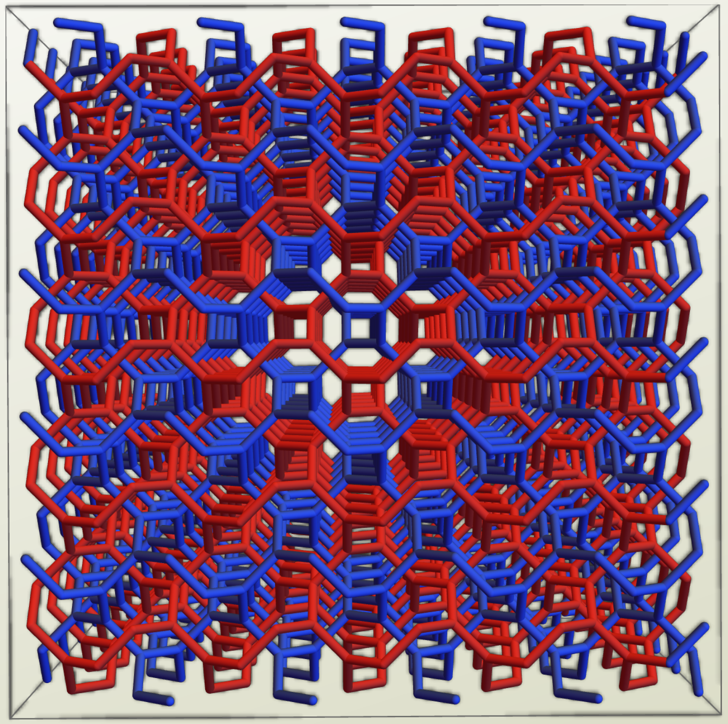
Loops in a double gyroid





Loops in a double gyroid



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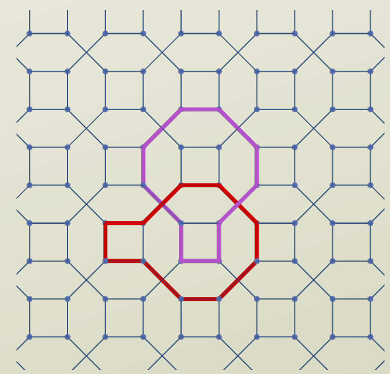

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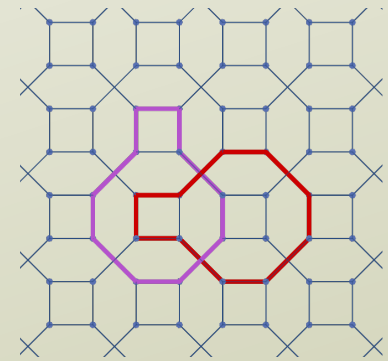

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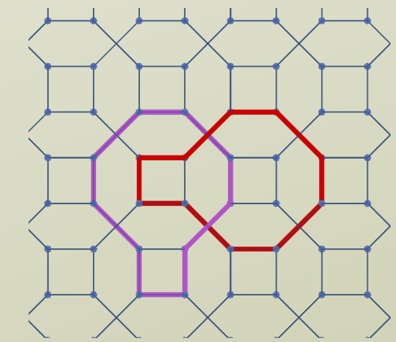

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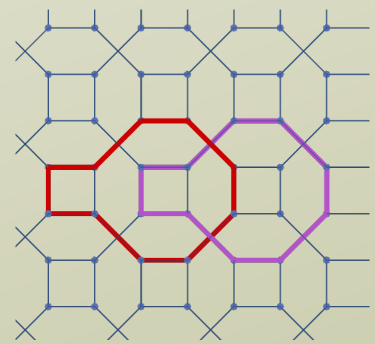

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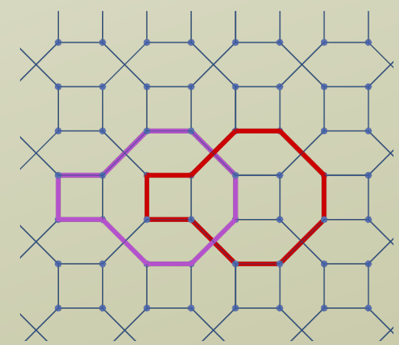

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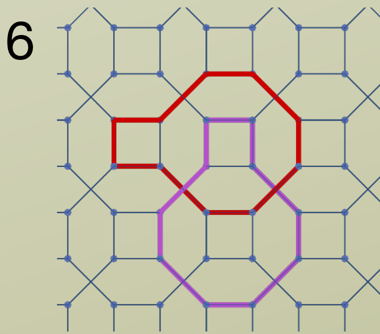

Each loop (within the bulk) is linked with 10 other loops

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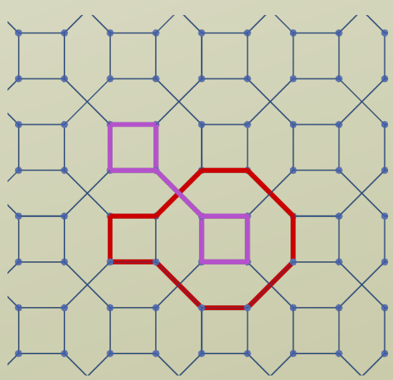

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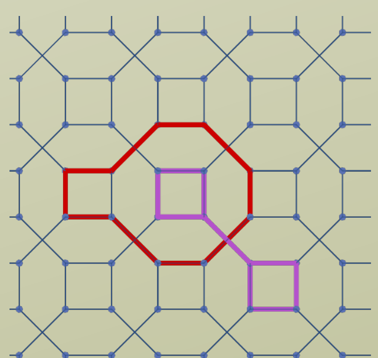

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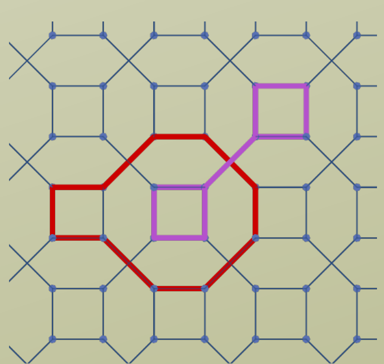

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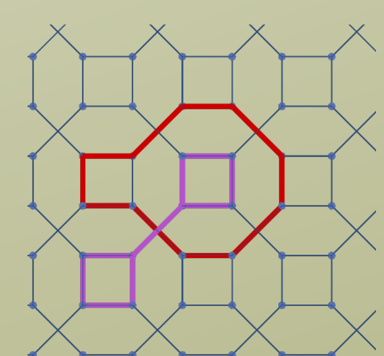
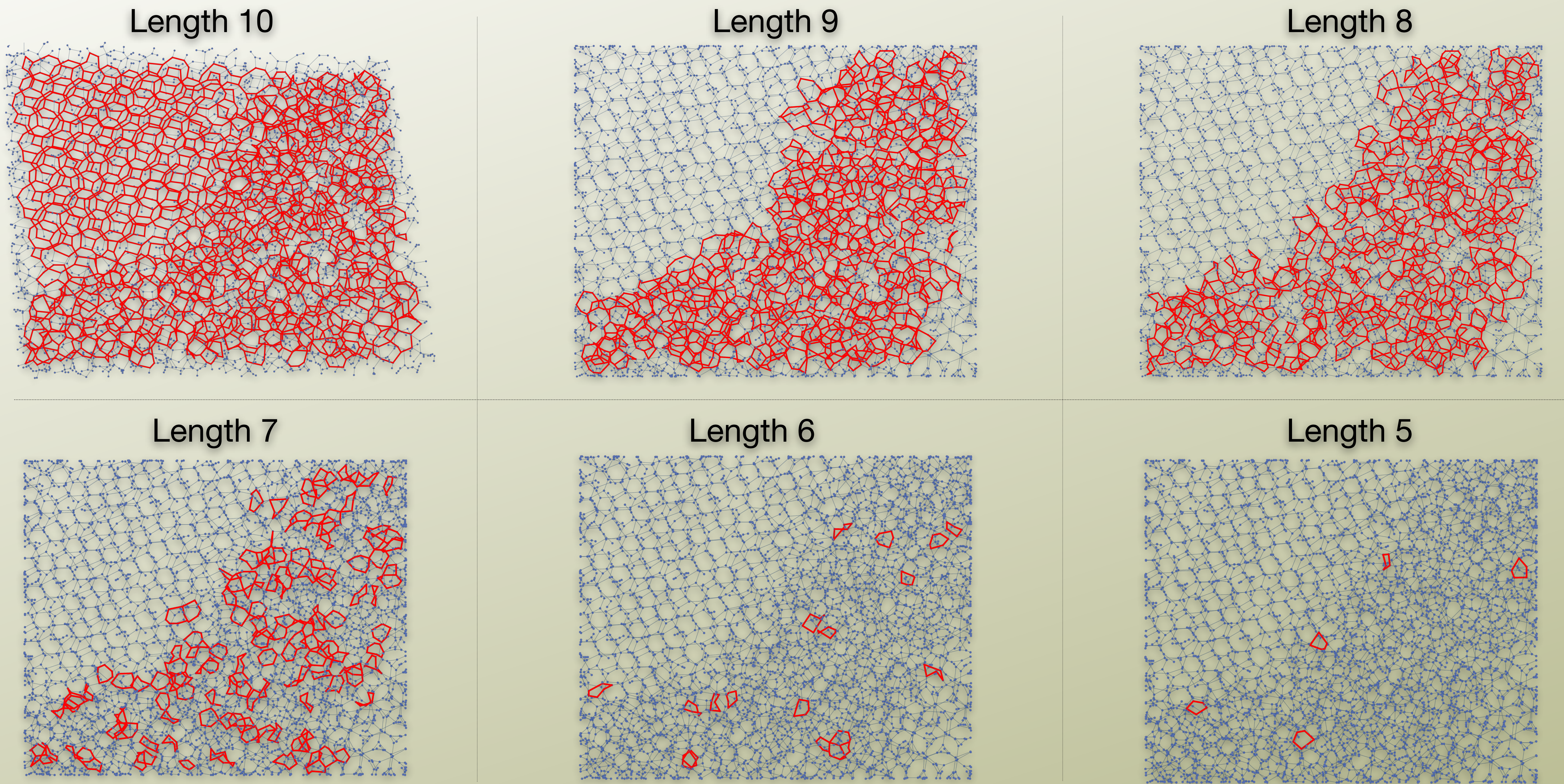
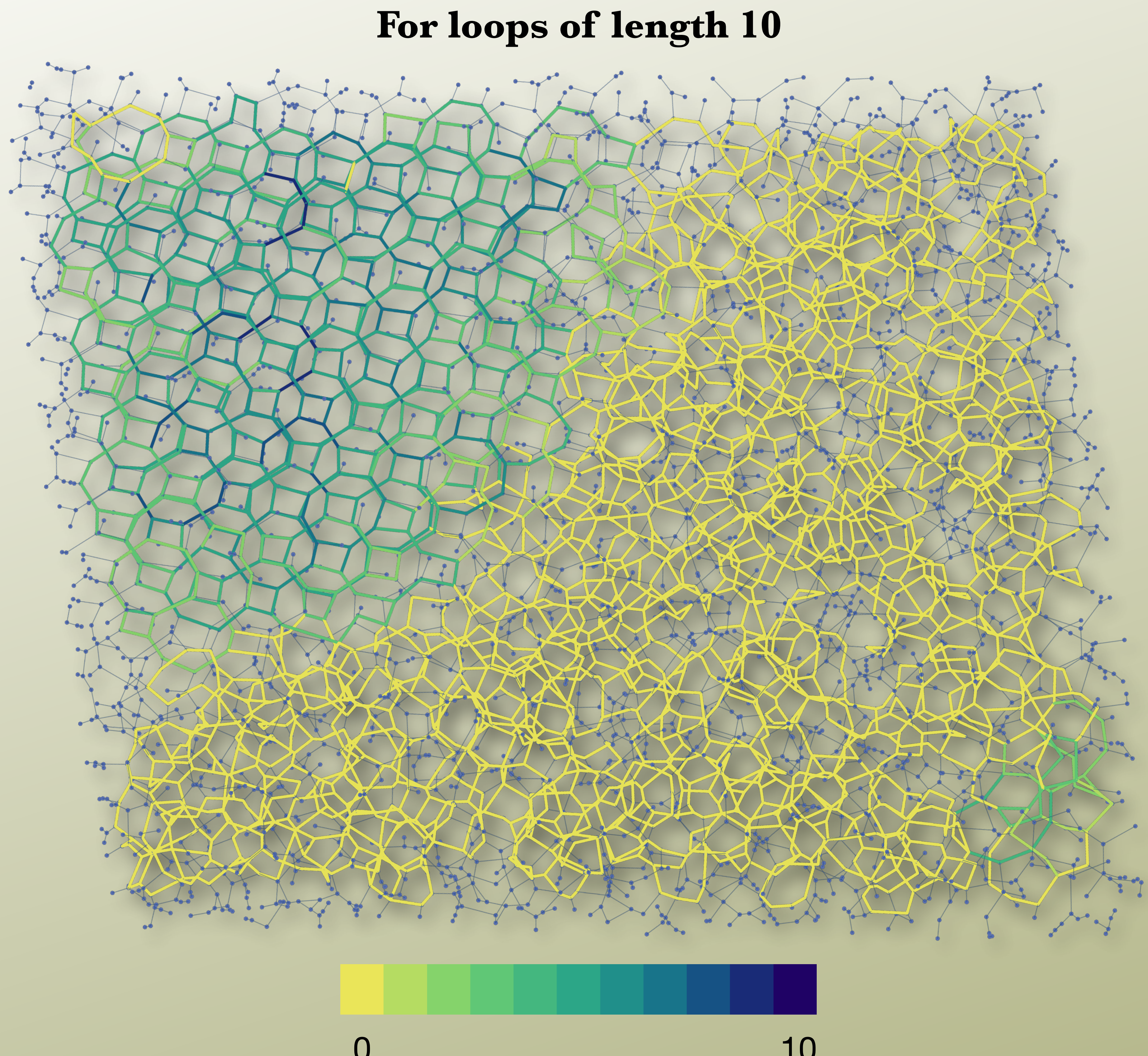


Loop Distribution



Linking Distribution



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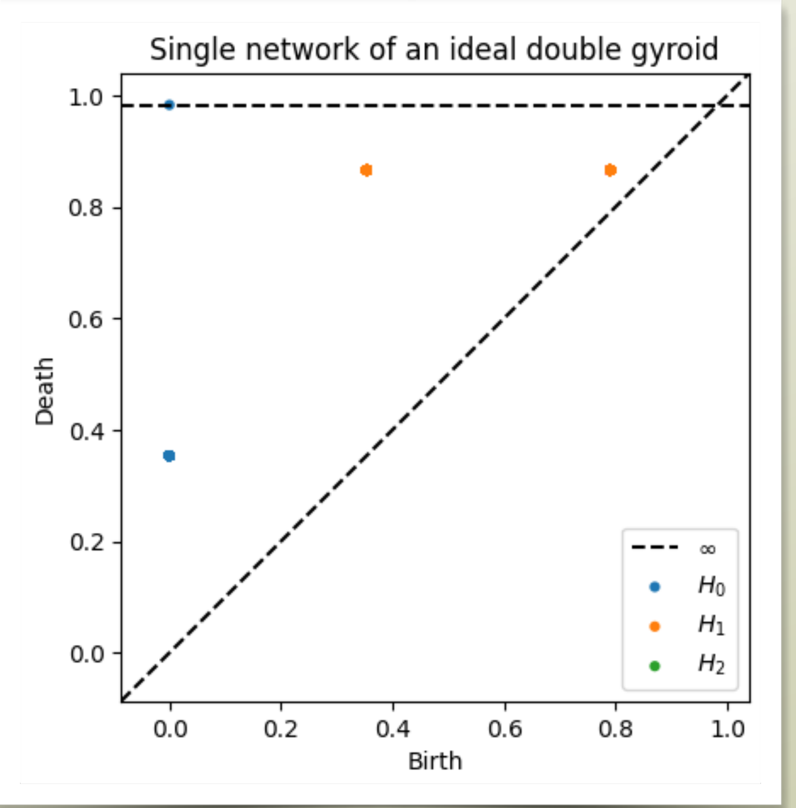
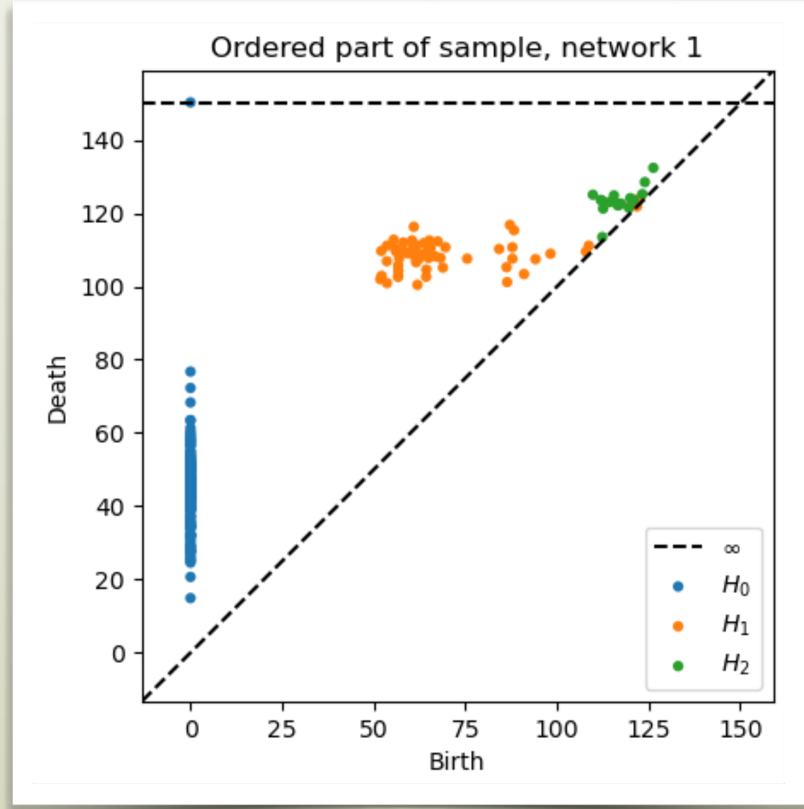
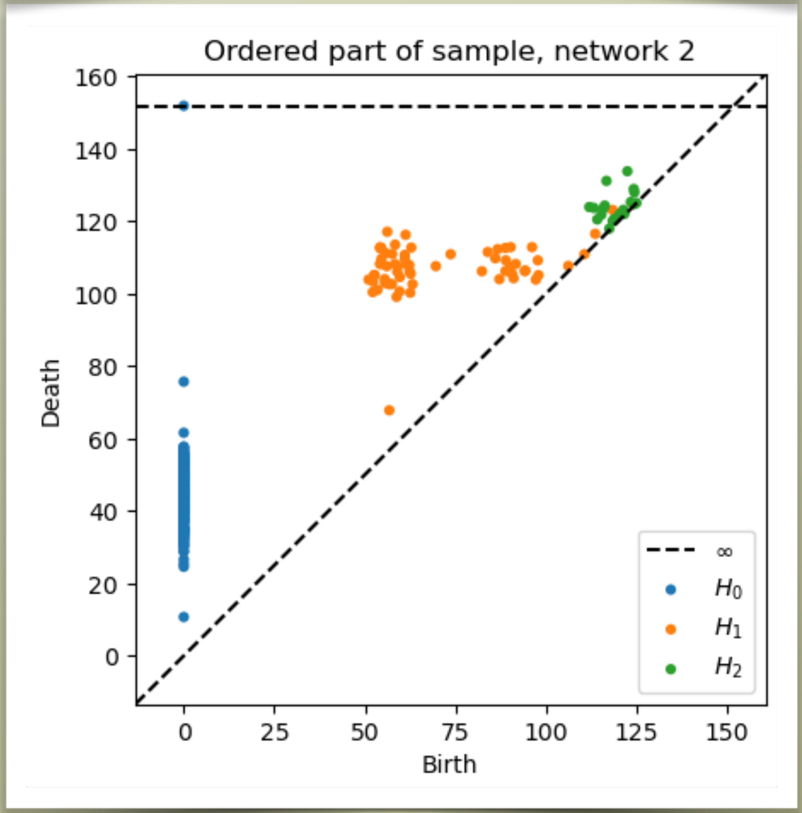
Persistent Homology







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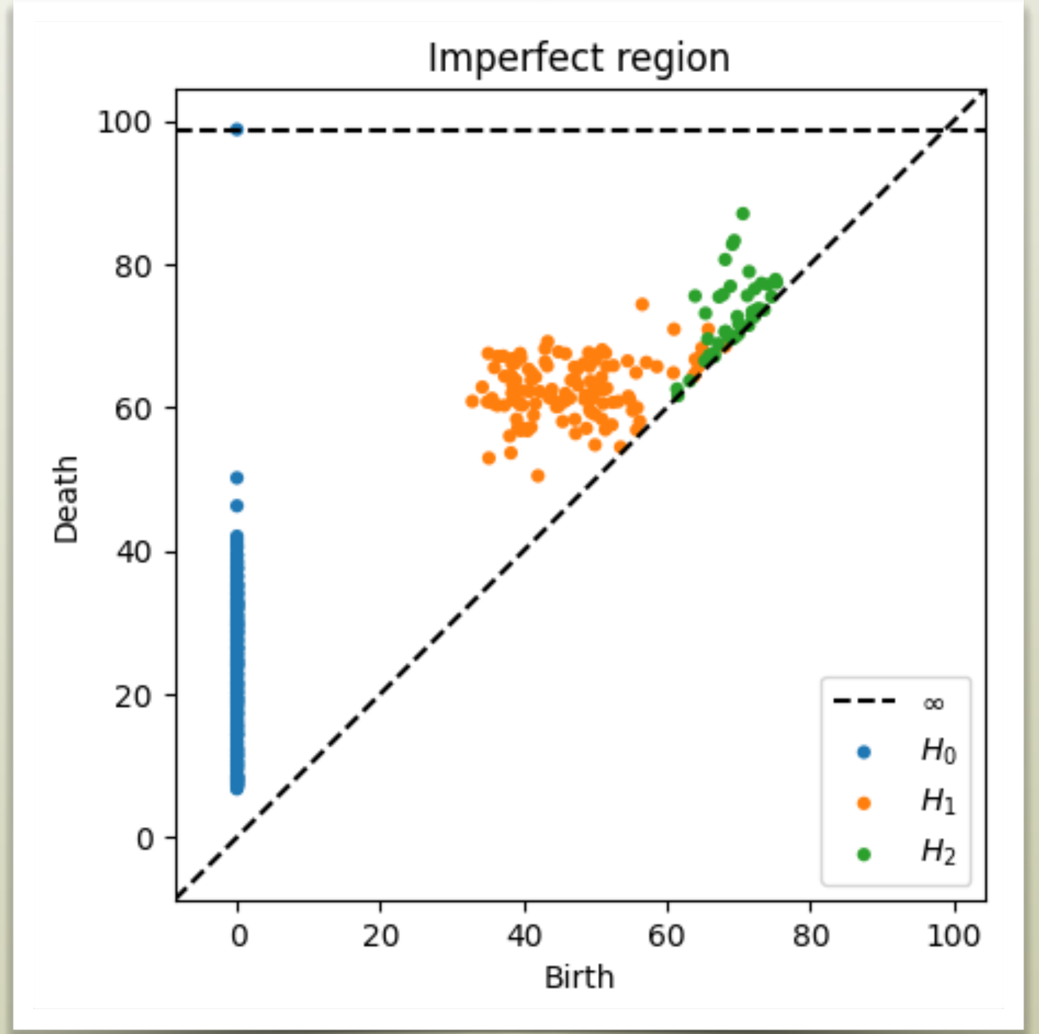
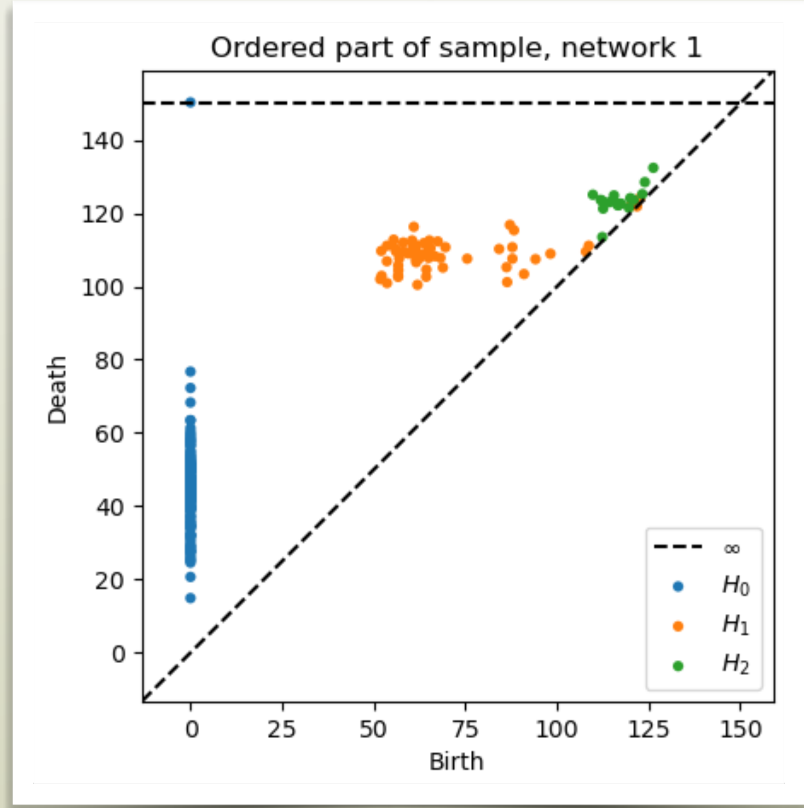
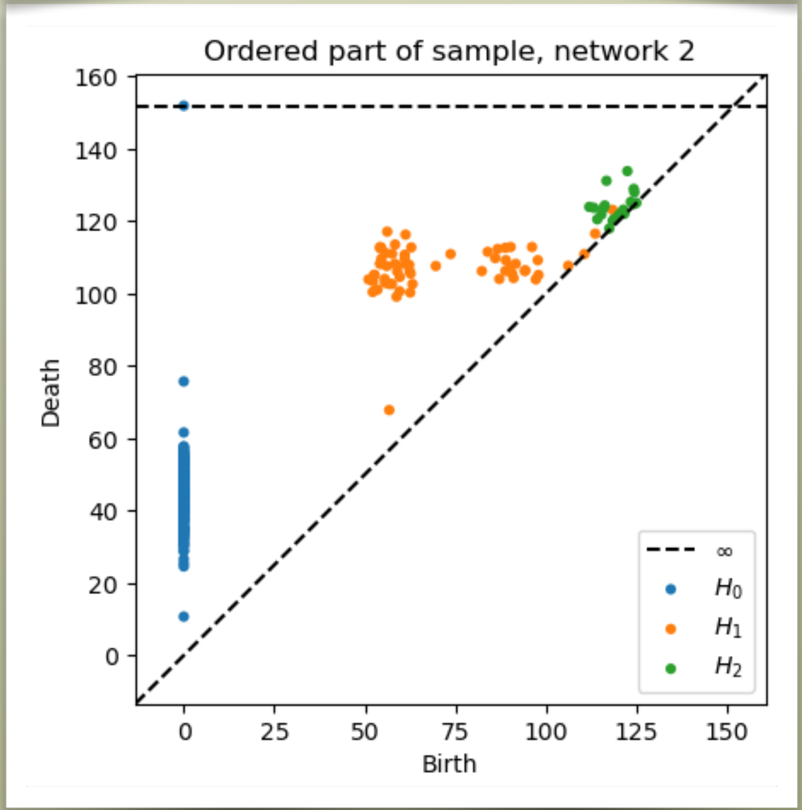
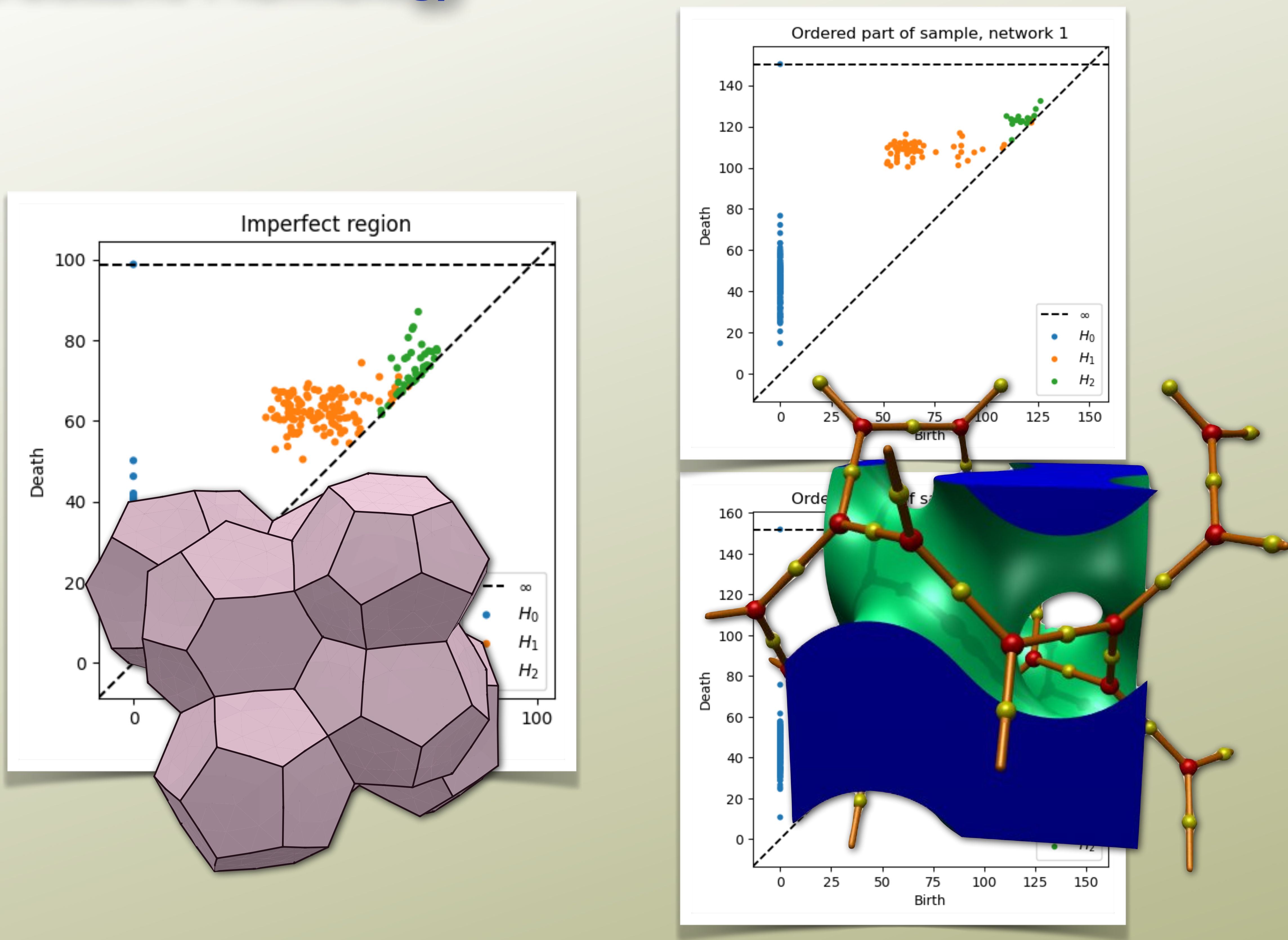






Persistent Homology



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