Some more problems

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1. Recall the definition of planes in general position from the discussion during the lecture. Let L be a set of $k+3$ planes in general position in $\mathbb{F}^3$, and let $S \subseteq \mathbb{F}^3$ be the set of points of intersections of subsets of size three of these planes. Show that S is an interpolating set for trivariate polynomials of degree k.

   Recall that S is an interpolating set if for every function $h : S \to \mathbb{F}$, there is a unique degree k trivariate polynomial P such that for every $a \in S$, $P(a) = h(a)$.

   Generalize this statement to hyperplanes in larger dimensional ambient space.

2. For every $\alpha \in \mathbb{F}$, let $v(\alpha) = (1, \alpha, \alpha^2)$ and $\lambda(\alpha) = \alpha^3$. Show that the planes $\{\Pi_\alpha : \alpha \in \mathbb{F}_q\}$ are in general position where $\Pi_\alpha := \{a \in \mathbb{F}^3 : \langle v(\alpha), a\rangle = \lambda(\alpha)\}$.

3. Use a random rotation of the planes constructed in the previous question to prove the lemma we used in the lecture - that the set U we had constructed has $10\varepsilon q$ planes in general position.

4. Let L be a set of $(1+\varepsilon)(k+3)$ planes in general position in $\mathbb{F}^3$ and let S be the points of intersections of subsets of size three of these planes. Let us consider the code defined as follows
$$C := \{(P(a))_{a \in S} : P \in \mathbb{F}[X], \deg(P) \leq k\}.$$
   What is the rate and distance of this code ? How does this compare to the rates and distance of bivariate Reed-Muller codes ?

5. Let $p_1, p_2, \ldots, p_n \in \mathbb{N}$ be n distinct prime numbers. Let $T_{d,n} = \{(p_1^i, p_2^i, \ldots, p_n^i) : i \in \{0, 2, \ldots, \binom{n+d}{d} - 1\}\}$. Show that for every non-zero n variate degree d polynomial $f(x_1 x_2, \ldots, x_n) \in \mathbb{C}[x_1, x_2, \ldots, x_n]$, there exists a point $\mathbf{b} = (b_1, b_2, \ldots, b_n) \in T_{d,n}$ such that $f(\mathbf{b}) \neq 0$.