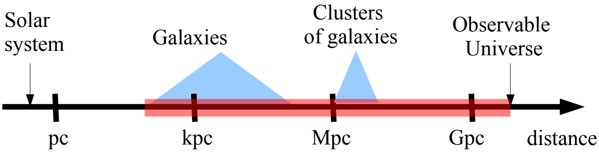
Stochastic Particle Production

Raghuveer Garani

What is dark matter?

Experimental evidences



What is dark matter?

The problem

DARK MATTER

$J =$?

Mass $m =$?
Mean life $\tau =$?

DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
?	?	?	?

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- No electric charge, no colour charge (Smith et al. '79, Perl et al. '01).
- Non-relativistic at the time of formation of the first structures (White, Frenk, Davis '83).
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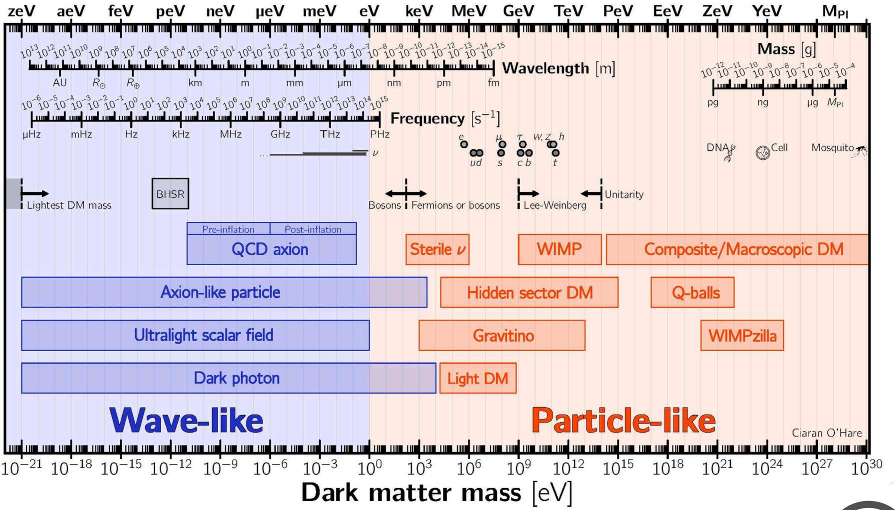
$\Longrightarrow$ Evidence for new particles/phenomena.

What is dark matter?

Plentitude of candidates but mass scale unknown!



What is dark matter?

How dark can dark matter be?

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How dark can dark matter be?

- All evidences $\rightarrow$ only gravitational interactions!

How to gravitationally produce dark matter?

Mechanisms: compute $\Omega_{\rm DM}$

- Gravitational Freeze-in Garny, Sandora, Sloth '16; Bernal, Dutra, Mambrini, Olive, Peloso '18
- Gravitational particle production Parker '69; Ford '76; Chung, Kolb, Riotto '98, Ema, Nakayama, Tang '16; Long, Kolb '24
- From perturbations:

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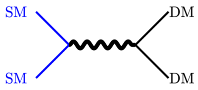
Gravitational Freeze-in



$$\mathcal{A} = \frac{1}{M_p^2 s}\left(T^{\rm SM}_{\mu\nu}T^{\rm DM}_{\alpha\beta}\eta^{\mu\alpha}\eta^{\nu\beta} - \frac{1}{2}T^{\rm SM}T^{\rm DM}\right)$$

- SM particles annihilate to DM: $\frac{1}{M_{\rm pl}} h_{\mu\nu} T^{\mu\nu}$
- Yield is controlled by reheating temperature:
$$Y_{\rm D} = 6 \times 10^{-6}\, c_D \left(\frac{T_R}{M_{\rm pl}}\right)^3 .$$
- $\Omega_{\rm DM} \approx 10^{-5} \frac{M}{3H_0^2 M_{\rm pl}^2}\left(\frac{T_R T_0}{M_{\rm pl}}\right)^3$

How to gravitationally produce dark matter?

Gravitational Freeze-in

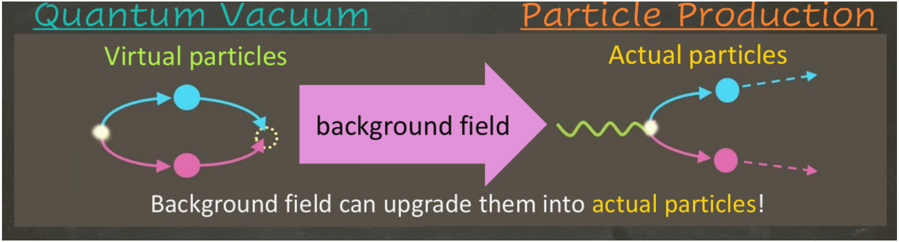
- Universal, depends only on central charge

$$M_{\rm DM} \sim \frac{1}{c_{\rm D}} 10^6 \mathrm{GeV} \left(\frac{10^{15}\mathrm{GeV}}{T_R} \right)^3$$

- Typical candidate Glueball Dark Matter Faraggi and Pospelov '02

Particle production from background fields

Example: Schwinger pair production

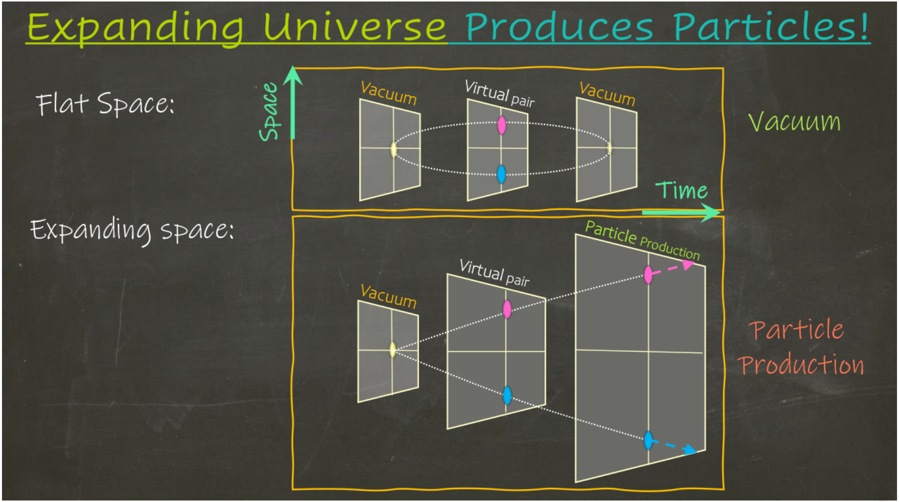


If work done by Lorentz force: $eE\lambda_{\text{compton}} = m_e$

$\implies$ Electron pairs created for $E \sim m_e^2 \approx 10^{18}$ V/m

Particle production from background fields

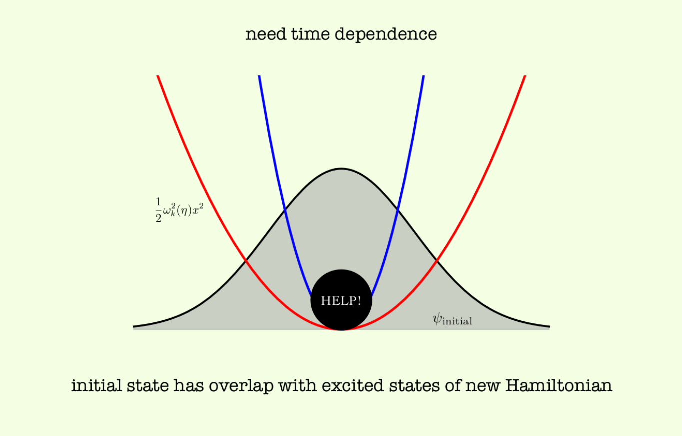
Background expansion



Credit: Azadeh Maleknejad

Gravitational Particle production

Building intuition: The harmonic oscillator



Credit: Andrea Tesi

Gravitational Particle production

Building intuition: The harmonic oscillator

- $\hat{H}(\tau) = \frac{1}{2}\hat{p}^2 + \frac{1}{2}\omega^2(\tau)\hat{x}^2$ $\qquad \hat{x} = \nu(\tau)a + \nu(\tau)a^\dagger$
- At all times $\ddot{\nu} + \omega^2(\tau)\nu = 0$ $\qquad \dot{\nu}\nu^\star - \dot{\nu}^\star\nu = -i$
- Bunch-Davis $a|0\rangle = 0$ and $\qquad \nu_0 = \frac{1}{\sqrt{2\omega}}e^{-i\omega\tau}$
- $\tau \to \infty$

$$\nu = \frac{\alpha(\tau)}{\sqrt{2\omega(\tau)}}e^{-i\omega\tau} + \frac{\beta(\tau)}{\sqrt{2\omega(\tau)}}e^{i\omega\tau}$$

Parker '69; Ford '76

Gravitational Particle production

Building intuition: The harmonic oscillator

- Occupation number per mode

$$\langle 0|a_k^\dagger a_k|0\rangle = |\beta_k(\tau)|^2$$

- Energy and number density

$$\frac{dn}{d\log k} = \frac{k^3}{2\pi^2}|\beta_k(\tau)|^2 \quad \frac{d\rho}{d\log k} = \frac{k^4}{2\pi^2}|\beta_k(\tau)|^2$$

- For QFT in curved space $\rightarrow$ gravitational particle production

How to gravitationally produce dark matter?

Gravitational particle Production: Massive fermion

- Massive fermion conformally coupled to the metric
- $ds^2 = a(\tau)^2 \eta_{\mu\nu} dx^\mu dx^\nu$
- $g_{\mu\nu}(x) \to \Omega^2(x) g_{\mu\nu}(x)$
- $\mathscr{L} = i\bar{\psi}\bar{\sigma}^\mu(\partial_\mu + \omega_\mu)\psi - \frac{M}{2}(\psi\psi + \bar{\psi}\bar{\psi})$ Wess and Bagger
- Rescale $\psi \to \chi/a^{3/2}$

Gravitational Particle Production

Massive fermion

- $i\bar{\sigma}^{\mu}\partial_{\mu}\chi = Ma\bar{\chi}$
- $\chi_0(\tau,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left[u_{\vec{k}}(\tau)e^{+i\vec{k}\cdot\vec{x}}a_{\vec{k}} + v_{\vec{k}}(\tau)e^{-i\vec{k}\cdot\vec{x}}b^{\dagger}_{\vec{k}}\right]$
- Mode equation:

$$\partial_{\tau}^2\nu_k + k^2\nu_k + a^2(\tau)M^2\nu_k = 0$$

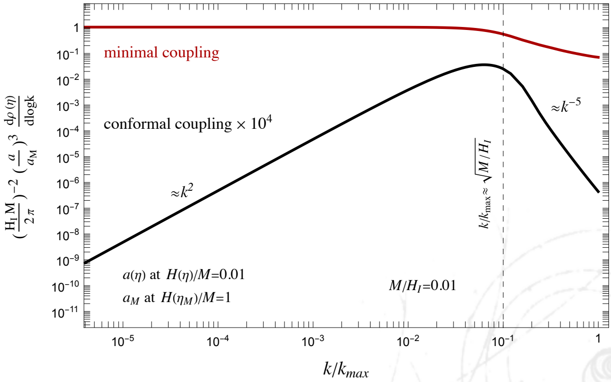
- Characteristic frequency $\omega^2(\tau) = k^2 + M^2a^2(\tau)$.
- Recall Energy and number density

$$\frac{dn}{d\log k} = \frac{k^3}{2\pi^2}|\beta_k(\tau)|^2 \quad \frac{d\rho}{d\log k} = \frac{k^4}{2\pi^2}|\beta_k(\tau)|^2$$

Gravitational Particle Production

Massive fermion

- Production peaked $k/a_M \sim H \sim M \implies$ suppressed by mass
- $\Omega_{\rm DM} = 10^{-2}\frac{Mk_M^3}{3M_{\rm pl}^2 H_0^2}$



Redi, Tesi '23; Long, Kolb '23

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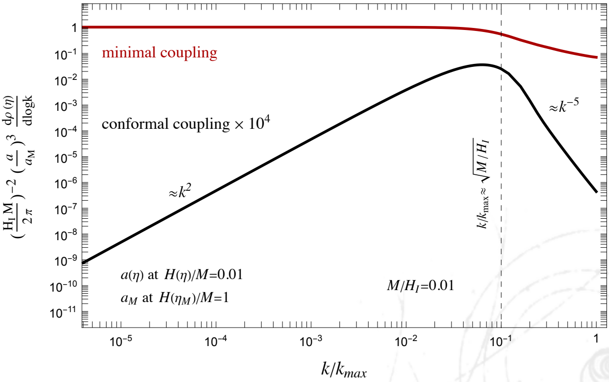

Redi, Tesi '23; Long, Kolb '23

- Can we make massless particles?

Stochastic Particle production [New]

Cosmological perturbations break Weyl invariance

- Production of conformally coupled particles $\to 0$ as $M \to 0$.
- GPP works since mass breaks Weyl invariance
- $M_{\rm pl}^2$ R breaks Weyl invariance
- Fluctuations of FLRW metric

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- $M_{\rm pl}^2$ R breaks Weyl invariance
- Fluctuations of FLRW metric
  - Curvature perturbations from inflation RG, Redi, Tesi 2408.15987
  - Tensor perturbations from phase transitions Maleknejad, Kopp '24

Stochastic Particle production [New]

Cosmological perturbations break Weyl invariance

- $ds^2 = a^2(\tau)\left((1+2\Phi)d\tau^2 - (1-2\Psi)dx^2\right)$
- $\psi \to \chi/a^{3/2}$ scalar factor dependence disappears
- Φ, Ψ functionsof $\tau, \vec{x}$
- No anisotropic stress $\Phi = \Psi$

Stochastic particle production

Massless fermions in curved space RG, Redi, Tesi 2408.15987

$$\gamma^a e^\mu_a(\partial_\mu + \frac{1}{8}\omega_\mu^{bc}[\sigma^b, \bar{\sigma}^c])\chi = 0$$

$$i\partial_\tau \chi_{\vec{k}} + (\vec{k}\cdot\vec{\sigma})\chi_{\vec{k}} = J_{\vec{k}}(\tau)\,.$$

$$J_{\vec{k}}(\tau) = \int \frac{d^3q}{(2\pi)^3}\left[2\Psi_{\vec{q}}\dot{\chi}_{\vec{\omega}} - \frac{i}{2}\Psi_{\vec{q}}(\vec{q}\cdot\vec{\sigma})\chi_{\vec{\omega}} + \frac{3}{2}\dot{\Psi}_{\vec{q}}\chi_{\vec{\omega}}\right]\,.$$

- RHS: source of Weyl breaking → produces particles
- Equation linear in field → quadratic Hamiltonian
- **New: apply Bogoliubov type calculation** (translations broken)

Stochastic particle production

Bogoliubov RG, Redi, Tesi 2408.15987

$$\chi_{\vec{k}}(\tau) = \chi_{0,\vec{k}}(\tau) + \int_{-\infty}^{\tau} d\tau' G^R_{\vec{k}}(\tau - \tau') J_{\vec{k}}(\tau')|_{\chi_{0,\vec{\omega}}}$$

$$G^R_{\vec{k}}(\tau) = i\theta(\tau)\left[P^-_{\vec{k}} e^{-ik\tau} + P^+_{\vec{k}} e^{ik\tau}\right] ,$$

$$\beta_{\vec{k}\vec{\omega}} = \int d\tau e^{-i(k+\omega)\tau} \Psi_{\vec{q}}(\tau)\, i(k-\omega)\, \xi^\dagger_{\vec{k},+} \xi_{\vec{\omega},-} \ .$$

- Notice $\beta \propto \Psi \to \langle \beta_k \rangle = 0$
- But $\langle |\beta_k|^2 \rangle \neq 0$ **!**

Stochastic particle production

Abundance of Dark Matter RG, Redi, Tesi 2408.15987

$$\langle \Psi_{\vec{q}}(\tau)\Psi^*_{\vec{q}'}(\tau')\rangle = (2\pi)^3\delta^3(\vec{q}-\vec{q}')\frac{2\pi^2}{q^3}\Delta_\Psi(q,\tau,\tau')\ .$$

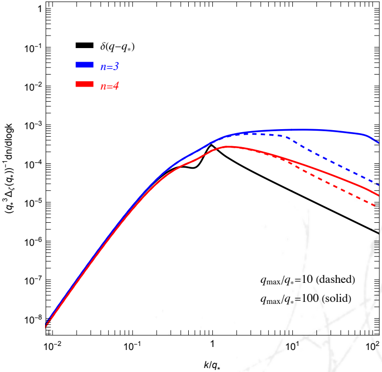
$$\langle|\beta_{\vec{k}}|^2\rangle = \frac{1}{2}\int d\tau\int d\tau'\int d(\log q)\int dx \times e^{-i(k+\omega)(\tau-\tau')}\Delta_\Psi(q,\tau,\tau')\mathcal{K}[k\,,q\,,x]\,,$$

- Well behaved
- Kernel $\mathcal{K}$ depends on spin: $\frac{(k-\omega)^2\left(q^2+2k\omega-k^2-\omega^2\right)}{4k\omega}$
- $\Delta_\Psi(q,\tau,\tau') = T(q,\tau)\,T(q,\tau')\Delta_\zeta(q)$

Stochastic particle production

The spectrum RG, Redi, Tesi 2408.15987

$$\frac{d(na^3)}{d\log k} = \frac{k^3}{4\pi^2}\int d(\log q)\, dx\, \Delta_\zeta(q)|\mathcal{I}(q, k+\omega)|^2\mathcal{K}[k\,, q\,, x] \;\;,$$



Stochastic particle production

Abundance of Dark Matter from curvature perturbations RG, Redi, Tesi 2408.15987

- $$na^3 = \frac{A}{4\pi^2}\int d(\log q)\, q^3 \Delta_\zeta(q)\,, \qquad A \approx 0.015\ .$$
- $$\Omega_{\rm DM}\big|_{\rm stochastic} \approx \frac{A}{4\pi^2}\frac{M\, q_*^3}{3M_{\rm pl}^2 H_0^2}\Delta_\zeta(q_*)\,.$$
- $$q_* \approx 10^{22}\,{\rm Mpc}^{-1}\left(\frac{10^6\,{\rm GeV}}{M}\right)^{\frac{1}{3}}\left(\frac{0.01}{\Delta_\zeta(q_*)}\right)^{\frac{1}{3}}\left(\frac{0.01}{A}\right)^{\frac{1}{3}}\ .$$

Stochastic particle production

Abundance of Dark Matter from curvature perturbations

- **Need a sizable primordial power spectrum**
- Need to happen at the last few e-foldings of inflation
- Related to ultra-slow roll scenarios
- Phase transition after production required to make them massive

Stochastic particle production

Summary: comparison to other mechanisms

- Gravitational freeze-in

$$\Omega_{\rm DM} \approx 4 \times 10^{-5} \frac{M k_R^3}{3M_{\rm Pl}^2 H_0^2} \,.$$

- Gravitational particle production

$$\Omega_{\rm DM} \approx 10^{-2} \frac{M\, k_M^3}{3M_{\rm pl}^2 H_0^2} \times \min\left[1\,, \frac{T_R}{\sqrt{M\, M_{\rm pl}}}\right],$$

- Stochastic particle production

$$\Omega_{\rm DM} \approx \frac{A}{4\pi^2} \frac{M\, q_*^3}{3M_{\rm pl}^2 H_0^2} \Delta_\zeta(q_*)\,.$$

Conclusions and Outlook

- Massless particles can be gravitationally produced from perturbations!
- The possibility that DM is coupled ONLY gravitationally is compelling

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Thank you for your attention!

Stochastic particle production