

Infinites, Small and Big

A guided mathematical adventure

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Let's Compare Sizes

Which of the following sets is larger?

$$S_1 = \{1, 2, 3\}$$

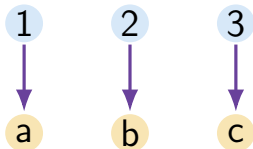
$$S_2 = \{a, b, c\}$$

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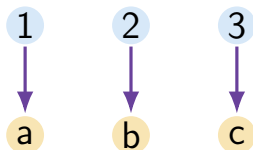
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Pretty boring question, right?

Let's spice this up.

Let's Try Cricket

Between these two cricketers,
who has hit more sixes in ODIs?



Virat Kohli



Jasprit Bumrah

Let's Try Cricket

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Kohli: **171 sixes**

Bumrah: **2 sixes**

How Did You Answer That?

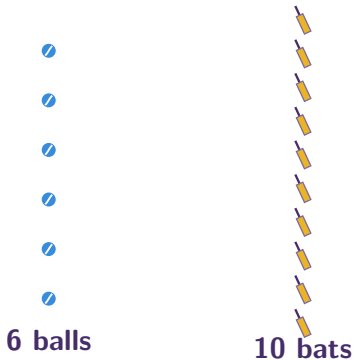
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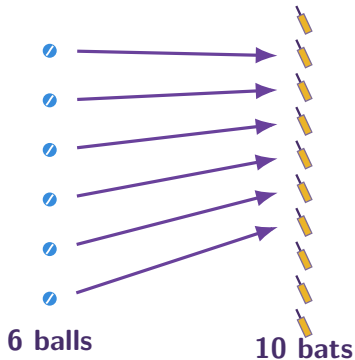
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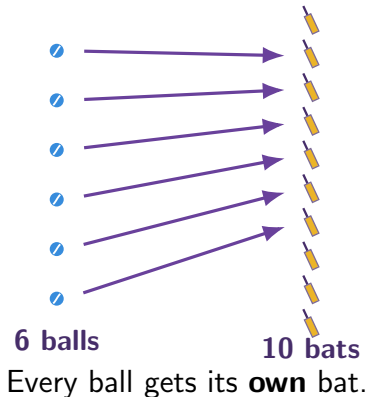
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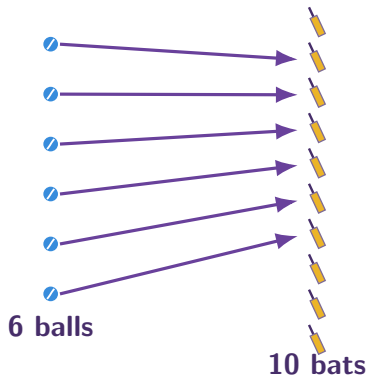
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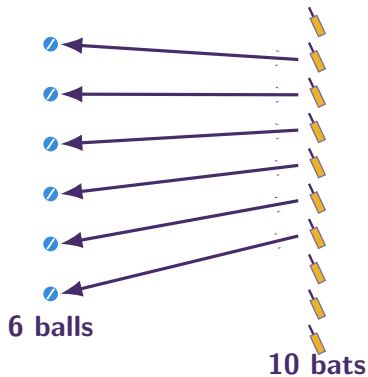
You cannot say that Kohli has hit fewer sixes than Bumrah!



What happens if we try to reverse the arrows?

Can We Go Backwards?

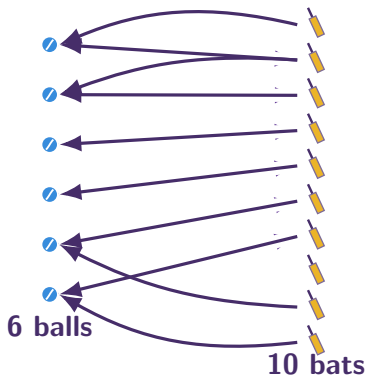
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Now we have a map from all ten bats to six balls.

Can We Go Backwards?

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Two bats have to land on the same ball.

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Motivated by this, we now make a definition.

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In many examples, we can do even better:
we can **pair up the two sets perfectly**.

Now Let's Do Something Weird

Which one is bigger?

$$\mathbb{N} = \{1, 2, 3, 4, 5, \dots\}$$

or

$$\text{EVEN} = \{2, 4, 6, 8, 10, \dots\}?$$

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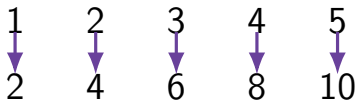
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Are you amused?

This is only the appetizer.

This and more awaits you.

Come to the Math Circle.