Pulsar timing arrays

Supposing the source is the inspiral of a super massive black hole binary:
what is the typical scale of the time variation of the metric perturbation?

$$f(\tau) = \frac{1}{\pi}\left(\frac{5}{256}\right)^{3/8} \frac{1}{(G\,M_c)^{5/8}\,\tau^{3/8}} \simeq 10^{-8}\,\mathrm{Hz}$$

for

$$M_c \simeq 10^9\,M_\odot$$

$$\tau = 4\times 10^4\,\mathrm{yrs}$$

Time to coalescence

Chirp mass

$$M_c = \frac{(m_1\,m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

GW varies on a scale of about 3 years

Period of the pulsar: millisecond

$$f_{\rm GW} P \ll 1$$

$$\Delta T = \frac{1}{2}\frac{\hat{u}^i\hat{u}^j}{1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}}\int_{t_e-\hat{\mathbf{k}}\cdot\mathbf{r_e}}^{t_e+L-\hat{\mathbf{k}}\cdot\mathbf{r_e}-L\hat{\mathbf{k}}\cdot\mathbf{u}} \mathrm{d}X\,[h_{ij}(X+P)-h_{ij}(X)]$$

Taylor expand

Relative change in the rate of the pulses measured on Earth because of the GW passing by:

$$\frac{\Delta T}{P} \simeq \frac{1}{2}\frac{\hat{u}^i\hat{u}^j}{1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}}\,[h_{ij}(t_e+L,\mathbf{r_o}) - h_{ij}(t_e,\mathbf{r_e})]$$

Earth term

Pulsar term

Pulsar timing arrays

Supposing the source is the inspiral of a super massive black hole binary: what is the typical scale of the time variation of the metric perturbation?

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Taylor expand

NB: this is the change in the frequency of the pulses due to the GWs, calculated between two successive geodesics, and NOT the redshift experienced by a photon on the same geodesic (usual gravitational redshift, depending on $\dot{h}_{ij}$)

However, the two expressions become the same in the limit of infinitesimal P

Pulsar timing arrays

Relative change in the rate of the pulses measured on Earth because of the GW passing by:

$$\frac{\Delta T}{P} \simeq \frac{1}{2}\frac{\hat{u}^i\hat{u}^j}{1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}}\left[h_{ij}(t_e+L,\mathbf{r_o})-h_{ij}(t_e,\mathbf{r_e})\right]$$

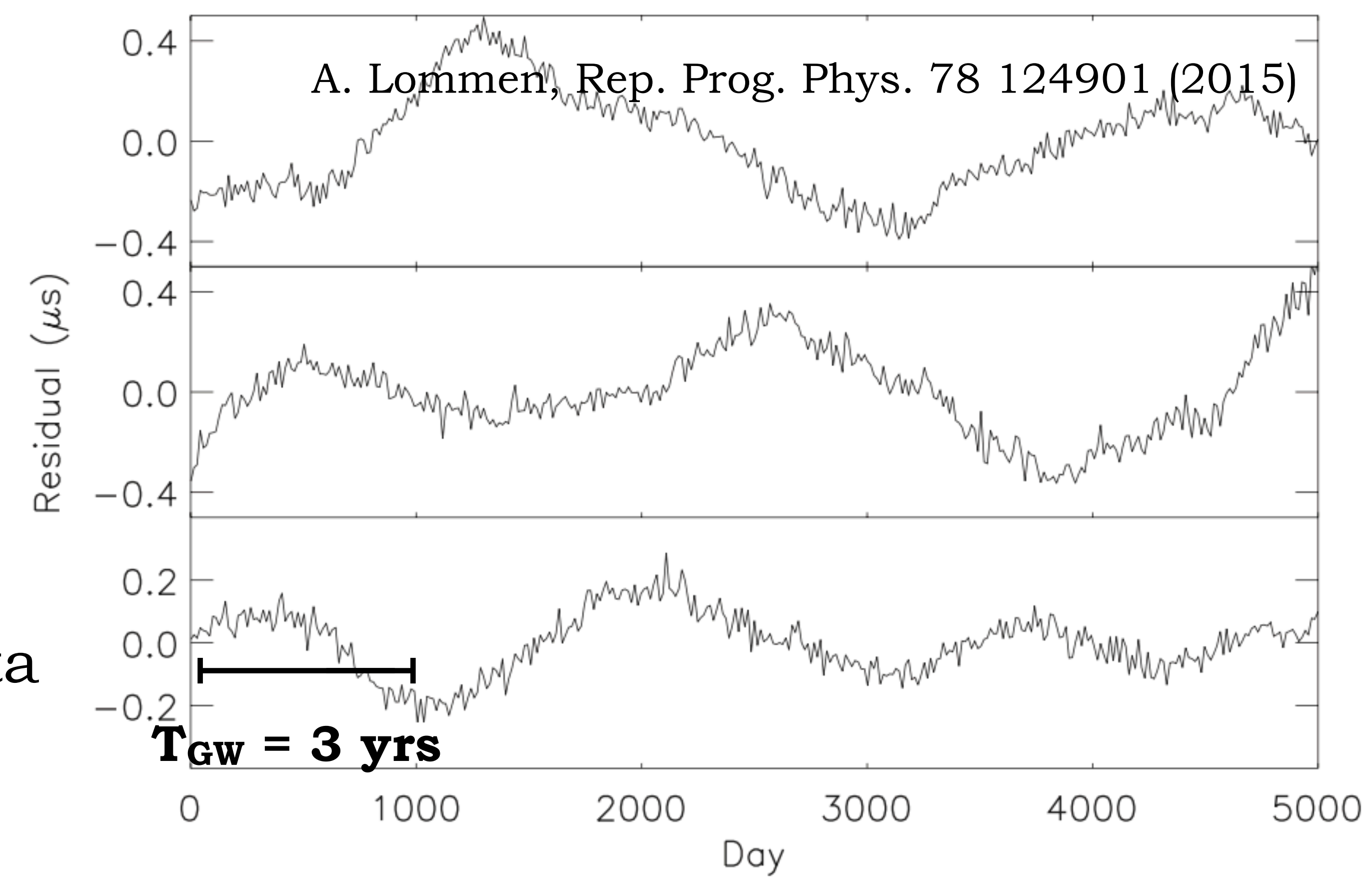
$$\sim 7\cdot 10^{-23}\,\frac{\mathrm{pc}}{d_L}\left(\frac{M_c}{M_\odot}\right)^{5/3}\left(\frac{f_{\mathrm{GW}}}{10^{-8}\,\mathrm{Hz}}\right)^{2/3} \simeq 7\cdot 10^{-16}$$

$M_c \simeq 10^9\, M_\odot$

$d_L = 100\,\mathrm{Mpc}$

Timing residuals: $R(T) = \int_{t_{\mathrm{ref}}}^{t_{\mathrm{ref}}+T} \mathrm{d}t\, \frac{\Delta T}{P}$

$$R(T_{\mathrm{GW}} = 3\,\mathrm{yrs}) \simeq 60\,\mathrm{nsec}$$



A GW with period of a few years induces a timing residual of order 100 nsec, the precision of pulsar monitoring!

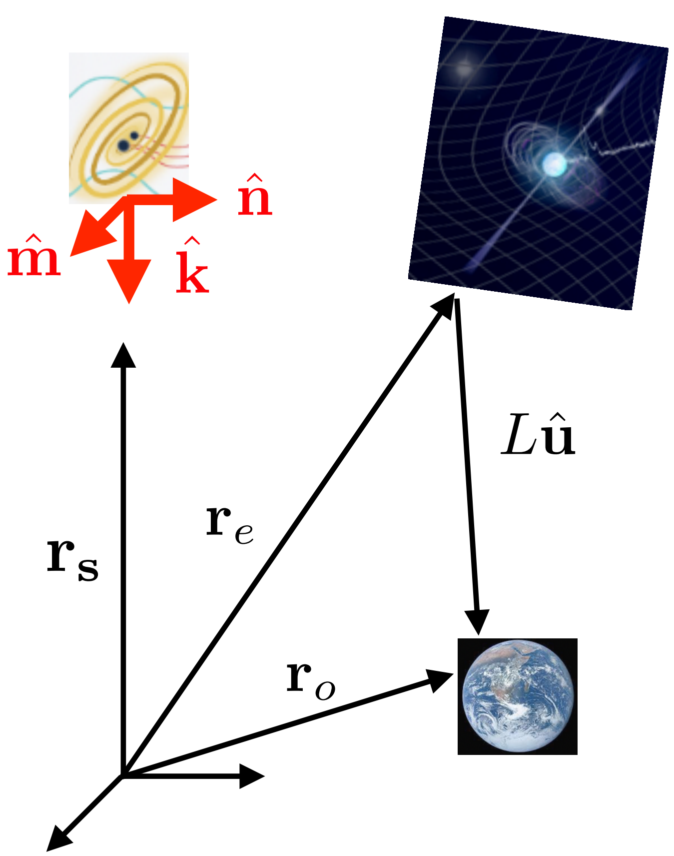
This renders the measurement possible, provided one has at least a few years of data

Pulsar timing arrays

HOWEVER! The signal from a single pulsar is very noisy: varying morphology of the pulses, propagation noise due to the dispersion by the interstellar medium, time referencing (time standards and solar system barycentre)…

Correlation between many pulsars to beat down the noise

$$\frac{\Delta T}{P} \simeq \frac{1}{2} \frac{\hat{u}^i \hat{u}^j}{1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{u}}} \left[h_{ij}(t_e + L, \mathbf{r_o}) - h_{ij}(t_e, \mathbf{r_e}) \right]$$



Earth term:
GW left the source at time

$$t_e + L - |\mathbf{r_o} - \mathbf{r_s}|$$

Pulsar term:
GW left the source at time

$$t_e - |\mathbf{r_e} - \mathbf{r_s}|$$

This term is different for each pulsar

In the correlation the Earth term in general dominates, but the pulsar term can create noise (unless one can determine the delay of each pulsar)

Pulsar timing arrays

Response of a pair of pulsars to a stochastic GW background

$$\langle R_a(T) R_b(T) \rangle = \int_{t_{\rm ref}}^{t_{\rm ref}+T} \mathrm{d}t' \int_{t_{\rm ref}}^{t_{\rm ref}+T} \mathrm{d}t'' \langle \frac{\Delta T}{P}(t')\Big|_a \frac{\Delta T}{P}(t'')\Big|_b \rangle$$

$$\frac{\Delta T}{P}(t')\Big|_a = \sum_r \int \frac{\mathrm{d}\mathbf{k}^3}{(2\pi)^3} h_r(\mathbf{k}) F_a^r(\hat{\mathbf{k}}) e^{-ik(t' - \hat{\mathbf{k}}\cdot\mathbf{r}_o)} \left[1 - e^{ikL_a(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}_a)}\right]$$

"Detector response" $F_a^r(\hat{\mathbf{k}}) = \dfrac{\hat{u}_a^i \hat{u}_a^j e_{ij}^r(\hat{\mathbf{k}})}{2(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}})}$

put Earth at origin to simplify

Earth term

Pulsar term

Use SGWB power spectrum

$$\langle h_r(\mathbf{k},\eta)\, h_p^*(\mathbf{q},\eta)\rangle = \frac{8\pi^5}{k^3}\,\delta^{(3)}(\mathbf{k}-\mathbf{q})\,\delta_{rp}\, h_c^2(k,\eta)$$

The Earth term dominates

$$\left[1 - e^{ikL_a(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}_a)}\right]\left[1 - e^{-ikL_b(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}_b)}\right] \simeq 1$$

$a \neq b$

and SMBHB not in pulsar direction

$$kL_a = \mathcal{O}(2\pi \cdot 10^{-8}\,\mathrm{Hz} \cdot 500\,\mathrm{pc}) = \mathcal{O}(3000) \gg 1$$

Pulsar timing arrays

Response of a pair of pulsars to a stochastic GW background

$$\langle R_a(T)R_b(T)\rangle = \int_{t_{\rm ref}}^{t_{\rm ref}+T} {\rm d}t' \int_{t_{\rm ref}}^{t_{\rm ref}+T} {\rm d}t'' \langle \frac{\Delta T}{P}(t')\Big|_a \frac{\Delta T}{P}(t'')\Big|_b \rangle$$

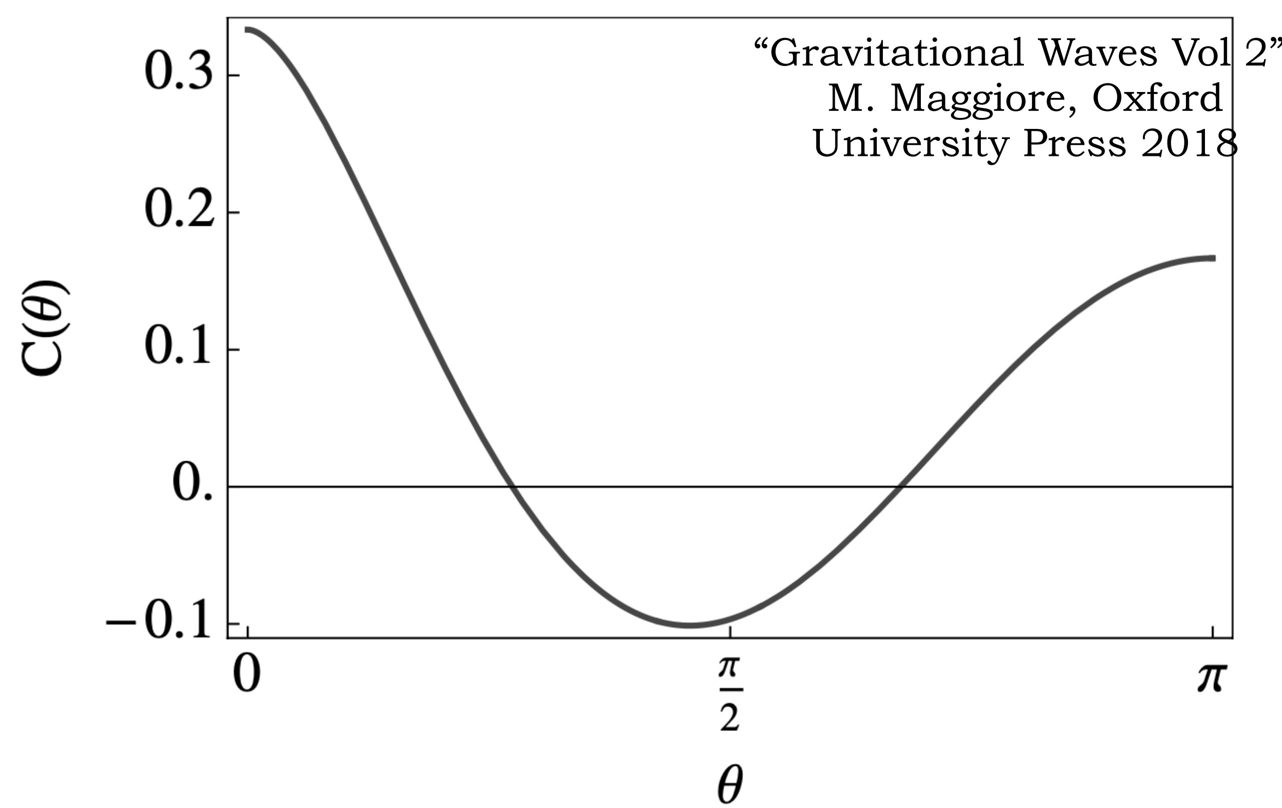
θ_{ab} angle between pulsars

$$= \mathcal{C}(\theta_{ab}) \int_0^\infty {\rm d}f\, \frac{h_c^2(f)}{(2\pi)^2 f^3} \left[1+\cos(2\pi f(T-t_{\rm ref})\right]$$

Hellings and Downs curve, characteristic of a GW signal because consequence of the quadrupolar nature of GWs

$$\mathcal{C}(\theta_{ab}) = \int \frac{{\rm d}\hat{\mathbf{k}}}{4\pi} \sum_r F_r^a(\hat{\mathbf{k}}) F_r^b(\hat{\mathbf{k}})$$
$$= \frac{1}{3} - \frac{1}{6}x_{ab} + x_{ab}\log(x_{ab})$$

$$x_{ab} = \frac{1}{2}(1-\cos\theta_{ab})$$



Pulsar timing arrays

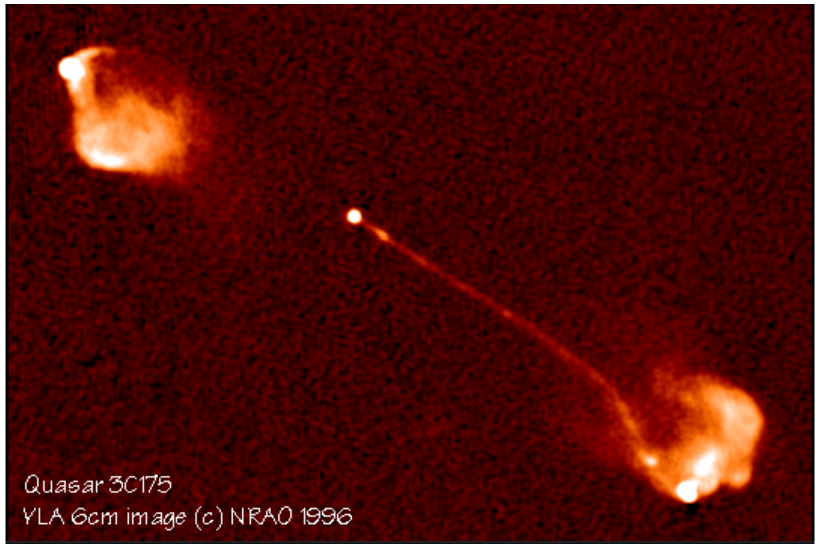
Observation of the *Hellings and Downs curve* is **smoking gun evidence** of GW detection

(not only SGWB but also from a single SMBHB - Cornish & Sesana arXiv:1305.0326)

time referencing errors generate a correlated noise but:

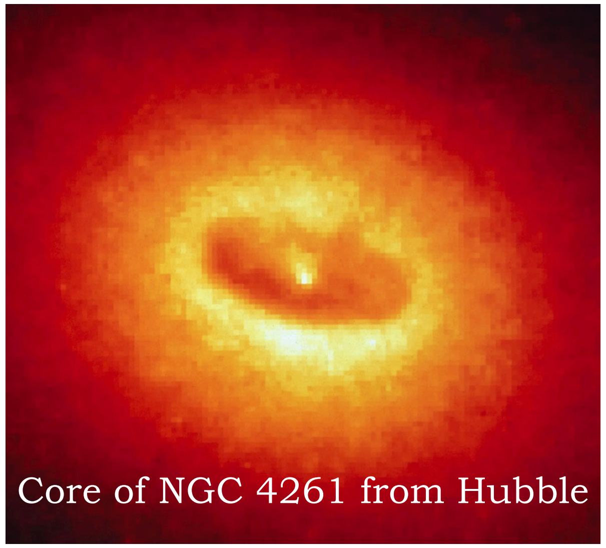
- noise from uncertainties on the time standards on Earth is independent on the pulsar angles
- noise from uncertainties on the solar system barycentre position take the form of a (rotating) dipole (dependent on the cosinus of the angle) -> can contaminate the quadrupole

A SGWB from SMBHBs is the best candidate source in PTA frequency band



What are SMBHBs?

- They have been observed in the core of galaxies and are the central engine of active galactic nuclei
- They can originate from the collapse of massive stars (~100 $M_\odot$) or gas clouds (~ 10^4 $M_\odot$), and then grow in mass through gas accretion and/or mergers following the collision of their host galaxies (but their origin is still to be confirmed, they can also be primordial…)
- JWST sees SMBHs up to very high redshift z ~ 11
- Their presence is linked to the formation of galaxies and matter structure in the Universe

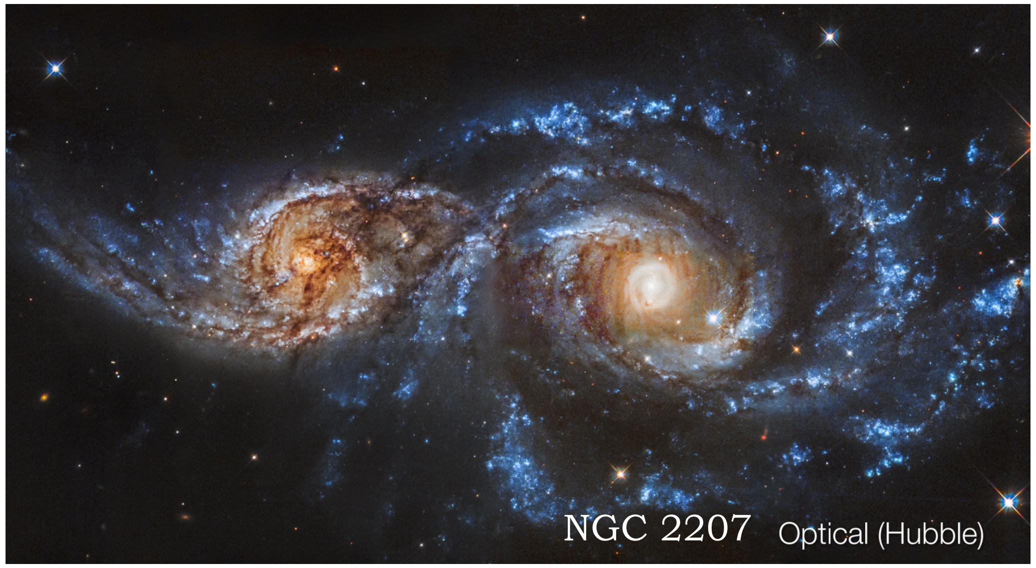

Core of NGC 4261 from Hubble

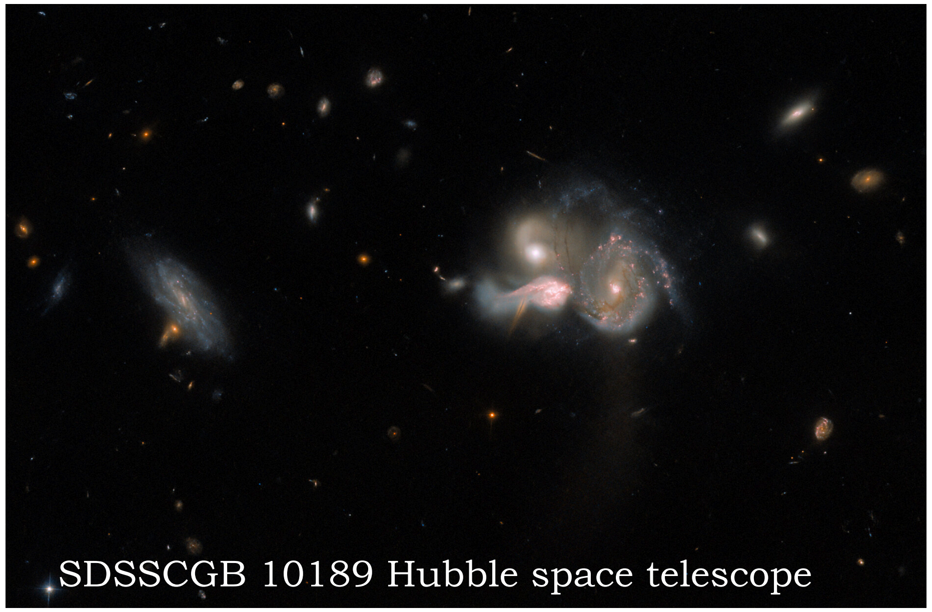
Pulsar timing arrays

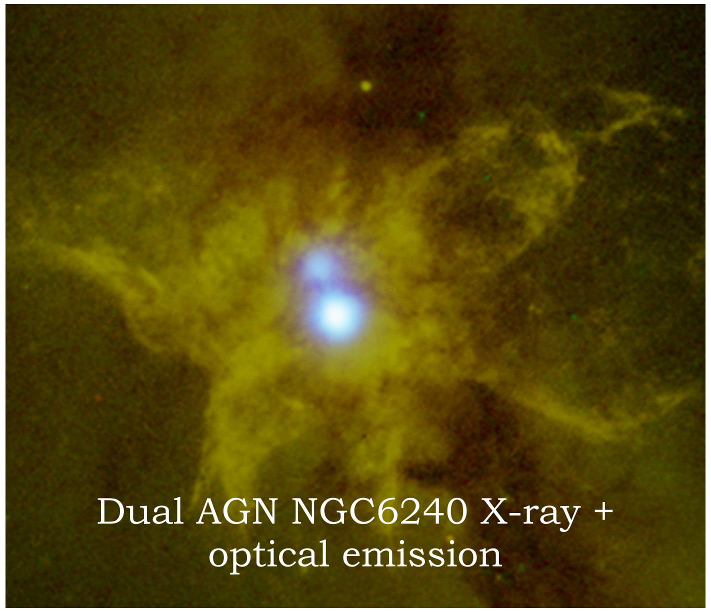
HOWEVER!

- To emit GWs, SMBHs must be paired in gravitationally bound binaries in the GW emitting regime: separation of ~ 0.01-0.001 pc
- Binaires can be formed after the collision of two galaxies: the MBH previously at the centre of galaxies get to ~ kpc separation (X-ray evidence from dual AGNs)
- Dynamical friction drives the two MBH towards the centre of the new galaxy until they form a bound binary
- 3-body interaction with the surrounding stars subsequently shrinks the binary to pc separation
- How to get them to the millipc separation necessary for GW emission and merger within one Hubble time? "LAST PARSEC PROBLEM"
- maybe more stars arrive, or there is gas drag from interaction with a circumbinary disk, and/or another MBH arrives…

If PTAs observe the SGWB from SMBHBs it means that SMBHBs exist and merge in the universe!


NGC 2207 Optical (Hubble)


SDSSCGB 10189 Hubble space telescope


Dual AGN NGC6240 X-ray + optical emission

Pulsar timing arrays

Prediction from SMBHBs formation scenarios

How does the SGWB from SMBHBs look like?

Characteristic strain: red spectrum

$$h_c(f) = A\left(\frac{f}{f_{\rm ref}}\right)^{-\alpha} \text{ with } \alpha = \frac{2}{3}$$

$A = \mathcal{O}(10^{-15})$ at

$f_{\rm ref} = 1\,{\rm yr}^{-1}$

Circular binary

Timing residuals power spectral density: Also red spectrum

$$S_{ab}(f) = \mathcal{C}(\theta_{ab})\Phi(f)$$

$$\Phi(f) = \frac{A^2}{(2\pi)^2} f_{\rm ref}^{-3}\left(\frac{f}{f_{\rm ref}}\right)^{-\gamma} \text{ with } \gamma = 2\alpha + 3 = \frac{13}{3}$$

Where does this spectral shape come from?

SGWB from a population of inspiralling binaries

$$h_c(f) = A\left(\frac{f}{f_{\rm ref}}\right)^{-\alpha} \text{ with } \alpha = \frac{2}{3}$$

in terms of the power spectrum of the GW energy density becomes

$$\Omega_{\rm GW}(f) = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f) = \Omega_{\rm GW}(f_{\rm ref})\left(\frac{f}{f_{\rm ref}}\right)^{2/3}$$

$$\frac{\rho_{\rm GW}^{\rm (tot)}}{\rho_c} = \int_0^\infty \frac{{\rm d}f}{f}\,\Omega_{\rm GW}(f) = \int {\rm d}\xi \int {\rm d}V_c \int {\rm d}\tau_c\, \frac{{\rm d}^3 N(z,\tau_c,\xi,\theta)}{{\rm d}\xi {\rm d}V_c {\rm d}\tau_c}\,\frac{\rho_{\rm GW}^{\rm (event)}}{\rho_c}$$

Parameters of the binary signal (essentially chirp mass)

Coming volume

Time to coalescence

Number density of GW sources (given within an astrophysical model for the binary population)

GW energy emitted by a single event

At the source
$$\frac{\rho_{\rm GW}^{\rm (event)}}{\rho_c} = \frac{1}{16\pi G\rho_c}\,\frac{\langle \dot h_+^2 + \dot h_\times^2\rangle}{(1+z)^4}$$

SGWB from a population of inspiralling binaries

$$\dot{h}_+(t_S) = \frac{4\pi^{2/3}}{a(t_S)r}(G\,M_c)^{5/3}\left(\frac{1+\cos^2\theta}{2}\right)\underbrace{\frac{\mathrm{d}[f^{2/3}(t_S)\cos(2\Phi(t_S))]}{\mathrm{d}t_S}}$$

In the limit of circular orbit with slowly varying radius

$$\dot{f}_S \ll f_S^2$$

$$\simeq -f^{2/3}(t_S)2\,\dot{\Phi}(t_S)\sin(2\Phi(t_S))$$

$$\dot{\Phi}(t_S) \rightarrow \pi f_S$$

$$\langle \dot{h}_+^2(t_S)\rangle = \frac{32}{a_S^2 r^2}(\pi G\,M_c)^{10/3}\left(\frac{1+\cos^2\theta}{2}\right)^2 f_S^{10/3}$$

$$\frac{\rho_{\rm GW}^{(\rm tot)}}{\rho_c} = \int \mathrm{d}\xi \int \mathrm{d}\tau_c \int \mathrm{d}z\,\frac{d_M^2}{H(z)}\,\frac{\mathrm{d}^3N(z,\tau_c,\xi,\theta)}{\mathrm{d}\xi\mathrm{d}V_c\mathrm{d}\tau_c}\,\frac{1}{16\pi G\rho_c(1+z)^4}$$

$$\frac{32}{a_S^2 r^2}(\pi G M_c f_S)^{10/3}\int \mathrm{d}\Omega\left[\left(\frac{1+\cos^2\theta}{2}\right)^2+\cos^2\theta\right]$$

$$d_M = a_0\,r$$

Extra factor $(1+z)^2$

$$\mathrm{d}V_c = \frac{d_M^2}{H(z)}\mathrm{d}\Omega\,\mathrm{d}z$$

$$= 16\pi/5$$

SGWB from a population of inspiralling binaries

Express the integral over time to coalescence in terms of frequency and change to frequency at the observer

$$\frac{\mathrm{d} f_S}{\mathrm{d}\tau_c} = \frac{96\pi^{8/3}}{5}(G\, M_c)^{5/3} f_S^{11/3}$$

$$f_S = f(1+z)$$

$$\frac{\rho_{\mathrm{GW}}^{(\mathrm{tot})}}{\rho_c} = \frac{\pi^{2/3}}{3\, G\rho_c}\int \frac{\mathrm{d}f}{f} f^{2/3} \int \mathrm{d}\xi \int \frac{\mathrm{d}z}{H(z)(1+z)^{4/3}}(GM_c)^{10/3}\frac{\mathrm{d}^3 N(z,\tau_c,\xi,\theta)}{\mathrm{d}\xi \mathrm{d}V_c \mathrm{d}\tau_c}$$

$$\frac{\rho_{\mathrm{GW}}^{(\mathrm{tot})}}{\rho_c} = \int_0^\infty \frac{\mathrm{d}f}{f}\,\Omega_{\mathrm{GW}}(f)$$

$$\Omega_{\mathrm{GW}}(f) = \Omega_{\mathrm{GW}}(f_{\mathrm{ref}})\left(\frac{f}{f_{\mathrm{ref}}}\right)^{2/3}$$

$$\Big\downarrow$$

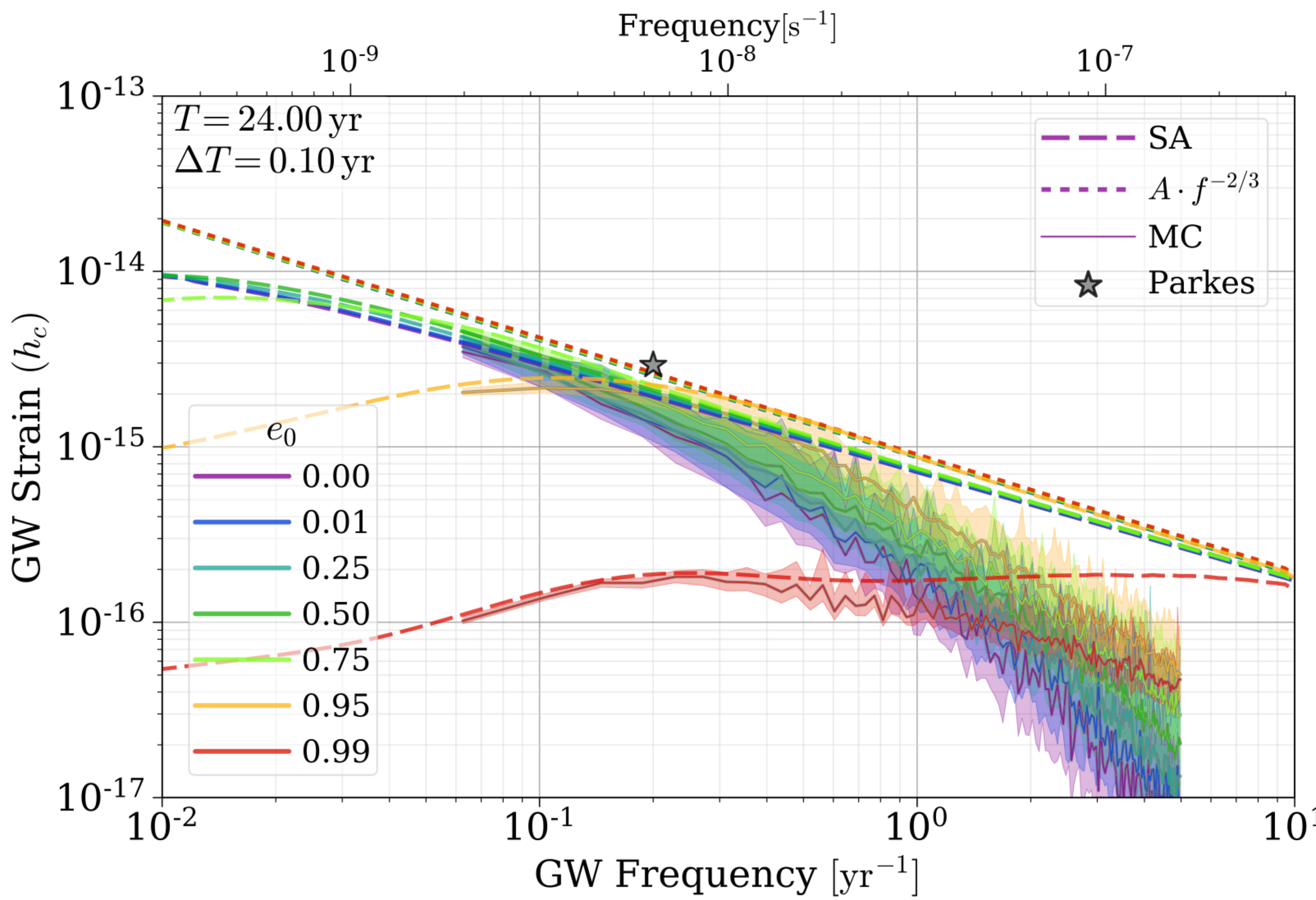
SGWB amplitude determined by the population characteristics and the cosmology

SGWB from a population of inspiralling binaries

The features of the SGWB power spectrum (amplitude A, slope α…) depend on the population characteristics such as the binary merger rate, its dependence with mass and redshift, the surrounding stellar density, the initial binary eccentricity…

- The assumption of homogeneous and isotropic SGWB isn't justified at high frequency: SMBHBs are less numerous, the SGWB slope is steeper, and discreteness starts to appear with spikes due to the loudest SMBHBs
- Interactions with the binary environment makes hardening stronger and suppresses SGWB power at low frequency
- Eccentricity enhances GW emission at higher frequencies

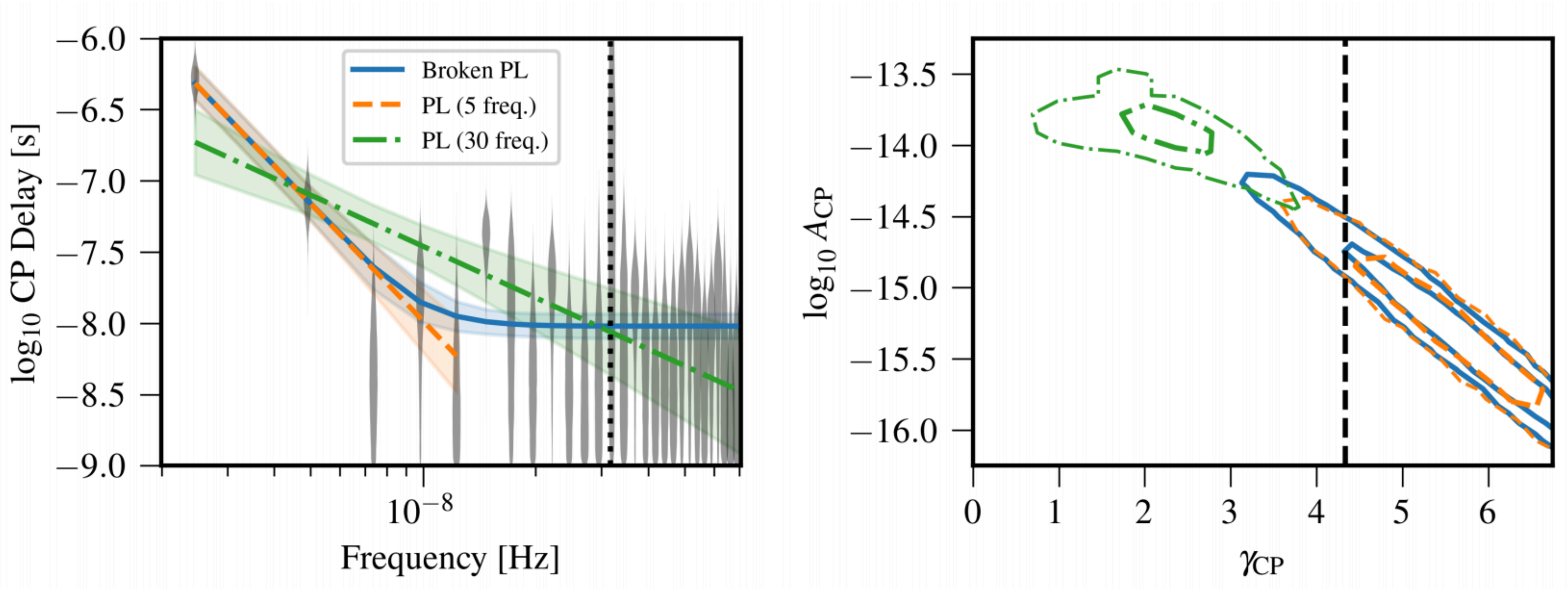
L.Z. Kelley et al, arXiv:1702.02180



Pulsar timing arrays

In 2020, NANOGrav (followed by EPTA and PPTA) has announced the presence of a common red noise in their 12.5 years data

$$\Phi(f) = \frac{A^2}{(2\pi)^2} f_{\rm ref}^{-3} \left(\frac{f}{f_{\rm ref}}\right)^{-\gamma} \quad \text{with} \quad \gamma = 2\alpha + 3 = \frac{13}{3}$$

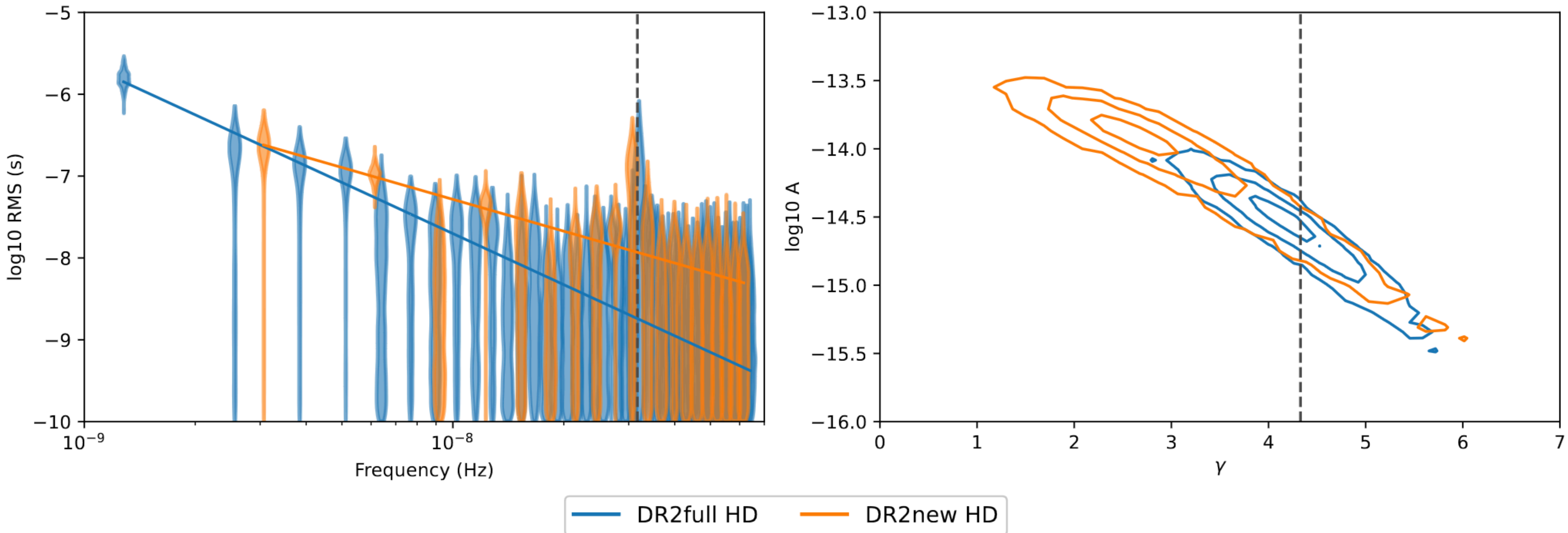


NANOGrav collaboration: arXiv:2009.04496

Pulsar timing arrays

Last year, all PTAs have confirmed the observation of a common red noise supplemented by evidence for the Hellings-Downs correlation

EPTA results:

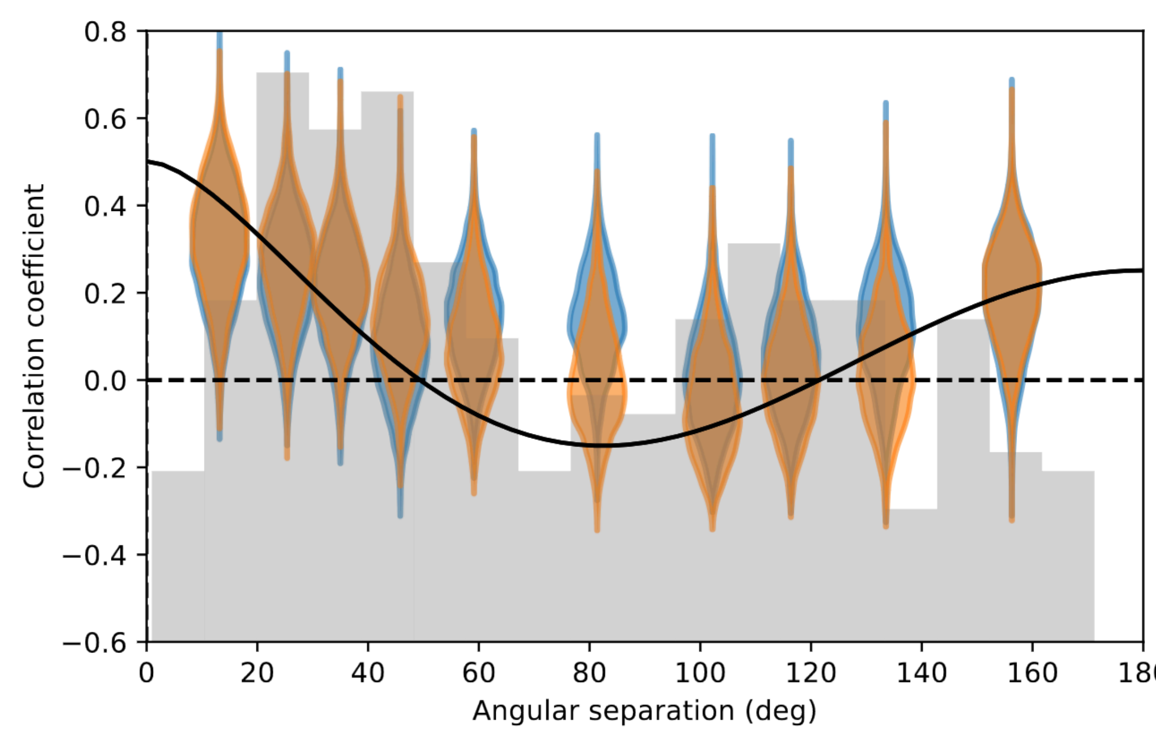


DR2new (10.3 yrs):

$$\log A = -13.94^{+0.23}_{-0.48} \quad \gamma = 2.71^{+1.18}_{-0.73} \quad \mathcal{B} = 60$$

DR2full (25 yrs):

$$\log A = -14.54^{+0.28}_{-0.41} \quad \gamma = 4.19^{+0.73}_{-0.63} \quad \mathcal{B} = 4$$



J. Antoniadis et al, arXiv:2306.16214

Pulsar timing arrays

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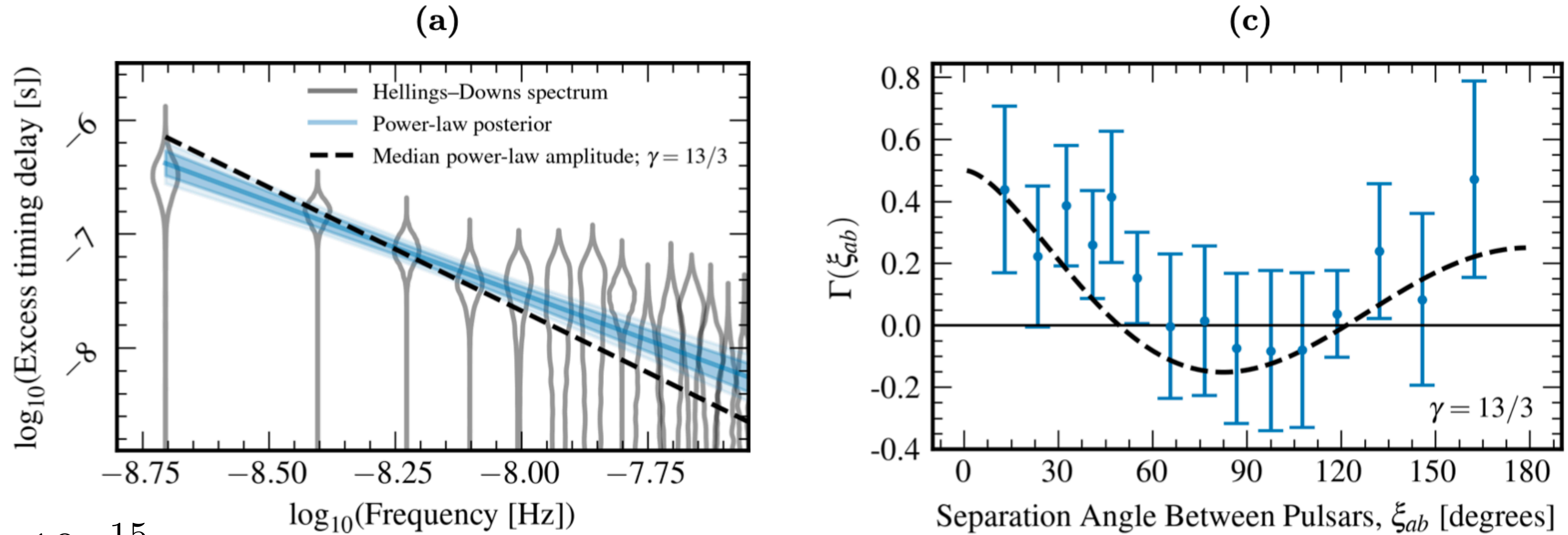
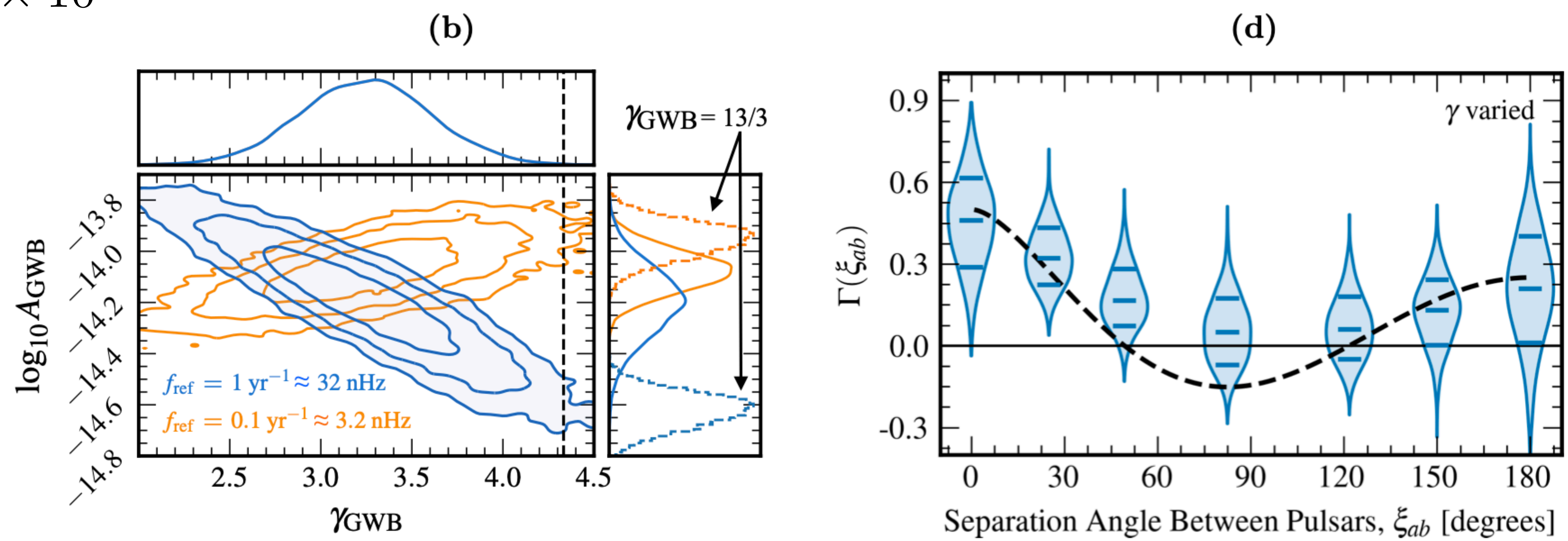
NANOGrav results:



$$A = -6.4^{+4.2}_{-2.7} \times 10^{-15}$$

$$\gamma = 3.2^{+0.6}_{-0.6}$$

$$\mathcal{B} = 226$$

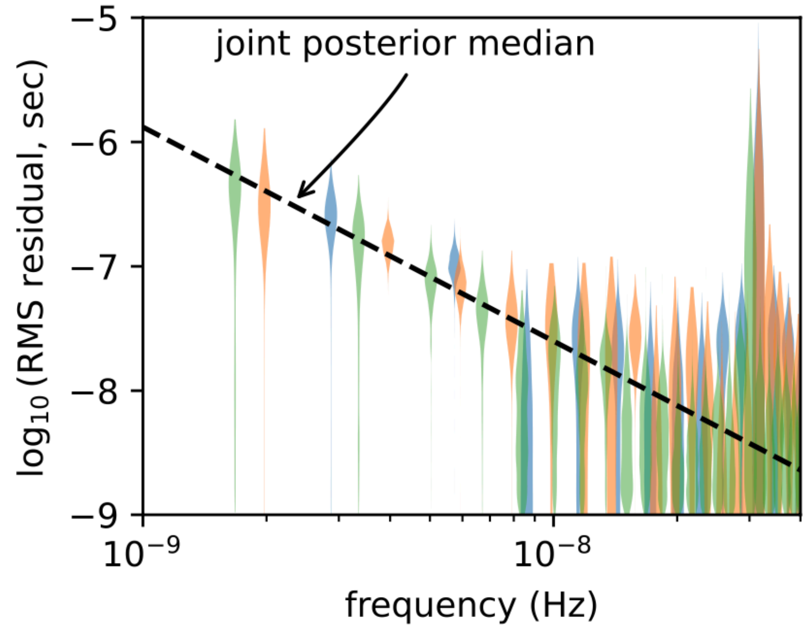


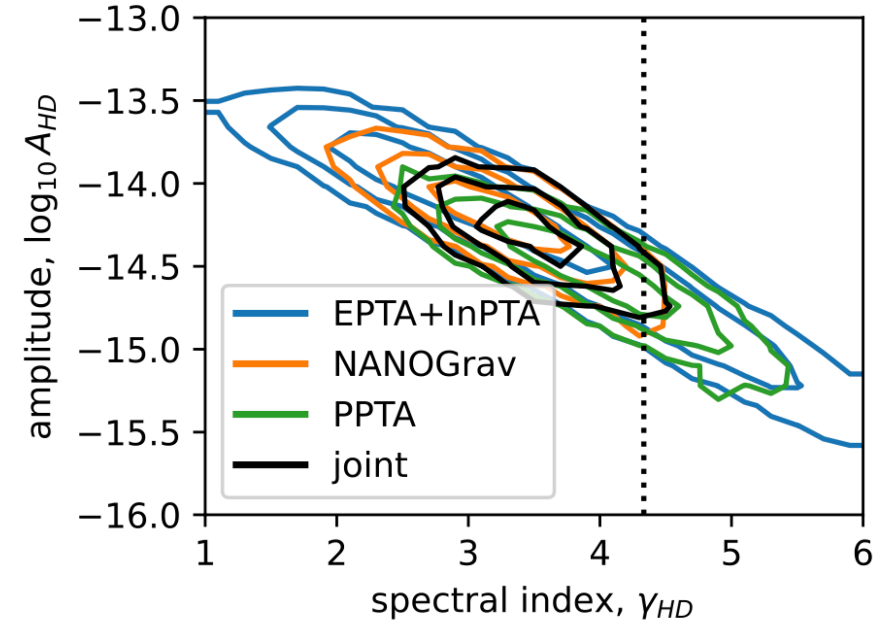
G. Agazie et al, arXiv:2306.16213

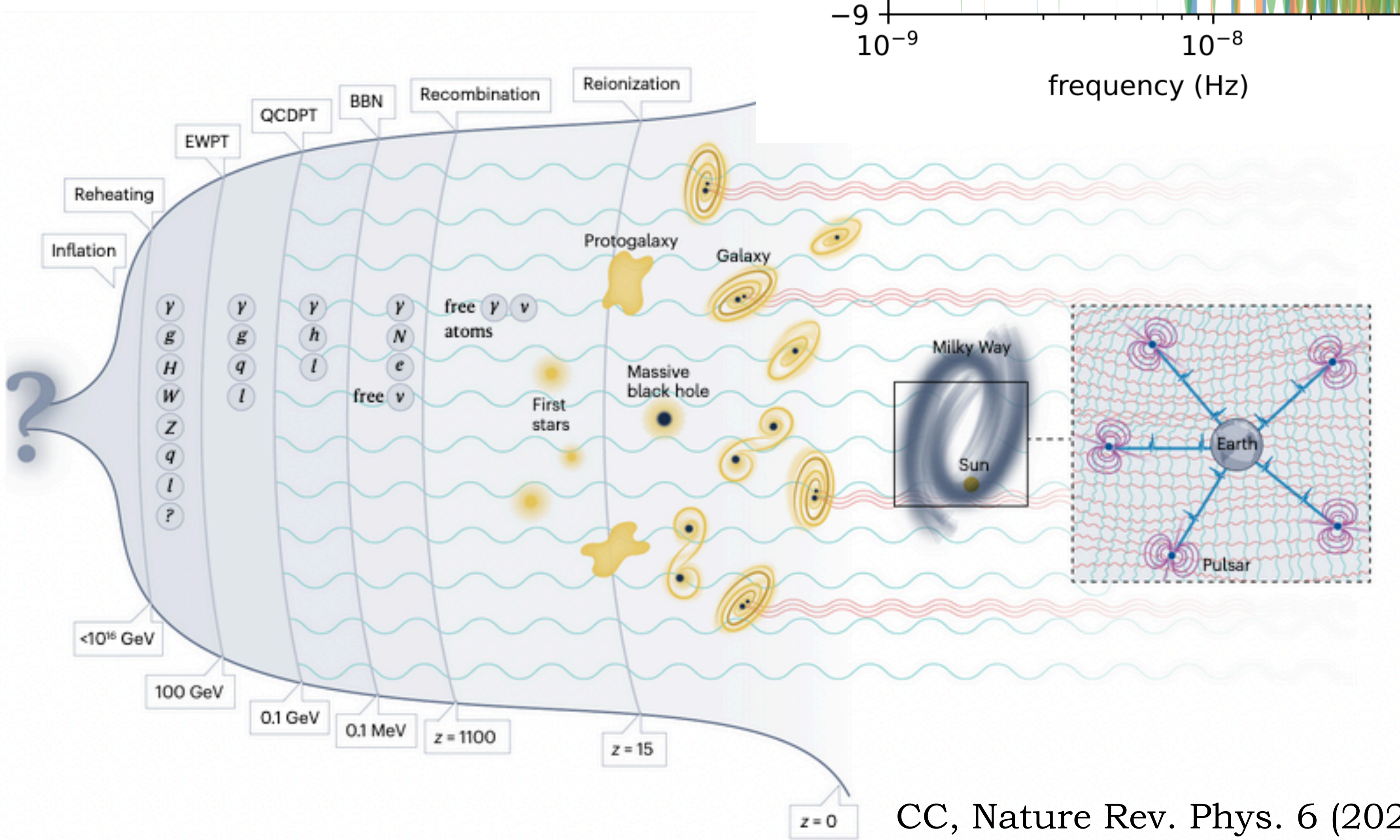
Pulsar timing arrays

- The slopes are shallower than 13/3 (but maybe the model isn't fully adapted…)
- The amplitude is consistent with the one from a SMBHBs SGWB
- All datasets are consistent within 1σ as shown by IPTA

IPTA Collaboration,
arXiv:2309.00693



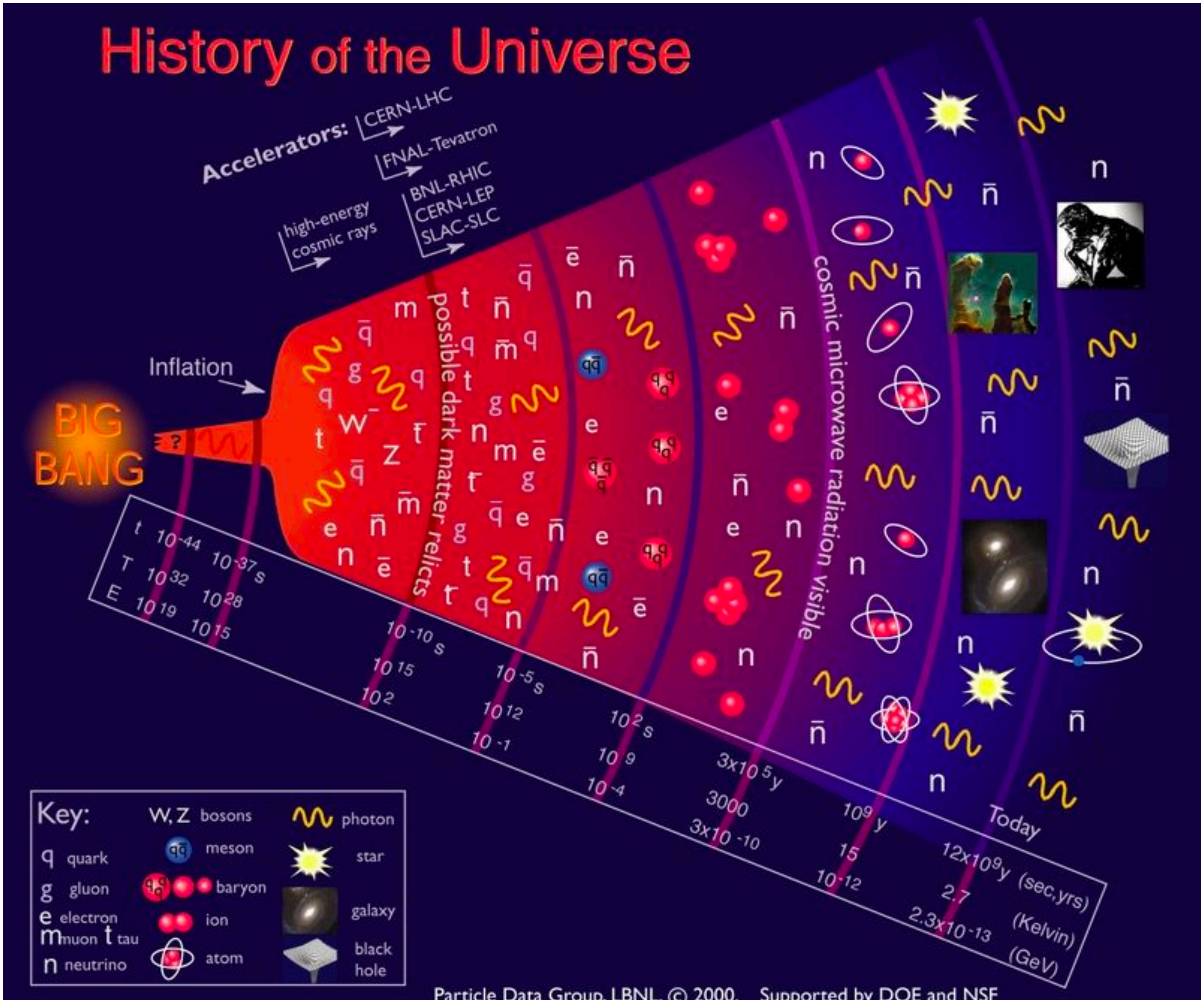




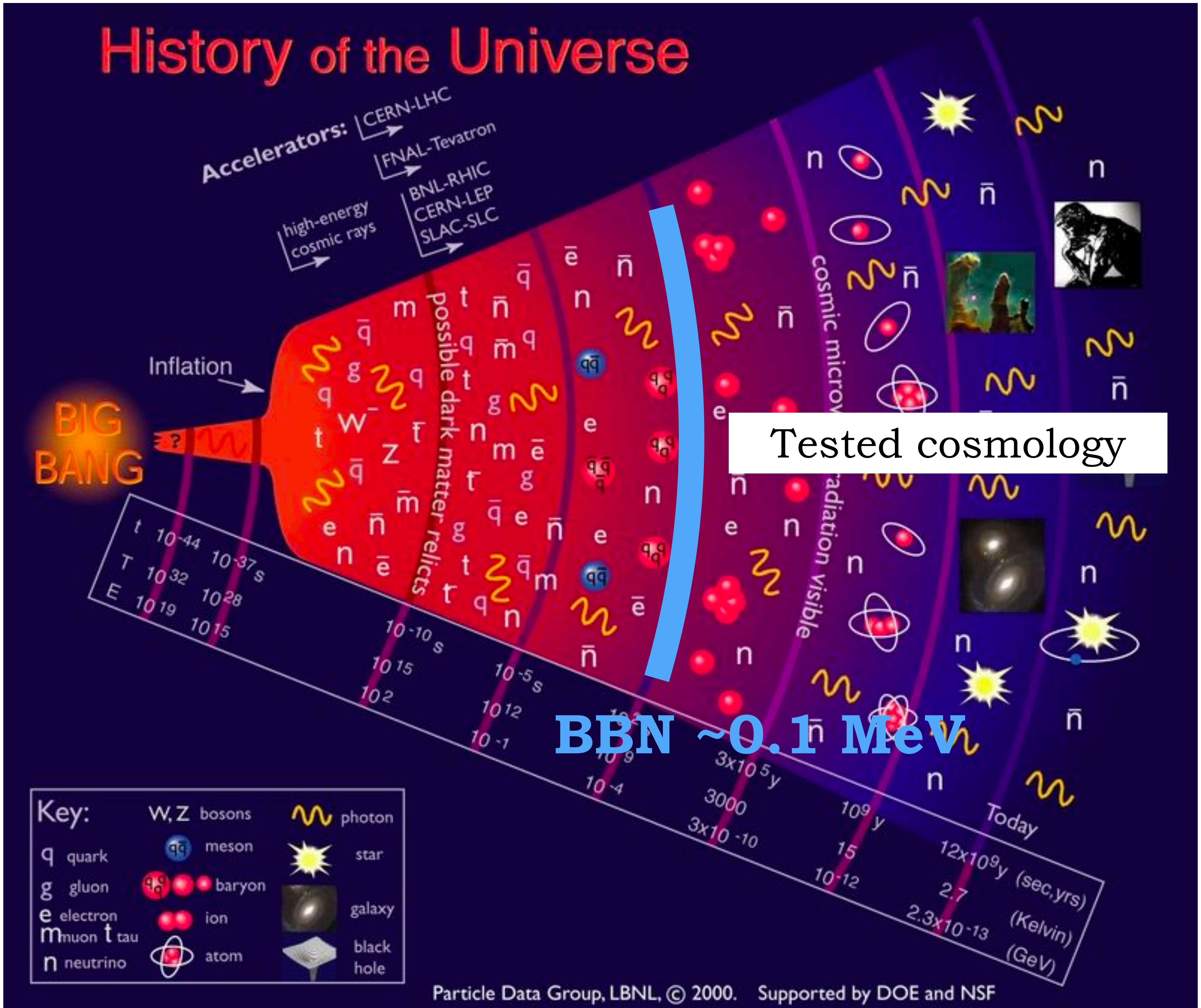
But the signal
could also be of
primordial origin…

CC, Nature Rev. Phys. 6 (2024), 5291-293

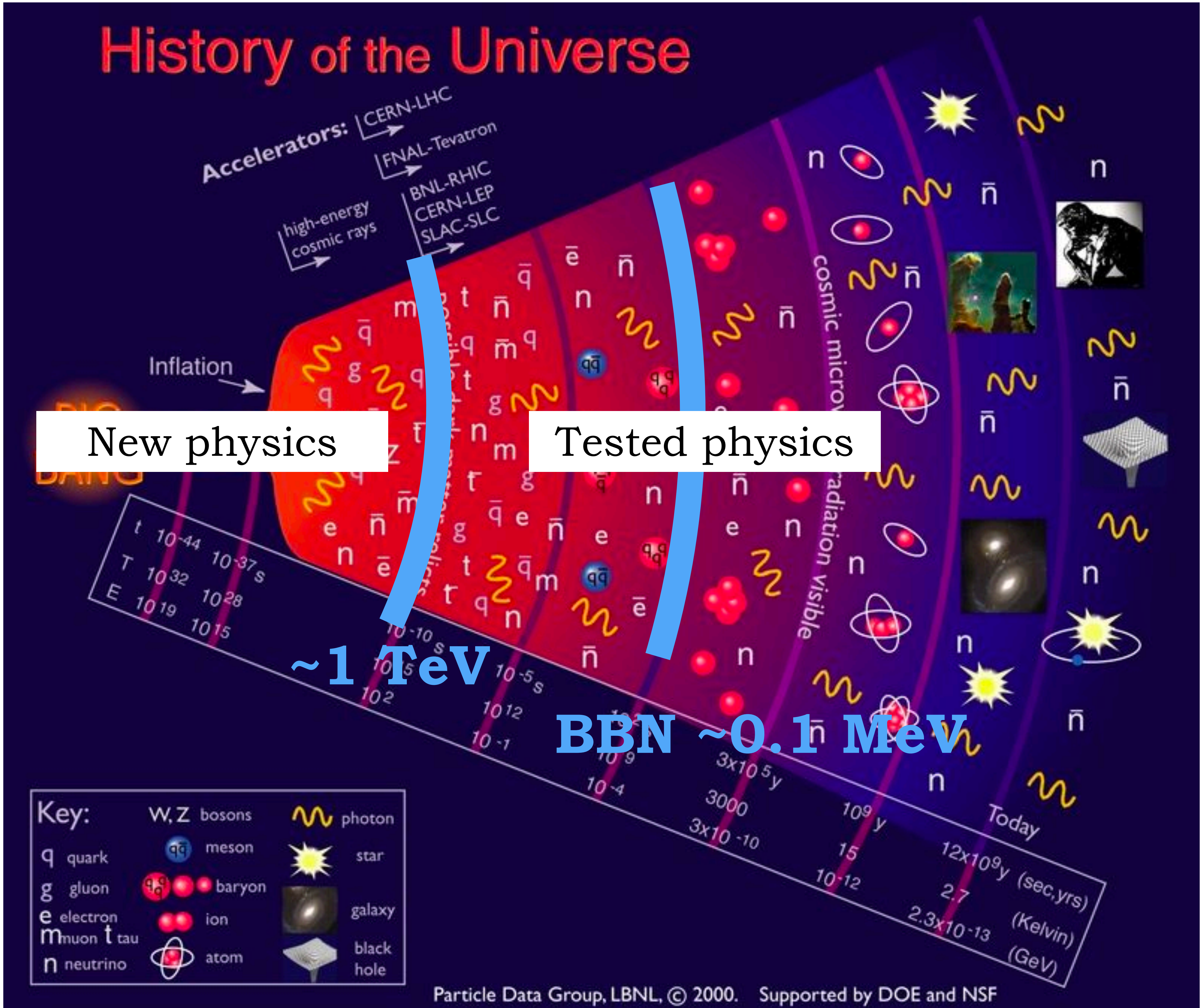
Examples of SGWB sources in the early universe



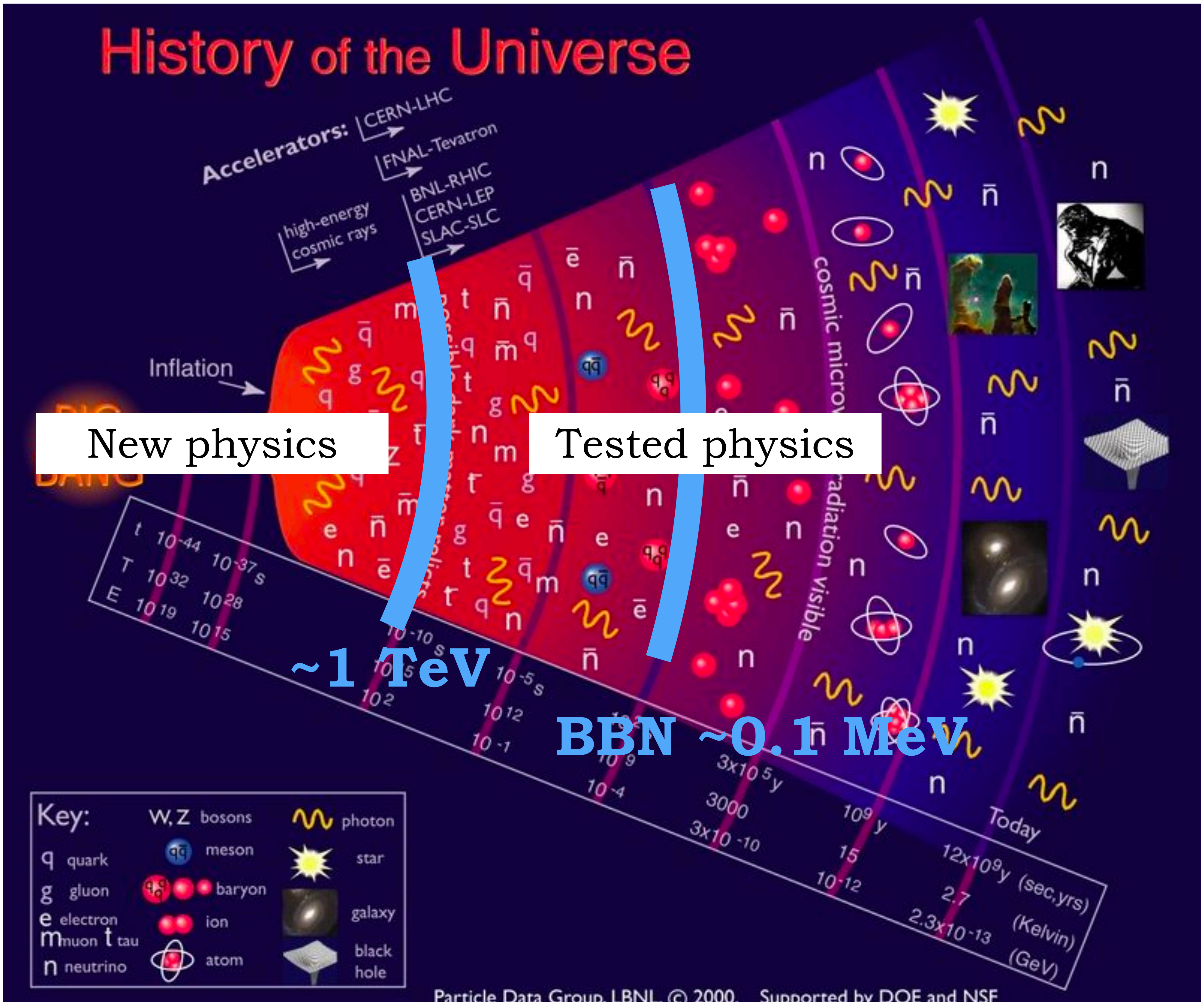
GWs can bring direct information from very early stages of the universe evolution, to which we have no direct access through em radiation —> amazing discovery potential



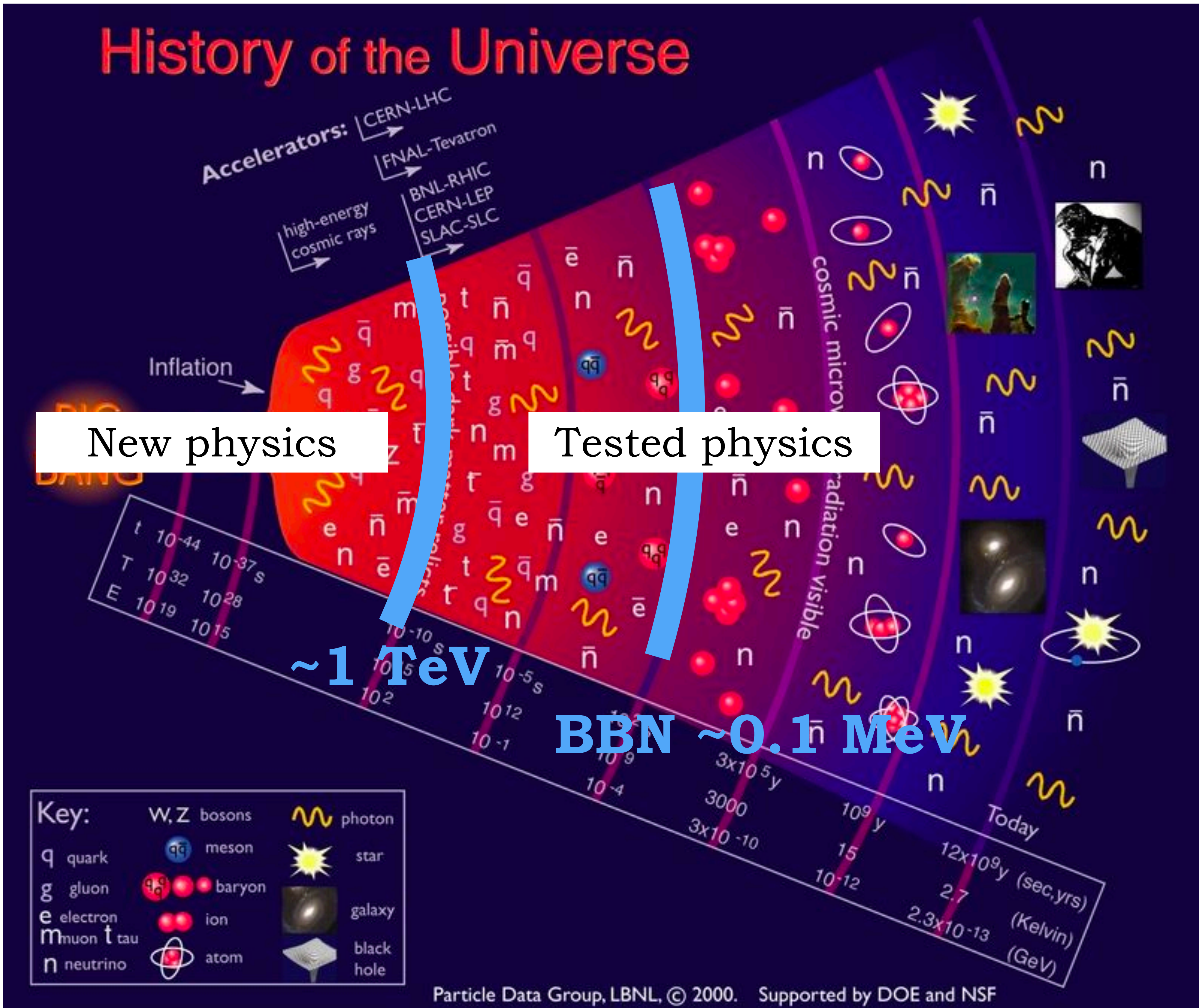
No guaranteed GW signal: predictions rely on untested phenomena, and are often difficult to estimate (non-linear dynamics, strongly coupled theories…)



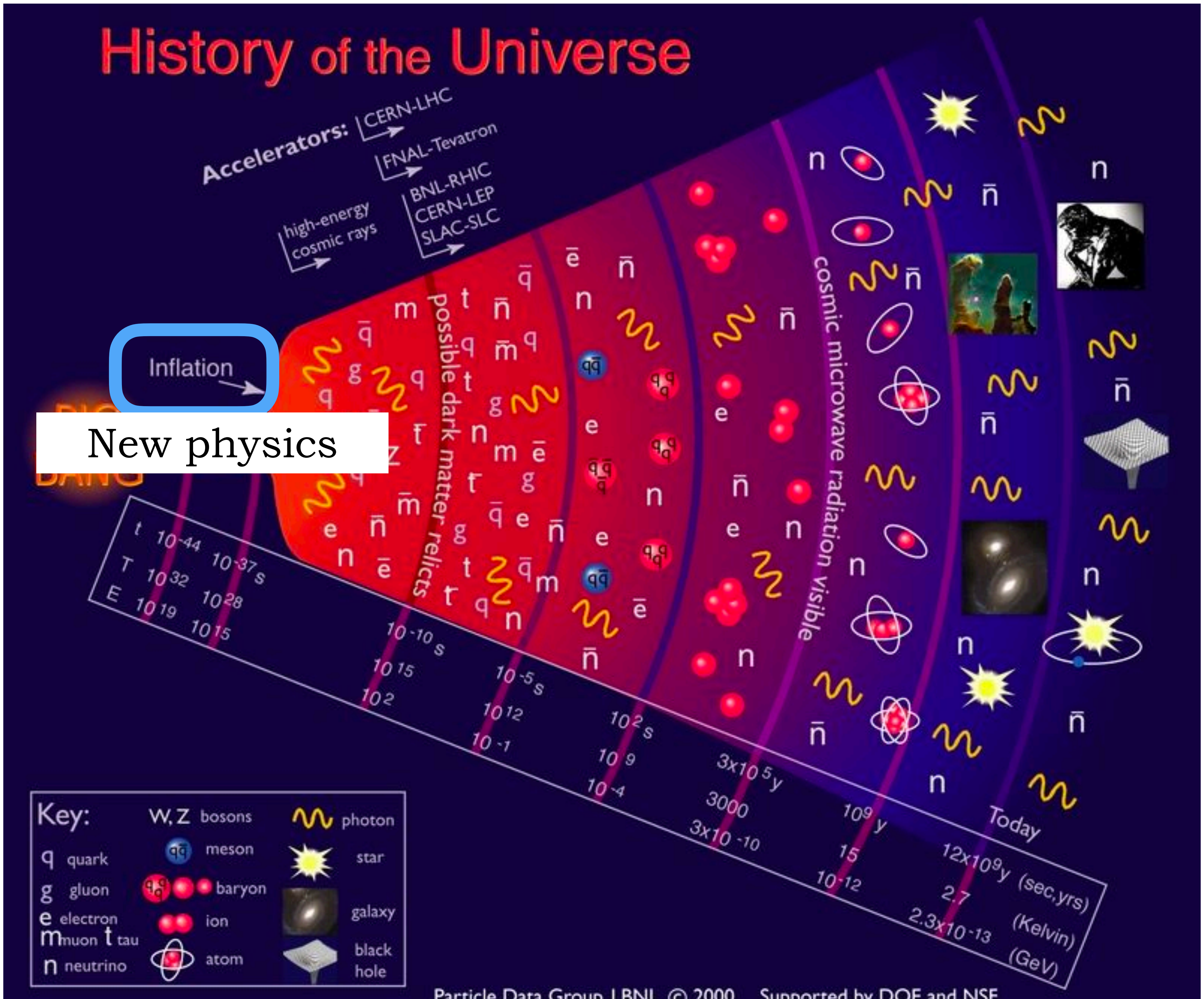
Many GW generation processes are related to PHASE TRANSITIONS



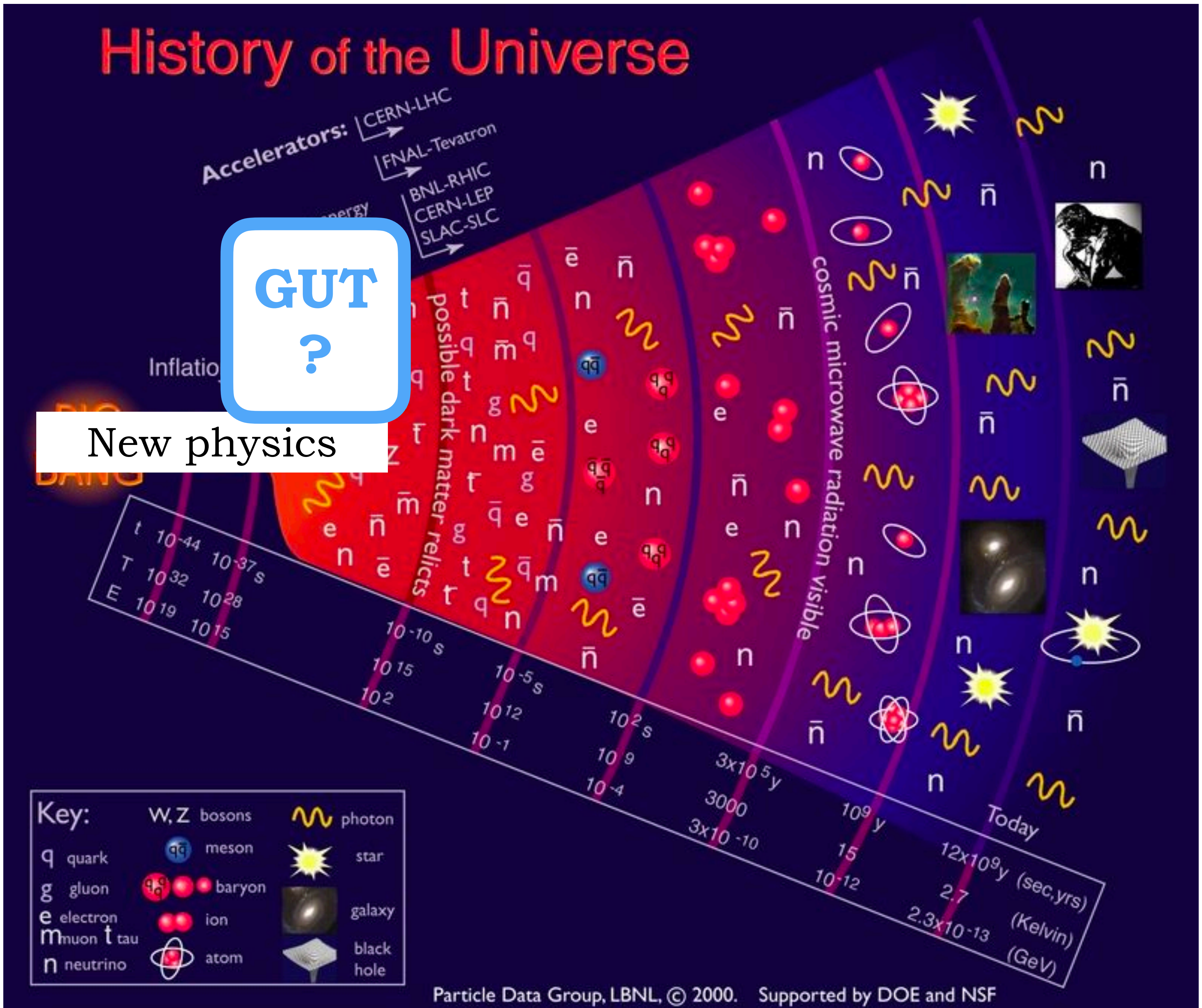
Phase transition: some field in the universe changes from one state to another, which has become more energetically favourable due to a change in external conditions (e.g. a change in temperature)



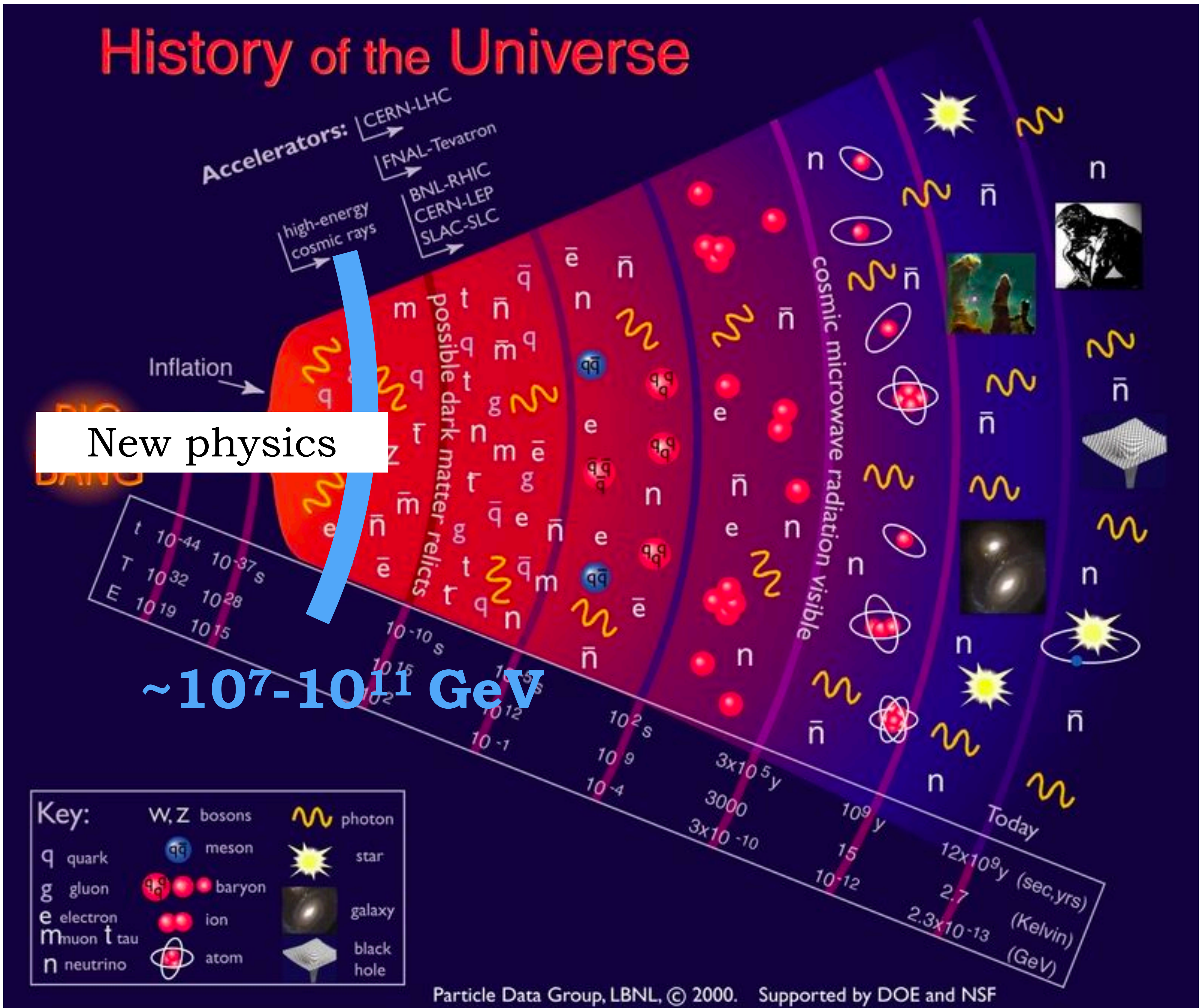
Inflation: phase transition of the Inflaton field



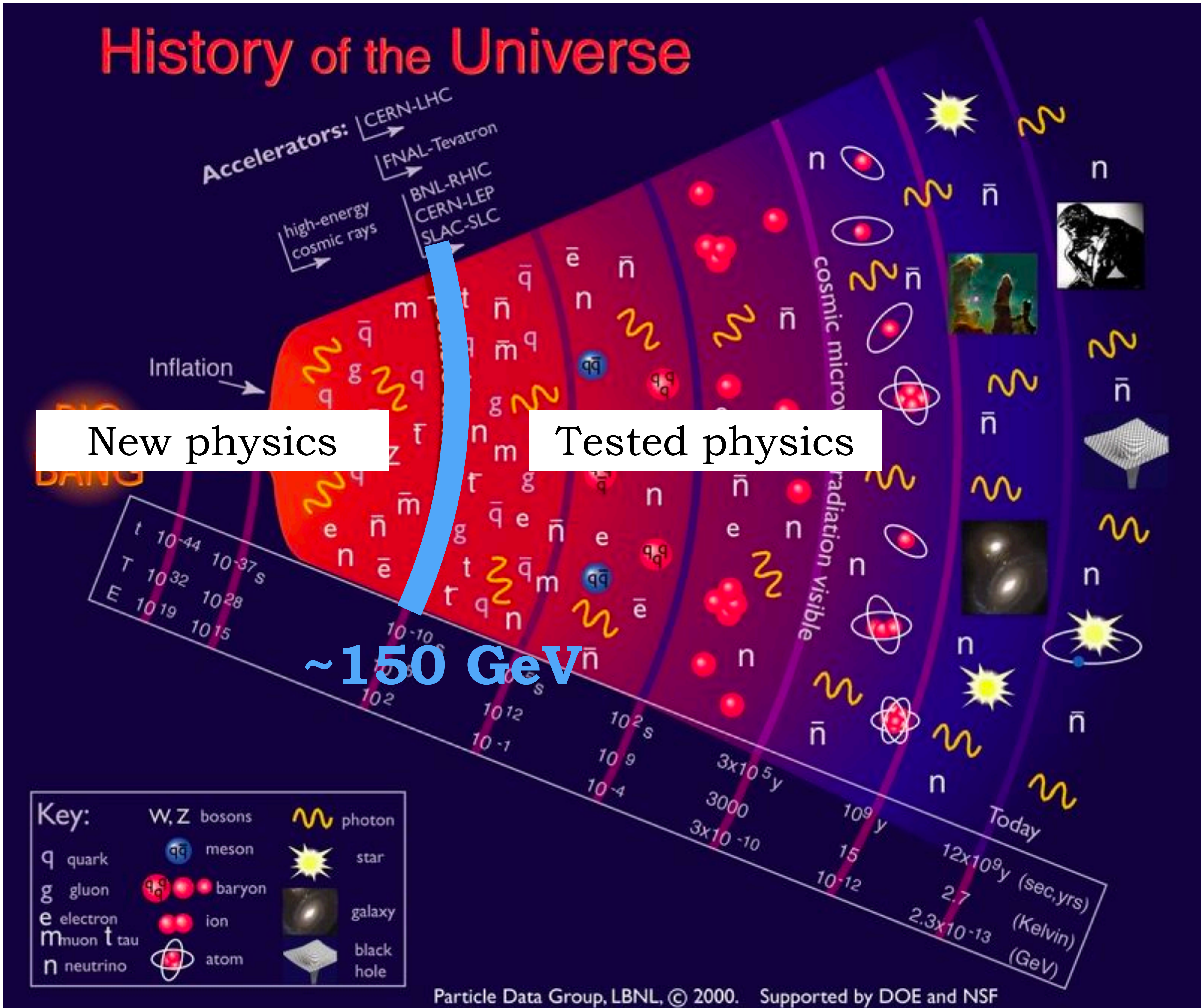
GUT phase transition or similar: related to the breaking of the symmetries of the high-energy theory describing the universe



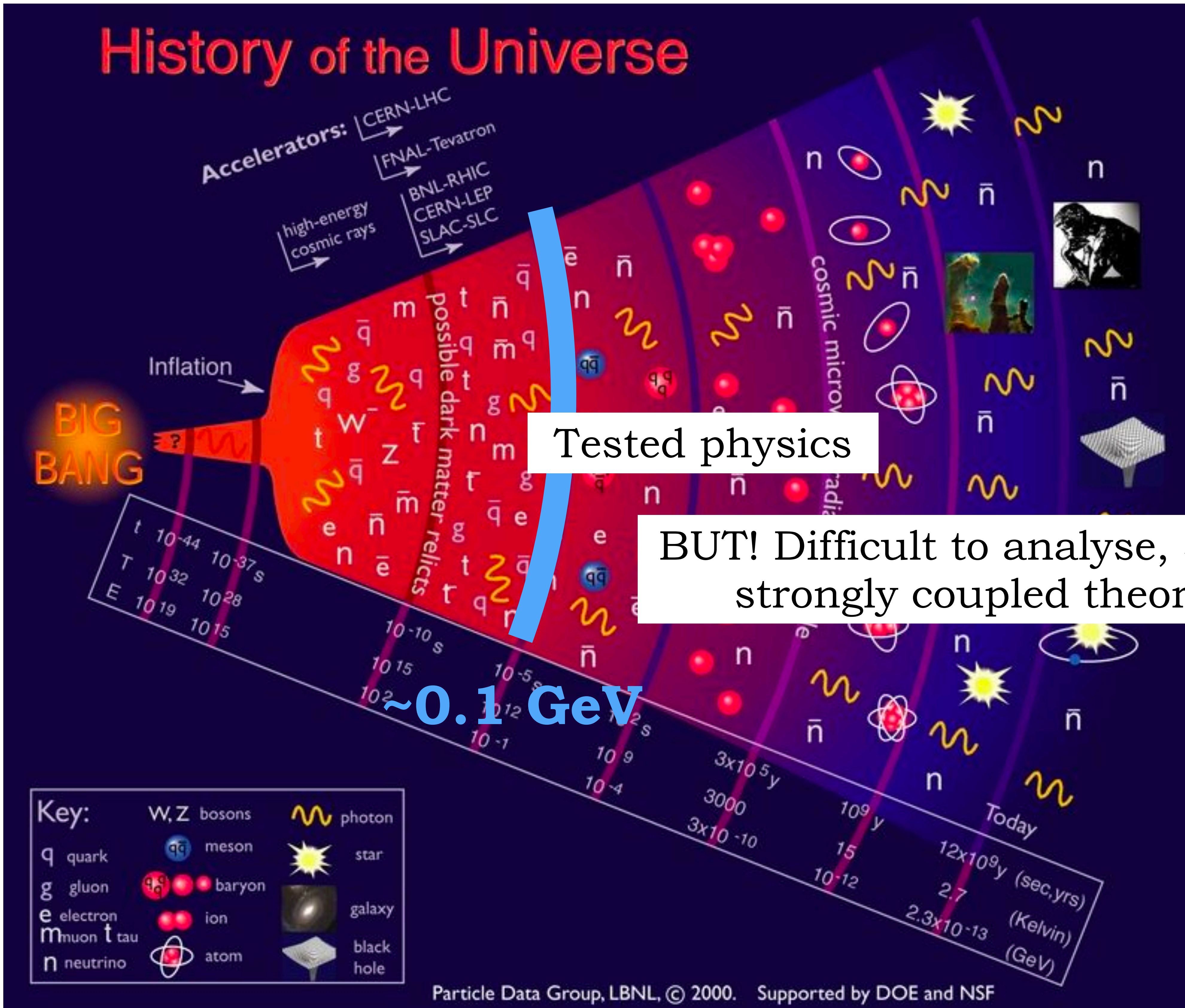
Peccei-Quinn phase transition: invoked to solve the strong CP problem



Electroweak phase transition: phase transition of the Higgs field, driven by the temperature decrease as the universe expands



QCD phase transition: phase transition related to the strong interaction, confinement of quarks into hadrons



Tested physics

BUT! Difficult to analyse, since strongly coupled theory

~0.1 GeV

Examples of GW sources in the early universe :

- irreducible SGWB from inflation
  - also sourced by second order scalar perturbations
- beyond the irreducible SGWB from inflation
  - particle production during inflation (scalar, gauge fields… coupled to the inflaton)
  - spectator fields
  - second order tensor from enhanced scalar perturbations, with primordial black holes
  - breaking symmetries (space-dependent inflaton, massive graviton)
  - modified gravity during inflation (massive GWs with $c \neq 1$)
  - …
- preheating and non-perturbative phenomena
  - parametric amplification of bosons/fermions
  - symmetry breaking in hybrid inflation
  - decay of flat directions
  - oscillons
  - …
- first order phase transition
  - true vacuum bubble collision
  - sound waves
  - (M)HD turbulence
  - …
- cosmic topological defects
  - irreducible SGWB from topological defect networks
  - decay of cosmic string loops
  - …

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