Pulsar timing arrays

Supposing the source is the inspiral of a super massive black hole binary:
what is the typical scale of the time variation of the metric perturbation?

$$f(\tau) = \frac{1}{\pi}\left(\frac{5}{256}\right)^{3/8} \frac{1}{(G\,M_c)^{5/8}\,\tau^{3/8}} \simeq 10^{-8}\,\mathrm{Hz}$$

for

$$M_c \simeq 10^9\,M_\odot$$

$$\tau = 4 \times 10^4\,\mathrm{yrs}$$

Time to coalescence

Chirp mass

$$M_c = \frac{(m_1\,m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

GW varies on a scale of about 3 years

Period of the pulsar: millisecond

$$f_{\mathrm{GW}}\,P \ll 1$$

$$\Delta T = \frac{1}{2}\frac{\hat{u}^i \hat{u}^j}{1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}}\int_{t_e-\hat{\mathbf{k}}\cdot\mathbf{r_e}}^{t_e+L-\hat{\mathbf{k}}\cdot\mathbf{r_e}-L\hat{\mathbf{k}}\cdot\mathbf{u}} \mathrm{d}X\,[h_{ij}(X+P)-h_{ij}(X)]$$

Taylor expand

Relative change in the rate of the pulses measured on Earth because of the GW passing by:

$$\frac{\Delta T}{P} \simeq \frac{1}{2}\frac{\hat{u}^i \hat{u}^j}{1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}}\,[h_{ij}(t_e+L,\mathbf{r_o}) - h_{ij}(t_e,\mathbf{r_e})]$$

Earth term | Pulsar term

Pulsar timing arrays

Supposing the source is the inspiral of a super massive black hole binary:
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Taylor expand

NB: this is the change in the frequency of the pulses due to the GWs, calculated between two successive geodesics, and NOT the redshift experienced by a photon on the same geodesic (usual gravitational redshift, depending on $\dot{h}_{ij}$)

However, the two expressions become the same in the limit of infinitesimal P

Pulsar timing arrays

Relative change in the rate of the pulses measured on Earth because of the GW passing by:

$$\frac{\Delta T}{P} \simeq \frac{1}{2}\frac{\hat{u}^i\hat{u}^j}{1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}}\left[h_{ij}(t_e+L,\mathbf{r_o})-h_{ij}(t_e,\mathbf{r_e})\right]$$

$$\sim 7\cdot 10^{-23}\,\frac{\mathrm{pc}}{d_L}\left(\frac{M_c}{M_\odot}\right)^{5/3}\left(\frac{f_{\mathrm{GW}}}{10^{-8}\,\mathrm{Hz}}\right)^{2/3} \simeq 7\cdot 10^{-16}$$

$M_c \simeq 10^9\, M_\odot$

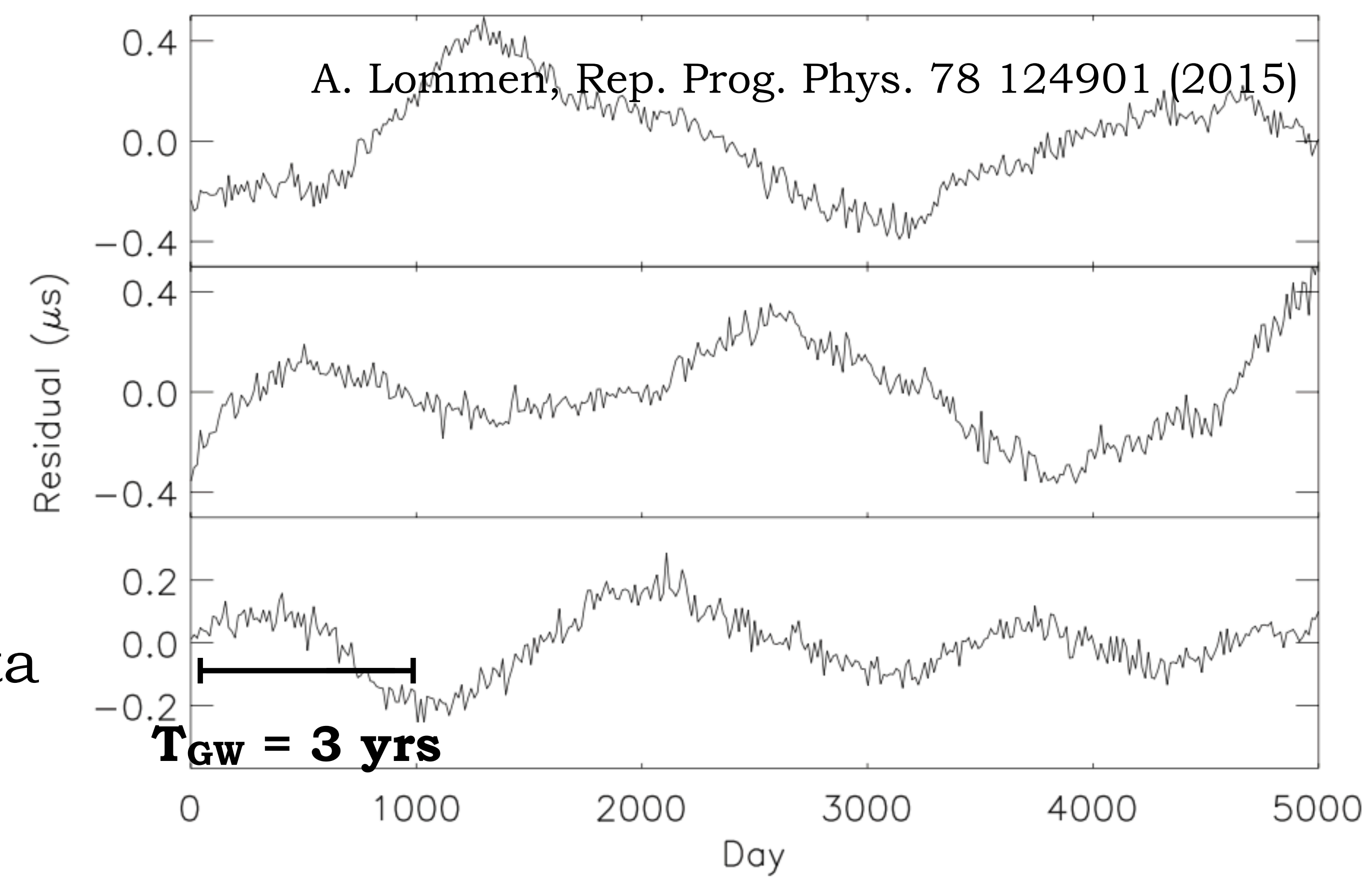
$d_L = 100\,\mathrm{Mpc}$

Timing residuals: $R(T) = \int_{t_{\mathrm{ref}}}^{t_{\mathrm{ref}}+T} \mathrm{d}t\, \frac{\Delta T}{P}$

$$R(T_{\mathrm{GW}} = 3\,\mathrm{yrs}) \simeq 60\,\mathrm{nsec}$$

A GW with period of a few years induces a timing residual of order 100 nsec, the precision of pulsar monitoring!

This renders the measurement possible, provided one has at least a few years of data

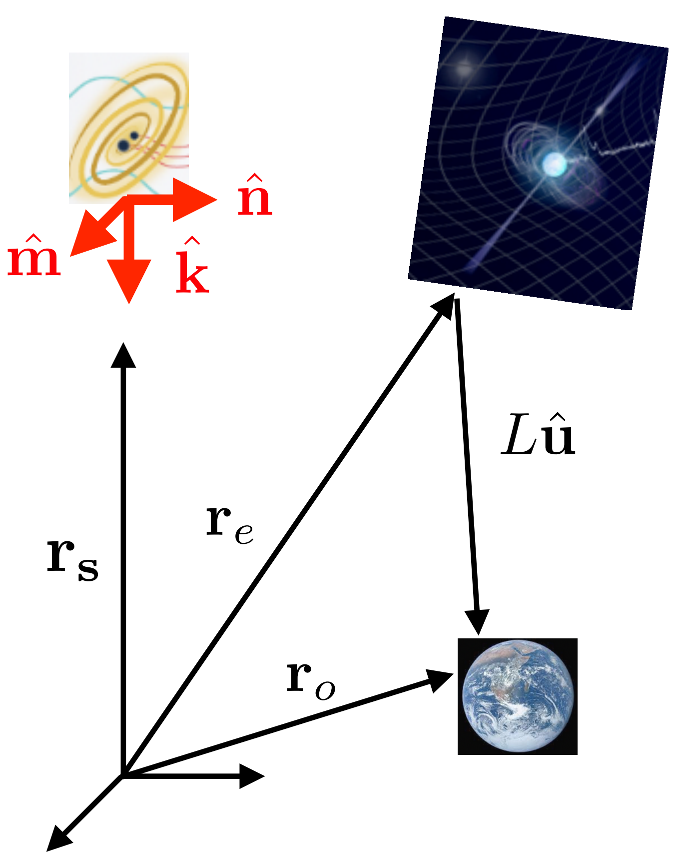


Pulsar timing arrays

HOWEVER! The signal from a single pulsar is very noisy: varying morphology of the pulses, propagation noise due to the dispersion by the interstellar medium, time referencing (time standards and solar system barycentre)...

Correlation between many pulsars to beat down the noise

$$\frac{\Delta T}{P} \simeq \frac{1}{2} \frac{\hat{u}^i \hat{u}^j}{1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{u}}} \left[h_{ij}(t_e + L, \mathbf{r_o}) - h_{ij}(t_e, \mathbf{r_e}) \right]$$



Earth term:
GW left the source at time

$$t_e + L - |\mathbf{r_o} - \mathbf{r_s}|$$

Pulsar term:
GW left the source at time

$$t_e - |\mathbf{r_e} - \mathbf{r_s}|$$

This term is different for each pulsar

In the correlation the Earth term in general dominates, but the pulsar term can create noise (unless one can determine the delay of each pulsar)

Pulsar timing arrays

Response of a pair of pulsars to a stochastic GW background

$$\langle R_a(T) R_b(T)\rangle = \int_{t_{\rm ref}}^{t_{\rm ref}+T} \mathrm{d}t' \int_{t_{\rm ref}}^{t_{\rm ref}+T} \mathrm{d}t'' \langle \frac{\Delta T}{P}(t')\Big|_a \frac{\Delta T}{P}(t'')\Big|_b \rangle$$

$$\frac{\Delta T}{P}(t')\Big|_a = \sum_r \int \frac{\mathrm{d}\mathbf{k}^3}{(2\pi)^3} h_r(\mathbf{k}) F_a^r(\hat{\mathbf{k}}) e^{-ik(t'-\hat{\mathbf{k}}\cdot\mathbf{r}_o)} \left[1 - e^{ikL_a(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}_a)}\right]$$

"Detector response" $F_a^r(\hat{\mathbf{k}}) = \dfrac{\hat{u}_a^i \hat{u}_a^j e_{ij}^r(\hat{\mathbf{k}})}{2(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}})}$

put Earth at origin to simplify

Earth term

Pulsar term

Use SGWB power spectrum

$$\langle h_r(\mathbf{k},\eta)\, h_p^*(\mathbf{q},\eta)\rangle = \frac{8\pi^5}{k^3}\, \delta^{(3)}(\mathbf{k}-\mathbf{q})\, \delta_{rp}\, h_c^2(k,\eta)$$

The Earth term dominates

$$\left[1 - e^{ikL_a(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}_a)}\right]\left[1 - e^{-ikL_b(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{u}}_b)}\right] \simeq 1$$

$a \neq b$

and SMBHB not in pulsar direction

$$kL_a = \mathcal{O}(2\pi \cdot 10^{-8}\,\mathrm{Hz} \cdot 500\,\mathrm{pc}) = \mathcal{O}(3000) \gg 1$$

Pulsar timing arrays

Response of a pair of pulsars to a stochastic GW background

$$\langle R_a(T)R_b(T)\rangle = \int_{t_{\rm ref}}^{t_{\rm ref}+T} {\rm d}t' \int_{t_{\rm ref}}^{t_{\rm ref}+T} {\rm d}t'' \langle \frac{\Delta T}{P}(t')\Big|_a \frac{\Delta T}{P}(t'')\Big|_b \rangle$$

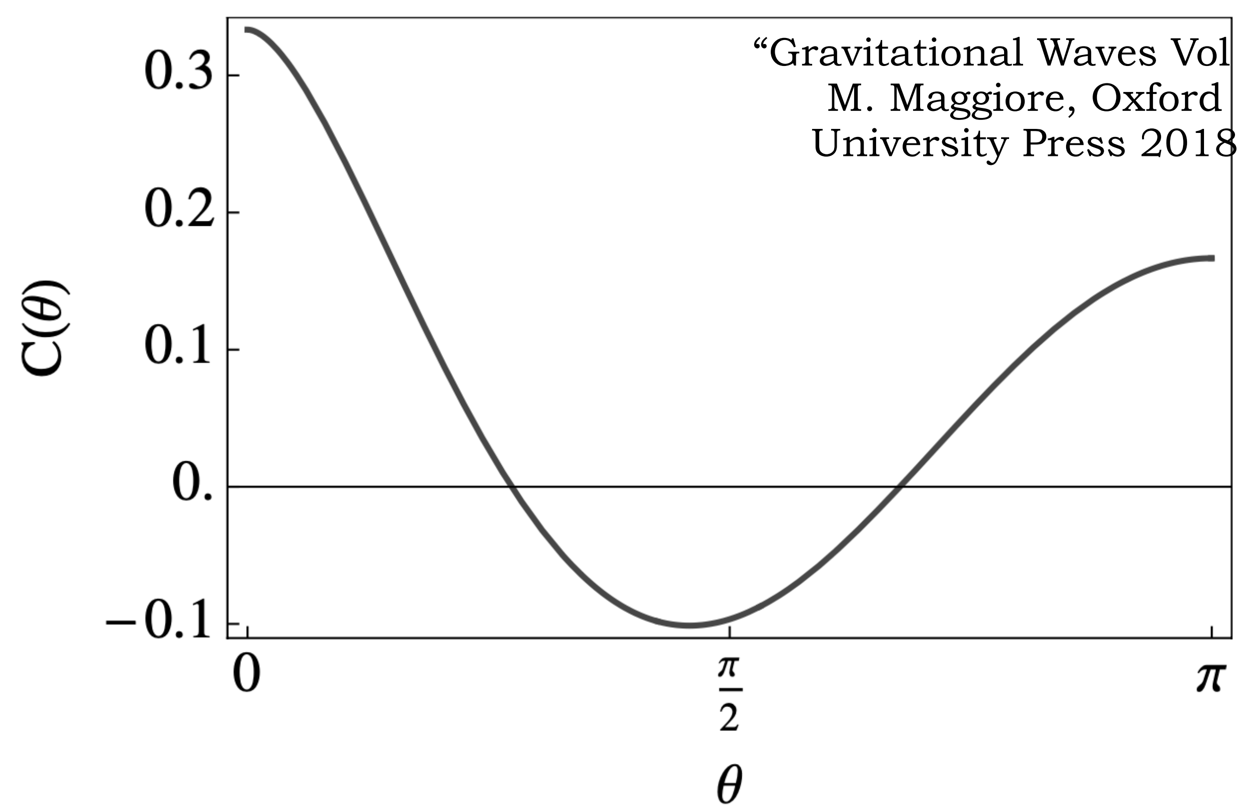
θ_{ab} angle between pulsars

$$= \mathcal{C}(\theta_{ab}) \int_0^\infty {\rm d}f\, \frac{h_c^2(f)}{(2\pi)^2 f^3} \left[1+\cos(2\pi f(T-t_{\rm ref})\right]$$

Hellings and Downs curve, characteristic of a GW signal because consequence of the quadrupolar nature of GWs

$$\mathcal{C}(\theta_{ab}) = \int \frac{{\rm d}\hat{\mathbf{k}}}{4\pi} \sum_r F_r^a(\hat{\mathbf{k}}) F_r^b(\hat{\mathbf{k}})$$
$$= \frac{1}{3} - \frac{1}{6} x_{ab} + x_{ab} \log(x_{ab})$$

$$x_{ab} = \frac{1}{2}(1-\cos\theta_{ab})$$



“Gravitational Waves Vol 2”, M. Maggiore, Oxford University Press 2018

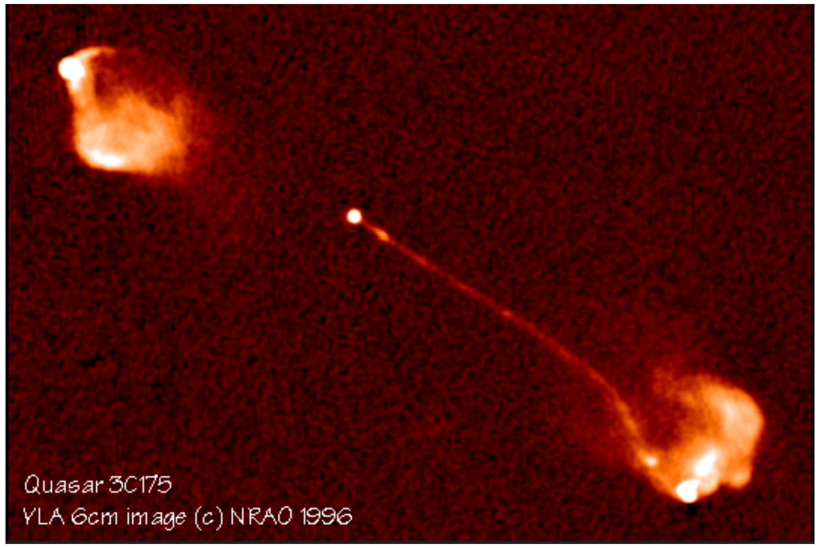
Pulsar timing arrays

Observation of the *Hellings and Downs curve* is **smoking gun evidence** of GW detection
(not only SGWB but also from a single SMBHB - Cornish & Sesana arXiv:1305.0326)

time referencing errors generate a correlated noise but:

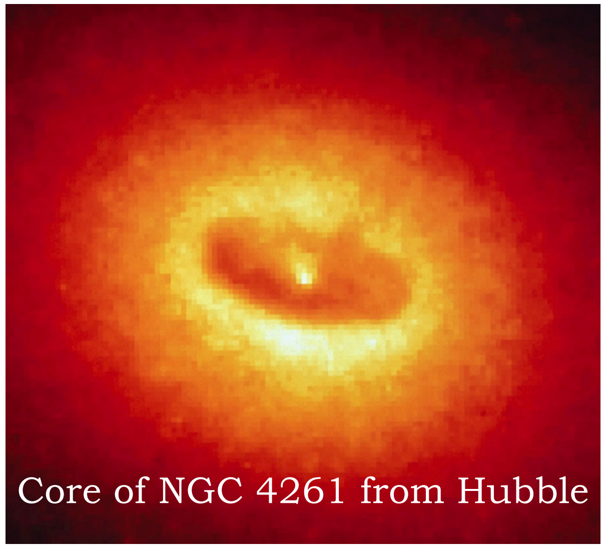
- noise from uncertainties on the time standards on Earth is independent on the pulsar angles
- noise from uncertainties on the solar system barycentre position take the form of a (rotating) dipole (dependent on the cosinus of the angle) -> can contaminate the quadrupole

A SGWB from SMBHBs is the best candidate source in PTA frequency band



What are SMBHBs?

- They have been observed in the core of galaxies and are the central engine of active galactic nuclei
- They can originate from the collapse of massive stars (~100 $M_\odot$) or gas clouds (~ 10^4 $M_\odot$), and then grow in mass through gas accretion and/or mergers following the collision of their host galaxies (but their origin is still to be confirmed, they can also be primordial…)
- JWST sees SMBHs up to very high redshift z ~ 11
- Their presence is linked to the formation of galaxies and matter structure in the Universe

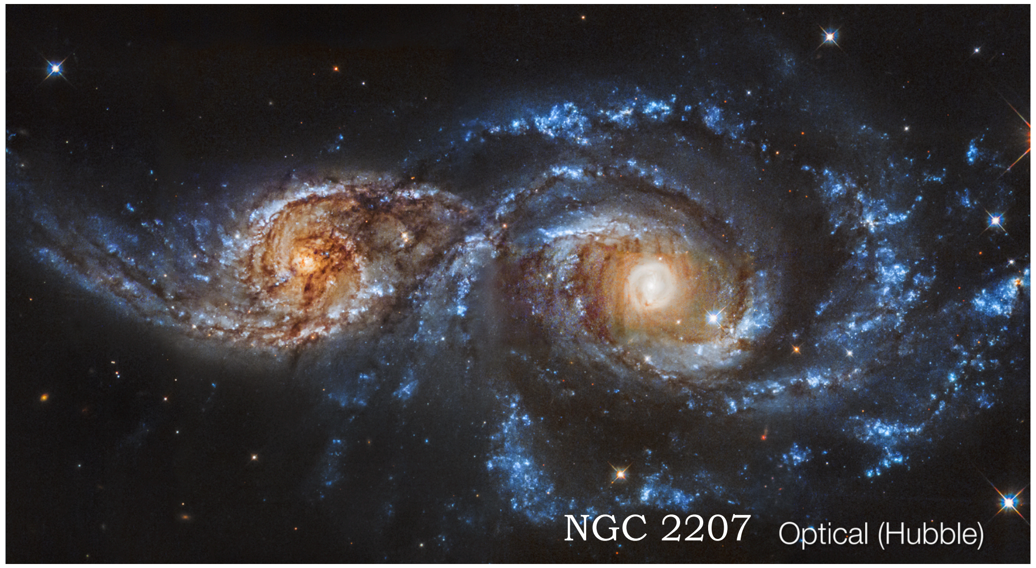


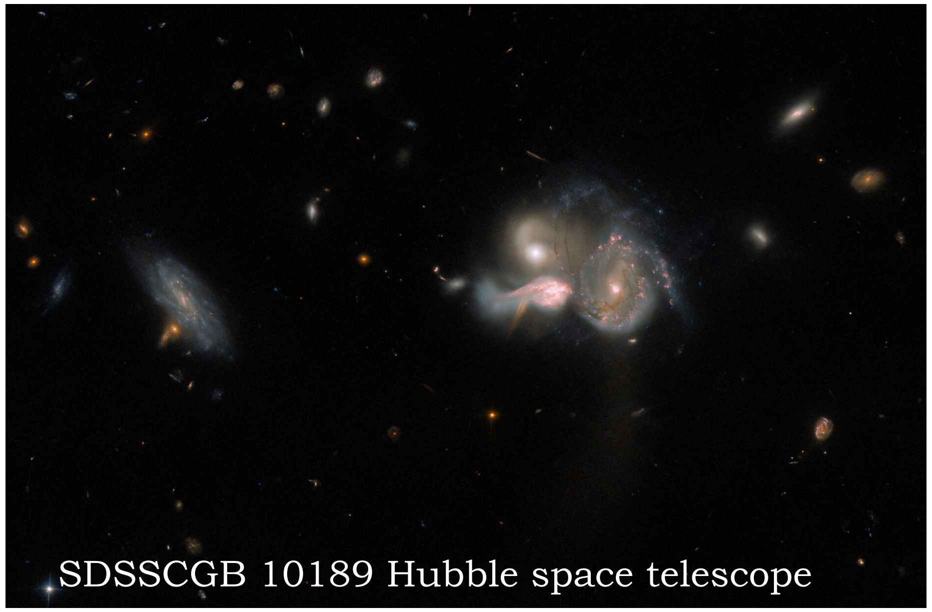
Pulsar timing arrays

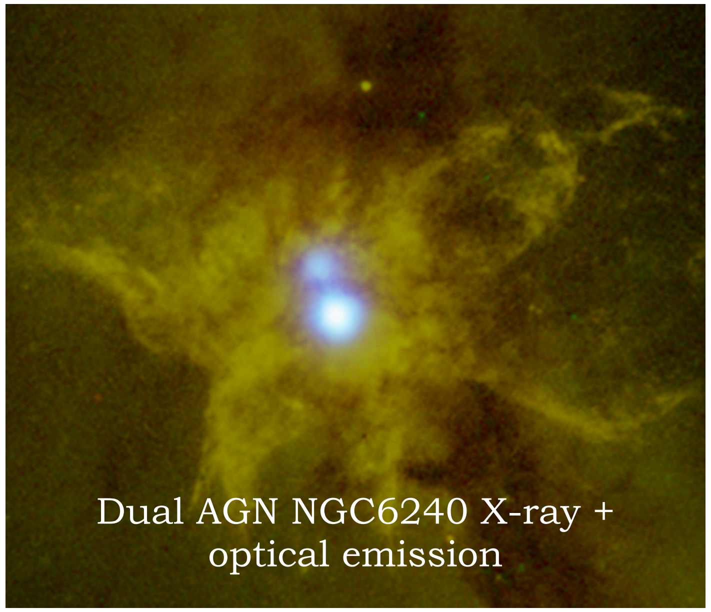
HOWEVER!

- To emit GWs, SMBHs must be paired in gravitationally bound binaries in the GW emitting regime: separation of ~ 0.01-0.001 pc
- Binaires can be formed after the collision of two galaxies: the MBH previously at the centre of galaxies get to ~ kpc separation (X-ray evidence from dual AGNs)
- Dynamical friction drives the two MBH towards the centre of the new galaxy until they form a bound binary
- 3-body interaction with the surrounding stars subsequently shrinks the binary to pc separation
- How to get them to the millipc separation necessary for GW emission and merger within one Hubble time? “LAST PARSEC PROBLEM”
- maybe more stars arrive, or there is gas drag from interaction with a circumbinary disk, and/or another MBH arrives…

If PTAs observe the SGWB from SMBHBs it means that SMBHBs exist and merge in the universe!







Pulsar timing arrays

Prediction from SMBHBs formation scenarios

How does the SGWB from SMBHBs look like?

Characteristic strain: red spectrum

$$h_c(f) = A \left(\frac{f}{f_{\rm ref}}\right)^{-\alpha} \quad \text{with} \quad \alpha = \frac{2}{3}$$

$A = \mathcal{O}(10^{-15})$ at

$f_{\rm ref} = 1\,{\rm yr}^{-1}$

Circular binary

Timing residuals power spectral density: Also red spectrum

$$S_{ab}(f) = \mathcal{C}(\theta_{ab})\Phi(f)$$

$$\Phi(f) = \frac{A^2}{(2\pi)^2} f_{\rm ref}^{-3} \left(\frac{f}{f_{\rm ref}}\right)^{-\gamma} \quad \text{with} \quad \gamma = 2\alpha + 3 = \frac{13}{3}$$

Where does this spectral shape come from?

SGWB from a population of inspiralling binaries

$$h_c(f) = A\left(\frac{f}{f_{\rm ref}}\right)^{-\alpha} \text{with} \quad \alpha = \frac{2}{3}$$

in terms of the power spectrum of the GW energy density becomes

$$\Omega_{\rm GW}(f) = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f) = \Omega_{\rm GW}(f_{\rm ref})\left(\frac{f}{f_{\rm ref}}\right)^{2/3}$$

$$\frac{\rho_{\rm GW}^{\rm (tot)}}{\rho_c} = \int_0^\infty \frac{{\rm d}f}{f}\,\Omega_{\rm GW}(f) = \int {\rm d}\xi \int {\rm d}V_c \int {\rm d}\tau_c\, \frac{{\rm d}^3 N(z,\tau_c,\xi,\theta)}{{\rm d}\xi {\rm d}V_c {\rm d}\tau_c}\, \frac{\rho_{\rm GW}^{\rm (event)}}{\rho_c}$$

- ${\rm d}\xi$: Parameters of the binary signal (essentially chirp mass)
- ${\rm d}V_c$: Coming volume
- ${\rm d}\tau_c$: Time to coalescence
- $\frac{{\rm d}^3 N}{{\rm d}\xi {\rm d}V_c {\rm d}\tau_c}$: Number density of GW sources (given within an astrophysical model for the binary population)
- $\frac{\rho_{\rm GW}^{\rm (event)}}{\rho_c}$: GW energy emitted by a single event

At the source

$$\frac{\rho_{\rm GW}^{\rm (event)}}{\rho_c} = \frac{1}{16\pi G\rho_c}\,\frac{\langle \dot h_+^2 + \dot h_\times^2\rangle}{(1+z)^4}$$

SGWB from a population of inspiralling binaries

$$\dot{h}_+(t_S) = \frac{4\pi^{2/3}}{a(t_S)r}(G\,M_c)^{5/3}\left(\frac{1+\cos^2\theta}{2}\right)\underbrace{\frac{\mathrm{d}[f^{2/3}(t_S)\cos(2\Phi(t_S))]}{\mathrm{d}t_S}}$$

In the limit of circular orbit with slowly varying radius

$$\dot{f}_S \ll f_S^2$$

$$\simeq -f^{2/3}(t_S)2\,\dot{\Phi}(t_S)\sin(2\Phi(t_S))$$

$$\dot{\Phi}(t_S) \to \pi f_S$$

$$\langle \dot{h}_+^2(t_S)\rangle = \frac{32}{a_S^2 r^2}(\pi G\,M_c)^{10/3}\left(\frac{1+\cos^2\theta}{2}\right)^2 f_S^{10/3}$$

$$\frac{\rho_{\mathrm{GW}}^{(\mathrm{tot})}}{\rho_c} = \int \mathrm{d}\xi \int \mathrm{d}\tau_c \int \mathrm{d}z \frac{d_M^2}{H(z)} \frac{\mathrm{d}^3 N(z,\tau_c,\xi,\theta)}{\mathrm{d}\xi \mathrm{d}V_c \mathrm{d}\tau_c} \frac{1}{16\pi G\rho_c(1+z)^4}$$

$$\frac{32}{a_S^2 r^2}(\pi G M_c f_S)^{10/3}\int \mathrm{d}\Omega\left[\left(\frac{1+\cos^2\theta}{2}\right)^2 + \cos^2\theta\right]$$

$$d_M = a_0\, r$$

Extra factor $(1+z)^2$

$$\mathrm{d}V_c = \frac{d_M^2}{H(z)}\mathrm{d}\Omega\,\mathrm{d}z$$

$$= 16\pi/5$$

SGWB from a population of inspiralling binaries

Express the integral over time to coalescence in terms of frequency and change to frequency at the observer

$$\frac{\mathrm{d}f_S}{\mathrm{d}\tau_c} = \frac{96\pi^{8/3}}{5}(G\,M_c)^{5/3} f_S^{11/3}$$

$$f_S = f(1+z)$$

$$\frac{\rho_{\mathrm{GW}}^{(\mathrm{tot})}}{\rho_c} = \frac{\pi^{2/3}}{3\,G\rho_c}\int \frac{\mathrm{d}f}{f} f^{2/3} \int \mathrm{d}\xi \int \frac{\mathrm{d}z}{H(z)(1+z)^{4/3}}(GM_c)^{10/3}\frac{\mathrm{d}^3N(z,\tau_c,\xi,\theta)}{\mathrm{d}\xi\mathrm{d}V_c\mathrm{d}\tau_c}$$

$$\frac{\rho_{\mathrm{GW}}^{(\mathrm{tot})}}{\rho_c} = \int_0^\infty \frac{\mathrm{d}f}{f}\,\Omega_{\mathrm{GW}}(f)$$

$$\Omega_{\mathrm{GW}}(f) = \Omega_{\mathrm{GW}}(f_{\mathrm{ref}})\left(\frac{f}{f_{\mathrm{ref}}}\right)^{2/3}$$

$\downarrow$

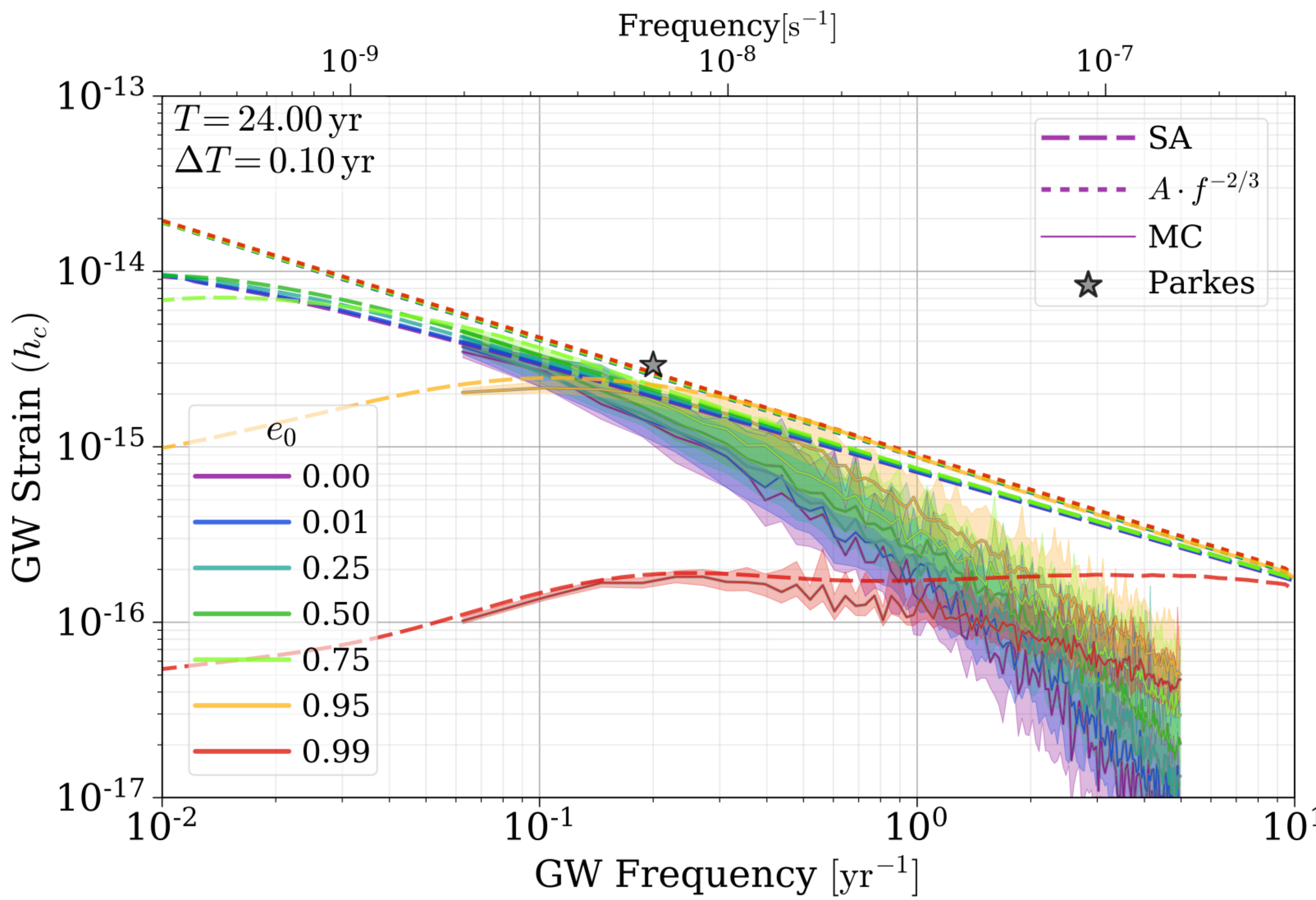
SGWB amplitude determined by the population characteristics and the cosmology

SGWB from a population of inspiralling binaries

The features of the SGWB power spectrum (amplitude A, slope α…) depend on the population characteristics such as the binary merger rate, its dependence with mass and redshift, the surrounding stellar density, the initial binary eccentricity…

- The assumption of homogeneous and isotropic SGWB isn't justified at high frequency: SMBHBs are less numerous, the SGWB slope is steeper, and discreteness starts to appear with spikes due to the loudest SMBHBs
- Interactions with the binary environment makes hardening stronger and suppresses SGWB power at low frequency
- Eccentricity enhances GW emission at higher frequencies

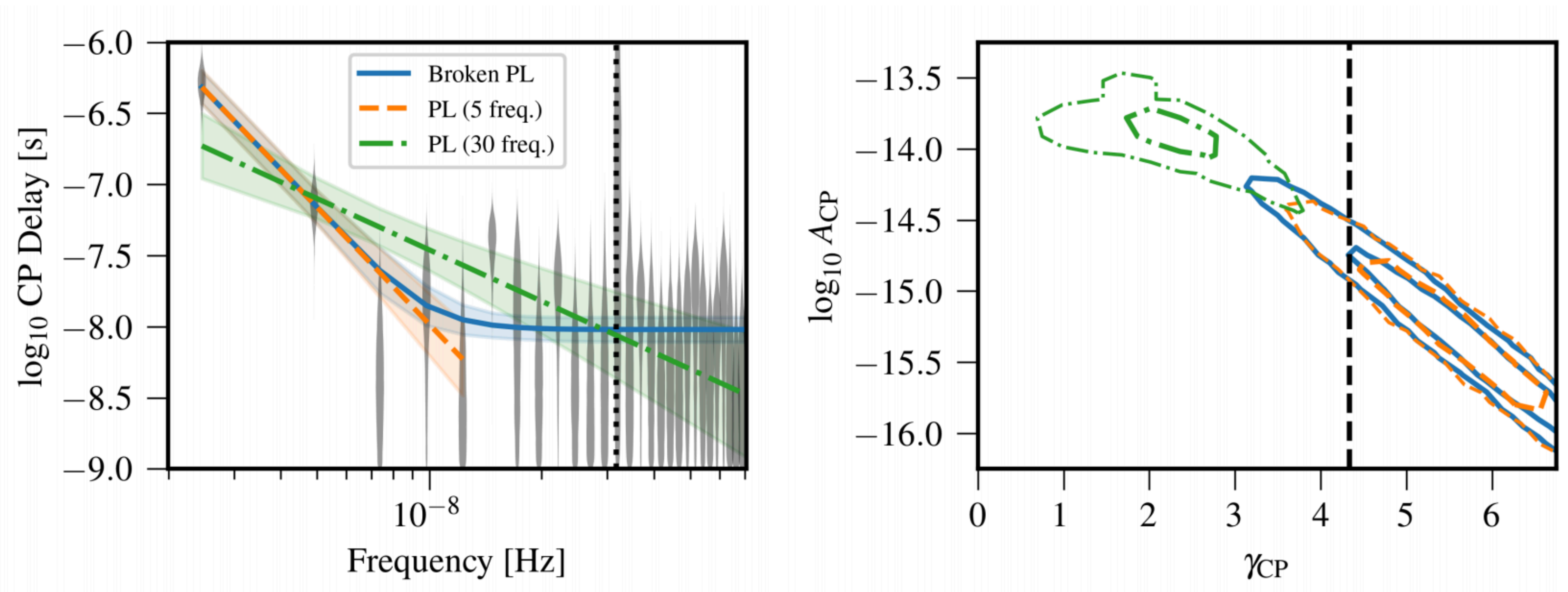
L.Z. Kelley et al, arXiv:1702.02180



Pulsar timing arrays

In 2020, NANOGrav (followed by EPTA and PPTA) has announced the presence of a common red noise in their 12.5 years data

$$\Phi(f) = \frac{A^2}{(2\pi)^2} f_{\mathrm{ref}}^{-3} \left(\frac{f}{f_{\mathrm{ref}}}\right)^{-\gamma} \quad \text{with} \quad \gamma = 2\alpha + 3 = \frac{13}{3}$$

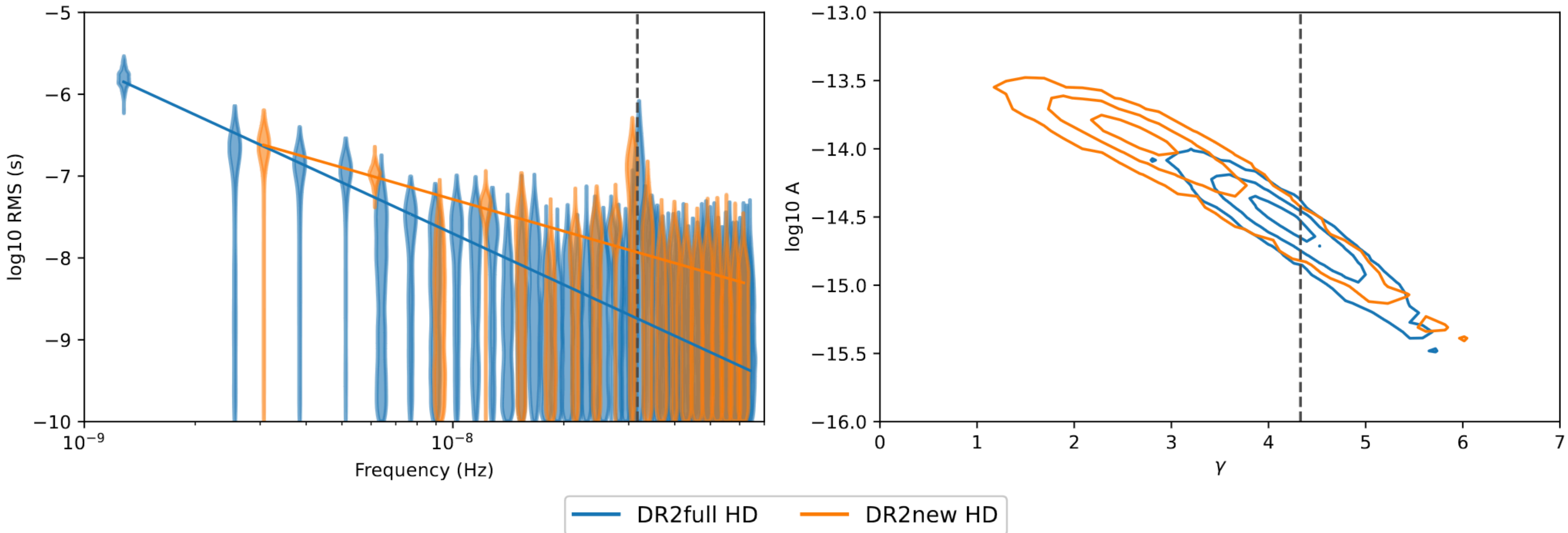


NANOGrav collaboration: arXiv:2009.04496

Pulsar timing arrays

Last year, all PTAs have confirmed the observation of a common red noise supplemented by evidence for the Hellings-Downs correlation

EPTA results:

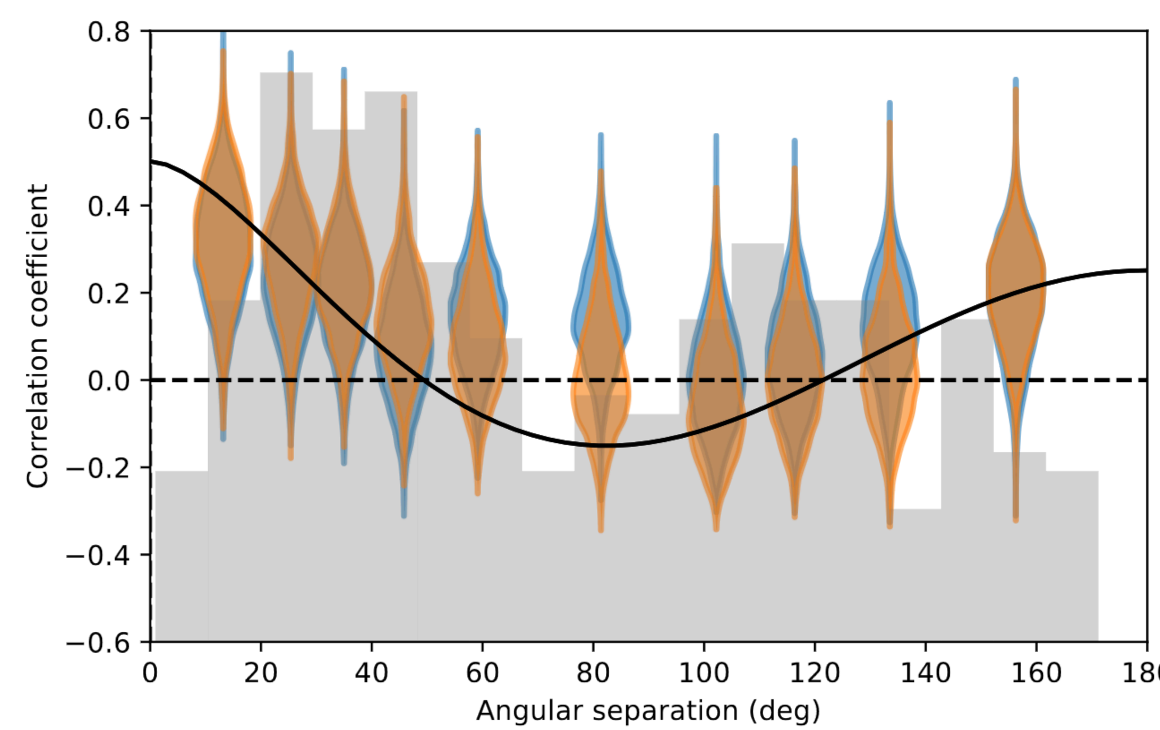


DR2new (10.3 yrs):

$$\log A = -13.94^{+0.23}_{-0.48} \quad \gamma = 2.71^{+1.18}_{-0.73} \quad \mathcal{B} = 60$$

DR2full (25 yrs):

$$\log A = -14.54^{+0.28}_{-0.41} \quad \gamma = 4.19^{+0.73}_{-0.63} \quad \mathcal{B} = 4$$



J. Antoniadis et al, arXiv:2306.16214

Pulsar timing arrays

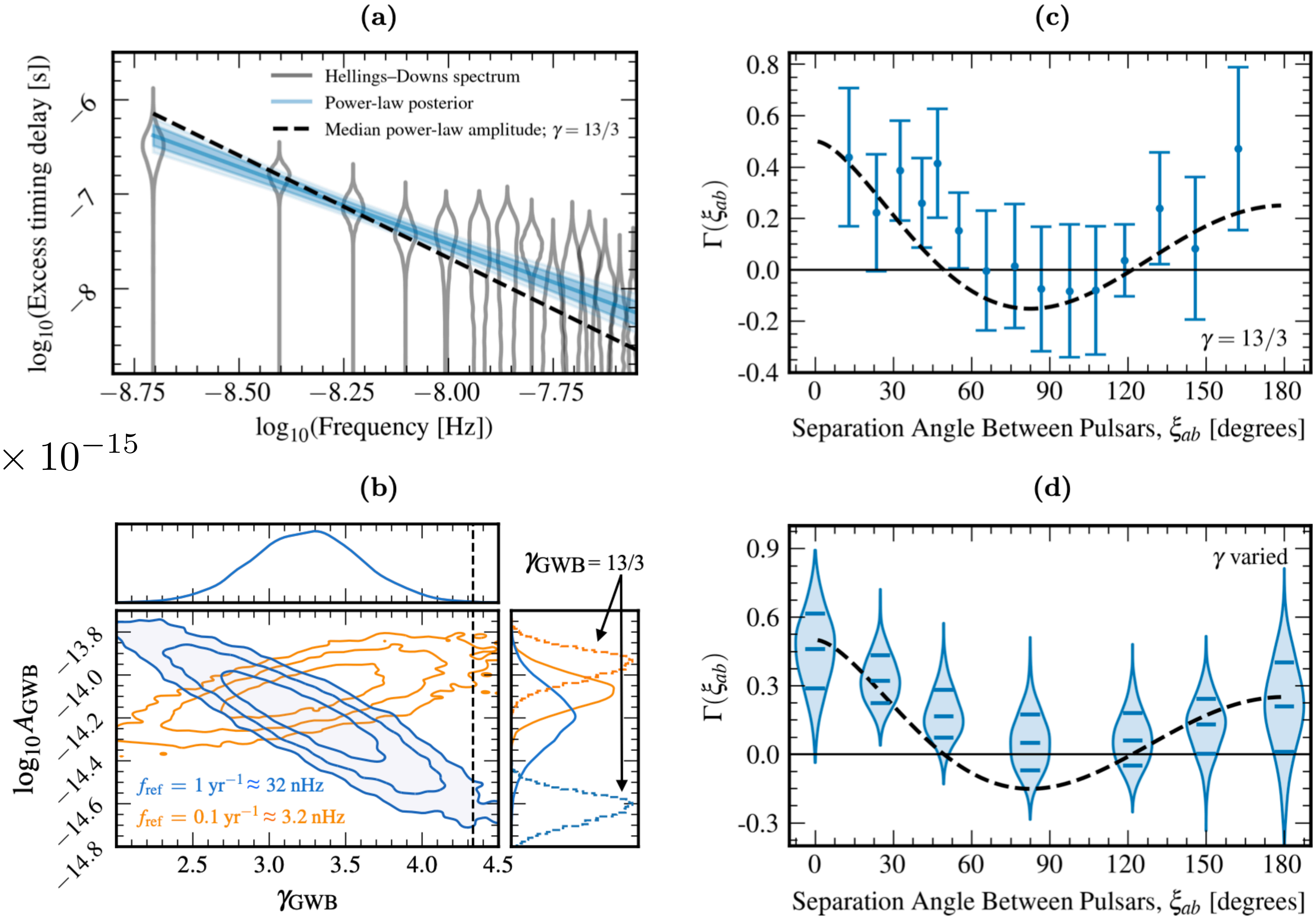
Last year, all PTAs have confirmed the observation of a common red noise supplemented by evidence for the Hellings-Downs correlation

NANOGrav results:

$$A = -6.4^{+4.2}_{-2.7} \times 10^{-15}$$

$$\gamma = 3.2^{+0.6}_{-0.6}$$

$$\mathcal{B} = 226$$

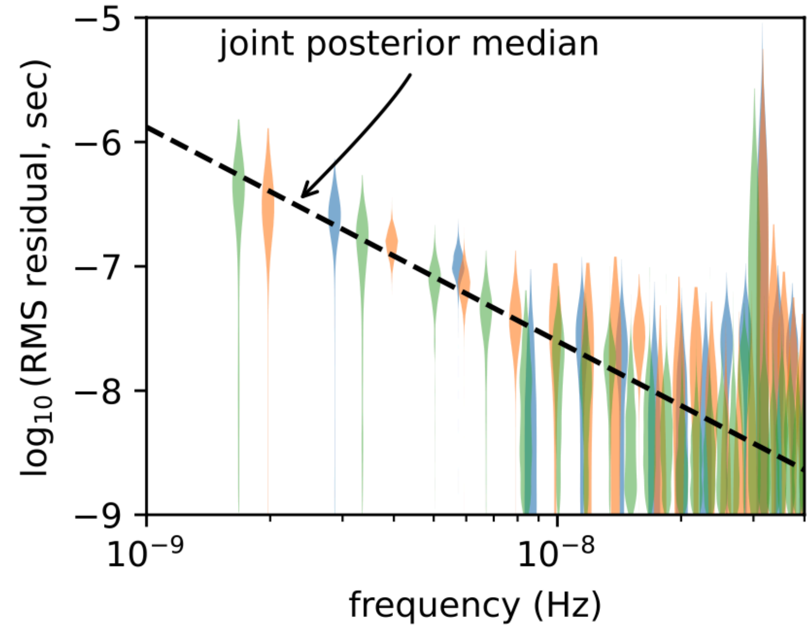


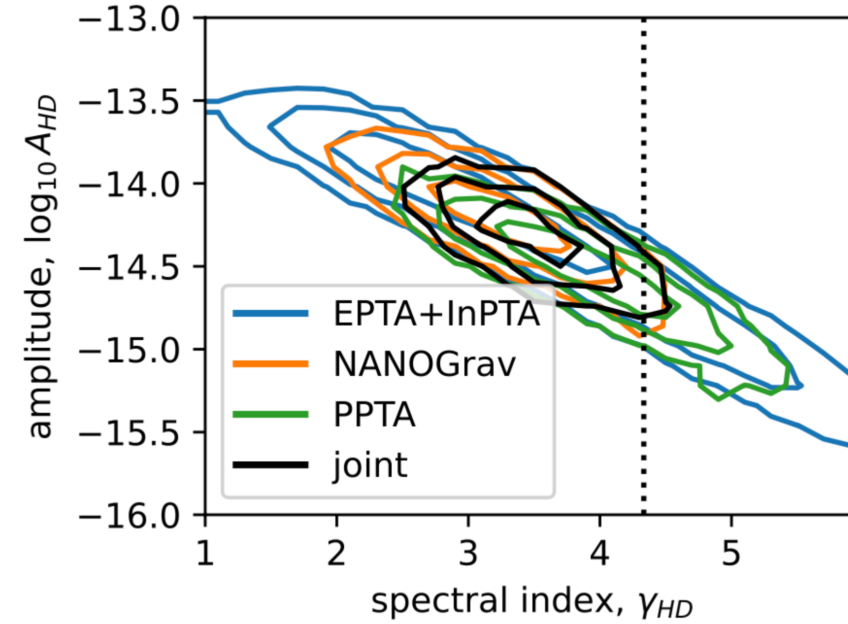
G. Agazie et al, arXiv:2306.16213

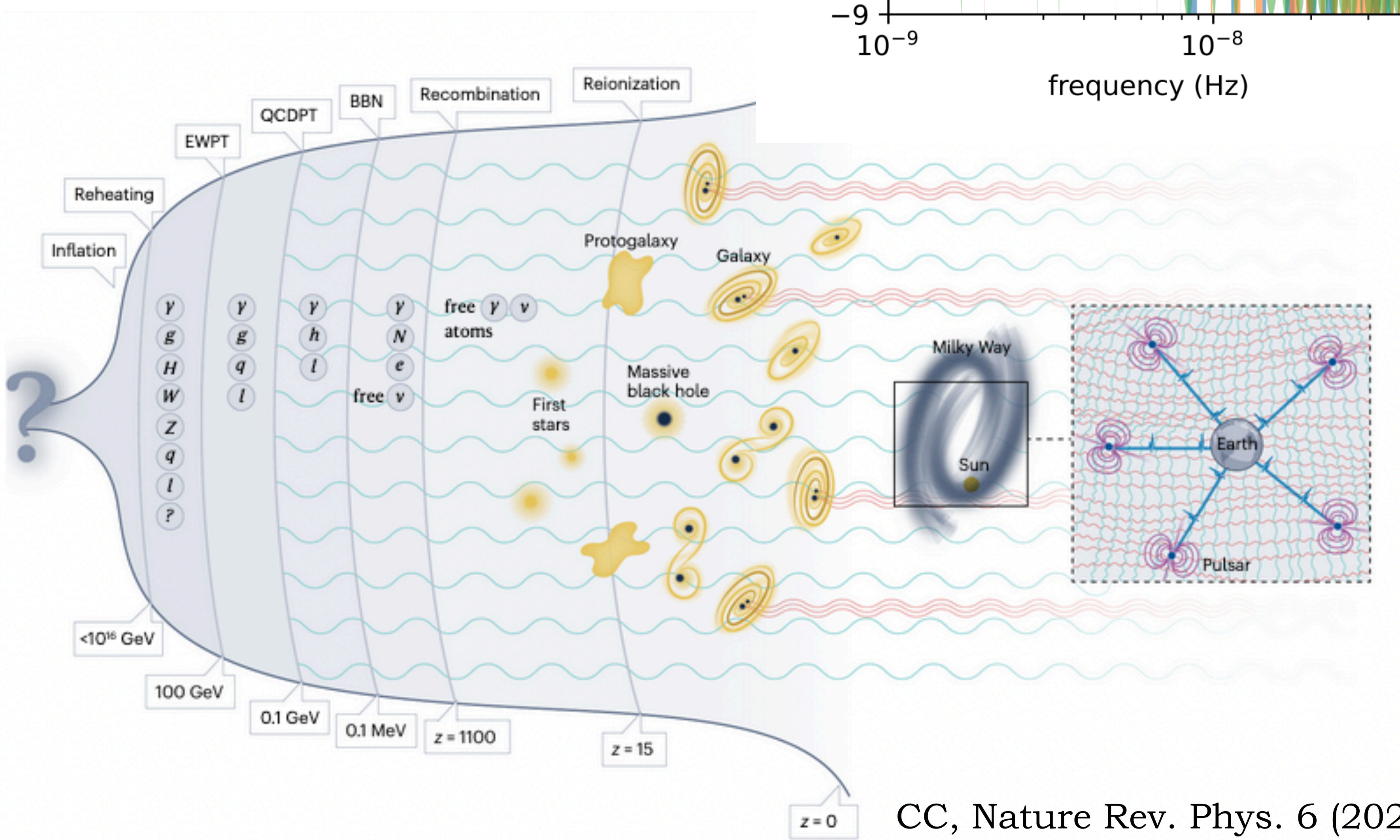
Pulsar timing arrays

- The slopes are shallower than 13/3 (but maybe the model isn't fully adapted…)
- The amplitude is consistent with the one from a SMBHBs SGWB
- All datasets are consistent within 1σ as shown by IPTA

IPTA Collaboration, arXiv:2309.00693



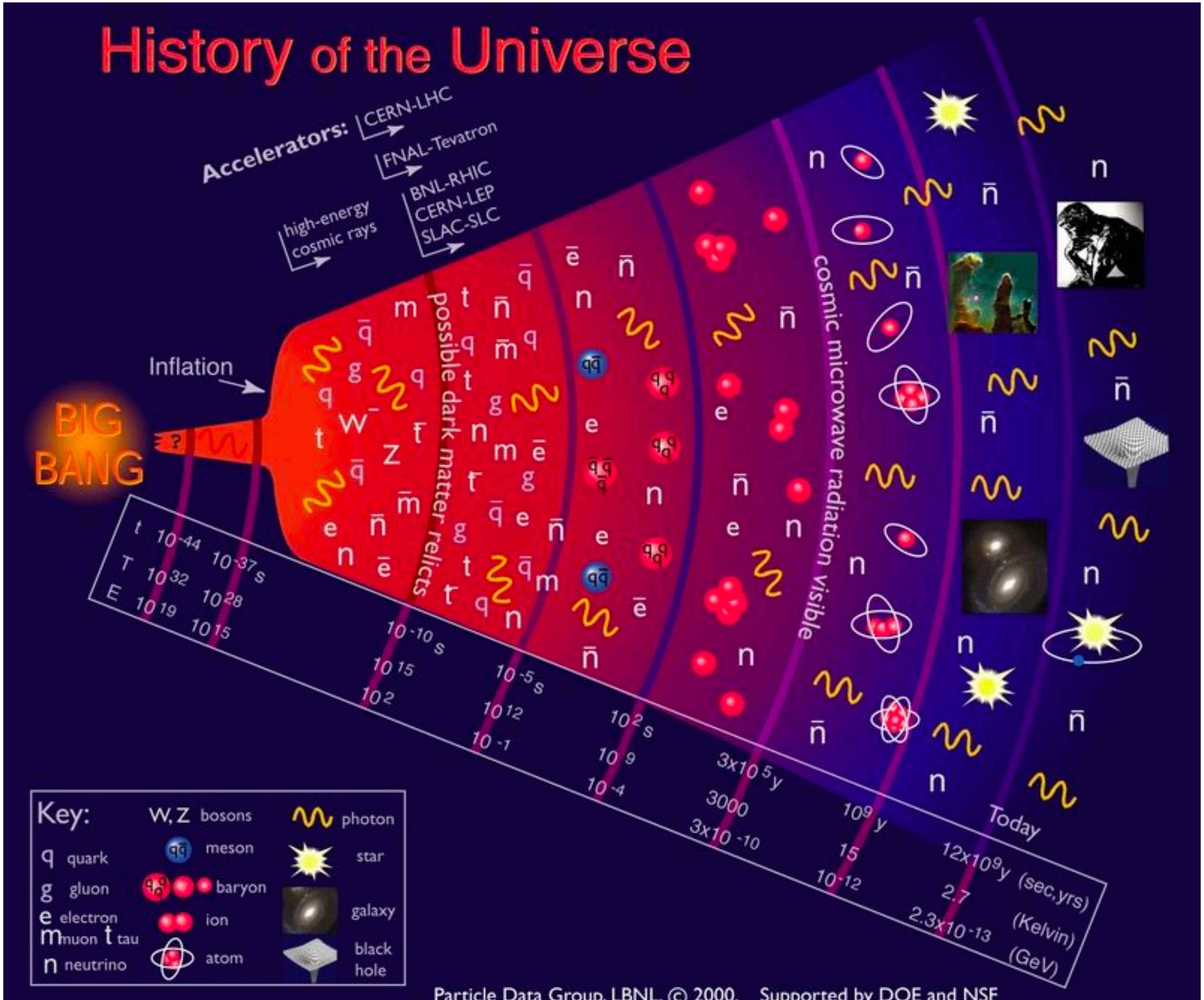




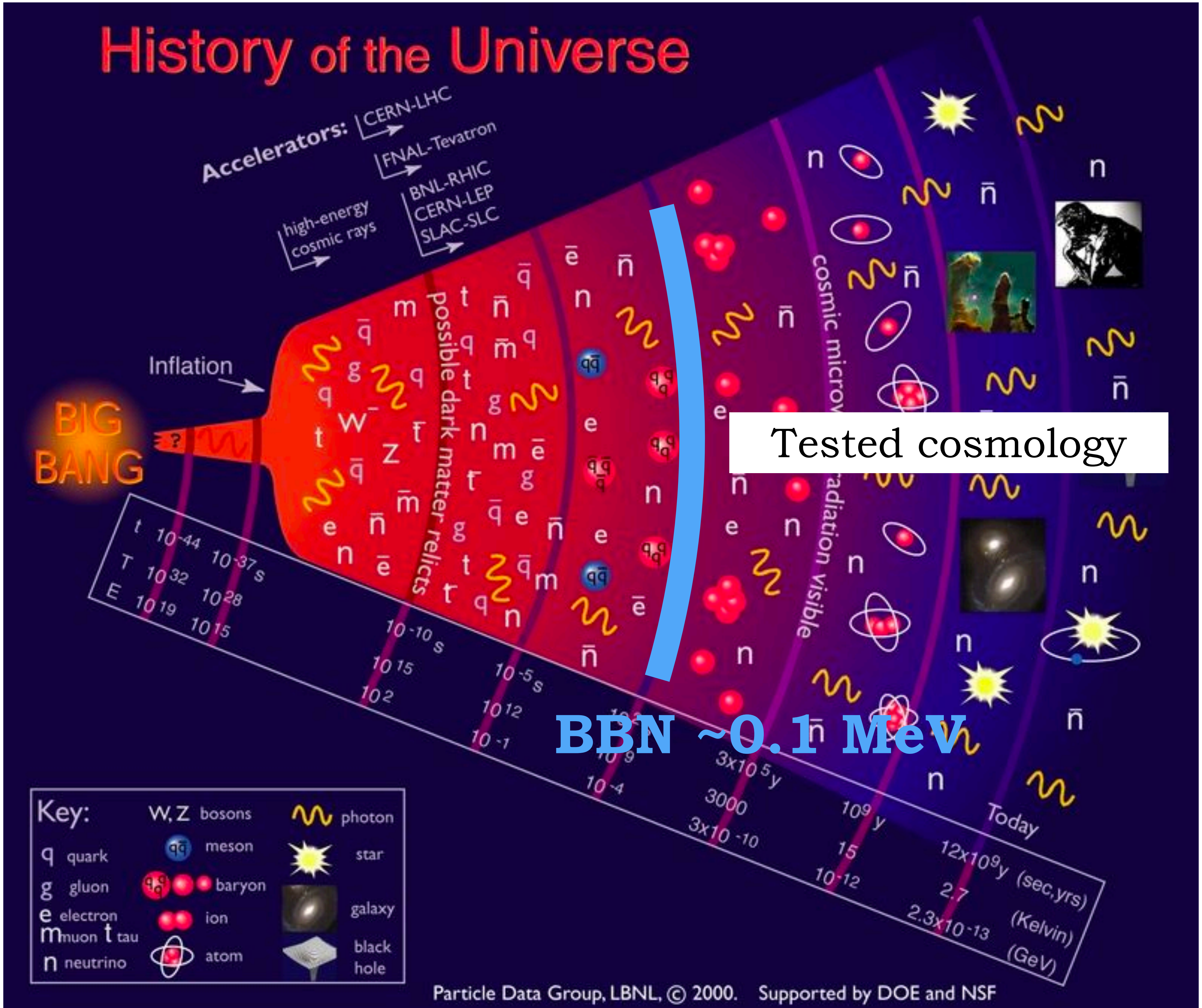
But the signal could also be of primordial origin…

CC, Nature Rev. Phys. 6 (2024), 5291-293

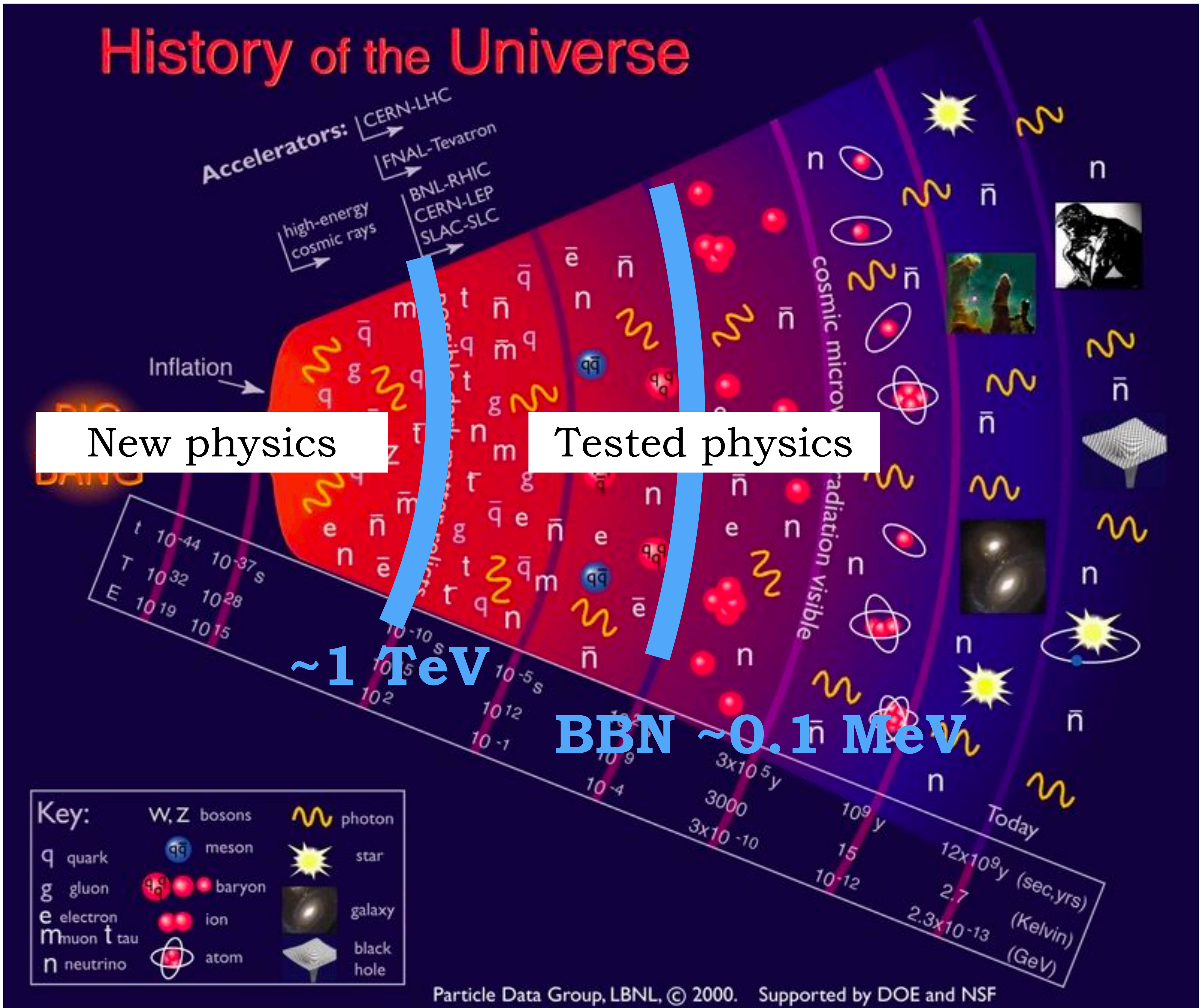
Examples of SGWB sources in the early universe



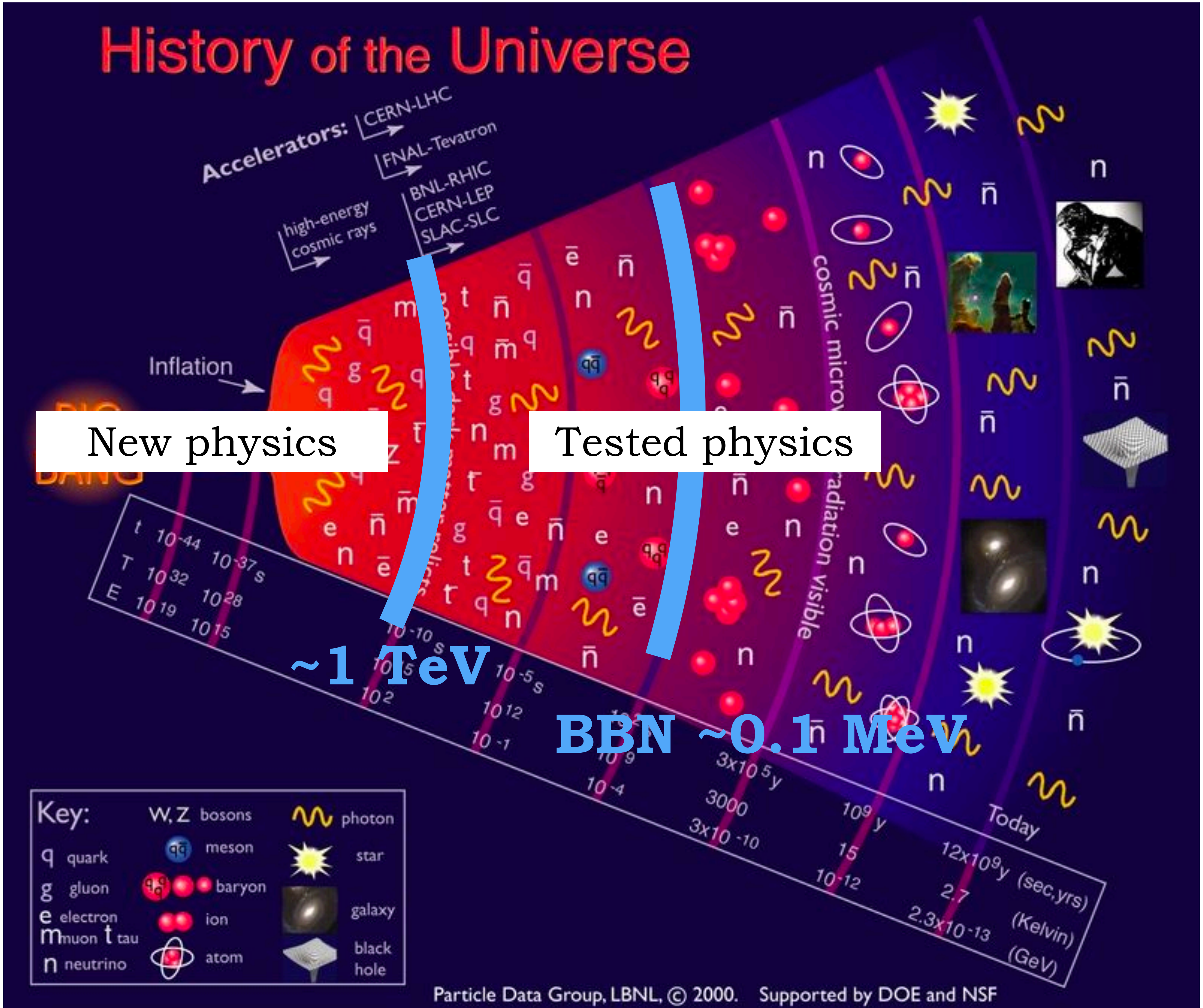
GWs can bring direct information from very early stages of the universe evolution, to which we have no direct access through em radiation —> amazing discovery potential



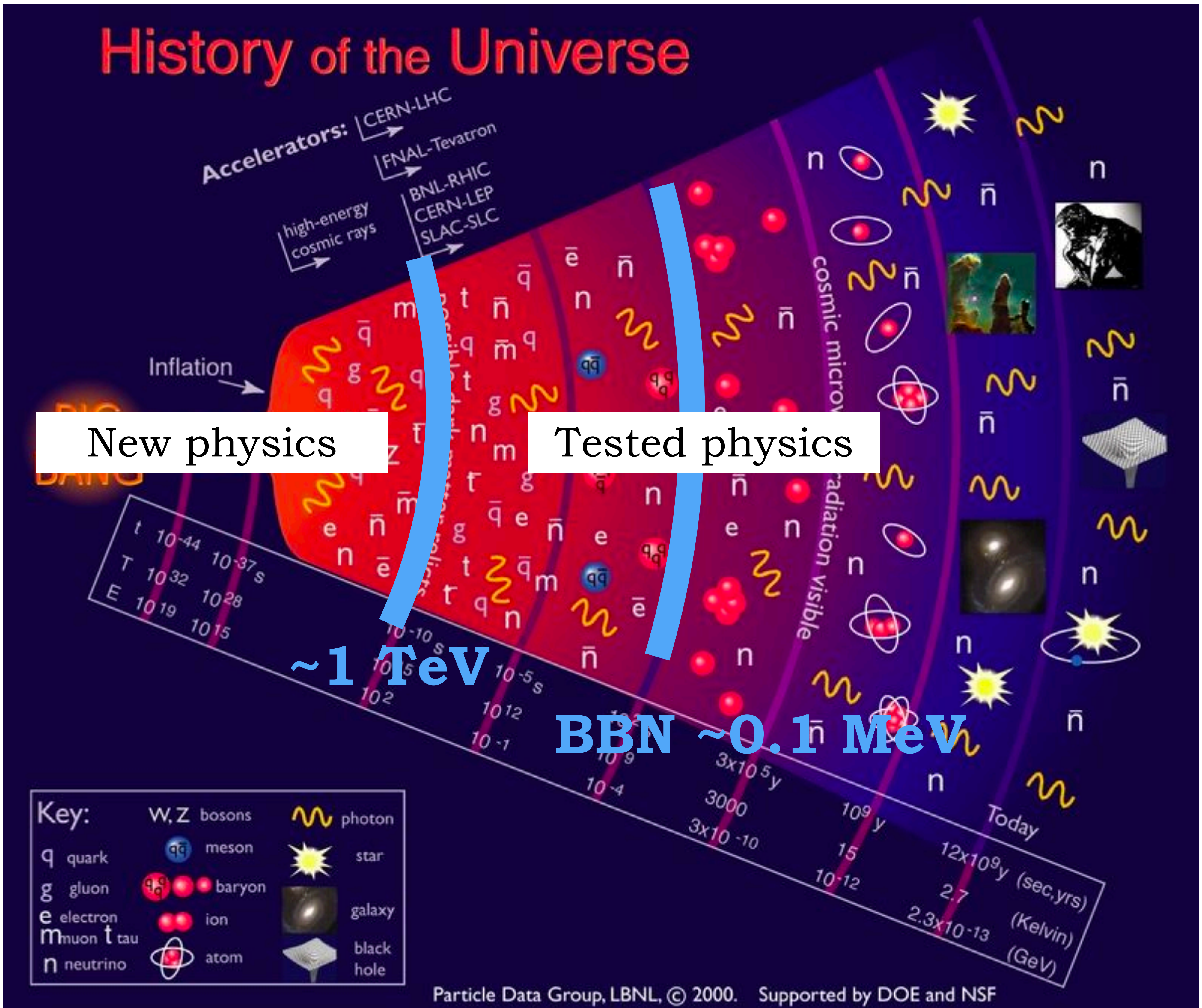
No guaranteed GW signal: predictions rely on untested phenomena, and are often difficult to estimate (non-linear dynamics, strongly coupled theories…)



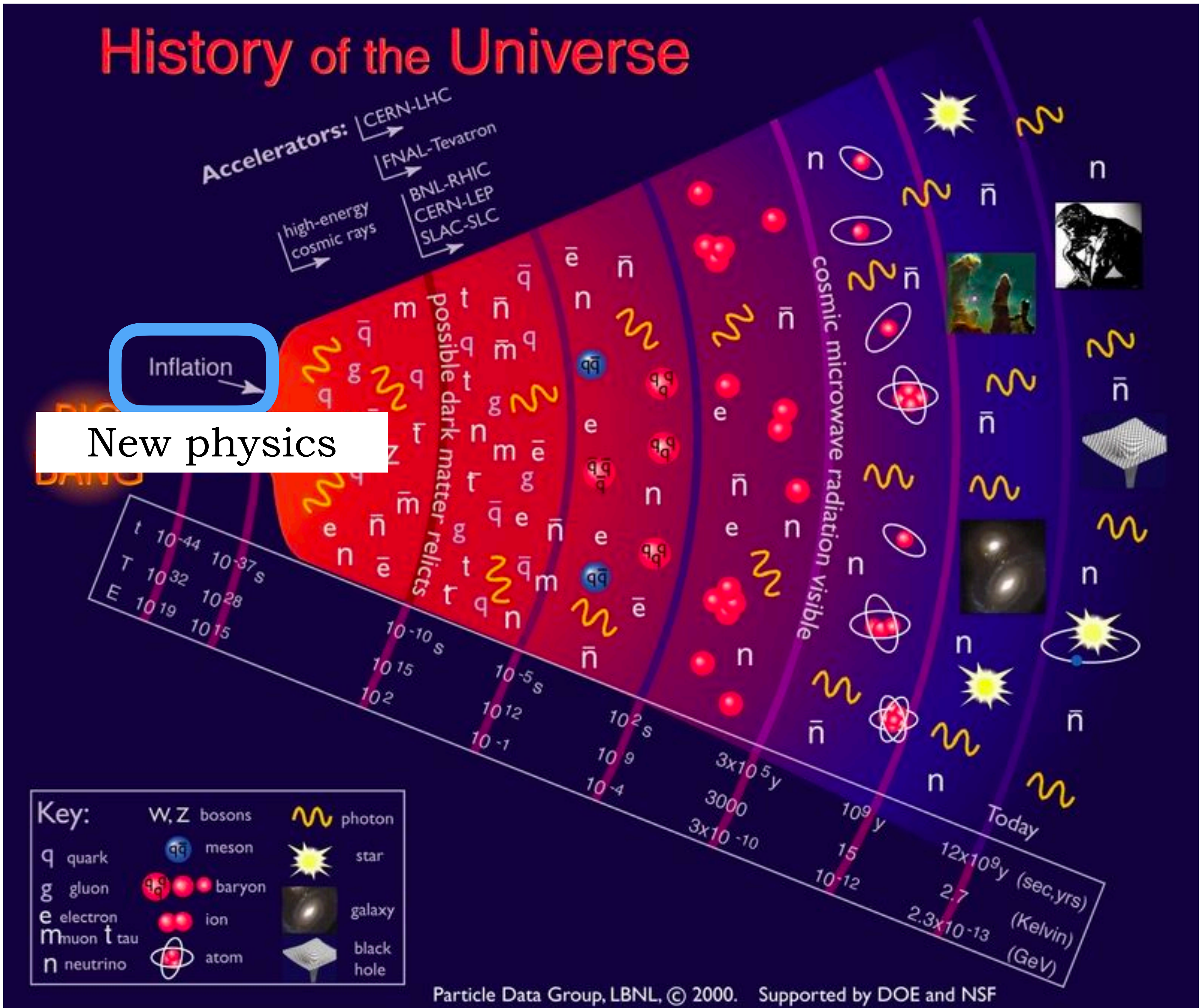
Many GW generation processes are related to PHASE TRANSITIONS



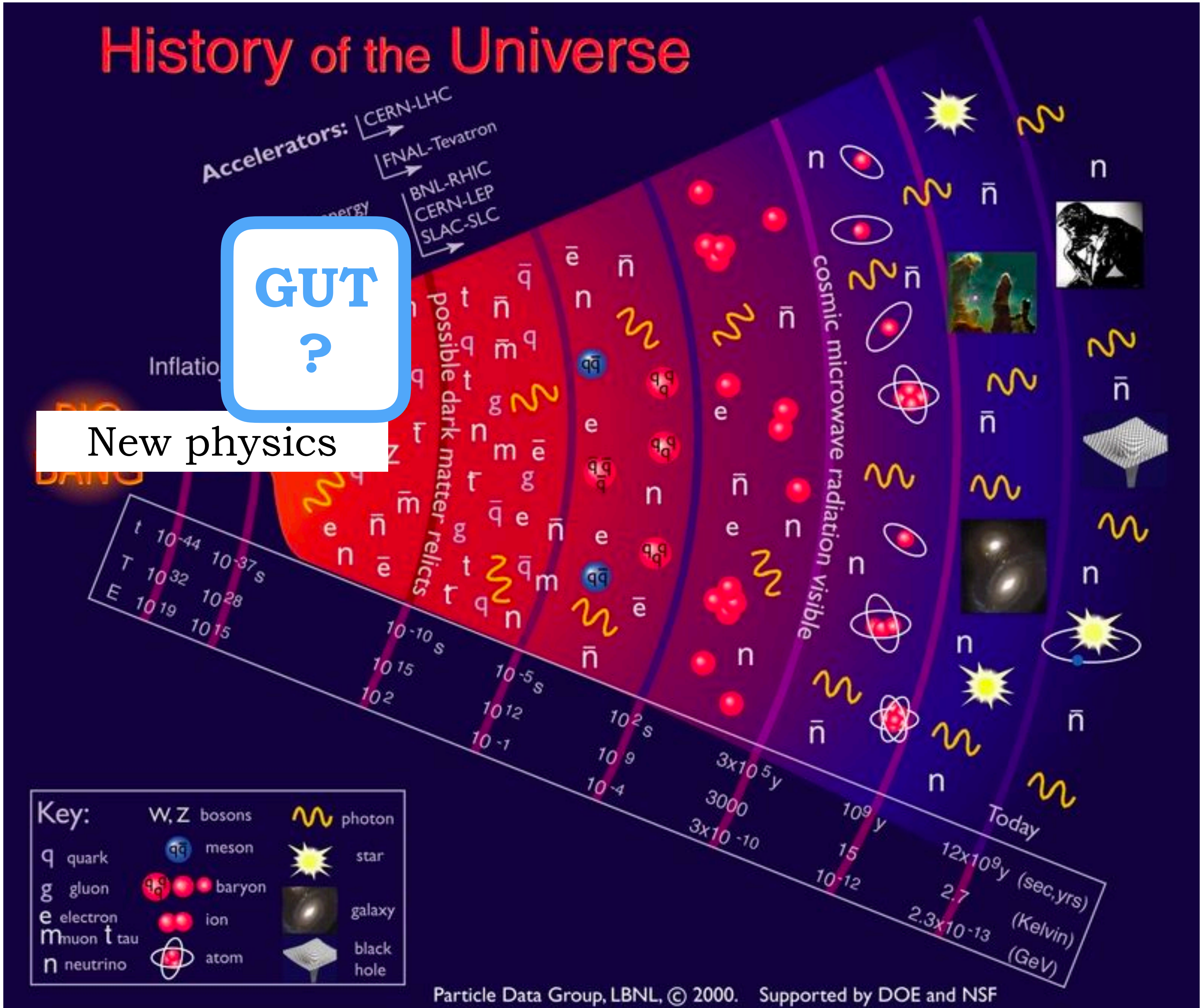
Phase transition: some field in the universe changes from one state to another, which has become more energetically favourable due to a change in external conditions (e.g. a change in temperature)



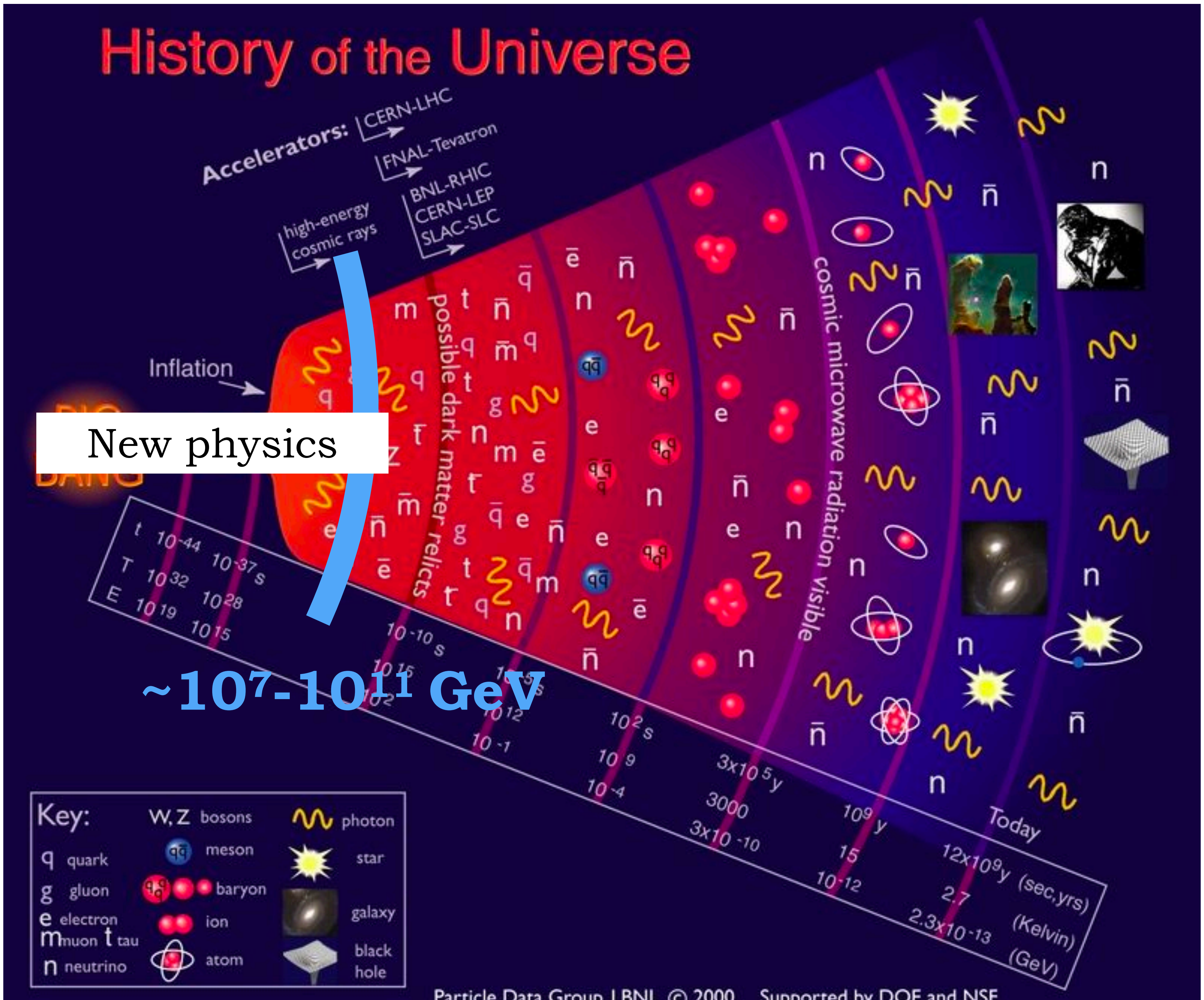
Inflation: phase transition of the Inflaton field



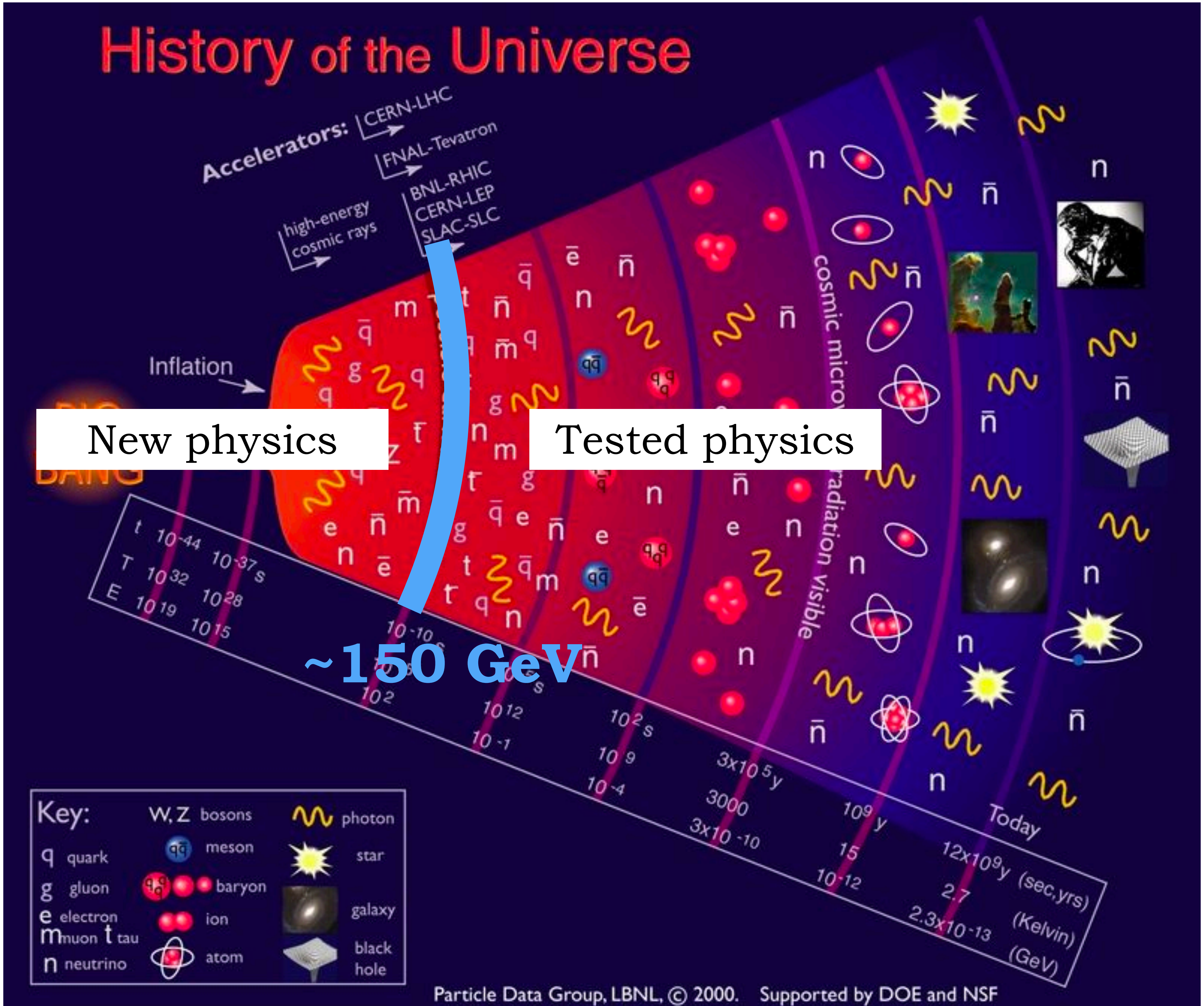
GUT phase transition or similar: related to the breaking of the symmetries of the high-energy theory describing the universe



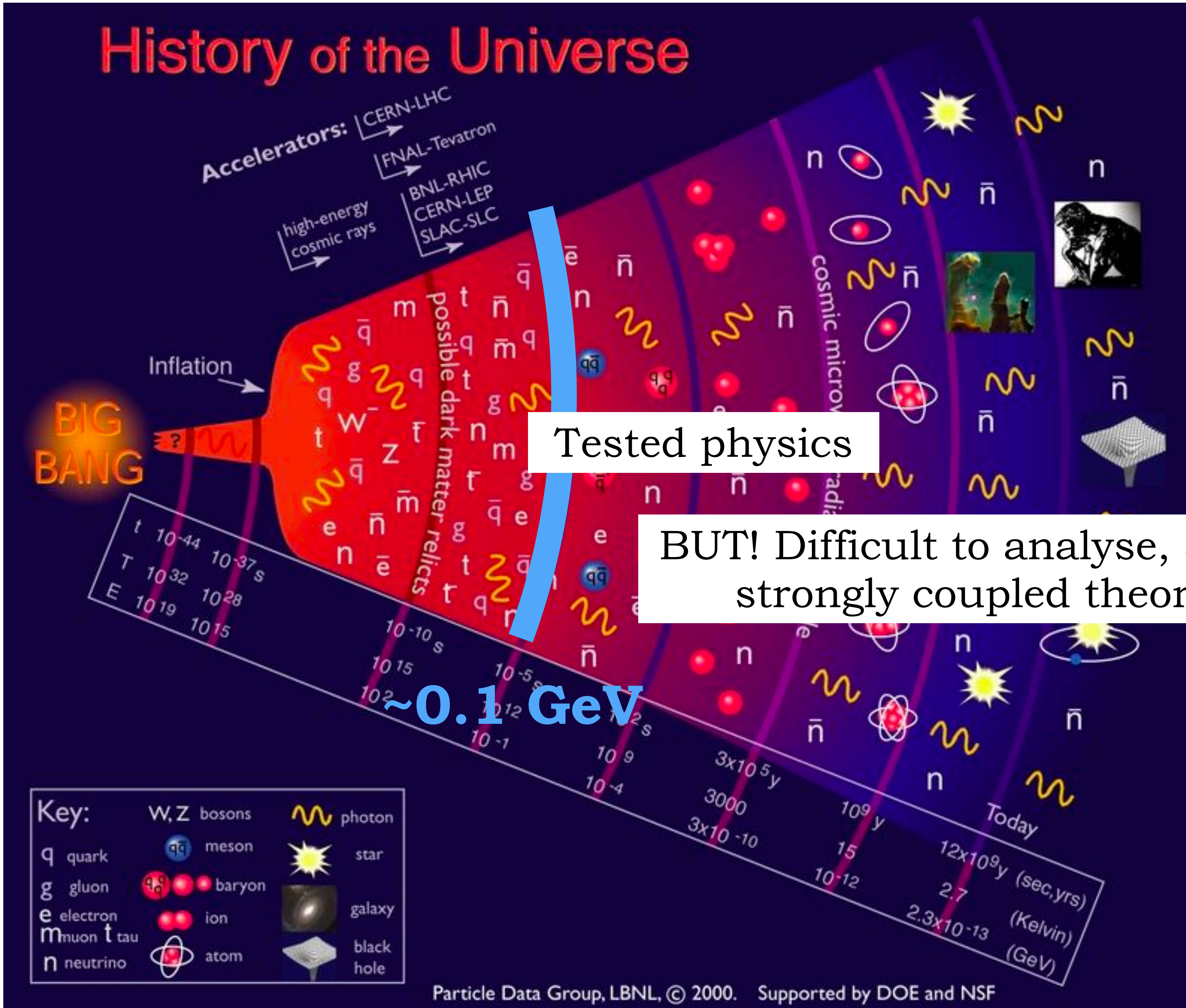
Peccei-Quinn phase transition: invoked to solve the strong CP problem



Electroweak phase transition: phase transition of the Higgs field, driven by the temperature decrease as the universe expands



QCD phase transition: phase transition related to the strong interaction, confinement of quarks into hadrons



Examples of GW sources in the early universe :

- irreducible SGWB from inflation
    - also sourced by second order scalar perturbations
- beyond the irreducible SGWB from inflation
    - particle production during inflation (scalar, gauge fields… coupled to the inflaton)
    - spectator fields
    - second order tensor from enhanced scalar perturbations, with primordial black holes
    - breaking symmetries (space-dependent inflaton, massive graviton)
    - modified gravity during inflation (massive GWs with $c \neq 1$)
    - …
- preheating and non-perturbative phenomena
    - parametric amplification of bosons/fermions
    - symmetry breaking in hybrid inflation
    - decay of flat directions
    - oscillons
    - …
- first order phase transition
    - true vacuum bubble collision
    - sound waves
    - (M)HD turbulence
    - …
- cosmic topological defects
    - irreducible SGWB from topological defect networks
    - decay of cosmic string loops
    - …

CC and Figueroa arXiv:1801.04268

SGWB from a stochastic source in the radiation era

$$h_r''(\mathbf{k},\eta) + 2\,\mathcal{H}\,h_r'(\mathbf{k},\eta) + k^2\,h_r(\mathbf{k},\eta) = 16\pi G\,a^2\,\Pi_r(\mathbf{k},\eta)$$

Possible sources of tensor anisotropic stress in the early universe:

- Scalar field gradients $\Pi_{ij} \sim [\partial_i\phi\partial_j\phi]^{TT}$
- Bulk fluid motion $\Pi_{ij} \sim [\gamma^2(\rho+p)v_iv_j]^{TT}$
- Gauge fields $\Pi_{ij} \sim [-E_iE_j - B_iB_j]^{TT}$
- Second order scalar perturbations, Π_{ij} from a combination of $\partial_i\Psi, \partial_i\Phi$
- …

The components of the anisotropic stress must be treated as
random variables
because we cannot access the detailed properties of the generation processes at the moment they operated

SGWB from a stochastic source in the radiation era

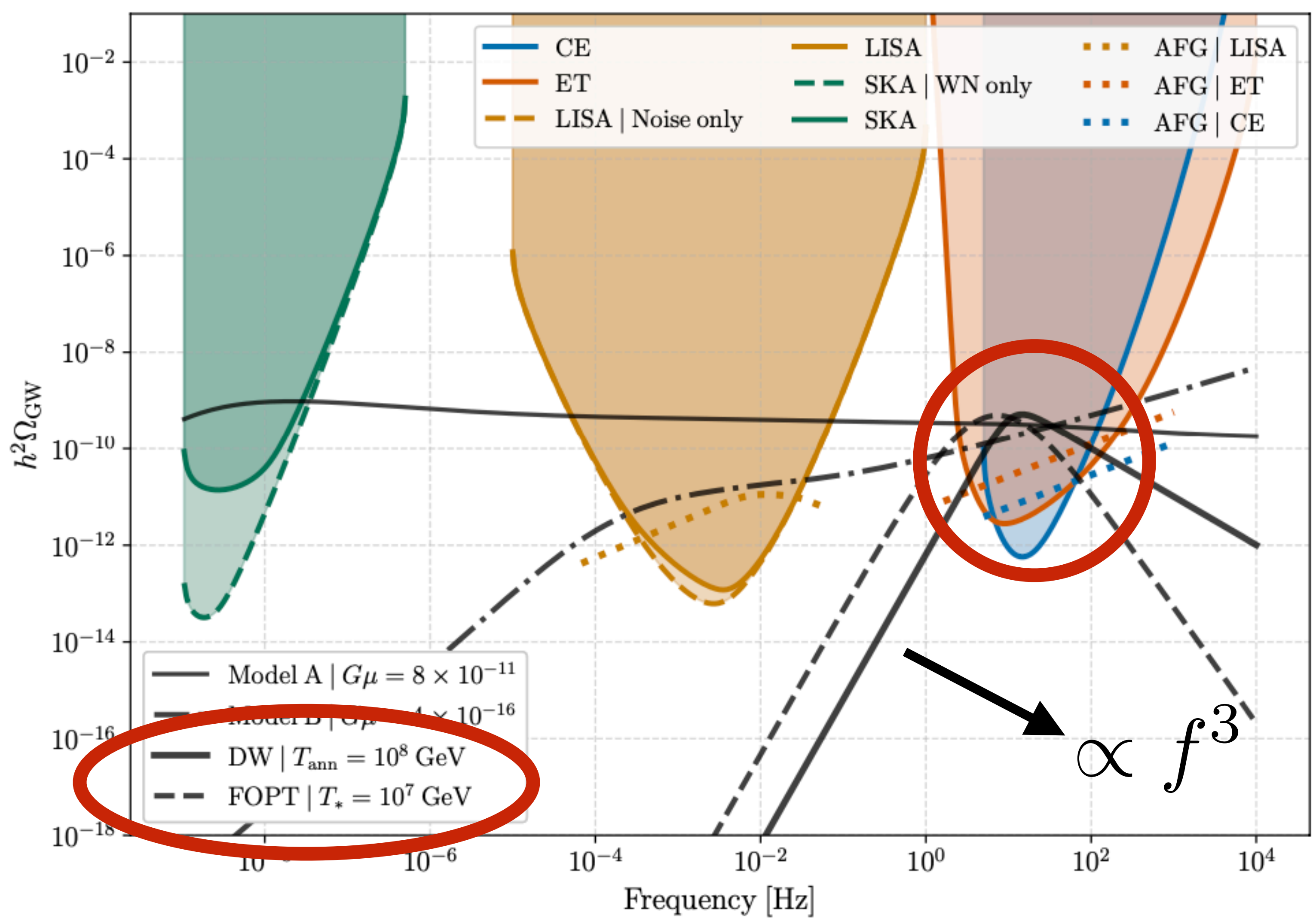
unequal time correlator of the anisotropic stress

$$\langle \Pi_r(\mathbf{k},\tau)\,\Pi_p^*(\mathbf{q},\zeta)\rangle = \frac{(2\pi)^3}{4}\,\delta^{(3)}(\mathbf{k}-\mathbf{q})\,\delta_{rp}\,\Pi(k,\tau,\zeta)$$

Anisotropic stress power spectral density at unequal time

We now proceed with **two approximate analytical solutions of the GW propagation equation**:

- **Fast source** operating for less than one Hubble time -> peaked SGWB power spectrum



SGWB from a stochastic source in the radiation era

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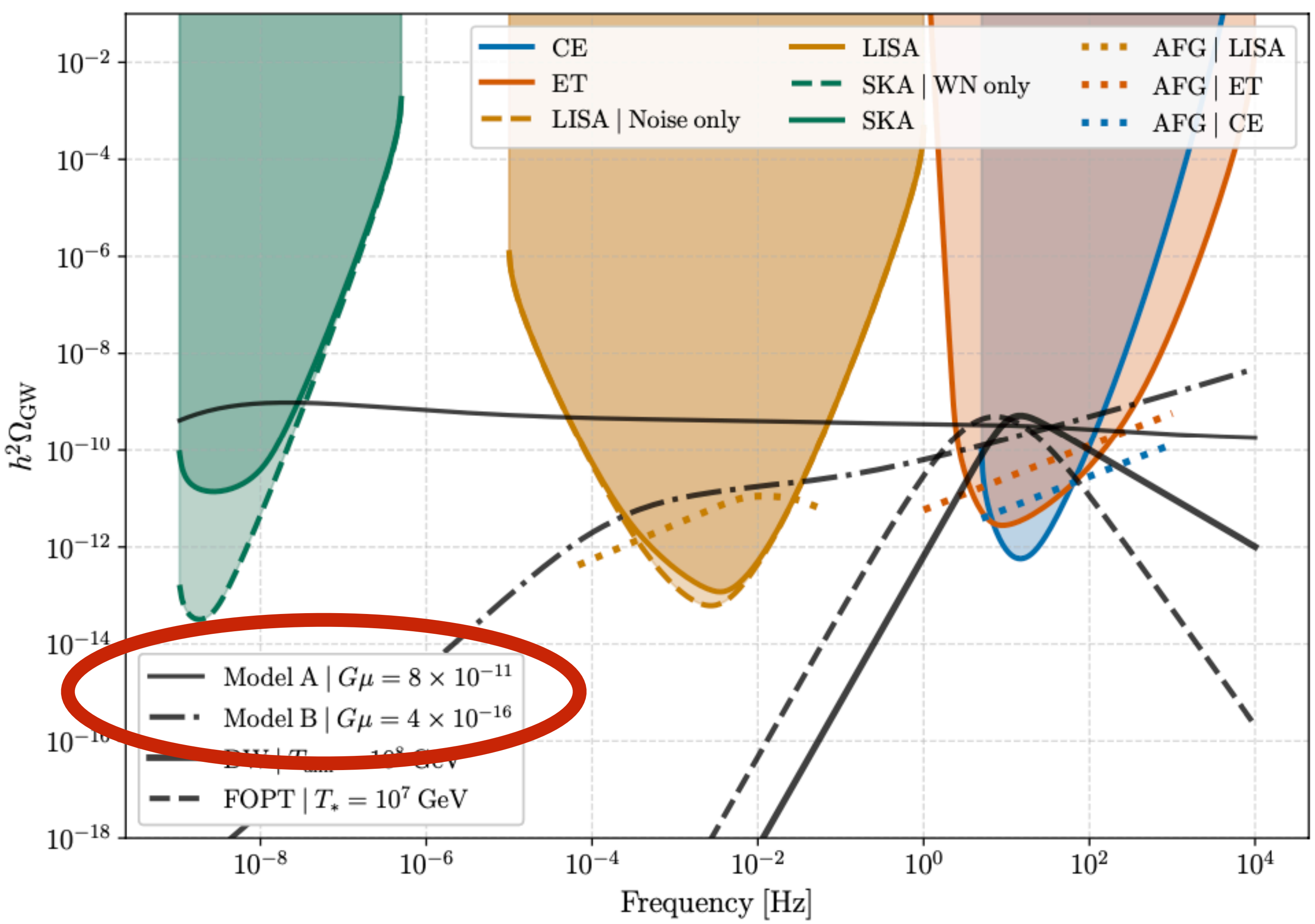
$$\langle \Pi_r(\mathbf{k},\tau)\,\Pi_p^*(\mathbf{q},\zeta)\rangle = \frac{(2\pi)^3}{4}\,\delta^{(3)}(\mathbf{k}-\mathbf{q})\,\delta_{rp}\,\Pi(k,\tau,\zeta)$$

Anisotropic stress power spectral density at unequal time

We now proceed with **two approximate analytical solutions of the GW propagation equation**:

- **Continuous source** operating for several Hubble times -> flat SGWB power spectrum



Possibility of coincident detection at several observatories

SGWB from a stochastic source in the radiation era

unequal time correlator of the anisotropic stress

$$\langle \Pi_r(\mathbf{k},\tau)\,\Pi_p^*(\mathbf{q},\zeta)\rangle = \frac{(2\pi)^3}{4}\,\delta^{(3)}(\mathbf{k}-\mathbf{q})\,\delta_{rp}\,\Pi(k,\tau,\zeta)$$

Anisotropic stress power spectral density at unequal time

Fast source operating in a time interval $\eta_{\rm fin}$ - $\eta_{\rm in}$ in the radiation dominated era

Typical example: first order phase transition

$$H_r^{\rm rad}(\mathbf{k},\eta>\eta_{\rm fin}) = A_r^{\rm rad}(\mathbf{k})\cos(k\eta) + B_r^{\rm rad}(\mathbf{k})\sin(k\eta)$$

Matching at $\eta_{\rm fin}$ with the homogeneous solution to find the GW signal today

$$A_r^{\rm rad}(\mathbf{k}) = \frac{16\pi G}{k}\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, a(\tau)^3\ \sin(-k\tau)\,\Pi_r(\mathbf{k},\tau),$$

$$B_r^{\rm rad}(\mathbf{k}) = \frac{16\pi G}{k}\int_{x_{\rm in}}^{x_{\rm fin}} d\tau\, a(\tau)^3\ \cos(k\tau)\,\Pi_r(\mathbf{k},\tau)$$

SGWB from a **FAST** stochastic source in the radiation era

GW amplitude power spectrum today for modes $k\eta_0 \gg 1$

$$\langle h_r(\mathbf{k},\eta_0)\, h_p^*(\mathbf{q},\eta_0)\rangle = \frac{1}{a_0^2}\left[\langle A_r(\mathbf{k})\, A_p^*(\mathbf{q})\rangle + \langle B_r(\mathbf{k})\, B_p^*(\mathbf{q})\rangle\right]$$
$$= 8\pi^5\, \delta^{(3)}(\mathbf{k}-\mathbf{q})\, \delta_{rp}\, \frac{h_c^2(k,\eta_0)}{k^3}$$

GW energy density power spectrum today for modes $k\eta_0 \gg 1$

$$\frac{d\rho_{\rm GW}}{d\log k} = \frac{k^2\, h_c^2(k,\eta_0)}{16\pi G\, a_0^2}$$

(freely propagating sub-Hubble modes)

$$\frac{d\rho_{\rm GW}}{d\log k}(k,\eta_0) = \frac{4}{\pi}\,\frac{G}{a_0^4} k^3 \int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, a^3(\tau) \int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\zeta\, a^3(\zeta)\, \cos[k(\tau-\zeta)]\, \Pi(k,\tau,\zeta)$$

SGWB from a **FAST** stochastic source in the radiation era

GW amplitude power spectrum today for modes $k\eta_0 \gg 1$

$$\langle h_r(\mathbf{k},\eta_0)\, h_p^*(\mathbf{q},\eta_0)\rangle = \frac{1}{a_0^2}[\langle A_r(\mathbf{k})\, A_p^*(\mathbf{q})\rangle + \langle B_r(\mathbf{k})\, B_p^*(\mathbf{q})\rangle]$$
$$= 8\pi^5\, \delta^{(3)}(\mathbf{k}-\mathbf{q})\, \delta_{rp}\, \frac{h_c^2(k,\eta_0)}{k^3}$$

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$$\frac{d\rho_{\rm GW}}{d\log k}(k,\eta_0) = \frac{4}{\pi}\frac{G}{a_0^4}k^3\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, \underbrace{\cancel{a^3(\tau)}}_{a_*^3} \int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\zeta\, \underbrace{\cancel{a^3(\zeta)}}_{a_*^3}\, \underbrace{\cancel{\cos[k(\tau-\zeta)]}}_{\simeq 1}\, \underbrace{\cancel{\Pi(k,\tau,\zeta)}}_{\Pi(k)}$$

SUPPOSE:

$\Delta\eta = \eta_{\rm fin} - \eta_{\rm in} \ll \mathcal{H}_*^{-1}$ $\qquad$ $k\eta_{\rm in} \ll 1$ $\qquad$ $\Pi(k,\tau,\eta)$ constant over $\Delta\eta$

SGWB from a **FAST** stochastic source in the radiation era

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

$$h^2\Omega_{\rm GW}(k,\eta_0) = \frac{3}{2\pi^2}\underline{h^2\Omega^0_{\rm rad}}\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}}(\underbrace{\Delta\eta\,\underline{\mathcal{H}_*}}_{\text{From the time integrals}})^2\left(\frac{\rho_\Pi}{\underline{\rho_{\rm rad}}}\right)^2(k\ell_*)^3\tilde{P}_{\rm GW}(k)$$

$$\Pi(k) = \rho_\Pi^2\tilde{P}_{\rm GW}(k)$$

From the time integrals

SGWB from a **FAST** stochastic source in the radiation era

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

$$h^2\Omega_{GW}(k,\eta_0) = \underbrace{\frac{3}{2\pi^2}h^2\Omega^0_{rad}}_{\mathcal{O}(10^{-9})} \underbrace{\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}}}_{\mathcal{O}(10^{-6})} \underbrace{(\Delta\eta\mathcal{H}_*)^2\left(\frac{\rho_\Pi}{\rho_{rad}}\right)^2}_{\mathcal{O}(10^{-3})} (k\ell_*)^3\tilde{P}_{GW}(k)$$

Value detected at PTA

Factor depending slightly on the generation epoch through the number of relativistic d.o.f.

Only slow, very anisotropic processes have the chance to generate detectable SGWB signals for sub-Hubble sources

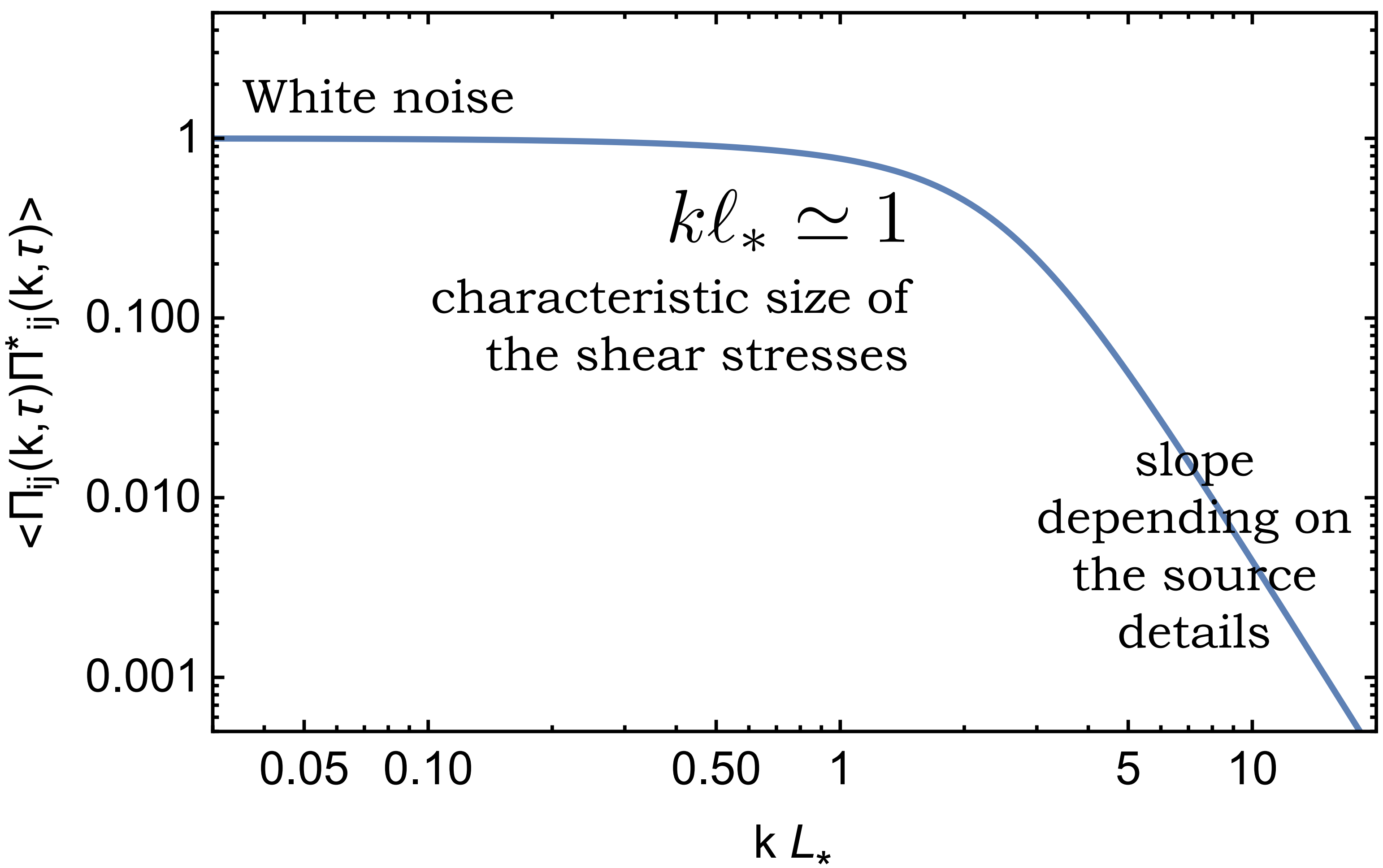
Value for detection at LISA

$\mathcal{O}(10^{-11})$ $\mathcal{O}(10^{-6})$ $\mathcal{O}(10^{-5})$

SGWB from a **FAST** stochastic source in the radiation era

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

$$h^2\Omega_{\rm GW}(k,\eta_0) = \frac{3}{2\pi^2} h^2\Omega^0_{\rm rad}\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}} (\Delta\eta\mathcal{H}_*)^2 \left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2 (k\ell_*)^3\tilde{P}_{\rm GW}(k)$$



Fast source: independent on k for large enough scales (uncorrelated)

$$\boxed{\ell_* \leq H_*^{-1}}$$

SGWB from a **FAST** stochastic source in the radiation era

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

$$h^2\Omega_{\rm GW}(k,\eta_0) = \frac{3}{2\pi^2} h^2\Omega^0_{\rm rad}\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}} (\Delta\eta\mathcal{H}_*)^2 \left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2 (k\ell_*)^3 \tilde{P}_{\rm GW}(k)$$

$$1/\eta_0 \ll \underbrace{k \ll \mathcal{H}_*}_{\text{Range of validity of the solution}} \ll \underbrace{1/(a_*\ell_*)}_{\text{Causality of the sourcing process}}$$

$$\boxed{\Omega_{\rm GW}(k) \propto (k\ell_*)^3}$$

SGWB from a **FAST** stochastic source in the radiation era

- Characteristic time of the source evolution $$\delta t_c = \frac{\ell_*}{v_{\rm rms}}$$
- Characteristic time of the GW production from the Green's function: $$\delta t_{\rm gw} \sim \frac{1}{k}$$
- GW production goes faster than source evolution for all relevant wave-numbers including the spectrum peak $$k > \frac{v_{\rm rms}}{\ell_*}$$
- One assumes that the source is constant in time for a finite time interval (which can be larger than the Hubble time) $$\delta t_{\rm fin} \sim \mathcal{N} \delta t_c$$
- One can then easily integrate to find the GW spectrum

$$\boxed{h^2\Omega_{\rm GW}(k,\eta_0) \propto h^2\Omega^0_{\rm rad}\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}}\left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2 (k\ell_*)^3 \tilde{P}_{\rm GW}(k)\begin{cases} \ln^2[1+\mathcal{H}_*\delta t_{\rm fin}] & \text{if } k\,\delta t_{\rm fin} < 1 \\ \ln^2[1+(k/\mathcal{H}_*)^{-1}] & \text{if } k\,\delta t_{\rm fin} \geq 1 \end{cases}}$$

A. Roper Pol et al, arXiv:2201.05630

SGWB from a **FAST** stochastic source in the radiation era

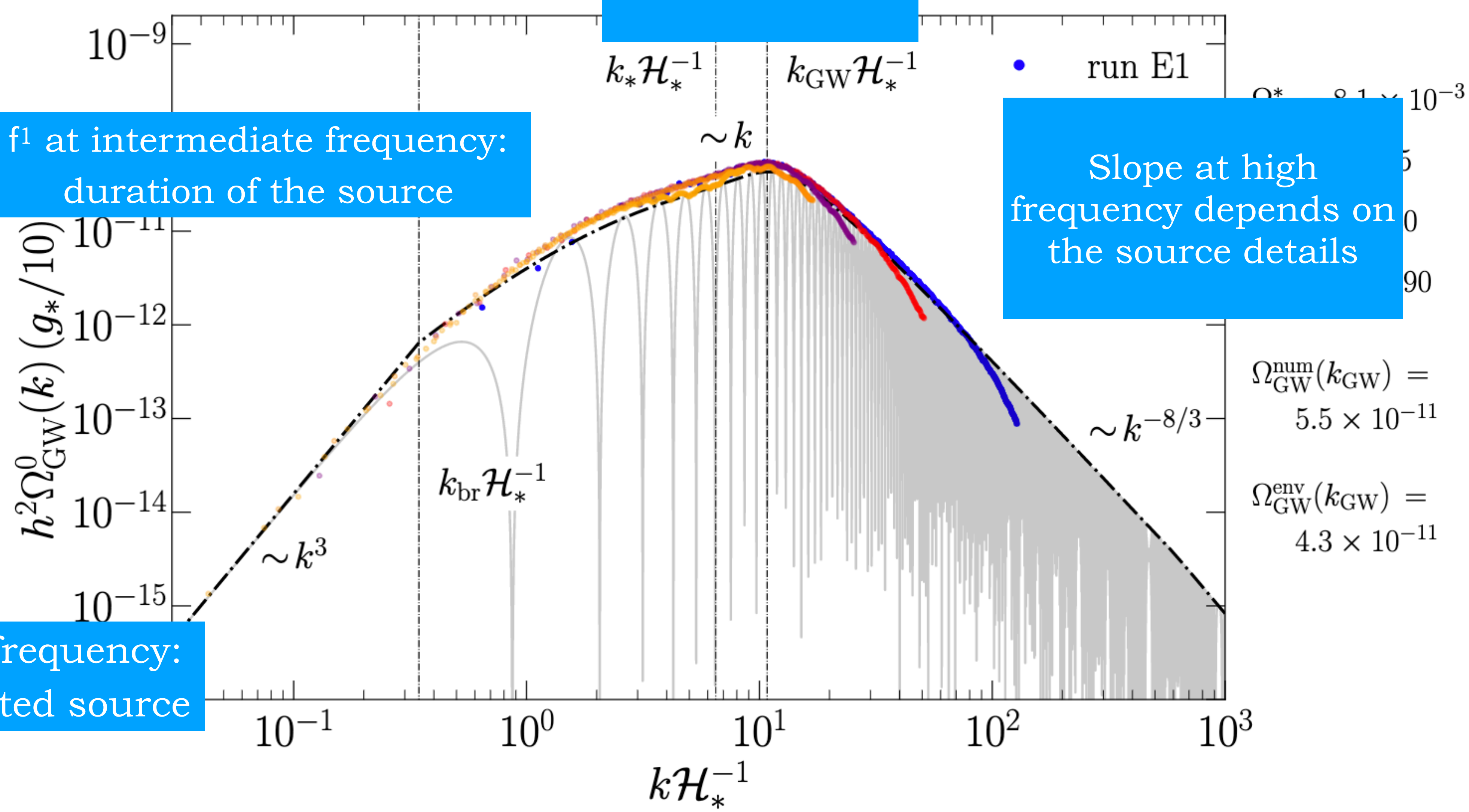
$$k_{\rm peak} \simeq 4\pi/\ell_*$$

$$\Omega_{\rm gw,\,peak} \propto \left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2 (\mathcal{H}_*\ell_*)^2$$

Transition from k^3 to k^1 at $k \simeq 1/\delta t_{\rm fin}$

Can be smoother if $\delta t_{\rm fin} > 1/\mathcal{H}_*$

Peak at f ~ $1/\ell_*$



f¹ at intermediate frequency: duration of the source

Slope at high frequency depends on the source details

f³ at low frequency: uncorrelated source

A. Roper Pol et al, arXiv:2201.05630

SGWB from a **CONTINUOUS** stochastic source in the radiation era

Typical example: topological defects

Suppose the source is operating continuously in the radiation dominated era

- No matching at the end time of the source
- No *free* sub-Hubble modes

$$H_r(\mathbf{k},\eta) = a\, h_r(\mathbf{k},\eta)$$

$$H_r^{\rm rad}(\mathbf{k},\eta) = \frac{16\pi G}{k}\int_{\eta_{\rm in}}^{\eta} d\tau\, a(\tau)^3\ \sin[k(\eta-\tau)]\,\Pi_r(\mathbf{k},\tau)$$

$${h_r}'(\mathbf{k},\eta) = \frac{16\pi G}{a(\eta)}\int_{\eta_{\rm in}}^{\eta} d\tau\, a(\tau)^3\ \cos[k(\eta-\tau)]\,\Pi_r(\mathbf{k},\tau)$$

$$\langle h_r'(\mathbf{k},\eta)\, {h_p'}^*(\mathbf{q},\eta)\rangle = \frac{8\pi^5}{k^3}\,\delta^{(3)}(\mathbf{k}-\mathbf{q})\,\delta_{rp}\, {h_c'}^2(k,\eta) \qquad \frac{d\rho_{\rm GW}}{d\log k} = \frac{{h_c'}^2(k,\eta)}{16\pi G\, a^2(\eta)}$$

$$\frac{d\rho_{\rm GW}}{d\log k}(k,\eta) = \frac{4}{\pi}\frac{G}{a^4}\,k^3\int_{\eta_{\rm in}}^{\eta} d\tau\, a(\tau)^3\int_{\eta_{\rm in}}^{\eta} d\zeta\, a(\zeta)^3\,\mathcal{G}(k,\eta,\tau,\zeta)\,\Pi(k,\tau,\zeta)$$

SGWB from a **CONTINUOUS** stochastic source in the radiation era

Typical example: topological defects

Suppose the source is operating continuously in the radiation dominated era

- Scaling (property of the topological defects network)
- Decays very fast in off-diagonal $k\tau \neq k\zeta$
- Decays as a power law on the diagonal $k\tau = k\zeta$

D. Figueroa et al, arXiv:1212.5458

$$\Pi(k,\tau,\zeta) = \frac{v^4}{\sqrt{\tau\zeta}} \frac{\mathcal{U}(k\tau,k\zeta)}{a(\tau)a(\zeta)}$$

$$\frac{d\rho_{\rm GW}}{d\log k}(k,\eta) = \frac{4}{\pi}\frac{G}{a^4}\,k^3 \int_{\eta_{\rm in}}^{\eta} d\tau\, a(\tau)^3 \int_{\eta_{\rm in}}^{\eta} d\zeta\, a(\zeta)^3\, \mathcal{G}(k,\eta,\tau,\zeta)\,\Pi(k,\tau,\zeta)$$

SGWB from a **CONTINUOUS** stochastic source in the radiation era

Typical example: topological defects

Suppose the source is operating continuously in the radiation dominated era

$$h^2\Omega_{\rm GW}(f) = \frac{32}{3} h^2\Omega_{\rm rad} \left(\frac{v}{M_{\rm Pl}}\right)^4 F_{\rm RD}^{[\mathcal{U}]}(\infty)$$

TODAY FLAT SPECTRUM
AT SUB-HORIZON MODES
IN THE RADIATION ERA

D. Figueroa et al, arXiv:1212.5458

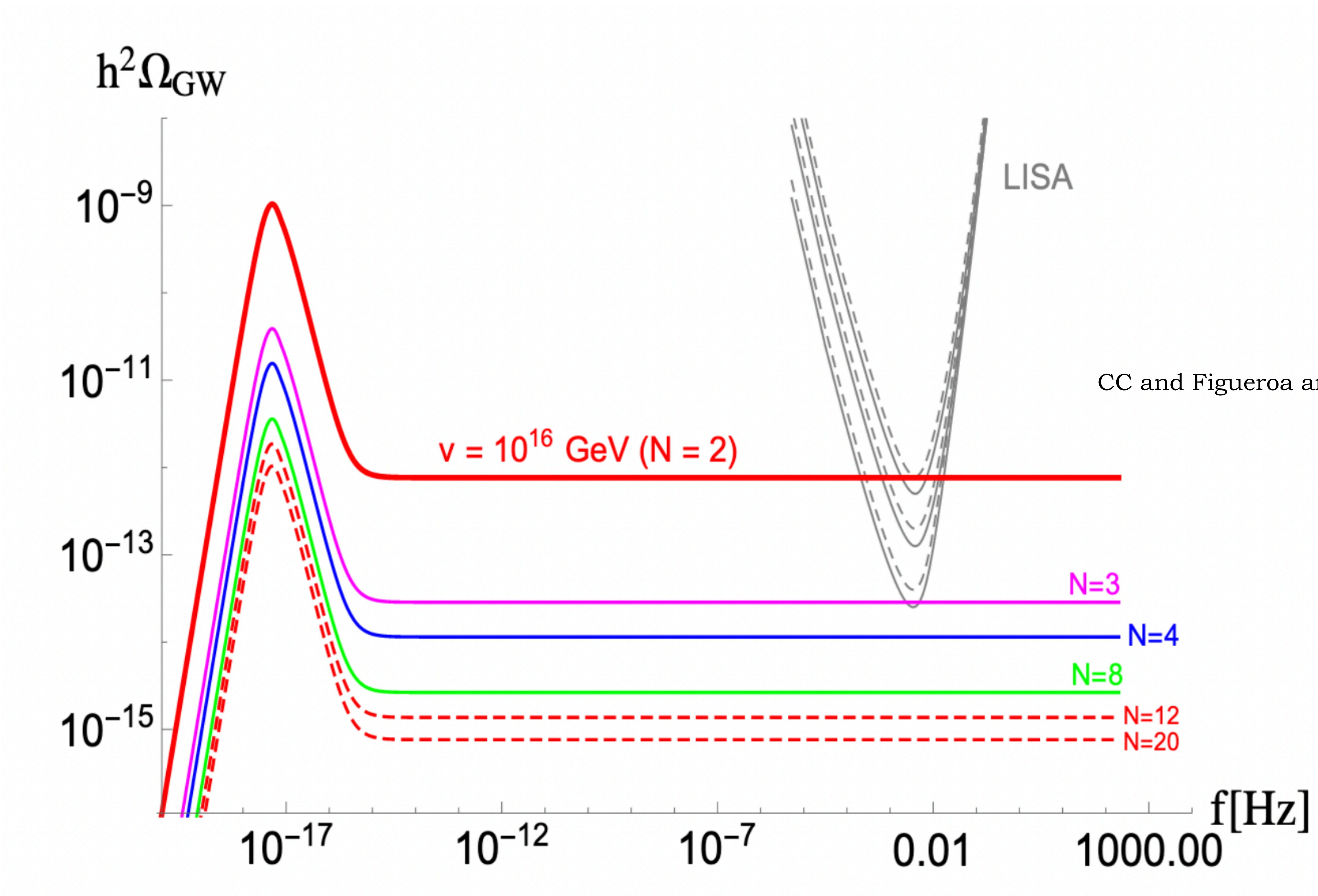
Progressively
independent on the
upper bound

$$\frac{d\rho_{\rm GW}}{d\log k}(k,\eta) = \frac{32}{3}\Omega_{\rm rad}\frac{\rho_c}{a^4}\left(\frac{v}{M_{\rm Pl}}\right)^4 \int_{x_{\rm in}}^{x} dx_1 \int_{x_{\rm in}}^{x} dx_2\, \sqrt{x_1 x_2}\,\mathcal{G}(x,x_1,x_2)\,\mathcal{U}(x_1,x_2)$$

D. Figueroa et al, arXiv:1212.5458

SGWB from a **CONTINUOUS** stochastic source in the radiation era

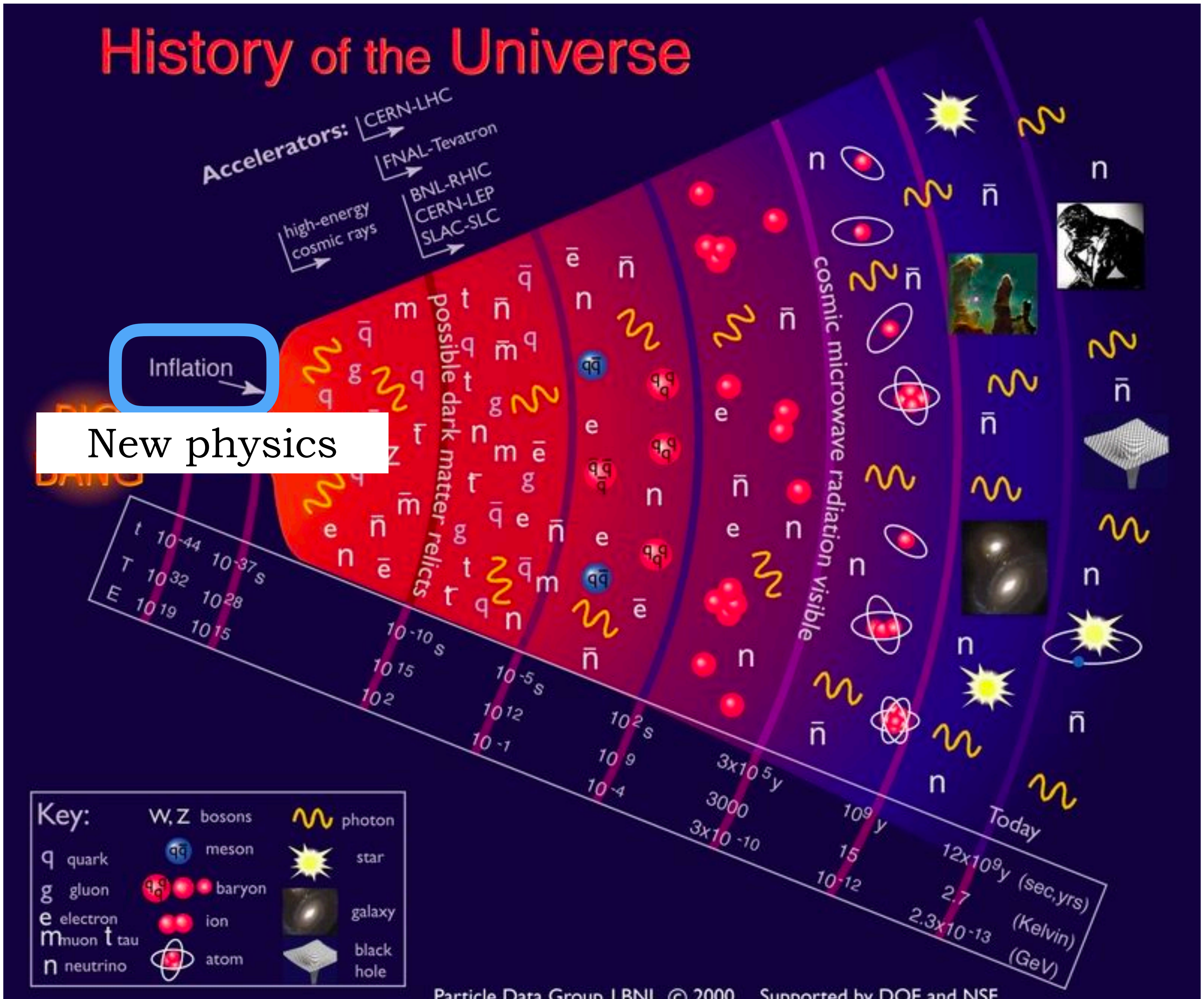
Typical example: topological defects



CC and Figueroa arXiv:1801.04268

Examples of signals

Inflation: phase transition of the Inflaton field

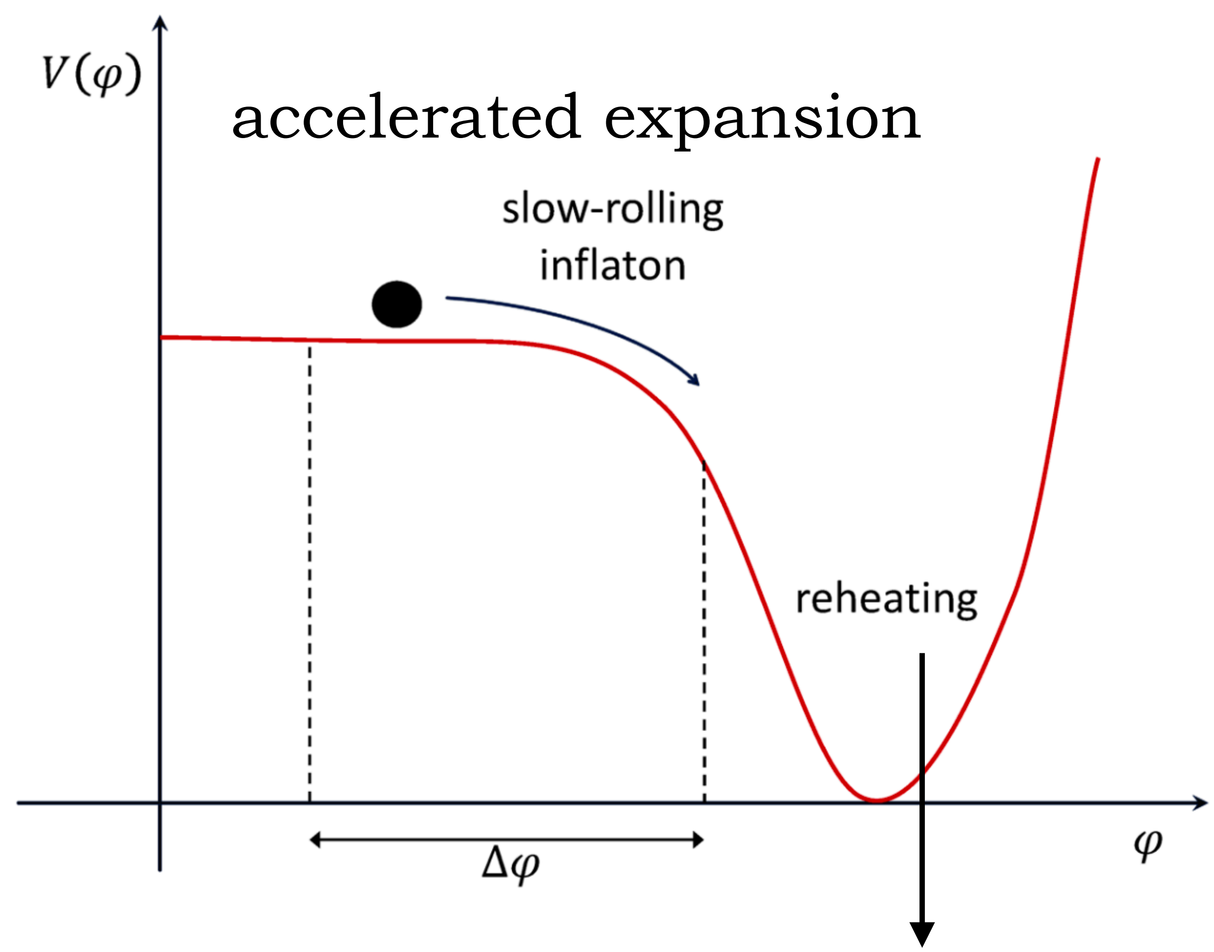


Inflation: phase transition of the Inflaton field

- Inflation is required in the present cosmological scenario as it *solves the flatness and horizon problems*, and provides *the seeds for the matter structure formation*
- It is the best model explaining *CMB black body spectrum, temperature anisotropies and polarisation*
- The universe at the end of inflation is characterised by *small metric fluctuations of quantum origin*, both *scalar* (of the order of 10^{-5}, providing density perturbations) and *tensor* (yet undetected, providing GWs)

Universe dominated by
a scalar field

$$\ddot{\varphi} + 3H\dot{\varphi} - \nabla^2\varphi + V'(\varphi) = 0$$



Generation of a thermodynamical state:
particles in thermal equilibrium,
radiation-dominated phase

Image credit: Guzzetti et al, arXiv:1605.01615

GW signal from inflation

Amplification of tensor metric vacuum fluctuations by the exponential expansion

$$h_r''(\mathbf{k},\eta) + 2\,\mathcal{H}\,h_r'(\mathbf{k},\eta) + k^2\,h_r(\mathbf{k},\eta) = 16\pi G\,a^2\,\cancel{\Pi_r(\mathbf{k},\eta)}$$

- ✓ canonically normalised free field $v_\pm = a\,M_{Pl}\,h_\pm$
- ✓ quantisation
- ✓ homogeneous wave equation: harmonic oscillator with *time dependent* frequency

$$v_\pm''(t) + (k^2 - a^2 H^2) v_\pm(t) = 0$$

$k \gg a\,H$ sub-Hubble modes

$$\omega^2(t) = k^2$$

free field in vacuum
zero occupation number

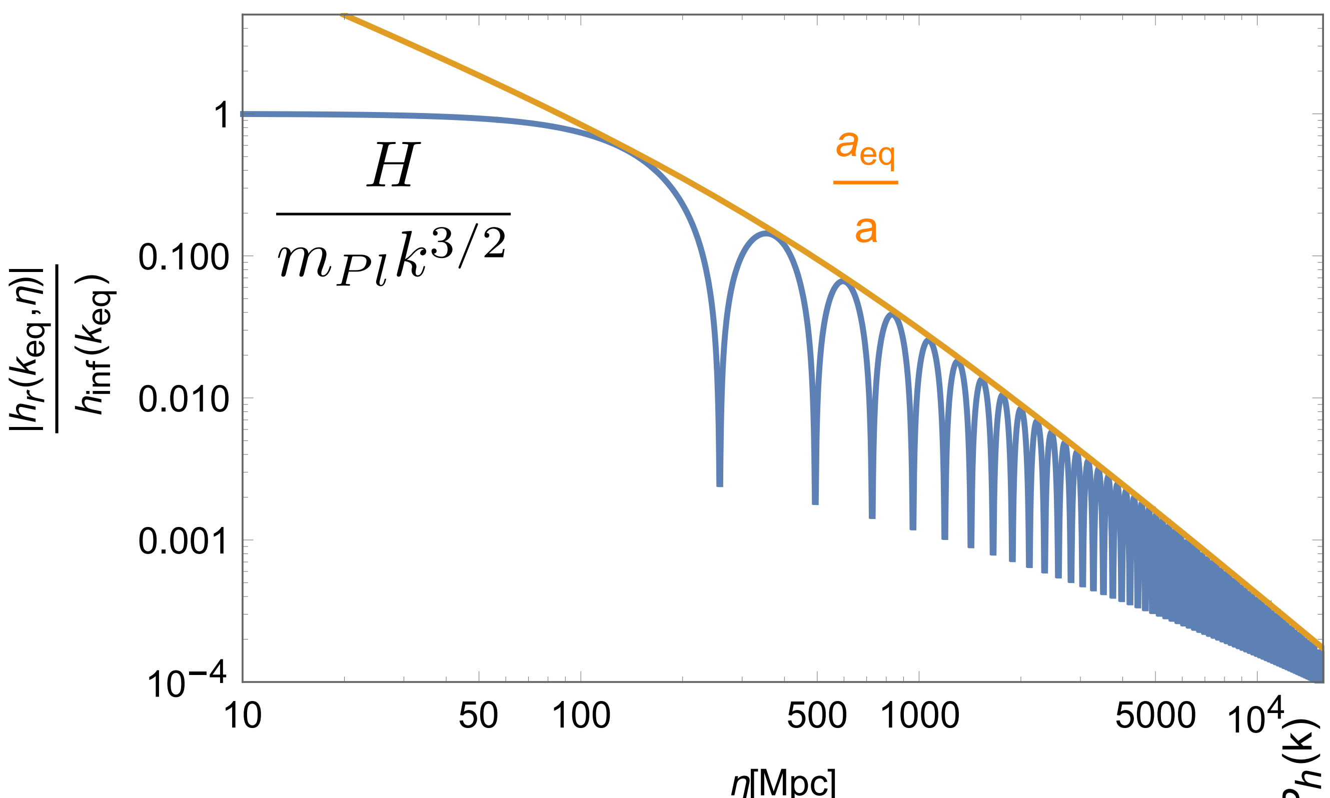
$k \ll a\,H$ super-Hubble modes

$$\omega^2(t) = -a^2 H^2$$

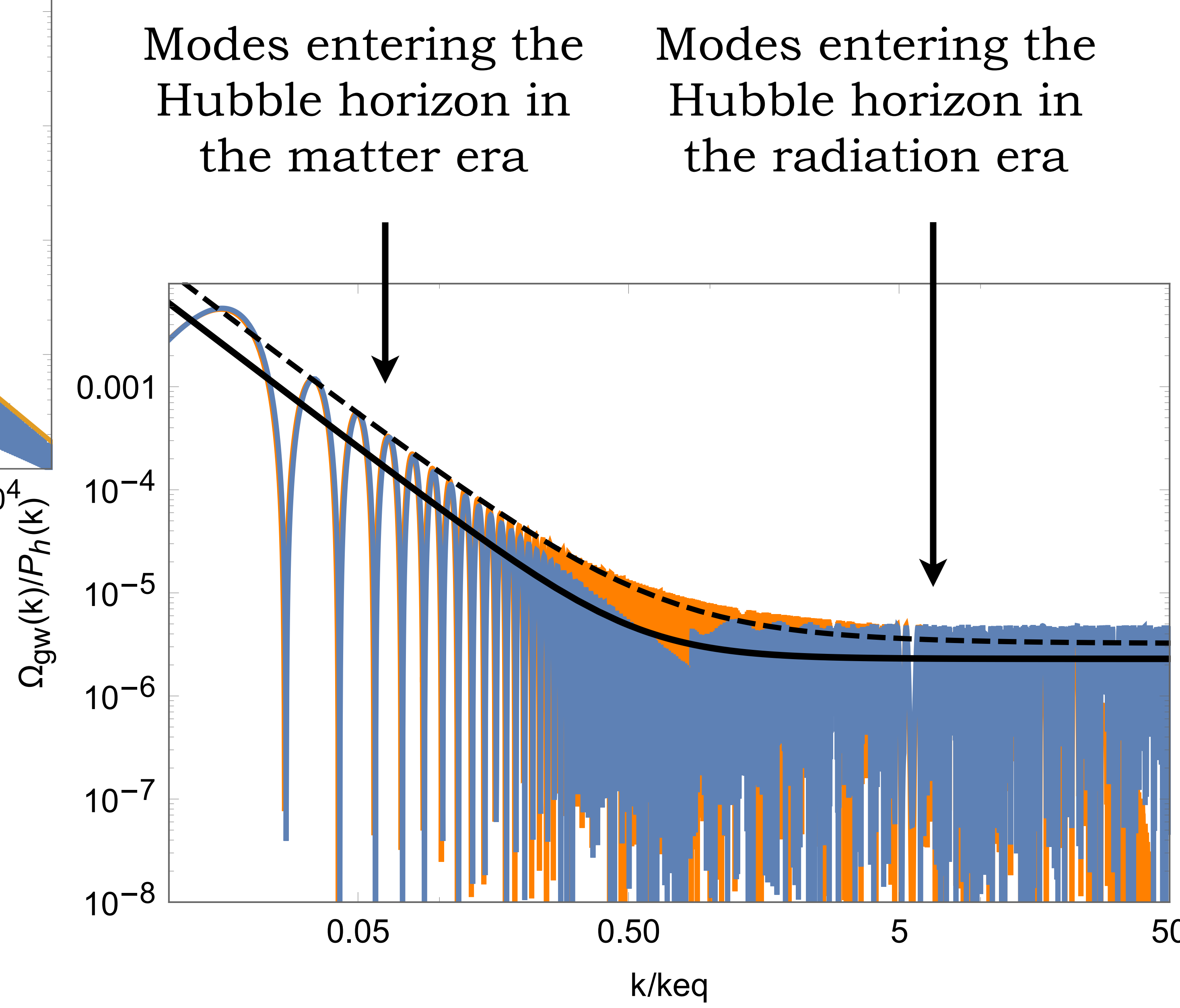
super-Hubble modes have very large occupation number

GW signal from (slow roll) inflation

- tensor spectrum $\mathcal{P}_h = \frac{2}{\pi}\frac{H^2}{m_{Pl}^2}\left(\frac{k}{aH}\right)^{-2\epsilon}$ $\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$
- transfer function from inflation to today, as modes re-enter the Hubble horizon



$$\Omega_{\rm GW}(k,\eta_0) = \frac{[T'(k,\eta_0)]^2}{12a_0^2H_0^2}\mathcal{P}_h(k)$$



GW signal from (slow roll) inflation

- tensor spectrum $\quad \mathcal{P}_h = \frac{2}{\pi}\frac{H^2}{m_{Pl}^2}\left(\frac{k}{aH}\right)^{-2\epsilon} \qquad \epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$

$$\boxed{\Omega_{\rm GW}(f) = \frac{3}{128}\,\Omega_{\rm rad}\, r\, \mathcal{P}_{\mathcal{R}}^{*}\left(\frac{f}{f_*}\right)^{n_T}\left[\frac{1}{2}\left(\frac{f_{\rm eq}}{f}\right)^2 + \frac{16}{9}\right]}$$

- tensor to scalar ratio $\quad r = \mathcal{P}_h/\mathcal{P}_{\mathcal{R}} \qquad r_* < 0.038$ Planck+BICEP+ACT+BAO limit

- scalar amplitude at CMB pivot scale $\quad \mathcal{P}_{\mathcal{R}}^{*} \simeq 2\cdot 10^{-9} \qquad k_* = \frac{0.05}{\rm Mpc}$

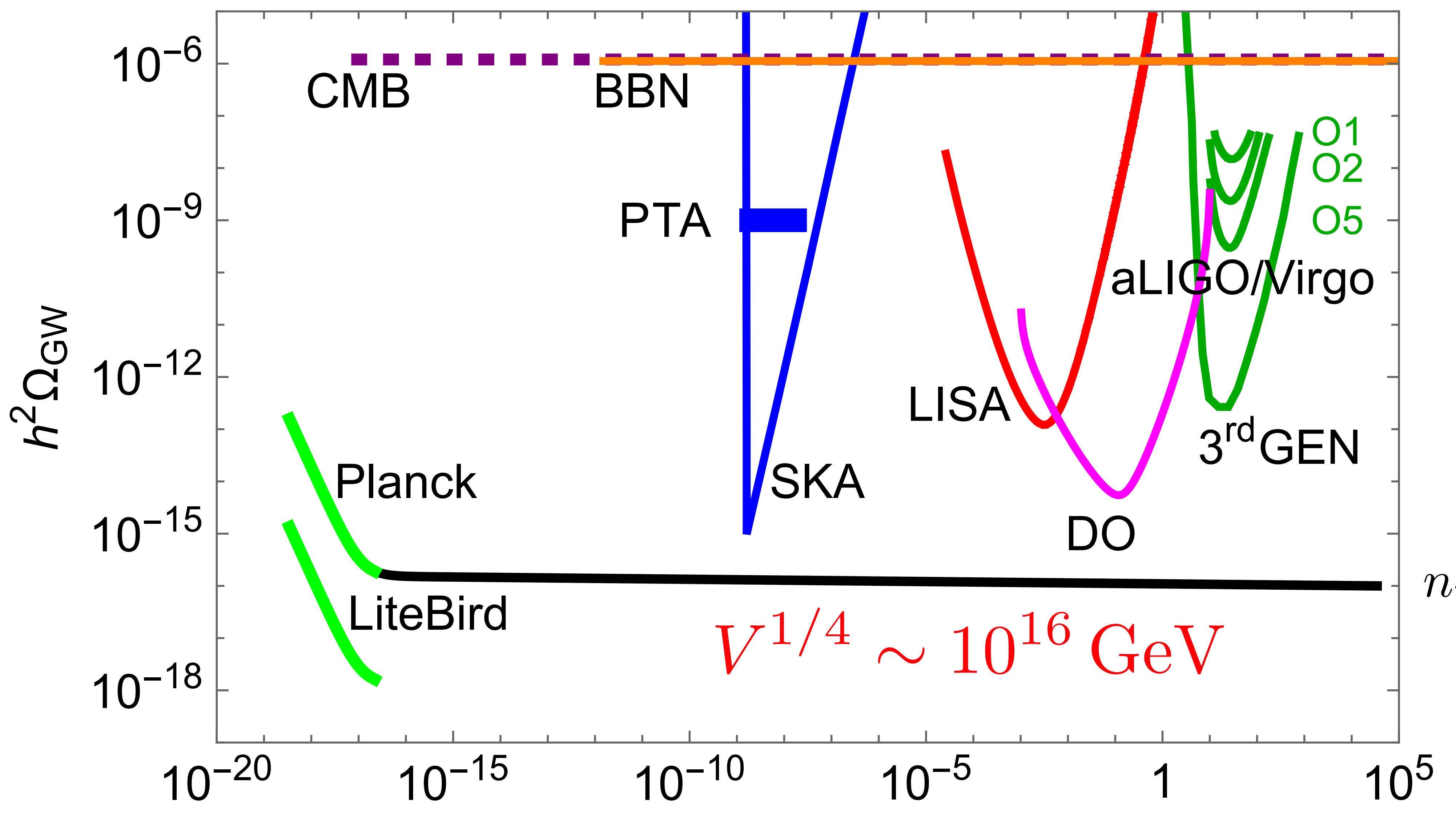
- GW signal extended in frequency: $\quad H_0 \leq f \leq H_{\rm inf}$

continuous sourcing of GW as modes re-enter the Hubble horizon

GW signal from (slow roll) inflation

Gw detectors offer the amazing opportunity to probe the inflationary power spectrum (and the model of inflation) down to the tiniest scales

BUT! The signal in the standard slow roll scenario is too low because of CMB observational bound



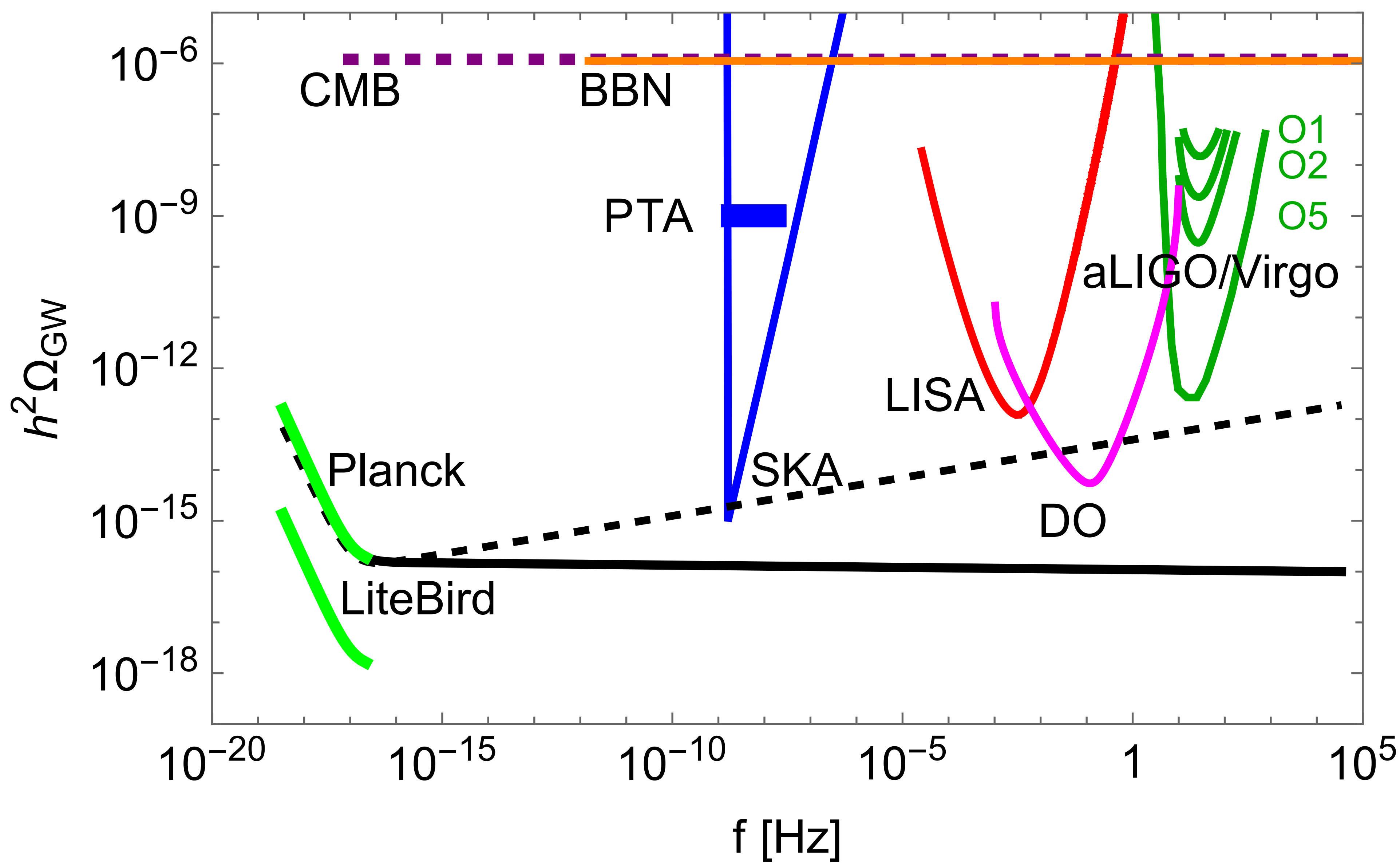
$n_T(\phi) \sim -2\epsilon(\phi)$

$$f \simeq 15\,\mathrm{mHz}\left(\frac{k}{10^{13}\,\mathrm{Mpc}^{-1}}\right)$$

GW signal from (non-standard) inflation

There is the possibility to enhance the signal going beyond the standard inflationary scenario: adding extra fields, modifying the inflaton potential, modifying the gravitational interaction, adding a phase with stiff equation of state...

$$H_r''(\mathbf{k},\eta) + \left(k^2 - \frac{a''}{a}\right) H_r(\mathbf{k},\eta) = \boxed{16\pi G\, a^3\, \Pi_r(\mathbf{k},\eta)}$$



Example: inflaton-gauge field coupling

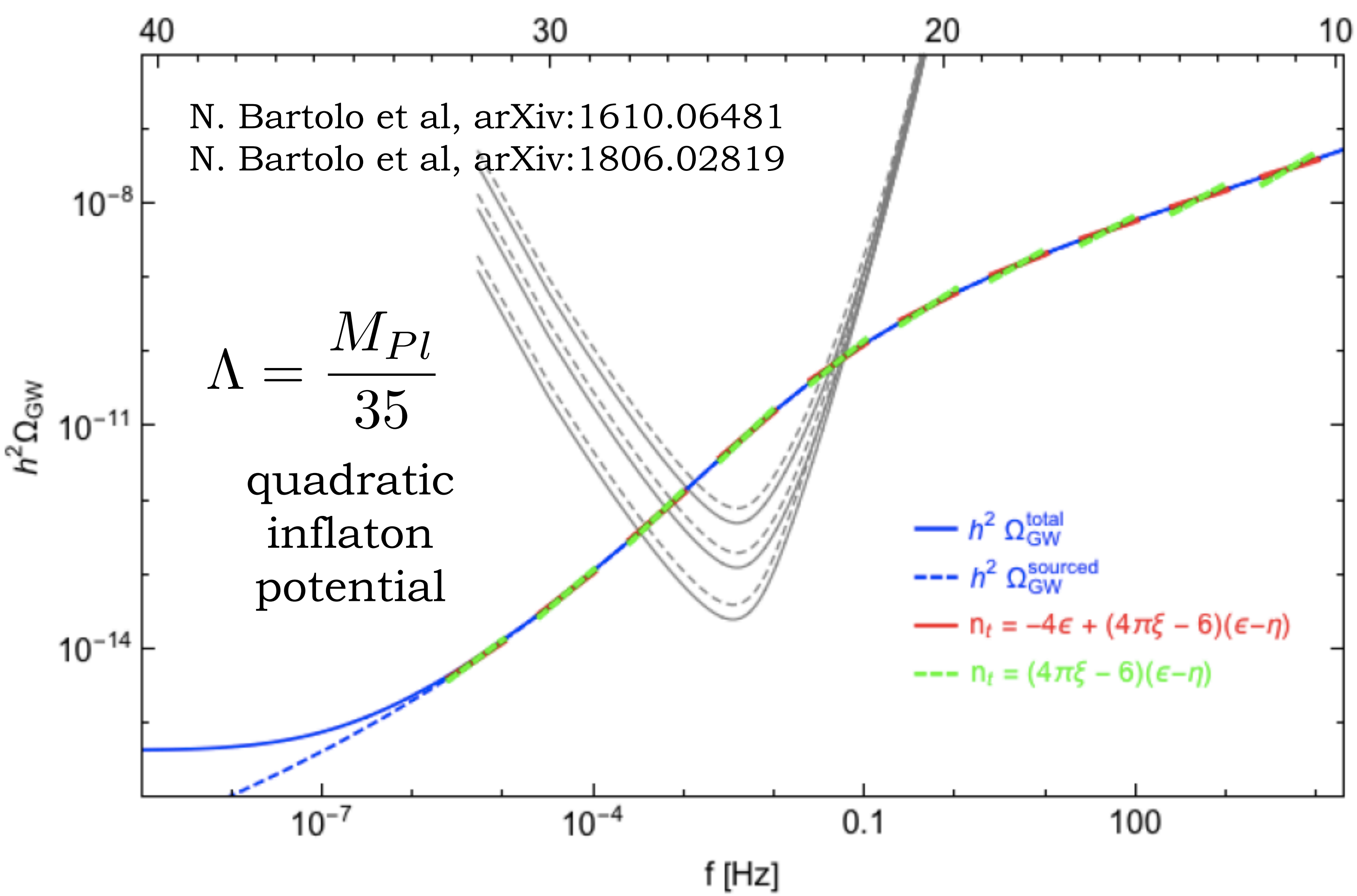
Add a term in the Lagrangian coupling pseudo-scalar inflaton to gauge fields

$$V(\phi) + \frac{\phi}{4\Lambda} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Production of gauge fields and consequently of GWs through the source

$$\Pi_{ij} \sim [-E_i E_j - B_i B_j]^{TT}$$

EXAMPLE IN THE LISA BAND:



OTHER SIGNATURES/ CONSTRAINTS: non-gaussianity, chirality, primordial black holes

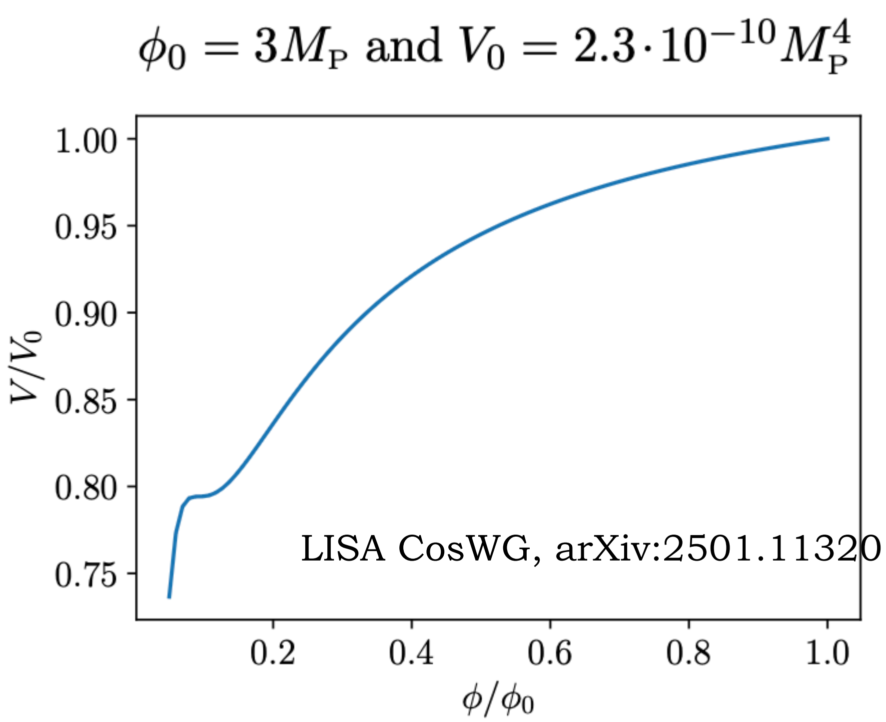
Predictions of the signal must be refined accounting for non-linearity of the system

Example: GW signal from second order scalar perturbations and associated primordial black holes

- At linear order in cosmological perturbation theory, scalar and tensor perturbations are decoupled and evolve separately, but *at second order they mix*
- Gradients in the scalar component *can source the tensor component at second order*: since the scalar fluctuations are order of 10^{-5}, the tensors are small $\partial_i \Psi, \partial_i \Phi$
- However, *if the scalar component is enhanced*, the induced tensor component can be important (e.g. from a phase of ultra slow-roll close to reheating)
- The enhanced scalar density fluctuations can collapse upon horizon reentry and produce *primordial black holes* whose properties are linked to those of the tensor spectrum

$$\Omega_{\rm GW}(f) = \Omega_{\rm rad} \int_0^\infty {\rm d}v \int_{|1-v|}^{1+v} {\rm d}u \, \mathcal{K}(u,v) \mathcal{P}_\mathcal{R}(uk) \mathcal{P}_\mathcal{R}(vk)$$

Second order in curvature perturbation



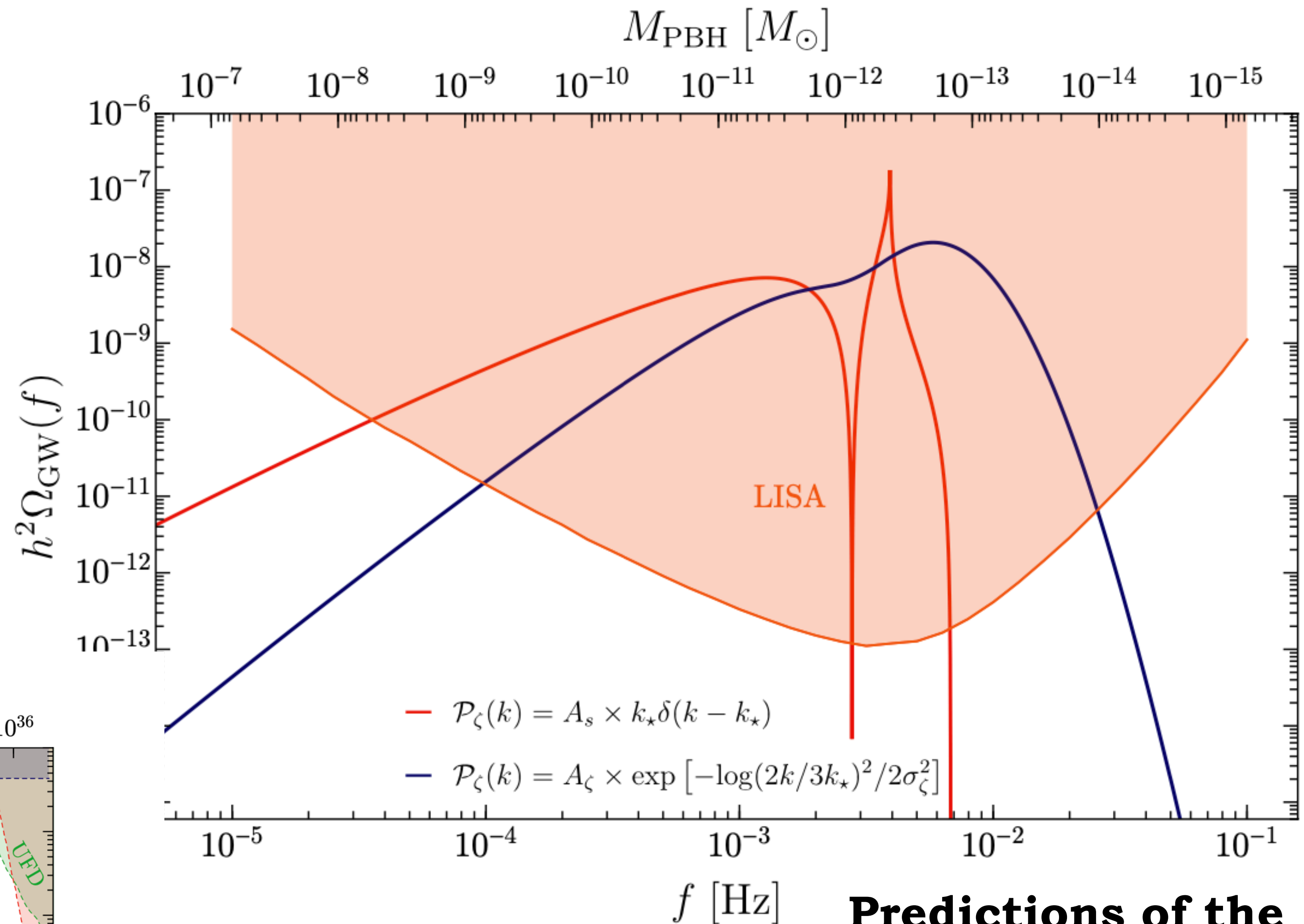
The signal depends on the shape of the curvature power spectrum, several phenomenological models are proposed

Example: GW signal from second order scalar perturbations and associated primordial black holes

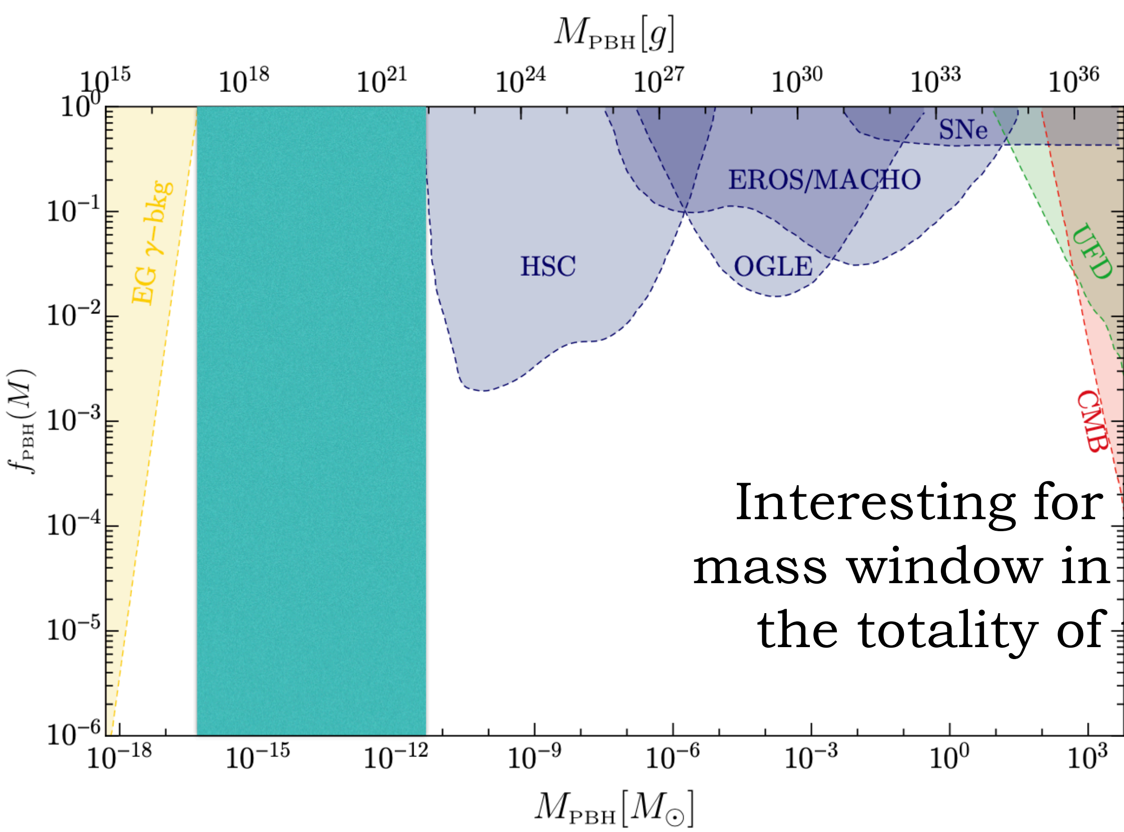
Source, Π_{ij} from a combination of

$$\partial_i \Psi, \partial_i \Phi$$

EXAMPLE OF SIGNAL IN THE LISA BAND:



Predictions of the signal must be refined accounting for non-gaussianity



Interesting for LISA: PBH in the mass window in which they can be the totality of the Dark Matter

$$\frac{M_H}{M_\odot} \simeq 7 \times 10^{-11} \frac{k}{10^{12}\ \mathrm{Mpc}^{-1}}$$

A. Riotto, https://indico.math.cnrs.fr/event/5766/contributions/5153/attachments/2801/3587/Paris2021.pdf

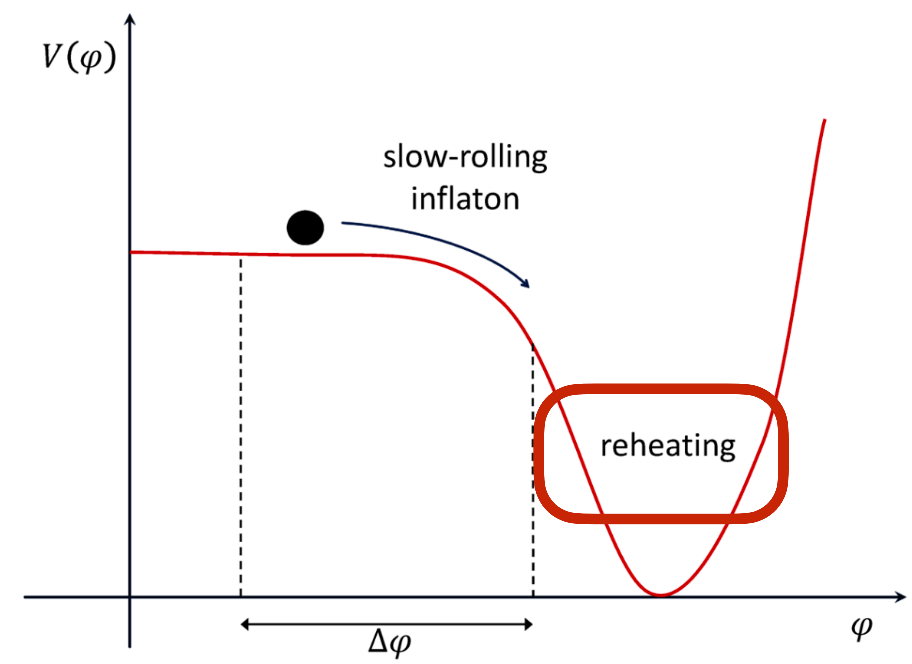
Example: resonant particle production at preheating

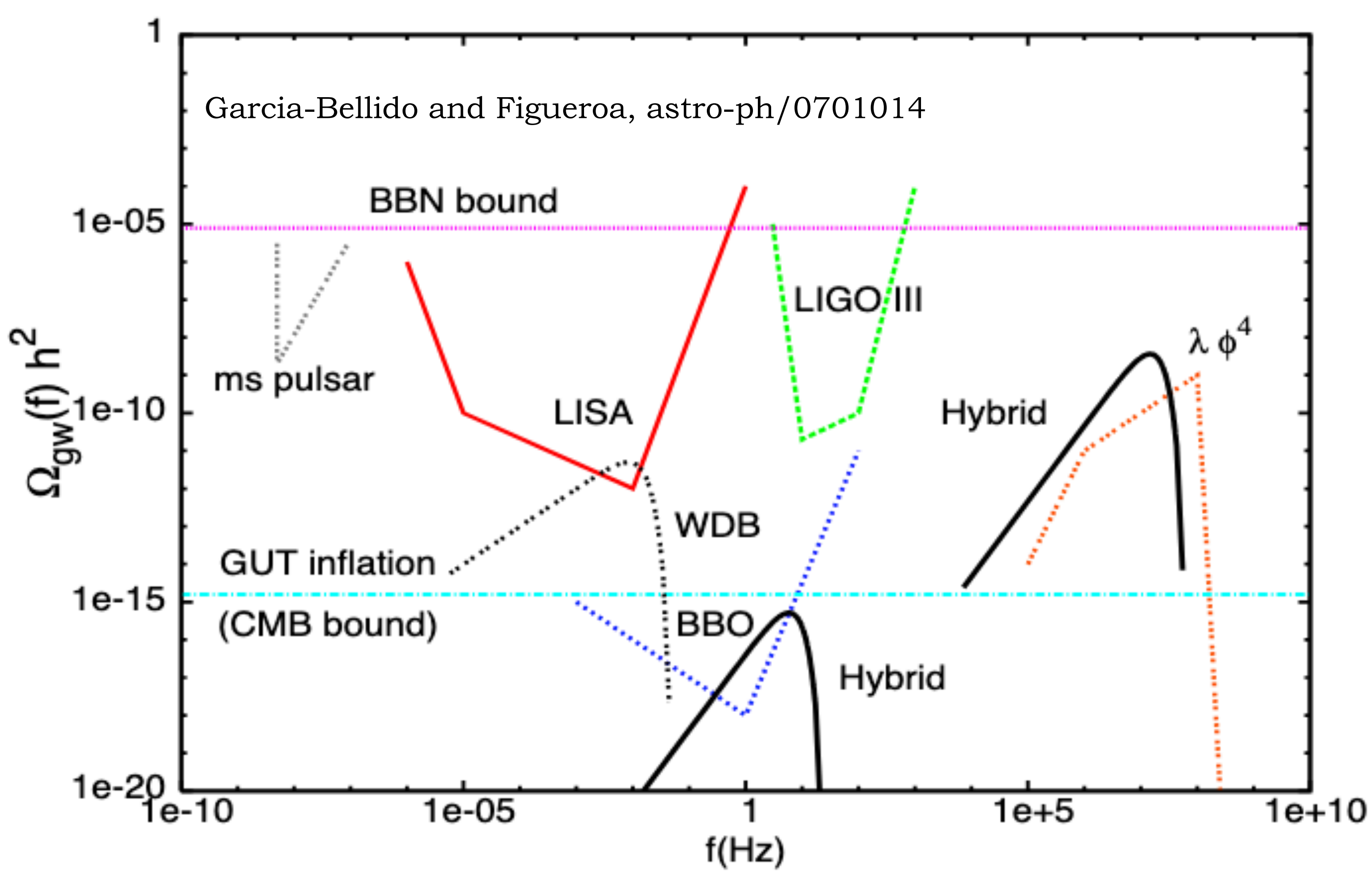
$$V(\phi) + \frac{1}{2}g^2\phi^2\chi^2$$

Kofman et al. arXiv:hep-ph/9704452

GW sourced from inhomogeneous field

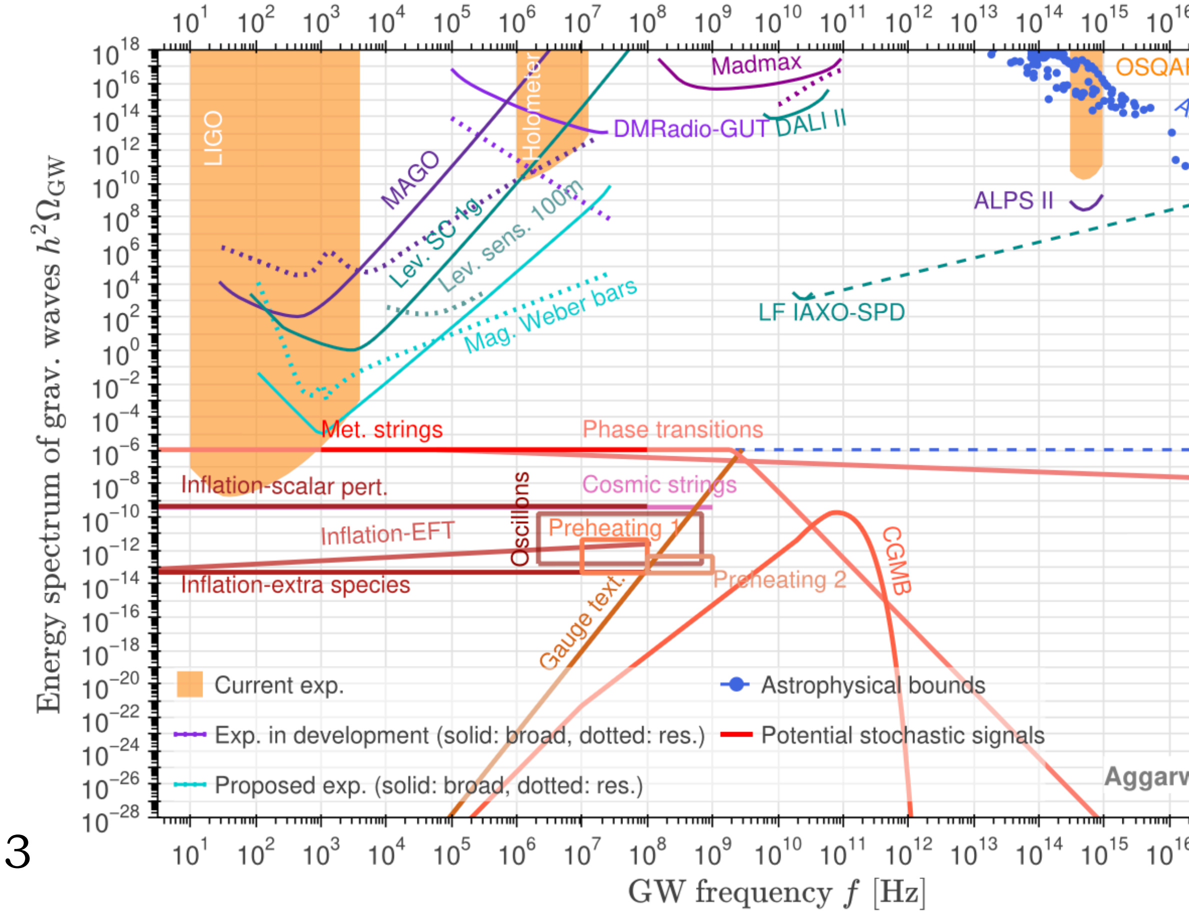
$$\Pi_{ij} \sim [\partial_i\chi\partial_j\chi]^{TT}$$





high frequency detectors up to 10^{18} GeV, sensitivity still above BBN and CMB bounds

Aggarwal et al, arXiv:2501.11723



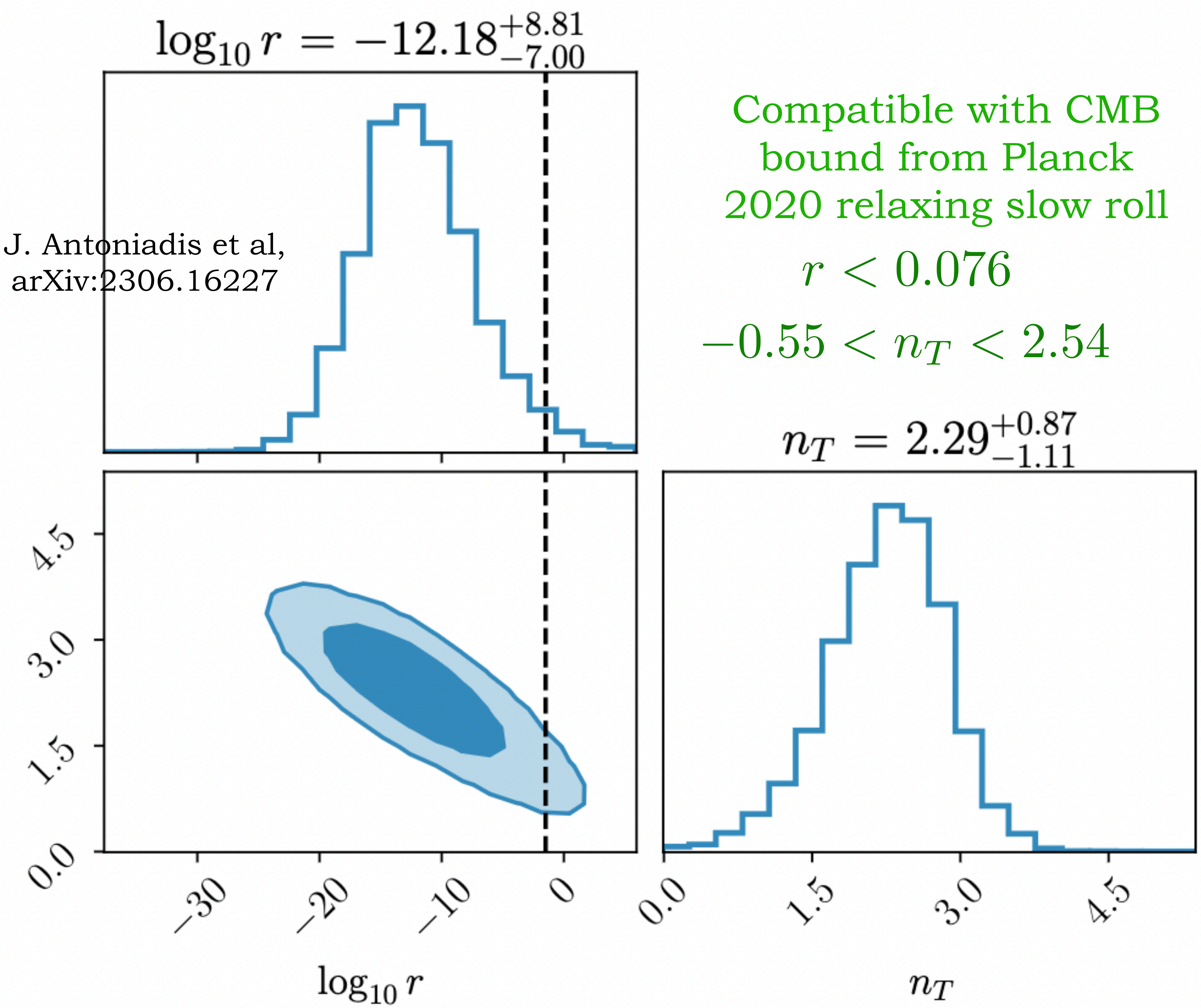
An example of possible detection at PTA?

$$\Omega_{\rm GW}(f) = \frac{3}{128}\,\Omega_{\rm rad}\, r\, \mathcal{P}^*_{\mathcal{R}} \left(\frac{f}{f_*}\right)^{n_T} \left[\frac{1}{2}\left(\frac{f_{\rm eq}}{f}\right)^2 + \frac{16}{9}\right]$$

$$\times \left(\frac{f}{f_{\rm RD}}\right)^{\frac{2(3w-1)}{3w+1}}$$

Very small value of r at CMB scale

J. Antoniadis et al, arXiv:2306.16227



Compatible with CMB bound from Planck 2020 relaxing slow roll

$$r < 0.076$$

$$-0.55 < n_T < 2.54$$

Would this be compatible with slow roll and a stiff equation of state? Marginally

$$\gamma = 5 - n_T + \frac{2(1-3w)}{3w+1}$$

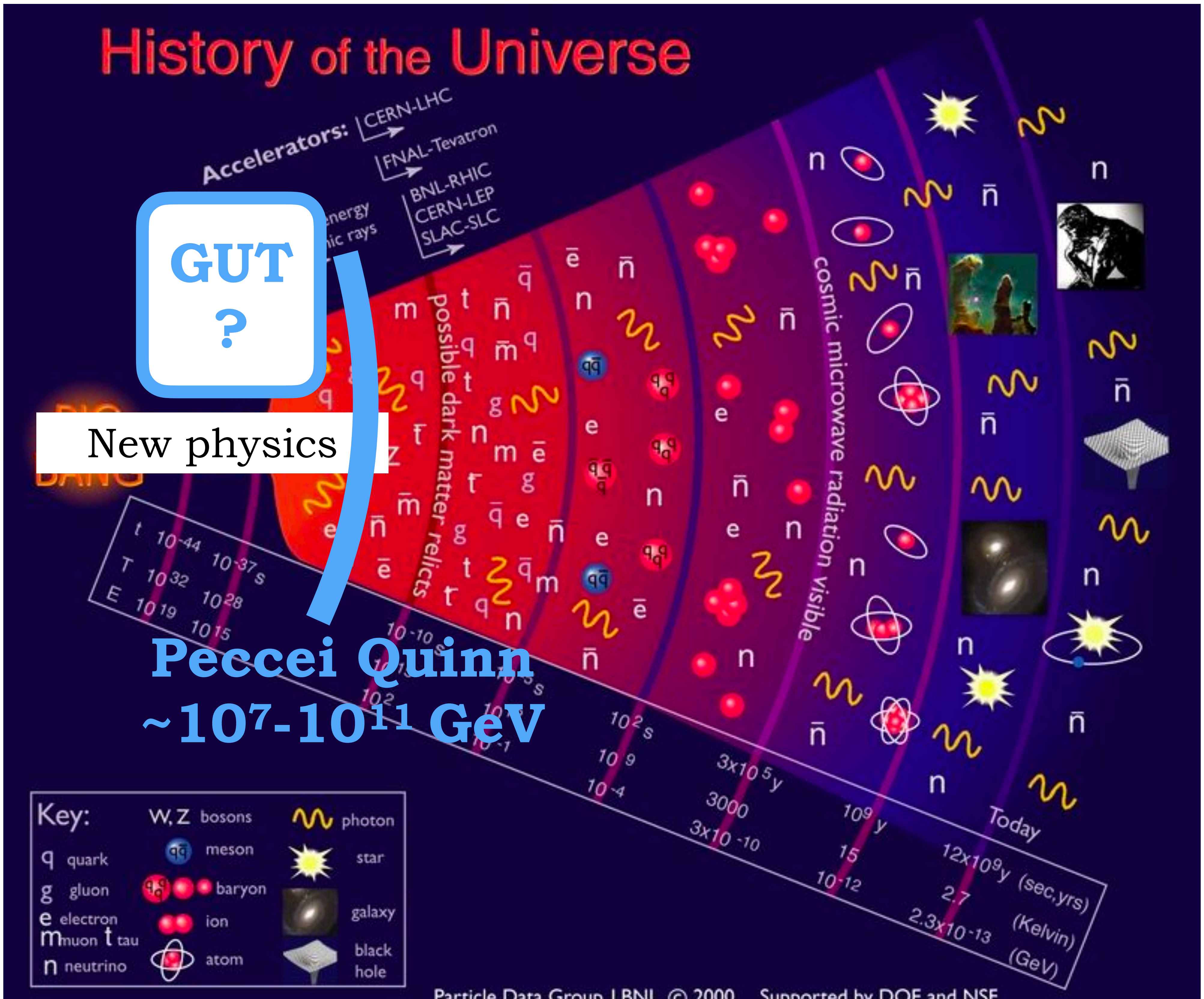
Bound between 0 and -2

$$\gamma_{\rm best\ fit} \simeq 2.7 \;\longrightarrow\; n_T \gtrsim 0.3$$

Strong degeneracy between the two parameters $n_T = -0.16\log_{10} r + 0.46$

phase transitions at high energy: GUT, Peccei-Quinn…

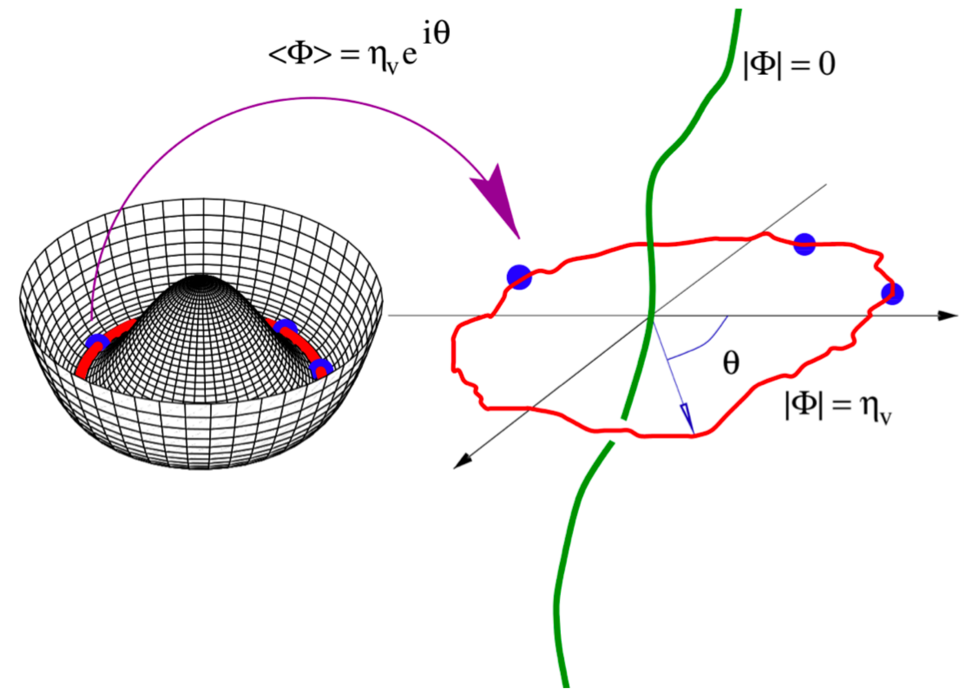
They can lead to topological defects sourcing GWs



Topological defects

- They can be formed by a symmetry breaking, whenever the system is symmetric under a group while the ground-state is invariant only under a subgroup, depending on the topology of the degenerate ground-state manifold

- The properties of the defects depend on the underlying action:
  - which kind of defects (strings, domain walls…)?
  - Are they stable?
  - thickness? energy per unit length/surface?
  - interaction (inter-commutation, presence of junctions…)?
  - additional degrees of freedom?
  - dynamics?



- As concentrations of high energy density with relativistic velocity they can be powerful sources of GWs, but they can also emit other kind of radiation (particles): which is the dominant decay mechanism? Can there be multi-messenger observational signatures? Does the network reach scaling?

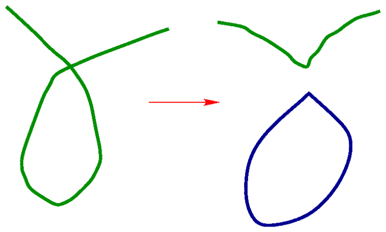
- Numerical simulations are necessary to model their evolution, but they need very large *dynamical range*, and should in principle contain the *gravitational back-reaction*

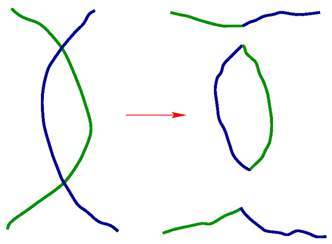
- Predictions of the **GW signal from topological defects is inevitably model dependent**

Topological defects

Local cosmic strings

- Formed e.g. at the breaking of a local U(1) symmetry at energy scale η
- All gravitational properties related to $G\mu \sim 10^{-6}\left(\frac{\eta}{10^{16}\ \mathrm{GeV}}\right)^2$
- Cosmic strings can be described by the Nambu Goto action: w ~ $1/\eta \ll \ell$
- Intercommutation generates loops with kinks and cusps
  -> loops self-intersects and fragment until they reach stable non-self-intersecting trajectories
  -> loop oscillations produce GWs
  -> loops decay with GW emission
  -> the network reaches scaling together with an associated SGWB

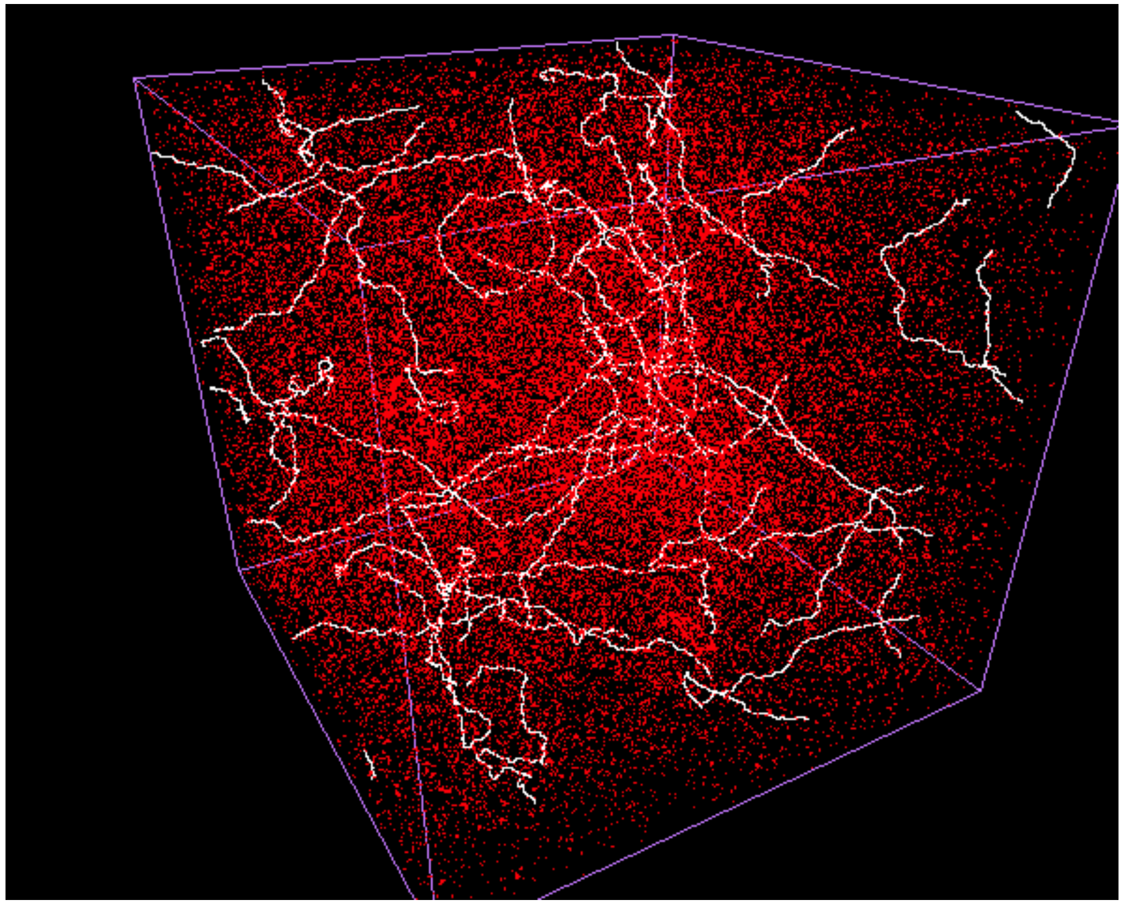




- *What is the GW power radiated by a loop? Does the loop also emit other kinds of radiation?*
- *What is the distribution of non-self-intersecting loops of given length at a given time?*
- *What is the role played by gravitational back-reaction on the loops?*

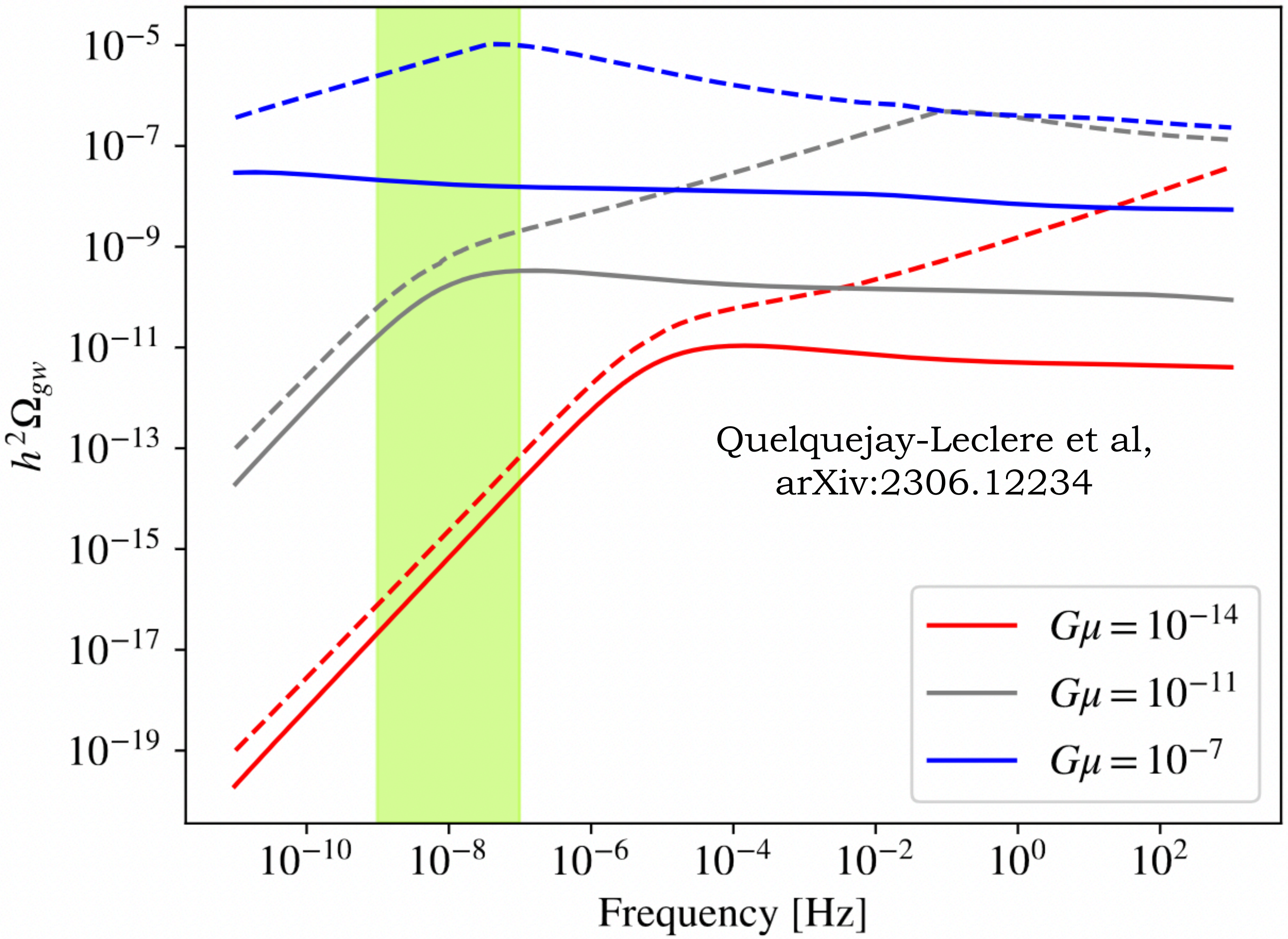
Topological defects

Local cosmic strings



https://curl.irmp.ucl.ac.be/~chris/strings.html

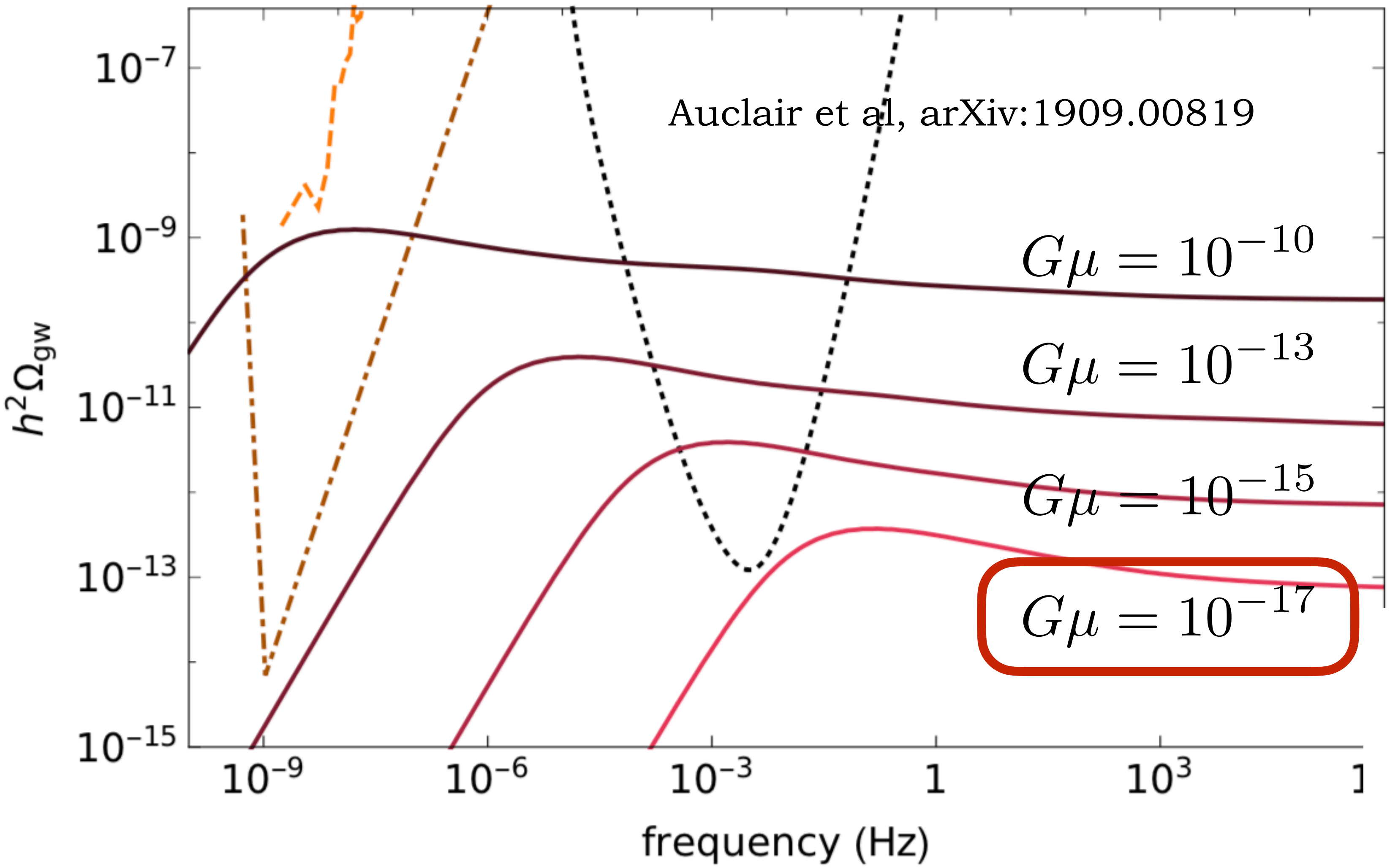
Different numerical simulations provide different predictions for the GW signal



Topological defects

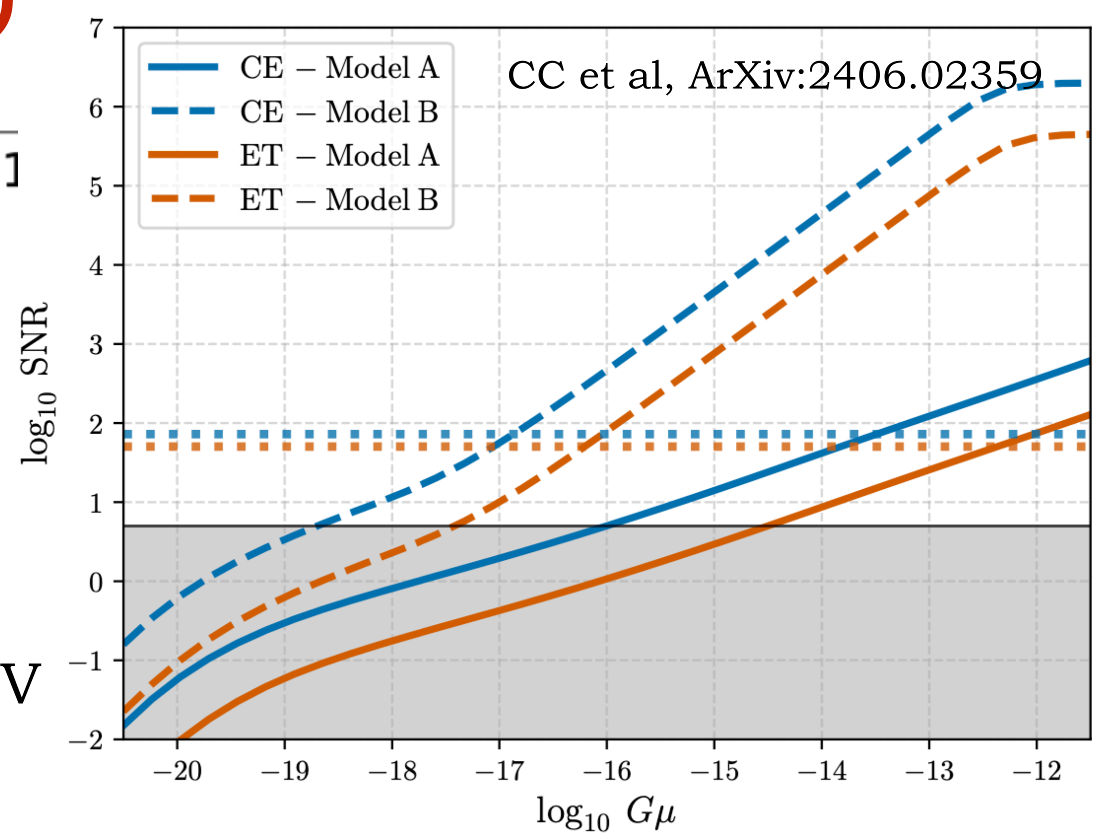
Local cosmic strings

One model of Nambu Goto local strings in the **LISA** band



LVK constraints:

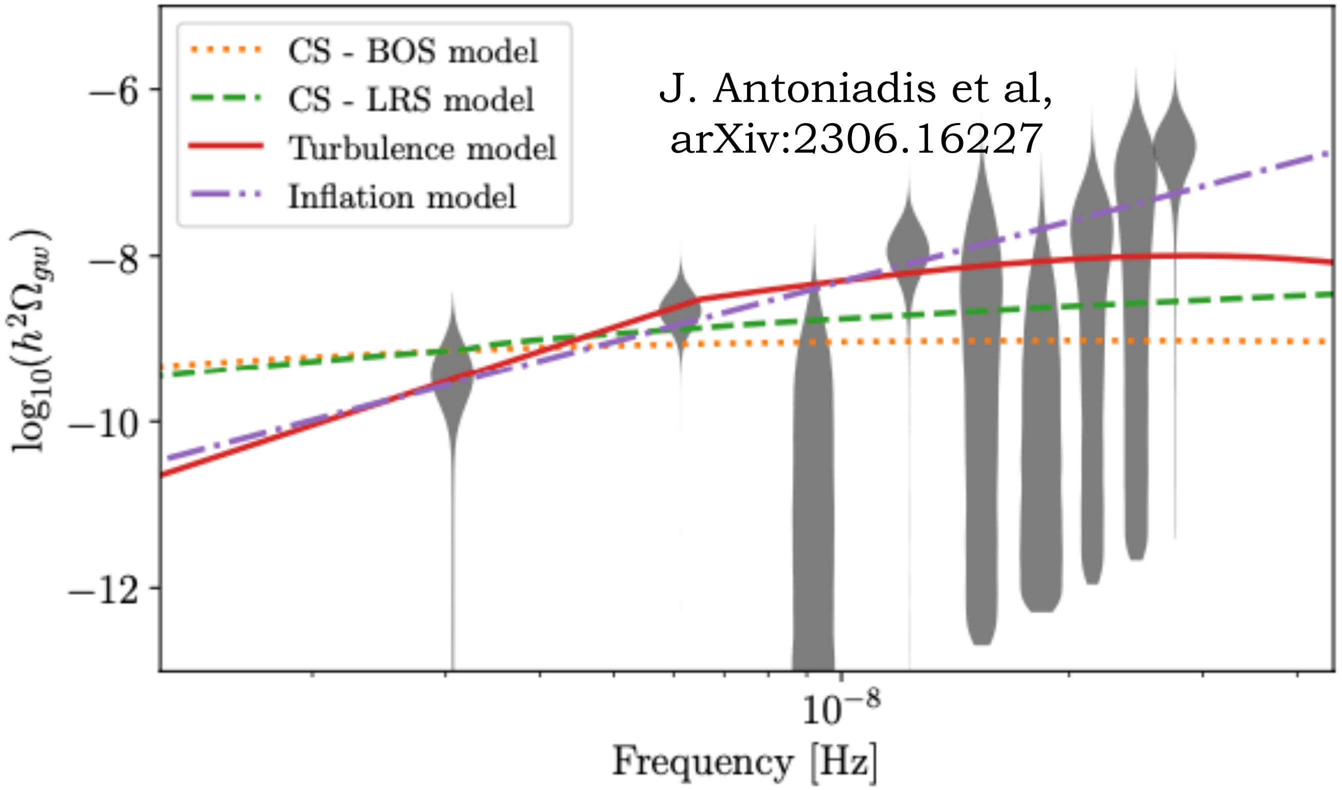
$$G\mu \lesssim 9.6 \cdot 10^{-9} \text{ (model A)}$$
$$G\mu \lesssim 10^{-15} \text{ (model B)}$$



3G detectors can probe scales down to $\eta \sim 10^{(11\text{-}12)}$ GeV

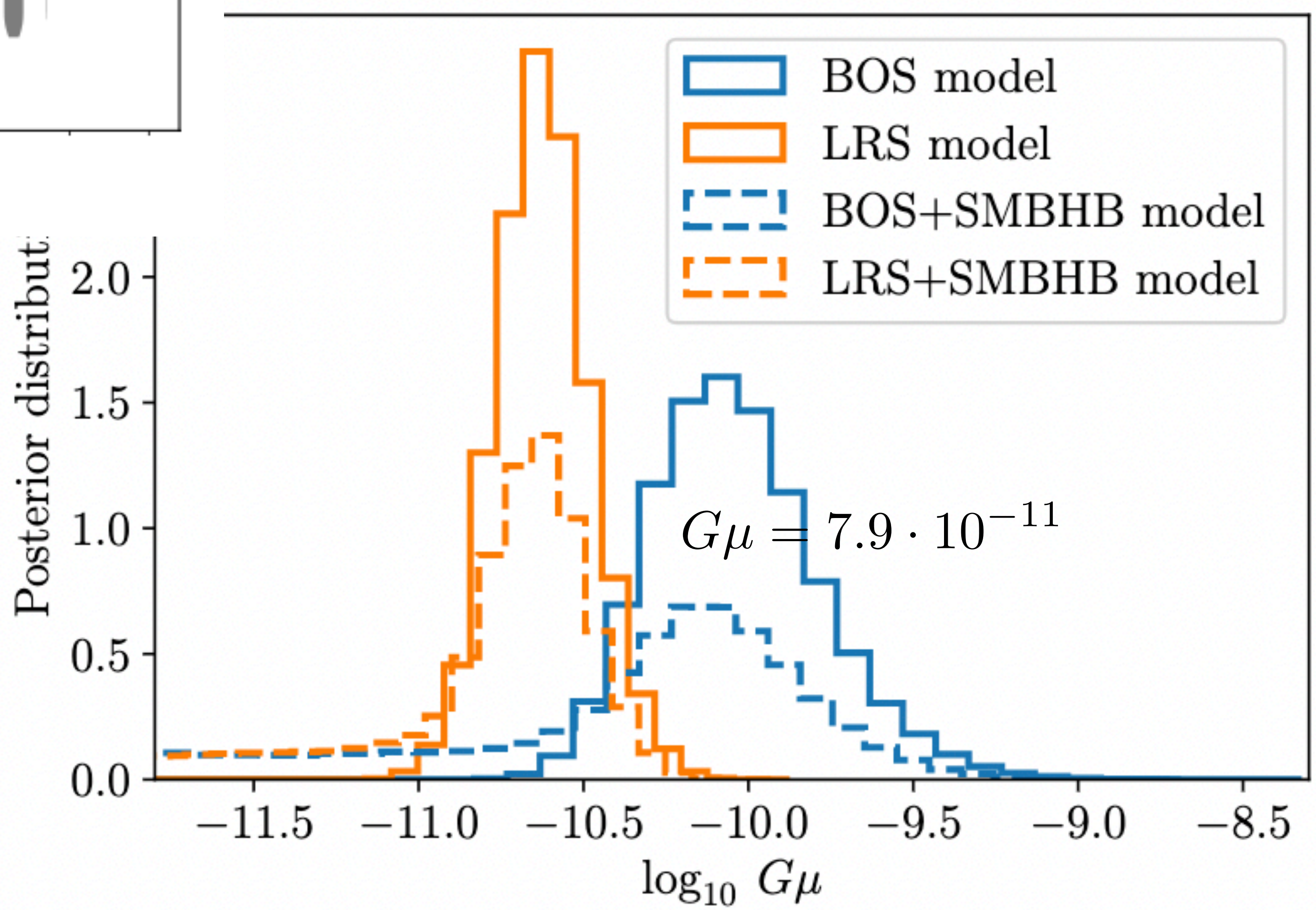
An example of possible detection at PTA?

Local cosmic strings



cosmic strings aren't a good fit to the data, because the spectrum is too shallow

$$\eta \sim M_{\rm Pl}\sqrt{G\mu} \sim 10^{14}\,\text{GeV}$$



The posteriors of the string tension and SMBHBs SGWB amplitude are strongly correlated: SMBHBs alone fit the data

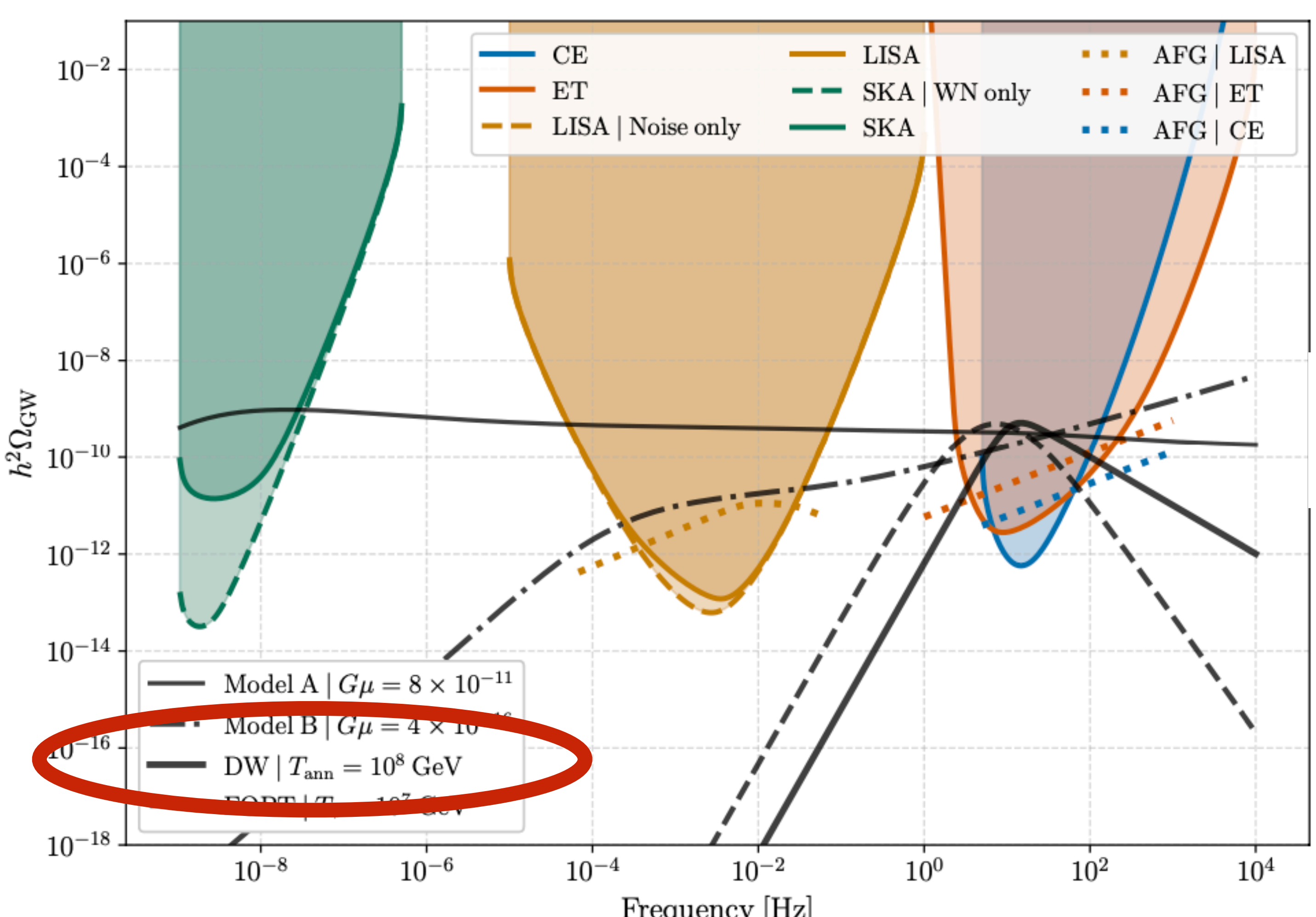
Topological defects

Domain walls

- Formed when a discrete symmetry is broken and the ground state is disconnected
- The network reaches a scaling regime by reconnection and radiation emission
- The energy density decays too slowly
  -> they can dominate the universe
  -> they need an annihilation mechanism

$$\rho_{\rm DW} \propto \sigma\, H$$

- Explicit breaking of the discrete symmetry -> vacuum non-degeneracy favouring the true vacuum $\quad H(T_{\rm ann}) \sim \Delta V/\sigma$



- GW emission can occur both during scaling and during annihilation

$$\alpha_{\rm ann} \equiv \left.\frac{\rho_{\rm DW}}{3H^2 M_p^2}\right|_{\rm ann}$$

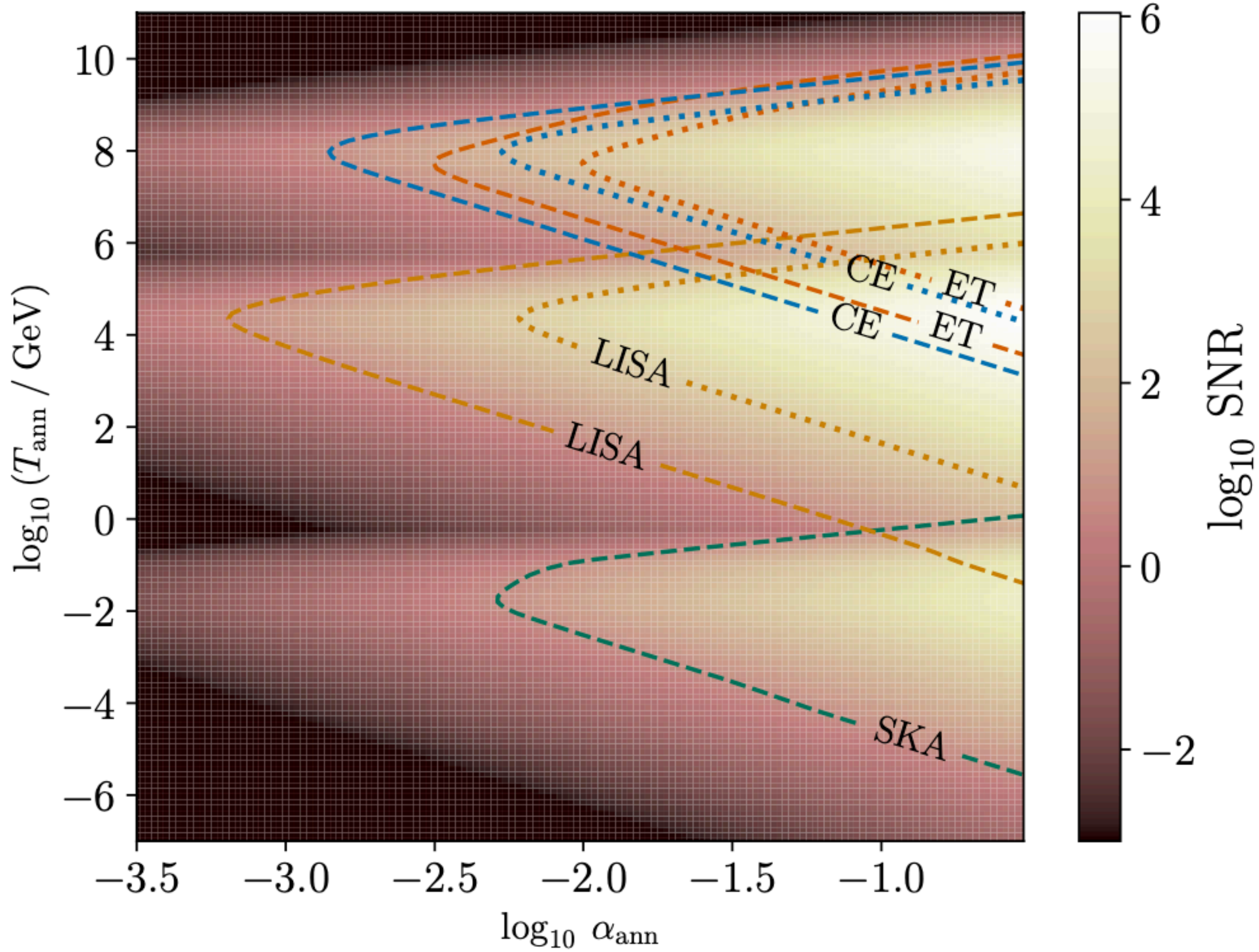
$$\Omega_{\rm GW,DW}(f)h^2 \simeq 10^{-10}\,\tilde{\epsilon}\left(\frac{10.75}{g_*(T_{\rm ann})}\right)^{\frac{1}{3}}\left(\frac{\alpha_{\rm ann}}{0.01}\right)^2 S\left(\frac{f}{f_p^0}\right)$$

$$f_p = H_{\rm ann}$$

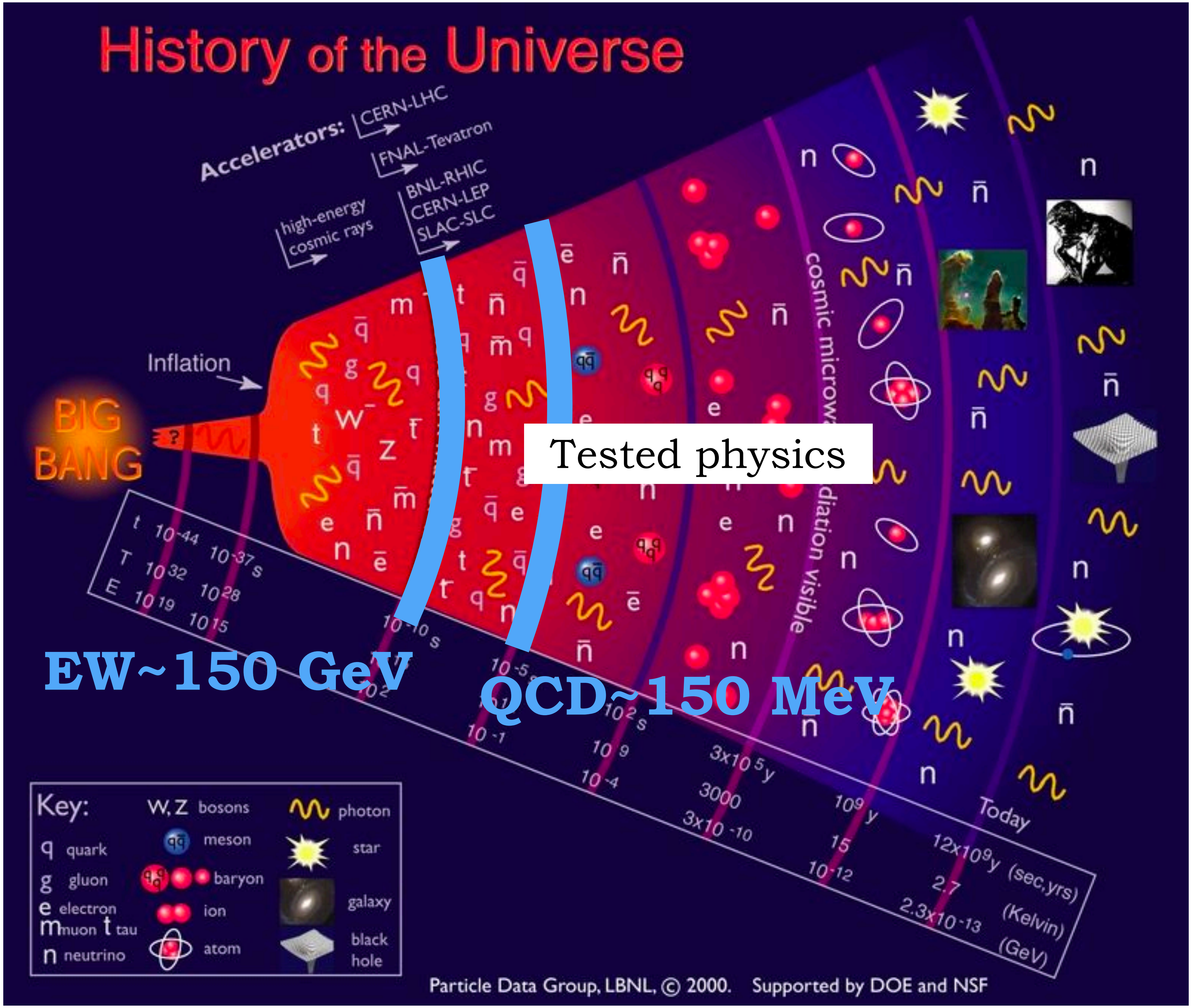
Topological defects

Domain walls

Assuming the spectral shape from numerical simulations (exponents 3 and -1), we have explored the reach of 3G detectors, LISA and PTA to DW parameter space



phase transitions predicted by the standard model of particle physics: electroweak and QCD

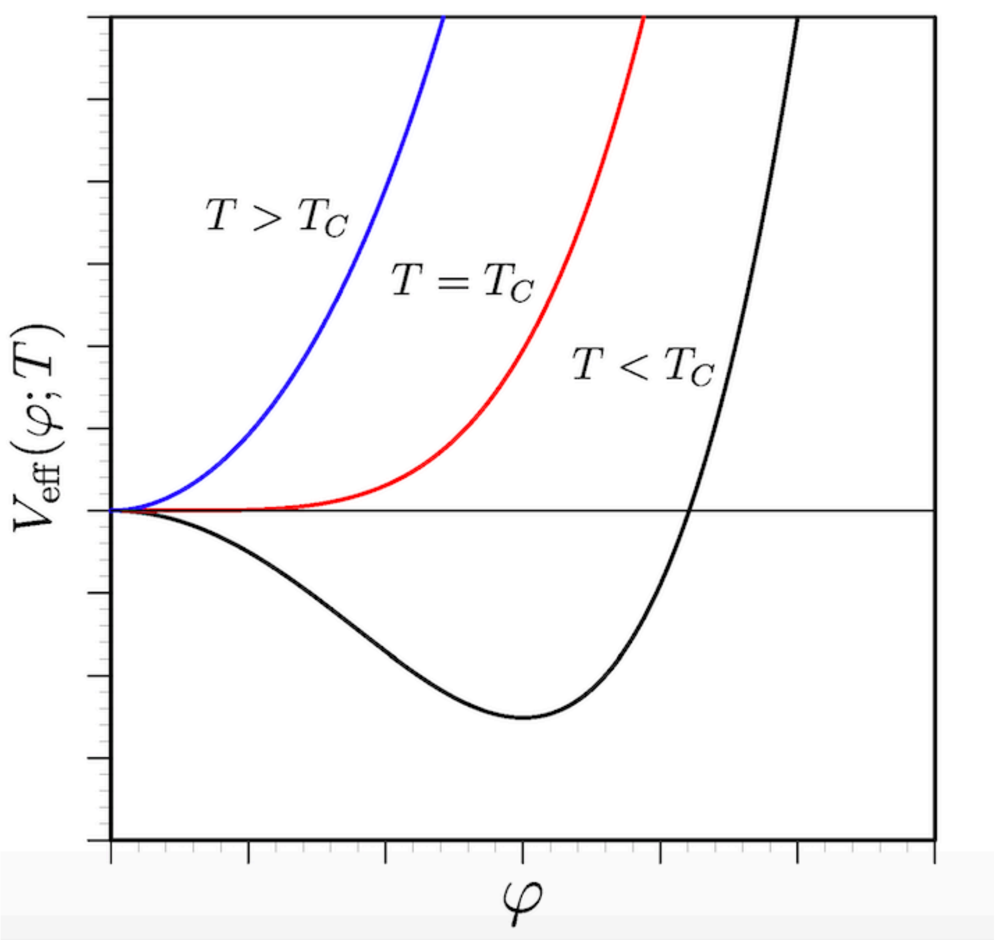


First order phase transitions

- We know that at least two PTs occurred in the universe, the EW one and the QCD one: according to the standard model, they are both **crossovers**

- However, *sizeable (detectable) GW generation requires a first order PT*, proceeding through the *nucleation of true vacuum bubbles*

Second order phase transition



First order phase transition

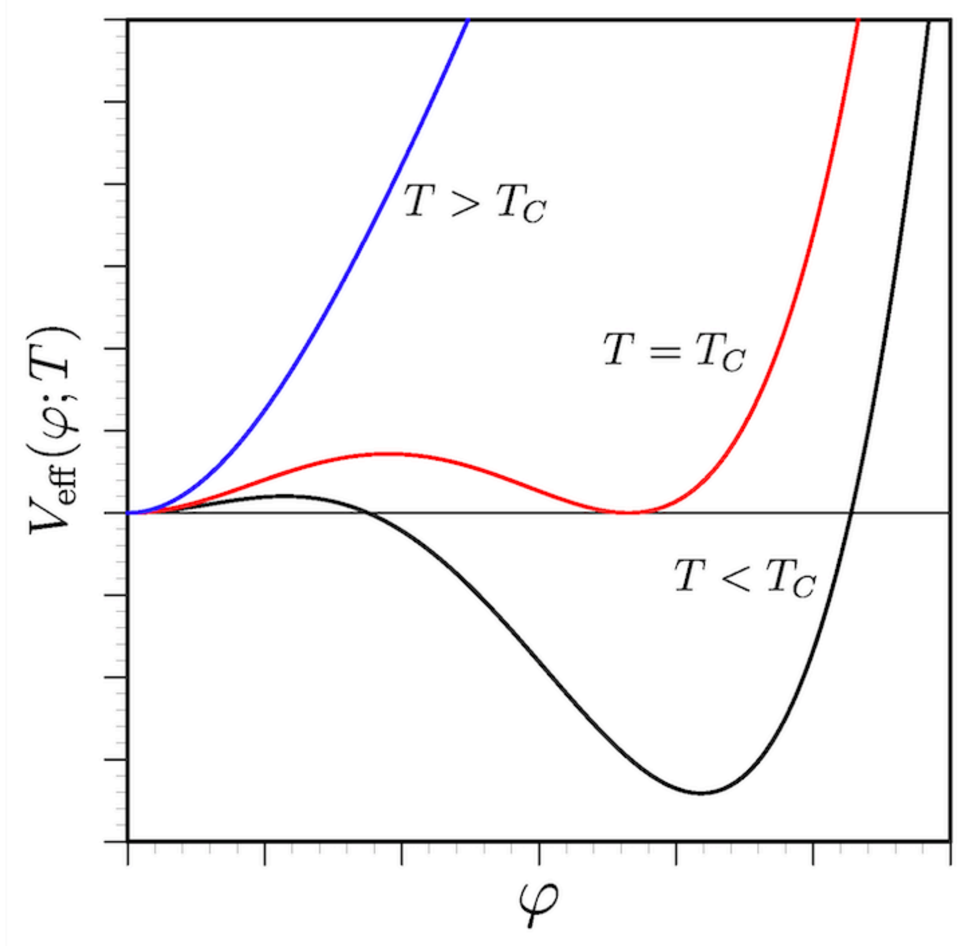


Image credit: E. Senaha, Symmetry 2020

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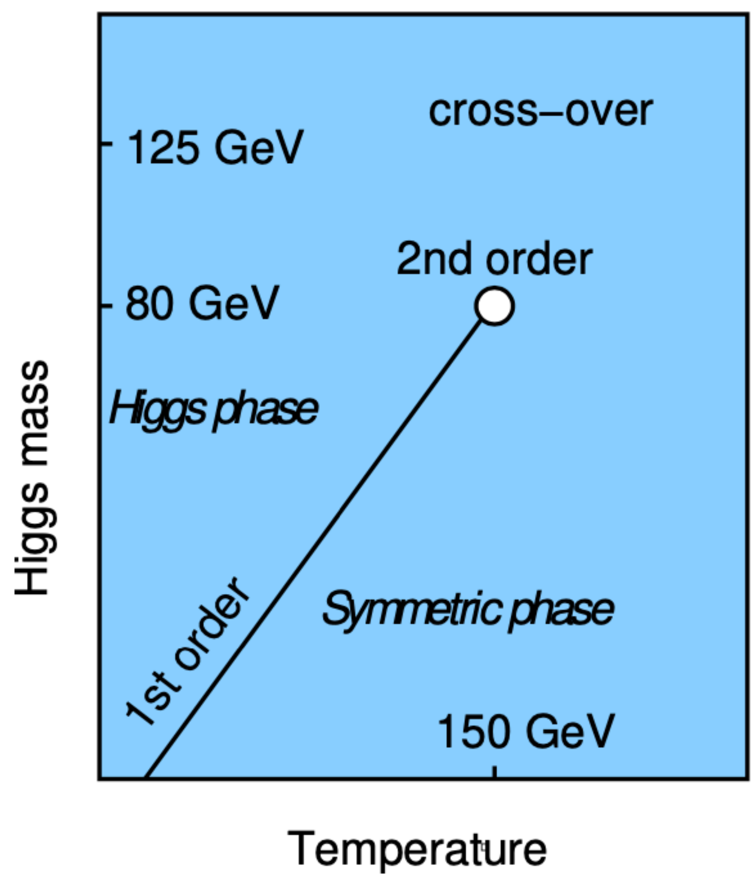
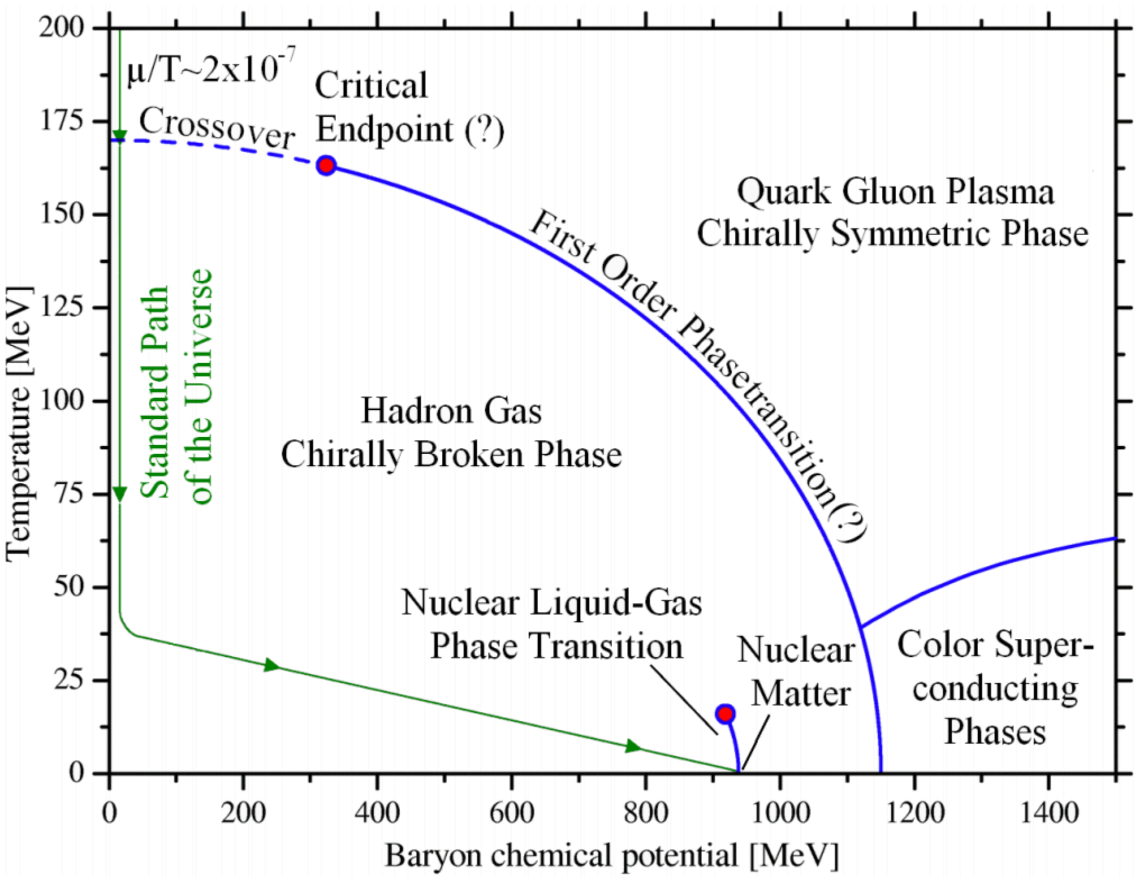
EWPT



M. Hindmarsh et al, arXiv:2008.09136

QCDPT



T. Boekel and J. Schaffner-Bielich, arXiv:1105.0832

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EWPT

might become first order in BSM EW sector extensions: SM + light scalars (SM+singlet, SUSY, 2HDM, composite Higgs…)

QCDPT

Depends on the conditions in the early universe: **might become first order if the lepton asymmetry in the universe is large**

D. Schwarz and Stuke, arXiv:0906.3434
M. Middeldorf-Wygas et al, arXiv:2009.00036

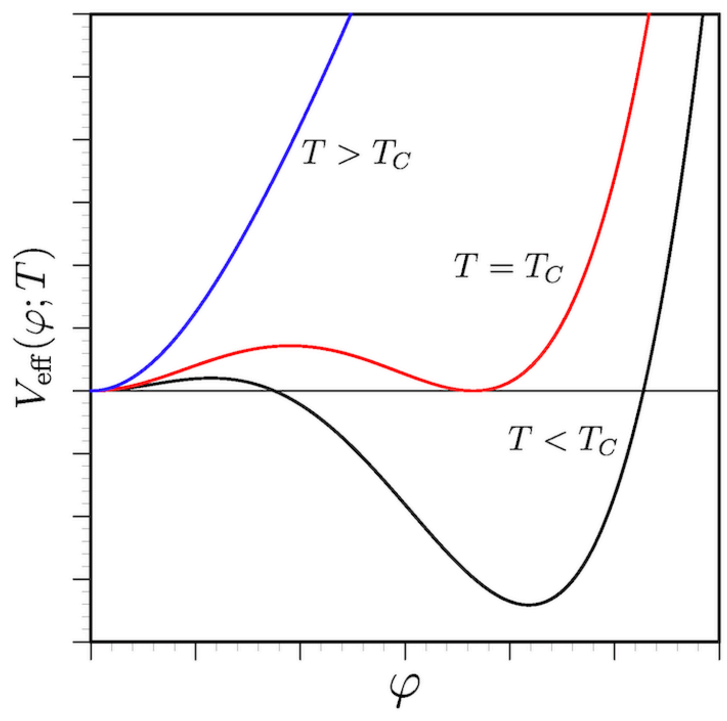
OTHER EXAMPLES OF POSSIBLE FOPTs:

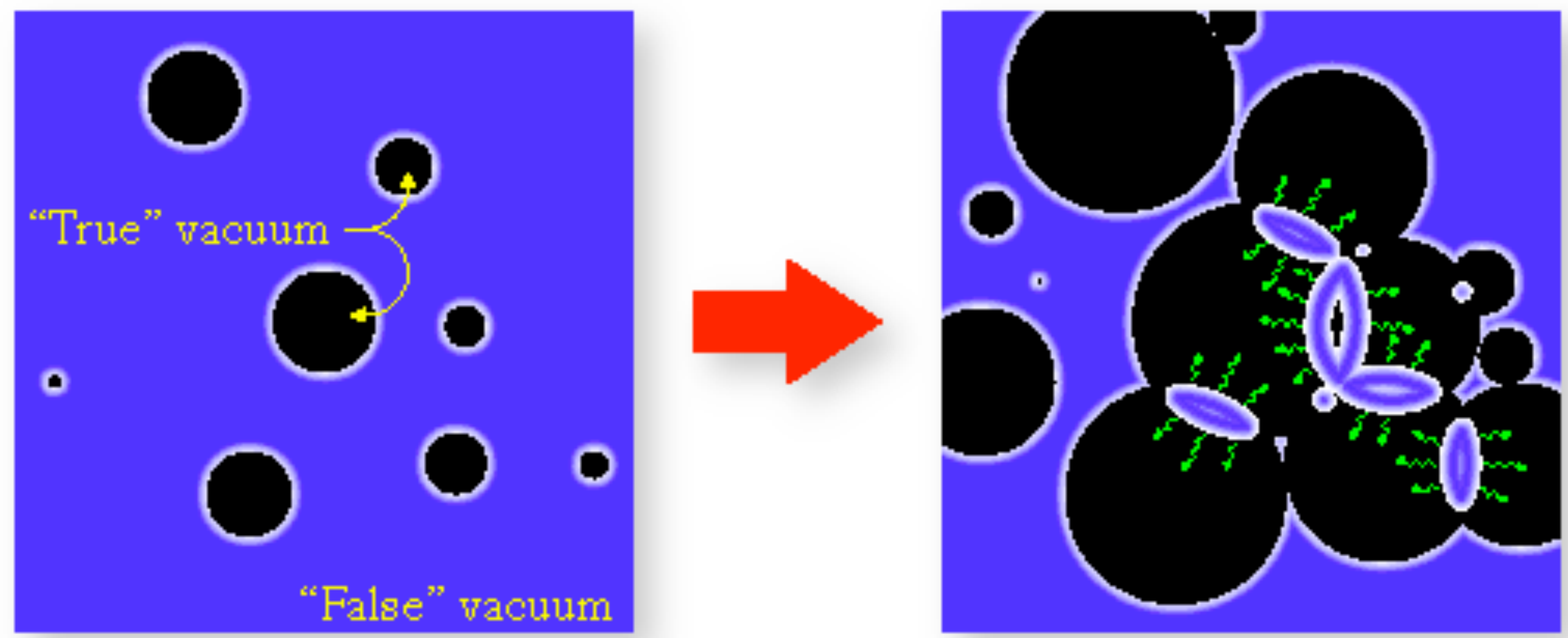
- **Effective approaches:** heavy new physic represented by higher dimensional operators
- **Conformal models:** e.g. conformal symmetry breaking with dilaton
- **New symmetries:** extend the SM with e.g. $U(1)_{B-L}$
- **Hidden sectors:** provide also dark matter candidates, PT can be as strong as one wants
- **Peccei Quinn** can be first order depending on the realisation

Opportunity to probe high energy physics scenarios beyond the standard model

First order phase transitions

Sources of tensor anisotropic stress (and thereby GWs) at a first order phase transition:





Several processes, rich phenomenology!

- Bubble collision (scalar field gradients) $\Pi_{ij}^{TT} \sim [\partial_i\phi\partial_j\phi]^{TT}$
- Bulk fluid motion $\Pi_{ij}^{TT} \sim [\gamma^2(\rho+p)v_iv_j]^{TT}$ sound waves and/or turbulence
- Electromagnetic fields $\Pi_{ij}^{TT} \sim [-E_iE_j - B_iB_j]^{TT}$

First order phase transitions

The signal depends on the following parameters

- The temperature of the FOPT T_*
- The amount of energy available in the source K, connected to the PT strength
- The size of the anisotropic stresses, connected to the bubble size $R_* = v_w/\beta$
- The bubble wall velocity v_w

$T_*, \ \alpha, \ \frac{\beta}{H_*}$ → Determined by the effective potential

v_w, K → Determined by the bubble expansion dynamics and interaction, and by the fluid dynamics (sound speed fixed)

If the PT is strong and non-linearities in the bulk fluid develop: fraction of kinetic energy in turbulent motions $\varepsilon = \frac{K_{\rm turb}}{K}$

Most of these parameters are known (at least in principle) given a PT model + numerical simulations of the fluid dynamics

numerical simulations are necessary to infer the GW signal because of non-linear dynamics and/or complicated fluid shells profiles and/or intrinsic randomness of the process

First order phase transitions

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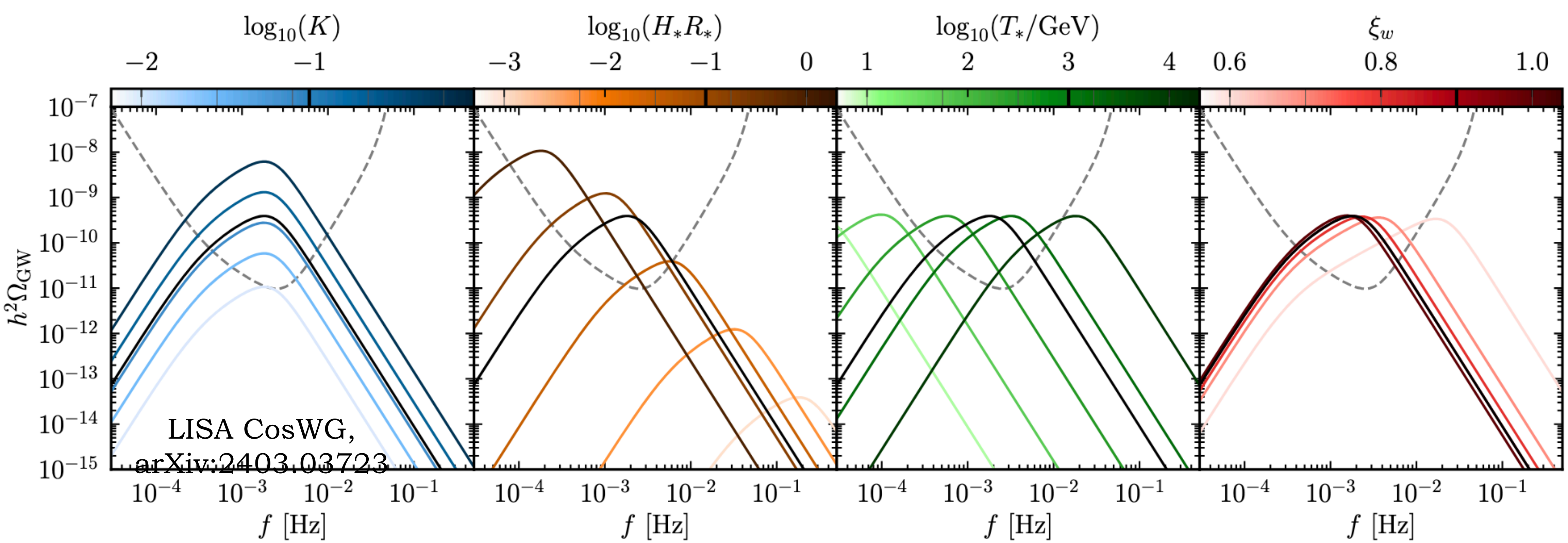


(b) sound waves (black: $K = 0.1$, $H_* R_* = 0.1$, $\xi_w = 0.9$, $T_* = 1\,\mathrm{TeV}$)

First order phase transitions

LIGO Virgo Kagra

$$1\,\mathrm{Hz} < f < 1000\,\mathrm{Hz} \quad \longrightarrow \quad 10^{6}\,\mathrm{GeV} \lesssim T_* \lesssim 10^{10}\,\mathrm{GeV}$$

CC et al, ArXiv:2406.02359

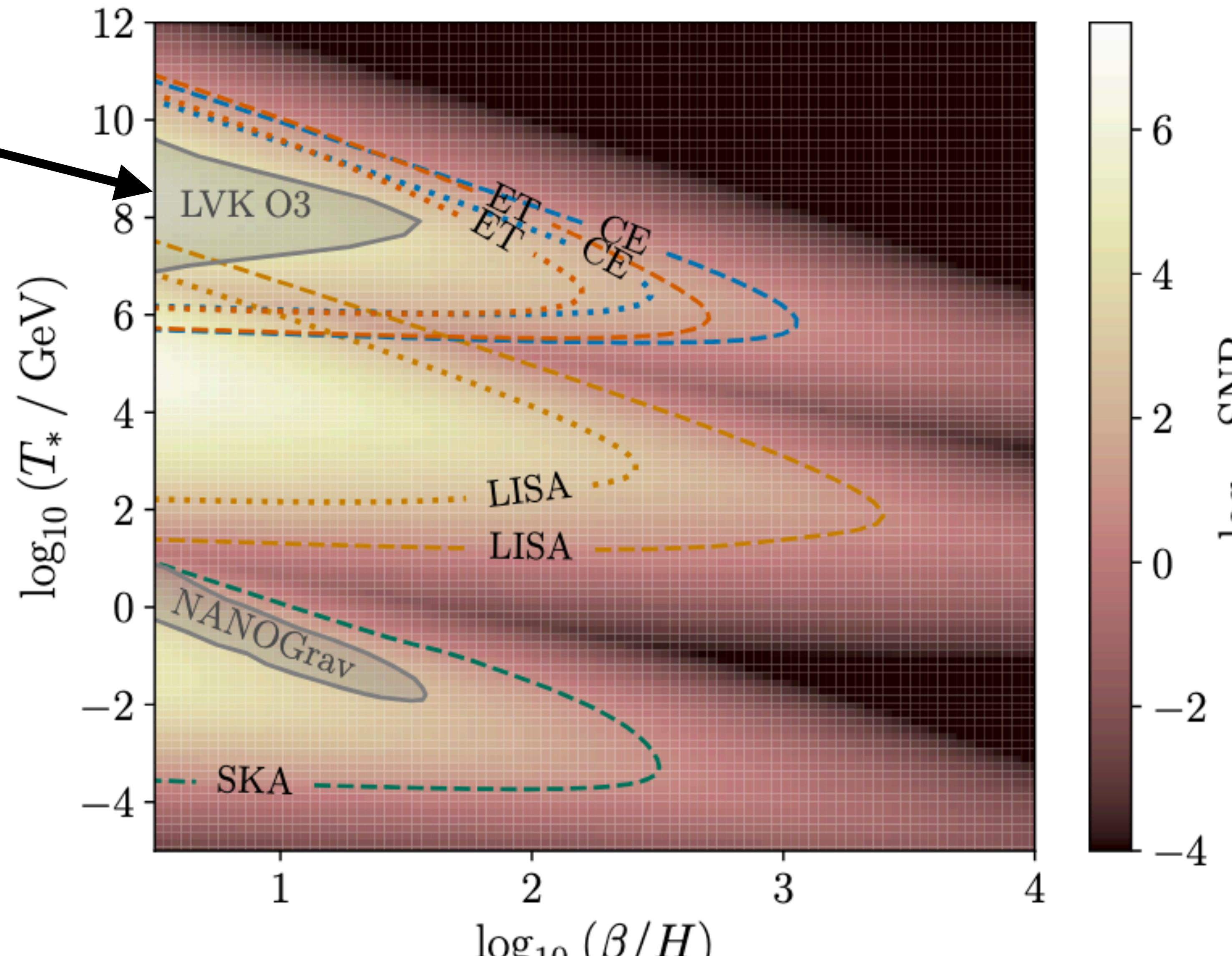
LVK constraints from non-detection

Badger et al, arXiv:2209.14707

Peccei-Quinn phase transition

$$T_{\mathrm{PQ}} \sim F_a$$

$$10^{7-8}\,\mathrm{GeV} \lesssim F_a \lesssim 10^{10-11}\,\mathrm{GeV}$$



Parameter to which the signal amplitude is *inversely* proportional

First order phase transitions

Pulsar Timing Arrays

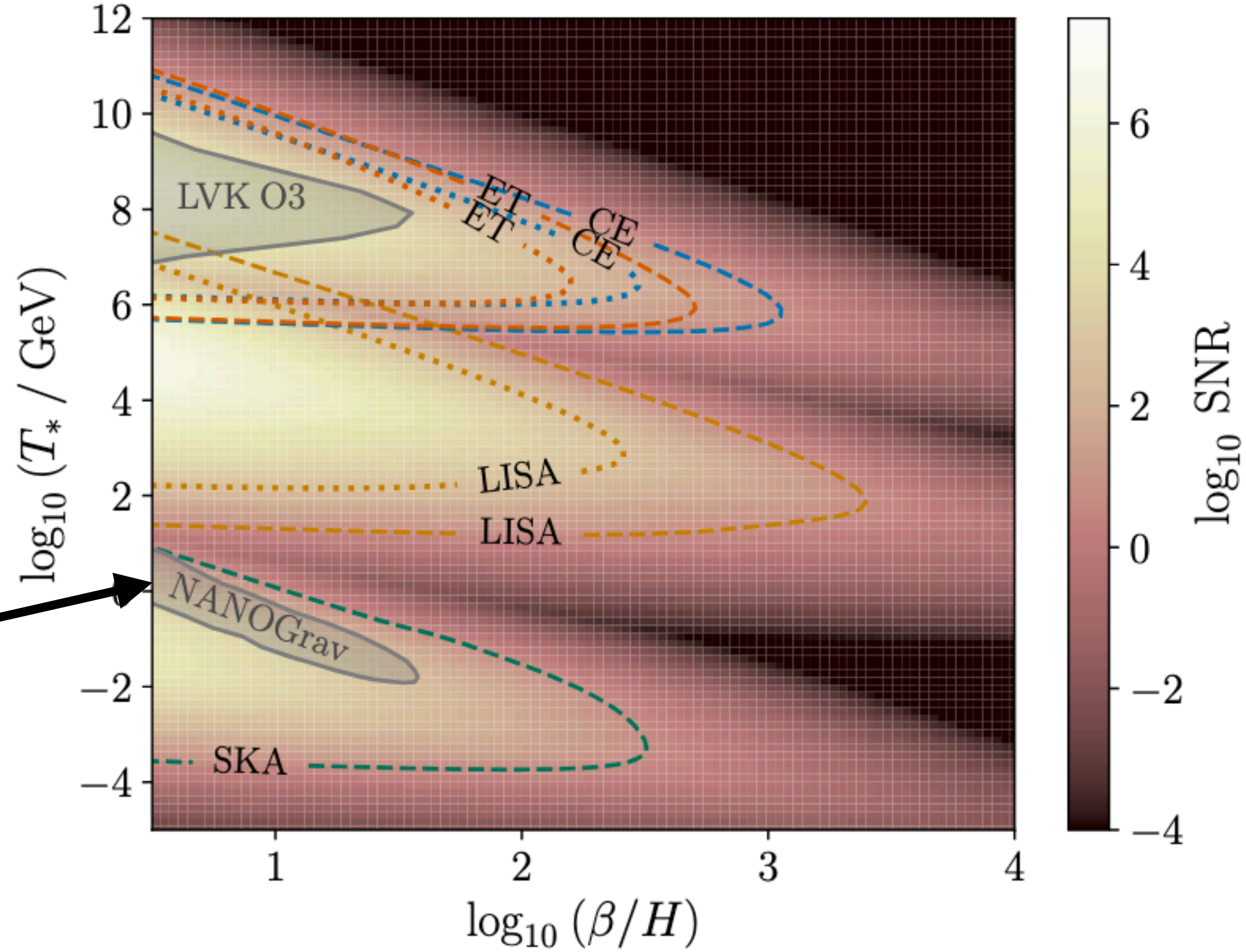
$$10^{-9}\,\mathrm{Hz} < f < 10^{-7}\,\mathrm{Hz} \quad \longrightarrow \quad 1\,\mathrm{MeV} \lesssim T_* \lesssim 1\,\mathrm{GeV}$$

CC et al, ArXiv:2406.02359

PTAs offer the possibility to probe the **QCD energy scale**

Parameter space region that could explain the measurement

Afzal et al arXiv:2306.16219



Parameter to which the signal amplitude is *inversely* proportional

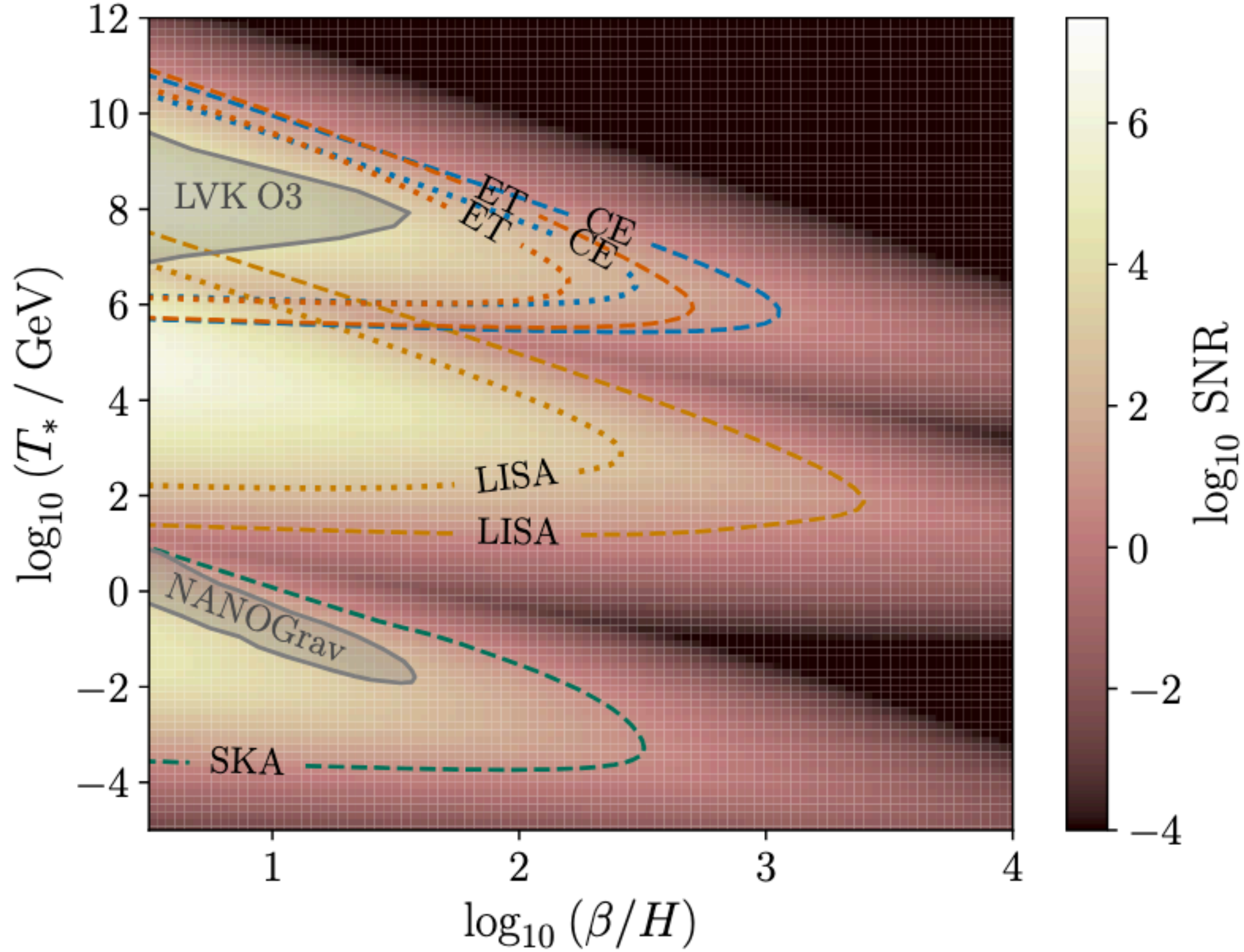
First order phase transitions

Laser interferometer space antenna

$$10^{-5}\,\mathrm{Hz} < f < 0.1\,\mathrm{Hz} \quad \longrightarrow \quad 10\,\mathrm{GeV} \lesssim T_* \lesssim 10^5\,\mathrm{GeV}$$

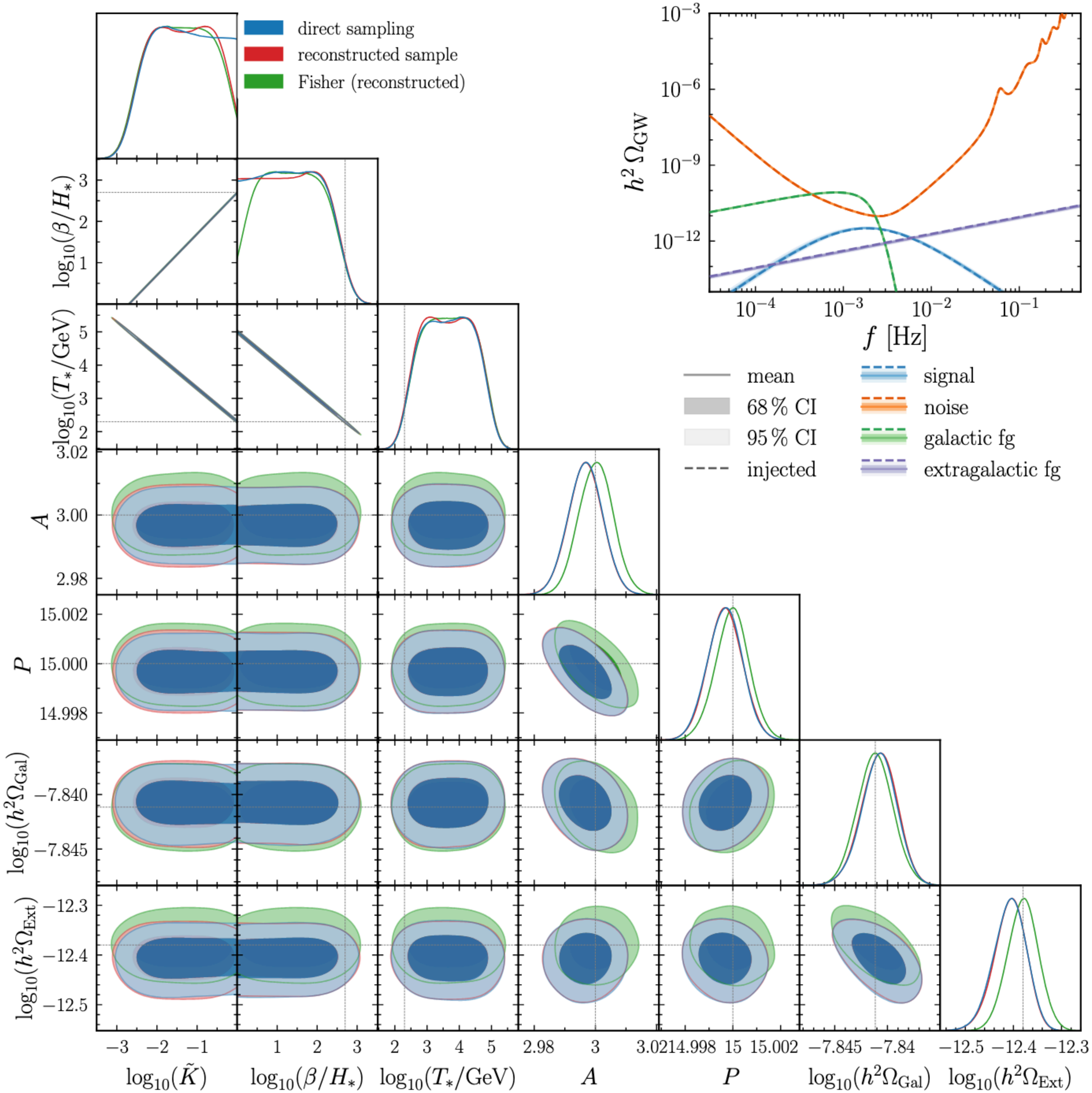
LISA offers the possibility to probe the **EW energy scale and beyond**

CC et al, ArXiv:2406.02359



Parameter to which the signal amplitude is *inversely* proportional

Examples of detectable signal from the EWPT

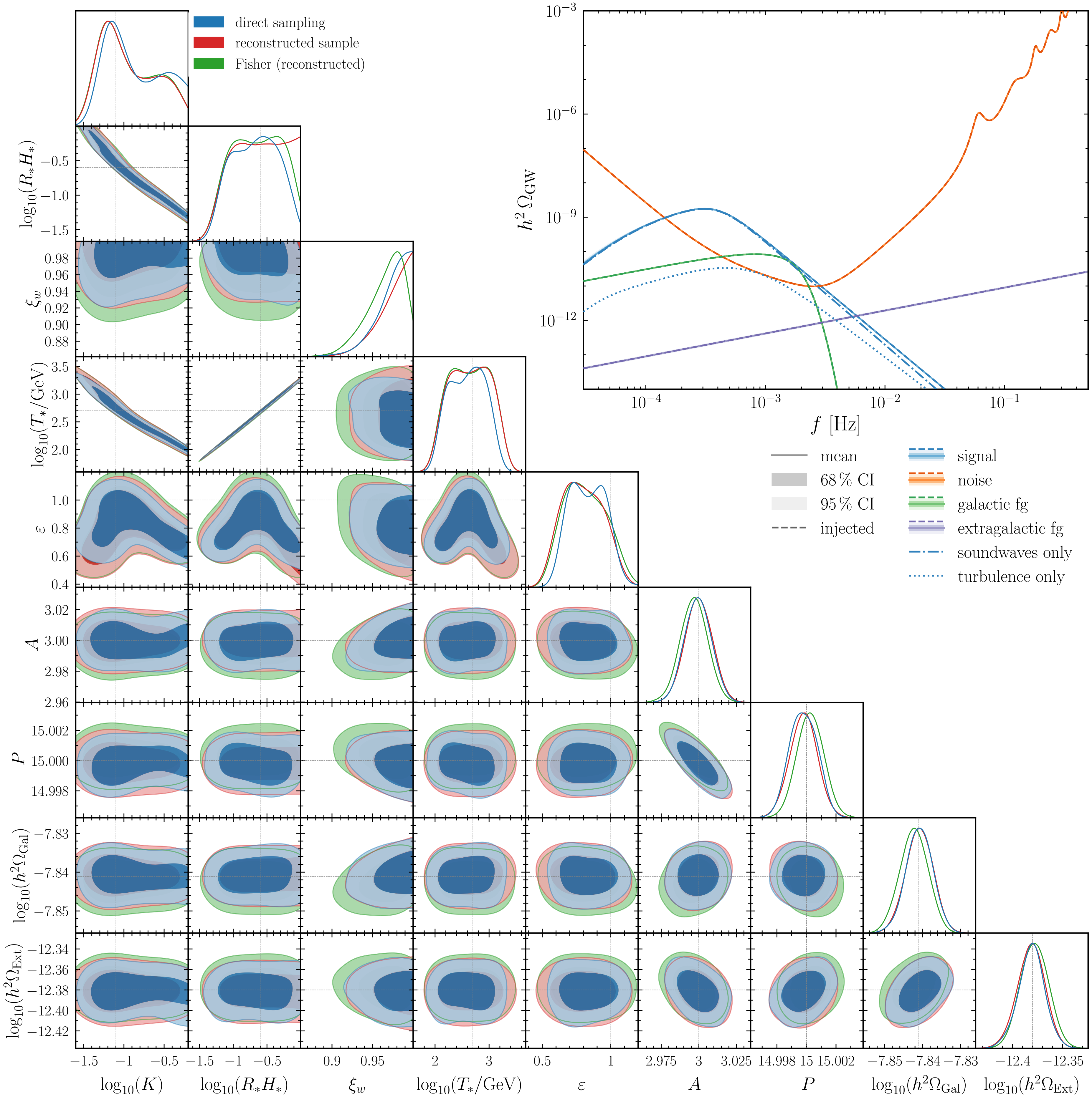


Template-based reconstruction of the *thermodynamic parameters* of the first order PT for **bubble collisions**

accounting for **foregrounds** and **assuming a two-parameters noise model**

LISA CosWG, arXiv:2403.03723

Examples of detectable signal from the EWPT



Template-based reconstruction of the *thermodynamic parameters* of the first order PT for **sound waves + turbulence**

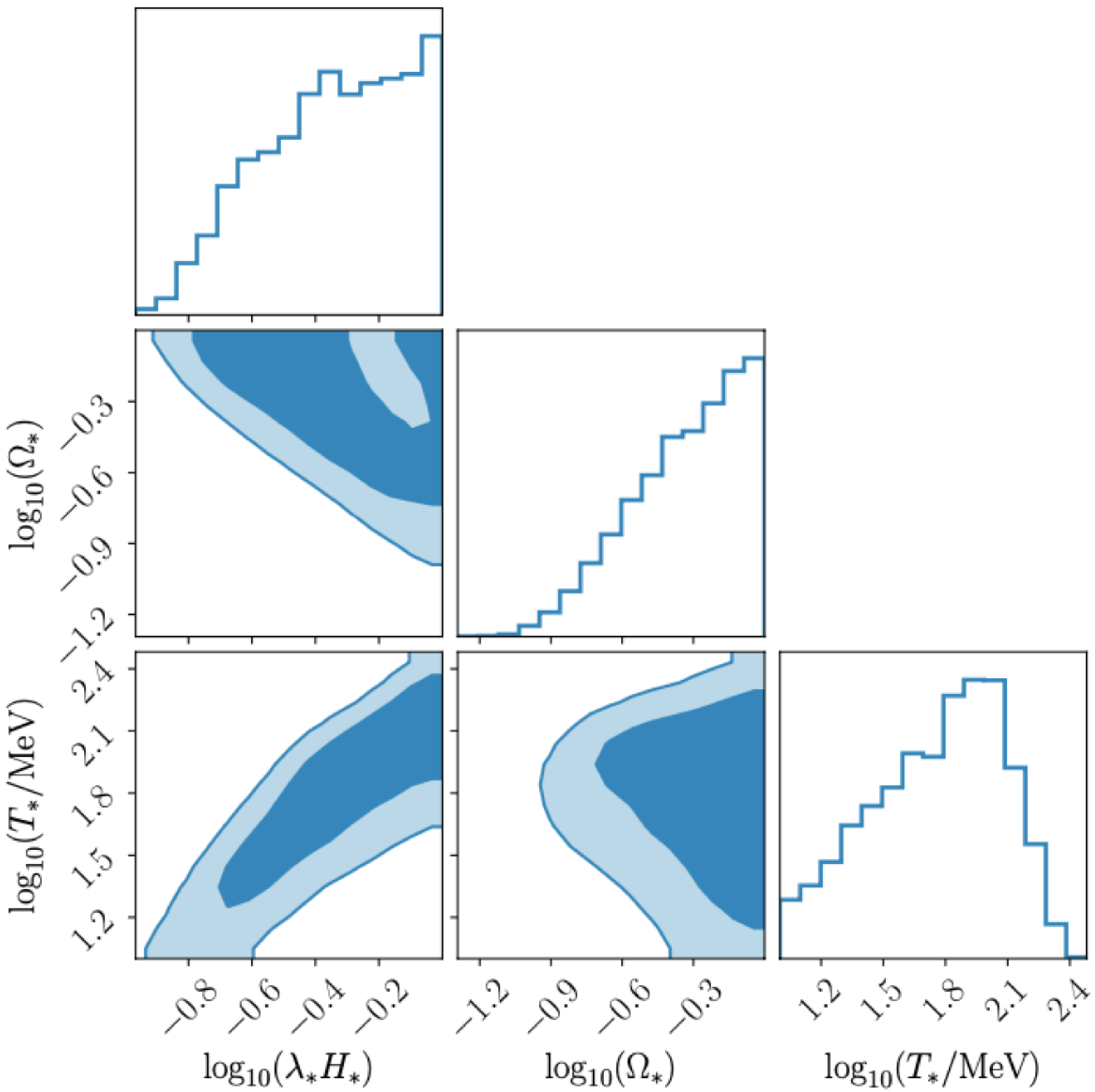
accounting for **foregrounds** and **assuming a two-parameters noise model**

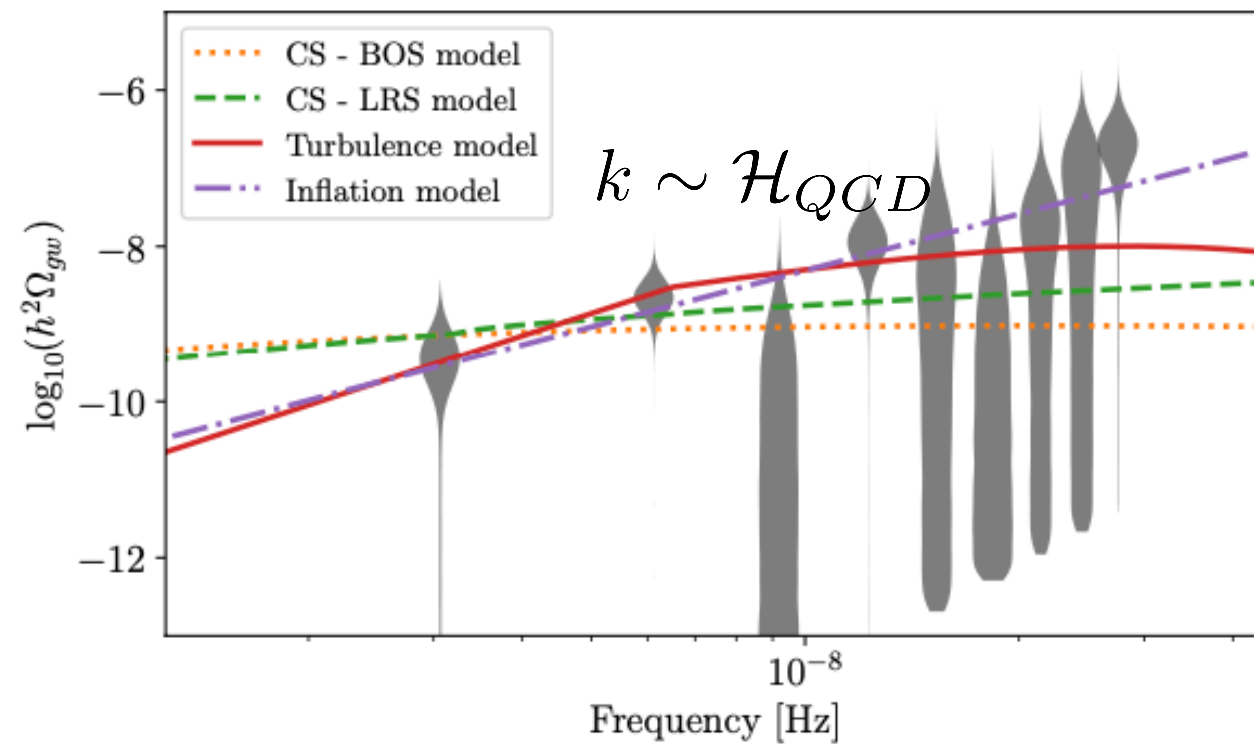
LISA CosWG, arXiv:2403.03723

An example of possible detection at PTA?

The PTA signal is compatible with GWs generated by MHD turbulence at the QCD scale

- T_* must be close to the QCD scale, the amount of energy available in anisotropic stress K must be high (at least 10% of the total energy density of the universe, and size of the anisotropic stresses $R_* = v_w/\beta$ must be close to the horizon





The the signal is fit with the low frequency tail, and the spectrum has a break at a scale comparable to the horizon at the QCD PT

To summarise:

- SGWB might reveal a powerful tool to probe the early universe and high energy physics
- The spectral shape must be predicted with good accuracy in order to disentangle the different sources (and also for foregrounds)
- General considerations about the characteristics of the spectral shape are possible in some cases, to pin down at least the class of SGWB sources
- Inflation: new physics but observationally compelling, extended GW signal in frequency, only accessible by CMB unless one goes beyond the standard slow roll scenario (there are well motivated scenarios!)
- Topological defects: amazing potential to probe high energy theory, but need to account for GW signal model dependent
- Electroweak PT: at the limit of tested physics, GW signal can be accessed/ constrained by LISA only for models beyond the standard model of particle physics
- QCD PT: tested physics but difficult to predict, GW signal can be accessed/ constrained by PTA only for models beyond the standard model of particle physics
- **SGWBs from the primordial universe might seem speculative but their potential to probe fundamental physics is great and amazing discoveries can be around the corner, especially after the PTA results!**