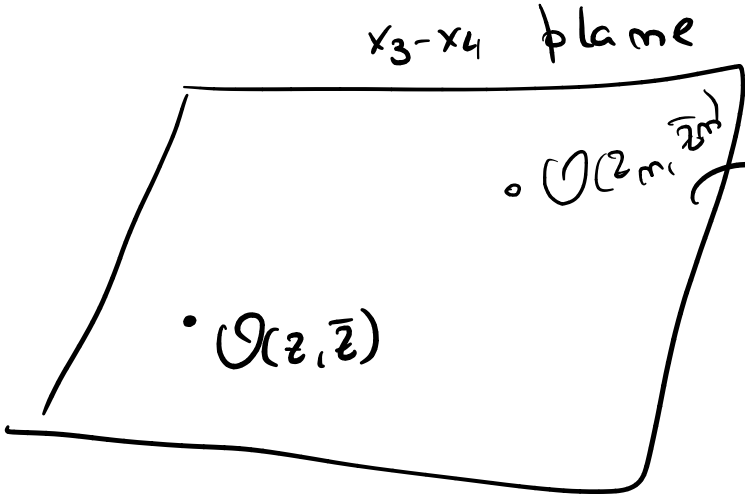
4d $\mathcal{N}=2$ SCFT



$\longrightarrow sl(2) \oplus \overline{sl(2|2)} \subset su(2,2|2)$

$\exists\ \mathbb{Q}$ s.t. $\mathbb{Q}^2 = 0$

$[\mathbb{Q}, sl(2)] = 0$

$[\mathbb{Q}, \#] = \widehat{sl(2)}$

$\longrightarrow$ consider cohomology of $\mathbb{Q}$

$\widehat{sl(2)}$

$$\hat L_{+1} = \bar L_{+1} - \bar z R_+$$
$$\hat L_{-1} = \bar L_{-1} + \cancel{\bar z} R_-$$
$$\hat L_0 = \bar L_0 - R$$

$\overline{sl(2)}$ $\quad$ $su(2)_R$

Cohomology of local ops at origin ($\mathbb{Q}_{i=1,2}$)

$\mathcal{O}(0) \leftrightarrow |\mathcal{O}\rangle$ in cohomology iff $\begin{cases} \Delta - \frac{1}{2}(j+\bar{j}) - R_D = 0 \\ -r + (j-\bar{j}) = 0 \end{cases}$

Schur operators

ops in particular short multiplets

if $\mathcal{O}(0)$ is conf. primary $[L_{+1}, \mathcal{O}(0)] = 0$

$L_{+1} \to K_3 - iK_4$

$$[\underline{L_{-1}}, \mathcal{O}(0)] = \partial\mathcal{O}(0) \qquad L_{-1} \to P_3 + iP_4$$

$$\partial_z = \frac{1}{2}(\partial_3 + i\partial_4)$$

$$[\underline{L_0}, \mathcal{O}(0)] = h\,\mathcal{O}(0) \to h = \frac{R_D + j + \bar{j}}{2} \in \frac{1}{2}\mathbb{Z}$$

$j + \bar{j} \to$ Dynkin label of rep

$$\mathcal{O}(z) = \left[e^{z L_{-1} + \bar{z}\hat{L}_{-1}}\, \mathcal{O}(0,0)\, e^{-z L_{-1} - \bar{z}\hat{L}_{-1}} \right]_{\mathbb{Q}_i}$$

twisted translated ops are in cohomology.

OPE at the level of cohomology

$$\mathcal{O}_1(z)\,\mathcal{O}_2(0) \sim \sum \frac{\lambda_{12k}}{z^{h_1+h_2-h_k}}\,\mathcal{O}_k(0) \left(+ \mathbb{Q}\text{-exact} \right)$$

single valid.

start from: 4d OPE ⇝ scale inv.

$SU(2)_R$ selection rules

⇒ meromorphic OPE algebra in 2d.

Vertex Operator algebra

$$V = \bigoplus_{n \in \frac{1}{2}\mathbb{Z}} V_n$$

→ space of Schur ops @ origin (the ones in $\mathbb{Q}$ coho)

$\frac{1}{2}$-$\mathbb{Z}$ graded vector space

$L_0 \rightsquigarrow h \in \frac{1}{2}\mathbb{Z} \quad h \geq 0$

id. $\Delta = R = \hat{j} = \bar{j} = 0 = n$ in coho. $\mathbb{1}$

$|0\rangle$

$$Y(|0\rangle, z) = 1$$

Y is map: $V \longrightarrow (\mathrm{End}\, V)[[z, \frac{1}{z}]]$

$$|0\rangle \longrightarrow Y(|0\rangle, z) = \sum_{n=-\infty}^{+\infty} \mathcal{O}_{-h-n} z^n$$

Laurent series

$\in \mathrm{End}(V)$

$$\lim_{z \to 0} Y(|0\rangle, z)|0\rangle = |0\rangle = \mathcal{O}_{-h}|0\rangle$$

$[L_0, \mathcal{O}] = h\mathcal{O}$

$\partial^m \mathcal{O}(0) \hookrightarrow \mathcal{O}_{-h-m}|0\rangle$

still need to show:

$sl(2) \rightsquigarrow$ Virasoro

i.e. show $\exists$ virasoro element $\to T(z)$

Virasoro enhancement

4d theory is local $\Rightarrow$ has $T^{4d}_{\mu\nu}$

$r=0$
$R=0$
$\Delta=4$
$\ell=2$

is not in cohomology.

same supermultiplet

$SU(2)_R$ current J^{3}_{μ} ← $SU(2)_R$

$\Delta=3, \; r=0$

$\ell=1$

$J^{11}_{+\dot{+}}(0)$ is a schur op $\Rightarrow$ in coho at origin

highest weight: $[R_+, J^{11}(0)] = 0$

$[R_-, J^{11}] = J^{12}$

$$T(z) := K\left[J^{11}_{+\dot{+}}(z,\bar{z}) - 2\bar{z}\,\mathcal{J}\, J^{12}_{+\dot{+}}(z,\bar{z}) + \bar{z}^2 \mathcal{J}^2 J^{22}_{+\dot{+}}(z,\bar{z}) \right]_{\mathbb{Q}}$$

$$\hat{L}_{-1} = \bar{L}_{-1} - \mathcal{J} R_- \qquad \bar{z}\,\hat{L}_{-1}$$

$$T(z)\,T(0) \sim \frac{6\, c_{4d}\, K^2}{\pi^4} \frac{1}{z^4} - \frac{i 2 K \mathcal{J}}{\pi^2} \frac{T(0)}{z^2} - \frac{i K}{\pi^2} \frac{\partial T(0)}{z} + \ldots$$

regular as $z \to 0$

$+(\mathbb{Q}\text{-exact terms})$

$h = 2$

4d superconformal current OPE

$$\langle T^{4d}_{\mu\nu}(x)\, T^{4d}_{\rho\sigma}(0) \rangle \propto c_{4d}\,(\text{kinematics})$$

Looks like 2d $T(z)$ ope if $k = \frac{i\pi^2}{J}$

$$\Rightarrow T(z)\,T(0) \sim \frac{c_{2d}/2}{z^4} + \frac{2T(0)}{z^2} + \frac{\partial T(0)}{z} + \cdots$$

$$\boxed{c_{2d} = -12\,c_{4d}} \leq 0$$

$c_{4d} \geq 0$ from 4d unitarity

$\Updownarrow$

modes of $T(z)$ obey Virasoro algebra

$\Rightarrow$ get non-unitary VOA:

- $h \geq 0$
- some states in VOA (inc. T) get a

negative norm.

$$T(z) = \sum_{n=-\infty}^{+\infty} z^{-n-2} \, \underline{L^T_n}$$

obey Vir.

$$[L^T_m , L^T_n] = (m-n) L^T_{m+n} + \frac{c_{2d}}{12} (m^3 - m) \delta_{m+n,0}$$

I still need to show $L^T_{\pm 1, 0}$ act geometrically so that $L_{0,\pm1} = L^T_{0,\pm1}$

$$\left.\begin{aligned} [L^T_0 , \mathcal{O}(0)] &= h \, \mathcal{O}(0) \\ [L^T_{-1} , \mathcal{O}(0)] &= \partial \mathcal{O}(0) \end{aligned}\right\}$$

We have Virasoro enhancement of the global $sl(2)$.

4d $\mathcal{N}=2$ SCFT $\longrightarrow$ VOA.

- conf. primaries $\longrightarrow$ $sl(2)$ primaries
  Schur ops. (quasi-primaries)

$[L_{+1}, \mathcal{O}(0)] = 0$

(conf. desc.)

$[L_{+m>1}, \mathcal{O}(0)] \neq 0$

not necess. Vir. primaries.

$\left([L_{-1}, X] + \begin{array}{c} sl(2) \\ \text{desc.} \end{array} \right)$

- Virasoro is there: $\mathcal{O}(0)$ schur. $\Rightarrow$

$\exists \quad [L_{-2}, \mathcal{O}(0)], \quad [L_{-3}, \mathcal{O}(0)], \quad \cdots \quad [L_{-2}^2 \mathcal{O}(0)], \cdots$

Virasoro relates diff 4d superconf. multiplets.

- $SU(2)_R$: VOA does not preserve R-grading.

- VOA is independent of exactly marginal def

– 1312.5344 Beem, ML, Liendo, Peelaers, Rastelli, van Rees.

- Flavor symm $\Rightarrow$ $\exists$ flavor current $\Rightarrow$ superconf primary is in coho

not in coho

susy

$\underline{\Delta = 2}$, $\underline{3}$ of $SU(2)_R$, $r=0$, scalar, adjoint of flavor algebra.

$$J^A(z)\, J^B(0) \sim \frac{k_{2d}}{2}\frac{1}{z^2} + i\,\frac{f^{AB}{}_C\, J^C(0)}{z} + \cdots$$

AKM current algebra

w/

$$\boxed{k_{2d} = -\frac{k_{4d}}{2}}$$

$\langle J_\mu^{4d\, SU(2)_R}\, J_\nu^{4d\, SU(2)_R}\rangle \propto k_{4d}$

Consequences for 4d physics.

- Assume 4d $\mathcal{N}=2$ SCFT, c_{4d}
- w/ flavor symmetry g_F

$\Rightarrow$ $T(z)\,T(0) \sim \cdots\cdots$ $\qquad c_{2d} = -12\, c_{4d} \leq 0$

AKM $\quad J^A(z)\, J^B(0) \sim \dfrac{k_{2d}}{z^2} + \dfrac{i f^{AB}{}_C\, J^C(0)}{z} + \cdots$

adj indices of g_F

$$(J^A J^B)(0) = \lim_{z\to 0} \left(J^A(z)\, J^B(0) - \text{singular} \right)$$

exists and has computable corr. fns

look at singlet (contract w/ δ_{AB}).

$$\underline{S(z)} = \underline{(J^A J^A)(z)} = \beta_1 T(z) + B(z)$$

$h=0$, singlet

$r=0$

from 4d $SU(2)_R$ current

"Higgs branch ϕ"

$$T(z)J(0) \sim \frac{J^A(0)}{z^2} + \frac{\partial J^A(0)}{z} + \ldots \Leftarrow \text{4d}$$

$$\langle T(0)\, S(z) \rangle = \frac{\dim g_F\, k_{2d}}{z^4}$$

$$\| $$

$$\underline{\beta_1} \langle \underline{T(0)\, T(z)} \rangle$$

$$\frac{c_{2d}}{2} \frac{1}{z^4}$$

$$B(z) = S(z) - \beta_1 T(z)$$

discovered its there

$$\langle \underline{B(z)\, B(0)} \rangle = \langle B(z)\, S(0) \rangle - \beta_1 \underbrace{\langle B(z)\, T(0) \rangle}_{=0} =$$

$$= \langle S(z)\, S(0) \rangle - \beta_1 \langle T(z)\, S(0) \rangle =$$

$$= 2 \dim g_F\, k_{2d}^2 \left[1 + \frac{h^\vee}{k_{2d}} - \frac{\dim g_F}{c_{2d}} \right] \geq 0$$

dual coxeter no of g_F

4d unitarity.

$$\boxed{\frac{\dim g_F}{c_{4d}} \geq \frac{24 h^\vee}{k_{4d}} - 12}$$

New unitarity bound

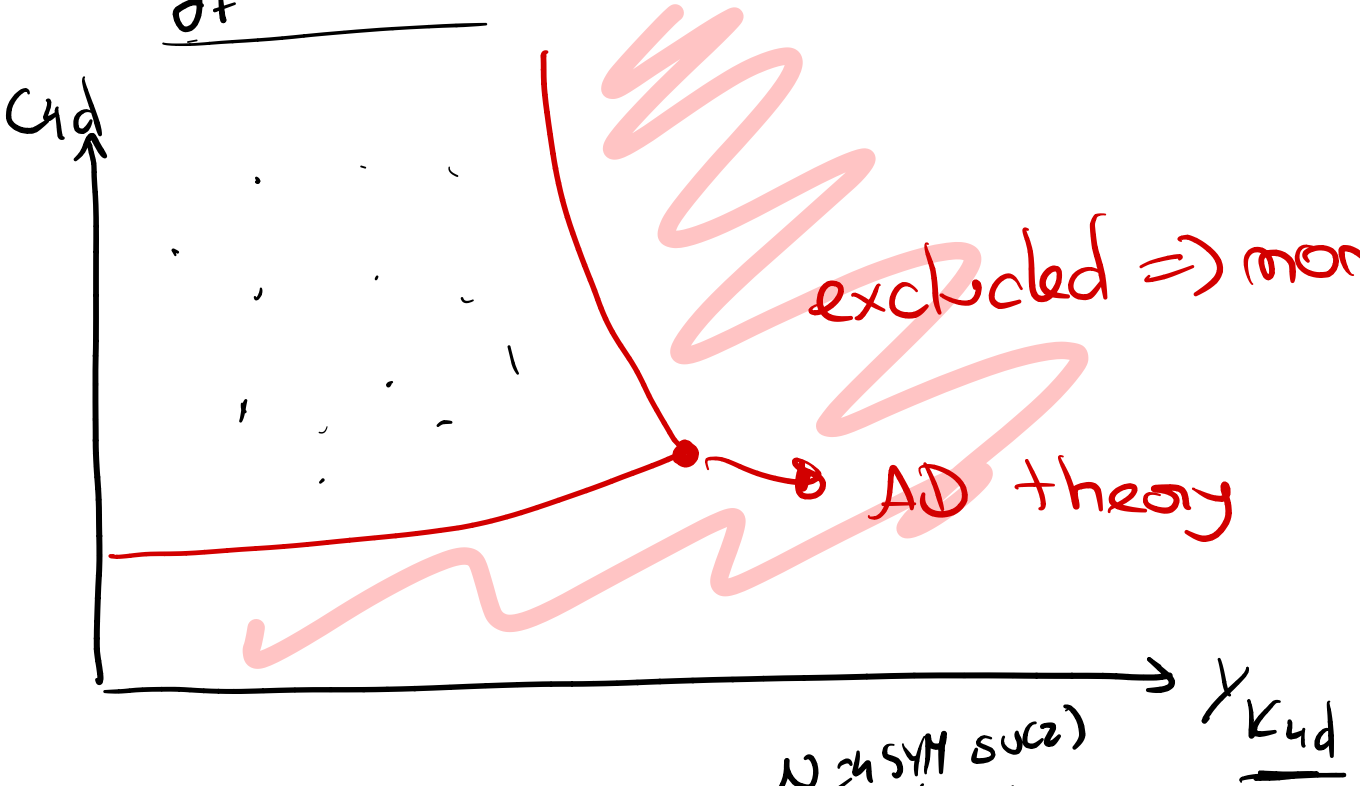
$T(z)\, T(0) \sim \#$

$J^A(z)\, J^B(0) \sim \#$

from 2d build diff. 4d multiplets and compute their norms. and demand right sign

4d unitarity

$g_F = SU(2)$



⇐ theory is interacting.

$\mathcal{N}=4$ SYM w/ $SU(2)$

$\mathcal{N}=4$: $a_{4d} = c_{4d} \geq \frac{3}{4}$ for interacting theories

$\mathcal{N}=3$: $a_{4d} = c_{4d} \geq \boxed{\frac{13}{24}}$