QCD at non-zero temperature and heavy quarks

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- Deconfinement and color screening
- Chiral transition at non-zero temperature and density
- Equation of state at zero density
- Taylor expansion: fluctuations and correlations of conserved charges and charm degrees of freedom across the chiral transition
- Bound states of heavy quarks
- Heavy quark diffusion coefficient from lattice QCD
- Complex heavy quark potential at $T>0$

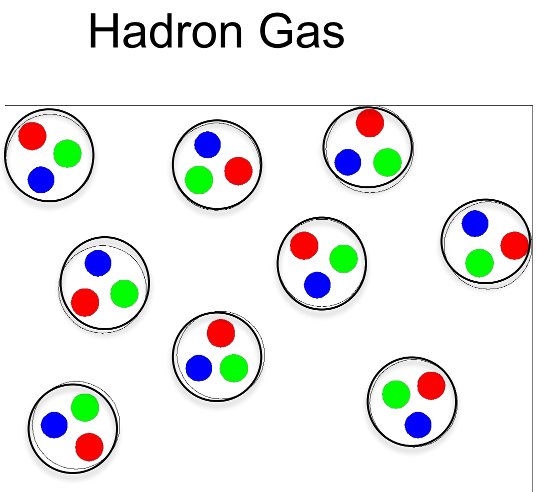
comparison with weak coupling calculations (HTL, EQCD, pNRQCD)

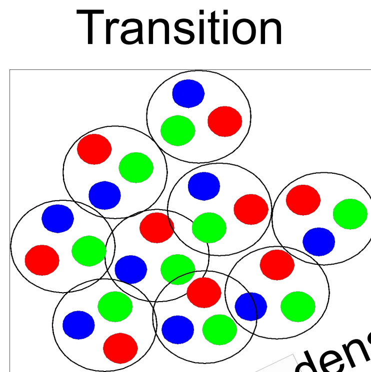
Comparison with Hadron Resonance Gas (HRG) model

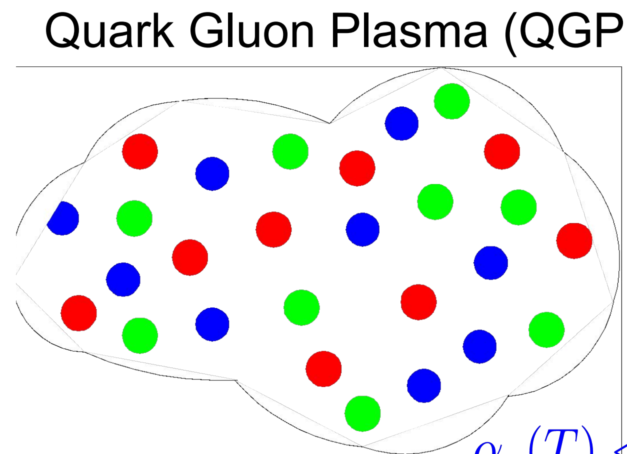
Hard probes in non-equilibrium QCD matter, ICTS, Bangalore, March 16-27,2026

Deconfinement at High Temperature and Density

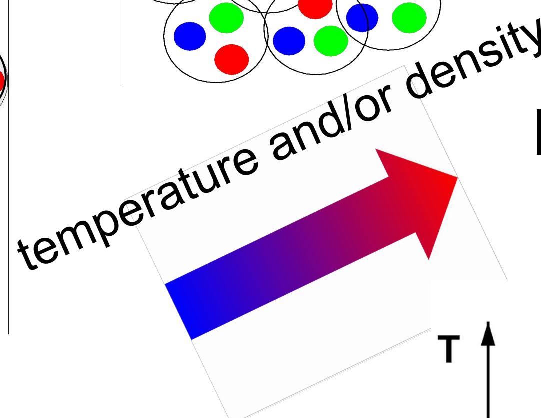
I. Ya. Pomeranchuk,
Doklady Akad. Nauk. SSSR 78 (1951) 889







$\alpha_s(T) \ll 1$



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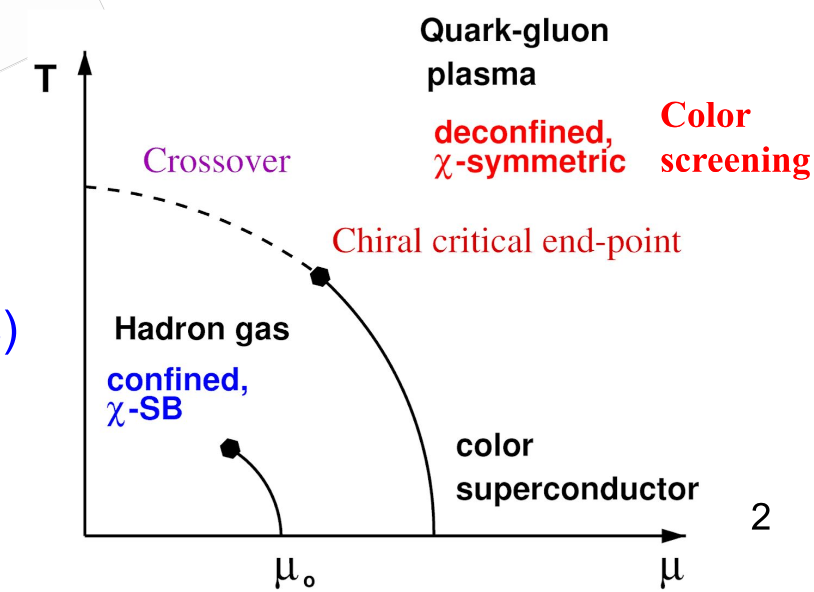
Why this is interesting ? :
basic properties of strong interaction

astrophysics (compact stars, transient objects)

cosmological consequences
(Early Universe few microseconds after Big Bang)



Symmetries of QCD in the vacuum and for T>0

Nobel Prize 2008

- Chiral symmetry : For light quarks $m_{u,d} << \Lambda_{QCD}$ QCD Lagrangian has

$SU_A(2)$ symmetry $\psi \rightarrow e^{i\phi^a T^a \gamma_5}\psi$ $\qquad \psi_{L,R} \rightarrow e^{i\phi^a_{L,R} T^a}\psi_{L,R}$

The vacuum breaks the symmetry $\langle\bar{\psi}\psi\rangle = \langle\bar{\psi}_L\psi_R\rangle + \langle\bar{\psi}_R\psi_L\rangle \neq 0$

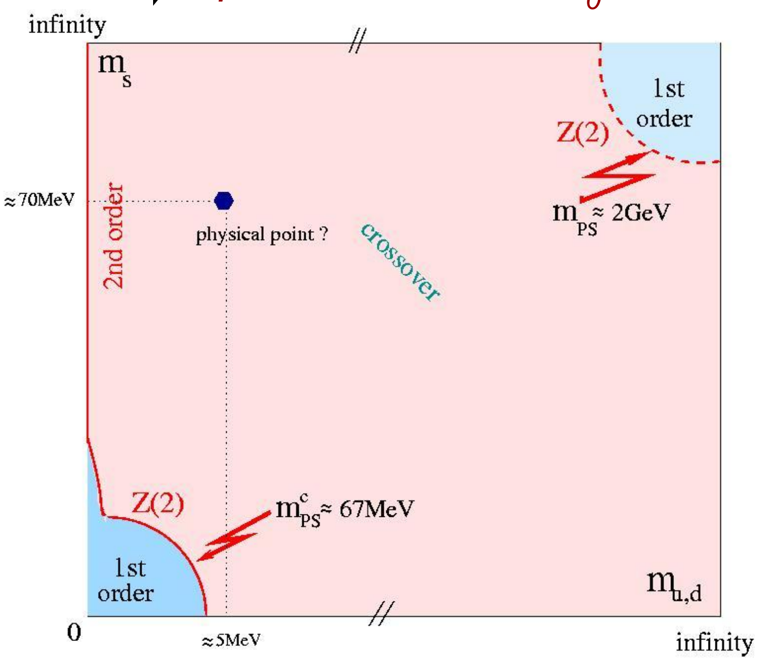
spontaneous symmetry breaking or Nambu-Goldstone realization of the symmetry

⇓

hadrons with opposite parity have very different masses, interactions between hadrons are weak at low E

$U_A(1)$ symmetry $\psi \rightarrow e^{i\phi\gamma_5}\psi$ is broken by anomaly (ABJ) : $\langle\partial^\mu j^a_\mu\rangle = -\frac{\alpha_s}{4\pi}\langle\epsilon^{\alpha\beta\gamma\delta}F^a_{\alpha\beta}F^a_{\gamma\delta}\rangle$

⇒ η' meson mass, π-a_0 mass difference



Pisarski, Wilczek, PD29 (1984) 338

$T \gg \Lambda_{QCD} : \langle\bar{\psi}\psi\rangle \simeq 0$, $U_A(1)$ symmetry ?

For large quark masses center symmetry there is an approximate center $Z(N)$ symmetry that is broken at high temperature and its breaking is associated with deconfinement

Evidence for 2^{nd} order transition in the chiral limit
=> universal properties of QCD transition:

$SU_A(2) \times SU_V(2) \sim O(4)$
relation to spin models

transition is a crossover for physical quark masses

Finite Temperature QCD and its Lattice Formulation

$$\langle O \rangle = \mathrm{Tr} O e^{-\beta H - \mu N} \qquad \beta = 1/T$$

evolution operator in imaginary time

$$\langle O \rangle = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} O e^{-\int_0^\beta d\tau d^3x \mathcal{L}_{QCD}}$$

$$A_\mu(0,\mathbf{x}) = A_\mu(\beta,\mathbf{x}) \quad \psi(0,\mathbf{x}) = -\psi(\beta,\mathbf{x})$$

Integral over functions → (Lattice) → integral with very large (but finite) dimension (> 1000)

$$\langle O \rangle = \int \prod_x dU_\mu(x) O(det D_q[U,m,\mu]) e^{-\sum_x S_G[U(x)]}, \; U_\mu(x) = e^{igaA_\mu(x)}$$

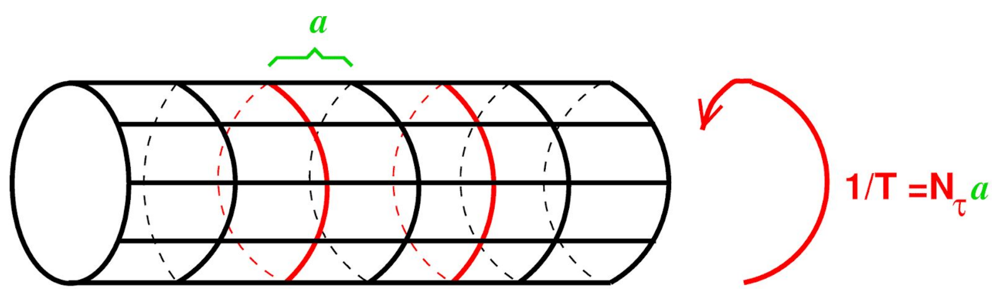
$\mu \neq 0$: $det D_q(U,m,\mu)$ complex → Monte-Carlo Methods
sign-problem

continuum limit

$N_\tau \to \infty$, N_σ / N_τ fixed

Costs : ?

$$\sim a^{-7} \sim N_\tau^7$$



Quenched QCD: $det D_q(U,m,\mu) = 1$

improved discretization schemes are needed : p4, asqtad, stout, HISQ

Meson correlators and Wilson loops

Meson states are created by quark bilinear operators:

[redacted]

Fixes the quantum number of of mesons, Γ is one of the Dirac matrices

Most often one considers point operators *x*=*y* and their correlation function:

[redacted]

decay constant

[redacted] ground state dominates

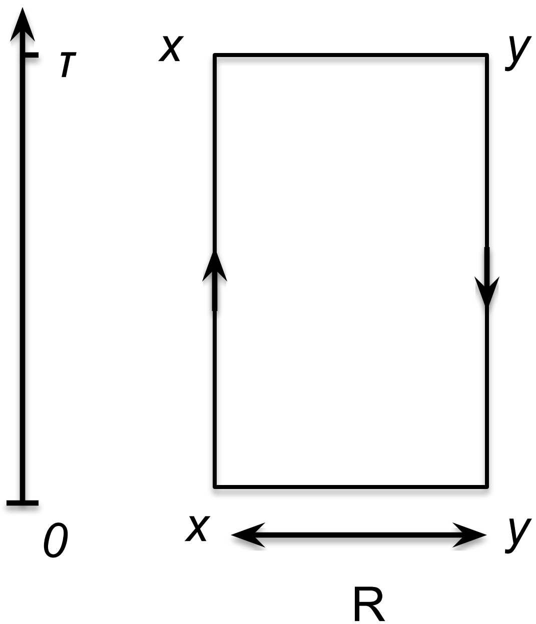
Consider static quarks : [redacted]

Formal solution: [redacted]

On lattice: [redacted]

Static meson correlation function functions after integrating out the static quark fields:

[redacted]



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Static quark anti-quark potential

$\tau \to \infty$ ground state $E_1 = V(R)$ dominates

$$V(R) = -\alpha/R + \sigma R$$

String tension

confinement

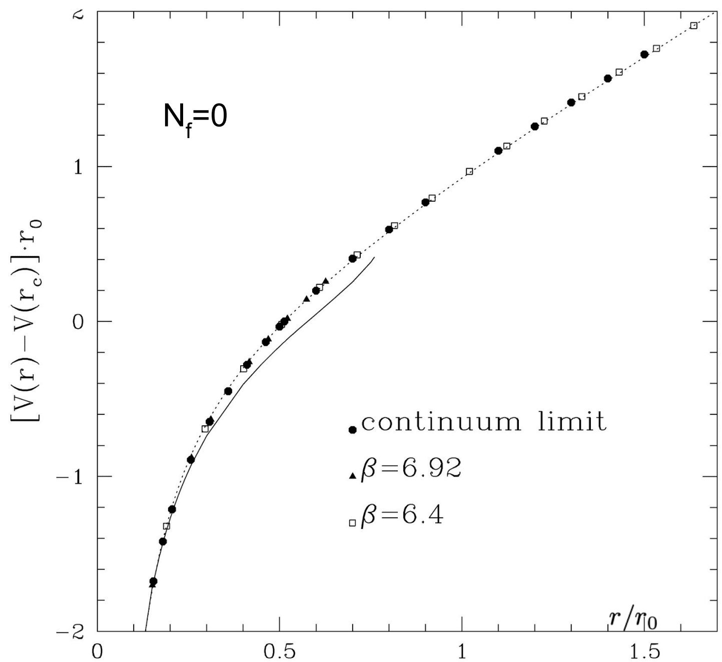
$$W(\tau, R) = \exp(-\sigma R \tau),$$

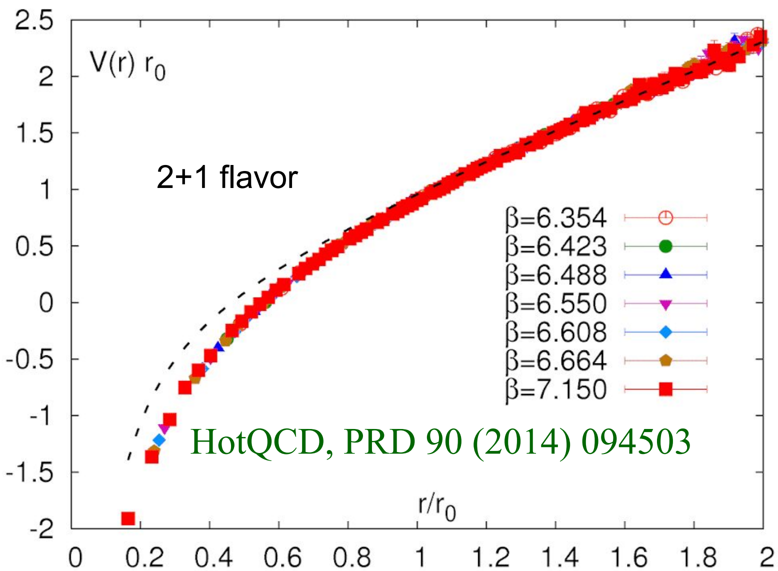
area law for large R and τ

n=2 and larger : hybrid potentials

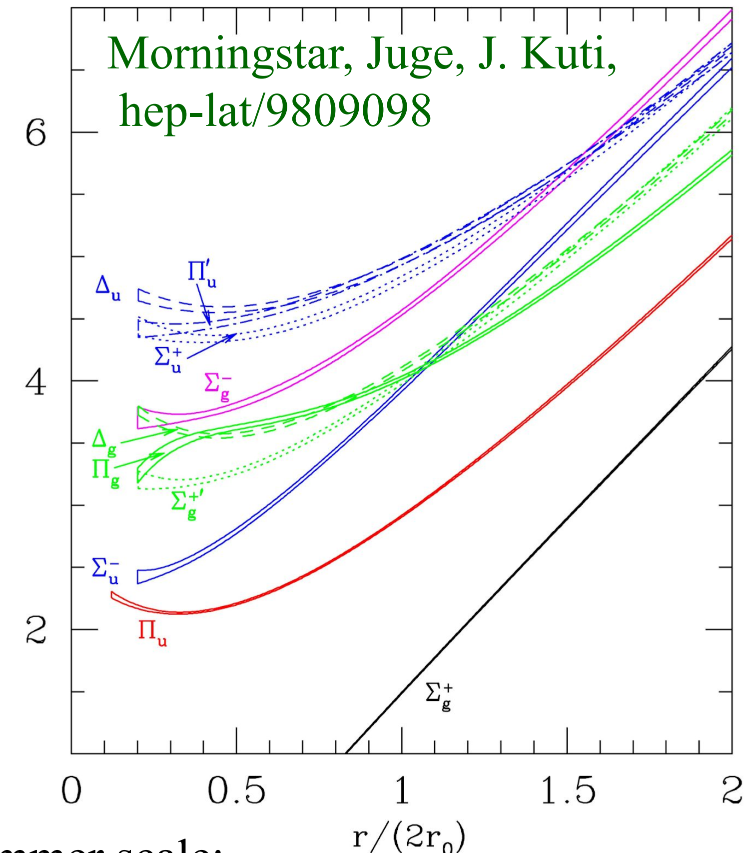
Numerial results on the static potential

Necco, Sommer, NPB 623 (02) 271





Hybrid potentials:



Sommer scale:

$$\left(r^2 \frac{dV_{q\bar{q}}(r)}{dr} \right)_{r=r_0} = 1.65, \;\; r_0 = 0.468(4)\,fm$$

$$\sqrt{\sigma} \simeq 470 \text{ MeV}$$

Center symmetry and deconfinement transition

Above the phase transition temperature Z(N) (center) symmetry of SU(N) gauge theory is broken
Quarks transform non-trivially under Z(N) symmetry group
=> static charges in fundamental representations can be screened by gluons !

Lattice set-up:

$U_\mu(\tau,x) = e^{igA_\mu(\tau,x)},\ N_\sigma^3 \times N_\tau,\ T = 1/(N_\tau a)$

Thermodynamic limit: $N_\sigma/N_\tau \to \infty$; Continuum limit : $N_\tau \to \infty$, T-fixed Temperature is set by $a \leftrightarrow \beta = 2N_c/g^2$; allowable gauge transformations: $U_\mu(x) \to \Omega(x+\mu)U_\mu(x)\Omega^\dagger(x)$

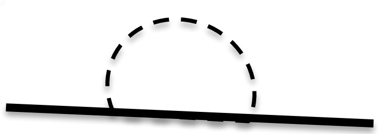
$$\Omega(0,\vec{x}) = \Omega(\beta,\vec{x})C, \quad C = e^{2\pi i n/N_c} I \quad \to Z(N) - \text{symmetry}$$

$$L(\vec{x}) = \frac{1}{N_c} \mathrm{tr} \prod_{\tau=1}^{N_\tau} U_0(\tau,\vec{x})$$

Polyakov loop is changed $L(\vec{x}) \to e^{2\pi n i/N_c} L(\vec{x})$

$< L > \neq 0 \ \to Z(N)$ spontaneously broken; $< L > = e^{-F_Q/T}$ -free energy of an isolated static quark is finite $\Rightarrow$ deconfinement

L is order parameter

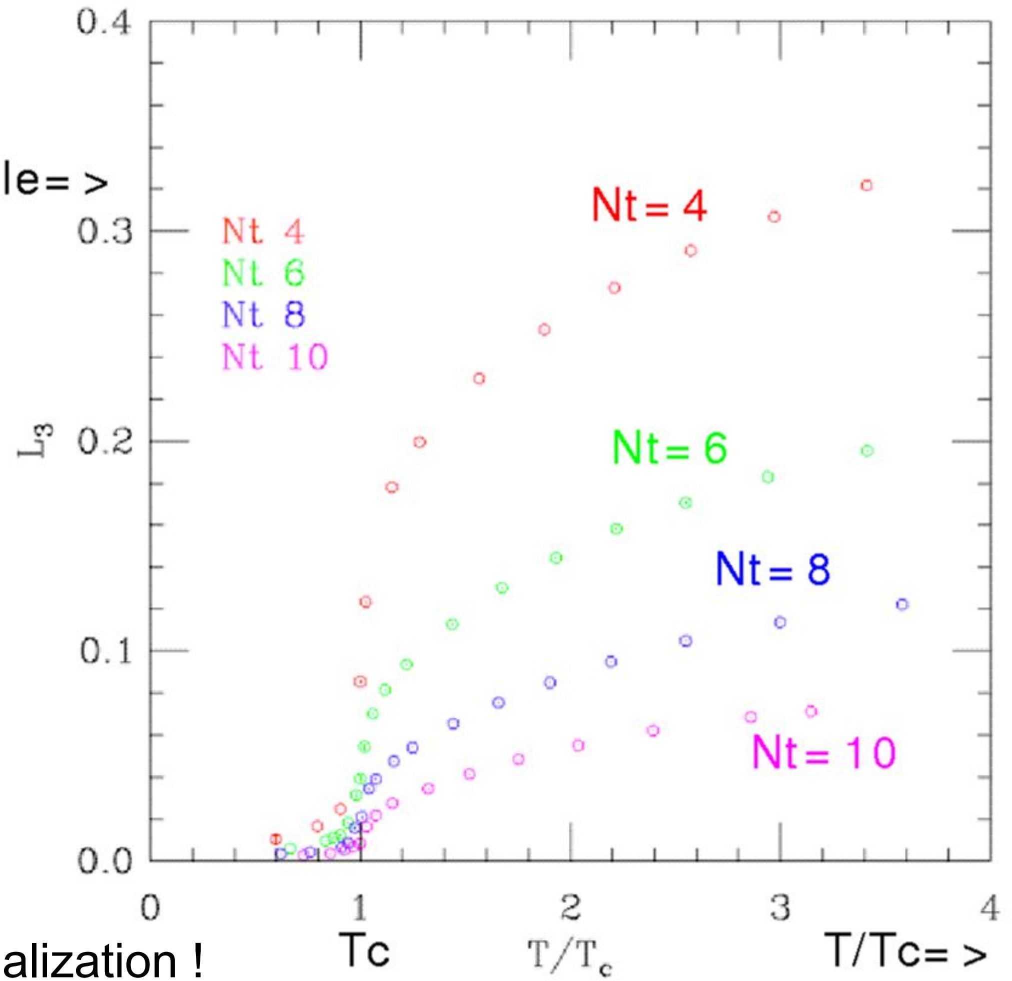


The free energy of static quark is infinite in the
Continuum limit due to linear *1/a* divergence => needs renormalization

Continuum limit for *L* ?

Dumitru et al, hep-th/0311223

Note scale=>
~ .3



Nt = # time steps.

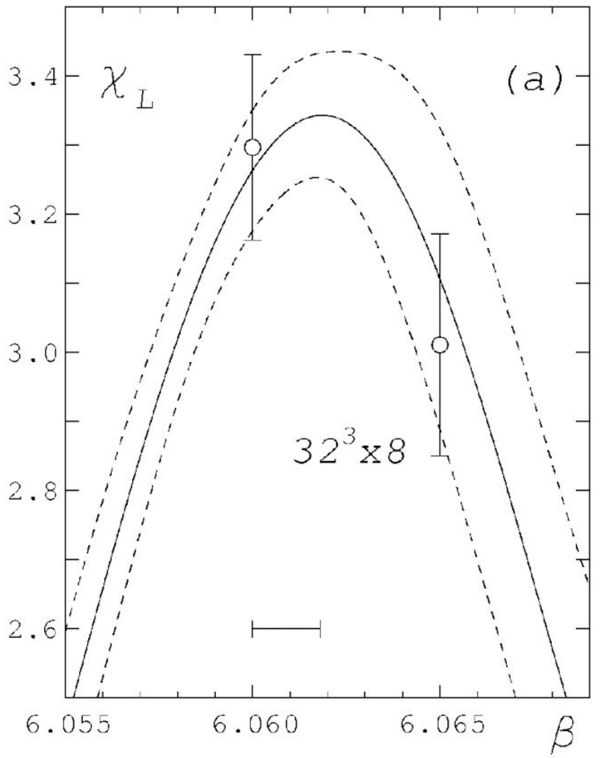
Bare loop vanishes as Nt -> ∞

needs renormalization !

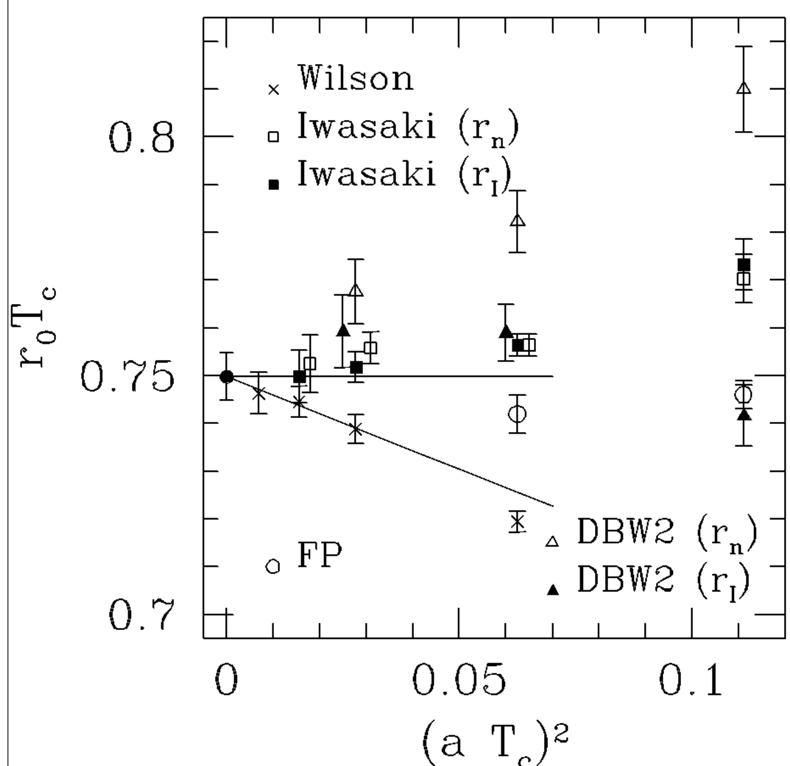
How to determine the deconfinement transition temperature ?

$$\frac{\chi_L}{T^2} = N_\sigma^3 \left(\langle L^2 \rangle - \langle L \rangle^2 \right) = \langle (\delta L)^2 \rangle \text{ has a peak at } \beta_c$$

Boyd et al., Nucl. Phys. B496 (1996) 167



Necco, Nucl. Phys. B683 (2004) 167



- Use different volumes and Ferrenberg-Swedsen re-weighting to combine information collected at different gauge couplings

Finite volume behavior can tell the order of the phase transition, e.g. for 1st order transition the peak height scales as spatial volume !

Correlator of Polyakov loops and deconfinement

The correlation function of Polyakov loops defines the free energy of static quark anti-quark pair (also an order parameter)

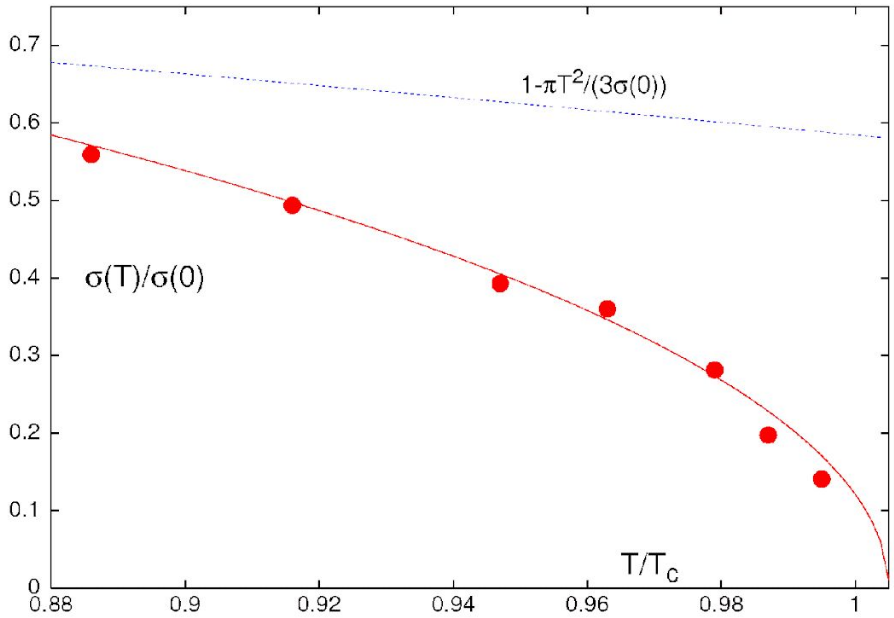
$T < T_c$:

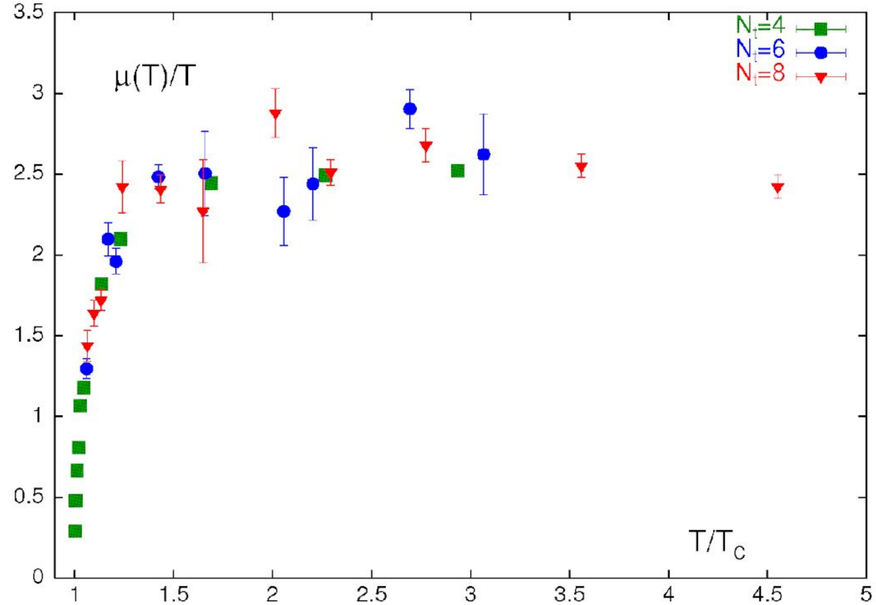
$$< L(r) L^\dagger(0) > \sim e^{-\sigma(T) r / T}$$

$T > T_c$:

$$\ln\left(\frac{< L(r) L^\dagger(0) >}{| < L > |^2}\right) \sim e^{-\mu(T) r}$$

Kaczmarek, Phys. Rev. D62 (2000) 034021



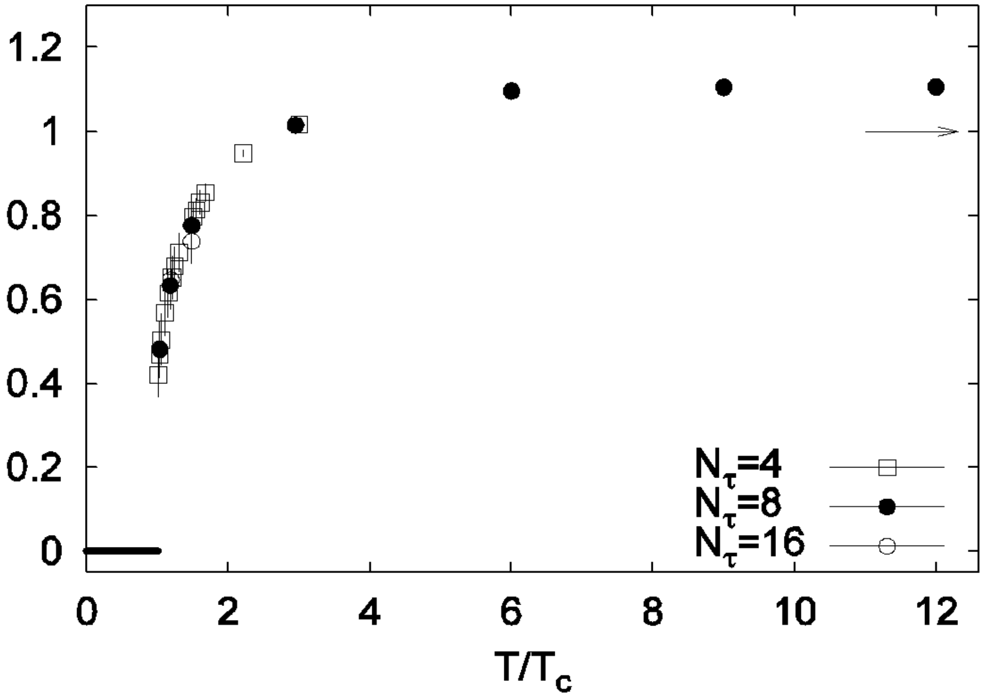


small inverse correlation length => weak 1st order phase transition SU(3) gauge theory is far from the large *N*-limit !

The renormalized Polyakov loop in pure glue theory

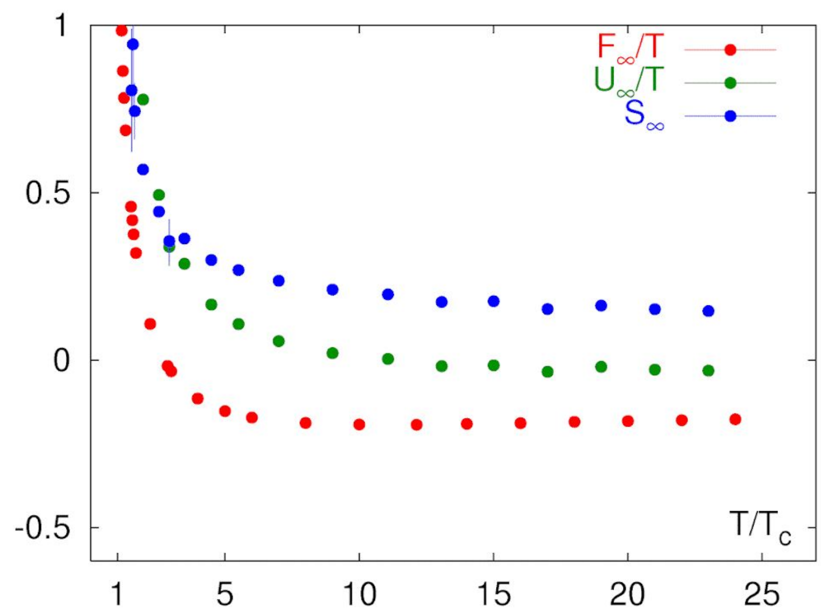
$$r \ll 1/T: \;\; F_{Q\bar{Q}}(r,T) = V(r, T=0) + T\ln 9$$
$$\Rightarrow \text{normalize the } Q\bar{Q} \text{ free energy to the } T=0 \text{ potential}$$

$$\lim_{r\to\infty} \langle L(r) L^\dagger(0)\rangle = \exp(-F_{Q\bar{Q}}(r\to\infty, T)/T) = \exp(-F_\infty/T) = |\langle L\rangle|^2, F_Q = F_\infty/2$$



$$L_{ren} = \exp(-F_\infty(T)/(2T))$$

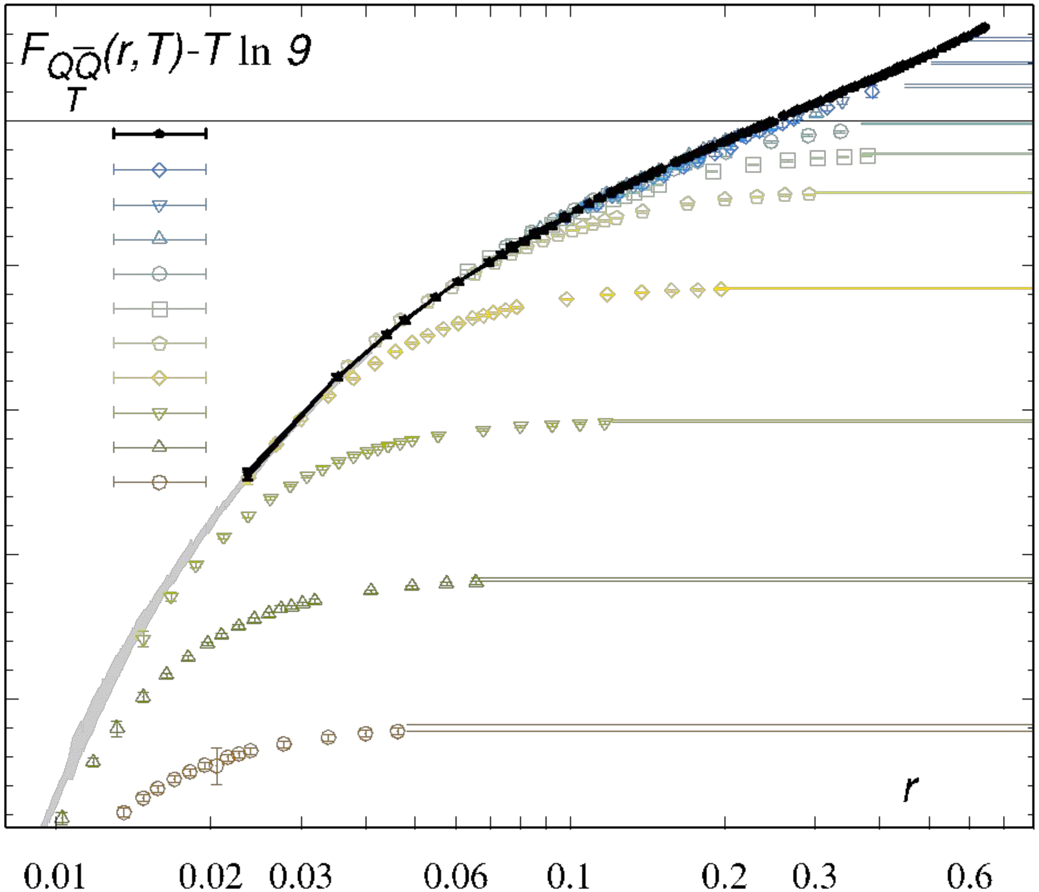
Kaczmarek et al, PLB 543 (2002) 41,
PRD 70 (2004) 074505, hep-lat/0309121



$$\text{LO}: F_Q = -TS_Q = -\frac{4}{3}\alpha_s m_D$$

Deconfinement and color screening in QCD

2+1 flavor QCD, continuum extrapolated, TUMQCD, PRD 98 (2018) 054511

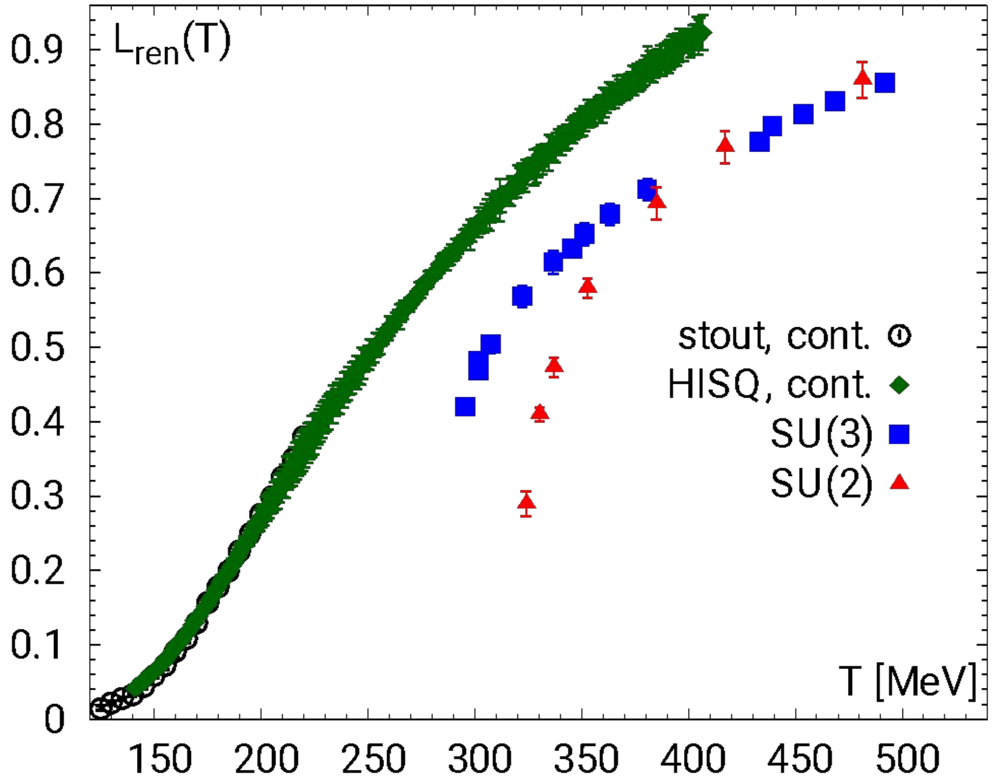


The free energy of static quark anti-quark pair agrees with the $T=0$ potential for $r \ll 1/T$

The free energy of static quark anti-quark pair is screened for $r>r_{scr}$ at any temperature

$r_{scr} \sim 1/T$ => Debye screening

Deconfinement and color screening in QCD (cont'd)



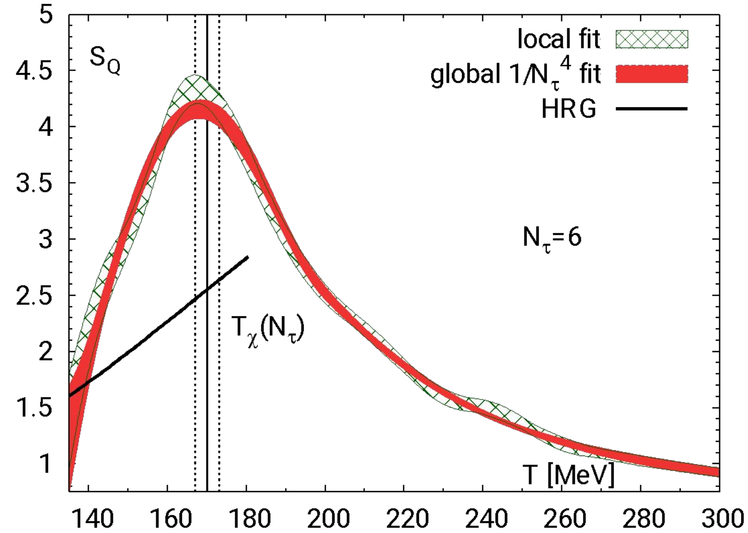
Pure glue ≠ QCD !

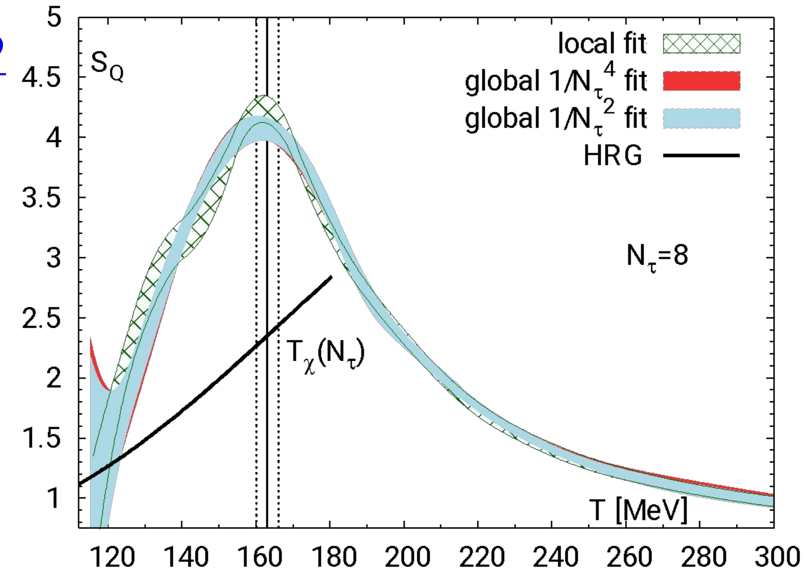
Deconfinement transition happens at lower temperature but the Polyakov loop behaves smoothlyaround T_c *, Z(3) symmetry plays no apparent role*

How to define deconfinement transition in QCD ?

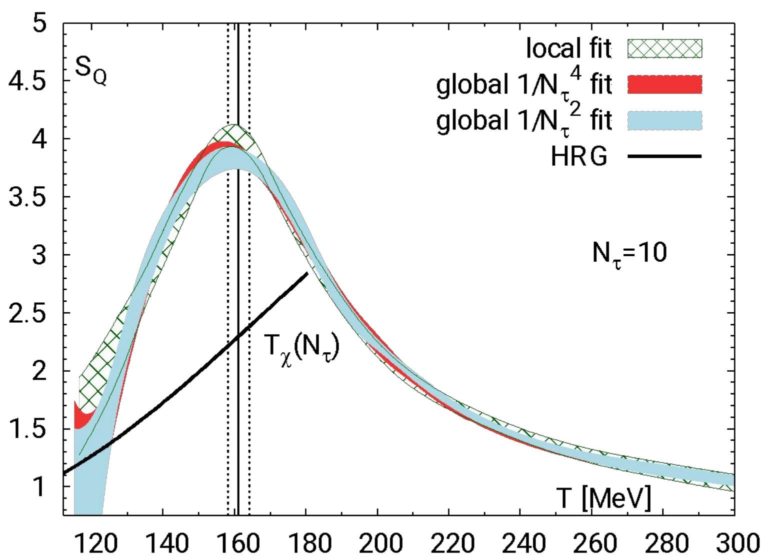
The entropy of static quark

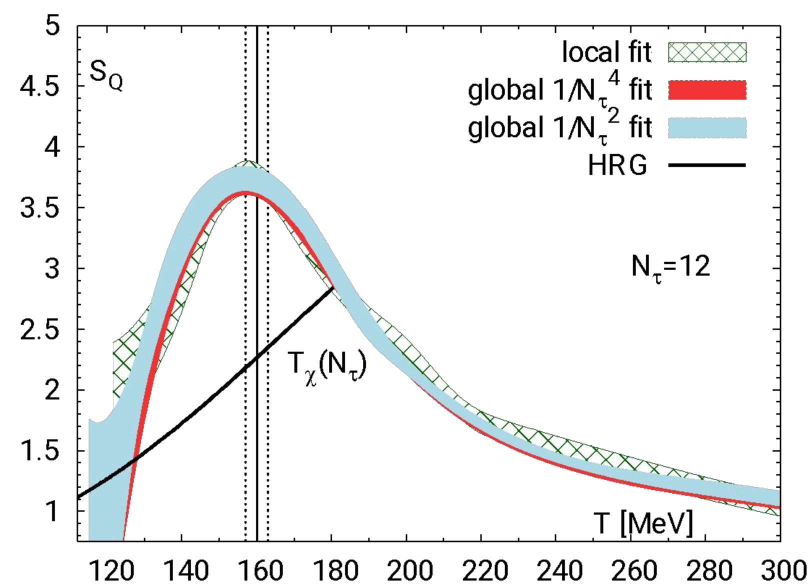
$$S_Q = -\frac{\partial F_Q}{\partial T}$$





TUMQCD, PRDD93 (2016) 114502



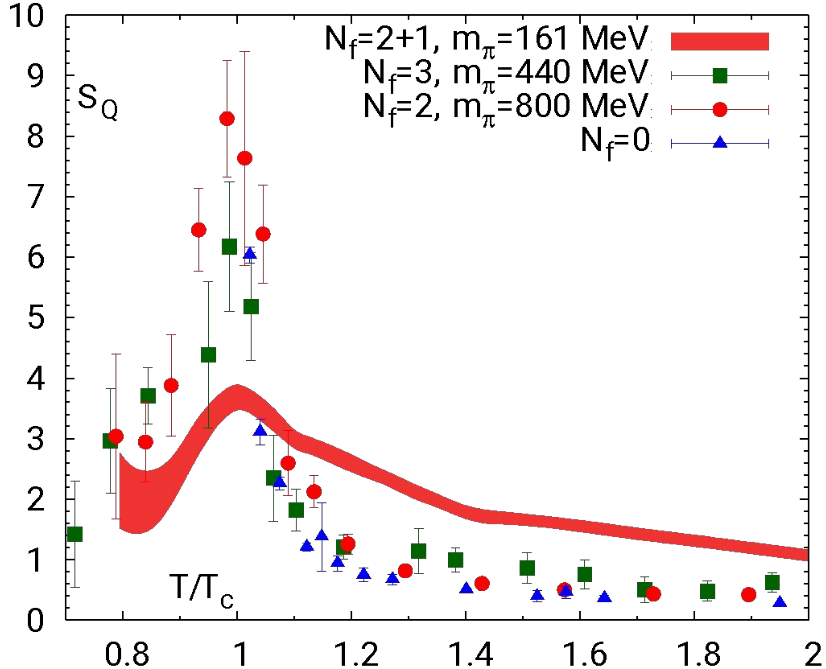


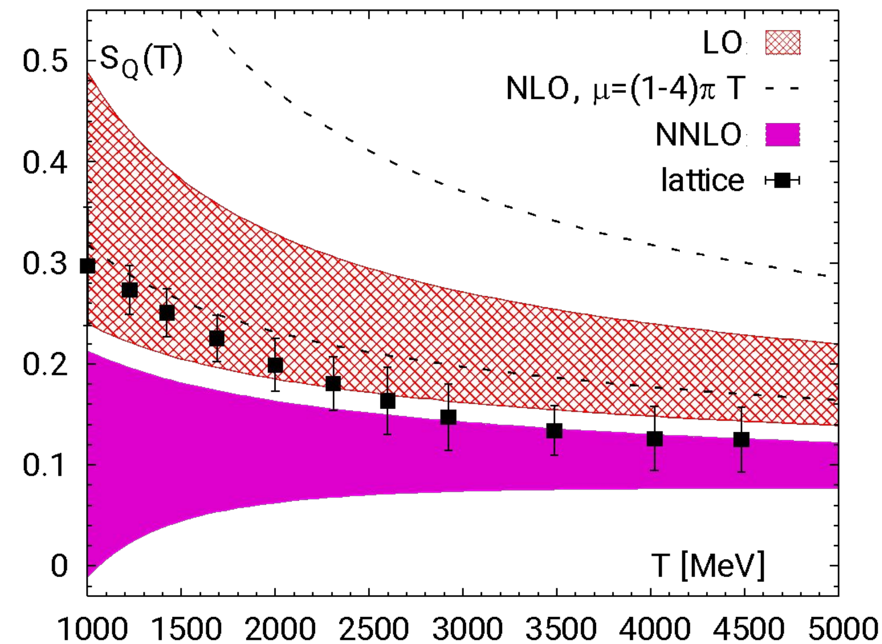
The onset of screening corresponds to peak is S_Q and its position coincides with T_c

The entropy of static quark

TUMQCD, PRDD93 (2016) 114502

$$S_Q = -\frac{\partial F_Q}{\partial T}$$





At low *T* the entropy S_Q increases reflecting the increase of states the heavy quark can be coupled to; at high temperature the static quark only "sees" the medium within a Debye radius, as *T* increases the Debye radius decreases and S_Q also decreases

The peak in the entropy is broader and smaller for smaller quark mass

Weak coupling (EQCD) calculations work for *T> 2000* MeV

Berwein et al, PRD 93 (2016) 034010

Free energy of a static quark anti-quark pair at high T

The work to separate the $Q\bar{Q}$ pair from distance r_1 to r_2 : $F_{Q\bar{Q}}(r_2) - F_{Q\bar{Q}}(r_1)$

Leading order in perturbation theory:

$$F_{Q\bar{Q}}(r,T) = -\frac{1}{9}\frac{\alpha_s^2}{r^2 T}\exp(-2m_D r) + F_\infty$$

In QED at leading order:

$$F_\infty = 2F_Q$$

$$F_{Q\bar{Q}}(r,T) = -\frac{\alpha}{r}\exp(-m_D r) + F_\infty$$

Conjecture: in QCD the work is reduced due to cancelation of color singlet and octet contribution

Fierz idenity: $\delta_{ij}\delta_{lk} = \frac{1}{N_c}\delta_{ik}\delta_{lj} + 2T^a_{ik}T^a_{lj}$

$$e^{-F_{Q\bar{Q}}(r,T)/T} = \frac{1}{9}e^{-F_S(r,T)/T} + \frac{8}{9}e^{-F_O(r,T)/T}$$

$$e^{-F_S(r,T)/T} = \frac{1}{N_c}\langle \mathrm{Tr}\mathcal{L}(r)\mathcal{L}^\dagger(0)\rangle,\ \mathcal{L}(r) = \prod_{x_0=0}^{N_\tau-1} U_0(x_0,\mathbf{r})$$

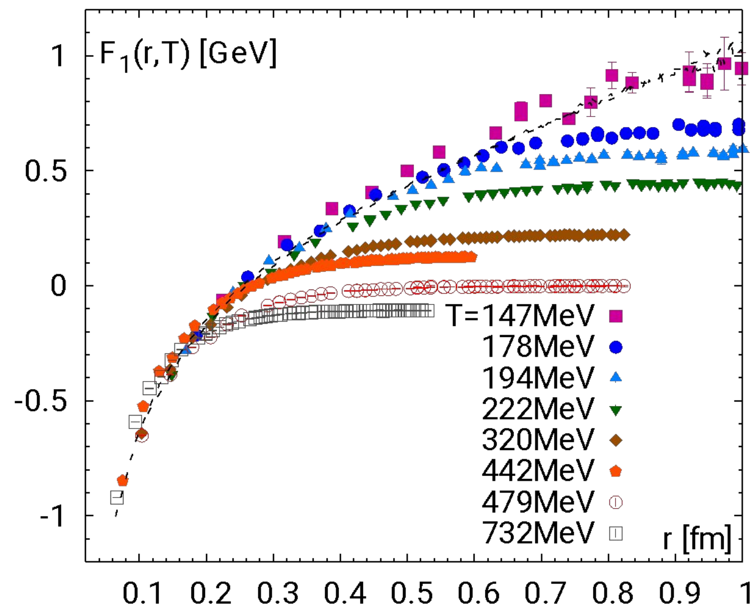
LO:

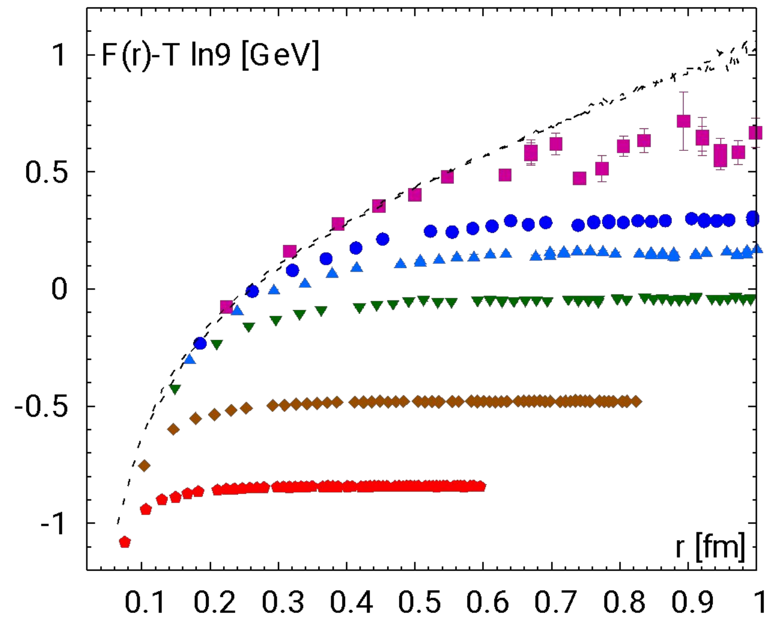
$$F_S = -\frac{4}{3}\frac{\alpha_s}{r}e^{-m_D r} + F_\infty,\ \ F_O = +\frac{1}{6}\frac{\alpha_s}{r}e^{-m_D r} + F_\infty$$

The LO result for $F_{Q\bar{Q}}$ is recovered

Free energy and singlet free energy

HISQ action, $24^3 \times 6$, $16^3 \times 4$ (high T) lattices, $m_\pi \simeq 160$ MeV

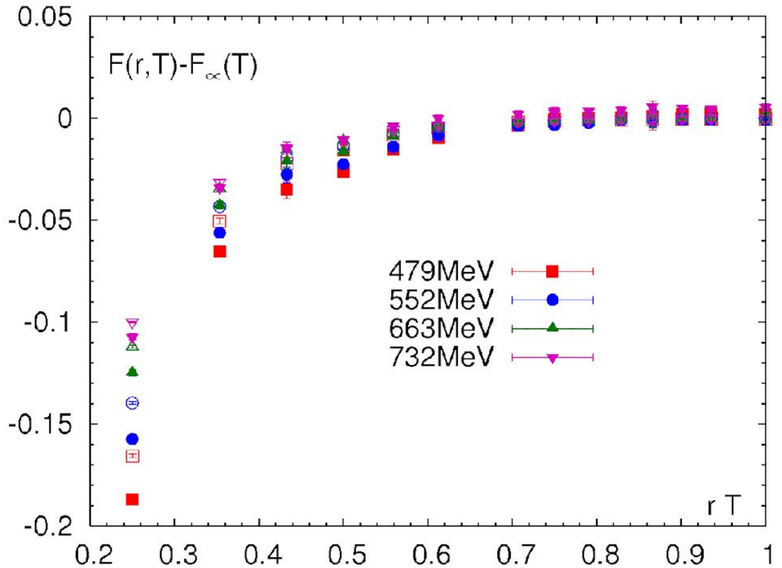




- The strong *T*-dependence for *T<200* MeV is not necessarily related to color screening

- The free energy has much stronger *T*-dependence than the singlet free energy due to the octet contribution

- At high T the temperature dependence of the free energy can be understood in terms of F_1 and Casimir scaling $F_1=-8\ F_8$



Free energy of a static quark anti-quark pair at high T (cont'd)

The definition of F_S requires gauge fixing.Is it possible to have a gauge invariant decomposition of $F_{Q\bar{Q}}$ into singlet and octet contributions ?

Yes, for $r \ll 1/T$ using pNRQCD

[redacted]

L_A is Polyakov loop in the adjoint representation

$$e^{-F_{Q\bar{Q}}(r,T)/T} = \frac{1}{9} e^{-f_s(r,T)/T} + \frac{8}{9} e^{-f_o(r,T)/T},$$

$$f_s = V_s(r) + \mathcal{O}(\alpha_s^2 r T^2), \;\; f_o = V_o(r) - \frac{N_c \alpha_s m_D}{2} + \mathcal{O}(\alpha_s^2 r T^2)$$

Brambilla et al, PRD 82 (2010) 074019

$$F_s = f_s + \mathcal{O}(\alpha_s^3 T) + \mathcal{O}(\alpha_s^2 r T^2), \;\; F_O = f_o + \mathcal{O}(\alpha_s^3 T) + \mathcal{O}(\alpha_s^2 r T^2)$$

Berwein et al, PRD 96 (2017) 014025

The naïve and the pNRQCD decomposition into singlet and octet agree

To calculate $F_{Q\bar{Q}}(r,T)$ and $F_S(r,T)$ for $r \sim 1/m_D$ another EFT, namely EQCD, should be used

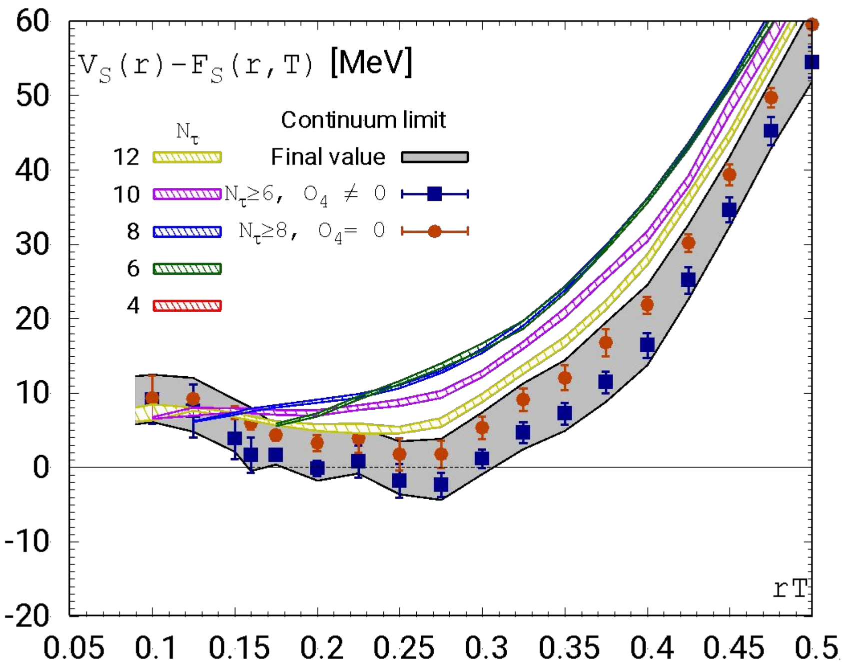
Free energy at short distances: lattice results vs. pNRQCD

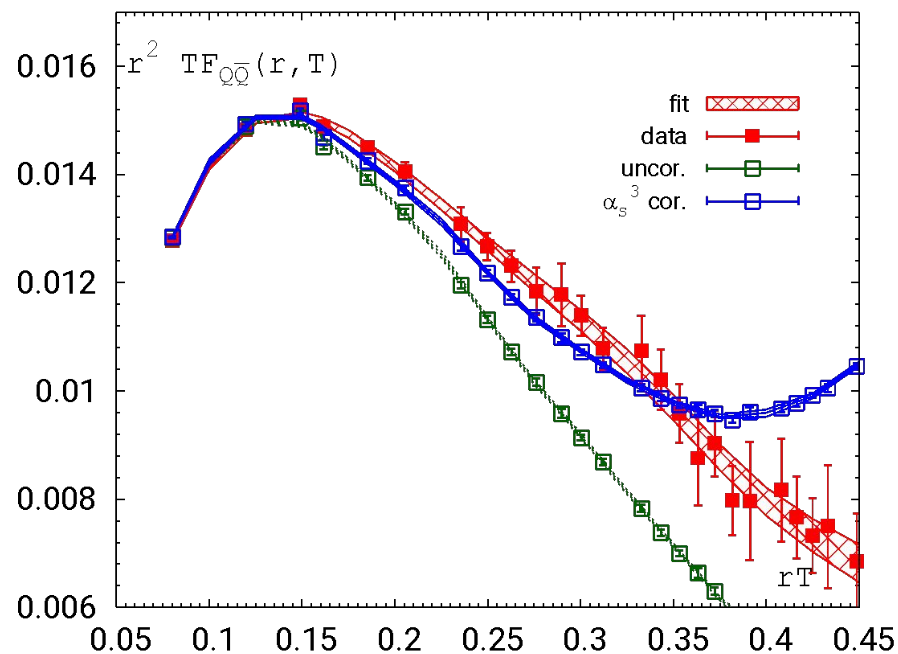
The difference between V_s and F_S is small as expected for $rT < 0.3$

Construct pNRQCD prediction for $F_{Q\bar{Q}}$ using the lattice data for F_S and V_s as proxy for f_s together with the relation:

$$f_o = -\frac{1}{8} f_s + \frac{3\alpha_s^3}{8r}\left(\frac{\pi^2}{4} - 3\right)$$

works !





T=407 MeV

TUMQCD, PRD 98 (2018) 054511

The interaction of static Q and $\bar{Q}$ is vacuum like for $rT < 0.3$

Free energy in the screening regime: lattice vs. weak coupling

NLO in EQCD results are available for $\bar{F}_i(r,T) = F_i(r,T) - F_\infty(T)$

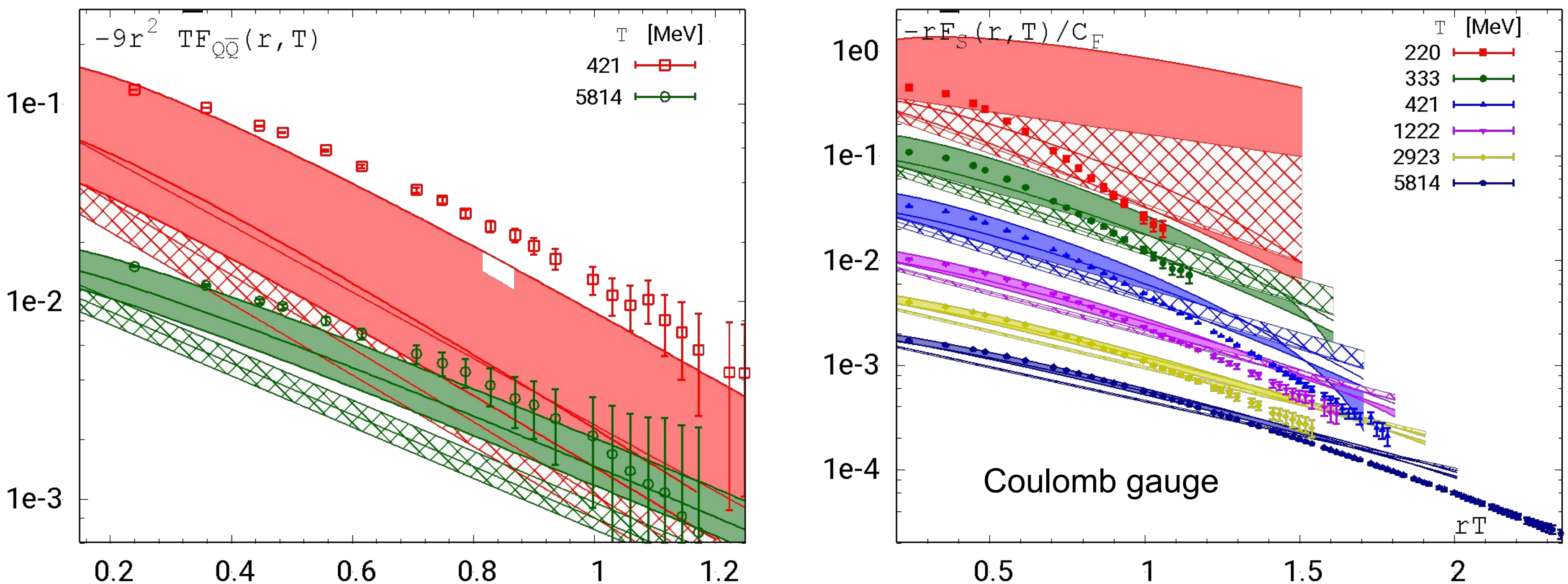
Nadkarni, PRD 33 (1986) 3738

$$F_{Q\bar{Q}}(r,T) = -\frac{\alpha_s^2 e^{-2m_D r}}{9r^2 T}(1+\alpha_s(\delta Z_1(\mu) + rTf(rm_D)))$$

Burnier et al, JHEP 01 (2010) 054

$$F_S(r,T) = -\frac{4\alpha_s e^{-m_D r}}{3r}(1+\alpha_s(\delta Z_1(\mu) + rTf_1(rm_D)))$$

TUMQCD, PRD 98 (2018) 054511



Lattice results are in reasonable agreement with NLO weak coupling result for *rT<0.6*, at larger distances, non-pertubtative effects (due to chromo-magnetic sector) are important