QCD at non-zero temperature and heavy quarks: Lecture 3

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Recap from lecture 2:

The chiral crossover in QCD is related to the chiral phase transition temperature in the limit of small quark masses.
Equation of state and fluctuations of conserved charges can be described by HRG below the crossover transition but for *T>300* MeV can be in terms of weak calculations (EQCD, HTL pert. theory),
Charm quark appear as new degrees of freedom above T_c but charm hadrons can still exist.

In this lecture: quarkonia and potential models, spectral functions, charmonia and bottomonia at T>0, complex static quark potential, heavy quark diffusion coefficient

Hard probes in non-equilibrium QCD matter, ICTS, Bangalore, March 16-27,2026

Quarkonia and potential models

$m_b, m_c \gg \Lambda_{QCD}$ $\Rightarrow$ non-relativistic bound states, analogs QED positronium

1-gluon exchange, $\alpha_s \sim 0.4$

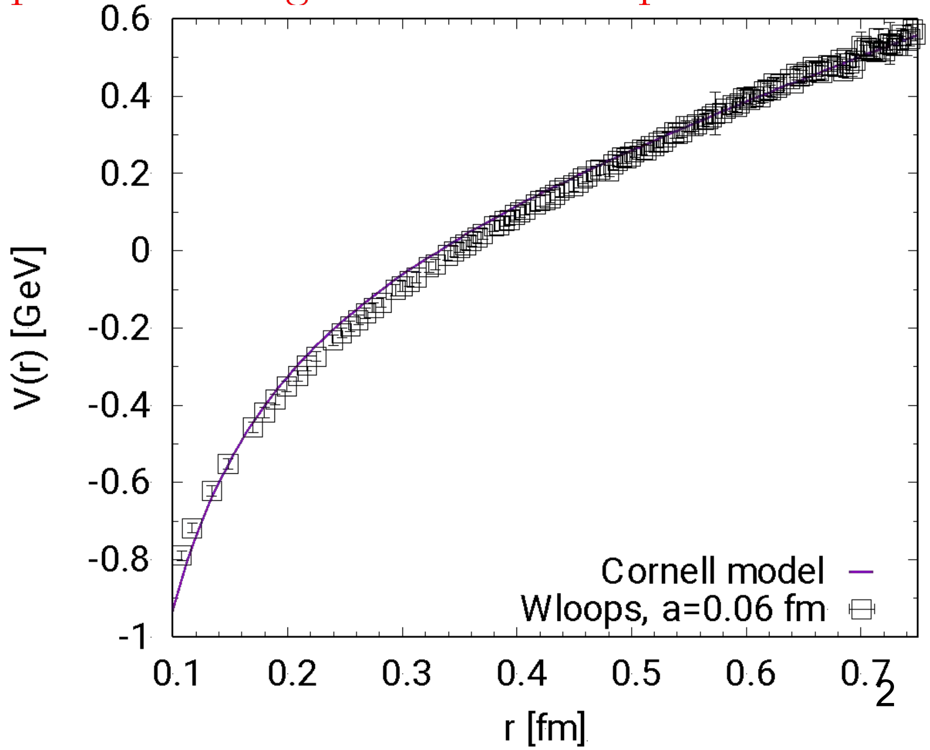
$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r + spin\ dep.$$

Confinement

Eichten et al, PRL 34 (75) 369, PRD 21 (80) 203

Very successful in describing charmonium and bottomonium spectrum below the the open charm and beauty threshold

Nevertheless nearly perfect agreement between the phenomenological and lattice potentials



Problems:

Running of α_s ?

Linear potential valid only for $r \gg 1$ fm,

$$V(r) = \sigma r - \frac{\pi}{12r} + \dots$$

Lüscher term

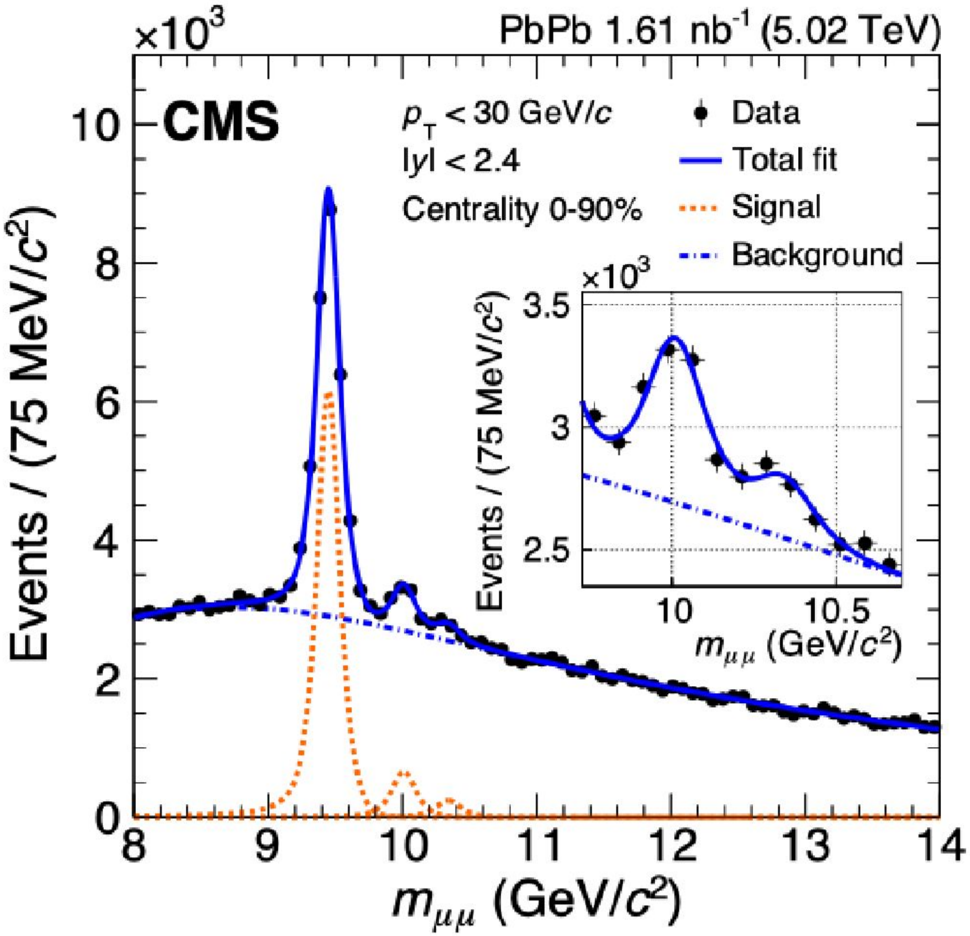
Soft gluon fields ??

Conjecture, Matsui and Satz, PLB 178 (86) 416

$$-\frac{4}{3}\frac{\alpha_s}{r} + \sigma r \rightarrow -\frac{4}{3}\frac{\alpha_s}{r}e^{-m_D r}, T > T_c$$

=>quarkonia melt

Quarkonia melting and suppression in heavy ion collision



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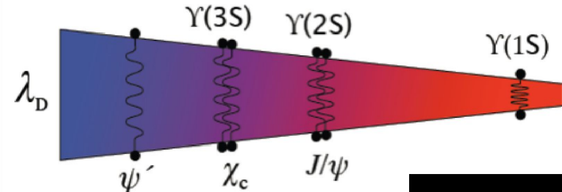

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Sequential meting pattern:

Smaller, more tightly bound quarkonia melt at higher temperature

Quarkonia in effective theory approach

$M \gg 1/r \sim Mv \gg Mv^2,\ M = m_{c,b}$ ➡ Effective theory (EFT) approach

Non-relativistic QCD (NRQCD) : EFT at scale $1/r$ (scale M is integrated out):

$$L_{NRQCD} = \psi^\dagger \left(iD_0 - \frac{D_i^2}{2M} \right) \psi + \chi^\dagger \left(iD_0 + \frac{D_i^2}{2M} \right) \chi + \ldots + \frac{1}{4} F_{\mu\nu}^2 + \bar{q}\gamma_\mu D_\mu q$$

Heavy quark fields are Pauli spinors, heavy pair creation is only present implicitly through higher dimension 4-fermion operators

Caswell, Lepage, PLB 167 (86) 437

potential NRQCD (pNRQCD): EFT at scale $E_{bin} \sim Mv^2$ (scale $1/r \sim Mv$ is integrated out):

$$L_{pNRQCD} = \int d^3\mathbf{r} \mathrm{Tr}\Big[S^\dagger \big[i\partial_0 - (\frac{-\nabla_r^2}{M} + V_s(r) + \ldots) \big] S + O^\dagger \big[iD_0 + \frac{-\nabla_r^2}{M} + V_o(r) + \ldots) \big] O \Big]$$

$$+ V_A(r) \mathrm{Tr}\Big[O^\dagger \mathbf{r} g \mathbf{E} S + S^\dagger \mathbf{r} g \mathbf{E} O \Big] + V_B(r) \mathrm{Tr}\Big[O^\dagger \mathbf{r} g \mathbf{E} O + O^\dagger O \mathbf{r} g \mathbf{E} \Big] +$$

$$\mathcal{O}(r^2, \frac{1}{M}) + \frac{1}{4} F_{\mu\nu}^2 + \bar{q}\gamma_\mu D_\mu q$$

Brambilla, Pineda, Soto, Vairo, NPB 566 (00) 275

$$S = S(\mathbf{r}, \mathbf{R}, t),\ O = O(\mathbf{r}, \mathbf{R}, t),\ E = E(\mathbf{R}, t)$$

Potentials are parameters of the EFT Largangian

Tree level $\leftrightarrow$ potential model $(i\partial_0 + \frac{\nabla_r^2}{M} - V_s(r)) S(\mathbf{r}, \mathbf{R}, t) = 0$

pNRQCD at non-zero temperature

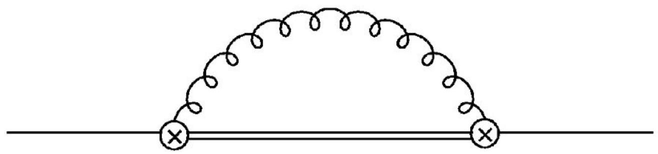
If $E_{bind} < T$ there are thermal contribution to the potentials

Brambilla, Ghiglieri, P.P., Vairo, PRD 78 (08) 014017

$r \ll 1/T \ll 1/m_D$

$$V_s(r) \to V_s(r) + \delta V_s(r,T)$$

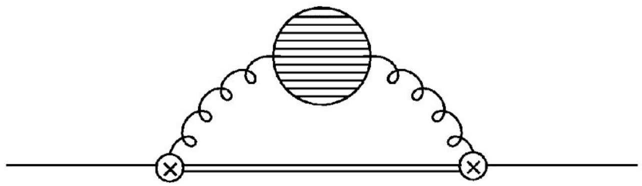
singlet-octet transition :



$\mathrm{Re}\delta V_s(r) \sim \alpha_s^2 T^2 r$

$\mathrm{Im}\delta V_s(r) \sim \alpha_s^3 T$

Landau damping :



$\mathrm{Re}\delta V_s(r,T) \sim \mathrm{Im}\delta V_s(r,T)$

$\sim \alpha_s T^3 r^2 \times \left(\frac{m_D}{T}\right)^n$

$r \gg 1/T$

[redacted] $\mathrm{Re}\delta V_s \sim g^2 T^3 r^2 \times \left(\frac{m_D}{T}\right)^3$ $\mathrm{Im}\delta V_s \sim g^2 T^3 r^2 \times \left(\frac{m_D}{T}\right)^2$

[redacted] $$V_s(r,T) = -C_F \frac{\alpha_s}{r} \exp(-m_D r) + i C_F \alpha_s T \frac{2}{r m_D} \int_0^\infty dx \frac{\sin(r m_D x)}{(x^2+1)^2}$$

Laine, Philipsen, Romatschke, Tassler, JHEP 073 (2007) 054

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The imaginary part of the potential is larger than the real part => quarkonium melting is determined by imaginary part of the potential and not by screening

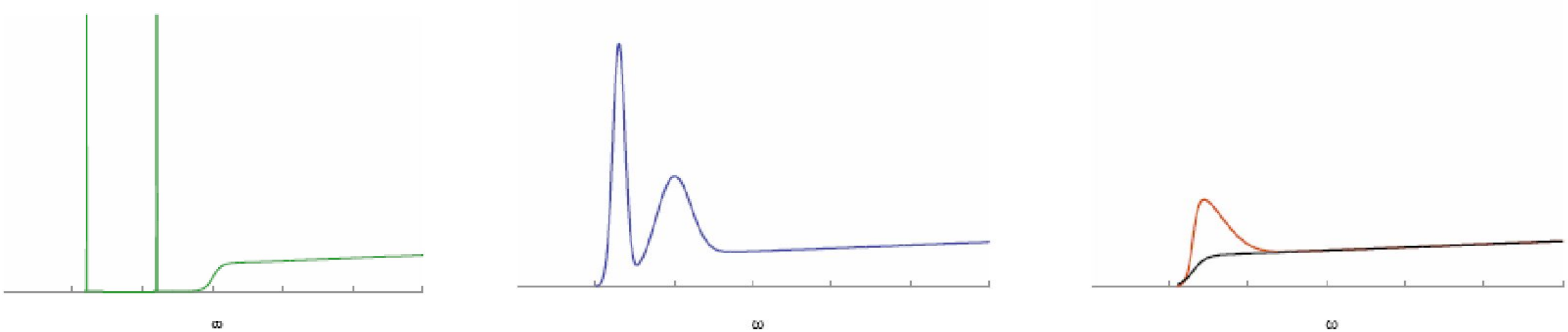
Meson correlators and spectral functions

Vacuum and in-medium properties as well as dissolution of mesons are encoded in the spectral functions:

$$\rho(\omega, p, T) = \frac{1}{2\pi} \mathrm{Im} \int_{-\infty}^{\infty} dt e^{i\omega t} \int d^3x e^{ipx} \langle [O(x,t), O(0,0)] \rangle_T, \quad O(x,t) \sim \bar{Q}(x,t) \Gamma Q(x,t)$$

Melting is seen as progressive broadening and disappearance of the bound state peaks

Modifications of quarkonium yields in heavy ion collisions Matsui and Satz, PLB 178 (1986) 416



$$G(\tau, T) = \sum_x \langle O(x,\tau) O(0,0) \rangle_T \quad \longleftrightarrow \quad G(\tau, T) = \int_0^{\infty} d\omega \rho(\omega, T) \frac{\cosh(\omega(\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

Consider large τ behavior of $C(\tau, T=0)$:

$$C(\tau, T) \sim \sum_n |\langle 0|O|n\rangle|^2 e^{-M_n \tau} \simeq f_1 e^{-M_1 \tau} + f_2 e^{-M_2 \tau} + \dots$$

$T > 0: \quad \tau < 1/T \Rightarrow \text{reconstruct } \rho(\omega, T)$

Temperature dependence of temporal charmonium correlators

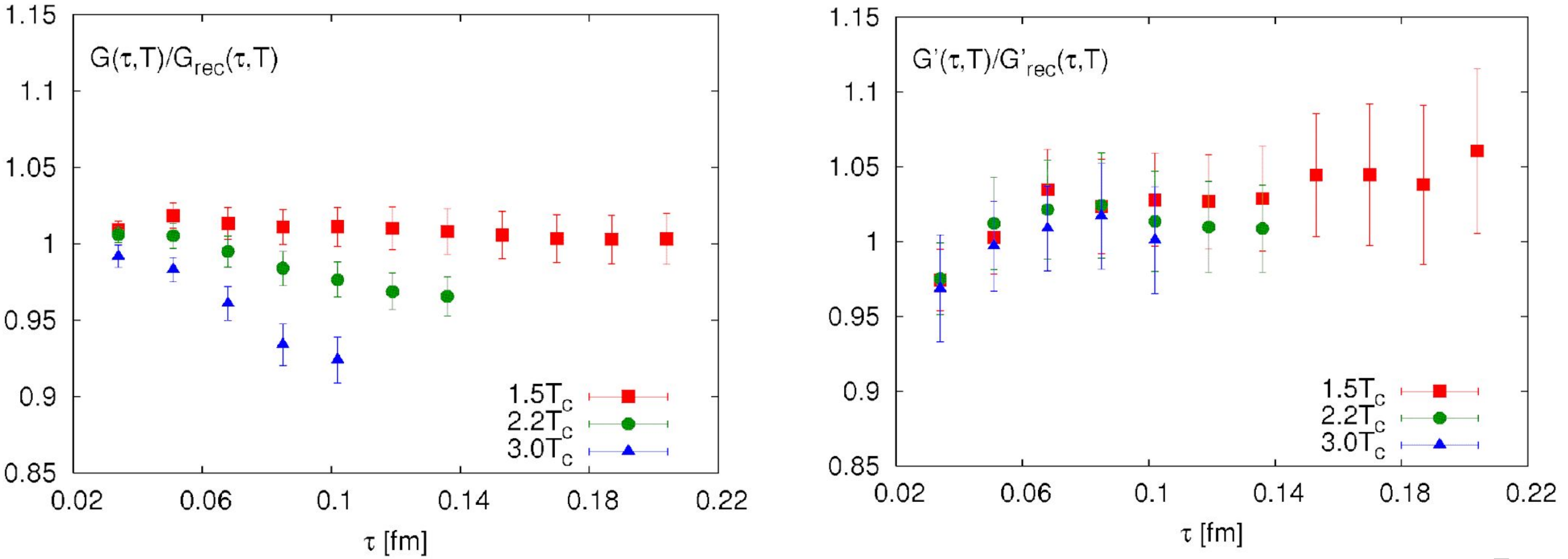
temperature dependene of $G(\tau, T)$

$$G(\tau,T) = \int_0^\infty d\omega \sigma(\omega,T) \frac{\cosh(\omega(\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

If there is no *T*-dependence in the spectral function, $G(\tau,T)/G_{rec}(\tau,T) = 1$

$$G_{rec}(\tau,T) = \int_0^\infty d\omega \sigma(\omega, T=0) \frac{\cosh(\omega \cdot (\tau - \frac{1}{2T}))}{\sinh(\omega/(2T))}$$

Pseudo-scalar $\Leftrightarrow$ $1S$ Scalar $\Leftrightarrow$ $1P$



Datta, Karsch, P.P., Wetzorke, PRD 69 (2004) 094507

Temporal vs spatial meson correlators

Spatial correlation functions can be calculated for arbitrarily large separations $z \to \infty$

$$G(z,T) = \int_0^{1/T} d\tau \int dx dy \langle O(\mathbf{x}, -i\tau) O(\mathbf{0}, 0) \rangle_T, \; G(z \to \infty, T) = A e^{-m_{scr}(T) z}$$

but related to the same spectral functions

$$G(z,T) = \int_{-\infty}^{\infty} e^{ipz} \int_0^{\infty} d\omega \frac{\sigma(\omega, p, T)}{\omega}$$

Low T limit :

$$\sigma(\omega, p, T) \simeq A_{mes} \delta(\omega^2 - p^2 - M_{mes}^2)$$

$$A_{mes} \sim |\psi(0)|^2 \to m_{scr}(T) = M_{mes}$$

$$G(z,T) \simeq |\psi(0)|^2 e^{-M_{mes}(T) z}$$

High T limit :

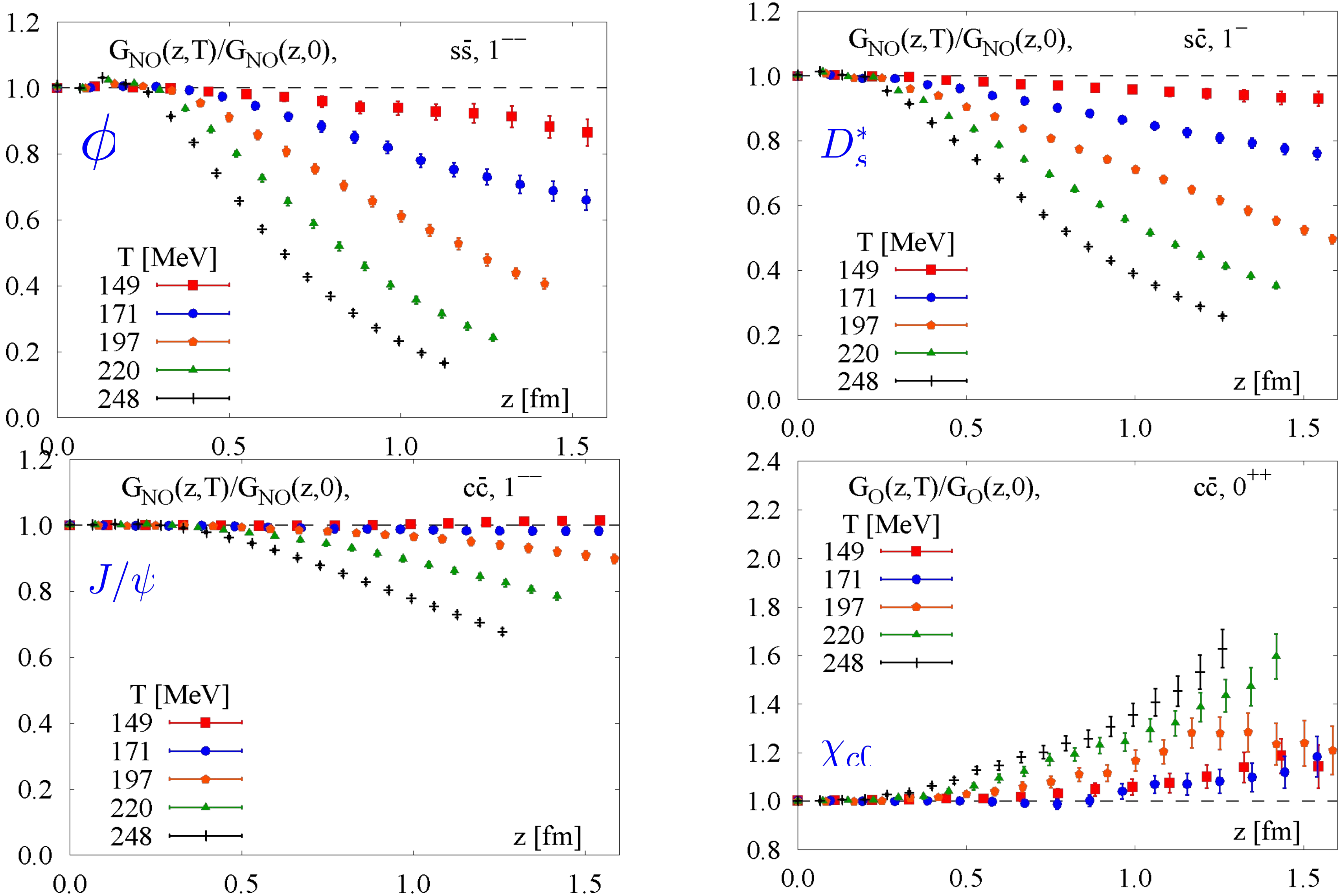
$$m_{scr}(T) = \sqrt{m_{q_1}^2 + (\pi T)^2} + \sqrt{m_{q_2}^2 + (\pi T)^2}$$

Temporal meson correlator only avalaible for $\tau T < ½$ and thus may not be very sensitive to In-medium modifications of the spectral functions; also require large N_τ (difficult in full QCD)

Spatial correlators can be studied for arbitrarily large separations and thus are more sensitive to the changes in the meson spectral functions; do not require large N_τ (easy in full QCD).

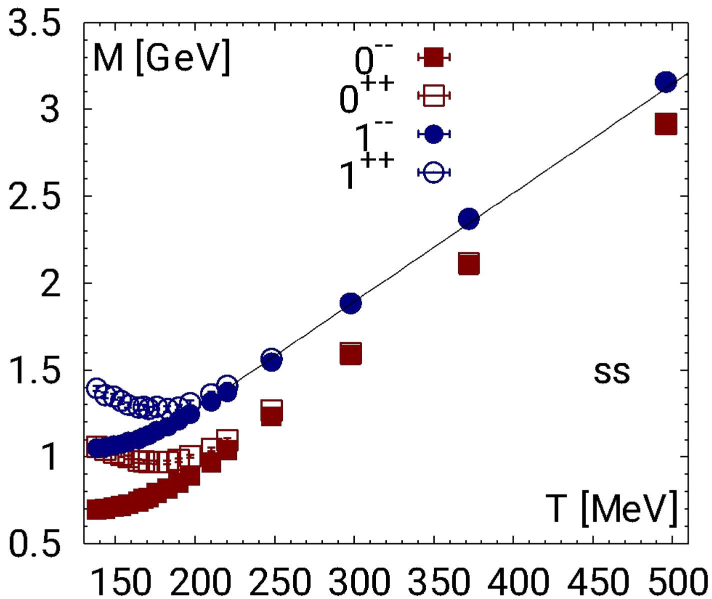
Lattice calculations: spatial meson correlators in 2+1 flavor QCD for ssbar, scbar and ccbar sectors using 48^3x12 lattices and highly improved staggered quark (HISQ) action (also suitable for charm quarks) , physical m_s and m_π=*160* MeV.

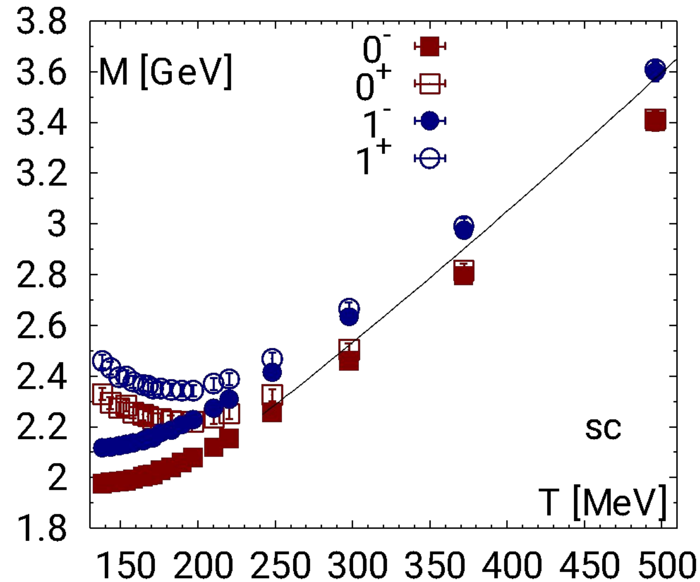
Temperature dependence of spatial meson correlators

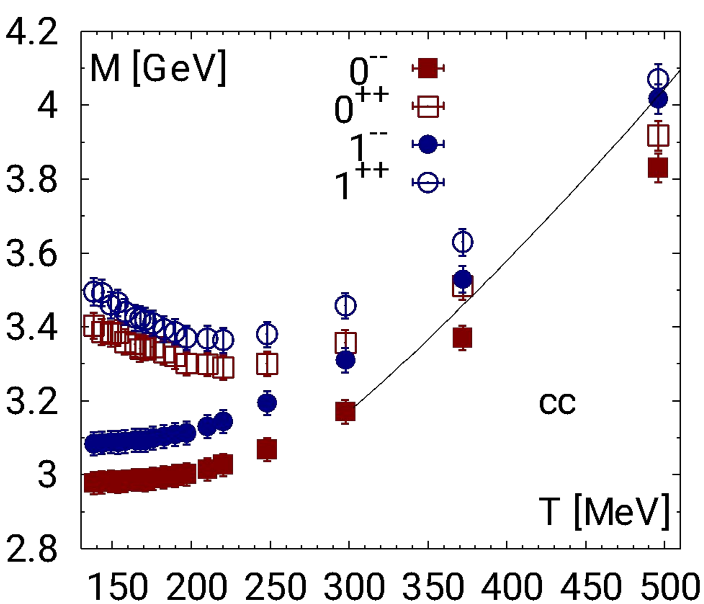


Medium modifications of meson correlators increase with *T*, but decrease with heavy quark content; larger for *1P* charmonium state than for *1S* charmonium state

Temperature dependence of meson screening masses







Qualitatively similar behavior of the screening masses for ssbar, scbar and ccbar sectors

Screening Masses of opposite parity mesons become degenerate at high T
(restoration of chiral and axial symmetry)

Screening masses are close to the free limit $2\,(m_q^{\,2}+(\pi T)^2)^{1/2}$ at $T>200$ MeV, $T>250$ MeV, $T>300$ MeV for ssbar, scbar and ccbar sectors, respectively.

Spatial meson correlators and bottomonium melting

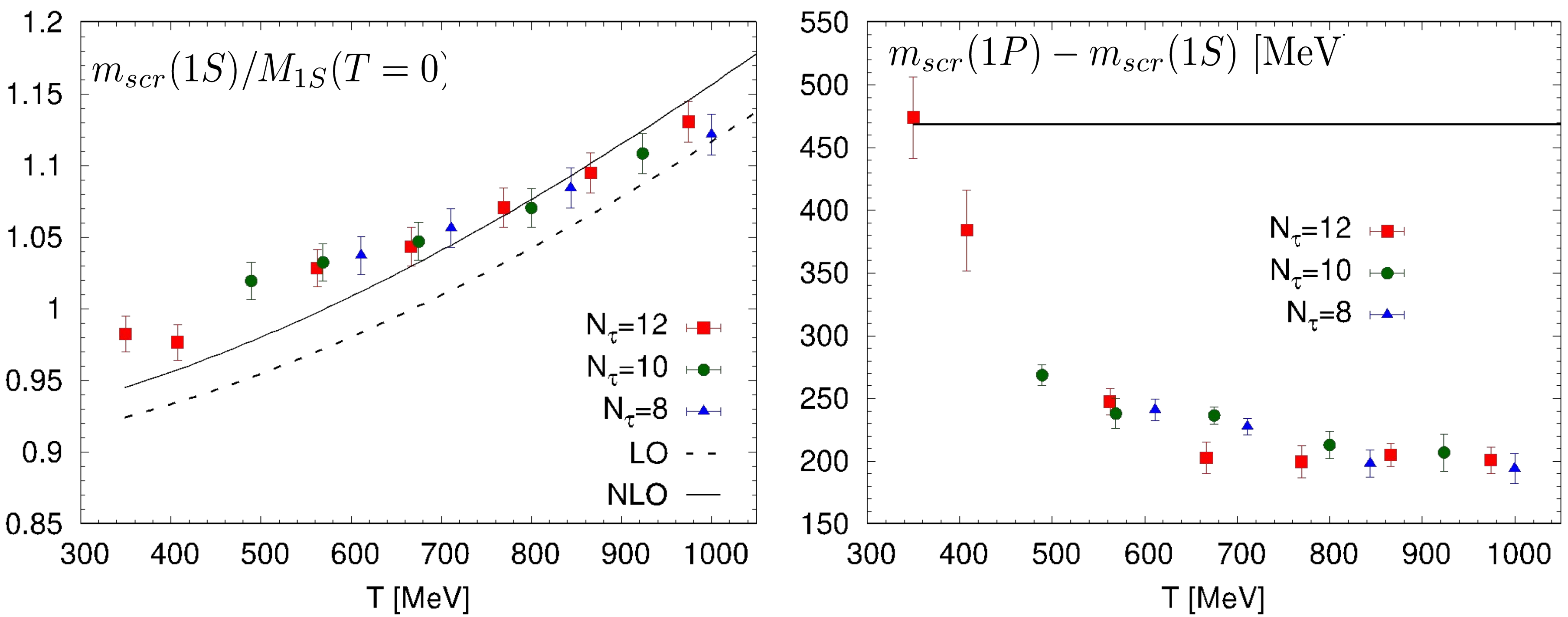
$$G(z,T) = \int_0^{1/T} d\tau \int dxdy \langle O(\mathbf{x}, -i\tau) O(\mathbf{0}, 0) \rangle_T, \; G(z \to \infty, T) = A e^{-m_{scr}(T) z}$$

$$\sigma(\omega, p, T) \simeq A_{mes} \delta(\omega^2 - p^2 - M_{mes}^2)$$

$$m_{scr}(T) = 2\sqrt{m_b^2 + (\pi T)^2}$$

$$A_{mes} \sim |\psi(0)|^2 \to m_{scr}(T) = M_{mes}$$

PP, Sharma, Weber, PRD 104 (2021) 054511



$$T_{melt}(\Upsilon(1S)) > 500 \text{ MeV}, \; T_{melt}(\chi_b(1P)) > 350 \text{ MeV}$$

Consistent with the previous estimates

NRQCD on the Lattice

Advantages: No large cutoff effects $\sim aM_b$, large τ range for $T > 0$

Inverse lattice spacing provides a natural UV cutoff for NRQCD,
provided $a^{-1} \leq 2M_Q$ (lattices cannot be too fine)

Quark propagators are obtained as initial value problem:

$$G_\alpha(\tau, T) = \int_{-\infty}^{\infty} d\omega \rho_\alpha(\omega, T) e^{-\omega\tau}$$

$$G_\psi(\mathbf{x}, t) = \langle \psi(\mathbf{x}, t)\, \psi^\dagger(\mathbf{0}, 0) \rangle$$

$$G_\psi(t) = K(t) G_\psi(t-1), \qquad G_\chi(\mathbf{x}, t) = -G_\psi^\dagger(\mathbf{x}, t)$$

$$K(t) = \left(1 - \frac{a\delta H|_t}{2}\right)\left(1 - \frac{aH_0|_t}{2n}\right)^n U_4^\dagger(t) \times \left(1 - \frac{aH_0|_{t-1}}{2n}\right)^n \left(1 - \frac{a\delta H|_{t-1}}{2}\right),$$

$$t = \tau/a, \ \ H_0 = \frac{-\Delta^{(2)}}{2M_b}, \ \ \delta H \sim v^4, \ v^6(\text{ spin} - \text{dep.})$$

@ Tree level

Meinel, PRD 82 (2010) 114502

masses are only defined up to *a*-dependent shift: $M_{\Upsilon(1S)} = E_{\Upsilon(1S)} + C_{\text{shift}}(a)$

Use kinetic mass instead: $E_{\Upsilon(1S)}(p) = E_{\Upsilon(1S)} + C_{\text{shift}}(a) + \dfrac{p^2}{2M^{kin}_{\Upsilon(1S)}}$

Tune M_b such that $M^{kin}_{\Upsilon(1S)} = M^{PDG}_{\Upsilon(1S)}$

NRQCD meson correlators

Point correlators:

Aarts et al (FASTUM) , Kim, PP, Rothkopf

$$G_p(t) = \sum_{\mathbf{x}} \langle O_p(t,\mathbf{x}) O_p(0,\mathbf{0}) \rangle,$$

$$O_p(t,\mathbf{x}) = \chi^\dagger(t,\mathbf{x}) \Gamma \psi(t,\mathbf{x}))$$

$$t = \tau/a$$

State	Irrep Λ^{PC}	Γ
η_b	A_1^{-+}	1
Υ	T_1^{--}	σ_j
h_b	T_1^{+-}	∇_j
χ_{b0}	A_1^{++}	$\boldsymbol{\sigma} \cdot \boldsymbol{\nabla}$
χ_{b1}	T_1^{++}	$(\boldsymbol{\sigma} \times \boldsymbol{\nabla})_j$
χ_{b2}	T_2^{++}	$\sigma_j \nabla_k + \sigma_k \nabla_j$

Extended correlators:

$$O_p(t,\mathbf{x}) \to O(t,\mathbf{x}) = \sum_{\mathbf{r}} \Psi(\mathbf{r}) \chi^\dagger(t,\mathbf{x}) \Gamma \psi(t,\mathbf{x}+\mathbf{r}))$$

$$\Psi(\mathbf{r}) \sim e^{-|\mathbf{r}|^2/\sigma^2}$$

or realistic wave-function

Optimized correlators: use several different extended meson operators with realistic wave functions and form orthogonal combinations

$$O_i \to \tilde{O}_\alpha = \Omega_{\alpha j} O_j, \; \langle \tilde{O}_\alpha(t) \tilde{O}^\dagger_\beta(0) \rangle \propto \delta_{\alpha,\beta}, \; i = 1, 2, 3, \ldots$$

Mixed correlators (Bethe-Salpeter amplitudes):

Bethe-Salpeter amplitude

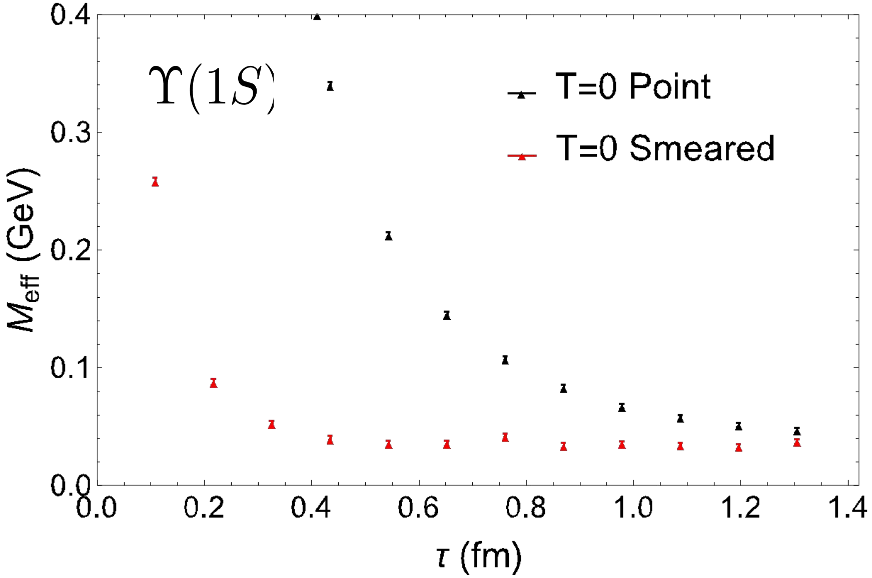
$$\tilde{G}^r_\alpha(t) = \sum_{\mathbf{x}} \langle O^r_{qq}(t,\mathbf{x}) \tilde{O}_\alpha(0,\mathbf{0}) \rangle \sim \phi_\alpha(r) e^{-E_\alpha t}, \; t \to \infty$$

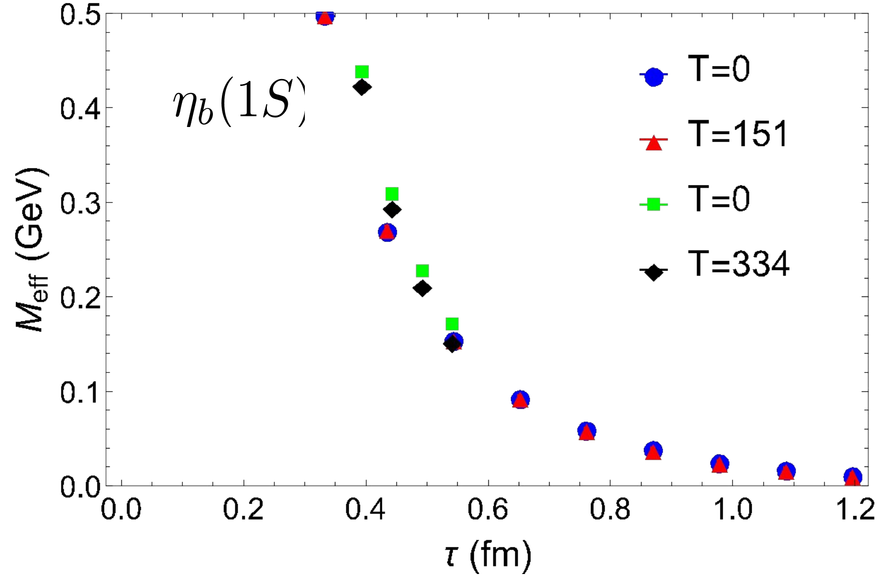
$$O^r_{qq}(t,\mathbf{x}) = \chi^\dagger(t,\mathbf{x}) \Gamma \psi(t,\mathbf{x}+\mathbf{r}))$$

Point operators vs. extended operators

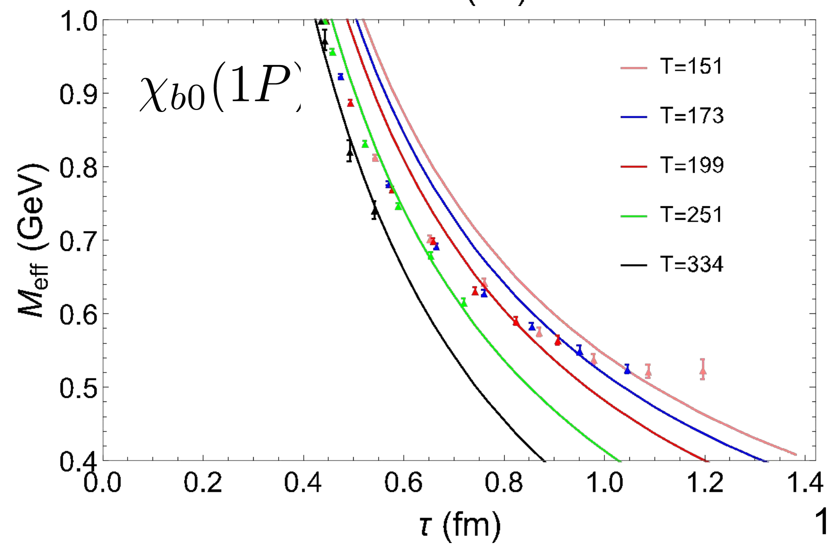
Larsen, Meinel, Mukherjee, PP, PRD100 (2019) 074506

$$M_{\rm eff}(\tau) = \frac{1}{a}\ln[G_\alpha(\tau)/G_\alpha(\tau+a)]$$





- The effective masses of point correlators do not show a plateau for $\tau < 1.2$ fm and have very small temperature dependence
- The small τ behavior of the effective masses is well described by perturbation theory for P-wave bottomonia
- The correlators of extended operators approach a plateau for $\tau < 1$ fm.

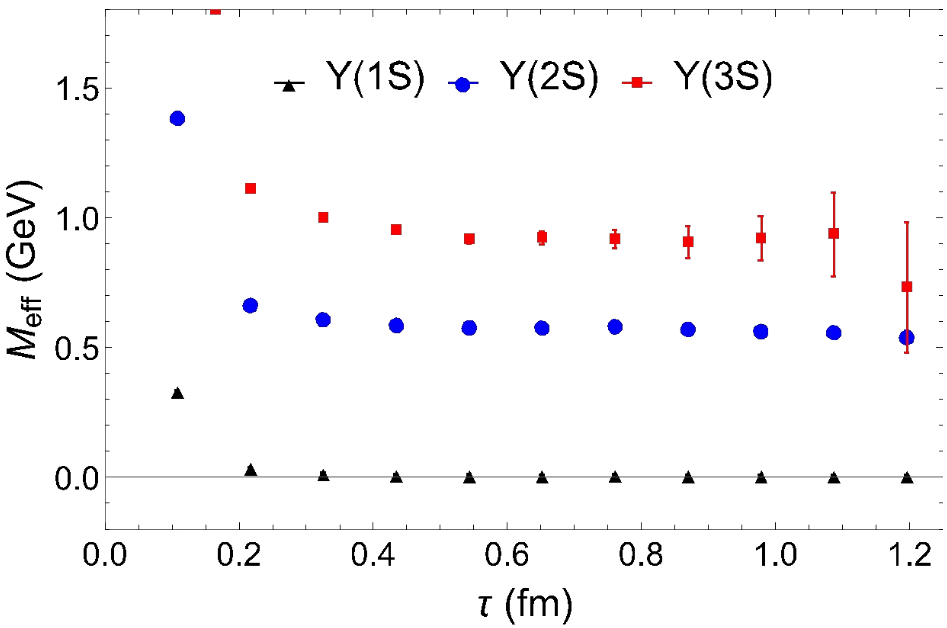


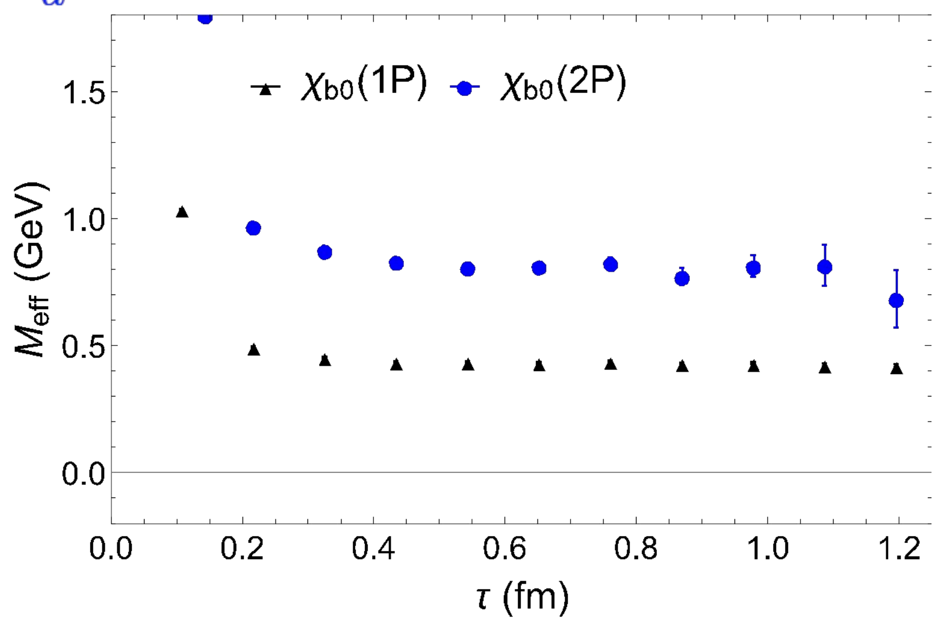
Correlators of Optimized Meson Operators at T=0

HISQ, $a = 0.109,\ 0.095,\ 0.083,\ 0.066,\ 0.060,\ 0.049$ fm, $48^3 \times 12$

Larsen, Meinel, Mukherjee, PP, PLB 800 (2020) 135119

$$M_{\rm eff}(\tau) = \frac{1}{a}\ln[G_\alpha(\tau)/G_\alpha(\tau+a)]$$





$$G_\alpha(\tau,T) = \int_{-\infty}^{\infty} d\omega \rho_\alpha(\omega,T) e^{-\omega\tau} \qquad \rho_\alpha(\omega,T) = \rho_\alpha^{\rm med}(\omega,T) + \rho_\alpha^{\rm high}(\omega)$$

$$\rho_\alpha^{\rm med}(\omega,T=0) = A_\alpha \delta(\omega - M_\alpha) \Rightarrow G_\alpha(\tau,T=0) = A_\alpha e^{-M_\alpha\tau} + G_\alpha^{\rm high}(\tau)$$

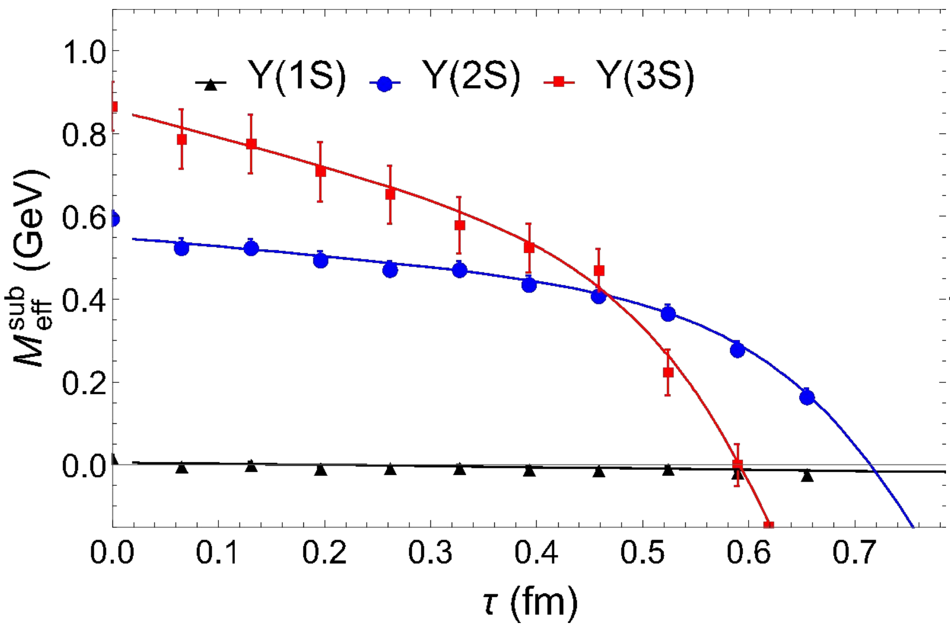
Determine A_α,M_α from single exponential fit for $\tau > 0.6$fm and then $G_\alpha^{\rm high}(\tau)$

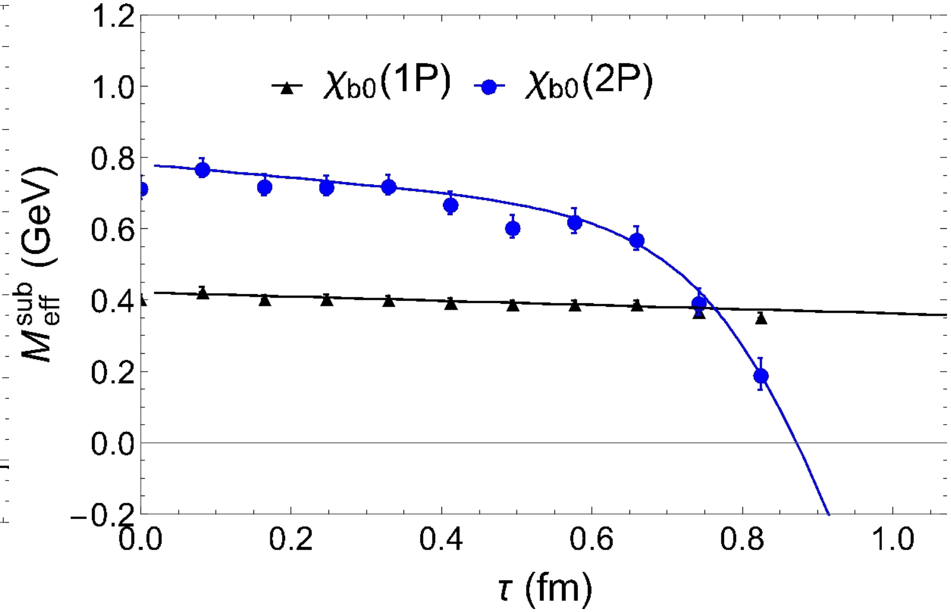
Correlators of Extended Meson Operators at T>0

Larsen, Meinel, Mukherjee, PP, PLB 800 (2020) 135119

Ding, Huang, Larsen, Meinel, Mukherjee, PP, Tang JHEP 05 (2025) 149

$$C_\alpha^{\text{sub}}(\tau,T) = C_\alpha(\tau,T) - C_\alpha^{\text{high}}(\tau) \Rightarrow aM_{\text{eff}}^{\text{sub}}(\tau,T) = \ln\left(C_\alpha^{\text{sub}}(\tau,T)/C_\alpha^{\text{sub}}(\tau+a,T)\right)$$





Fit $M_{\text{eff}}^{\text{sub}}(\tau,T)$ using a simple Ansatz:

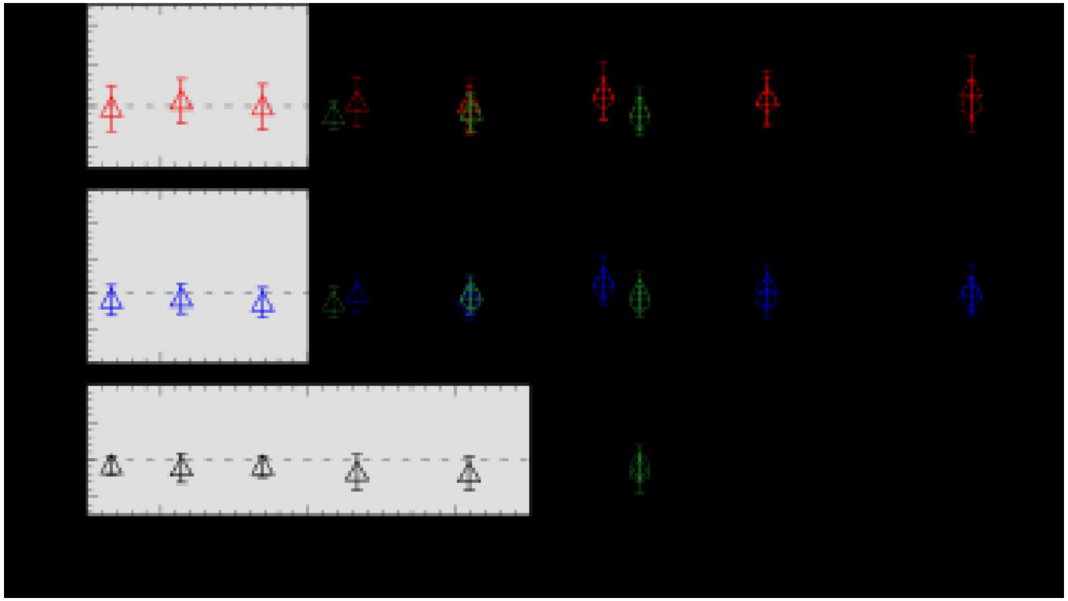
$$\rho_\alpha^{\text{med}}(\omega,T) = A_\alpha^{\text{low}}(T)\,\delta\left(\omega-\omega_\alpha^{\text{low}}(T)\right) + A_\alpha(T)\frac{1}{\pi}\frac{\Gamma_\alpha(\omega,T)}{(\omega-M_\alpha(T))^2+\Gamma_\alpha^2(\omega,T)}$$

Low energy tail

$\Gamma(\omega\simeq M_\alpha,T)\simeq\Gamma_0$, and $\Gamma(\omega,T)\to 0$ for $|\omega-M_\alpha(T)|\gg\Gamma_0$

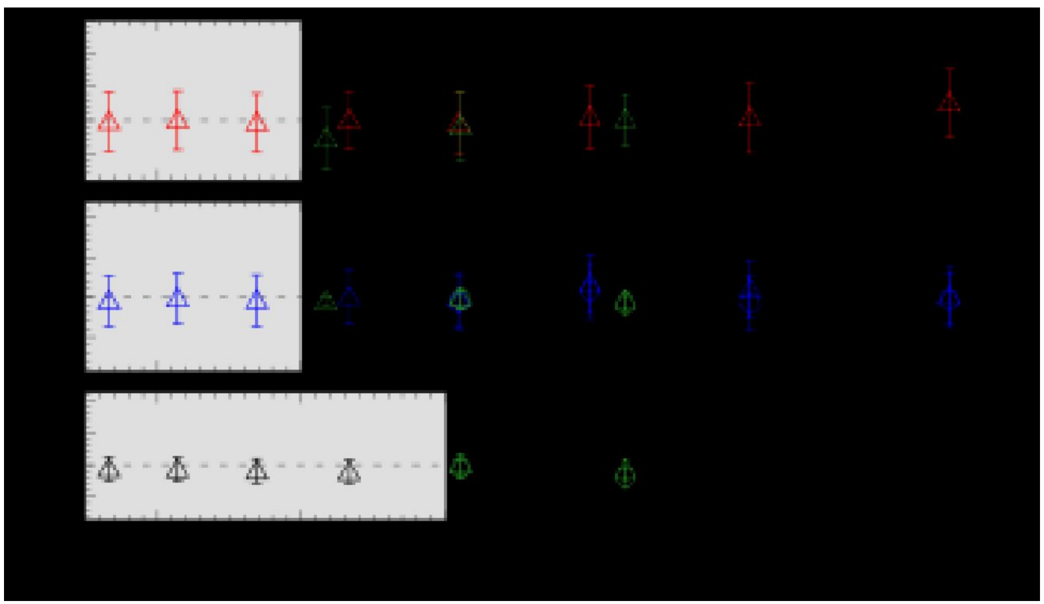
$\Rightarrow M_\alpha(T),\Gamma_\alpha(T)$

Thermal mass shift of bottomonium



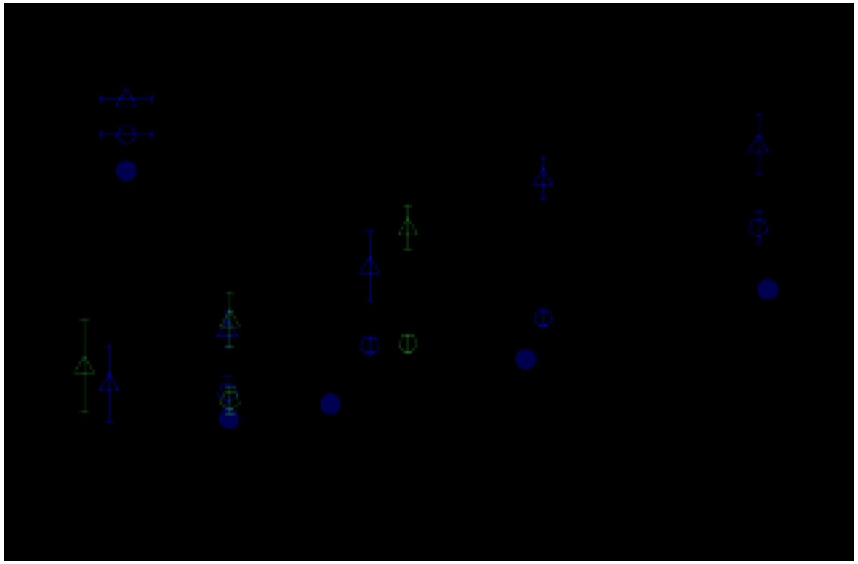
$$\Delta M_\alpha(T) = M_\alpha(T) - M_\alpha(T=0)$$

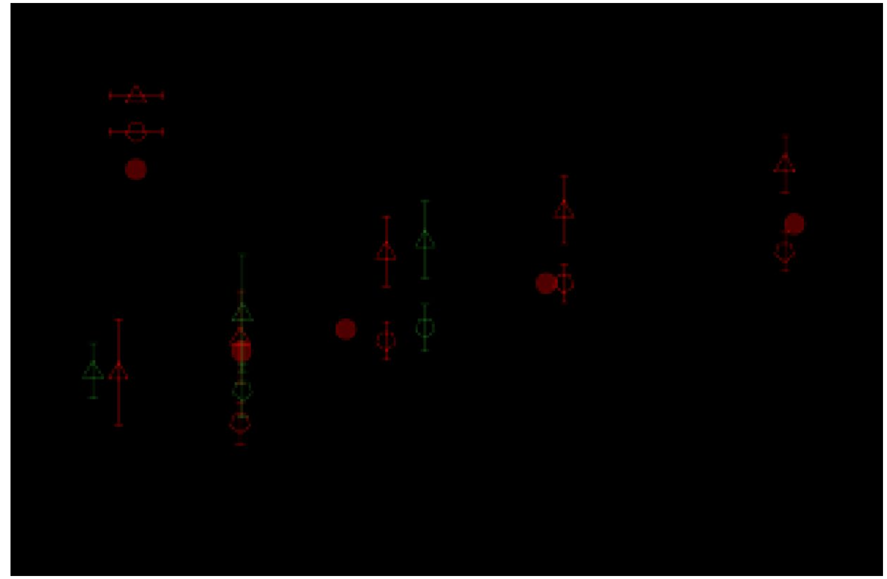
No shift in the bottomonium masses
=>. very small screening,
Ding, Huang, Larsen, Meinel, Mukherjee,
PP, Tang JHEP 05 (2025) 149

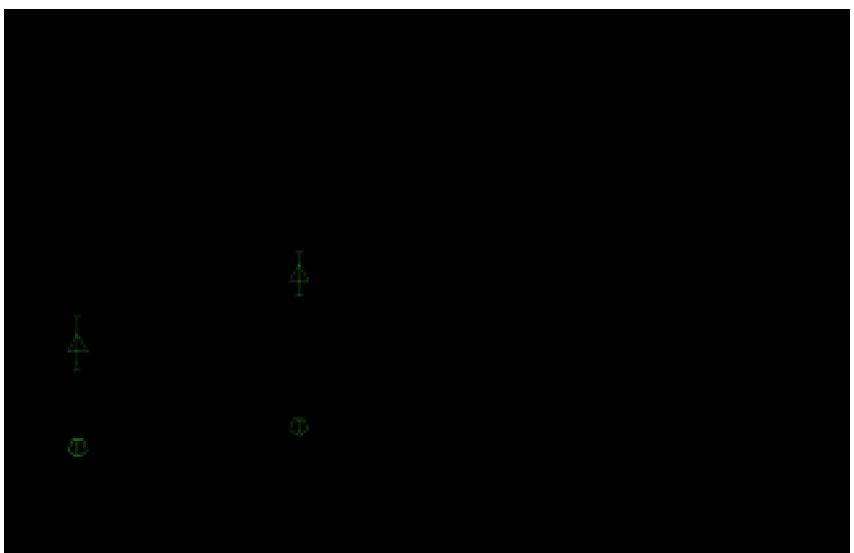


Thermal width of bottomonium

Ding, Huang, Larsen, Meinel, Mukherjee, PP, Tang JHEP 05 (2025) 149







Significant thermal width for all bottomonium states that increases with T

Some sensitivity of the width to the form of the spectral peak

Quark anti-quark potential at $T>0$

Conjecture, Matsui and Satz, PLB 178 (86) 416 $-\frac{4}{3}\frac{\alpha_s}{r}+\sigma r \to -\frac{4}{3}\frac{\alpha_s}{r}e^{-m_D r}, T>T_c$

Extending pNRQCD to $T>0$: the potential is complex, the real part can have thermal correction but is not necessarily screened, except when $r \sim 1/m_D$

Based on weak coupling

Laine, Philipsen, Romatschke, Tassler, JHEP 03 (06) 054
Brambilla, Ghiglieri, PP, Vairo, PRD 78 (08) 014017

Calculate the potential non-perturbatively on the lattice by considering Wislon loops of size $r \times \tau$ at $T>0$

$$W(r,\tau,T)=\int_{-\infty}^{\infty}\rho_r(\omega,T)e^{-\omega\tau}$$

If potential at $T>0$ exists the $\rho_r(\omega,T)$ should have a well defined peak at $\omega \simeq \mathrm{Re}V(r,T)$, and the width of the peak is $\mathrm{Im}V(r,T)$

Rothkopf, Hatsuda, Sasaki, PRL 108 (2012) 162001

Challenge: reconstruct $\rho_r(\omega,T)$

$$\rho_r(\omega,T=0)=\delta(\omega-V(r))+\sum_n \delta(\omega-E_n(r))$$

Hybrid potentials,
pairs of static-light mesons ..

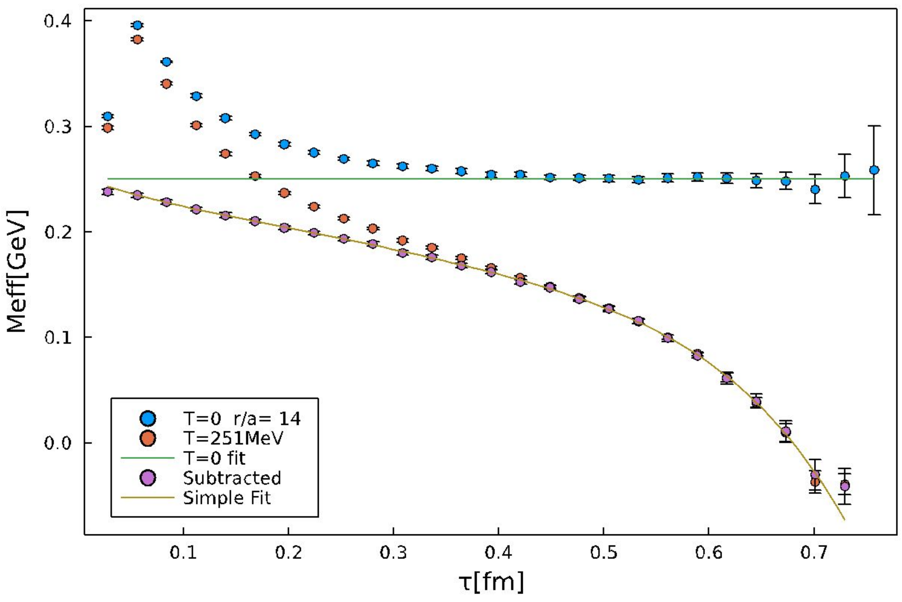
Calculations on fine lattices

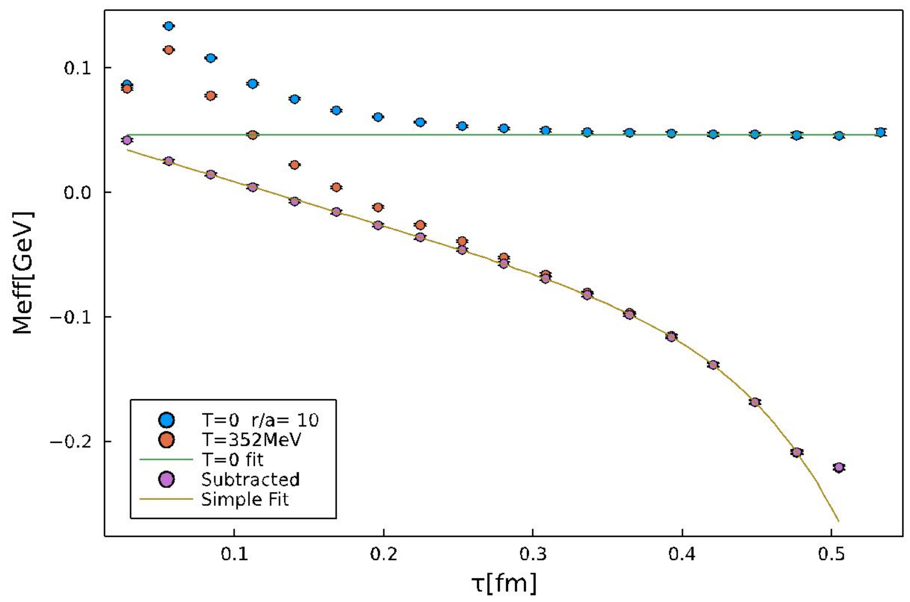
$2+1$ f QCD, $m_\pi = 300$ MeV $\quad T = 126,\ 196,\ 220,\ 252,\ 294,\ 354$ MeV

$a = 0.028$ fm, $96^3 \times N_\tau, N_\tau = 56,\ 36,\ 32,\ 28,\ 24,\ 20$

Gradient flow for noise reduction: $\sqrt{8\tau_F}T = 0.04 - 0.05$

$$m_{eff}(r,\tau,T) = -\partial_\tau \log(W(\tau,r,T)) \simeq \frac{1}{a} \ln \frac{W(r,\tau,T)}{W(r,\tau+a,T)}$$

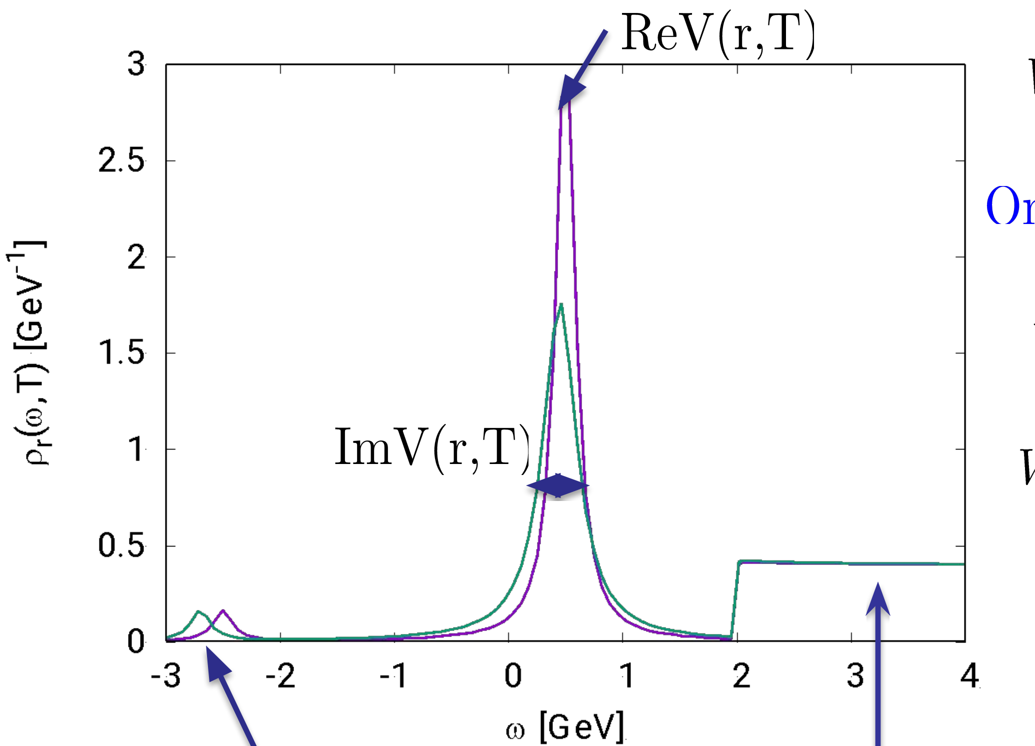




- No plateau at $T>0$ in m_{eff} at $T>0$
- Only tiny T-dependence for small τ

HotQCD, PRD 109 (2024) 074504

Spectral function and the subtracted correlators



$$\rho_r(\omega,T) = \rho_r^{tail}(\omega,T) + \rho_r^{peak}(\omega,T) + \rho_r^{high}(\omega)$$

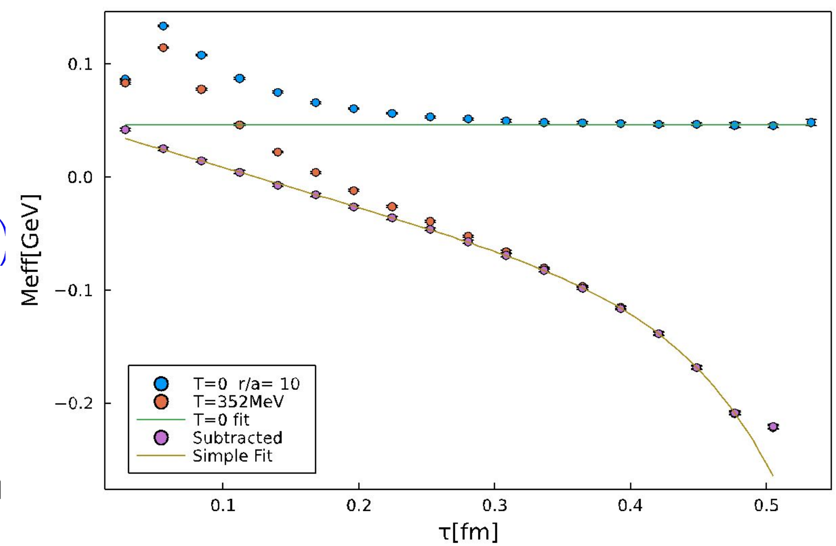
See, Bala et al (HotQCD), PRD 105 (2022) 054513

$$W^{high}(r,\tau) = \int_{-\infty}^{\infty} d\omega \rho_r^{high}(\omega) e^{-\omega\tau}$$

On the lattice:

$$W^{high}(r,\tau) = W(r,\tau,T=0) - A_0 \exp(-V(r)\tau)$$

$$W^{sub}(r,\tau,T) = W(r,\tau,T) - W^{high}(r,\tau)$$



HotQCD, PRD 109 (2024) 074504

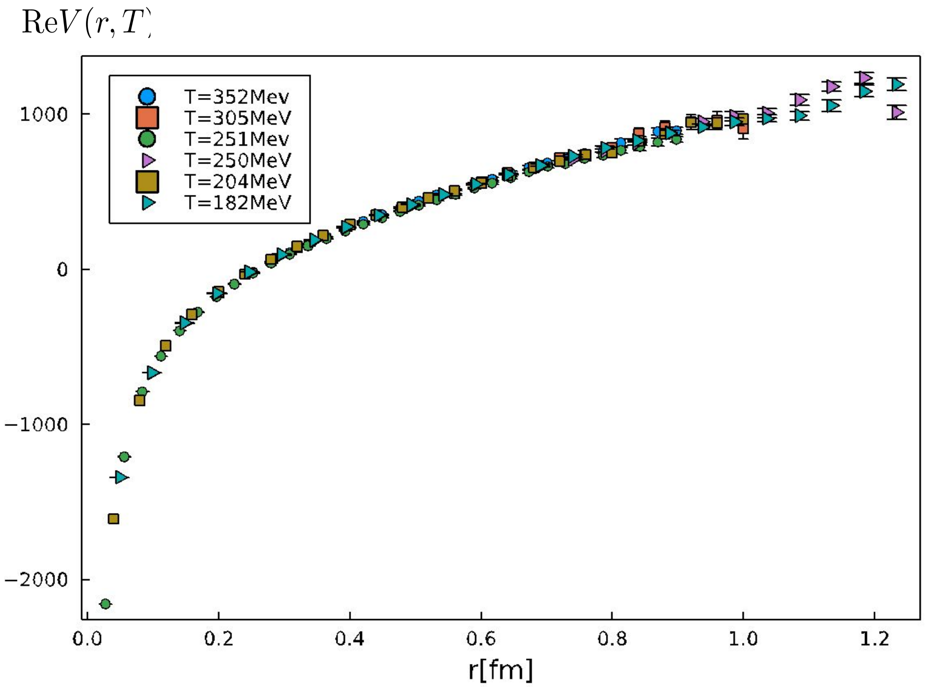
Cumulants of $W^{sub}(r,\tau,T)$ carry information about T-dependent part of the spectral function

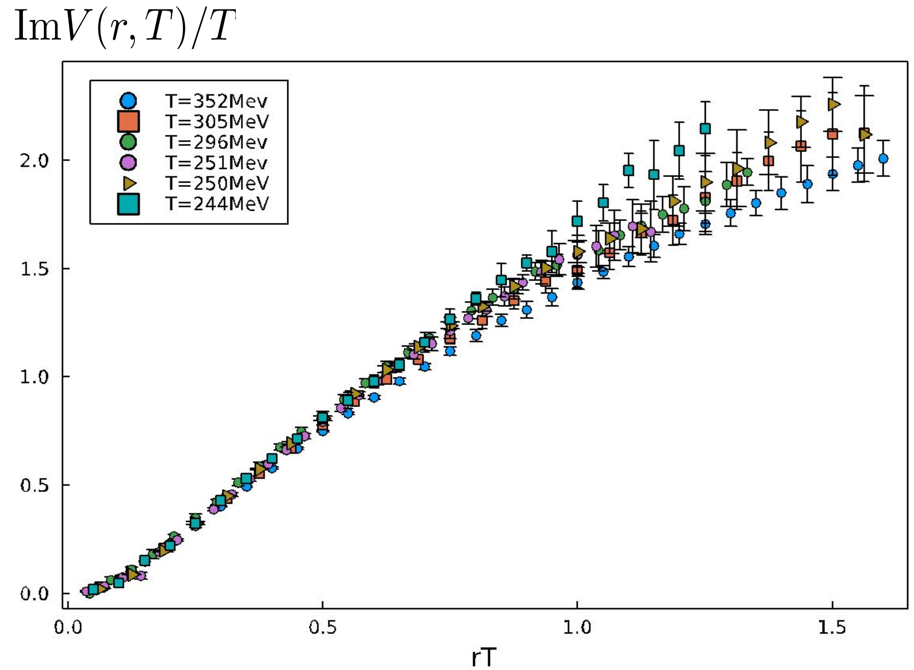
m_{eff} for the subtracted correlator has milder τ-dependence, which is approximately linear

Model spectral function and the complex potential

$$\rho_r^{peak}(\omega,T)=\frac{A}{\pi}\frac{\Gamma(\omega,r,T)}{(\omega-\mathrm{Re}V(r,T))^2+\Gamma^2(\omega,r,T)}\qquad \Gamma(\omega,r,T)=\begin{cases}\Gamma_0(r,T) & -2\Gamma_0<\omega<2\Gamma_0\\ 0 & n \text{ otherwise}\end{cases}$$

$$\rho_r^{tail}(\omega,T)=A^{tail}\delta(\omega-E^{tail})$$





$\mathrm{Re}V(r,T)$ shows tiny temperature dependence and no hint of screening

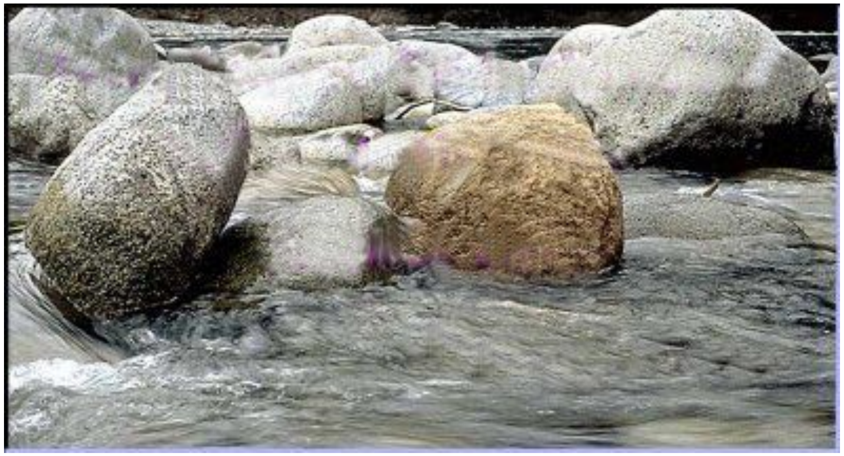
$\mathrm{Im}V(r,T)$ increases with rT and is proportional to T

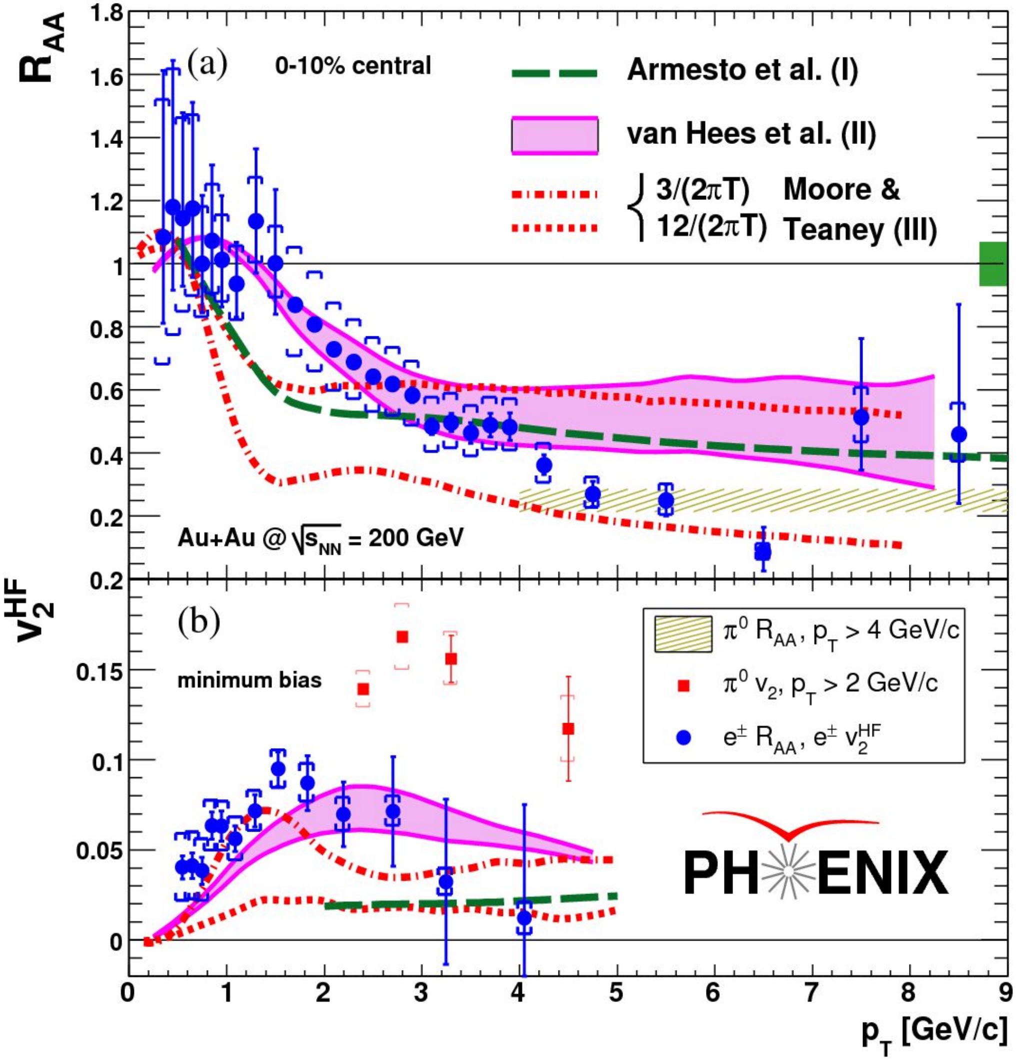
HotQCD, PRD 109 (2024) 074504

Flow of heavy quarks in quark gluon plasma

Heavy quarks ($M_c \sim 1.5$ *GeV*) flow in the strongly coupled QGP

Analogy from Jamie Nagle





$$t_{rel}^{heavy} \sim \frac{M_c}{T} t_{rel}^{light} \Rightarrow$$ Langevin dynamics:

$$\frac{dx^i}{dt} = \frac{p^i}{M}, \frac{dp^i}{dt} = \xi^i(t) - \eta p^i,$$

$$\langle \xi^i(t) \xi^j(t') \rangle = \kappa \delta^{ij} \delta(t - t')$$

$$\eta = \frac{\kappa}{2MT}, \quad D = \frac{T}{M\eta}$$

Current-current correlators and heavy quark diffusion coefficient

$$\rho_V^{\mu\nu}(\omega) \equiv \int_{-\infty}^{\infty} dt e^{i\omega t} \int d^3x \left\langle [\hat{J}^\mu(t,\vec{x}), \hat{J}^\nu(0,\vec{0})] \right\rangle$$

$$\partial_t p_i = -\eta p_i + f_i(t),$$

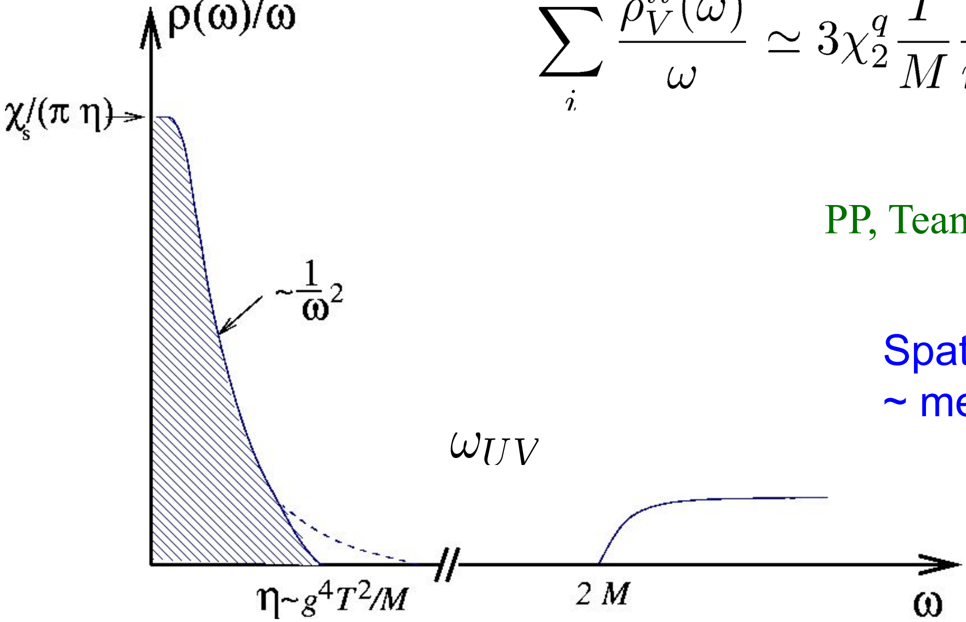
$$\langle f_i(t) f_j(t') \rangle = \kappa \delta_{i,j} \delta(t-t')$$

$$\kappa = 2MT\eta = 2T^2/D_s$$

Momentum diffusion coefficient

$$\sum_i \frac{\rho_V^{ii}(\omega)}{\omega} \simeq 3\chi_2^q \frac{T}{M} \frac{\eta}{\eta^2+\omega^2}, \quad \omega < \omega_{UV}, \quad \eta = \frac{T}{M}\frac{1}{D_s}$$

drag constant



PP, Teaney, PRD 72 (2006) 014508

Spatial diffusion constant
~ mean free path (weak coupling)

$$D_s \sim \frac{1}{g^4 T}$$

area under the peak ~ $\chi_2^q \frac{T}{M}$

heavy quark coefficient ~ width of the peak

For large quark mass the transport peak is very narrow even for strong coupling and its difficult to reconstruct it accurately from Euclidean correlator calculated on the lattice

Heavy quark diffusion and lattice QCD

Obtain the momentum heavy quark transport coefficient through the force correlator

$$\langle f_i(t) f_j(t)\rangle = \langle E_i(t) E_j(t')\rangle + \frac{1}{3}\langle \mathbf{v}^2\rangle \langle \delta_{ij} B_k(t) B_k(t') - B_i(t') B_j(t)\rangle \qquad \langle \mathbf{v}^2\rangle = \frac{3T}{M}$$

$$t \to i\tau$$

Can be rigorously derived in Heavy Quark Effective Theory

Casalderrey-Solana, Teaney, PRD 74 (2006) 085012; Caron-Huot, Laine, Moore, JHEP 0904 ('09) 053

Bouttefeux, Laine, JHEP 12 (2020) 150

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{ReTr}\left[U(\beta,\tau)\, gE_i(\tau,\vec{0})\, U(\tau,0)\, gE_i(0,\vec{0})\right]\right\rangle}{\left\langle \mathrm{ReTr}[U(\beta,0)]\right\rangle} \qquad \kappa_E = \lim_{\omega\to 0} \frac{2T}{\omega}\rho_E(\omega)$$

$$G_B(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{ReTr}\left[U(\beta,\tau)\, gB_i(\tau,\vec{0})\, U(\tau,0)\, gB_i(0,\vec{0})\right]\right\rangle}{\left\langle \mathrm{ReTr}[U(\beta,0)]\right\rangle} \qquad \kappa_B = \lim_{\omega\to 0} \frac{2T}{\omega}\rho_B(\omega)$$

$$G_{E,B}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho_{E,B}(\omega)\frac{\cosh\left(\tau - \frac{1}{2T}\right)\omega}{\sinh\frac{\omega}{2T}}$$

$$\kappa = \kappa_E + \frac{2}{3}\langle \mathbf{v}^2\rangle \kappa_B$$

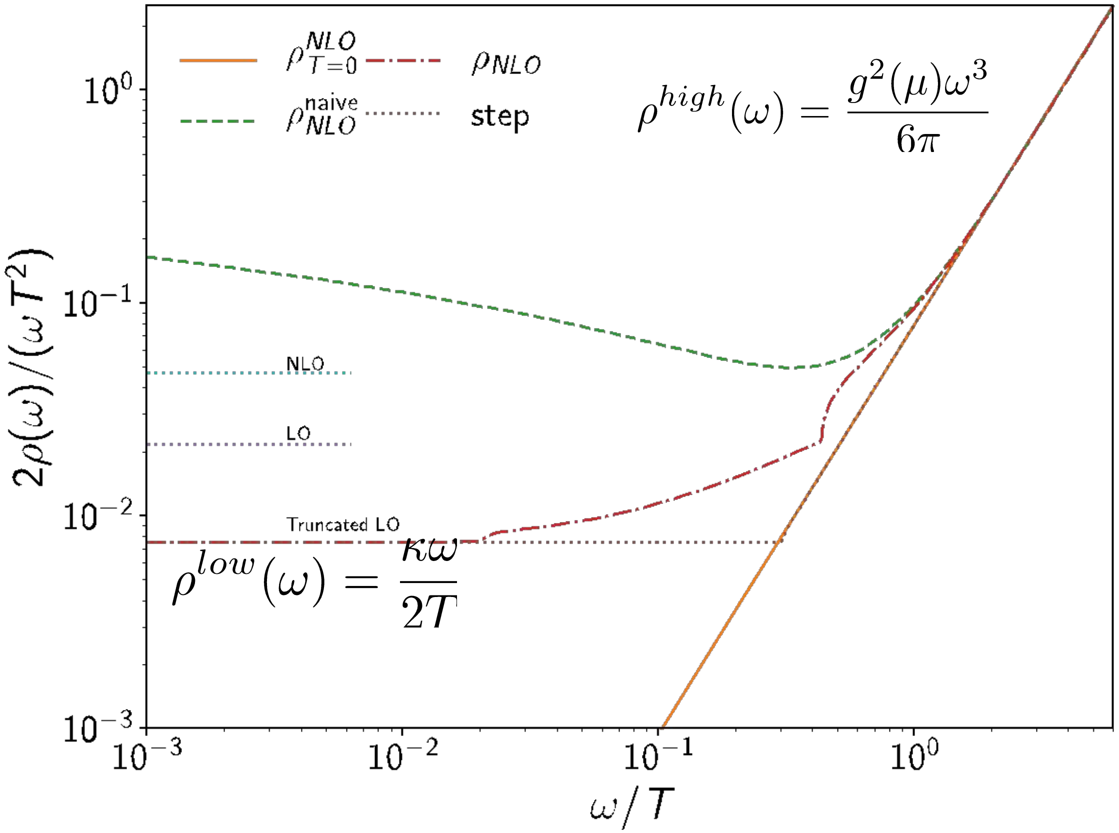
Extracting momentum diffusion coefficient from the lattice

Challenge 1: obtain precise results for chromo-electric and chromo-magnetic (very noisy)

⇒ Noise reduction by gradient flow method

$$A_\mu(x) \to B_\mu(\tau_F, x) \qquad \partial_{\tau_F} B_\mu(\tau_F, x) = -g_0^2 \frac{\delta S_{\rm YM}[B]}{\delta B_\mu(\tau_F, x)}$$
$$B_\mu(0, x) = A_\mu(x)$$

Challenge 2: reconstruct the spectral function from the Euclidean time lattice correlator

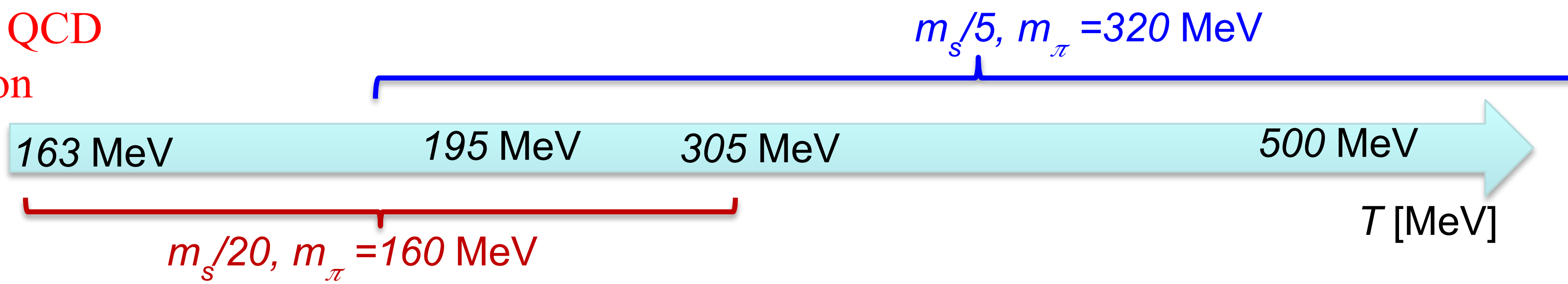


⇒ use known large and small energy behavior of the spectral

Parameterize $\rho(\omega, T)$ as smooth interpolation between $\rho^{low}(\omega, T)$ and $\rho^{high}(\omega)$, and treat κ as well as the additional nuisance parameters of interpolation as fit parameters

Chromo electric and magnetic correlators on the lattice

2+1 flavor QCD
HISQ action

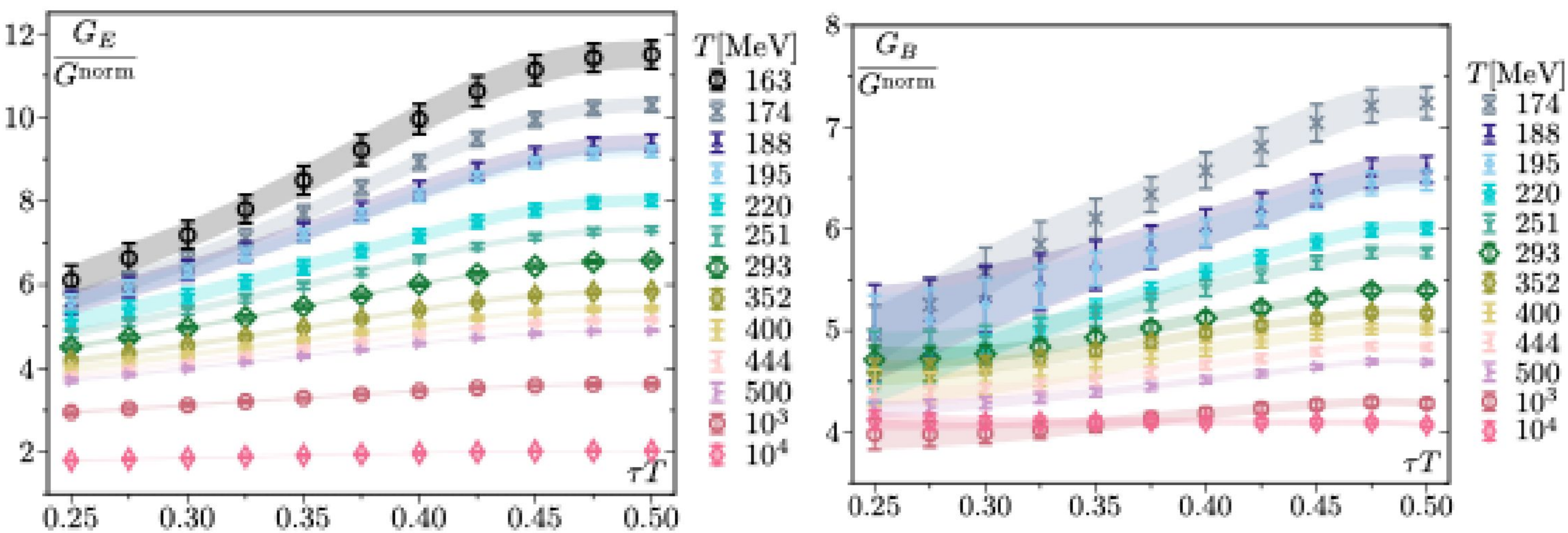


Improved gradient (Zeuthen) flow

HotQCD, JHEP 09 (2025) 180

Gauge fields are smeared in the radius $\sqrt{8\tau_F}$ $\quad a < \sqrt{8\tau_F} < \tau/3$

[redacted]



[redacted]

Analysis and modeling the chromo-electric correlator

Analysis of the chromo-electric correlator:

- Extrapolate the lattice results on the chromo-electric correlator to the continuum limit
- Perform the zero flow time extrapolation

HotQCD, PRL 130 (2023) 231902

Fits to model spectral function:

$$\rho^{low}(\omega,T)=\frac{\kappa\omega}{2T} \qquad \rho^{high}(\omega)=\rho^{LO,NLO}(\omega)$$

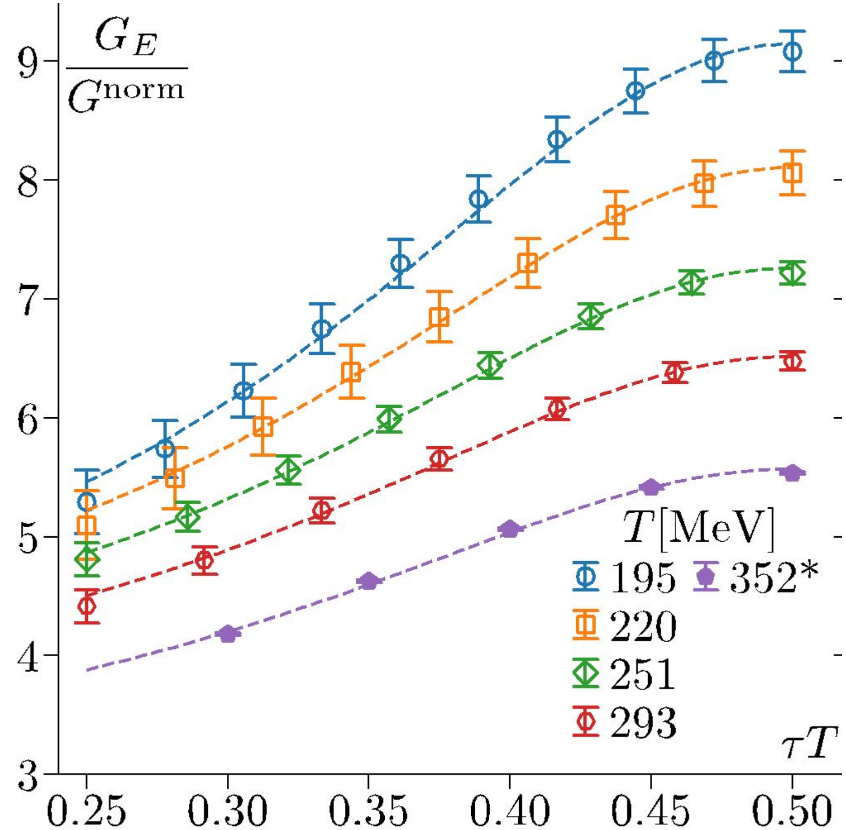
$$\rho^{max}(\omega,T)=max(\rho^{low}(\omega,T),\rho^{high}(\omega))$$

$$\rho^{smax}(\omega,T)=\sqrt{(\rho^{low})^2+(\rho^{high})^2}$$

$$\rho^{pow}(\omega,T)=\rho^{low}(\omega,T),\ \omega\le\omega_{IR}$$

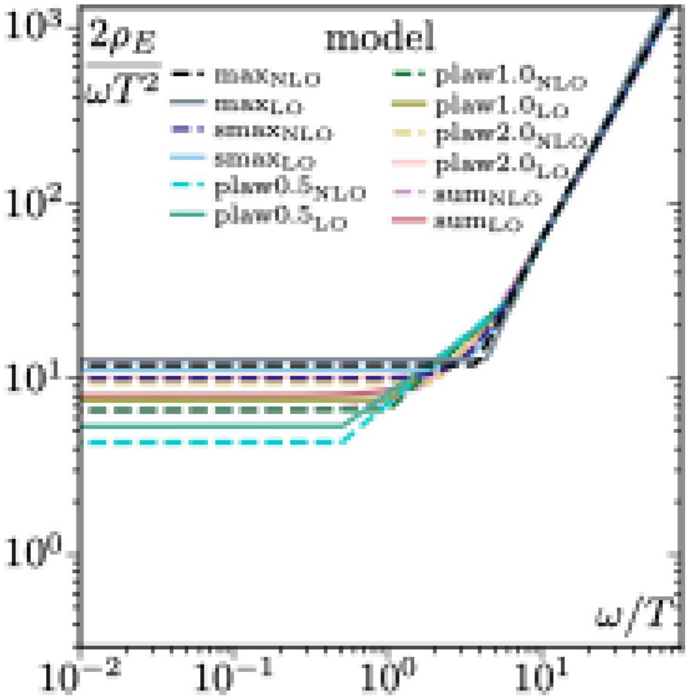
$$\rho^{pow}(\omega,T)=A\omega^{\alpha},\ \omega_{IR}<\omega<\omega_{UV}$$

$$\rho^{pow}(\omega)=\rho^{high}(\omega),\ \omega\ge\omega_{UV}$$



Analysis and modeling the chromo-electric correlator

Fits to model spectral function: $\rho^{low}(\omega,T)=\frac{\kappa\omega}{2T}$ $\rho^{high}(\omega)=\rho^{LO,NLO}(\omega)$



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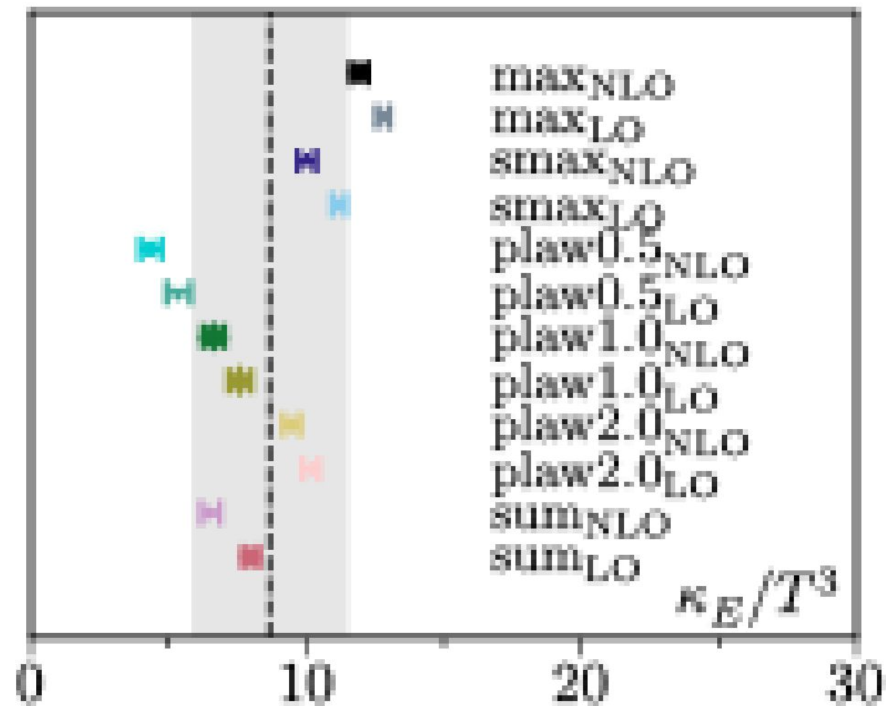


$$\rho^{max}(\omega,T)=max(\rho^{low}(\omega,T),\rho^{high}(\omega))$$

$$\rho^{smax}(\omega,T)=\sqrt{(\rho^{low})^2+(\rho^{high})^2}$$

$$\rho^{pow}(\omega,T)=\rho^{low}(\omega,T),\ \omega\le\omega_{IR}$$

$$\rho^{pow}(\omega,T)=A\omega^{\alpha},\ \omega_{IR}<\omega<\omega_{UV}$$

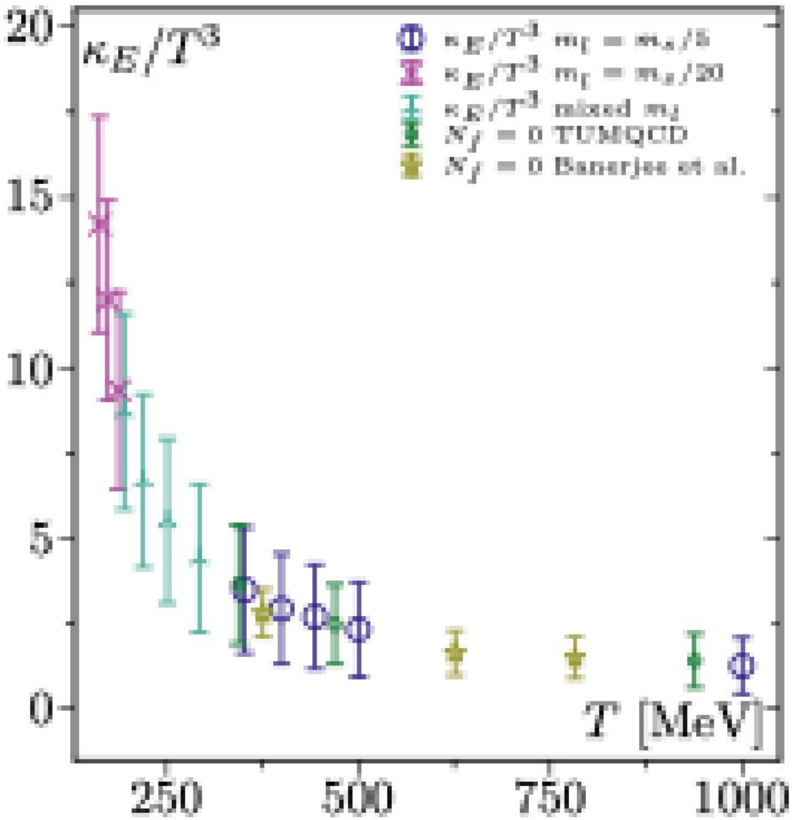
$$\rho^{pow}(\omega)=\rho^{high}(\omega),\ \omega\ge\omega_{UV}$$

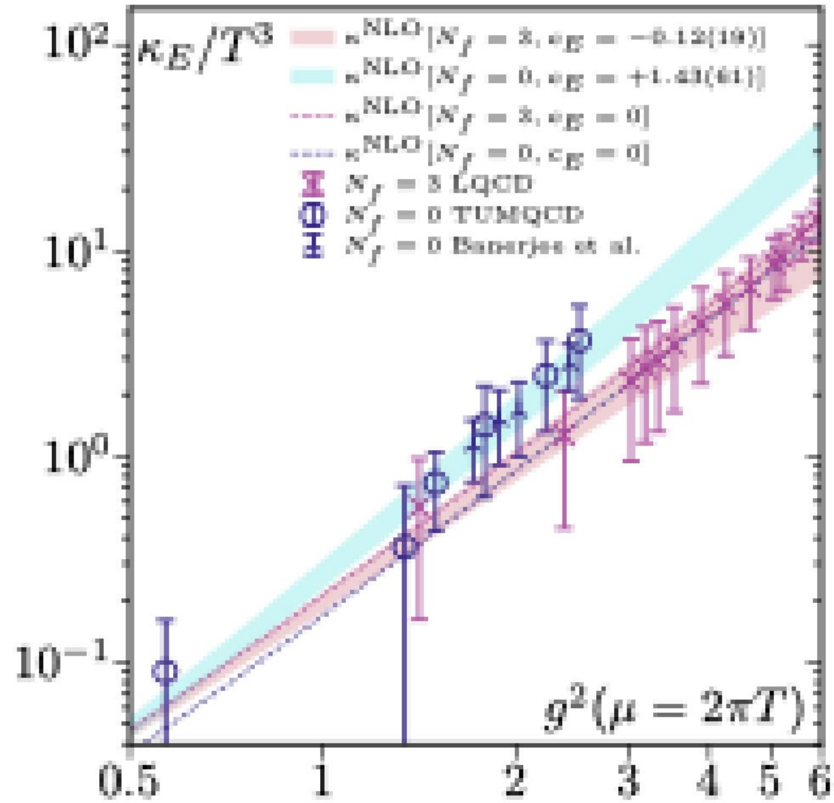
[redacted]

Model averaging

$$\kappa_E/T^3=8.7^{+2.9}_{-2.8}$$

Temperature dependence of the chromo-electric momentum diffusion coefficient



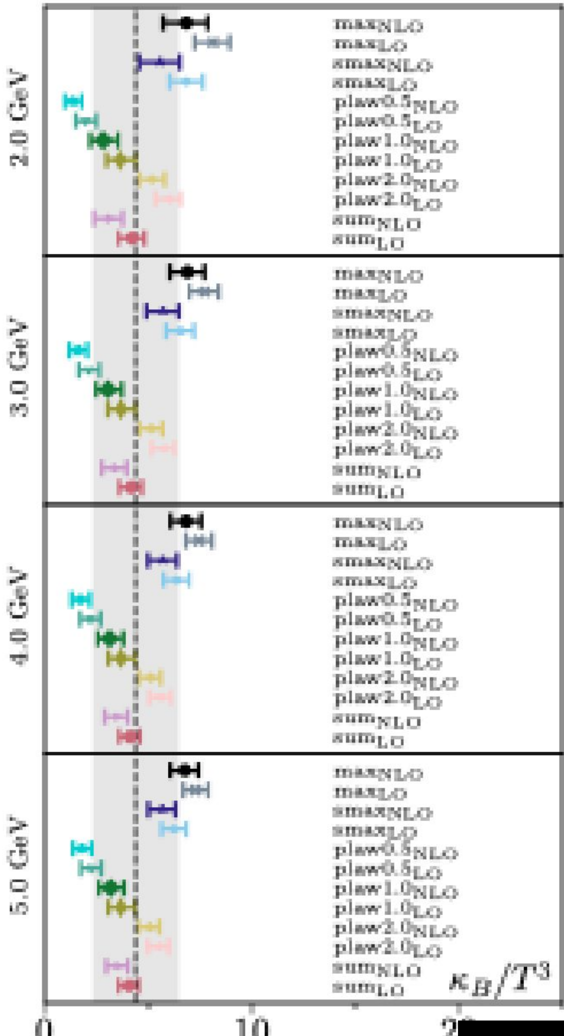


The temperature dependence chromo-electric diffusion coefficient follows the weak coupling expectation

$$\kappa_E(T) = \frac{g^4 C_F T^3}{18\pi}\left(\left[N_c + \frac{N_f}{2}\right]\left[\ln\frac{2T}{m_D} + \xi\right] + \frac{N_f \ln 2}{2} + 2.3302 N_c m_D + c_E g^2\right)$$

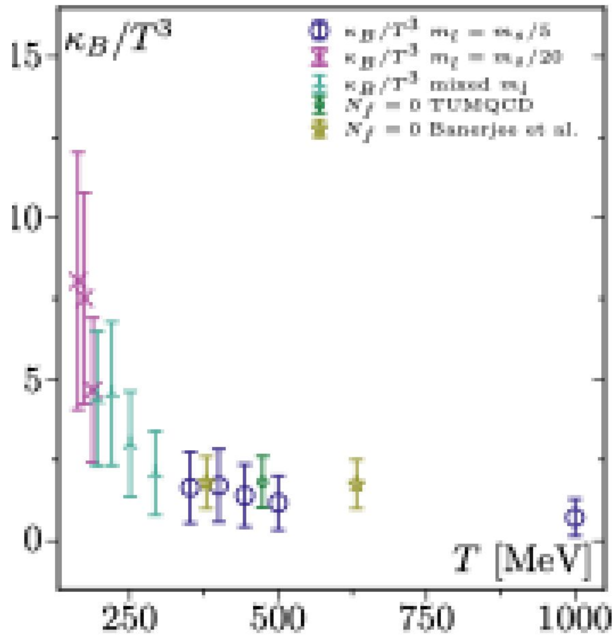
Temperature dependence of the chromo-magnetic momentum diffusion coefficient

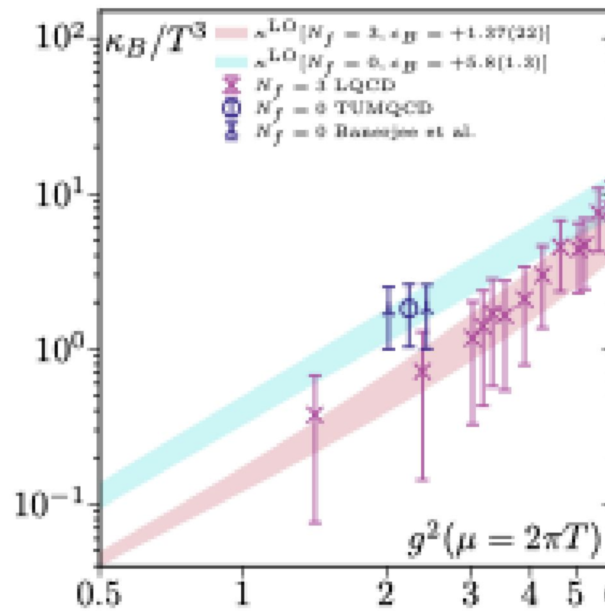
[redacted]



Model averaging

$$\kappa_B/T^3 = 4.7^{+2.1}_{-2.1}$$



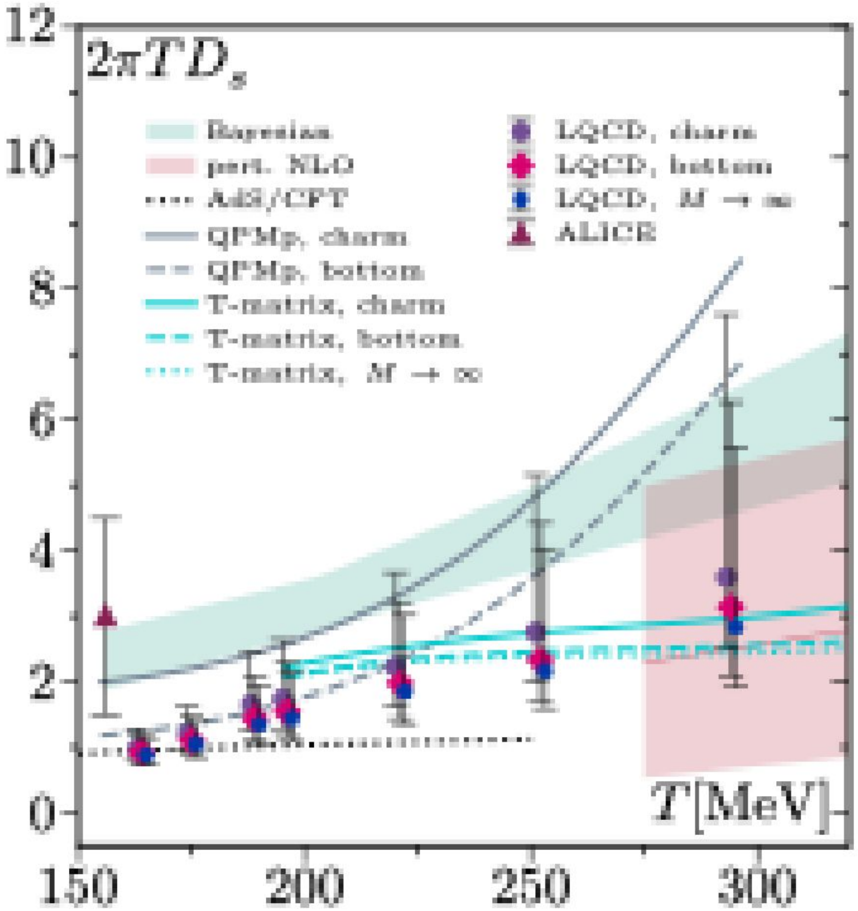


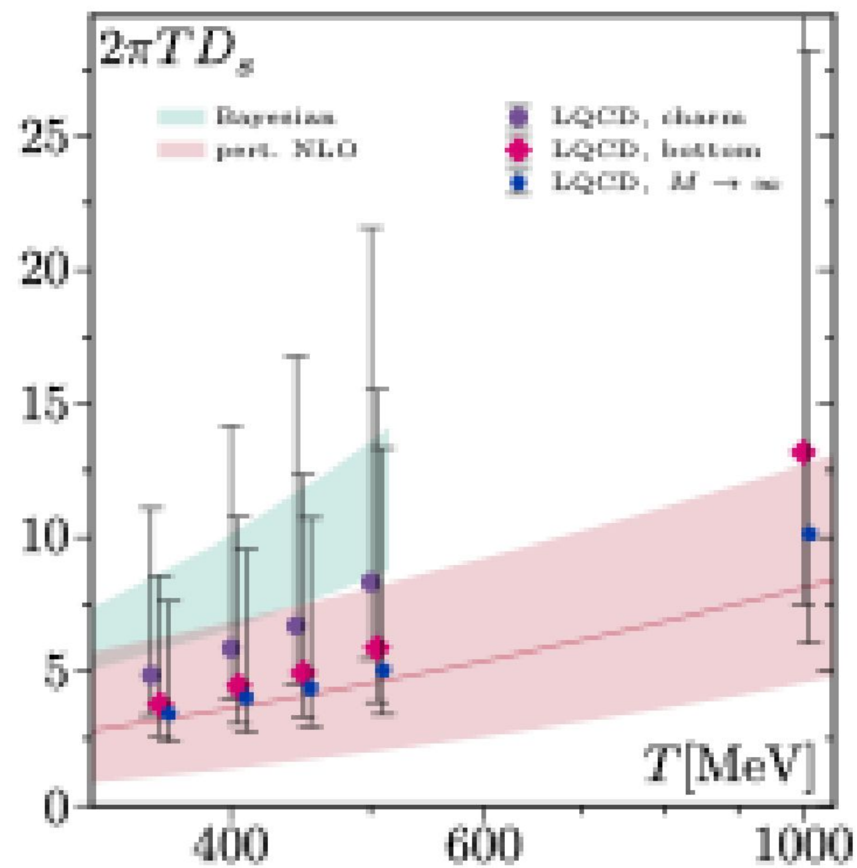
$$\kappa_B(T) = \frac{g^4 C_F T^3}{18\pi}\left(\left[N_c + \frac{N_f}{2}\right]\ln\frac{1}{g^2} + c_B\right)$$

Heavy Quark Diffusion and Lattice QCD

HotQCD, JHEP 09 (2025) 180

$$D_s = \frac{2T^2}{\kappa_E + \frac{2}{3}\langle \mathbf{v}^2 \rangle \kappa_B} \frac{\langle \mathbf{p}^2 \rangle}{3MT}$$





small close to T_c as expected in strongly coupled QGP

agrees with weak coupling results at high T

Back-up:

HotQCD, PRL 130 (2023) 231902