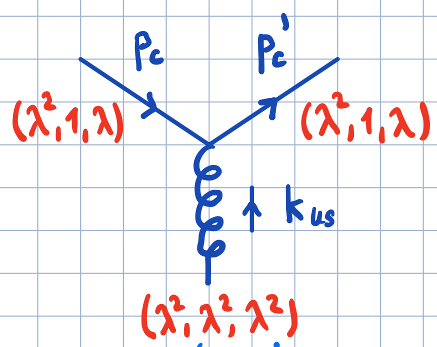
Coupling to ultra-soft gluons:

Besides the collinear Lagrangian, the leading-order SCET Lagrangian contains interactions of collinear fields with the component $n \cdot A_{us}$ of the ultra-soft gluon field:

$$\mathcal{L}_{c+us} = \bar{\xi}_n \frac{\bar{n}\!\!\!/}{2} g_s \, n \cdot A_{us} \, \xi_n \quad + \text{(pure glue terms)}$$

Let us look at the structure of these interactions in more detail:



power suppressed compared with collinear momenta

In the method of regions it is important that we expand the Feynman integrands consistently to leading order in λ. This means that we should expand:

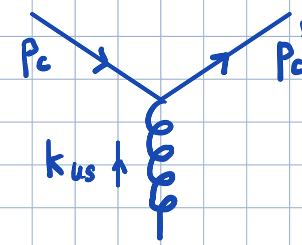
$$p_c'^{\mu} = p_c^{\mu} + k_{us}^{\mu} = (\overset{\lambda^2}{n \cdot p_c} + \overset{\lambda^2}{n \cdot k_{us}}) \frac{\bar{n}^{\mu}}{2} + (\overset{1}{\bar{n} \cdot p_c} + \overset{\lambda^2}{\bar{n} \cdot k_{us}}) \frac{n^{\mu}}{2} + \overset{\lambda}{p_{c\perp}^{\mu}} + \overset{\lambda^2}{k_{us\perp}^{\mu}}$$

$$= (n \cdot p_c + n \cdot k_{us}) \frac{\bar{n}^{\mu}}{2} + \bar{n} \cdot p_c \frac{n^{\mu}}{2} + p_{c\perp}^{\mu} + \text{higher orders}$$

$$\longrightarrow p_c^{\mu} + k_{us+}^{\mu} \; ; \quad k_+^{\mu} \equiv n \cdot k \, \frac{\bar{n}^{\mu}}{2}$$

must expand away!

This implies that 4-momentum is not conserved at the vertex:



$$p_c' = p_c + k_{us+}$$

To implement this rule at the Lagrangian level, we must perform a <u>multipole expansion</u> of the ultra-soft fields whenever they interact with collinear fields:

$$x^\mu = \underbrace{n\cdot x \,\frac{\bar{n}^\mu}{2}} + \underbrace{\bar{n}\cdot x\,\frac{n^\mu}{2}} + x_\perp^\mu$$

$$\equiv \underset{1}{x_+^\mu} + \underset{\lambda^{-2}}{x_-^\mu} + \underset{\lambda^{-1}}{x_\perp^\mu}$$

(since $x\cdot p_c \sim 1$) ← in interactions with collinear fields

$$\Rightarrow \quad \phi_{us}(x) = \phi_{us}(x_-) + \overset{\lambda^{-1}}{x_\perp}\cdot \overset{\lambda^{2}}{\partial_\perp}\,\phi_{us}(x_-) + \left(\underset{1}{x_+}\cdot\underset{\lambda^2}{\partial_-} + \underset{\lambda^{-2}}{\frac{x_\perp^\mu x_\perp^\nu}{2}}\,\underset{\lambda^2}{\partial_\mu^\perp}\,\underset{\lambda^2}{\partial_\nu^\perp}\right)\phi_{us}(x_-) + \dots$$

↳ generates series of higher-order terms in λ

At leading order in λ, the correct form of the effective Lagrangian thus contains:

$$\boxed{\mathcal{L}_{c+us}(x) = \bar{\xi}_n(x)\,\frac{\bar{n}}{2}\,g_s\, n\cdot A_{us}(\overset{!}{x_-})\,\xi_n(x) + (\text{pure glue terms})}$$

For the vertex shown above, the action $\int d^4x\, \mathcal{L}_{c+us}(x)$ generates:

$$\int d^4x\; e^{i(p_c'\cdot x - p_c\cdot x - \overbrace{k_{us}\cdot x_-}^{\frac{1}{2}\bar{n}\cdot x\, n\cdot k_{us} = x\cdot\left(\frac{\bar{n}}{2}\, n\cdot k_{us}\right) = x\cdot k_{us+}})}$$

$$= \int d^4x\; e^{i(p_c'\cdot x - p_c\cdot x - k_{us+}\cdot x)} = (2\pi)^4\, \delta^{(4)}(p_c' - p_c - k_{us+}) \quad ✓$$

Leading-order SCET Lagrangian:

Collecting our results, and adding back the anti-collinear sector, we obtain:

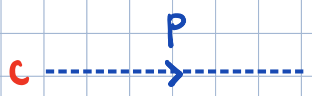
$$
\begin{aligned}
\mathcal{L}_{SCET} = {} & \left.\begin{aligned} & \bar{\xi}_n \frac{\not{\bar{n}}}{2}\, i n\cdot D_c\, \xi_n(x) \\ & + (\bar{\xi}_n\, i\not{D}_c^{\perp} W_c)(x)\, \frac{\not{\bar{n}}}{2}\, i\int_{-\infty}^{0} dt\, (W_c^{\dagger}\, i\not{D}_c^{\perp}\, \xi_n)(x+t\bar{n}) \end{aligned}\right\} \mathcal{L}_c \\
& + \left[\text{same with } n\leftrightarrow\bar{n},\; c\leftrightarrow\bar{c}\right] && \mathcal{L}_{\bar{c}} \\
& + \bar{q}_{us}\, i\not{D}_{us}\, q_{us}(x) && \mathcal{L}_{us} \\
& + \bar{\xi}_n(x)\, \frac{\not{\bar{n}}}{2}\, g_s\, n\cdot A_{us}(x_-)\, \xi_n(x) && \mathcal{L}_{c+us} \\
& + \bar{\xi}_{\bar{n}}(x)\, \frac{\not{n}}{2}\, g_s\, \bar{n}\cdot A_{us}(x_+)\, \xi_{\bar{n}}(x) && \mathcal{L}_{\bar{c}+us} \\
& + (\text{pure glue terms}) && \text{same structure as above}
\end{aligned}
$$

Note the important fact that ultra-soft quarks do not interact with collinear fields at leading order!

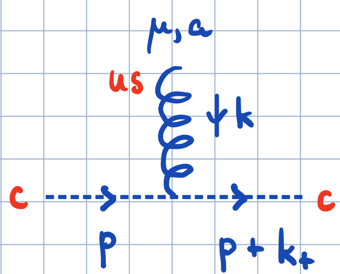
In principle, it is possible to go to higher orders in the expansion in λ ($\rightarrow$ a topic of intensive current research), but we will focus on the leading terms in this course.

Feynman rules:

$$\frac{\not n \not{\bar n}}{4}\not p\frac{\not{\bar n}\not n}{4} = \frac{\not n}{2}\,\bar n\cdot p$$

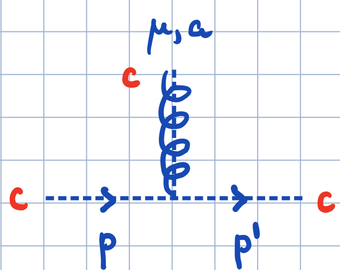


$$= \frac{\not n}{2}\,\frac{i\,\bar n\cdot p}{n\cdot p\;\bar n\cdot p + p_\perp^2 + i0} = P_n\,\frac{i\not p}{p^2+i0}\,\bar P_n$$

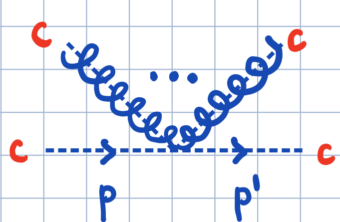


$$= i g_s t^a\, n^\mu\, \frac{\not{\bar n}}{2}$$

(momentum not conserved)



$$= i g_s t^a \left(\underset{\lambda^0}{n^\mu} + \underset{\lambda^1}{\frac{\gamma_\perp^\mu \not p_\perp}{\bar n\cdot p}} + \underset{\lambda^1}{\frac{\not p_\perp' \gamma_\perp^\mu}{\bar n\cdot p'}} - \underset{\lambda^2}{\bar n^\mu \frac{\not p_\perp' \not p_\perp}{\bar n\cdot p'\;\bar n\cdot p}} \right) \frac{\not{\bar n}}{2}$$



$=$ complicated

plus pure glue and ghost vertices

Due to the presence of the Wilson lines, there are vertices involving any number of collinear gluons!