Hard matching corrections:

The 2-jet current carries a hard momentum transfer $q^2 = -Q^2$. Unlike for the SCET Lagrangian, we thus expect that there should be a non-trivial Wilson coefficient $C_V(Q^2)$ accounting for the effects of hard gluons, which have been integrated out, i.e.:

$$\boxed{\bar{\psi}\gamma^\mu\psi(0) \longrightarrow \underbrace{C_V(Q^2)}_{\text{hard quantum fluctuations}} \, (\bar{\xi}_{\bar{n}} W_{\bar{c}})(0)\, \gamma_\perp^\mu \, (W_c^\dagger \xi_n)(0)}$$

This is indeed the correct relation, but the question arises how such a dependence of the Wilson coefficient on $q^2 = (p_c - p_{\bar{c}})^2$ can arise, since q^2 depends on the momenta of light particles in the low-energy EFT.

This question is connected with another worry we had when constructing SCET: the presence of fields $\bar{n}\cdot A_c \sim \lambda^0$, $n\cdot A_{\bar{c}} \sim \lambda^0$ with unsuppressed power counting. In fact, the infinite set

$$\left[\bar{\xi}_{\bar{n}}\, (i n\cdot D_{\bar{c}})^{\dagger m_1} W_{\bar{c}}\right](0)\; \gamma_\perp^\mu \left[W_c^\dagger\, (i\bar{n}\cdot D_c)^{m_2}\, \xi_n\right](0)$$

of gauge-invariant operators, with $m_1, m_2 \in \mathbb{N}_0$, can equally well arise in the matching condition for the vector current at leading power in λ. Using the relations

$$W_c^\dagger \, i\bar{n}\cdot D_c \, W_c = i\bar{n}\cdot\partial$$

$$W_{\bar{c}}^\dagger \, (in\cdot D_{\bar{c}})^\dagger \, W_{\bar{c}} = (i\,n\cdot\partial)^\dagger$$

(see p. 8, lecture 5)

these operators can be rewritten as:

$$[\bar{\xi}_{\bar{n}} W_{\bar{c}}](0)\,(-in\cdot\overleftarrow{\partial})^{m_1}\,\gamma_\perp^\mu\,(i\bar{n}\cdot\partial)^{m_2}\,[W_c^\dagger \xi_n](0)$$

$$\equiv \mathcal{O}_{m_1 m_2}(0)$$

The most general leading-order matching relation is thus of the form:

$$\bar{\psi}\gamma^\mu\psi(0) \to \sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty} c_{m_1 m_2}\,\mathcal{O}_{m_1 m_2}(0)$$

An equivalent way of writing this result uses the <u>non-local</u> expression:

$$\bar{\psi}\gamma^\mu\psi(0)$$

$$\to \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \; \tilde{C}_V(t_1,t_2)\,[\bar{\xi}_{\bar{n}} W_{\bar{c}}](t_1 n)\,\gamma_\perp^\mu\,[W_c^\dagger \xi_n](t_2\bar{n})$$

Using the Taylor series

$$f(t\bar{n}) = e^{t\bar{n}\cdot\partial_x} f(x)\Big|_{x=0} = \sum_{m=0}^{\infty} \frac{t^m}{m!} (\bar{n}\cdot\partial)^m f(0)$$

we find:

$$c_{m_1 m_2} = \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \; \tilde{C}_V(t_1,t_2) \, \frac{(-it_1)^{m_1}}{m_1!} \, \frac{(it_2)^{m_2}}{m_2!}$$

We can now use the fact that the large component of the total (anti-) collinear momentum of each jet (or in each sector) is fixed by kinematics. Let us call these momenta $\bar{n}\cdot\mathcal{P}_c$ and $n\cdot\mathcal{P}_{\bar{c}}$. We can then use translational invariance to write:

$$\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \; \tilde{C}_V(t_1,t_2) \, [\bar{\xi}_{\bar{n}} W_{\bar{c}}](t_1 n) \, \gamma_\perp^\mu \, [W_c \xi_n](t_2 \bar{n})$$

$$= \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \; \tilde{C}_V(t_1,t_2) \; e^{it_1 n\cdot\mathcal{P}_{\bar{c}}} \; e^{-it_2 \bar{n}\cdot\mathcal{P}_c}$$

$$\times (\bar{\xi}_{\bar{n}} W_{\bar{c}})(0) \, \gamma_\perp^\mu \, (W_c^\dagger \xi_n)(0)$$

$$\equiv C_V(n\cdot\mathcal{P}_{\bar{c}}, \bar{n}\cdot\mathcal{P}_c) \, (\bar{\xi}_{\bar{n}} W_{\bar{c}})(0) \, \gamma_\perp^\mu \, (W_c^\dagger \xi_n)(0)$$

Type - III reparameterization invariance requires that the coefficient C_V can only depend on the product of its two arguments:

$$n\cdot\mathcal{P}_{\bar{c}}\;\bar{n}\cdot\mathcal{P}_c \;\underset{\text{as eigenvalue}}{\simeq}\; -q^2 = Q^2$$

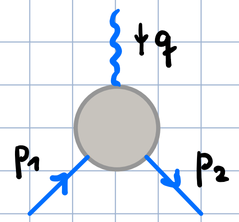
Hence, we have derived the matching condition shown on p. 1.

Note:

In the literature the objects $\bar{n}\cdot\mathcal{P}_c$ and $n\cdot\mathcal{P}_{\bar{c}}$ are often called "label operators". These operators project out the large components of the sum of all (anti-) collinear particles in a given process.

VI. The Sudakov Form Factor in SCET

We now return to the case of the off-shell Sudakov form factor in QCD,



$|q^2| \gg |p_i^2|$
$(m_i = 0)$

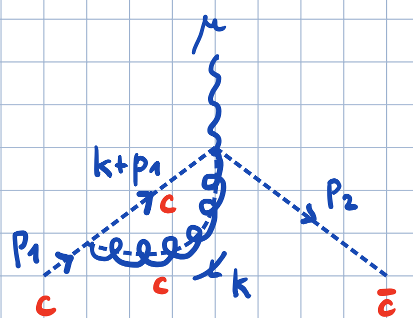
this time performing the calculation in SCET (and not ignoring the numerator terms). We thus evaluate the quark matrix element

$$C_V(Q^2) \; \langle q(p_2) | \, (\bar{\xi}_{\bar{n}} W_{\bar{c}})(0) \, \gamma_\perp^\mu \, (W_c^\dagger \xi_n)(0) \, | q(p_1) \rangle$$

at one-loop order, where $C_V(Q^2) = 1 + \mathcal{O}(\alpha_s)$ is the hard matching coefficient.

Collinear contribution:

The SCET Feynman rules allow for a single one-loop diagram:



(we work in Feynman gauge $\xi = 1$)

We find:

$$D_c = \int \frac{d^D k}{(2\pi)^D}\, \bar{u}_{\bar{n}}(p_2)\, \gamma_\perp^\mu\, \frac{\not{n}}{2}\, \frac{i\,\bar{n}\cdot(k+p_1)}{(k+p_1)^2+i0}$$

$$\times\, i g_s t_a \left(n^\alpha + \frac{\gamma_\perp^\alpha \not{p}_{1\perp}}{\bar{n}\cdot p_1} + \frac{(\not{k}_\perp+\not{p}_{1\perp})\gamma_\perp^\alpha}{\bar{n}\cdot(k+p_1)} - \bar{n}^\alpha \frac{(\not{k}_\perp+\not{p}_{1\perp})\not{p}_{1\perp}}{\bar{n}\cdot(k+p_1)\ \bar{n}\cdot p_1} \right) \frac{\not{\bar{n}}}{2}\, u_n(p_1)$$

$$\times\, \underbrace{(-g_s t_b)\,\frac{\bar{n}^\beta}{\bar{n}\cdot k}}_{\langle k|\, W_c^\dagger\, |0\rangle}\ \frac{(-i g_{\alpha\beta})}{k^2+i0}\, \delta_{ab}$$

(cf. p. 3, Lecture 6, part 3)

$$= -i\, C_F\, g_s^2\, \bar{u}_{\bar{n}}(p_2)\, \overbrace{\gamma_\perp^\mu \frac{\not{n}\not{\bar{n}}}{4}\, u_n(p_1)}^{=\gamma_\perp^\mu}\ \overbrace{n\cdot\bar{n}}^{2}$$

$$\times \int \frac{d^D k}{(2\pi)^D}\, \frac{\bar{n}\cdot(k+p_1)}{(k^2+i0)\,[(k+p_1)^2+i0]\ \bar{n}\cdot k}$$

If we multiply numerator and denominator by $n\cdot p_2$ we get:

$$\frac{\bar{n}\cdot(k+p_1)}{\bar{n}\cdot k} \;\rightarrow\; \frac{\bar{n}\cdot(k+p_1)\ n\cdot p_2}{\bar{n}\cdot k\ n\cdot p_2} = \frac{2k\cdot p_{2+} + 2p_{1-}\cdot p_{2+}}{2k\cdot p_{2+}}$$

Apart from the factor $2k\cdot p_{2+}$ in the numerator (which we had previously ignored), the loop integral coincides with the integral I_c for the collinear region on page 1 of Lecture 4. Evaluating the integral leads to:

$$D_c = \bar{u}_{\bar{n}}(p_2)\,\gamma_\perp^\mu\, u_n(p_1)\,\frac{C_F\alpha_s}{4\pi}$$
$$\times\left[\frac{2}{\epsilon^2}+\frac{2}{\epsilon}\ln\frac{\mu^2}{p_1^2}+\frac{2}{\epsilon}+\ln^2\frac{\mu^2}{p_1^2}+2\ln\frac{\mu^2}{p_1^2}+4-\frac{\pi^2}{6}\right]$$

We work in the $\overline{MS}$ scheme, where $\mu^{2\epsilon}\longrightarrow\mu^{2\epsilon}(4\pi)^{-\epsilon}e^{\epsilon\gamma_E}$.

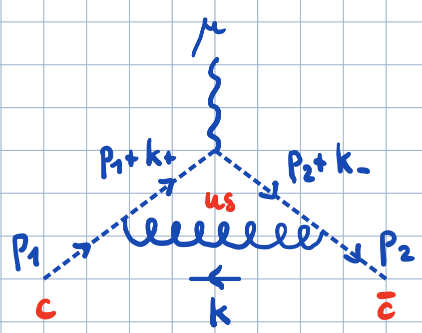
Anti-collinear contribution:

An analogous calculation leads to:

$$D_{\bar{c}} = \bar{u}_{\bar{n}}(p_2)\,\gamma_\perp^\mu\, u_n(p_1)\,\frac{C_F\alpha_s}{4\pi}$$
$$\times\left[\frac{2}{\epsilon^2}+\frac{2}{\epsilon}\ln\frac{\mu^2}{p_2^2}+\frac{2}{\epsilon}+\ln^2\frac{\mu^2}{p_2^2}+2\ln\frac{\mu^2}{p_2^2}+4-\frac{\pi^2}{6}\right]$$

Ultra-soft contribution:

The SCET Feynman rules allow for a single one-loop diagram:



(we work in Feynman gauge $\xi = 1$)

Recall that 4-momentum is not conserved at these vertices.

We find:

$$D_{us} = \int \frac{d^Dk}{(2\pi)^D} \, \bar{u}_{\bar{n}}(p_2) \, i g_s t_a \, \bar{n}^\alpha \frac{\not{n}}{2} \frac{\not{\bar{n}}}{2} \frac{i\, \overbrace{n\cdot(k_- + p_2)}^{=0}}{(k_- + p_2)^2 + i0} \gamma_\perp^\mu$$

$$\times \frac{\not{n}}{2} \frac{i\, \overbrace{\bar{n}\cdot(k_+ + p_1)}^{=0}}{(k_+ + p_1)^2 + i0} \, i g_s t_b \, n^\beta \frac{\not{\bar{n}}}{2} u_n(p_1) \frac{(-i g_{\alpha\beta})}{k^2 + i0} \delta_{ab}$$

$$= -i C_F g_s^2 \, \bar{u}_{\bar{n}}(p_2) \overbrace{\frac{\not{n}\not{\bar{n}}}{4} \gamma_\perp^\mu \frac{\not{n}\not{\bar{n}}}{4}}^{\gamma_\perp^\mu} u_n(p_1) \, \overbrace{n\cdot\bar{n}}^{2}$$

$$\times \int \frac{d^Dk}{(2\pi)^D} \frac{\bar{n}\cdot p_1 \; n\cdot p_2}{(k^2 + i0)(2k_+\cdot p_1 + p_1^2 + i0)(2k_-\cdot p_2 + p_2^2 + i0)}$$

$$\uparrow$$

$$\frac{2 p_{1-}\cdot p_{2+}}{(k^2 + i0)(2k\cdot p_{1-} + p_1^2 + i0)(2k\cdot p_{2+} + p_2^2 + i0)}$$

This expression coincides with the integral I_{us} for the ultra-soft region on page 4 of Lecture 4. Evaluating the integral one finds:

$$D_{us} = \bar{u}_{\bar{n}}(p_2) \gamma_\perp^\mu u_n(p_1) \frac{C_F \alpha_s}{4\pi}$$

$$\times \left[-\frac{2}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{\mu^2 Q^2}{p_1^2 p_2^2} - \ln^2 \frac{\mu^2 Q^2}{p_1^2 p_2^2} - \frac{\pi^2}{2} \right]$$

Wave-function renormalization:

$$Z_q^{1/2}\, Z_{\bar q}^{1/2}\, \bar u_{\bar n}(p_2)\, \gamma_\perp^\mu\, u_n(p_1)$$

same as in QCD

In Feynman gauge, the WFR factor for an off-shell quark with momentum p^μ is:

some constant

$$Z_q = 1 + \frac{C_F \alpha_s}{4\pi}\left(-\frac{1}{\epsilon} - \ln\frac{\mu^2}{-p^2 - i0} - c\right)$$

We thus obtain:

$$\bar u_{\bar n}(p_2)\, \gamma_\perp^\mu\, u_n(p_1) \left\{ 1 + \frac{C_F \alpha_s}{4\pi}\left[-\frac{1}{\epsilon} - \frac{1}{2}\left(\ln\frac{\mu^2}{P_1^2} + \ln\frac{\mu^2}{P_2^2}\right) - c\right]\right\}$$

One-loop SCET matrix element:

Adding up all pieces, we find at one-loop order:

$$\langle q(p_2)|\, (\bar\xi_{\bar n} W_{\bar c})(0)\, \gamma_\perp^\mu\, (W_c^\dagger \xi_n)(0)\, |q(p_1)\rangle$$

$$= \bar u_{\bar n}(p_2)\, \gamma_\perp^\mu\, u_n(p_1)$$

$$\times \left\{ 1 + \frac{C_F \alpha_s}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\left(\ln\frac{\mu^2}{P_1^2} + \ln\frac{\mu^2}{P_2^2} - \ln\frac{\mu^2 Q^2}{P_1^2 P_2^2}\right) + \frac{3}{\epsilon} \right.\right.$$

$$+ \ln^2\frac{\mu^2}{P_1^2} + \ln^2\frac{\mu^2}{P_2^2} - \ln^2\frac{\mu^2 Q^2}{P_1^2 P_2^2}$$

$$\left.\left. + \frac{3}{2}\ln\frac{\mu^2}{P_1^2} + \frac{3}{2}\ln\frac{\mu^2}{P_2^2} + 8 - c - \frac{5\pi^2}{6}\right]\right\}$$