The crumpled phase of active tethered membranes

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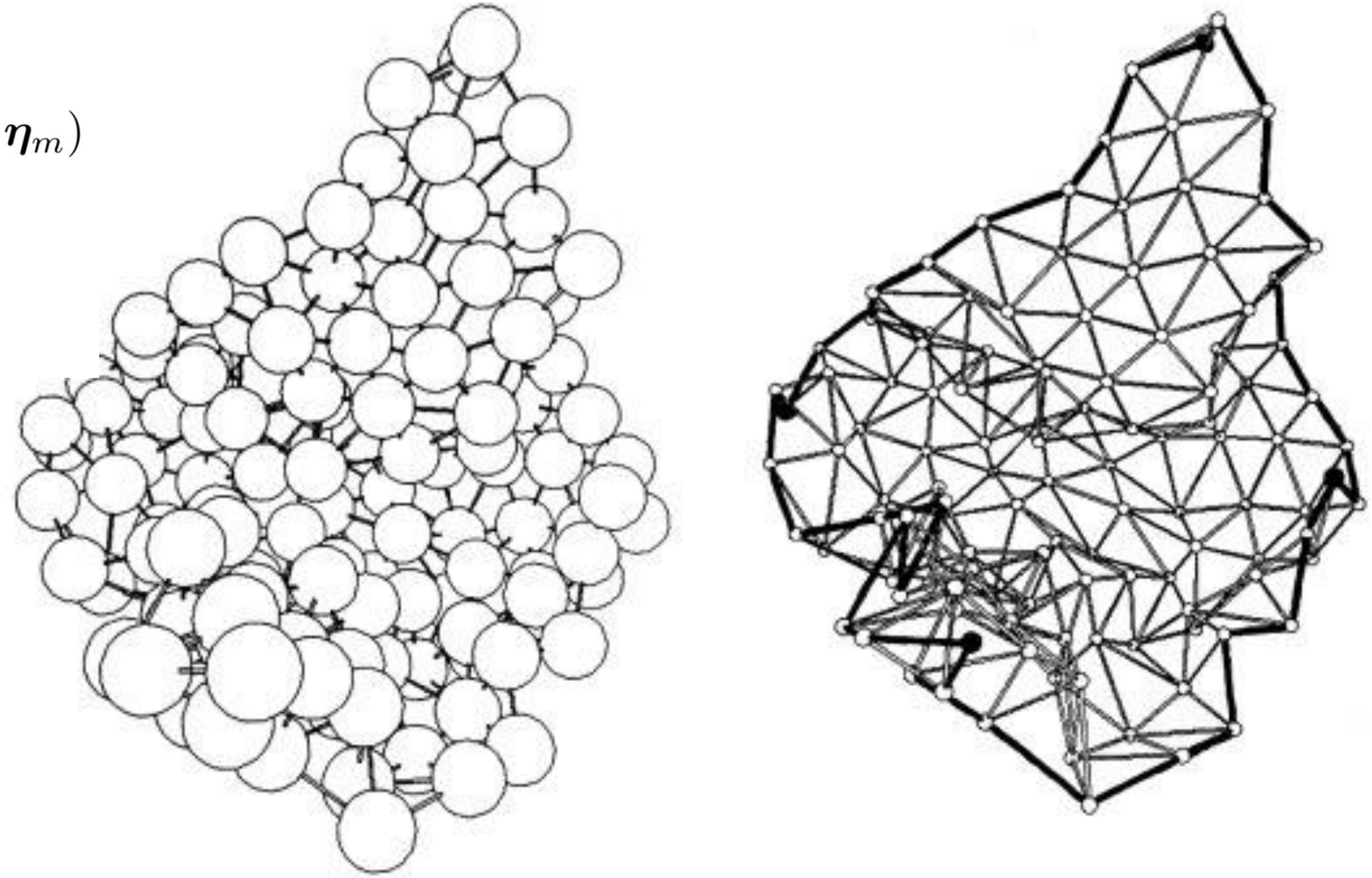
Tethered Membranes

$$U = K \sum_{<ij>} (r_{ij} - \sigma)^2 + \kappa \sum_{<lm>} (1 - \boldsymbol{\eta}_l \cdot \boldsymbol{\eta}_m)$$
$$+ 4\varepsilon \sum_{ij} \left[\left(\frac{\sigma}{r_{ij}} \right)^{12} - \left(\frac{\sigma}{r_{ij}} \right)^{6} + \frac{1}{4} \right],$$



Measuring dimension

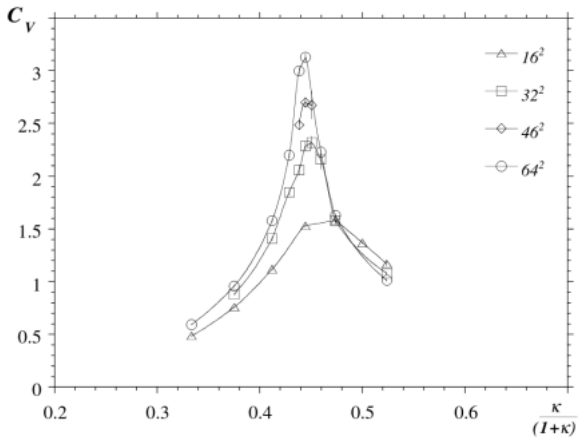
D: dimension of polymerized system

$R_g \sim L^\nu \sim N^{\nu/D}$

$d_H = D/\nu$

Ideal Membranes

$$U = K \sum_{<ij>} (r_{ij} - \sigma)^2 + \kappa \sum_{<lm>} (1 - \boldsymbol{\eta}_l \cdot \boldsymbol{\eta}_m)$$
$$\cancel{+ 4\varepsilon \sum_{ij} \left[\left(\frac{\sigma}{r_{ij}} \right)^{12} - \left(\frac{\sigma}{r_{ij}} \right)^{6} + \frac{1}{4} \right]},$$



Bowick et al. Journal de Physique (1996)

Second-order phase transition

$$\nu = \begin{cases} 0 & \text{if } \kappa/k_B T < 0.3 \quad \text{(Crumpled)} \\ 1 & \text{if } \kappa/k_B T > 0.3 \quad \text{(Flat)} \end{cases}$$

Flat phase is stabilized by renormalized bending constant $\kappa(q) \sim q^{-1}$

Measuring dimension

D: dimension of polymerized system

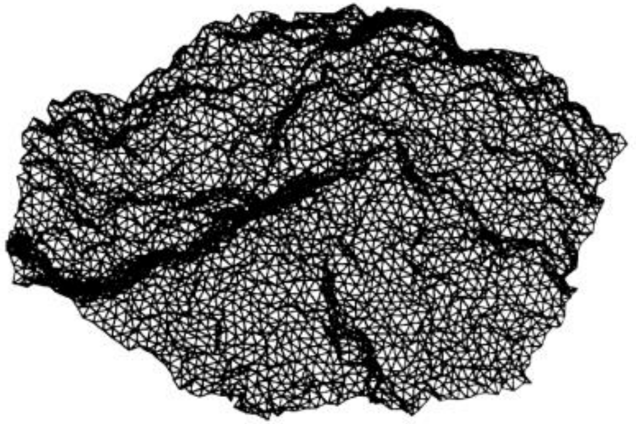
$$R_g \sim L^{\nu} \sim N^{\nu/D}$$
$$d_H = D/\nu$$

Self-avoiding membranes

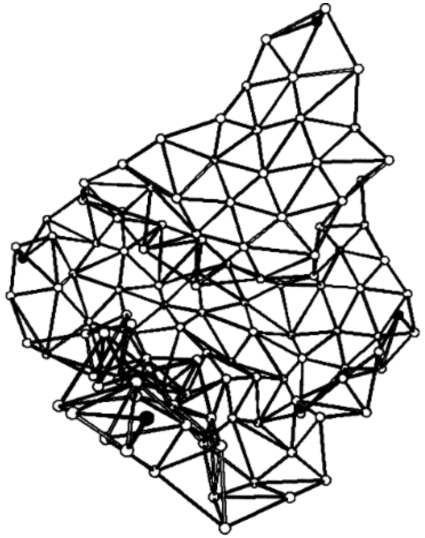
$$U = K \sum_{<ij>} (r_{ij} - \sigma)^2 + \cancel{\kappa \sum_{<lm>} (1 - \mathbf{n}_l \cdot \mathbf{n}_m)}$$
$$+ 4\varepsilon \sum_{ij} \left[\left(\frac{\sigma}{r_{ij}} \right)^{12} - \left(\frac{\sigma}{r_{ij}} \right)^{6} + \frac{1}{4} \right],$$

$$R_g \sim L^{\nu_F} \qquad \nu_F = \frac{D+2}{d+2}$$
$$d_H = D/\nu_F$$

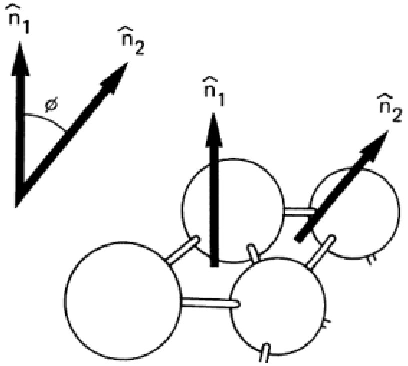
D: dimension of polymerized system
d: embedding dimension
$\nu_F = 0.6$ (polymers)
$\nu_F = 0.8$ (tethered membranes)



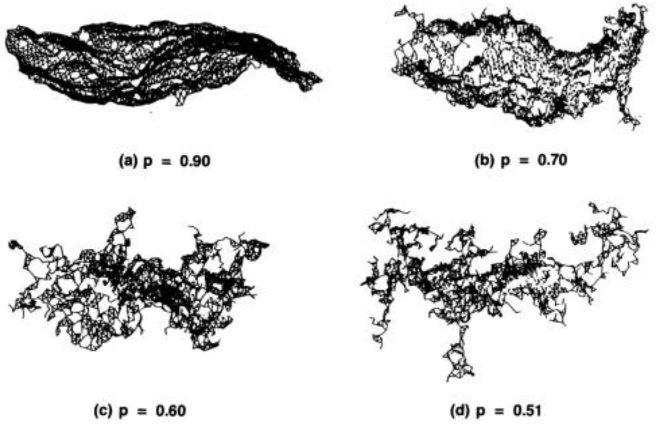
Search for the crumpled phase



Kantor, Kardar and Nelson,
Physical Review Letters (1986)

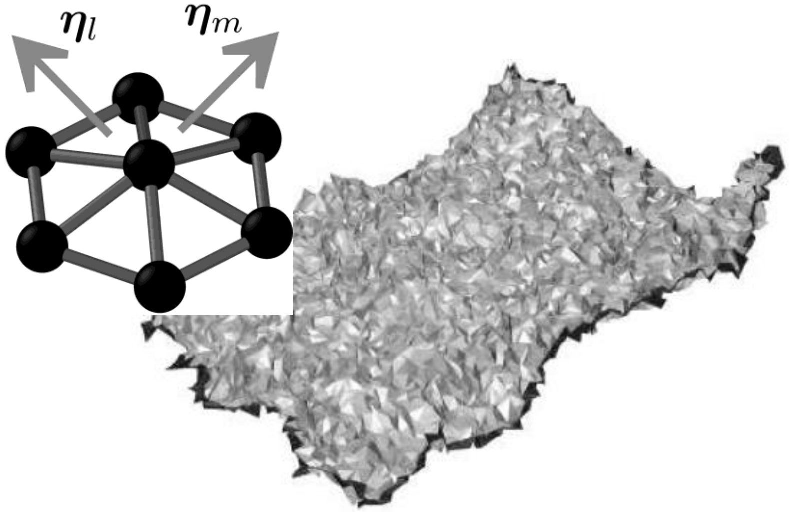


Abraham and Nelson, Journal de Physique (1990)

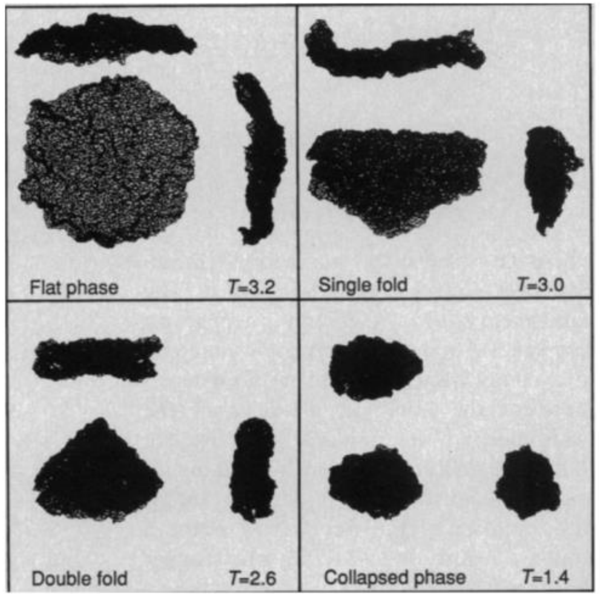


Grest and Murat. *Journal de Physique* (1990)

Abraham, Rudge and Plischke
Physical Review Letters (1989)



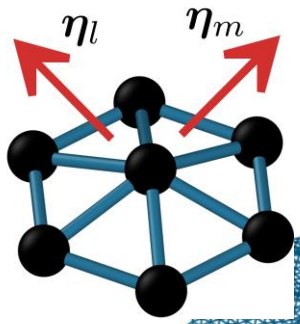
Bowick et al., The European Physical Journal E (2001)

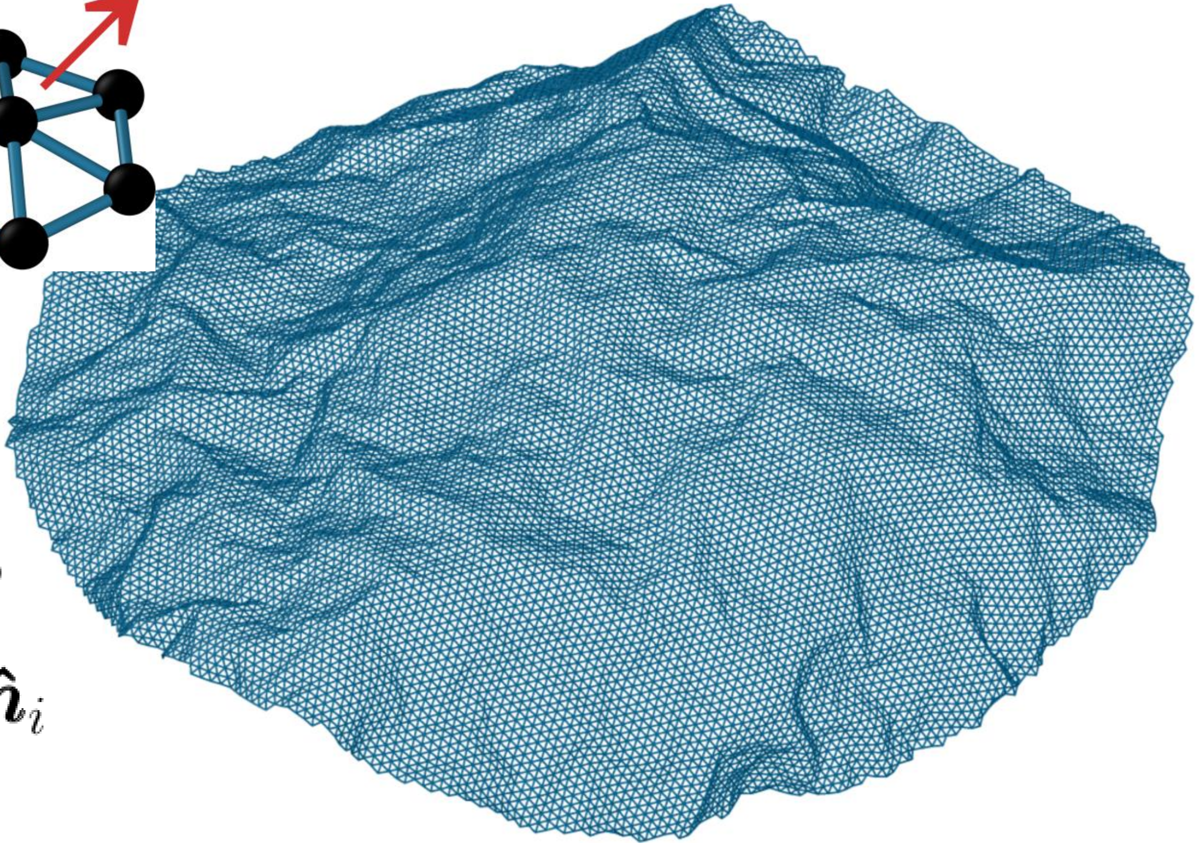


Abraham and Kardar,
Science (1991)

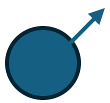
Active Tethered Membranes

$$U = K \sum_{<ij>} (r_{ij} - \sigma)^2 + \kappa \sum_{<lm>} (1 - \boldsymbol{\eta}_l \cdot \boldsymbol{\eta}_m)$$
$$+ 4\varepsilon \sum_{ij} \left[\left(\frac{\sigma}{r_{ij}} \right)^{12} - \left(\frac{\sigma}{r_{ij}} \right)^{6} + \frac{1}{4} \right],$$





$$\frac{d\boldsymbol{r}_i(t)}{dt} = \frac{1}{\gamma}\boldsymbol{f}_i + v_p \hat{\boldsymbol{n}}_i(t) + \sqrt{2D}\,\boldsymbol{\xi}(t),$$
$$\frac{d\hat{\boldsymbol{n}}_i(t)}{dt} = \sqrt{2D_r}\,\boldsymbol{\xi}_r(t) \times \hat{\boldsymbol{n}}_i(t),$$



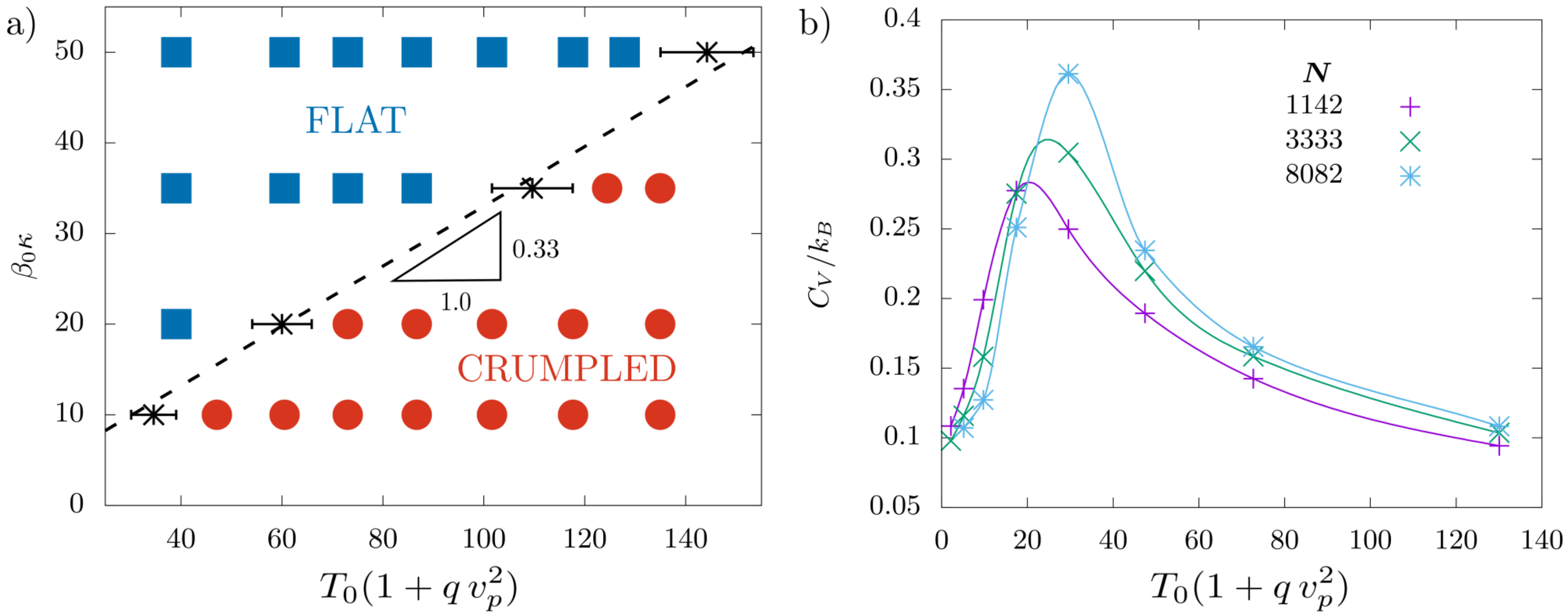
Gandikota, M. C. & A. Cacciuto *Soft Matter* (2023)

Ideal active membranes map to an effective higher temperature

$$T^{\mathrm{eff}} = T_0(1 + q\,Pe^2)$$

with constant q=1/42

N=3333

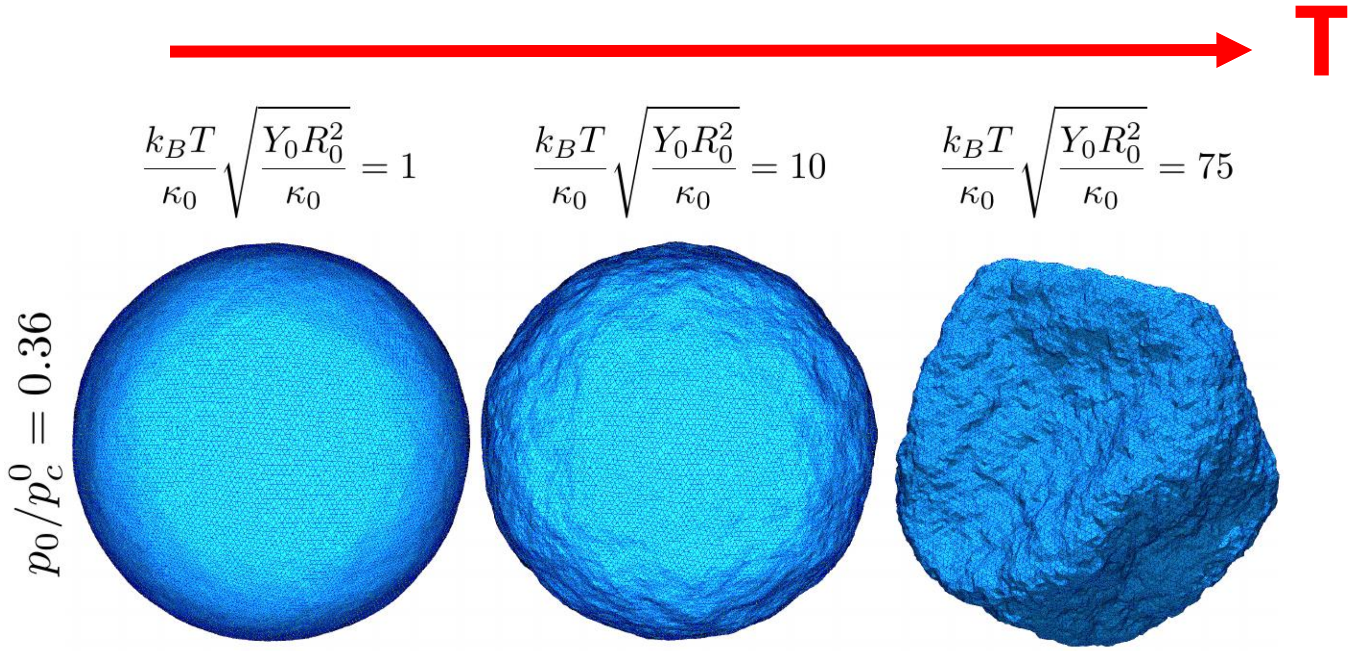


Gandikota, M. C. & A. Cacciuto *Soft Matter* (2023)

No **qualitative** difference between thermal and active tethered membranes

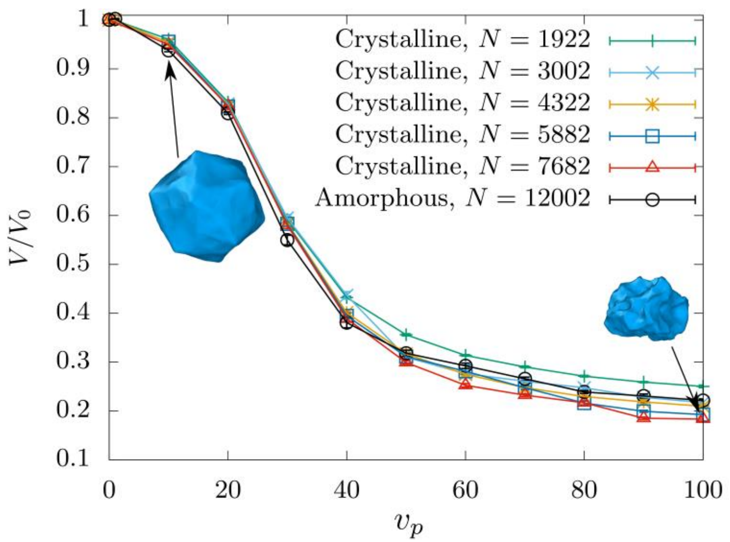
What about shells?

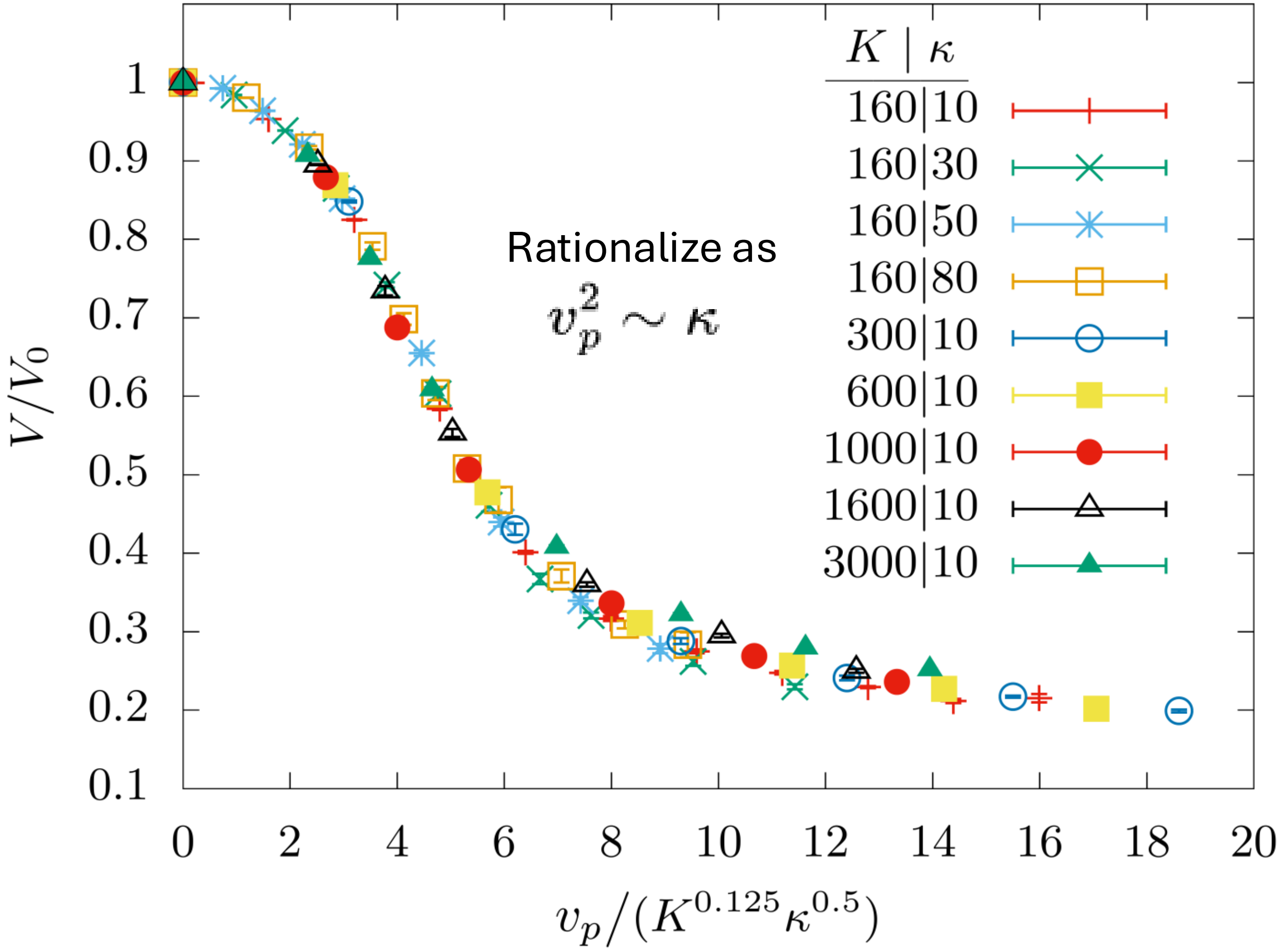
$$p_c^0 = 4\frac{\sqrt{\kappa_0 Y_0}}{R^2}$$



Košmrlj, Andrej, and David R. Nelson *Physical Review X* 7.1 (2017)

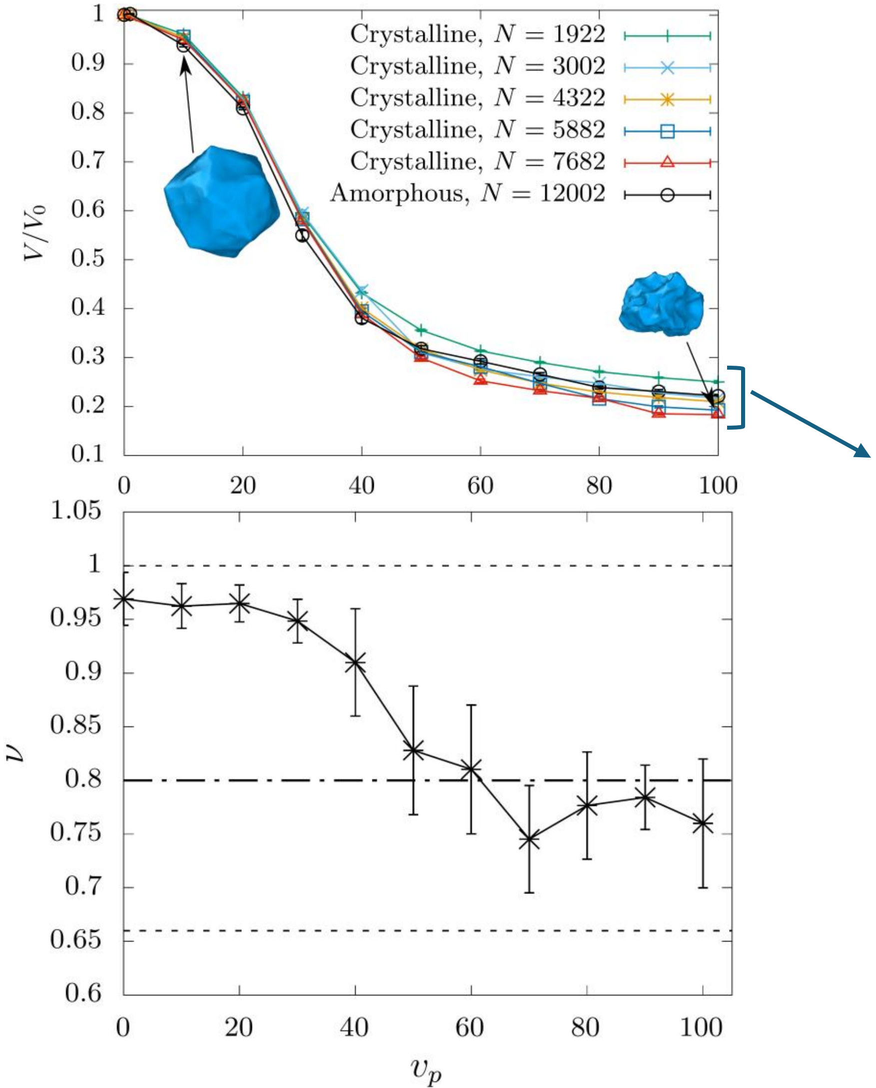
Self-avoiding active shells crumple

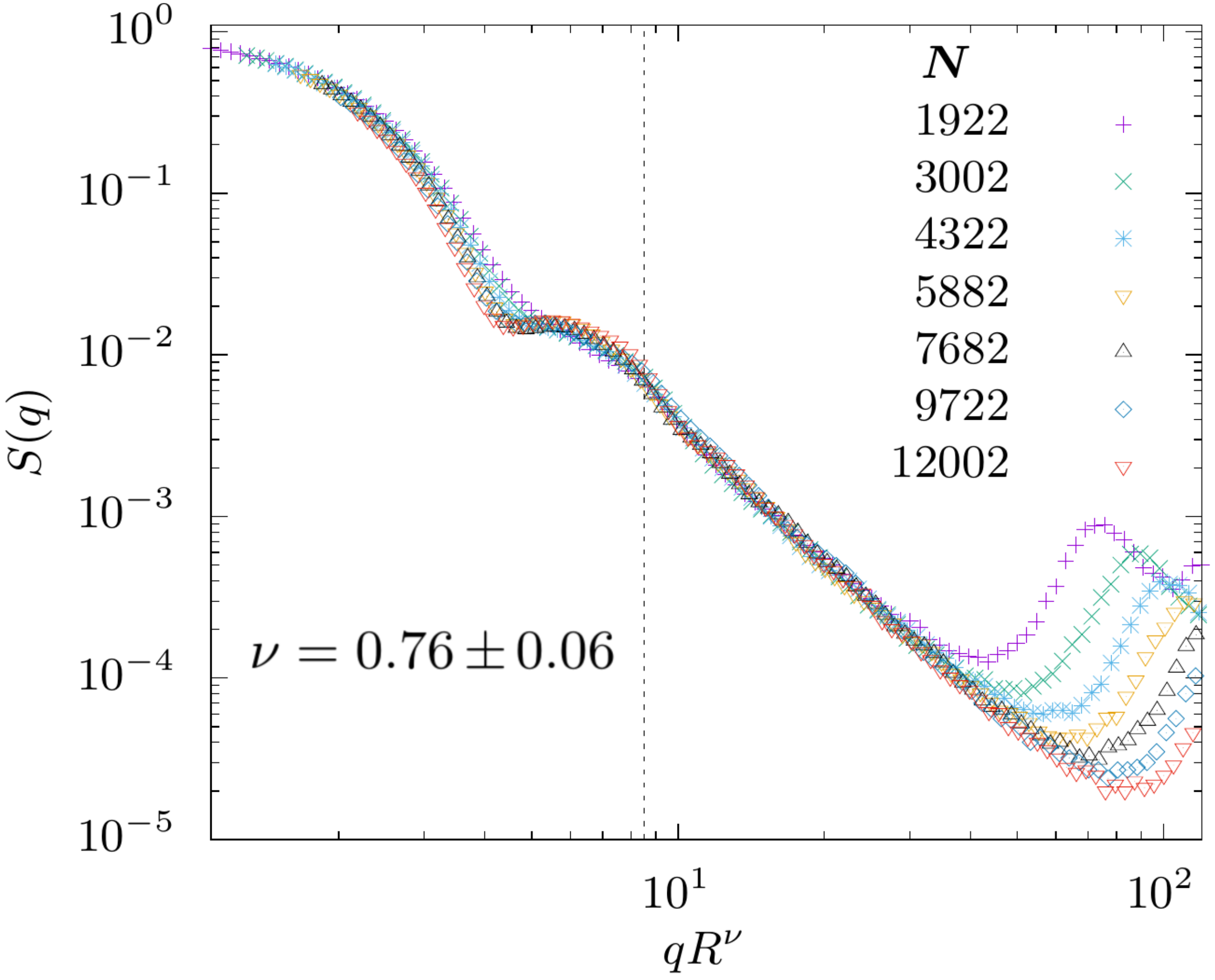




Gandikota, M. C., Das, S., & Cacciuto, A. *Soft Matter* 20.17 (2024)

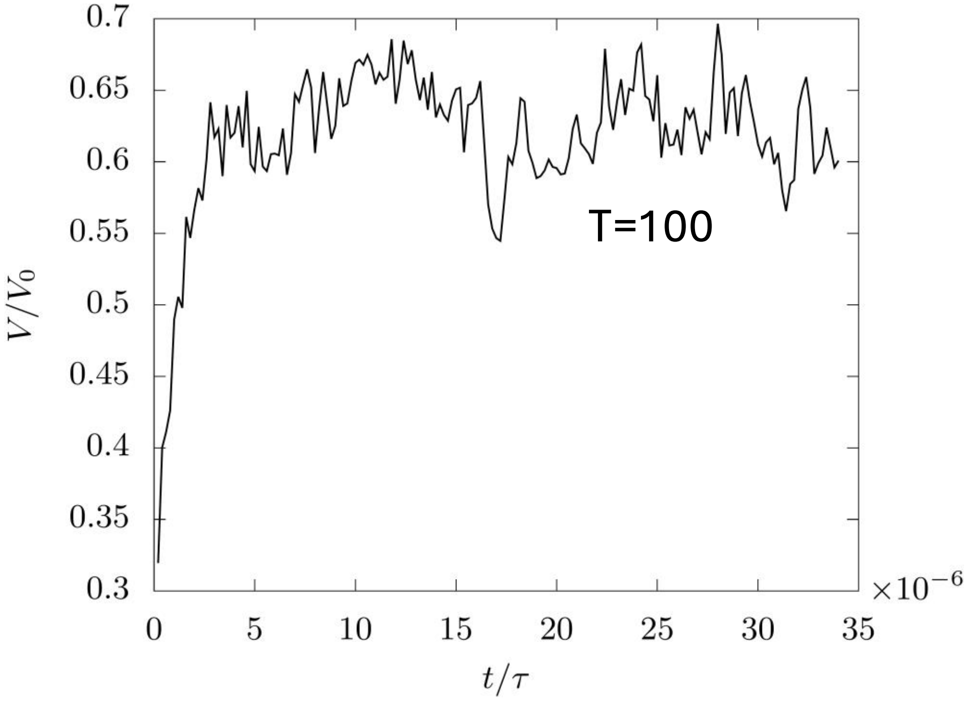
Flory-like phase at high activity



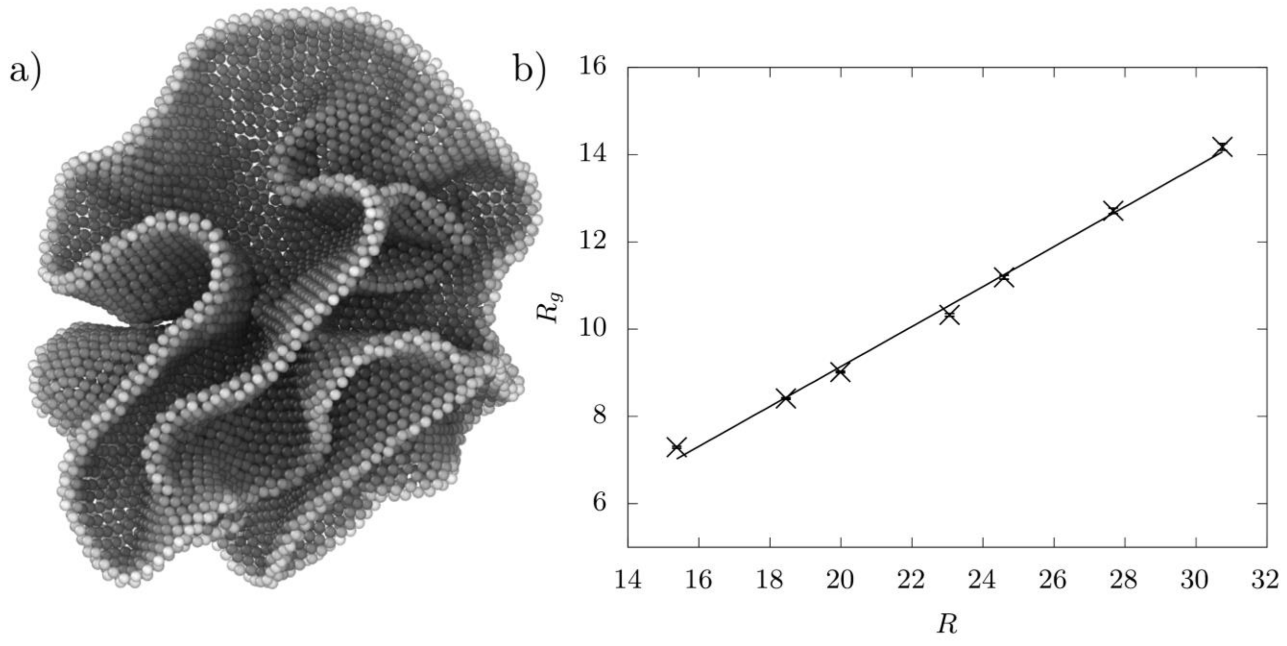


Temperature/pressure unlike activity

High temperature



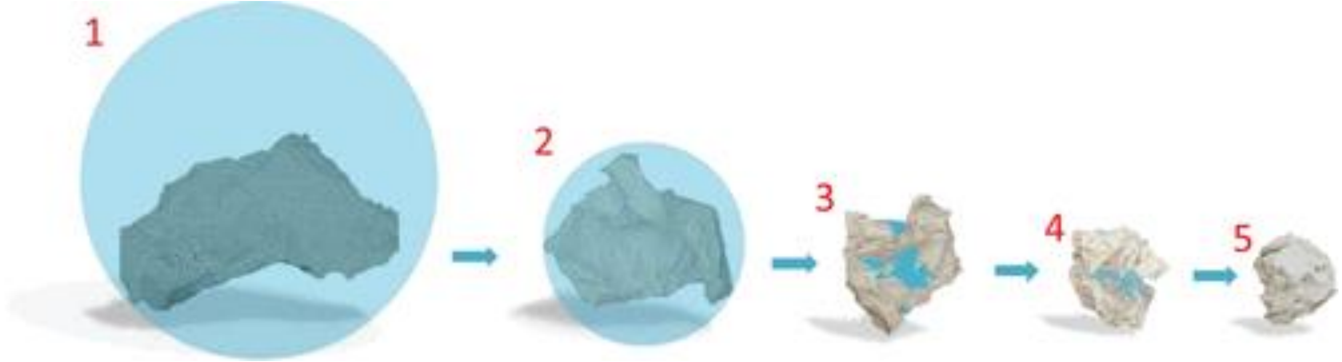
High inward pressure



Crumpling via non-thermal effects

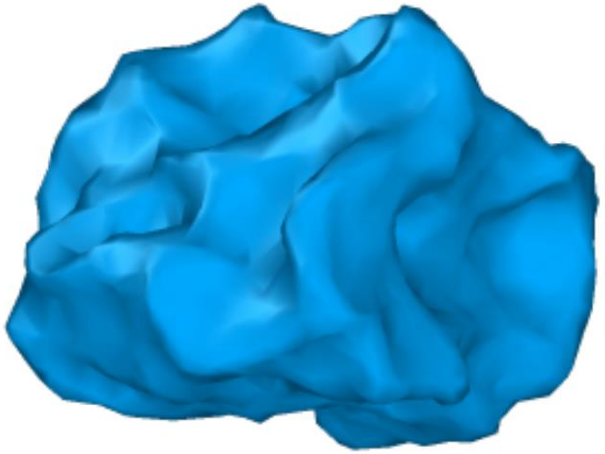
Crushing paper

Rapid dehydration of Graphene-oxide nanopaper



Nano Lett. 2012, 12, 486–489

Active shells



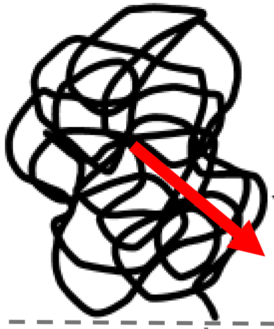
Thank You!

IDEAL/Phantom

Self avoiding

- Flexible Polymer

$\nu = 1/2$



$\nu = 1/3$ Bad solvent

$\nu = 3/5$ Good solvent

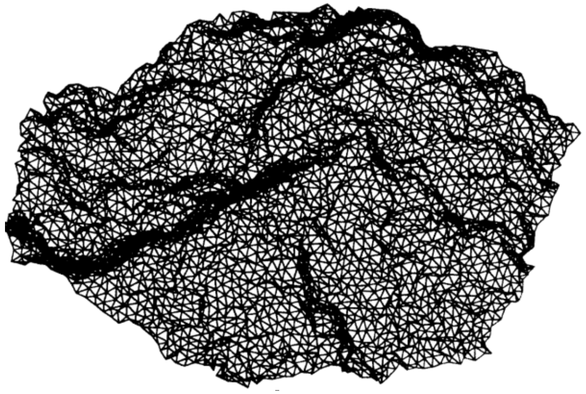
- Elastic Sheet

$\nu = 0$ $R_g \sim \log L$ $\kappa < 0.3\, k_B T$

$\nu = 1$ $\kappa > 0.3\, k_B T$

Coupling between in-plane and out-of-plane modes renormalizes bending rigidity as

$\kappa(q) \sim q^{-1}$



$\nu = 1/3$ Bad solvent

$\nu = 1$ Good solvent $\forall T$

No crumpled phase!

Flory calculations predict a high temperature phase with

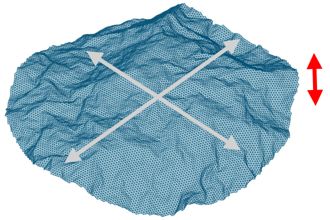
$\nu = (D+2)/(d+2)$

$\nu = 4/5 \; (D = 2, d = 3)$

Bowick, Mark J., and Alex Travesset *Physics Reports* 344.4-6 (2001).

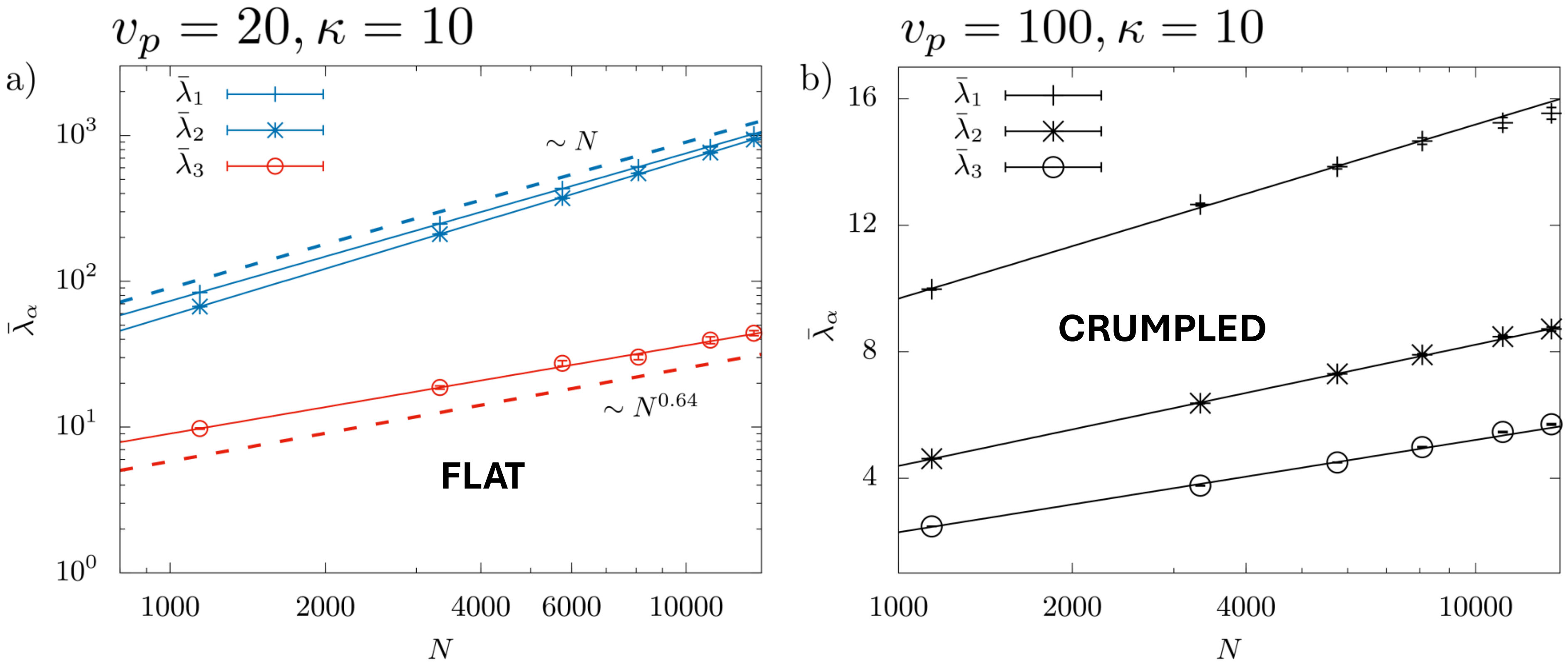
Ideal Active membranes

Shape tensor $S_{\alpha\beta} = \frac{1}{2N^2}\sum_{i,j=1}^{N}(r_{i\alpha}-r_{j\alpha})(r_{i\beta}-r_{j\beta})$



Eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3$

$\lambda_1 + \lambda_2 + \lambda_3 = R_g^2$

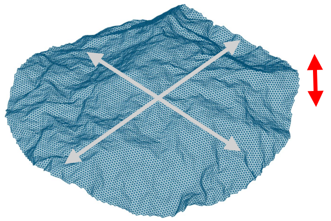


Gandikota, M. C. & A. Cacciuto *Soft Matter* (2023)

Self-avoiding active membranes remain flat

Shape tensor

$$S_{\alpha\beta} = \frac{1}{2N^2} \sum_{i,j=1}^{N} (r_{i\alpha} - r_{j\alpha})(r_{i\beta} - r_{j\beta})$$



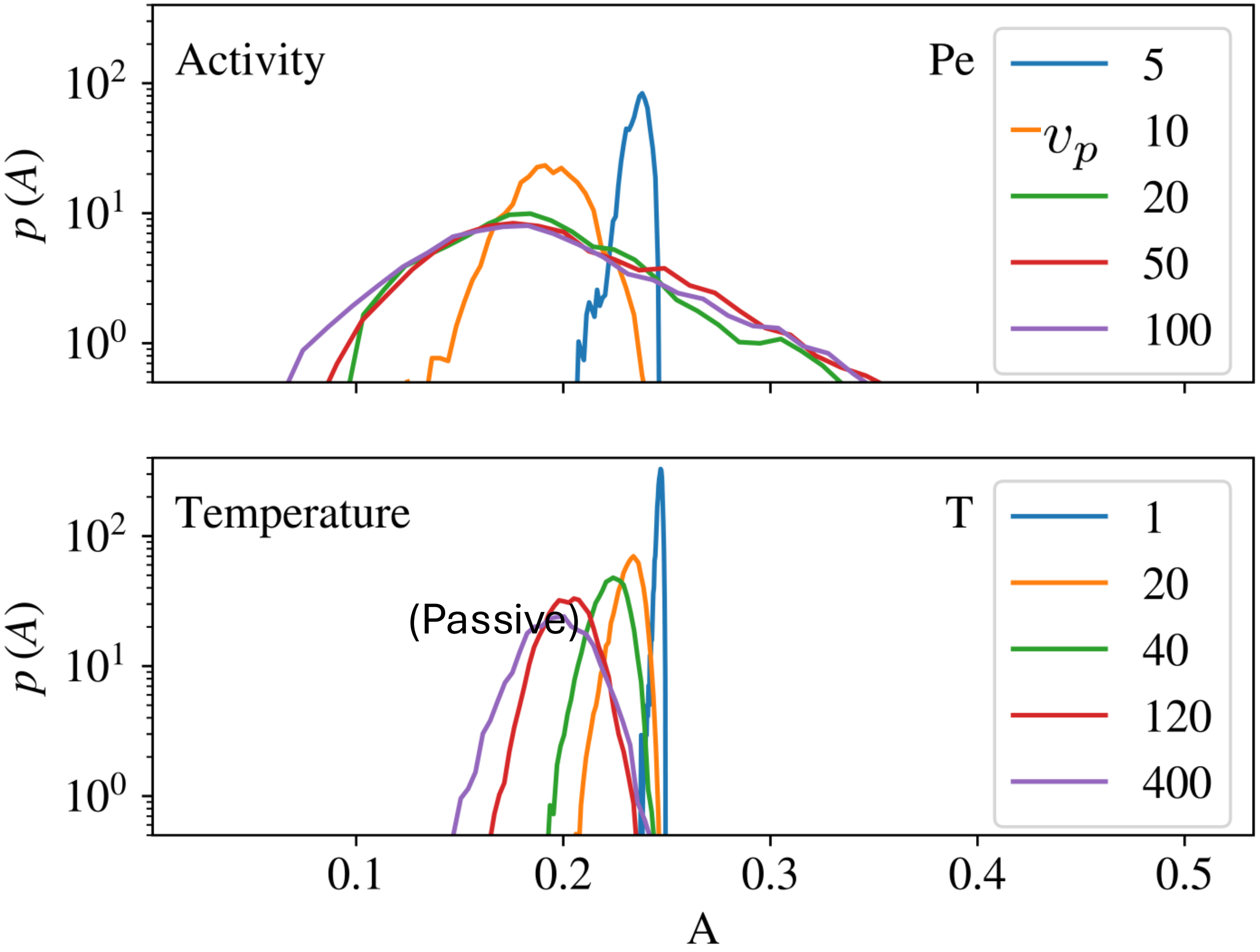
Eigenvalues

$$\lambda_1 \geq \lambda_2 \geq \lambda_3$$

Asphericity

$$A = \frac{3}{2} \frac{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}{(\lambda_1 + \lambda_2 + \lambda_3)^2} - \frac{1}{2}$$

0 -> isotropic
0.25 -> flat
1 -> linear



Gandikota, M. C. & A. Cacciuto *Soft Matter* 19.28 (2023)