Dynamics of fluctuation correlation in periodically driven classical system

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Aritra Kundu, **Atanu Rajak**, and Tanay Nag, PRB 104, 075161 (2021)

Periodically Driven systems

- Periodic drive in isolated many-body systems: *Novel dynamic phases*

- Problem: A generic many-body system (chaotic) heats up

(D'Alessio, Polkovnokov, Ann. Phys. 333, 19-33 (2013); D'Alessio, Rigol, PRX 4, 041048 (2014); Lazarides et al, PRE 90, 012110 (2014); Rusomanno et al, JSTAT P08030 (2015))

- Exponentially suppressed heating rate: *"Prethermalization"*

(Choudhury & Mueller PRA 2014 - Bukov *etal* PRL 2015 - Abanin *etal* PRL 2015 – Goldman *etal* PRA 2015 - Chandra & Sondhi PRB 2016-Mori *etal* PRL 2016- Mallayya *etal* PRL 2019)

Exponential suppressed heating

For quantum spin systems with a local norm bound

Using perturbative argument and Floquet Magnus expansion

Prethermal time-scale, $\tau^{*} \sim e^{\frac{A\Omega}{J}}$

A is unitless parameter, J is the energy bound

Abanin , et al. PRL 115, 256803(2015), Mori, et al. PRL 116, 120401 (2016)

Experimental realization: Rubio-Abadal *etal* PRX 2020-Peng *etal* Nat. Phys. 2021

Limitation of above studies:

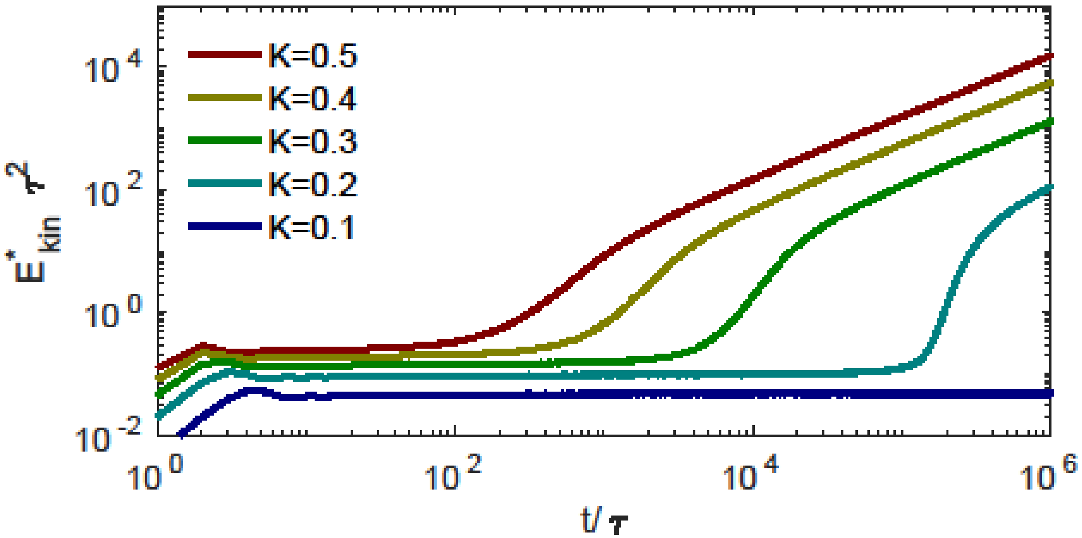
1. Finite-size effects
2. Finite bandwidth

Can we overcome these issues?

Coupled kicked rotors

$$H = \sum_{j=1}^{N} \left[\frac{p_j^2}{2} - \kappa \cos(\phi_j - \phi_{j+1}) \sum_{n=-\infty}^{+\infty} \delta(t - n\tau) \right].$$

Always diffusive
Kaneko & Konishi (1989)
Chirikov & Vecheslavov (1997)



$$E_{kin}(t) = \frac{1}{N}\sum_{j=1}^{N}\langle \frac{p_j^2(t)}{2} \rangle$$

$$K = \kappa\tau$$

$$T^* = \frac{\kappa}{1.066\tau} = \frac{0.9831K}{\tau^2}$$

Lifetime , $t^* \sim e^{1/T^*}$

"Statistical Floquet Prethermalization"

Rajak, Citro, Dalla Torre, JPA (2018)
Rajak, Dana, Dalla Torre, PRB (R) (2019)
Sadia, Dalla Torre, **Rajak**, PRB (2022)

Beyond average?

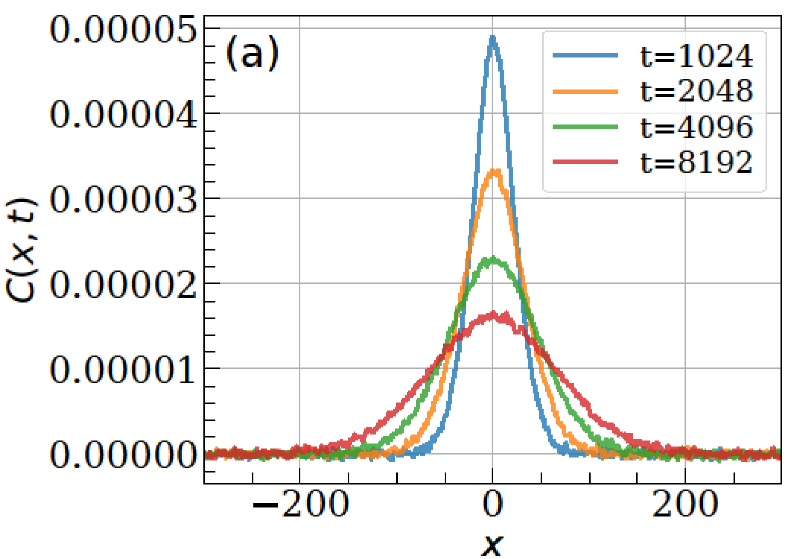
Spatiotemporal Fluctuation Correlation

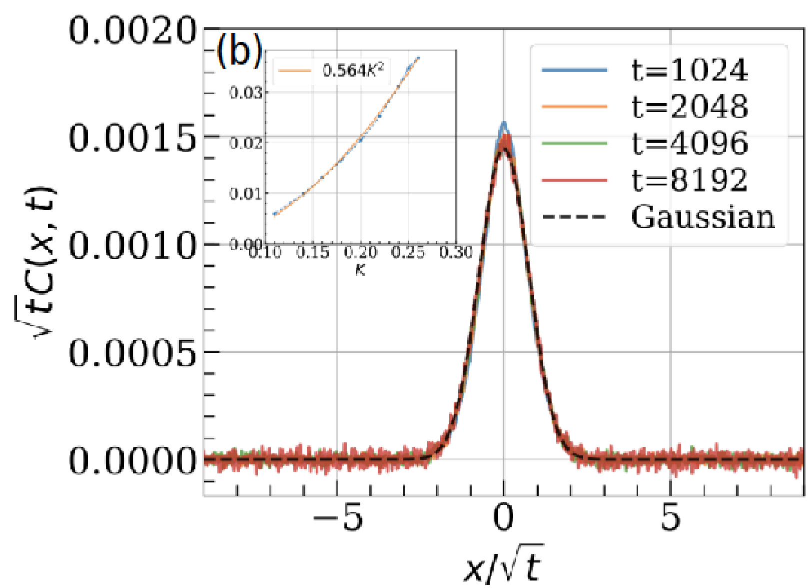
$$C(i,j,t,t_w) = \frac{1}{4}\left[\langle p_i^2(t)p_j^2(t_w)\rangle - \langle p_i^2(t)\rangle\langle p_j^2(t_w)\rangle\right]$$

For isolated static systems, $C \sim \frac{1}{t^\gamma} f(\frac{x}{t^\gamma})$

Spohn, J. Stat. Phys. (2014),
Das et al., PRE (2014)
Bastianello et al., SciPost Phys. (2018)
Nardis, Bernard, and Doyon, PRL (2018)
Spohn, J. Phys. A (2020)

Prethermal Regime: $K = 0.14, t > t_w$





$$A_K = \sum_x C(x,t) \approx \frac{0.4402K^2}{\tau^4}$$

$$C(x,t) = A_k t^{-\frac{1}{2}} f(x t^{-\frac{1}{2}})$$
$$f(y) = (1/\sqrt{2\pi D})e^{-y^2/2D}$$

Hydrodynamic picture

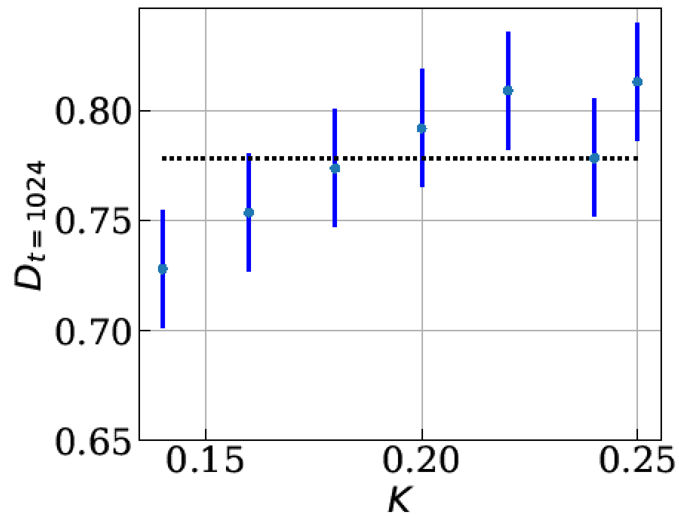
Hydrodynamic equation similar to the un-driven case

$$\partial_t u(x,t) \cong \frac{D}{2}\partial_x^2 u(x,t) + B\partial_x \xi(x,t)$$

Local energy fluctuation: $u(x,t) = \frac{1}{2}\left(p^2(x,t) - \langle p^2(x,t)\rangle\right)$

Noise: $\langle \xi(x,t)\xi(x',t')\rangle = \delta_{x,x'}\delta(t-t')$

$$C(x,t-t_w) = \frac{\langle u(t_w)\rangle^2}{\sqrt{2\pi D(t-t_w)}} e^{-\frac{x^2}{2D(t-t_w)}}$$

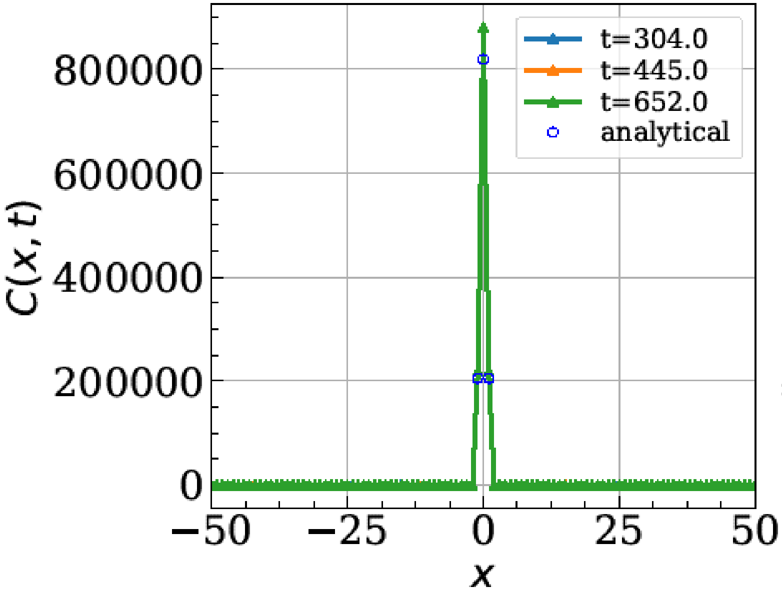


Fluctuation dissipation relation: $DC \sim B^2$

$C \sim K^2, B \sim K$ □ $D \sim K^0$

Spatiotemporal Fluctuation Correlation

Heating Regime: $K = 3$ and $t_w = 142$



Independent rotors model:

$$\left\langle \sin(r_i(n))\sin\left(r_j(m)\right)\right\rangle = \frac{1}{2}\delta_{n,m}\delta_{i,j}$$

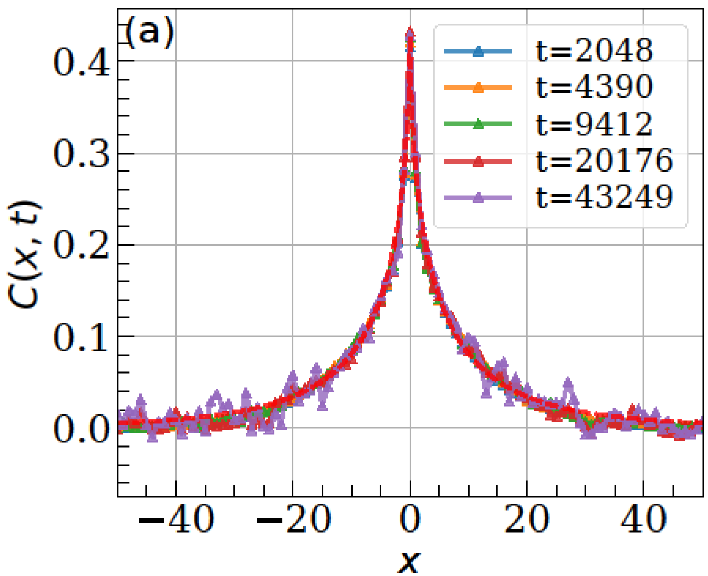
$$\left\langle \sin(r_i(n))\sin\left(r_j(n')\right)\sin(r_k(m))\sin(r_l(m'))\right\rangle$$
$$= \frac{1}{4}\left(\delta_{n,n'}\delta_{m,m'}\delta_{i,j}\delta_{k,l} + \delta_{n,m}\delta_{n',m'}\delta_{i,k}\delta_{j,l} + \delta_{n,m'}\delta_{n',m}\delta_{i,l}\delta_{j,k}\right)$$
$$+ \frac{3}{8}\delta_{n,n'}\delta_{m,m'}\delta_{m,n}\delta_{i,j}\delta_{k,l}\delta_{i,k}$$

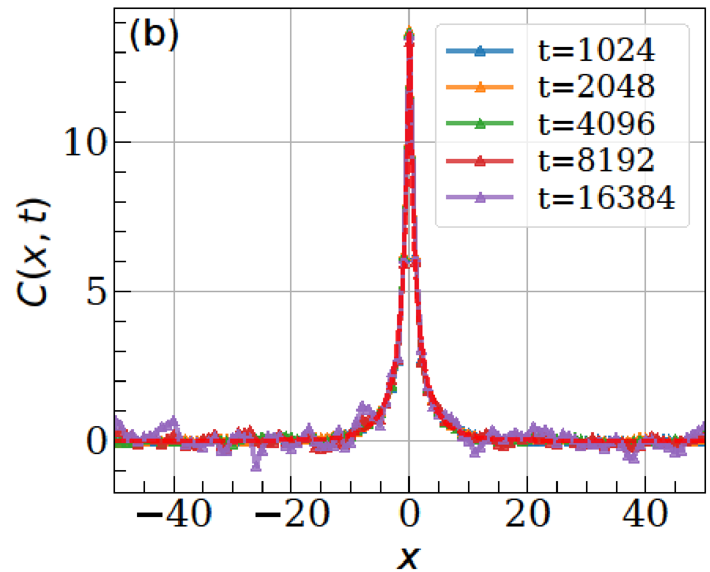
$$C(i,j,t,t_w) = K^4\left(\frac{t_w^2}{2}\delta_{i,j} + \frac{t_w^2}{8}\delta_{i,j+1} + \frac{t_w^2}{8}\delta_{i,j-1}\right) + O(t_w)$$

Correlations are frozen both in space and time

Spatiotemporal Fluctuation Correlation

Crossover Regime: (a) $K = 0.14, t_w = 142$; $(b) K = 0.7, t_w = 64$



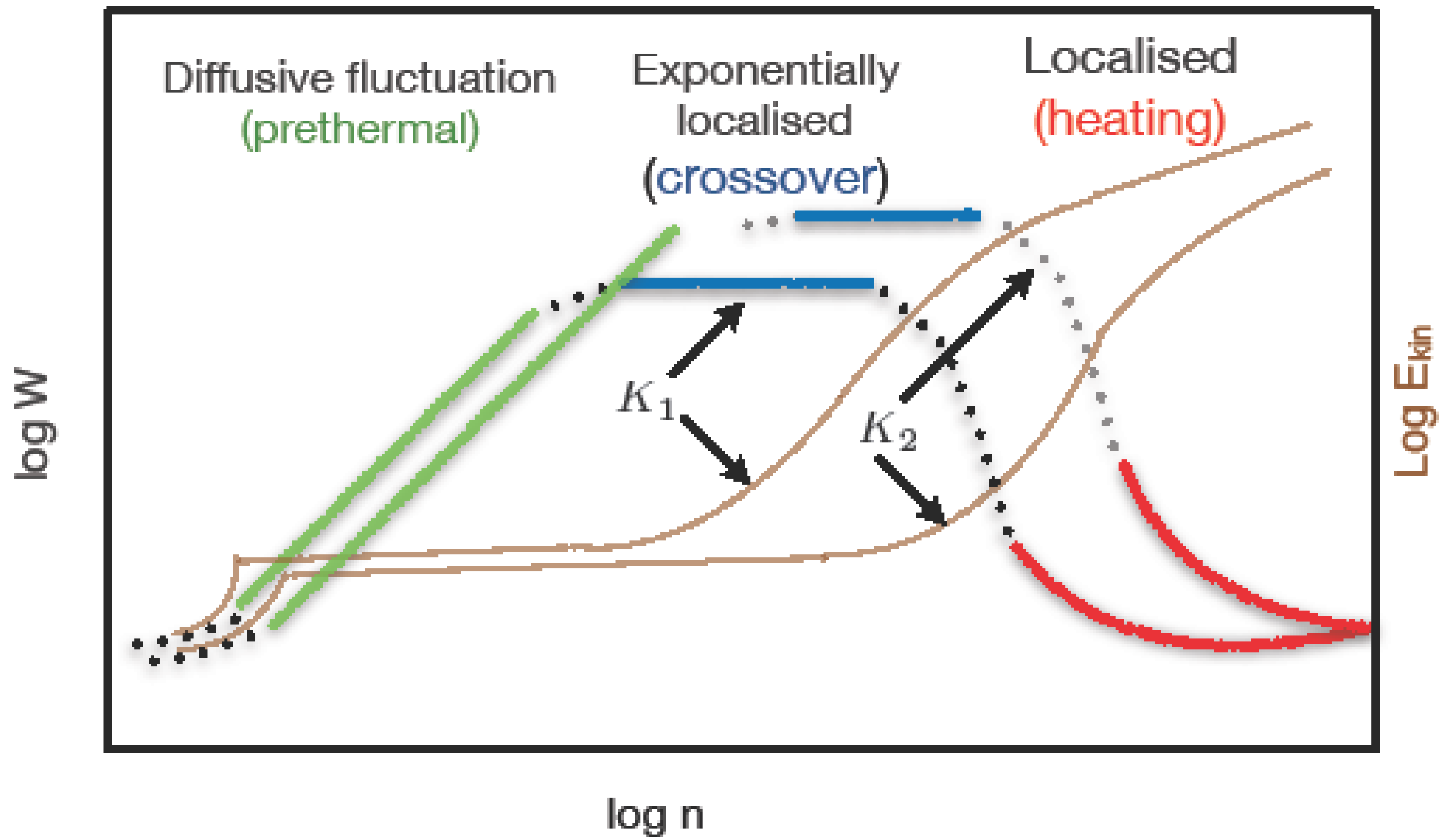


Stretched Exponential, $C(x,t) \sim e^{-\alpha|x|^{\beta}}$

$\beta \sim 0.63[0.65]$, almost independent of K

Correlations are frozen in time but not space

Dynamical Phases: Schematic diagram



$$W = \frac{\sum_x x^2 C(x,t)}{\sum_x C(x,t)}$$

Conclusions

- Quasi-static nature of prethermal phases is supported by fluctuation correlations
- Fluctuation correlations show distinct behavior in different dynamic regimes
- Hydrodynamic picture of prethermal phases

Thank You

High-frequency limit

For $\Omega = \frac{2\pi}{\tau} \gg 1$, the average Hamiltonian

$$H^* = \frac{1}{\tau}\int_0^\tau H(t)dt = \sum_{j=1}^{N}\left[\frac{p_j^2}{2} - \frac{\kappa}{\tau}\cos(\phi_j - \phi_{j+1})\right]$$

H and H^* are invariant under $\phi_j \rightarrow \phi_j + \chi$

The angular momentum of the center of mass

$$P = \frac{1}{N}\sum_{j=1}^{N} p_j$$

P is an exact constant of the motion

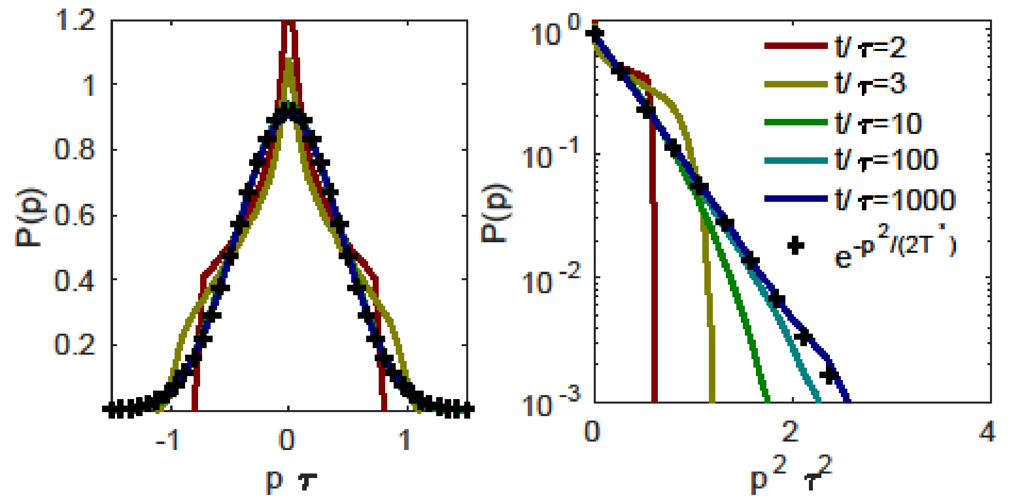
Generalized Gibbs ensemble

The prethermal state is described by

$$P^*(\{p_j,\phi_j\}) = \frac{1}{Z} exp\left[-\frac{H^*(\{p_j,\phi_j\})}{T^*} + \gamma P(\{p_j\})\right] \qquad \bar{p} = \frac{\gamma T^*}{N}$$

H^* contains terms of p_j and ϕ_j separately

$$P^*(\{p_j\}) = Z^{-1}\prod_{j=1}^{N} exp\left[-\frac{(p_j-\bar{p})^2}{2T^*}\right]$$



M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, PRL **98**, 050405 (2007).
A. Lazarides, A. Das, and R. Moessner, PRL **112**, 150401 (2014).

Average energy

$$E^* = \sum_{j=1}^{N} \frac{\langle p_j^2 \rangle_*}{2} - \frac{\kappa}{\tau} \sum_{j=1}^{N} \langle \cos(\phi_j - \phi_{j+1}) \rangle_*$$

Here $\langle O \rangle_* = \int \prod_{j=1}^{N} O(\{p_j, \phi_j\}) P^*(\{p_j, \phi_j\}) dp_j d\phi_j$

$$E^* = E_K + E_I = \frac{N}{2}(T^* + \bar{p}^2) - \frac{N\kappa}{\tau} \frac{I_1(\epsilon)}{I_0(\epsilon)}$$

Where $I_n(\epsilon)$ is the modified Bessel function of order n

$$\epsilon = \kappa / \tau T^*$$

Initial conditions: $p_j = \bar{p}$ and ϕ_j are homogenously distributed from 0 to 2π

$$E_0 = \frac{N\bar{p}^2}{2}$$

Temperature of prethermal state

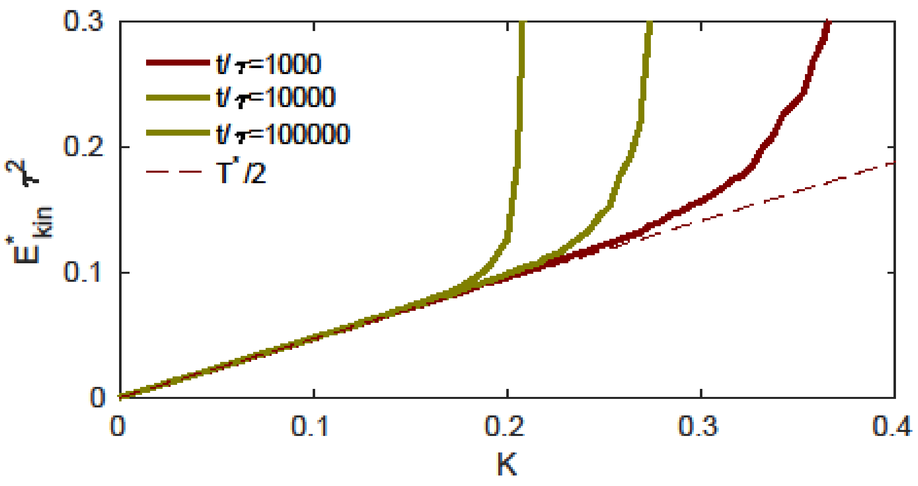
The energy of the initial state (E_0) =
The average energy of the prethermal state (E^*)

$$2\epsilon I_1(\epsilon) = I_0(\epsilon)$$

Numerical solution of the equation:

$$\epsilon = 1.066$$

$$T^* = \frac{\kappa}{1.066\tau} = \frac{0.9831K}{\tau^2}$$



For $\bar{p} = 0, E_{kin} = \frac{T^*}{2}$

$$E^*_{kin}\tau^2 = \frac{\tau^2 T^*}{2} = 0.469K$$

Many-body resonance

$$H = \sum_{j=1}^{N} \left[\frac{p_j^2}{2} - \kappa \cos(\phi_j - \phi_{j+1}) \sum_{n=-\infty}^{+\infty} \delta(t - n\tau) \right].$$

Consider the system close to the integrable one, i.e., $\kappa \to 0$

The condition for a primary resonance with unperturbed frequencies

$$\left| \sum_{j=1}^{N} m_j \omega_j - M\Omega \right| \lesssim \sqrt{\kappa/\tau}$$

$\{m'_j\}_{j'=1}^{N}$ are integer vectors, with $m_j = 1$ and $m_{j+1} = -1$ for some j

For our system, $\dot{\phi}_j = \omega_j = p_j$

$$|p_j - p_{j+1} - M\Omega| \lesssim \sqrt{\kappa/\tau}$$

B. Chirikov and V. Vecheslavov, J. Stat. Phys. **71**, 243 (1993), J. Exp. Theor. Phys. **85**, 616 (1997).

Probability of escape

$$P^{(M)} = \int_{-\sqrt{\kappa/\tau}}^{\sqrt{\kappa/\tau}} dx P^*(p_j - p_{j+1} \; - M\Omega = \mathrm{x})$$

$$\approx 2\sqrt{\frac{\kappa}{\tau}} P^*(p_j - p_{j+1} = M\Omega) \qquad P^*(\{p_j\}) = Z^{-1}\prod_{j=1}^{N} exp\left[-\frac{(p_j - \bar{p})^2}{2T^*}\right]$$

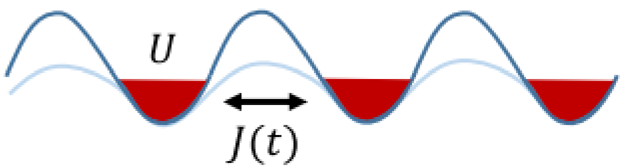
$$= \left(\frac{\kappa}{\pi T^* \tau}\right)^{\frac{1}{2}} \exp\left(-\frac{M^2\Omega^2}{4T^*}\right)$$

Using $T^* = 0.9381\frac{K}{\tau^2}$ and $\Omega = \frac{2\pi}{\tau}$

$$P^{(M)} = \left(\frac{1}{0.9381\pi}\right)^{1/2} \exp\left(-\frac{M^2\pi^2}{0.9381K}\right)$$

Time to escape from prethermal state $\sim 1/P^{(M)}$

Possible Experimental realization of the model



$$\bar{H} = \sum_{j=1}^{N} U n_j^2 + J(t)(\psi_j^\dagger \psi_j + \mathrm{H}.c.)$$

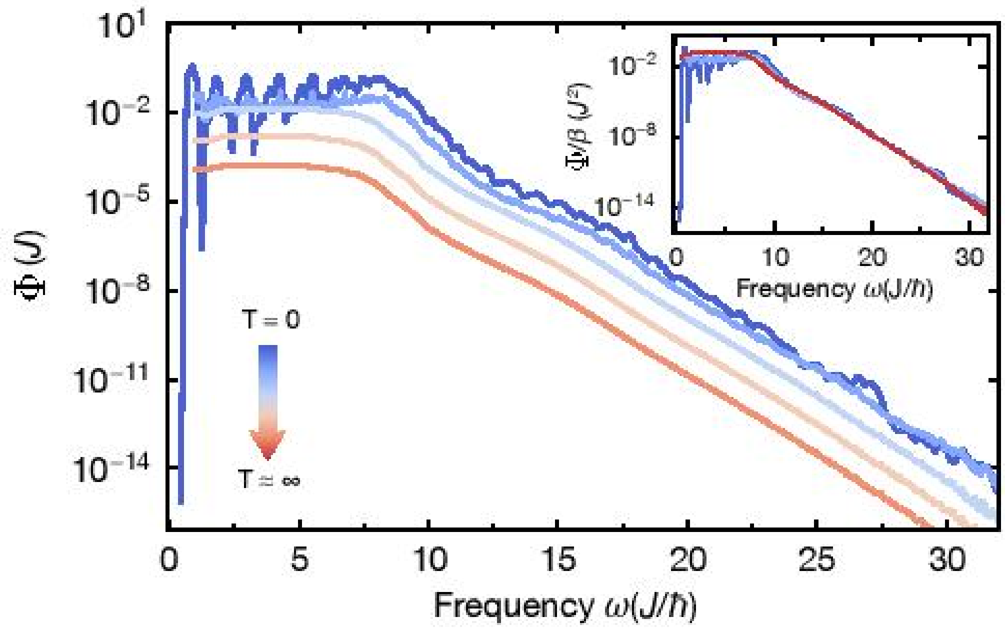
In the limit, $n = \langle n_j \rangle \gg 1$

$$\psi_j = \sqrt{n} e^{i\phi_j}$$

$$\bar{H} = \sum_{j=1}^{N} U n_j^2 + 2J(t) n \cos(\phi_j - \phi_{j+1})$$

A kicking potential, $J(t) = J_0 \Delta(t)$

Rajak, Dana, Dalla Torre, PRB Rapid (2019)



Exponentially suppressed heating rate!

Rubio Abadal et al (Bloch group), PRX (2020)

Prethermalization (longer times)